Light $U(1)$s in Heterotic String Models

Thesis submitted in accordance with the requirements of
the University of Liverpool for the degree of Doctor in Philosophy
by

Viraf M. Mehta

September 2013

Abstract

We present the construction of heterotic string models built using the free fermionic formulation, and focus on how additional $U(1)$s may arise. We motivate an anomaly free combination, $U(1)_\zeta$, as a proton lifetime preserving symmetry external to a left–right symmetric gauge group. This same combination is found to nullify lepton number in $U(1)_{B-L}$ to form a leptophobic combination, also in left–right symmetric models, which we compare to other leptophobic $U(1)$ combinations constructed in the context of different gauge groups.

We then accommodate $U(1)_\zeta$ as a proton lifeguard symmetry in an effective field theory. We present a comparative study of how gauge coupling unification constraints may be satisfied when $SO(10) \times U(1)_\zeta \not\subset E_6$ and when the $U(1)_\zeta$ charges do have an E_6 embedding. We show that without such an embedding, current values of $\sin^2\theta_W\,(M_Z)$ and $\alpha_3\,(M_Z)$ rule these models out. We go on to discuss how viable string models with this property included may be constructed.

Declaration

I hereby declare that all work described in this thesis is the result of my own research unless reference to others is given. None of this material has previously been submitted to this or any other university. All work was carried out in the Theoretical Physics Division of the Department of Mathematical Sciences, University of Liverpool, U.K. during the period of October 2009 until September 2013.

Publication List

This thesis contains material that has appeared in the following publications by the author:

- A. E. Faraggi and V. M. Mehta, Proton Stability and Light Z' Inspired by String Derived Models, *Physical Review D* **84** (2011), 086006, `arXiv:1106.3082[hep-ph]`.

- A. E. Faraggi and V. M. Mehta, Leptophobic Z' in Heterotic–String Derived Models, *Physics Letters B* **703** (2011), 567, `arXiv:1106.5422[hep-ph]`.

- A. E. Faraggi and V. M. Mehta, Proton Stability, Gauge Coupling Unification, and a Light Z' in Heterotic–String Models, *Physical Review D* **88** (2013), 025006, `arXiv:1304.4230[hep-ph]`.

Acknowledgements

During my time in Liverpool, I've had the pleasure of working with and meeting some great friends and colleagues. The memories made here will stay with me forever and I have many people to thank. Firstly, my supervisor, Alon Faraggi, for the ideas and directions from which this work spawned.

A great deal of thanks to John Gracey and Dave Muskett, to who I am hugely indebted and my gratitude to Ian Jack and Thomas Mohaupt for helping me get to this stage, despite the bumpy road along the way. Thank you also to Tim Jones and Steve Downing for the squash games and for letting me win sometimes.

I'd like to thank my friends Owen Vaughan, Gary Soar, Ruofan Liao, Adriano Lo Presti, Elisa Manno, Conor Smyth, Aaron Roberts, Aaron Bundock, Katie Walters, Rob Purdy, Panos Athanasopoulos, Josh Davies, David Errington, Tomàš Ježo, Paul Dempster, Jaclyn Bell and Stephen Jones for making the time spent in Liverpool most enjoyable. I'll miss coffee time. I am very grateful to my officemate and good friend Dr William Walters; the journey would have been less fun had I endured it with anyone else.

To my family, I dedicate a special mention: the past 4 years have been the hardest of our lives but it has made us better, stronger people. We've grown up a lot during this time and this new sense of perspective will help us make our world a better place. Zenobia, Shahnavaz and Mum: I love you all.

Finally, to my darling Katie; your unwavering support and dedication helped me through these difficult years. The path that lies before us will be long and I can think of no better person to have by my side for its duration.

Gratias pro memoria.

Contents

For you, Dad. I miss you.

Chapter 1

Introduction

The recent discovery of the Higgs boson at the LHC [1, 2] lends further credence to the hypothesis that the Standard Model (SM) provides a viable effective parameterization of all subatomic interactions. However, there are many observed results that simply cannot be reproduced within this framework. The unification with gravity and the quantization of electromagnetic charges are but two of these unanswerable questions within the framework of the SM. Current energies being explored at the LHC could lead to interesting new physics beyond the Standard Model (BSM).

1.1 Proton Stability

The main aim of this thesis is to provide a viable solution to the problem of proton stability in a class of supersymmetric extensions of the Standard Model originating in the heterotic string, while accommodating other nuances of the SM, *e.g.* light neutrinos, three generations and a light Higgs. We propose an additional gauge symmetry, external to that of the SM gauge group, $SU(3)_C \times SU(2)_L \times U(1)_Y$, that, should it remain unbroken to sufficiently low energy, may suppress proton decay mediating operators (PDMOs) that commonly appear in these scenarios. With current limits at $\tau_p \gtrsim 10^{34}$ years [3] the question of proton stability is at the forefront of testing BSM physics.

In the SM, accidental global symmetries conserve baryon and lepton number at the renormalizable level. PDMOs may be induced at dimension-6, *i.e.*

$$\frac{1}{\Lambda_p^2} Q_L Q_L Q_L L_L, \tag{1.1}$$

which, from the current bound on the proton lifetime, indicate that the SM is an effective field theory (EFT) below a cutoff, $\Lambda_p \sim 10^{16}$GeV.

Many extensions of the SM that have been proposed to address other issues, in particular the hierarchy problem, introduce a cut–off at the TeV scale. Such extensions consequently induce proton decay at an unacceptable rate. For example, in supersymmetric extensions of the SM, operators violating baryon and lepton number are induced at the renormalizable level. These are given by

$$\begin{gathered} Q_L L_L d_L^c, \\ u_L^c d_L^c d_L^c, \end{gathered} \tag{1.2}$$

where each of the fields represents a chiral supermultiplet. One must then rely on some ad hoc global or discrete symmetries, to forbid the unwanted terms. For example, in the MSSM, R–parity is invoked to forbid these operators. R–parity is a $\mathbb{Z}_2$ symmetry of matter states. The charges are given by a linear combination of SM quantum numbers:

$$Q_R = (-1)^{3(B-L)+2s}, \tag{1.3}$$

where B and L are the baryon and lepton numbers, and s the spin. R–parity is used to distinguish between SM states and their superpartners, with the SM fermions having charge $+1$ and their supersymmetric scalar partners charge -1. This also results in the lightest superpartner being a dark matter candidate as it must be stable. When looking for a unified theory, one must also consider the high energy regime, *i.e.* $\mathcal{O}\left(M_{\text{Planck}}\right)$. Attempts have recently been made to derive R–symmetries of this type from string models [4–7]. However, it is expected that only local symmetries survive quantum gravity effects [8] and so in this thesis we do not consider such symmetries. Instead we investigate an alternative appealing proposition for the suppression of PDMOs: the existence of an abelian gauge symmetry beyond that of the SM gauge group. Allowing the SM matter states to be charged under this additional gauge symmetry, we may forbid PDMOs, which are then only induced at the symmetry's breaking scale. For the extra symmetry to provide adequate suppression of the unwanted terms, it has to exist at a mass scale within reach of contemporary particle accelerators [9, 10].

The simplest abelian extension one may construct to prohibit PDMOs is by gauging baryon minus lepton number, $U(1)_{B-L}$, which naturally arises in $SO(10)$ Grand Unified Theories (GUTs). To date, many $SO(10)$–based models have been constructed following the various symmetry breaking patterns of this rank-5 Lie group, both in the context of a top–down approach, *e.g.* the heterotic string (see *e.g.* [11] and references within) and in a bottom–up field theory construction [12–16] in 4- and 5-dimensions.

Gauged–$(B-L)$ (GBL) in $SO(10)$ has the advantage of being an anomaly free symmetry. That is, as each of the SM generations (plus a right–handed neutrino) is embedded in a single $SO(10)$ spinorial representation, the **16**, the only two $U(1)$ combinations free of gauge and gravitational anomalies are $U(1)_{B-L}$ and $U(1)_Y$ (or any linear combinations thereof). In the literature, the phenomenology of this minimal gauge extension of the SM has been widely explored (see *e.g.* [17] and references therein). $U(1)_{B-L}$ has also been discussed within the context of the free fermionic models of the heterotic string in Standard–like (SL) models [18–20] and in extra dimensional heterotic string models [21]. In [22], it was shown that sufficiently low neutrino masses could not be produced in SL models built in the free fermionic construction.

The requirement of light neutrino masses necessitates that lepton number is broken. In bottom–up $SO(10)$ grand unified models, one can use the **126** representation, which breaks lepton number by two units and leaves an unbroken symmetry, which still forbids the dimension-4 PDMOs. However, the **126** representation, in general, does not arise in perturbative string models [23]. This implies that lepton number is broken by unit–one carrying fields and thus, the dangerous dimension-4 PDMOs are generated. Specifically, in $SO(10)$, these operators are contained in the $\mathbf{16}^4$ term,

$$\begin{gathered} Q_L L_L d^c_L \nu^c_L, \\ u^c_L u^c_L d^c_L \nu^c_L, \end{gathered} \tag{1.4}$$

where ν^c_L is the SM singlet field, *i.e.* the CP–conjugate of ν_L, and gets a vev of the order of the GUT scale. Additionally, the $\mathbf{16}^4$ gives rise to the dimension-5 terms contained in

$$\begin{gathered} Q_L Q_L Q_L L_L, \\ u^c_L d^c_L d^c_L e^c_L, \end{gathered} \tag{1.5}$$

which are not forbidden by $U(1)_{B-L}$. It is therefore apparent that gauged baryon minus lepton number by itself is not sufficient to guarantee proton stability. Other local gauge symmetries, possibly in conjunction with $U(1)_{B-L}$, are needed to ensure proton stability [24, 25]. In this thesis, our discussion focusses on a specific $U(1)$ combination that incorporates $U(1)_{B-L}$ but allows for phenomenologically viable neutrino masses and forbids PDMOs up to dimension-6. The necessary requirements and conditions for allowing this $U(1)$ to be light are discussed in Section 1.3.

1.2 Unification

Another remarkable feature of the MSSM, as well as being the minimal extension to the SM that allows for SUSY and thus alleviates the electroweak–Planck hierarchy problem, is that, at one–loop, the SM gauge couplings unify, *i.e.* extrapolating the current experimental data for $\alpha_3(M_Z)$, $\alpha_2\,(M_Z)$ and $\alpha_1\,(M_Z)^*$, we find unification at $M_{\text{GUT}} \sim 2 \cdot 10^{16}$GeV [26–28].

However, as we are constructing a string–inspired model, string–scale unification is expected. In the heterotic string the scale at which coupling unification is predicted is $M_S \sim 5 \cdot 10^{17}$GeV [29]. In our analysis we vary the unification scale in this range, *i.e.* $M_{\text{GUT}} \leq \mu \leq M_S$. We see the effect of this variation on our low–energy obervables, $\sin^2\theta_W\,(M_Z)$ and $\alpha_3\,(M_Z)$, in Figure 1.1. As μ moves away from the MSSM unification scale, M_{GUT}, and toward the string scale, M_S, we notice that the values of $\sin^2\theta_W\,(M_Z)$ and $\alpha_3\,(M_Z)$ move away from their experimental results. The factor of 20 discrepancy between these unification scales was discussed in [30] and it was concluded that intermediate matter thresholds contributed enough to overcome its effect, allowing string unification in a wide class of realistic free fermion heterotic string models. In the analysis of our string–inspired model, we will look to accommodate intermediate scales in order for gauge coupling unification at the string scale to occur. It has also been demonstrated that nonperturbative effects arising in heterotic M–theory [31] can push the unification scale down to the MSSM unification scale [32]. As our additional $U(1)$ will take charge assignments that satisfy

$^*\alpha_2$ and $\alpha_1 \equiv \frac{5}{3}\alpha_Y$ are extracted from $\sin^2\theta_W$ and α_{EM}.

heterotic string constraints, *i.e.* an E_8 embedding, we expect a similar model based in the heterotic M–theory regime to have equivalent charge assignments. Thus, allowing variation of the unification scale, from M_{GUT} to M_S, may also account for nonperturbative effects.

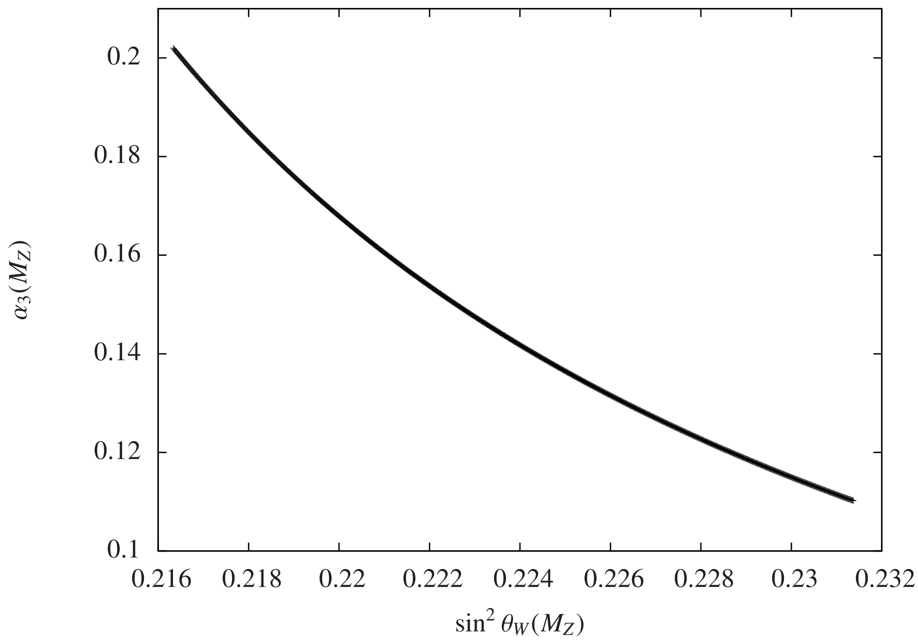

Figure 1.1: $\sin^2\theta_W(M_Z)$ vs. $\alpha_3(M_Z)$ with $2 \cdot 10^{16} \lesssim \mu \lesssim 5.27 \cdot 10^{17}$ GeV. The current limits are $\sin^2\theta_W(M_Z)\big|_{\overline{\mathrm{MS}}} = 0.23116 \pm 0.00012$ and $\alpha_3(M_Z) = 0.1184 \pm 0.0007$[33]. Here μ runs from right to left.

1.3 Low–scale $U(1)$

Additional abelian spacetime vector bosons beyond those that mediate the $SU(3) \times SU(2) \times U(1)_Y$ subatomic interactions are abundant in extensions of the SM. Their existence, in both GUTs and string theories, have been amply discussed in the literature [34–36], with the most appealing abelian extensions arising in $SO(10)$ and E_6 [37–42]. The embedding of the SM states* in three generations of the spinorial **16**

*with three right–handed neutrinos

representation strongly hints at the realisation of $SO(10)$ in nature, while E_6 GUTs go a step further by accommodating both matter and Higgs states in a common representation, the **27**. These GUT groups may be reproduced in the heterotic string regime and broken directly to the SM or to subgroups with additional abelian factors [43–46]. A class of three generation heterotic-string models that produce these GUT embeddings are the free fermion models of [18, 19, 47–51], which correspond to compactifications on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds [52–58] due to the relation of bosons and fermions in two dimensions,

$$\psi^\mu + i\chi^\mu =: e^{iX^\mu} : \quad . \tag{1.6}$$

This will be the formulation used to discuss our heterotic string models and we review their structure in Chapter 2. The discussion of additional $U(1)$s in the context of free fermion models has included those that act as proton lifeguards [9, 10, 25, 59] as well as other motivations from potential BSM signatures [60, 61] and supersymmetry breaking [62].

Recently a leptophobic U(1) was motivated due to an excess in the W + 2 jets channel detected at CDF [63]. The absence of such an enhancement in the dilepton channel, as well as constraints arising from direct production at LEPII, TeVatron and LHC searches necessitates suppressed couplings to leptons. This posed an interesting problem as most GUT and string theory models will produce extra bosons with unsuppressed coupling to both leptons and baryons. It is, therefore, of interest to examine how a leptophobic Z' can arise [60, 64]. From a bottom–up approach, one can simply gauge the baryon number, $U(1)_B$. This exercise has been undertaken [65, 66] and within Type-I string theories a gauged $U(1)_B$ may indeed arise. However, in GUTs and string models, abelian extensions of the Standard Model typically have unsuppressed couplings to leptons.

The abundance of $U(1)$ symmetries in GUTs, and the string models from which they originate, does not necessarily result in their existence at accessible energies. For example, much of the discussion of $U(1)$ symmetries as proton lifeguards in free fermion models has focussed on the requirements necessary for their existence at the string–scale in addition to accommodating the constraints coming from SM data. These requirements include:

PDMOs – the dimension-4, -5 and -6 proton decay mediating operators must be forbidden;

Light neutrinos – lepton number violation must be allowed in order for a seesaw–mechanism to be realised;

Yukawa couplings – the SM fermions must still form the necessary couplings with the electroweak Higgs doublets in order to generate the correct mass terms upon breaking of $SU(2)_L \times U(1)_Y$;

Family universal – to avoid flavour changing neutral currents, and to avoid generation–dependent couplings that may induce rapid proton decay, we demand family universality;

Anomaly freedom – to build a consistent effective field theory (EFT) that can describe low–scale physics, while allowing for an additional abelian gauge symmetry accessible at current experiments, requires the theory to be anomaly free.

Building a string–inspired model that satisfies these constraints does not guarantee that its low–energy EFT will do so. In the examples mentioned previously, the existence of the desired symmetry in explicit string constructions guarantees anomaly freedom of the additional $U(1)$, yet facilitating satisfaction of these properties in a phenomenologically viable toy–model can prove to be difficult. In this thesis we construct a string–inspired field theory model that takes into account the ingredients, in particular the string charge assignments, from the string–derived models to explore some phenomenological properties of the extra $U(1)$.

1.4 Inspiration from strings

The models that we use in this thesis are constructed in the free fermionic formulation (FFF) of the heterotic string. In the extra–dimensional construction of the heterotic string [67–69], comprising of a supersymmetric left–moving sector and a purely bosonic right–moving sector, it is predicted that there exist ten spacetime dimensions, six of which are compactified on some 6-dimensional manifold, $\mathcal{M}^6$. However, the FFF of the heterotic string is constructed directly in four dimensions, removing the necessity of some geometrical interpretation for the additional dimensions. The additional degrees of freedom are thought of as fermions that freely propagate on the string worldsheet, the two–dimensional surface mapped out by the string propagating through time.

In the geometrical construction of the heterotic string, as the right–moving sector is purely bosonic, there is a mismatch between the number of dimensions of the left– and right–moving sectors. That is, the left–moving sector corresponds to a superstring sector, *i.e.* a 10-dimensional theory, whereas the right–moving sector corresponds to a bosonic string sector, *i.e.* a 26-dimensional theory. This discrepancy in dimension is resolved by compactifying the sixteen additional dimensions of the bosonic sector on some internal 16-dimensional torus. This torus corresponds directly to an even, self–dual lattice, Γ, which is equivalent to the root lattice of $E_8 \times E_8$ or the weight lattice of $\frac{\mathrm{Spin}(32)}{\mathbb{Z}_2}$.

In extra dimensional models, the additional dimensions are thought to be of microscopic scale (or even Planck), invisible to current detectors. The additional dimensions, though, contain a lot of information that dictates the physics of the four dimensions that we see. This is because in the extra dimensional construction of heterotic string models, the ten–dimensional theory fixes the gauge and matter content. Thus, the description of $\mathcal{M}^6$ becomes very important in determining unified string models of particle physics.

However, with the absence of extra dimensions, physics in the FFF of the heterotic string is not determined by specifying the geometry of additional spacetime dimensions. In fact, to specify models in the FFF, one only requires two ingredients: the phases picked up by the worldsheet fermions; and the generalised GSO phases (see 2.1 and proceeding discussion). Realistic unified string models are, therefore, readily constructed within this framework. Due to their accessibility, one can do vast searches through string vacua [70] with relative ease. New techniques are being adapted for geometrical constructions but for now, comprehensive scans of possible models in string vacua are restricted by our knowledge of $\mathcal{M}^6$ geometries [71, 72].

With access to the string vacua in 4-dimensions in the FFF of the heterotic string, the charge assignments of our states, gauge structure and matter content of our low–scale models are readily available, while the preservation of string–scale physics, *i.e.* modular invariance, conformal invariance, *etc.*, is already built in.

Outline

In this thesis, we discuss the phenomenological effects of an additional abelian gauge symmetry, external to an $SO(10)$ GUT group. This $U(1)$ is generated by a linear combination of the Cartan generators of the 8-dimensional visible gauge group in the heterotic string construction, and an example is constructed that forbids proton decay mediating operators up to dimension-6 while allowing for light neutrinos via a seesaw mechanism. Another example has suppressed couplings to leptons to form a leptophobic Z'.

Chapter 2

In this chapter we introduce important concepts with regard to a particular construction of the heterotic string: the free fermionic formulation. We give examples of how GUT models are built within this framework and how GUT representations are decomposed under low–scale gauge groups that eventually break to the SM gauge group, $SU(3)_C \times SU(2)_L \times U(1)_Y$. We discuss the string formation of additional $U(1)$s and motivate two examples: a proton lifeguard combination and a leptophobic $U(1)$. We also outline the stringy origins of the matter we will use to build our string–inspired model when constructing a specific model accommodating our proton–protecting $U(1)$.

Chapter 3

Using the rules and techniques of the previous chapter, we discuss a particular $U(1)$ combination originally motivated as a solution to the Wjj anomaly found at the CDF detector back in 2011: a leptophobic $U(1)$. We summarise previous examples of leptophobic $U(1)$s and their respective gauge group embeddings. We then present a new combination that features in models with the left–right symmetric breaking pattern of $SO(10)$. The analysis carried out in this chapter featured in [61].

Chapter 4

Here we specify a model whose attributes allow the suppression of proton decay mediating operators up to dimension-6. We build a spectrum, starting with the MSSM, that satisfies the string charge assignments. We present the spectra above and below an intermediate $SU(2)_R$ symmetry breaking scale and also the respective superpotentials. The analysis done in this chapter featured in [73].

Chapter 5

Having specified a model in the previous chapter, we now look at phenomenological constraints that can be applied to our string–inspired model. Specifically we look at how both proton lifetime limits and gauge coupling unification constrain our model. For the GCU analysis, we present a comparison of two classes of models: $SO(10) \times U(1)_\zeta \not\subset E_6$ and $SO(10) \times U(1)_\zeta \subset E_6$. This allows us to discuss the benefits and difficulties in each. The analysis done in this chapter featured in [74].

Chapter 6

To accompany the analysis and conclusions of the previous chapter, we present a "recipe" for constructing the $SO(10) \times U(1)_\zeta \subset E_6$ model within the free fermionic formulation of the heterotic string. This model must accommodate the properties of our proton protecting $U(1)$ while, in principle, allowing for gauge coupling unification and LRS intermediate gauge structure. The discussion in this chapter also featured in [74].

Chapter 7

To conclude we summarise our results and discuss possible future projects that may extend this work.

Appendix A

In the first appendix, we present the root vectors in the basis we use to construct $SO(10)$. We use these to build the **16** representation explicitly.

Appendix B

Here we present the model building rules applicable to the free fermionic formulation of the heterotic string.

Chapter 2

Model Building & Free Fermionic Formulation

In this review chapter, we present the formalism we use to construct our string models. It treats the extra degrees of freedom, commonly compactified as extra dimensions, as free fermions on the string worldsheet and, thus, these are string models constructed directly in four dimensions. In this section we review the construction and structure of the free fermionic formulation; the framework within which we build our models. We then discuss the various semi–realistic gauge groups and matter representations that have been explored in the literature.

From a two–dimensional perspective, bosons and fermions are equivalent, with real fermions and bosons carrying conformal weight $\frac{1}{2}$ and 1 respectively. Thus, we may construct a worldsheet theory for any type of string in either the bosonic or fermionic language. Here and for the remainder of this thesis, we focus on models originating from the heterotic string in the fermionic language. We note here that, due to the equivalence between bosons and fermions in two–dimensions, there is a subtle correspondence between free fermionic models and those constructed in the bosonic language. This is beyond the scope of this thesis, but has been explored in [52–58].

2.1 Free Fermionic Formulation

As we are now restricting ourselves to only the four dimensions that have been observed in nature, we induce a conformal anomaly on the worldsheet, *i.e.* the trace over all conformal states in our string theory is now non–zero. This is rectified by including freely propagating fermions as worldsheet degrees of freedom. Requiring a cancellation of the conformal anomaly gives, in the light–cone gauge, 18 worldsheet Majorana–Weyl fermions in the supersymmetric sector and 44 worldsheet Majorana–Weyl fermions in the bosonic sector. We also have the superpartner of the bosonic coordinates in the supersymmetric sector and the bosonic coordinates themselves in both sectors. In the right–moving, bosonic sector, we actually complexify 32 fermionised real degrees of freedom; these 16 complex fermions define our gauge structure. This corresponds to the 16-dimensional internal torus of the compactified heterotic string, corresponding to an $E_8 \times E_8$ root lattice or $\frac{\text{Spin}(32)}{\mathbb{Z}_2}$ weight lattice. Due to the non–geometrical interpretation of these models, the additional 12 Majorana–Weyl fermions, bosons corresponding to the compactified dimensions in the bosonic language, allow us to increase the rank of our overall gauge group up to 22. However, as we will see later, these will play the role of a 'counting' operator for the number of generations in our semi–realistic models. A full list of worldsheet fields is given in Table 2.1.

All of the physics is contained within the one–loop partition function, *i.e.* the vacuum–to–vacuum amplitude, and thus gives us access to the full theory,

$$\mathcal{Z} = \sum_{\mathbf{a},\mathbf{b}} c\begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix} Z \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix}. \tag{2.1}$$

Contributing to this are the phases of the fermions around the non–contractible cycles of our toroidal worldsheet (see Figure 2.1), described by the 64-component

	Label	Description
Left–moving	X^μ	Bosonic coordinates with spacetime index, $\mu = 0, \ldots, 3$
	ψ^μ	Majorana–Weyl superpartners of the bosonic coordinates with spacetime index
	$\chi^{1,\ldots,6}$	Majorana–Weyl superpartners to the six compactified dimensions
	$y^{1,\ldots,6}, w^{1,\ldots,6}$	Majorana–Weyl fermions that correspond to the bosons describing the six compactified dimensions in the bosonic formulation
Right–moving	$\overline{X}^\mu$	Bosonic coordinates with spacetime index
	$\overline{y}^{1,\ldots,6}, \overline{w}^{1,\ldots,6}$	Majorana–Weyl fermions that correspond to the bosons describing the six compactified dimensions in the orbifold formulation
	$\overline{\psi}^{1,\ldots,5}, \overline{\eta}^{1,2,3}$	Complex fermions that describe the visible gauge sector
	$\overline{\phi}^{1,\ldots,8}$	Complex fermions that describe the hidden gauge sector

Table 2.1: States that describe our worldsheet, where we have separated the internal freely propagating fermions from the spacetime coordinates. As shown, we have 18 in the left–moving, supersymmetric sector and 44 in the right–moving, bosonic sector in the light–cone gauge.

vectors **a** and **b**. In fact, along with the generalised GSO (GGSO) coefficients,

$$c\begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix}, \tag{2.2}$$

we have the tools to describe our models in full. However, as is generic in string model building, one must also take into consideration an overcounting over the *spin–structures i.e.* the phases picked up on parallel transport around the loops. In other words, our theory must remain modular invariant. This was considered in [75, 76] and we follow the rules laid out in these papers, known henceforth as the *ABK rules* to construct our models. We reproduce these rules in Appendix B for reference.

As mentioned above, to specify the models we must specify the phases, $\alpha(f)$, picked by the worldsheet fermions when transported along the non–contractible

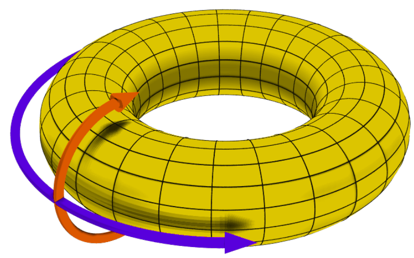

Figure 2.1: When parallel transported around either of the two non–contractible loops of the torus, the fermions pick up a phase, $\alpha(f)$, and the bosonic coordinates remain invariant.

loops of the torus, *i.e.*

$$f \to -e^{i\pi\alpha(f)}f, \quad \alpha(f) \in (-1, 1], \tag{2.3}$$

where the minus sign is simply convention and f corresponds to our worldsheet fermions. Applying different boundary conditions to each of our fermions will correspond to different models in the FFF. These models are generated by a set of basis vectors, $\mathbf{b}_k$, describing the transformation properties of the 64 worldsheet fermions and span a finite additive group

$$\Xi = \sum_{i}^{k} n_i \mathbf{b}_i \tag{2.4}$$

$$\simeq \mathbb{Z}_{N_i} \oplus \cdots \oplus \mathbb{Z}_{N_k}, \tag{2.5}$$

where $n_i = 0, \ldots, N_i - 1$. In our construction, we will see that this additive set consists of

$$\Xi = \mathbb{Z}_2^7 \oplus \mathbb{Z}_4, \tag{2.6}$$

where the $\mathbb{Z}_2$ corresponds to a basis vector with $\alpha(f) = 0, 1$, *i.e.* antiperiodic or periodic fermions only, and the $\mathbb{Z}_4$ a basis vector with some fermions picking up a complex phase, *i.e.* $\alpha(f) = \frac{1}{2}$.

The physical massless states in the Hilbert space of a given sector, $\alpha \in \Xi$, are then obtained by acting on $|0\rangle_\alpha$ with the worldsheet bosonic and fermionic mode operators, with frequencies $\nu_b, \nu_f,\ \nu_{f^*}$, and by subsequently applying the GGSO projections,

$$\left\{ e^{i\pi(\mathbf{b}_i \cdot F_\alpha)} - \delta_\alpha c^* \begin{pmatrix} \alpha \\ \mathbf{b}_i \end{pmatrix} \right\} |s\rangle_\alpha = 0 \tag{2.7}$$

to preserve modular invariance where δ_α describes the spacetime statistics of the sector α, *i.e.*

$$\begin{aligned} \delta_\alpha &= e^{i\pi\alpha(\psi^\mu)} \\ &= \pm 1 \end{aligned} \tag{2.8}$$

i.e. $\delta_\alpha = -1$ if ψ^μ is periodic in the sector α and thus the state is a spacetime fermion, and $\delta_\alpha = +1$ if ψ^μ is antiperiodic in the sector α, resulting in a spacetime boson. Also we have that

$$(\mathbf{b}_i \cdot F_\alpha) \equiv \left\{ \sum_{\substack{\text{real+complex} \\ \text{left}}} - \sum_{\substack{\text{real+complex} \\ \text{right}}} \right\} (\mathbf{b}_i(f) F_\alpha(f)), \tag{2.9}$$

where $F_\alpha(f)$ is a fermion number operator counting each mode of f once (and if f is complex, f^* minus once). All physical states must satisfy the Virasoro condition,

$$M_L^2 = -\frac{1}{2} + \frac{\alpha_L \cdot \alpha_L}{8} + \sum \nu_L \qquad = \qquad M_R^2 = -1 + \frac{\alpha_R \cdot \alpha_R}{8} + \sum \nu_R \tag{2.10}$$

where $\alpha = (\alpha_L | \alpha_R) \in \Xi$ and the massless states are $M_L^2 = M_R^2 = 0$.

Thus, the states that are allowed in the Hilbert space are

$$\mathcal{H} = \bigoplus_{\alpha \in \Xi} \prod_{i=1}^{k} \left\{ e^{i\pi(\mathbf{b}_i \cdot F_\alpha)} = \delta_\alpha c^* \begin{pmatrix} \alpha \\ \mathbf{b}_i \end{pmatrix} \right\} \mathcal{H}_\alpha. \tag{2.11}$$

Those that do not satisfy (2.7) and thus do not appear in $\mathcal{H}$ are said to be *projected out*.

For a sector consisting of periodic complex fermions only, the vacuum is a spinor, $|\pm\rangle$, representing the Clifford algebra of the corresponding zero modes, f_0 and f_0^*, which have fermion number $F(f) = 0, -1$ respectively. In addition, the Cartan subalgebra of our rank–22 group is $U(1)^{22}$, generated by the right–moving currents, $\overline{ff}^*$. For each complex fermion, f, the $U(1)$ charges correspond to

$$Q(f) = \frac{1}{2}\alpha(f) + F(f). \tag{2.12}$$

The representation (2.12) shows that $Q(f)$ is identical with the worldsheet fermion numbers, $F(f)$, for worldsheet fermions with Neveu–Schwarz boundary conditions, $\alpha(f) = 0$, and is $F(f) + \frac{1}{2}$ for those with Ramond boundary conditions, $\alpha(f) = 1$. The charges for the $|\pm\rangle$ spinor vacua are $\pm\frac{1}{2}$.

2.2 Model–building

As we are building our string models within the heterotic string regime, we will now discuss the correspondence between the standard compactification methods used and their description in the FFF.

The gauge structure in the geometrical interpretation comes from an internal 16-dimensional torus in the right–moving sector. This sector is purely bosonic and so, in order for the conformal anomaly to be cancelled it must be a 26–dimensional sector. However, this is in disagreement with the left–moving sector which can be described by a 10-dimensional superstring. This discrepancy requires the additional sixteen dimensions to be purely internal. In fact, the only even, self–dual 16-dimensional torus on which a consistent theory may be reproduced corresponds to the root lattice of $E_8 \times E_8$ or the weight lattice of $\frac{\mathrm{Spin}(32)}{\mathbb{Z}_2}$ as first demonstrated in [67–69].

In the FFF of the heterotic string, the gauge structure can be described by any of the right–moving free fermions, in general. In fact, the simplest case is that all 44 fermions are periodic. Allowing the left–movers to also remain invariant under parallel transport around a and b forms a 64-vector with all the worldsheet fermions being periodic. This is known as the $\mathbb{1}$ vector and generates an $SO(44)$ gauge group, the starting point of all NAHE–based models. As shown in Appendix B and [75–77], models built in the FFF all begin with the $\mathbb{1}$ vector. Below we will briefly discuss the construction of the NAHE set [78], focussing on the visible gauge and matter sectors. As our main aim is to explore specific models rather than generalities in the construction, we specify our gauge structure to be described by sixteen complex right–moving fermions, $\overline{\psi}^{1,\ldots,5}, \overline{\eta}^{1,2,3}, \overline{\phi}^{1,\ldots,8}$, where:

- $\overline{\phi}^{1,\cdots,8}$ generate the rank eight hidden gauge group;
- $\overline{\psi}^{1,\cdots,5}$ generate the $SO(10)$ GUT gauge group;
- $\overline{\eta}^{1,2,3}$ generate the three remaining $U(1)$ generators in the Cartan subalgebra of the observable rank eight gauge group.

A combination of these three $U(1)$ currents plays the role of the proton lifeguard [73] and a linear combination of these will also be shown to cancel the lepton number component of $U(1)_{B-L}$ such that an effective baryon number gauge symmetry results. As we will demonstrate further, the different patterns of symmetry breaking come from different boundary conditions on these generators of the gauge groups.

2.3 The NAHE set

There are two broad classes of free fermionic models that have been studied in the literature: the first are models that utilise the NAHE set of boundary condition basis vectors, which we discuss here; the second are the models spanned in the classification of [57, 79, 80]. The two classes differ in that the first allows and uses complexified internal fermions from the set $\{y, w|\overline{y}, \overline{w}\}$, resulting in additional $U(1)$ gauge symmetries, whereas such fermions have not been incorporated in the second class to date. The treatment of the sixteen complex worldsheet fermions that generate the gauge degrees of freedom is identical in the two classes of models. As both the extra proton safeguarding $U(1)$ and the leptophobic $U(1)$ symmetry arise exclusively from these worldsheet fermions, the two classes are identical in respect to the extra $U(1)$s of interest here. To date, the majority of phenomenological studies of free fermionic models are NAHE–based [78] with the notable exception being the exophobic* Pati–Salam vacua of [81–83]. For definiteness, we discuss the NAHE–based models and provide a brief discussion of how the different gauge structures of these models are derived. This will highlight necessary techniques of symmetry breaking/enhancement which will become useful later on. The first stage in the construction of these models consists of the $\mathbb{1}$ vector, mentioned before, where,

$$\mathbb{1} = \left\{\psi^{\mu}, \chi^{1,\dots,6}, y^{1,\dots,6}, w^{1,\dots,6} \,|\, \overline{y}^{1,\dots,6}, \overline{w}^{1,\dots,6}, \overline{\psi}^{1,\dots,5}, \overline{\eta}^{1,2,3}, \overline{\phi}^{1,\dots,8}\right\}. \tag{2.13}$$

We are using the convention of any fermions present in $\{\dots\}$ transforming trivially under parallel transport and $|$ separates right–movers, indicated also with bars, from left–movers. In the case of the $\mathbb{1}$ vector, the right–moving fermions are indistinguish-

*$i.e.$ models without exotics. Exotics are states that have fractional electromagnetic charge and are stable at low energies.

able and so generate an $SO(2n)$, where n is the number of complex fermions. In this case $n = 22$ and so we have the gauge bosons of $SO(44)$ originating in the **0**–sector, or the NS–sector, *i.e.*

$$\psi^\mu \overline{\phi}^a \overline{\phi}^b |0\rangle_{\rm NS} \qquad \text{for } \ a, b = 1, \ldots, 44. \tag{2.14}$$

At this stage the spectrum also contains the gravity multiplet, a tachyon and no supersymmetry. The gravity multiplet is actually a model–independent feature and can be shown to be present in all string models built in the FFF, independent of specifying basis vectors.

We then add the vector

$$\mathbf{S} = \left\{\psi^\mu, \chi^{1,\ldots,6}\right\}. \tag{2.15}$$

The **S**–sector is now also massless and is, in fact, a Ramond vacuum due to the presence of only complex periodic fermions; we may complexify real fermions with equivalent boundary conditions in the spin–structures as

$$\begin{aligned} \psi^\mu &= \frac{1}{\sqrt{2}}\left(\psi^1 + i\psi^2\right) & \chi^{34} &= \frac{1}{\sqrt{2}}\left(\chi^3 + i\chi^4\right) \\ \chi^{12} &= \frac{1}{\sqrt{2}}\left(\chi^1 + i\chi^2\right) & \chi^{56} &= \frac{1}{\sqrt{2}}\left(\chi^5 + i\chi^6\right) \end{aligned} \tag{2.16}$$

or their conjugates $\lambda^{*\,ab} = \frac{1}{\sqrt{2}}\left(\lambda^a - i\lambda^b\right)$. As mentioned previously these carry $U(1)$ charge, $\pm\frac{1}{2}$, and so we introduce a combinatorial notation

$$\left[\begin{pmatrix} 4 \\ 0 \end{pmatrix} + \cdots + \begin{pmatrix} 4 \\ 4 \end{pmatrix}\right] \tag{2.17}$$

where the combinatorial factor counts the number of $|-\rangle$ in the degenerate vacuum of a given state. Our GGSO projection $c\begin{pmatrix} \mathbb{1} \\ \mathbf{S} \end{pmatrix} = \pm 1$ allows only an odd (-1) or even (+1) number of $|-\rangle$. Our choice is even, *i.e.* $c\begin{pmatrix} \mathbb{1} \\ \mathbf{S} \end{pmatrix} = +1$. The $\mathbf{S}$–sector contains the gravitini,

$$\left[\begin{pmatrix} 4 \\ \text{even} \end{pmatrix}\right] \partial\overline{X}^{\mu}|0\rangle_{\mathbf{S}}, \tag{2.18a}$$

and the gaugini,

$$\left[\begin{pmatrix} 4 \\ \text{even} \end{pmatrix}\right] \overline{\phi}^{a}\overline{\phi}^{b}|0\rangle_{\mathbf{S}}, \tag{2.18b}$$

of our spectrum and we see that we have $\mathcal{N} = 4$ spacetime SUSY. As of yet, neither of our sectors, $\mathbf{0}$ or $\mathbf{S}$, contain matter states.

2.3.1 Adding $\mathbf{b}_1$

To complete the NAHE set, we add the basis vectors $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{b}_3$,

$$\mathbf{b}_1 = \left\{\psi^{\mu}, \chi^{12}, y^{3,\dots,6} \,|\, \overline{y}^{3,\dots,6}, \overline{\psi}^{1,\dots,5}, \overline{\eta}^{1}\right\}, \tag{2.19a}$$

$$\mathbf{b}_2 = \left\{\psi^{\mu}, \chi^{34}, y^{1,2}, w^{5,6} \,|\, \overline{y}^{1,2}, \overline{w}^{5,6}, \overline{\psi}^{1,\dots,5}, \overline{\eta}^{2}\right\}, \tag{2.19b}$$

$$\mathbf{b}_3 = \left\{\psi^{\mu}, \chi^{56}, w^{1,\dots,4} \,|\, \overline{w}^{1,\dots,4}, \overline{\psi}^{1,\dots,5}, \overline{\eta}^{3}\right\}. \tag{2.19c}$$

These act in similar fashion and so it will be demonstrative to explore one of these, say $\mathbf{b}_1$, in more detail and simply show the highlights of the other two. We first consider how this acts on our gauge bosons in the $\mathbf{0}$–sector. From (2.8) we see that

$$\delta_{\mathbf{b}_1} = -1. \tag{2.20}$$

This implies that for a state to survive the GGSO projections, pairs of internal fermions forming bosonic states with ψ^{μ} in the NS–sector must satisfy,

$$\mathbf{b}_1 \cdot F_{\text{NS}} = 0 \bmod 2. \tag{2.21}$$

As we can see, the states that survive and correspond to gauge bosons are

$$\psi^\mu \left\{\overline{y}^{3,\dots,6}, \overline{\psi}^{1,\dots,5}, \overline{\eta}^1\right\} \left\{\overline{y}^{3,\dots,6}, \overline{\psi}^{1,\dots,5}, \overline{\eta}^1\right\} |0\rangle_{\text{NS}} \simeq SO(16); \tag{2.22a}$$

$$\psi^\mu \left\{\overline{y}^{1,2}, \overline{w}^{1,\dots,6}, \overline{\eta}^{2,3}, \overline{\phi}^{1,\dots,8}\right\} \left\{\overline{y}^{1,2}, \overline{w}^{1,\dots,6}, \overline{\eta}^{2,3}, \overline{\phi}^{1,\dots,8}\right\} |0\rangle_{\text{NS}} \simeq SO(28). \tag{2.22b}$$

Therefore our gauge group has broken from

$$SO(44) \to SO(16) \times SO(28). \tag{2.23}$$

In the **S**–sector, we find that two of our gravitini are projected out,

$$\left[\begin{pmatrix} 4 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix}\right] \partial\overline{X}^\mu |0\rangle_{\mathbf{S}} \to \underbrace{\begin{pmatrix} 2 \\ 1 \end{pmatrix}}_{\psi^\mu, \chi^{12}} \underbrace{\left[\begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \end{pmatrix}\right]}_{\chi^{34}, \chi^{56}} \partial\overline{X}^\mu |0\rangle_{\mathbf{S}}, \tag{2.24}$$

resulting in $\mathcal{N} = 4 \to \mathcal{N} = 2$ spacetime SUSY. We also have four generations of the **128** spinor representation of $SO(16)$ coming from the $\mathbf{b}_1$–sector: the vacuum of the $\mathbf{b}_1$ sector is Ramond, *i.e.* a degenerate $|\pm\rangle$, and so we have

$$\left[\begin{pmatrix} 12 \\ 0 \end{pmatrix} + \cdots + \begin{pmatrix} 12 \\ 12 \end{pmatrix}\right], \tag{2.25}$$

i.e. 4096 states. The GGSO projection, $c\begin{pmatrix} \mathbb{1} \\ \mathbf{b}_1 \end{pmatrix}$ is chosen such that only the even states remain, *i.e.*

$$\left[\begin{pmatrix} 12 \\ \text{even} \end{pmatrix}\right] = 2048 \text{ states}. \tag{2.26}$$

This can be decomposed into the **128** and $\overline{\mathbf{128}}$ of $SO(16)$ as

$$\underbrace{\left[\begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \end{pmatrix}\right]}_{\psi^\mu, \chi^{12}} \left\{ \underbrace{\left[\begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \end{pmatrix}\right]}_{y^{3,\dots,6}} \underbrace{\left[\begin{pmatrix} 8 \\ \text{even} \end{pmatrix}\right]}_{\overline{y}^{3,\dots,6}, \overline{\psi}^{1,\dots,5}, \overline{\eta}^1} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} \left[\begin{pmatrix} 8 \\ \text{odd} \end{pmatrix}\right] \right\} \tag{2.27}$$

where

$$\left[\begin{pmatrix} 8 \\ \text{even} \end{pmatrix}\right] \simeq \mathbf{128} \tag{2.28a}$$

and

$$\left[\begin{pmatrix} 8 \\ \text{odd} \end{pmatrix}\right] \simeq \overline{\mathbf{128}}. \tag{2.28b}$$

As stated, we have four generations of each representation coming from the $\mathbf{b}_1$–sector. In the $(\mathbf{b}_1 + \mathbf{S})$–sector we have the superpartners of these states. In fact, $\mathbf{S}$ is our SUSY generator, thus any states that appear in the sector α will have superpartners in $\alpha + \mathbf{S}$.

We will now see how this decomposes under $\mathbf{b}_2$ and $\mathbf{b}_3$, along with the gauge bosons in the NS–sector.

2.3.2 Addition of $\mathbf{b}_2$ and $\mathbf{b}_3$

We will find that adding $\mathbf{b}_2$ and $\mathbf{b}_3$ results in the gauge group $SO(10) \times SO(6)^3 \times E_8'$ with $\mathcal{N} = 1$ spacetime SUSY and that the vacuum contains forty–eight multiplets in the $\mathbf{16}$ chiral representation of $SO(10)$, sixteen coming from each of $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$.

Gauge bosons At the level of $\{\mathbb{1}, \mathbf{S}, \mathbf{b}_1\}$, we had the gauge bosons in (2.22) generating $SO(16) \times SO(28)$, with 4 generations of the $\mathbf{128}$ and $\overline{\mathbf{128}}$ of $SO(16)^*$. We also had $\mathcal{N} = 2$ SUSY. Adding $\mathbf{b}_2$ in the NS sector decomposes the $SO(16)$ gauge bosons as,

$$\psi^\mu \left\{\overline{y}^{3,\ldots,6}, \overline{\eta}^1\right\} \left\{\overline{y}^{3,\ldots,6}, \overline{\eta}^1\right\} |0\rangle_{\text{NS}} \simeq SO(6)_1, \tag{2.29a}$$

$$\psi^\mu \left\{\overline{\psi}^{1,\ldots,5}\right\} \left\{\overline{\psi}^{1,\ldots,5}\right\} |0\rangle_{\text{NS}} \simeq SO(10), \tag{2.29b}$$

*As well as our spinor representations, we have scalars that transform under both gauge groups. We do not include these in our demonstrative analysis.

i.e. $SO(16) \rightarrow SO(10) \times SO(6)_1$. In a similar fashion, the $SO(28)$ is broken to $SO(22) \times SO(6)_2$, with the generating bosons,

$$\psi^{\mu}\left\{\overline{y}^{1,2},\overline{w}^{5,6},\overline{\eta}^{2}\right\}\left\{\overline{y}^{1,2},\overline{w}^{5,6},\overline{\eta}^{2}\right\}|0\rangle_{\text{NS}} \simeq SO(6)_2, \tag{2.30a}$$

$$\psi^{\mu}\left\{\overline{w}^{1,\ldots,4},\overline{\eta}^{3},\overline{\phi}^{1,\ldots,8}\right\}\left\{\overline{w}^{1,\ldots,4},\overline{\eta}^{3},\overline{\phi}^{1,\ldots,8}\right\}|0\rangle_{\text{NS}} \simeq SO(22). \tag{2.30b}$$

The $\mathbf{b}_3$ then splits the $\{\overline{w}^{1,\ldots,4},\overline{\eta}^{3}\}$ and the $\left\{\overline{\phi}^{1,\ldots,8}\right\}$ resulting in $SO(22) \rightarrow SO(6)_3 \times SO(16)'$. The $SO(6)_i$ are flavour symmetries as the different generations are charged under each different $SO(6)$. We will see that our proton lifetime preserving $U(1)$ will originate in these and that we may form a linear combination that is family universal,

$$U(1)_{\zeta} = U(1)_1 + U(1)_2 + U(1)_3 \tag{2.31}$$

The sector, ξ, formed by the combination

$$\xi = \mathbb{1} + \mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3 \equiv \left\{\phi^{1,\ldots,8}\right\}, \tag{2.32}$$

generates

$$\left[\begin{pmatrix} 8 \\ \text{even} \end{pmatrix}\right] \simeq \mathbf{128}. \tag{2.33}$$

However, unlike the $\mathbf{b}_1$–sector, the massless states of ξ in the left–moving sector are not degenerate; they are obtained by acting on the vacuum with a fermionic oscillator. Therefore,

$$\psi^{\mu}\left[\begin{pmatrix} 8 \\ \text{even} \end{pmatrix}\right]|0\rangle_{\xi} \tag{2.34}$$

transform as gauge bosons. These combine with the **120** in the NS–sector and form the adjoint of E_8. Therefore, the hidden $SO(16)'$ gauge group is enhanced to E_8

and

$$SO(10) \times SO(6)^3 \times E_8' \tag{2.35}$$

is the overall gauge group at the level of the NAHE set.

Chiral Matter Earlier we saw that we have four generations of the **128** and $\overline{\mathbf{128}}$ of $SO(16)$ originating in $\mathbf{b}_1$, at the level of the set $\{\mathbb{1}, \mathbf{S}, \mathbf{b}_1\}$. With the addition of $\mathbf{b}_2$, we must now decompose as $SO(10) \times SO(6)_1$ representations. Considering the GGSO projections,

$$c\begin{pmatrix} \mathbb{1} \\ \mathbf{b}_1 \end{pmatrix} = c\begin{pmatrix} \mathbb{1} \\ \mathbf{b}_2 \end{pmatrix} = -1, \tag{2.36}$$

we find that (2.27) decomposes as

$$\overbrace{\left\{\begin{pmatrix} 2 \\ 0 \end{pmatrix}}^{\psi^\mu,\chi^{12}} \overbrace{\left[\begin{pmatrix} 4 \\ \text{even} \end{pmatrix}\right]}^{y^{3,\dots,6}|\overline{y}^{3,\dots,6}} \overbrace{\left[\begin{pmatrix} 5 \\ \text{even} \end{pmatrix}\right]}^{\overline{\psi}^{1,\dots,5}} + \begin{pmatrix} 2 \\ 2 \end{pmatrix}\left[\begin{pmatrix} 4 \\ \text{odd} \end{pmatrix}\right]\left[\begin{pmatrix} 5 \\ \text{odd} \end{pmatrix}\right]\right\} \overbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}^{\overline{\eta}^1} \tag{2.37a}$$

$$+\left\{\begin{pmatrix} 2 \\ 2 \end{pmatrix}\left[\begin{pmatrix} 4 \\ \text{even} \end{pmatrix}\right]\left[\begin{pmatrix} 5 \\ \text{odd} \end{pmatrix}\right] + \begin{pmatrix} 2 \\ 0 \end{pmatrix}\left[\begin{pmatrix} 4 \\ \text{odd} \end{pmatrix}\right]\left[\begin{pmatrix} 5 \\ \text{even} \end{pmatrix}\right]\right\}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.37b}$$

where

$$\left[\begin{pmatrix} 5 \\ \text{even} \end{pmatrix}\right] \simeq \mathbf{16} \tag{2.38}$$

with

$$\left[\begin{pmatrix} 5 \\ 0 \end{pmatrix}\right] \tag{2.39}$$

being the highest weight. Also,

$$\left[\begin{pmatrix} 5 \\ \text{odd} \end{pmatrix}\right] \simeq \overline{\mathbf{16}} \tag{2.40}$$

with

$$\left[\begin{pmatrix} 5 \\ 5 \end{pmatrix}\right] \tag{2.41}$$

the highest weight in this case. We explicitly construct the **16** representation in this basis in Appendix A. For $\mathbf{b}_2$ and $\mathbf{b}_3$, the corresponding fermions are $4 = \{y^1 y^2, w^5 w^6, \overline{y}^1\overline{y}^2, \overline{w}^5\overline{w}^6\}$, $2 = \{\psi^\mu, \chi^{34}\}$, $5 = \{\overline{\psi}^{1,\ldots,5}\}$ and $1 = \{\overline{\eta}^2\}$ and $4 = \{w^{1,\ldots,4}, \overline{w}^{1,\ldots,4}\}$, $2 = \{\psi^\mu, \chi^{56}\}$, $5 = \{\overline{\psi}^{1,\ldots,5}\}$ and $1 = \{\overline{\eta}^3\}$ respectively and so we have a total of 48 generations of the **16** and $\overline{\mathbf{16}}$ representations, sixteen coming from each of the $\mathbf{b}_i$–sectors. As we can see, the multiplicative factor determining the number of **16**s comes from the boundary conditions on

$$\left\{y^i, w^i | \overline{y}^i, \overline{w}^i\right\}. \tag{2.42}$$

This was discussed in depth in [84, 85]. We also notice that, at the level of the NAHE set, half the generations per $\mathbf{b}_i$ carry charge of the opposite sign to the other half of the **16** representations under the $U(1)$ combination, (2.31). As we will see, upon breaking $SO(10)$ using further basis–vectors at the string scale, we are able to project out half the states of each representation resulting in a complete representation being formed by states with Q_ζ of opposite sign. This is dependent on the symmetry breaking pattern down from $SO(10)$ and our choices of GGSO projections, as outlined in the next section. The GGSO projections that we have used for the NAHE set are summarised as:

$$\begin{array}{c} \\ \mathbb{1} \\ \mathbf{S} \\ \mathbf{b}_1 \\ \mathbf{b}_2 \\ \mathbf{b}_3 \end{array} \begin{array}{c} \begin{array}{ccccc} \mathbb{1} & \mathbf{S} & \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 \end{array} \\ \begin{pmatrix} 1 & 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 \end{pmatrix} \end{array} \tag{2.43}$$

2.4 Beyond the NAHE set

The second stage consists of adding three or four basis vectors, typically denoted by $\{\alpha, \beta, \gamma\}$, to the NAHE set. The additional basis vectors reduce the number of generations to three and break the four dimensional gauge symmetry. In this section we explore the various patterns of $SO(10)$ breaking and how they are realised in the FFF. We also look at how the matter representations are decomposed for our various symmetry breaking patterns.

2.4.1 Visible Gauge–Symmetry Breaking Patterns

At the level of the NAHE set, the GUT group in the visible sector is $SO(10)$, generated by the worldsheet fermions $\overline{\psi}^{1,\ldots,5}$. Assigning different boundary conditions to these fermions, in a way consistent with the string constraints outlined in Appendix B, leads to various symmetry breaking patterns at the string scale, M_S. We will outline those commonly seen in semi–realistic free fermionic models below. When breaking to the maximal subgroups, the flipped $SU(5)$ (FSU5) and Pati–Salam (PS) breaking patterns, we use a single basis vector to break $SO(10)$ at the string scale. However, we may also employ more than one basis vector to break the GUT group further at M_S, removing the need for an intermediate GUT scale, M_{GUT}; the standard–like (SL) and left–right symmetric (LRS) models being the most commonly seen in the literature [18, 19, 51].

Flipped $SU(5)$

As the fermions generating our $SO(10)$ GUT group are complex, we may assign them rational boundary conditions [75, 76] corresponding to the fermions picking up a complex phase, *i.e.* $\mathbb{Z}_4$ boundary conditions.

In order to break $SO(10) \to SU(5) \times U(1)$ we take

$$\alpha \left\{ \overline{\psi}^{1,\dots,5} \right\} = \left\{ \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \right\}. \tag{2.44}$$

Assuming $\delta_\alpha = -1 \Leftrightarrow \alpha(\psi^\mu) = 1$, we require the $\{\overline{\psi}\}\{\overline{\psi}\}$ pairs to satisfy

$$\alpha \cdot F_{\text{NS}} = 0 \bmod 2 \tag{2.45}$$

in order for the states to survive the GGSO projection. Therefore, the only combinations permitted are

$$\psi^\mu \overline{\psi}^{i*} \overline{\psi}^{j} |0\rangle_{\text{NS}} \tag{2.46a}$$

$$\psi^\mu \overline{\psi}^{i} \overline{\psi}^{j*} |0\rangle_{\text{NS}}. \tag{2.46b}$$

We can easily see that we now have 25 generators: 20 for $i \neq j$ and 5 Cartan generators, where $i = j$. Thus we have a rank–5, dimension–25 group, *i.e.*

$$U(5) \simeq SU(5) \times U(1). \tag{2.47}$$

The FSU5 models were first constructed in string–models in [48, 78, 86] and further phenomenological analysis was done in [87, 88].

Pati–Salam

To break $SO(10) \to SO(6) \times SO(4)$ makes use of only periodic boundary conditions for the $SO(10)$ generators, *i.e.* $\mathbb{Z}_2$ boundary conditions,

$$\alpha \left\{ \overline{\psi}^{1,\dots,5} \right\} = \{11100\}. \tag{2.48}$$

Again assuming $\delta_\alpha = -1 \Leftrightarrow \alpha(\psi^\mu) = 1$, we require (2.45) to hold for the $\{\overline{\psi}\}\{\overline{\psi}\}$ pairs and so only

$$\psi^\mu \overline{\psi}^{i} \overline{\psi}^{j} |0\rangle_{\text{NS}} \tag{2.49}$$

for $i,j=1,2,3$ or $i,j=4,5$ survive. The GGSO projections remove 24 generators from our spectrum and we are only left with 21: 15 originating from $i,j=1,2,3$ and 6 from $i,j=4,5$. Therefore, our gauge group is now a product of a rank–3, dimension–15 group, *i.e.* $SO(6)\simeq SU(4)$, and a rank–2, dimension–6 group, *i.e.* $SO(4)\simeq SU(2)\times SU(2)$, and so the resulting breaking is

$$SO(10)\xrightarrow{\alpha} SU(4)\times SU(2)\times SU(2). \tag{2.50}$$

Standard–like

Here we apply both breaking patterns demonstrated above in two separate basis vectors,

$$\alpha\left\{\overline{\psi}^{1,\ldots,5}\right\}=\left\{\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}\right\} \tag{2.51a}$$

$$\beta\left\{\overline{\psi}^{1,\ldots,5}\right\}=\{11100\}\,. \tag{2.51b}$$

As we saw previously, following the application of α, 25 generators remain:

$$i\neq j\left\{\begin{array}{c}\psi^\mu\overline{\psi}^{i*}\overline{\psi}^{j}|0\rangle_{\rm NS}\\ \\ \psi^\mu\overline{\psi}^{i}\overline{\psi}^{j*}|0\rangle_{\rm NS}\end{array}\right\}\text{20 generators,} \tag{2.52a}$$

$$i=j\;\{\;\psi^\mu\overline{\psi}^{i*}\overline{\psi}^{j}|0\rangle_{\rm NS}\;\}\text{ 5 Cartan generators.} \tag{2.52b}$$

Applying β projects out 12 gauge bosons and only the combinations $i,j=1,2,3$ and $i,j=4,5$ survive. Thus, the product of a rank–3, dimension–9 group, *i.e.* $U(3)\simeq SU(3)\times U(1)$, and a rank–2, dimension–4 group, *i.e.* $U(2)\simeq SU(2)\times U(1)$, remain. Our symmetry breaking pattern at the string–scale is

$$SO(10)\xrightarrow{\alpha,\beta} SU(3)\times U(1)\times SU(2)\times U(1). \tag{2.53}$$

Left–right Symmetric

Similarly, we can break $SO(10) \to SU(3) \times SU(2) \times SU(2)$ by the application of two basis vectors,

$$\alpha \left\{ \overline{\psi}^{1,\dots,5} \right\} = \{11100\} \tag{2.54a}$$

$$\beta \left\{ \overline{\psi}^{1,\dots,5} \right\} = \left\{ \frac{1}{2}\frac{1}{2}\frac{1}{2}00 \right\}. \tag{2.54b}$$

The application of α to the NS sector of the NAHE set gives, as was shown previously, the PS gauge group, *i.e.* $SO(6) \times SO(4)$. Applying β projects out 6 generators from the 15 that generate $SO(6)$ and leaves the $SO(4)$ untouched. We are now left with a rank–3, dimension–9 group, *i.e.* $U(3)$, as seen earlier. Therefore our visible gauge group at the string–scale is

$$SO(10) \xrightarrow{\alpha,\beta} SU(3) \times U(1) \times SU(2) \times SU(2). \tag{2.55}$$

$\mathbf{SU(4) \times SU(2) \times U(1)}$

Alternatively, we can instead assign $\overline{\psi}^{4,5}$ complex boundary conditions, *i.e.*

$$\beta \left\{ \overline{\psi}^{1,\dots,5} \right\} = \left\{ 000\frac{1}{2}\frac{1}{2} \right\}. \tag{2.56}$$

This would result in

$$SO(10) \xrightarrow{\alpha,\beta} SU(4) \times SU(2) \times U(1) \tag{2.57}$$

as 2 gauge bosons would be projected out from the 6 that generate $SO(4)$. Here the $SO(6)$ generators remain untouched.

However, the $SU(4) \times SU(2) \times U(1)$ (SU421) models do not provide phenomenologically viable vacua in which to construct realistic effective field theories [89].

2.4.2 Matter

So far, viable three generation models with $SU(5) \times U(1)$ [78], $SO(6) \times SO(4)$ [49, 50], $SU(3) \times SU(2) \times U(1)^2$ [18–20], or $SU(3) \times SU(2)^2 \times U(1)$ [51, 90], $SO(10)$ subgroups have been constructed. Three chiral generations arise from the sectors $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{b}_3$, and are decomposed under the final $SO(10)$ subgroup below. The flavour $SO(6)^3$ groups are broken to products of $U(1)^n$ with $3 \le n \le 9$. This comes from assigning different boundary conditions to $\{y^{3,\dots,6} \,|\, \overline{y}^{3,\dots,6}\}$ in $\mathbf{b}_1$, $\{y^{1,2}, w^{5,6} \,|\, \overline{y}^{1,2}, \overline{w}^{5,6}\}$ in $\mathbf{b}_2$ and $\{w^{1,\dots,4} \,|\, \overline{w}^{1,\dots,4}\}$ in $\mathbf{b}_3$, than to our right–moving complex fermions $\overline{\eta}^i$ in $\mathbf{b}_i$, which generate the $U(1)_{1,2,3}$ factors.

Above, we have shown the origin of the **16** representation of $SO(10)$ in free fermion models. Here we present how these are decomposed under the various subgroups.

FSU5 Breaking $SO(10) \to SU(5) \times U(1)$ decomposes the **16** as

$$\mathbf{16} \to \mathbf{1}_5 + \mathbf{10}_1 + \overline{\mathbf{5}}_{-3}, \tag{2.58}$$

corresponding to

$$\left[\begin{pmatrix} 5 \\ 0 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} 5 \\ 4 \end{pmatrix}\right] \to \underbrace{\begin{pmatrix} 5 \\ 0 \end{pmatrix}}_{\mathbf{1}_5} + \underbrace{\begin{pmatrix} 5 \\ 2 \end{pmatrix}}_{\mathbf{10}_1} + \underbrace{\begin{pmatrix} 5 \\ 4 \end{pmatrix}}_{\overline{\mathbf{5}}_{-3}}, \tag{2.59}$$

where the $U(1)$ charges are $\mathrm{Tr}\,[U(5)]$. These representations then break to the SM states in the usual way:

$$\begin{pmatrix} 5 \\ 0 \end{pmatrix} \rightarrow \underbrace{\begin{pmatrix} 3 \\ 0 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \end{pmatrix}}_{e^c}, \tag{2.60a}$$

$$\begin{pmatrix} 5 \\ 2 \end{pmatrix} \rightarrow \underbrace{\begin{pmatrix} 3 \\ 2 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \end{pmatrix}}_{d^c} + \underbrace{\begin{pmatrix} 3 \\ 1 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix}}_{Q_L} + \underbrace{\begin{pmatrix} 3 \\ 0 \end{pmatrix}\begin{pmatrix} 2 \\ 2 \end{pmatrix}}_{\nu^c}, \tag{2.60b}$$

$$\begin{pmatrix} 5 \\ 4 \end{pmatrix} \rightarrow \underbrace{\begin{pmatrix} 3 \\ 2 \end{pmatrix}\begin{pmatrix} 2 \\ 2 \end{pmatrix}}_{u^c} + \underbrace{\begin{pmatrix} 3 \\ 3 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix}}_{L_L}, \tag{2.60c}$$

with

$$Y = \frac{1}{3}\,\mathrm{Tr}\left[U(3)_C\right] + \frac{1}{2}\,\mathrm{Tr}\left[U(2)_L\right], \tag{2.61}$$

where

$$\mathrm{Tr}\left[U(3)_C\right] = \frac{3}{2}\left(B-L\right) \equiv Q_C \tag{2.62}$$

and

$$\mathrm{Tr}\left[U(2)_L\right] = 2T_{3_R} \equiv Q_L. \tag{2.63}$$

This combination is a universal combination for NAHE–based models. Before breaking to the SM, the FSU5 model may also go via an intermediate breaking at the string scale,

$$SU(5) \times U(1) \rightarrow U(3)_C \times U(2)_L \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \tag{2.64}$$

i.e. via the SL gauge group mentioned earlier. Alternatively, a heavy Higgs mechanism may be employed to break FSU5 to the SM directly.

PS Writing our **16** in combinatorial notation, decomposed under the PS gauge group,

$$SO(6) \times SO(4) \simeq SU(4)_C \times SU(2)_L \times SU(2)_R. \tag{2.65}$$

we have

$$\left[\binom{5}{0}+\binom{5}{2}+\binom{5}{4}\right] \rightarrow \overbrace{\left[\binom{3}{1}+\binom{3}{3}\right]\binom{2}{1}}^{F_L} \tag{2.66a}$$

$$+\underbrace{\left[\binom{3}{2}+\binom{3}{0}\right]\left[\binom{2}{2}+\binom{2}{0}\right]}_{F_R} \tag{2.66b}$$

i.e. $\mathbf{16} \rightarrow (\mathbf{4},\mathbf{2},\mathbf{1}) + (\overline{\mathbf{4}},\mathbf{1},\mathbf{2})$. This will break to the SM states via the LRS gauge group

$$SU(4)_C \times SU(2)_R \times SU(2)_L \rightarrow SU(3)_C \times SU(2)_R \times SU(2)_L \times U(1)_C \tag{2.67}$$

as

$$\left[\binom{3}{1}+\binom{3}{3}\right]\binom{2}{1} \rightarrow \overbrace{\binom{3}{1}\binom{2}{1}}^{Q_L}+\overbrace{\binom{3}{3}\binom{2}{1}}^{L_L}, \tag{2.68a}$$

$$\begin{aligned}\left[\binom{3}{2}+\binom{3}{0}\right]\left[\binom{2}{2}+\binom{2}{0}\right] \rightarrow &\overbrace{\binom{3}{2}\left[\binom{2}{2}+\binom{2}{0}\right]}^{Q_R} \\ &+\underbrace{\binom{3}{0}\left[\binom{2}{2}+\binom{2}{0}\right]}_{L_R},\end{aligned} \tag{2.68b}$$

where (2.68b) then decomposes to the SM states,

$$Q_R \sim \left(\overline{\mathbf{3}}, \mathbf{2}, \mathbf{1}, -\frac{1}{2}, \frac{1}{2}\right) \rightarrow \overbrace{\left(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}\right)}^{u_L^c} + \overbrace{\left(\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3}\right)}^{d_L^c}, \tag{2.69a}$$

$$L_R \sim \left(\mathbf{1}, \mathbf{2}, \mathbf{1}, \frac{3}{2}, \frac{1}{2}\right) \rightarrow \underbrace{(\mathbf{1}, \mathbf{1}, 0)}_{\nu_L^c} + \underbrace{(\mathbf{1}, \mathbf{1}, 1)}_{e_L^c}. \tag{2.69b}$$

The breaking of $SO(10) \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_C$ is via the PS gauge group, and it is clear from (2.66), the states in F_L and F_R must have charges of opposite sign under the $U(1)_\zeta$ combination in (2.31), whereas breaking $SO(10)$ via FSU5 does not give these charge assignments. In fact, taking this route of symmetry breaking pattern restricts the charge of all the states in the **16** to be of the same sign as either (2.37a) or (2.37b) are projected out.

In order to elucidate the $U(1)_{1,2,3}$ charges of the matter states in the free fermionic models, it is instructive to extend the $SO(10)$ symmetry, at the level of the NAHE set, to E_6. This is achieved by adding to the NAHE set the basis vector [91, 92],

$$\mathbf{x} \equiv \left\{\overline{\psi}^{1,\cdots,5}, \overline{\eta}^{1,2,3}\right\}. \tag{2.70}$$

2.4.3 Addition of x

As there are 8 periodic right–moving complex fermions, the vacua in the **x**-sector are degenerate, $|\pm\rangle$. Thus, before the GGSO projections, we have 256 states,

$$\left[\begin{pmatrix} 8 \\ 0 \end{pmatrix} + \cdots + \begin{pmatrix} 8 \\ 8 \end{pmatrix}\right]. \tag{2.71}$$

Once we apply the GGSO projections

$$c\begin{pmatrix} \mathbf{x} \\ \mathbb{1} \end{pmatrix} = c\begin{pmatrix} \mathbf{x} \\ \mathbf{S} \end{pmatrix} = \pm 1, \tag{2.72}$$

half of these states are projected out. We are therefore left with either

$$\left[\begin{pmatrix} 8 \\ \text{even} \end{pmatrix}\right] \tag{2.73a}$$

or

$$\left[\begin{pmatrix} 8 \\ \text{odd} \end{pmatrix}\right]. \tag{2.73b}$$

For our choice of GGSO,

$$c\begin{pmatrix} \mathbf{x} \\ \mathbb{1} \end{pmatrix} = c\begin{pmatrix} \mathbf{x} \\ \mathbf{S} \end{pmatrix} = -1 \tag{2.74}$$

we keep (2.73a). As we saw earlier, the NS-sector contains the gauge bosons that generate

$$SO(10) \times U(1)^3 \times SO(16)' \tag{2.75}$$

and the gauge bosons in the ξ-sector enhance the hidden $SO(16)' \to E_8'$.

The gauge bosons coming from the $\mathbf{x}$-sector transform as the **16** and $\overline{\mathbf{16}}$ of $SO(10)$,

$$\overbrace{\left[\begin{pmatrix} 5 \\ \text{even} \end{pmatrix}\right]}^{\overline{\psi}^{1,\dots,5}}\overbrace{\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]}^{\overline{\eta}^{1,2,3}} + \left[\begin{pmatrix} 5 \\ \text{odd} \end{pmatrix}\right]\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right], \tag{2.76}$$

and enhance the $SO(10) \times U(1) \to E_6$ where the $U(1)$ combination is given by (2.31). Adding the $\mathbf{x}$ basis–vector at the level of the NAHE set, the gauge group is enhanced to $SO(4)^3 \times E_6 \times U(1)^2 \times E_8'$ with $\mathcal{N} = 1$ space–time supersymmetry. There are 24 generations in the **27** representation of E_6, eight from each twisted sector. In the free fermionic construction these are the sectors ($\mathbf{b}_1$;$\mathbf{b}_1$+$\mathbf{x}$),($\mathbf{b}_2$;$\mathbf{b}_2$+$\mathbf{x}$) and ($\mathbf{b}_3$;$\mathbf{b}_3$+$\mathbf{x}$), where the sectors $\mathbf{b}_i$ produce the spinorial **16** of $SO(10)$, as shown previously, and the sectors $(\mathbf{b}_i{+}\mathbf{x})$ produce the vectorial $(\mathbf{10}+\mathbf{1})+\mathbf{1}$ representations in the decomposition of the $\mathbf{27}$ representation of E_6

$$\mathbf{27} = \mathbf{16}_{\pm\frac{1}{2}} + \mathbf{10}_{\mp 1} + \mathbf{1}_{\pm 2} \tag{2.77}$$

under $SO(10)\times U(1)$. The additional "$\mathbf{1}$" arising in the $(\mathbf{b}_i{+}\mathbf{x})$ sectors is an E_6 singlet. These are obtained by acting on the vacuum with the oscillators of the complex worldsheet fermions $\left\{\overline{\psi}^{1,\dots,5}\overline{\eta}^{i}\right\}$, which have Neveu–Schwarz boundary conditions in the sectors $\mathbf{b}_i{+}\mathbf{x}$.

The vacuum of the sectors $\mathbf{b}_i$ contain twelve periodic fermions, with each periodic fermion giving rise to a two dimensional degenerate vacuum $|+\rangle$ and $|-\rangle$ with fermion numbers 0 and -1, respectively. After applying the GGSO projections, we can write the degenerate vacuum of the sector $\mathbf{b}_1$ in combinatorial form:

$$\left[\begin{pmatrix}4\\0\end{pmatrix}+\begin{pmatrix}4\\2\end{pmatrix}+\begin{pmatrix}4\\4\end{pmatrix}\right]\left\{\begin{pmatrix}2\\0\end{pmatrix}\left[\begin{pmatrix}5\\0\end{pmatrix}+\begin{pmatrix}5\\2\end{pmatrix}+\begin{pmatrix}5\\4\end{pmatrix}\right]\begin{pmatrix}1\\0\end{pmatrix}\right. \tag{2.78a}$$

$$\left.+\begin{pmatrix}2\\2\end{pmatrix}\left[\begin{pmatrix}5\\1\end{pmatrix}+\begin{pmatrix}5\\3\end{pmatrix}+\begin{pmatrix}5\\5\end{pmatrix}\right]\begin{pmatrix}1\\1\end{pmatrix}\right\} \tag{2.78b}$$

where $4=\{y^3y^4,y^5y^6,\overline{y}^{3,4},\overline{y}^{5,6}\}$, $2=\{\psi^\mu,\chi^{12}\}$, $5=\left\{\overline{\psi}^{1,\dots,5}\right\}$ and $1=\{\overline{\eta}^1\}$. We notice that half of the states from (2.27) are projected out due to our choice of

$$c\begin{pmatrix}\mathbf{x}\\ \mathbf{b}_1\end{pmatrix} = -1 \tag{2.79}$$

The first term in square brackets counts the degeneracy of the multiplets, being eight in this case. The two terms in the curly brackets correspond to the two CPT conjugated components of a Weyl spinor. The first term among those corresponds to the $\mathbf{16}$ spinorial representation of $SO(10)$, and fixes the space–time chirality properties of the representation, whereas the second corresponds to the CPT conjugated anti–spinorial $\overline{\mathbf{16}}$ representation.

The charge under the $U(1)$ symmetry generated by $\overline{\eta}_1$ is determined by its vacuum state, being a Ramond state in the $|+\rangle$ vacuum for the degenerate vacuum in (2.78a). Hence, in this case the $U(1)_1$ charge is $+\frac{1}{2}$. Similar vacuum structure is obtained for the sectors $\mathbf{b}_2$ and $\mathbf{b}_3$ with $\{\chi^{34}, y^{1,2}, w^{5,6}|\overline{y}^{1,2}, \overline{w}^{5,6}, \overline{\eta}^2\}$ and $\{\chi^{56}, w^{1,\ldots,4}|\overline{w}^{1,\ldots,4}, \overline{\eta}^3\}$ respectively.

The $\mathbf{10}+\mathbf{1}$ in the $\mathbf{27}$ of E_6 are obtained from the sector $\mathbf{b}_i$+$\mathbf{x}$. The effect of adding the vector $\mathbf{x}$ to the sectors $\mathbf{b}_i$ is to replace the periodic boundary conditions for $\left\{\overline{\psi}^{1,\ldots,5}, \overline{\eta}^i\right\}$ with periodic boundary conditions for $\overline{\eta}^{j,k}$ with $i \neq j \neq k$ and $i, j, k \in \{1, 2, 3\}$. Consequently, massless states from the sectors $\mathbf{b}_i$+$\mathbf{x}$ are obtained by acting on the vacuum with a fermionic oscillator.

If the space–time vector bosons that enhance the $SO(10) \times U(1)$ symmetry to E_6 are projected out, either the spinorial $\mathbf{16}$ or the vectorial $(\mathbf{10}+\mathbf{1})+\mathbf{1}$, survive the GGSO projections at a given fixed point. By breaking the degeneracy with respect to the internal fermions $\{y, w|\overline{y}, \overline{w}\}$ we can obtain spinorial and vectorial representations from the twisted sectors at different fixed points. A classification of symmetric free fermionic heterotic string models along these lines was done in [57, 79, 80, 93].

When the $SO(10)\times U(1)$ symmetry is enhanced to E_6, the charges of the spinorial $\mathbf{16}$, the vectorial $\mathbf{10}$ and the singlet $\mathbf{1}$, under the $U(1)_\zeta$, are fixed by the E_6 symmetry, as shown in (2.77). When the E_6 symmetry is broken by the GGSO projections, *i.e.* the $\mathbf{x}$–sector bosons are projected out, the $U(1)_\zeta$ charges are not restricted by the E_6 embedding, and can take either sign. The $U(1)$ symmetry that serves as the proton lifeguard is a combination of the three $U(1)$ symmetries generated by the worldsheet complex fermions $\overline{\eta}^{1,2,3}$. The states from each of the sectors $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{b}_3$ are charged with respect to $U(1)_1$, $U(1)_2$ and $U(1)_3$, respectively. Consequently, the

$U(1)$ combination in (2.31) is family universal. This is also true of the leptophobic combination as it is just a linear combination of $U(1)_\zeta$ and $U(1)_{B-L}$.

2.5 Light $U(1)s$

In the string derived models of [18–20, 48, 49, 78], the $U(1)_{1,2,3}$ are anomalous. Therefore, $U(1)_\zeta \equiv U(1)_A$ is also anomalous and must be broken near the string scale. In the string derived left–right symmetric models of [51], $U(1)_{1,2,3}$ are anomaly free and hence the combination $U(1)_\zeta$ is also anomaly free. It is this property of these models which allows this $U(1)$ combination to remain unbroken.

It is instructive to study the characteristics of $U(1)_\zeta$ in the left–right symmetric string derived models [51], versus those of $U(1)_A$ in the string derived models of [19, 49, 78, 94]. We note that both $U(1)_\zeta$ as well as $U(1)_A$ are obtained from the same combination of complex right–moving worldsheet currents $\overline{\eta}^{1,2,3}$. The distinction between the two cases, as we describe below, is due to the charges of the SM states, arising from the sectors $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{b}_3$, under this combination.

The periodic boundary conditions of the worldsheet fermions $\overline{\eta}^i$ ensures that the fermions from each sector $\mathbf{b}_i$ are charged with respect to one of the $U(1)_i$ symmetries. In the models of [18–20, 48, 49, 78] the charges of a given $\mathbf{b}_i$ generation under $U(1)_i$ is of the same sign, whereas in the models of [51] they differ. In general, the distinction is by the breaking of $SO(10)$ to either $SU(5)\times U(1)$ or $SO(6)\times SO(4)$. In the former case they will always have the same sign, whereas in the latter they may differ. This can be clearly seen in (2.37). The crucial point is that the PS breaking pattern allows the terms carrying odd and even combinatorial factors to come with opposite charges under $U(1)_j$. This results from the GGSO projection of the basis vector γ on the states arising from the sectors $\mathbf{b}_i$ fixing the vacuum of $\overline{\eta}^i$ with opposite chirality in the two terms of (2.66). The reason being that the combinatorial factor with respect to $\overline{\psi}^{1,\ldots,3}$ is odd in the first term and even in the second, with the γ projection that utilises (2.48) being blind to $\overline{\psi}^{4,5}$. On the other hand, in the models that utilise (2.44), the γ projection is not blind to $\overline{\psi}^{4,5}$, and consequently, the vacuum of $\overline{\eta}^i$ is fixed with the same chirality for all the states arising from the sector $\mathbf{b}_i$.

Thus, in models that descend from $SO(10)$ via the $SU(5)\times U(1)$ breaking pattern the charges of a generation from a sector $\mathbf{b}_i$, where $i = 1, 2, 3$, under the corresponding symmetry $U(1)_i$, are either $+\frac{1}{2}$ or $-\frac{1}{2}$ for all the states from that sector. In contrast, in the left–right symmetric string models, the corresponding charges, up to a sign, are

$$Q_j(\mathbf{2}_L) = +1/2; \qquad Q_j(\mathbf{2}_R) = -1/2, \tag{2.80}$$

i.e. the charges of the $SU(2)_L$ doublets have the opposite sign from those of the $SU(2)_R$ doublets. In fact, this is the reason that, in contrast to the FSU5 and the SL string models, in the LRS models, the $U(1)_i$ symmetries are not part of the anomalous $U(1)$ symmetry. This arises because the $SO(10)$ symmetry is not enhanced to E_6. If the NAHE symmetry is extended to E_6, the spinorial **16** states with the "wrong" $U(1)_\zeta$ charge are projected out, as is the case in the FSU5 and SL models, as well as the PS models constructed to date.

The LRS model given in Table 2.2 and (2.81) is an example of an explicit string model that exhibits this property. The full massless spectrum of this model, as well as the superpotential up to quintic order, are given in [51]. We note that the boundary conditions on $\left\{\overline{\phi}^{1,\ldots,8}\right\}$ correspond to a breaking pattern in the hidden sector which may result in interesting physics. However, we will not discuss this further in this thesis. We refer the interested reader to [51] for the full gauge group of the model, including the hidden sector.

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\overline{\psi}^{1,\ldots,5}$					$\overline{\eta}^1$	$\overline{\eta}^2$	$\overline{\eta}^3$	$\overline{\phi}^{1,\ldots,8}$							
α	0	0	0	0	1	1	1	0	0	0	0	0	1	1	1	1	0	0	0	0
β	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	1	1	0	0
γ	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0

	y^3y^6	$y^4\overline{y}^4$	$y^5\overline{y}^5$	$\overline{y}^3\overline{y}^6$	y^1w^5	$y^2\overline{y}^2$	$w^6\overline{w}^6$	$\overline{y}^1\overline{w}^5$	w^2w^4	$w^1\overline{w}^1$	$w^3\overline{w}^3$	$\overline{w}^2\overline{w}^4$
α	1	0	0	0	0	0	1	1	0	0	1	1
β	0	0	1	1	1	0	0	0	0	1	0	1
γ	0	0	1	0	1	0	0	1	0	1	0	0

$$
\begin{aligned}
&c\begin{pmatrix}\alpha\\ \mathbb{1}\end{pmatrix} = -\,c\begin{pmatrix}\alpha\\ \mathbf{S}\end{pmatrix} = -c\begin{pmatrix}\alpha\\ \mathbf{b}_1\end{pmatrix} = -c\begin{pmatrix}\alpha\\ \mathbf{b}_2\end{pmatrix} = -c\begin{pmatrix}\alpha\\ \mathbf{b}_3\end{pmatrix} = \\
&c\begin{pmatrix}\beta\\ \mathbb{1}\end{pmatrix} = -\,c\begin{pmatrix}\beta\\ \mathbf{S}\end{pmatrix} = -c\begin{pmatrix}\beta\\ \mathbf{b}_1\end{pmatrix} = -c\begin{pmatrix}\beta\\ \mathbf{b}_2\end{pmatrix} = c\begin{pmatrix}\beta\\ \mathbf{b}_3\end{pmatrix} = \\
&i\,c\begin{pmatrix}\gamma\\ \mathbb{1}\end{pmatrix} = -\,c\begin{pmatrix}\gamma\\ \mathbf{S}\end{pmatrix} = c\begin{pmatrix}\gamma\\ \mathbf{b}_1\end{pmatrix} = c\begin{pmatrix}\gamma\\ \mathbf{b}_2\end{pmatrix} = c\begin{pmatrix}\gamma\\ \mathbf{b}_3\end{pmatrix} = \\
&-c\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = -\,c\begin{pmatrix}\alpha\\ \gamma\end{pmatrix} = c\begin{pmatrix}\beta\\ \gamma\end{pmatrix} = 1
\end{aligned}
\tag{2.81}
$$

Table 2.2: LRS Model 1 boundary conditions. Below are the GGSO coefficients, where the GGSO coefficients for the NAHE set were already shown in (2.43). Taken from [51].

2.5.1 Proton Lifeguard

The preservation of the $U(1)$ combination

$$U(1)_\zeta = U(1)_1 + U(1)_2 + U(1)_3$$

as an anomaly free symmetry is the key to keeping it as an unbroken low–scale symmetry. The left–right symmetric string models admit cases without any anomalous $U(1)$ symmetry; free of any gauge and gravitational anomalies. We note that there may exist string models in the classes of [19, 49, 78, 94] in which $U(1)_\zeta$ is anomaly free. This may be the case in the so–called self–dual vacua of spinor vector duality. In [79] a duality symmetry was uncovered in the space of fermionic $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetric orbifolds under the exchange of the total number of twisted spinor plus anti–spinor and twisted vector representations of $SO(10)$. The self–dual models are the models in which the total number of spinors and anti–spinors is equal to the total number of vector representations. The self–dual models are free of any $U(1)$ anomalies. Thus, in such self–dual models with three light chiral generations the $U(1)_\zeta$ combination is anomaly free and can remain unbroken below the string scale. Such quasi–realistic self–dual string models, with an anomaly free $U(1)_\zeta$, have not been constructed to date.

Here, we discuss the stringy origins of the SM matter and additional matter that may be included in our effective theory to allow $U(1)_\zeta$ to remain anomaly free. In addition to the three light SM generations arising from the twisted sectors, $\mathbf{b}_i$, the string models contain additional states arising from the twisted or untwisted sectors. The additional spectrum is in general highly model dependent. Later we will fix our string–inspired model by fitting it with additional states that are compatible with the string charge assignments.

The twisted sectors can produce additional states that arise from spinorial representations of the underlying $SO(10)$ symmetry with charges $\pm\frac{1}{2}$ under $U(1)_i$. The original $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold that underlies the free fermionic models has forty–eight fixed points. Additional states may also arise from different fixed points.

Sectors that contain the basis vectors that break the $SO(10)$ gauge symmetry, generically produce exotic states with fractional Q_{EM} that must obtain a sufficiently high mass. We note the existence of exophobic heterotic string models in which fractionally charged states appear only in the massive string spectrum [83]. We do not consider these states further here.

As shown previously, the twisted sectors $\mathbf{b}_i$+$\mathbf{x}$ produce states that transform in the vectorial representations of the underlying $SO(10)$ GUT symmetry. A twisted sector that produces $SO(10)$ vectorial representations does not exist in the model of Table 2.2. An alternative model that gives rise to twisted states in the vectorial representation of $SO(10)$ is given in Table 2.3.
The sector $\mathbf{b}_1$+$\mathbf{b}_2$+$\alpha + \beta$ in the additive group, Ξ, spanned by this basis gives rise to twisted vectorial $SO(10)$ representations. In this sector, the charges under the $U(1)_\zeta$ are fixed by the vacuum of the $\overline{\eta}_1$ and $\overline{\eta}_2$.

In the left–right symmetric models, the twisted sectors $\mathbf{b}_i$+$\mathbf{x}$ produce states that transform as $SU(2)_L \times SU(2)_R$ bi-doublets with the $U(1)_\zeta$ charge assignments

$$(\mathbf{1}, \mathbf{2}, \mathbf{2}, 0, \pm 1) \tag{2.83}$$

as well as colour triplets. The $U(1)_\zeta$ charges of these colour triplets is dependent upon the γ projection and there are several possibilities. If the twisted plane produces bi–doublets with $Q_\zeta = +1$, then the γ projection dictates that any colour triplet arising from that sector is neutral under $U(1)_\zeta$. In this case, we must take the colour triplets to have the charges

$$(\mathbf{3}, \mathbf{1}, \mathbf{1}, -1, 0) \quad + \quad \left(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1}, +1, 0\right) \tag{2.84}$$

The vectorial states arising from the twisted sector also depend, however, on the specific choice of basis vectors and the pairing of the worldsheet fermions from

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\overline{\psi}^{1,\ldots,5}$					$\overline{\eta}^1$	$\overline{\eta}^2$	$\overline{\eta}^3$	$\overline{\phi}^{1,\ldots,8}$							
α	0	0	0	0	1	1	1	0	0	0	0	0	1	1	1	1	0	0	0	0
β	0	0	0	0	1	1	1	0	0	0	0	0	1	1	1	1	0	0	0	0
γ	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0

	y^3y^6	$y^4\overline{y}^4$	$y^5\overline{y}^5$	$\overline{y}^3\overline{y}^6$	y^1w^5	$y^2\overline{y}^2$	$w^6\overline{w}^6$	$\overline{y}^1\overline{w}^5$	w^2w^4	$w^1\overline{w}^1$	$w^3\overline{w}^3$	$\overline{w}^2\overline{w}^4$
α	1	0	0	0	0	0	1	1	0	0	1	1
β	0	0	1	1	1	0	0	0	0	1	0	1
γ	0	0	1	0	1	0	0	1	0	1	0	0

$$
\begin{aligned}
& c\begin{pmatrix} \alpha \\ \mathbb{1} \end{pmatrix} = -\,c\begin{pmatrix} \alpha \\ \mathbf{S} \end{pmatrix} = -c\begin{pmatrix} \alpha \\ \mathbf{b}_1 \end{pmatrix} = -c\begin{pmatrix} \alpha \\ \mathbf{b}_2 \end{pmatrix} = -c\begin{pmatrix} \alpha \\ \mathbf{b}_3 \end{pmatrix} = \\
& c\begin{pmatrix} \beta \\ \mathbb{1} \end{pmatrix} = -\,c\begin{pmatrix} \beta \\ \mathbf{S} \end{pmatrix} = -c\begin{pmatrix} \beta \\ \mathbf{b}_1 \end{pmatrix} = -c\begin{pmatrix} \beta \\ \mathbf{b}_2 \end{pmatrix} = -c\begin{pmatrix} \beta \\ \mathbf{b}_3 \end{pmatrix} = \\
& i\,c\begin{pmatrix} \gamma \\ \mathbb{1} \end{pmatrix} = -\,c\begin{pmatrix} \gamma \\ \mathbf{S} \end{pmatrix} = c\begin{pmatrix} \gamma \\ \mathbf{b}_1 \end{pmatrix} = c\begin{pmatrix} \gamma \\ \mathbf{b}_2 \end{pmatrix} = c\begin{pmatrix} \gamma \\ \mathbf{b}_3 \end{pmatrix} = \\
& c\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = -\,c\begin{pmatrix} \alpha \\ \gamma \end{pmatrix} = c\begin{pmatrix} \beta \\ \gamma \end{pmatrix} = 1
\end{aligned}
\tag{2.82}
$$

Table 2.3: LRS Model 2 boundary conditions and GGSO projection coefficients for the additional basis vectors, $\{\alpha, \beta, \gamma\}$, where the GGSO coefficients for the NAHE set were already shown in (2.43). Taken from [51].

the set $\{y, w|\overline{y}, \overline{w}\}$ into complex fermions. There exist choices of basis sets that produce vectorial states from none, one, two or three of the twisted planes. To date, only models of the first and second class have been studied in detail, where the example in Table 2.2 belongs to the first kind, and the example in Table 2.3 belongs to the second. If the twisted plane produces both electroweak doublets and color triplets, the γ–projection dictates that they have ± 1 and vanishing $U(1)_\zeta$ charges, respectively, and vice versa.

An alternative possibility, is that more than one twisted plane produces states in vectorial $SO(10)$ representations. In this case, one plane can produce bi–doublets and a second plane produces the colour triplets. Here, the charges of the twisted colour triplets are not correlated with those of the bi–doublets and we can obtain twisted vectorial colour triplets with charges

$$(\mathbf{3},\mathbf{1},\mathbf{1},+1,-1) \quad + \quad \big(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},-1,+1\big) \tag{2.85a}$$

or

$$(\mathbf{3},\mathbf{1},\mathbf{1},+1,+1) \quad + \big(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},-1,-1\big)\,. \tag{2.85b}$$

Electroweak Higgs bidoublets may also arise from the untwisted sector. However, in this case the γ GGSO projection dictates that they are neutral under $U(1)_\zeta$,

$$H_0 = (\mathbf{1},\mathbf{2},\mathbf{2},0,0) = \begin{pmatrix} H^u_+ & H^d \\ H^u & H^d_- \end{pmatrix}. \tag{2.86}$$

The untwisted Higgs bidoublet is the one that forms invariant leading mass terms with the Standard Model matter states, due to the fact that the Q_L and Q_R multiplets carry opposite $U(1)_\zeta$ charge.

The string models may also produce $SO(10)$ singlets, which carry $U(1)_\zeta$ charges. The singlets can arise from the Neveu–Schwarz untwisted sector and twisted sectors that produce vectorial representations, like the sector $\mathbf{b}_1+\mathbf{b}_2+\alpha+\beta$. The $U(1)_\zeta$ charges are fixed according to the following rules:

In the untwisted sector these states arise by acting on the vacuum with two oscillators, $\overline{\eta}^i$ and $\overline{\eta}^j$. Their $U(1)_\zeta$ charges are fixed by the γ projection according to the sign of δ_γ in (2.7), being zero for $\delta_\gamma=+1$ *i.e.* $\gamma\,(\psi^\mu)=0$ and ±2 for $\delta_\gamma=-1$, *i.e.* $\gamma\,(\psi^\mu)=1$. As the twisted sector we consider, for concreteness, the sector

$\mathbf{b}_1+\mathbf{b}_2+\alpha+\beta$. The singlets from that sector are obtained by acting on the vacuum with $\overline{\eta}^3$, or with an oscillator of a real fermion from the set $\{\overline{y}\,\overline{w}\}$, which are not periodic in $\mathbf{b}_1+\mathbf{b}_2+\alpha+\beta$. The $U(1)_\zeta$ charges are again fixed by the γ projection. The γ GGSO projection phase in this sector can be either ± 1 or $\pm i$. As we have seen, depending on this GGSO phase and the type of state, the $U(1)_\zeta$ charges in this sector can be ± 2, ± 1 or zero. Therefore, we can have a combination of singlets with charges $+2$ and $+1$, as well as singlets with vanishing $U(1)_\zeta$ charge.

2.5.2 Leptophobic $U(1)$

At the stage of the NAHE set, the gauge group reads

$$SO(10) \times SO(6)^3 \times E_8'$$

where the additional basis vectors, $\{\alpha, \beta, \gamma\}$, break the $SO(10)$ GUT group as detailed in Section 2.4. The flavour $SO(6)^3$ symmetries are broken to $U(1)^{3+n}$ with $n = 0, \cdots, 6$. The first three, denoted by $U(1)_i$, arise from the world–sheet currents $\overline{\eta}^i\overline{\eta}^{i^*}$ where $i = 1, 2, 3$. These three $U(1)$ symmetries are present in all the three generation free fermionic models which use the NAHE set and we saw a linear combination of these form the proton lifetime preserving $U(1)_\zeta$. Additional horizontal $U(1)$ symmetries, denoted by $U(1)_j$ with $j = 4, 5, ...$, arise by pairing two real fermions from the sets

$$\left\{\overline{y}^{3,\dots,6}\right\}, \qquad \left\{\overline{y}^{1,2}, \overline{w}^{5,6}\right\}, \qquad \left\{\overline{w}^{1,\dots,4}\right\}, \tag{2.87}$$

as mentioned previously. The final observable gauge group depends on the number of such pairings. The model, introduced in [60], is based on a Standard–like gauge symmetry and there are three such pairings, $\{\overline{y}^3\overline{y}^6, \overline{y}^1\overline{w}^5, \overline{w}^2\overline{w}^4\}$, which generate three additional $U(1)$ symmetries, denoted by $U(1)_{4,5,6}$. It is important to note that the existence of these three additional $U(1)$ currents is correlated with a superstring doublet–triplet splitting mechanism [20, 95]. Due to these extra $U(1)$ symmetries, the colour triplets from the NS sector are projected out of the spectrum by the GGSO projections while the electroweak doublets remain in the light spectrum. The full massless spectrum and charges are given in [60].

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\overline{\psi}^{1,\ldots,5}$					$\overline{\eta}^1$	$\overline{\eta}^2$	$\overline{\eta}^3$	$\overline{\phi}^{1,\ldots,8}$							
α	0	0	0	0	1	1	1	0	0	0	0	0	1	1	1	1	0	0	0	0
β	0	0	0	0	1	1	1	0	0	0	0	0	1	1	1	1	0	0	0	0
γ	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0

	y^3y^6	$y^4\overline{y}^4$	$y^5\overline{y}^5$	$\overline{y}^3\overline{y}^6$	y^1w^6	$y^2\overline{y}^2$	$w^5\overline{w}^5$	$\overline{y}^1\overline{w}^6$	w^1w^3	$w^2\overline{w}^2$	$w^4\overline{w}^4$	$\overline{w}^1\overline{w}^3$
α	1	1	1	0	1	1	1	0	1	1	1	0
β	0	1	0	1	0	1	0	1	1	0	0	0
γ	0	0	1	1	1	0	0	0	0	1	0	1

$$
\begin{aligned}
c\begin{pmatrix}\alpha\\ \mathbb{1}\end{pmatrix} &= c\begin{pmatrix}\alpha\\ \mathbf{S}\end{pmatrix} = c\begin{pmatrix}\alpha\\ \mathbf{b}_1\end{pmatrix} = -c\begin{pmatrix}\alpha\\ \mathbf{b}_2\end{pmatrix} = -c\begin{pmatrix}\alpha\\ \mathbf{b}_3\end{pmatrix} = \\
c\begin{pmatrix}\beta\\ \mathbb{1}\end{pmatrix} &= c\begin{pmatrix}\beta\\ \mathbf{S}\end{pmatrix} = -c\begin{pmatrix}\beta\\ \mathbf{b}_1\end{pmatrix} = -c\begin{pmatrix}\beta\\ \mathbf{b}_2\end{pmatrix} = c\begin{pmatrix}\beta\\ \mathbf{b}_3\end{pmatrix} = \\
-c\begin{pmatrix}\gamma\\ \mathbb{1}\end{pmatrix} &= c\begin{pmatrix}\gamma\\ \mathbf{S}\end{pmatrix} = i\,c\begin{pmatrix}\gamma\\ \mathbf{b}_1\end{pmatrix} = c\begin{pmatrix}\gamma\\ \mathbf{b}_2\end{pmatrix} = i\,c\begin{pmatrix}\gamma\\ \mathbf{b}_3\end{pmatrix} = \\
c\begin{pmatrix}\alpha\\ \beta\end{pmatrix} &= -\,i\,c\begin{pmatrix}\alpha\\ \gamma\end{pmatrix} = i\,c\begin{pmatrix}\beta\\ \gamma\end{pmatrix} = 1
\end{aligned}
\tag{2.88}
$$

Table 2.4: Standard–like model boundary conditions and GGSO coefficients, with others specified by modular invariance. This model accommodates a leptophobic $U(1)$. Taken from [60].

For some choices of the additional basis vectors there may exist a combination

$$\mathbf{X} = n_\alpha\alpha + n_\beta\beta + n_\gamma\gamma \tag{2.89}$$

for which $\mathbf{X}_L \cdot \mathbf{X}_L = 0$ and $\mathbf{X}_R \cdot \mathbf{X}_R \neq 0$ that produces additional space–time vector bosons, just as $\mathbf{x}$ did previously. In the example given in Table 2.4, these may come from $\mathbf{X} = 1 + \alpha + 2\gamma$ and transform as triplets under $SU(3)_C$ with $U(1)$ charges. Thus, they enhance the SL gauge group to

$$SU(4)_C \times SU(2)_L \times U(1)_{C'} \times U(1)_L \tag{2.90}$$

with the $U(1)$ generator of $SU(4)_C$ being the leptophobic combination discussed in [60, 61]. Here the $U(1)_{C'}$ is a combination of $U(1)_C$, generated by (2.62), and $U(1)_{7'}$, generated by the worldsheet current $\overline{\phi}^1\overline{\phi}^1 - \overline{\phi}^8\overline{\phi}^8$, and $U(1)_L$ is generated by (2.63). Forming anomaly free, leptophobic, abelian symmetries in free fermionic heterotic string models is highly non–trivial and model–specific. We give a specific example later in this work, based on the SL symmetry breaking pattern [60] and find that the LRS symmetry breaking pattern may also produce a suitable symmetry.

2.6 Normalization of $U(1)$s

An important consideration when investigating phenomenological aspects of string models is the proper normalization of $U(1)$s. As detailed in Sections 2.5.1 and 2.5.2, we are interested in the different phenomenological implications of light $U(1)$s. As we have seen, different boundary conditions coming from the basis vectors $\{\alpha, \beta, \gamma\}$ result in different symmetry breaking patterns. A common feature of these models, however, is the occurrence of additional $U(1)$ symmetries.

As all of the models come from a single GUT symmetry, there are model–independent features one may exploit within the context of a given model. For example, gauge coupling unification. Taking the LRS breaking pattern as our model, with each gauge group there is an associated coupling constant, *i.e.* $\alpha_3, \alpha_{2R}, \alpha_{2L}$ and

α_C. Just as with the gauge couplings of the MSSM, discussed in Section 1.2, these run with energy scales. In fact, they are expected to unify at some high scale as they are embedded within an $SO(10)$ GUT, with a single gauge coupling expected to continue running beyond this, α_G, that will unify at the string scale with α_ζ, the coupling of our $U(1)_\zeta$, external to the $SO(10)$ GUT.

In GUT constructions the normalization of abelian generators is fixed by their embedding in non–abelian groups. This results in non–abelian gauge couplings, α_{NA}, being unchanged, *i.e.* they are already normalized. In the case of abelian gauge groups, their respective gauge couplings, α_A, will require some normalization constant to satisfy

$$\alpha_G \equiv \alpha^i_{NA} = k_i \alpha^i_A. \tag{2.91}$$

However, in string theory the non–abelian symmetry is not manifest, and the proper normalization of the $U(1)$ currents is obscured. The $U(1)$ normalization in string models that utilise a worldsheet conformal field theory construction is fixed by their contribution to the conformal dimensions of physical states. Also, one has the choice of normalizing the non–abelian gauge couplings too. This is due to the relationship between low–energy effective gauge symmetries and their descriptions as worldsheet Kač–Moody algebras. One may rewrite the standard one–loop renormalization group equations (RGEs),

$$\alpha_i^{-1}\left(M_{\text{GUT}}\right) = \alpha_i^{-1}\left(\mu\right) - \frac{\beta_i}{2\pi}\log\left(\frac{M_{\text{GUT}}}{\mu}\right) + \Delta_i^{(\text{GUT})} \tag{2.92}$$

in a string model as [96]

$$\frac{4\pi}{\alpha_i\left(\mu\right)} = k_i\frac{4\pi}{\alpha_{\text{string}}} + \beta_i \log\frac{M^2_{\text{string}}}{\mu^2} + \Delta_i^{(\text{string})}, \tag{2.93}$$

where k_i are the relevant normalization constants, β_i are the one–loop β–functions, Δ_i are the heavy unification–scale thresholds and $\alpha_{\text{string}} \equiv \alpha_i\left(M_{\text{string}}\right)$. The difference is due to the unification with gravity in string theory [97] *i.e.*

$$8\pi \frac{G_N}{\alpha'} = g_i^2 k_i = g_{\text{string}}^2. \tag{2.94}$$

The k_i featuring in (2.91) are the Kač–Moody levels of the gauge group. These are related to the normalization of the generators, which is a subtle issue in the analysis of gauge coupling unification in string models. The procedure for fixing the normalization of $U(1)$s was outlined in [30, 98] and we repeat it here for completeness.

In unified string models, the Kač–Moody level of non–abelian group factors is always one. In the FFF of the heterotic string, a given $U(1)$ current, U, in the Cartan subalgebra of the four dimensional gauge group, is a combination of the simple right–moving worldsheet currents

$$U(1)_f \equiv f^* f, \tag{2.95}$$

corresponding to individual worldsheet fermions, f. U then takes the form

$$U = \sum_f a_f \, U(1)_f, \tag{2.96}$$

where the a_f are model dependent coefficients. Each $U(1)_f$ is normalized to one, so that $\langle U(1)_f, U(1)_f \rangle = 1$, and each of the linear combinations must also be normalized to one. The proper normalization coefficient for the linear combination U is given by

$$N = \left(\sum_f a_f^2 \right)^{-\frac{1}{2}}, \tag{2.97}$$

and the properly normalized $U(1)$ current is, thus, given by $\hat{U}(1) = N \cdot U$.

In general, the Kač–Moody level, k, of a $U(1)$ generator can be deduced from the operator product expansion between two of the $U(1)$ currents, and is given by

$$k = 2N^{-2} = 2\sum_f a_f^2. \tag{2.98}$$

The result is generalised to $k = \sum_i a_i^2 k_i$ when the $U(1)$ is a combination of several $U(1)$s with different normalizations. This procedure is used to determine the Kač–Moody level, k_1, of the weak–hypercharge generator, as well as that of any other $U(1)$ combination in the effective low–energy field theory.

In the LRS heterotic string models, the $SO(10)$ symmetry is broken to $SU(3)_C \times U(1)_C \times SU(2)_L \times SU(2)_R$, where the combinations of worldsheet currents

$$\frac{1}{3}\left(\overline{\psi}_1^*\overline{\psi}_1 + \overline{\psi}_2^*\overline{\psi}_2 + \overline{\psi}_3^*\overline{\psi}_3\right) \tag{2.99}$$

and

$$\frac{1}{2}\left(\overline{\psi}_4^*\overline{\psi}_4 + \overline{\psi}_5^*\overline{\psi}_5\right) \tag{2.100}$$

generate $U(1)_C$ and T_{3_R}, respectively, where the latter is the diagonal generator of $SU(2)_R$. The weak–hypercharge is then given by

$$U(1)_Y = T_{3_R} + \frac{1}{3}U(1)_C. \tag{2.101}$$

In our analysis of gauge coupling unification, the symmetry of $SU(2)_R$ is incorporated at the M_R scale, where above this scale the multiplets are in representations of the LRS gauge group and below the M_R scale they are in SM representations. The weak-hypercharge coupling relation is given by

$$\frac{1}{\alpha_1(M_R)} = \frac{1}{\alpha_{2R}(M_R)} + \frac{k_C}{9}\frac{1}{\alpha_{\hat{C}}(M_R)} = \frac{1}{\alpha_{2R}(M_R)} + \frac{2}{3}\frac{1}{\alpha_{\hat{C}}(M_R).} \tag{2.102}$$

Here we have used (2.98) to find that the Kač–Moody level of $U(1)_C$ is $k_C = 6$. Again using (2.98) we find that $k_1 = \frac{5}{3}$ as expected. This reproduces the expected result at the unification scale

$$\sin^2\theta_W\,(M_S) = \frac{1}{1+k_1} \equiv \frac{3}{8}. \tag{2.103}$$

Chapter 3

A Leptophobic $U(1)$ from the Heterotic String

In the previous chapter we outlined the general approach to model building in the free fermionic formulation of the heterotic string. In the coming chapters, we specify models that exhibit properties that may be advantageous for phenomenological purposes. These include a leptophobic $U(1)$ symmetry. These form an interesting problem, as additional abelian gauge symmetries that arise in string and GUT models, generically couple to leptons and baryons equally. The interest in leptophobic symmetries arose, for example, as a possible explanation for the 3.2σ excess in the Wjj channel at CDF, back in June 2011 [63] and continues to be of interest in collider searches.

3.1 Standard–like models

The string construction of such a symmetry was briefly outlined in Section 2.5.2 and also in [60]. The key to understanding how the leptophobic $U(1)$ arises in the model of [60] are the charges of the matter states from the sectors $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{b}_3$ under the flavour $U(1)_j$ with $j = 4, 5, 6$. For example, the charges of the states from the sector $\mathbf{b}_1$ are given in Table 3.1 and are similar for the states from the sectors $\mathbf{b}_2$ and $\mathbf{b}_3$. With these charge assignments, the quarks are charged with respect to the

Field	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Q_L	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
u^c_L	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
d^c_L	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
L_L	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
e^c_L	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
ν^c_L	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0

Table 3.1: 1 generation of MSSM matter, originating in the $\mathbf{b}_1$–sector, with charges under $U(1)_i$, with $i = 1, \ldots, 6$. As presented in [60].

following combination

$$U(1)_B = \frac{1}{3}U(1)_C - (U(1)_4 + U(1)_5 + U(1)_6), \tag{3.1}$$

whereas the leptons are neutral with respect to it. Hence, this combination is a family universal, leptophobic $U(1)$ symmetry. In the model of [60] additional space–time vector bosons arise from the sector $\mathbf{X} = \mathbb{1} + \alpha + 2\gamma$ in which $\mathbf{X}_L \cdot \mathbf{X}_L = 0$ and $\mathbf{X}_R \cdot \mathbf{X}_R = 8$. The additional vector bosons transform as triplets of $SU(3)_C$ and enhance it to $SU(4)_C$, where the $U(1)$ combination given by

$$U(1)_{B'} = U(1)_B - U(1)_7 + U(1)_9, \tag{3.2}$$

is the $U(1)$ generator of the enhanced $SU(4)$ symmetry. Here, U_7 and U_9 arise from the world–sheet complex fermions $\overline{\phi}^1$ and $\overline{\phi}^8$ that generate the hidden E_8' symmetry. In this model the $U(1)_{1,2,3}$ symmetries are anomalous, with $\mathrm{Tr}[U(1)_1] = 24$, $\mathrm{Tr}[U(1)_2] = 24$ and $\mathrm{Tr}[U(1)_3] = 24$. Hence, the family universal combination of these three $U(1)$ is anomalous, whereas the two family non–universal combinations are anomaly free. The $U(1)_{4,5,6,7,9}$ are, however, anomaly free. Hence, the leptophobic $U(1)$ combination is anomaly free and can remain, in principle, unbroken down to low scales.

The existence of a leptophobic, family universal and anomaly free $U(1)$ is highly non–trivial and not generic in phenomenological heterotic string models. To demonstrate that this is indeed the case, we examine the model of [19, 99]. The sectors $\mathbf{b}_i$ produce the three chiral generations that are charged with respect to the same flavour symmetries, but differ from the corresponding charges in the model of [60]. For example, the states from the sector $\mathbf{b}_1$ carry the charges in Table 3.2. We observe that e_L^c ad L_L have like–sign charges under $U(1)_4$. Since they carry opposite sign charges under $U(1)_C$, $U(1)_4$ cannot be used to cancel the $B-L$ charge for both these states. Since they also carry like–sign charges under $U(1)_1$, a leptophobic, family universal $U(1)$ cannot be made from these $U(1)$ symmetries. The model of [19, 99] preserves the cyclic S_3 permutation of the NAHE set. Hence, a similar charge assignment is obtained in the sectors $\mathbf{b}_2$ and $\mathbf{b}_3$. In this model the flavour

Field	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Q_L	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
u_L^c	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
d_L^c	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
L_L	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
e_L^c	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
ν_L^c	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0

Table 3.2: 1 generation of MSSM matter, originating in the $\mathbf{b}_1$–sector, with charges under $U(1)_i$, with $i = 1, \ldots, 6$, for Standard–like models akin to [19, 99].

symmetries $U(1)_{4,5,6}$ are anomalous. Therefore, their combination with $U(1)_C$ is not anomaly free and must be broken.

Taking another example of the Standard–like gauge symmetry breaking pattern, we consider the model of [18, 100]. In this model the states from the sector $\mathbf{b}_1$ carry the $U(1)$ charges detailed in Table 3.3. In this sector the combination given in (3.1)

Field	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Q_L	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
u^c_L	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
d^c_L	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
L_L	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
e^c_L	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
ν^c_L	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0

Table 3.3: 1 generation of MSSM matter, originating in the $\mathbf{b}_1$–sector, with charges under $U(1)_i$, with $i = 1, \ldots, 6$, for the SL models of [18, 100].

is leptophobic. However, the states from the sector $\mathbf{b}_2$ have charges as in Table 3.4, which differ making (3.1) family non–universal. Also, due to the charges of the leptonic states, the combination is no longer leptophobic and so is now unsuitable.

Furthermore, the flavour symmetries are anomalous in this model and, consequently, as is the combination given in (3.1). So, is the existence of a leptophobic $U(1)$ combination therefore a peculiarity of the model of [60]? As seen from the charge assignments in Table 3.1, the key is that the charges of the left– and right–handed fields differ in sign with respect to $U(1)_{4,5,6}$ in the sectors $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{b}_3$ respectively. This model preserves the cyclic permutation symmetry of the NAHE set and, therefore, the $U(1)$ combination in (3.1) is family universal. The $U(1)_{4,5,6}$ are also anomaly free in the model of [60] and, therefore, their combination with

Field	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Q_L	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0
u^c_L	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0
d^c_L	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0
L_L	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0
e^c_L	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0
ν^c_L	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0

Table 3.4: 1 generation of MSSM matter, originating in the $\mathbf{b}_2$–sector, with charges under $U(1)_i$, with $i = 1, \ldots, 6$.

$U(1)_{B-L}$ is also anomaly free. In this model the gauge symmetry is enhanced by space–time vector bosons arising from the twisted sector, $\mathbf{X}$. However, we can envision a more systematic classification, along the lines of [79, 83], and that the extra bosons can be projected out from the spectrum in vacua that resemble the properties of this model. In such a case, the leptophobic $U(1)$ will arise without enhancement.

3.2 Left–right symmetric models

As seen from the other two examples provided by the models in [19, 99] and [18, 100], the existence of a family universal, anomaly free, leptophobic $U(1)$ combination in heterotic string vacua is highly non–trivial. A class of models that reproduces the conditions for the existence of such a $U(1)$ combination are the left–right symmetric models of [51]. However, in this case the $U(1)$ symmetries that are combined with $U(1)_{B-L}$ are not the flavour $U(1)_{4,5,6}$, but rather the $U(1)_{1,2,3}$. This possibility is particular to the left–right symmetric heterotic string models, and is not applicable in the other quasi–realistic free fermionic models, in which the $SO(10)$ symmetry is broken to the flipped $SU(5)$, $SO(6) \times SO(4)$ or $SU(3) \times SU(2) \times U(1)^2$ subgroups.

The reason is that, in these cases, the charges of all the states from a given sector $\mathbf{b}_j$ are the same with respect to $U(1)_j$ with $j = 1, 2, 3$. This situation arises because the states from the sectors $\mathbf{b}_j$ in these models preserve the E_6 charge assignment under the decomposition $E_6 \to SO(10) \times U(1)$. A further consequence is that the $U(1)$ combination which arises from E_6 becomes anomalous in these models [101].

On the other hand, in the left–right symmetric models, the GGSO projection that breaks

$$SO(10) \to SU(3) \times U(1) \times SU(2)^2$$

dictates that the $U(1)_{1,2,3}$ charges of the left–handed fields, Q_L and L_L, differs in sign from those of the right–handed fields, Q_R and L_R, just as we discussed previously. Their charges with respect to $U(1)_{4,5,6}$ may, or may not differ in sign and are model–specific. Hence, for example, in the first model of [51], described in Table 2.2, we find the charges for the sector $\mathbf{b}_1$, detailed in Table 3.5, with similar charges under $U(1)_{2,3}$ for the states from the sectors $\mathbf{b}_2$ and $\mathbf{b}_3$ respectively. The $U(1)$ combination

Field	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Q_L	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
u^c_L	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
d^c_L	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
L_L	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
e^c_L	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
ν^c_L	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0

Table 3.5: 1 generation of MSSM matter, originating in the $\mathbf{b}_1$–sector, with charges under $U(1)_i$, with $i = 1, \ldots, 6$ for the LRS model of Table 1 in [51].

given by

$$U(1)_B = \frac{1}{3}U(1)_C - U(1)_1 - U(1)_2 - U(1)_3, \tag{3.3}$$

is family universal, anomaly free and leptophobic. In the left–right symmetric models, the $U(1)_{1,2,3}$ are anomaly free due to the specific symmetry breaking pattern and consequent charge assignments, whereas $U(1)_{4,5,6}$ may be anomalous or anomaly free in different models. The left–right symmetric free fermionic heterotic string models therefore provide a second example that produces a potentially viable leptophobic $U(1)$ at low scales. In both cases, it is seen that the mechanism that yields a leptophobic $U(1)$ symmetry involves the existence of a combination of flavour $U(1)$ symmetries that nullifies the lepton number component of $U(1)_{B-L}$.

The left–right symmetric models produce examples that are completely free of any gauge or gravitational anomalies. Specifically, all $U(1)$ symmetries in these models are anomaly free. Hence, any combinations of the $U(1)$ symmetries, including the leptophobic combination, are anomaly free.

Chapter 4

String–Inspired Models

In this chapter, we specify a model that forbids proton decay mediating operators up to dimension-6, requiring any states that feature in our low–energy effective field theory (EFT) to satisfy the string constraints presented in Section 2.2. A more detailed discussion specific to this construction was outlined in Section 2.5.1. In this model, we assume the $SO(10) \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_C$ GUT symmetry breaking occurs at the string scale, using the mechanism specified in Chapter 2. We emphasise here that this model exhibits an extra $U(1)$ that satisfies the constraints outlined in Section 1.3 and is external to the $SO(10)$ GUT symmetry. This $U(1)_\zeta$ symmetry forbids the dimension-4, -5 and -6 PDMOs at the string scale and, by keeping the $U(1)$ sufficiently light, we may adequately suppress these operators, induced after the $U(1)$ is broken. The $U(1)$ arises as an anomaly free symmetry in the string models but we need to ensure that it remains anomaly free in the low–energy EFT. For this purpose, we construct a string–inspired model that takes into account the $U(1)$ charges of the Standard Model matter states as they arise in the string model, and we augment the model with additional states, compatible with the string charge assignments, to render the spectrum of the string–inspired EFT anomaly free. The analysis done in this section was first presented in [73].

4.1 Low-scale construction

The MSSM matter states originate in the **16** of $SO(10)$ (with the right–handed neutrino, ν^c) and decompose under the left–right symmetric gauge group in the representations shown in Table 4.1. The electroweak Higgs states lie in a bidoublet representation of the LRS gauge group and originate in the **10** representation of $SO(10)$ (also shown in Table 4.1). This bidoublet is neutral under both $U(1)_C$ and $U(1)_\zeta$.

$SO(10)$	Field	$SU(3)_C$	$SU(2)_L$	$SU(2)_R$	Q_C	Q_ζ
$\mathbf{16}^i$	Q_L^i	$\mathbf{3}$	$\mathbf{2}$	$\mathbf{1}$	$+\frac{1}{2}$	$-\frac{1}{2}$
	Q_R^i	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$
	L_L^i	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$-\frac{3}{2}$	$-\frac{1}{2}$
	L_R^i	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$+\frac{3}{2}$	$+\frac{1}{2}$
$\mathbf{10}$	H_0	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	0	0

Table 4.1: MSSM with $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_C \times U(1)_\zeta$ quantum numbers, with $i = 1, 2, 3$ for the three generations. The representations of $SO(10)$ from where the states originate are also shown.

In order for us to construct a consistent low–scale model, we must ensure our EFT is anomaly free. We now analyse where the gauge and gravitational anomalies could originate and, should any arise, augment our field content with fields satisfying the string charge assignments, as alluded to in Section 2.5.1, such that the EFT becomes anomaly free.

Anomalies

When a classical symmetry is broken by quantum corrections it is said to be anomalous. This only becomes problematic for gauge symmetries; should a global symmetry be anomalous upon quantization, processes that were previously forbidden, may be induced. However, should a gauge symmetry be anomalous, this could have ramifications for unitarity and the renormalizability of our theory. Thus, in order for our additional $U(1)$ to be an allowed gauge symmetry of our theory, we must ensure that it is free of anomalies. In the string–derived model of [51], the $U(1)_\zeta$ is anomaly free as it includes the full massless string–scale spectrum. Only including the representations in Table 4.1, one would not expect, *a priori*, $U(1)_\zeta$ to be free of gravitational and gauge anomalies. As discussed in Section 1.3, only $U(1)_{B-L}$ and $U(1)_Y$ form anomaly free combinations with a complete spinor representation of $SO(10)$.

To construct a consistent model with our LRS gauge group and additional $U(1)$, we need to consider the following anomaly diagrams (shown in Figure 4.1):

$\mathcal{A}_1 : (SU(3)^2_C \times U(1)_\zeta) -$ Only quarks are summed over in this diagram and we find that, for Table 4.1, it is anomaly free.

$\mathcal{A}_2 : (SU(2)^2_L \times U(1)_\zeta) -$ Due to our charge assignment, the left-handed quark and lepton fields have the same sign resulting in an anomaly. In fact, $\mathcal{A}_2^{SM} = -2$.

$\mathcal{A}_3 : (SU(2)^2_R \times U(1)_\zeta) -$ Again, because of our charge assignment for right-handed quark and lepton fields there is a resulting anomaly. In fact, $\mathcal{A}_3^{SM} = +2$.

$\mathcal{A}_4 : (U(1)^2_C \times U(1)_\zeta) -$ All fermions are summed over in this diagram. As left- and right-handed fields have charges of opposite sign under $U(1)_\zeta$, it is found to be anomaly free.

$\mathcal{A}_5 : \left(U(1)_C \times U(1)^2_\zeta\right) -$ Again all fermions are summed over in this diagram. It is also found to be anomaly free due to the opposite-signed $U(1)_\zeta$ charges for left- and right-handed doublets.

$\mathcal{A}_6 : \left(U(1)_\zeta^3\right)$ – Again we sum over all fermions and the diagram is found to be anomaly free.

$\mathcal{A}_7 : (U(1)_\zeta \times \text{Gravity})$ Here we also sum over all fermions. Due to our choices of Q_ζ, this diagram is obviously anomaly free.

As mentioned previously, $U(1)_{B-L}$ is anomaly free for the MSSM states and three generations of right–handed neutrinos, *i.e.* the complete **16** representation of $SO(10)$. Thus, we must only check for anomalies in diagrams involving a $U(1)_\zeta$ vertex, all of which are shown in Figure 4.1. These one–loop diagrams are induced above the $SU(2)_R$ breaking scale. At this stage the spectrum possesses mixed anomalies in the $SU(2)_L^2 \times U(1)_\zeta$ and $SU(2)_R^2 \times U(1)_\zeta$ diagrams, $\mathcal{A}_2$ and $\mathcal{A}_3$ respectively. In the string vacua, these anomalies are cancelled by additional states that arise in the full string spectrum. As shown in [51] the $U(1)_\zeta$ is anomaly free and in fact the LRS gauge group is entirely free of gauge and gravitational anomalies. However, the additional spectrum in the string vacua is highly model dependent. We, therefore, judicially augment the spectrum in Table 4.1 with additional states that cancel the $SU(2)_{L,R}^2 \times U(1)_\zeta$ mixed anomalies and are compatible with the string charges. This guarantees that any combination of the $U(1)$ generators in the Cartan subalgebra of the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Z'}$ gauge group is anomaly free. That is, it guarantees that any $U(1)_{Z'}$ arising from this group is anomaly free at the low scale. To obtain a spectrum which is free of the mixed anomalies we look to add states charged under each of the anomalous symmetries, *i.e.* $SU(2)_L, SU(2)_R$ and $U(1)_\zeta$, that also leave the other diagrams anomaly free.

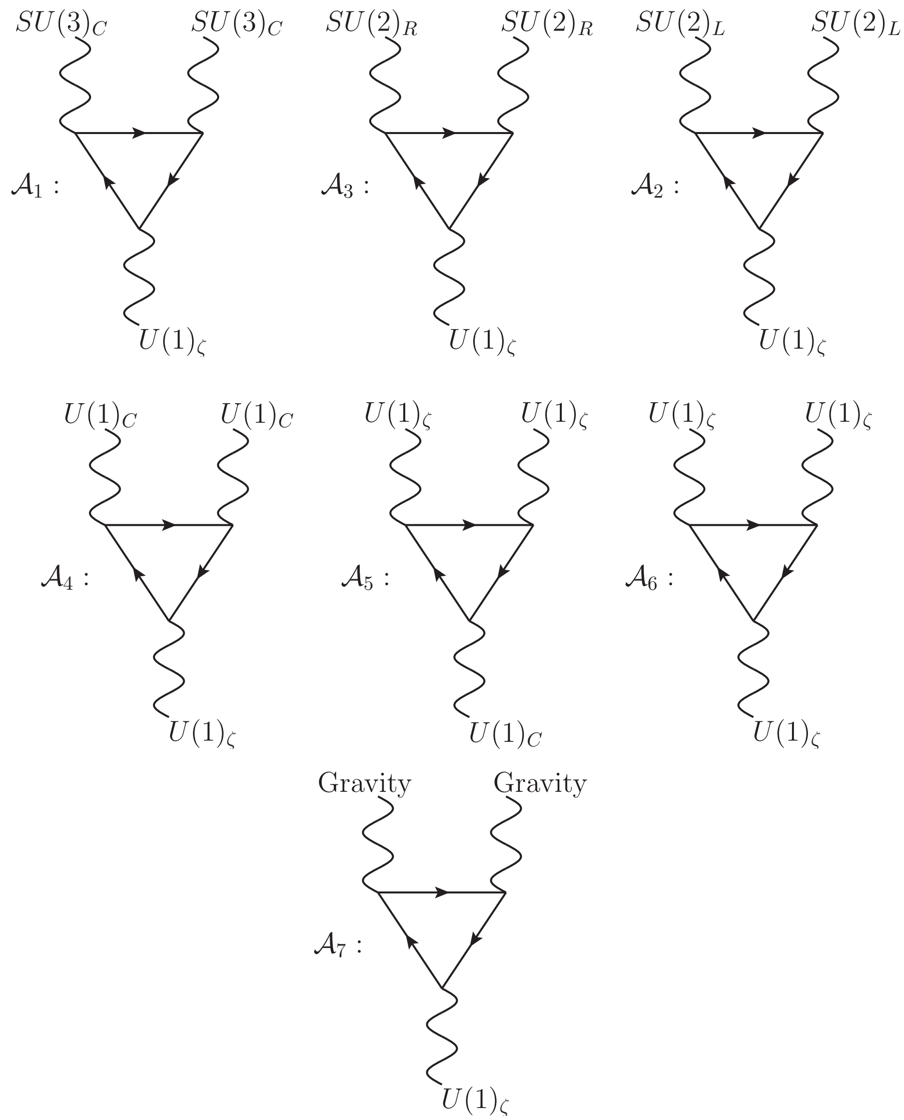

Figure 4.1: The relevant diagrams considered in our analysis of the anomalies. These are only those that occur at the high scale, however including anomaly cancelling states that agree with the string charge assignments negates anomalies in any symmetry that is a combination of the Cartan generators.

Anomaly Cancelling Doublets

If we try and mimic E_6 by adding states originating in the **10**, *i.e.* by "completing" the **27** representation of E_6, we would require states transforming as

$$H_1 \sim (\mathbf{1}, \mathbf{2}, \mathbf{2}, \pm 1, \pm 1)\,. \tag{4.1}$$

Given that

$$\mathcal{A}_2 = -2; \tag{4.2a}$$

$$\mathcal{A}_3 = +2, \tag{4.2b}$$

clearly means that bidoublets are ruled out as anomaly cancelling doublets (ACDs). This leaves us with the option to add states originating in the **16**. These must have $Q_\zeta = \pm\frac{1}{2}$ and, as $\mathcal{A}_1 = 0$, must be added with vector–like charges under $SU(3)$ or as colour singlets. We choose the latter for simplicity. This leaves us with the option of lepton–like states

$$H_L^i \sim \left(\mathbf{1}, \mathbf{2}, \mathbf{1}, \pm\frac{3}{2}, +\frac{1}{2}\right) \qquad\qquad H_R^i \sim \left(\mathbf{1}, \mathbf{1}, \mathbf{2}, \mp\frac{3}{2}, -\frac{1}{2}\right) \tag{4.3}$$

where $i = 2, \ldots, 2n$. In order to satisfy (4.2), $n = 4$. However, these induce anomalies in diagrams involving a $U(1)_C$ vertex. Adding states with vector–like Q_C charges but with Q_ζ of the same sign should resolve this problem. In fact, adding

$$\begin{aligned}
H_L^{ij} &= \left(\mathbf{1}, \mathbf{2}, \mathbf{1}, +\frac{3}{2}, +\frac{1}{2}\right),\\
H_L'^{ij} &= \left(\mathbf{1}, \mathbf{2}, \mathbf{1}, -\frac{3}{2}, +\frac{1}{2}\right),\\
H_R^{ij} &= \left(\mathbf{1}, \mathbf{1}, \mathbf{2}, -\frac{3}{2}, -\frac{1}{2}\right),\\
H_R'^{ij} &= \left(\mathbf{1}, \mathbf{1}, \mathbf{2}, +\frac{3}{2}, -\frac{1}{2}\right),
\end{aligned} \tag{4.4}$$

where $j = 1, 2$ and $i = 1, 2, 3$, the spectrum becomes free of all anomalies. Thus, we now have an EFT free of gravitational and gauge anomalies.

Heavy Higgs States

In addition to the ACDs, heavy Higgs states in vectorlike representations are needed to break the $SU(2)_R \times U(1)_C \times U(1)_\zeta \rightarrow U(1)_Y \times U(1)_{Z'}$ at some intermediate scale below M_S. These are

$$\mathcal{H}_R + \overline{\mathcal{H}}_R = \left(\mathbf{1}, \mathbf{1}, \mathbf{2}, +\frac{3}{2}, \mp\frac{1}{2}\right) + \left(\mathbf{1}, \mathbf{1}, \mathbf{2}, -\frac{3}{2}, \pm\frac{1}{2}\right). \tag{4.5}$$

These pick up a vev in the neutral directions, $\mathcal{N}_R$ and $\overline{\mathcal{N}}_R$, which leave the combinations

$$Y = T_{3_R} + \frac{1}{3}Q_C, \tag{4.6}$$

$$Q_{Z'} = \frac{1}{5}Q_C - \frac{2}{5}T_{3_R} \pm Q_\zeta, \tag{4.7}$$

unbroken at the low scale. In fact, in generic $SO(10)$ GUT models, the weak hypercharge is given by (4.6), where T_{3_R} is the diagonal generator of $SU(2)_R$ and

$$Q_C = \frac{3}{2}Q_{B-L},$$

as described in (2.62). The sign of $Q_\zeta\left(\mathcal{H}_R\right)$ and $Q_\zeta\left(\overline{\mathcal{H}}_R\right)$ will result in a different $U(1)_{Z'}$ combination shown in (4.7) and thus, different phenomenology for each model. We will focus on the linear combination resulting from the charge assignments

$$\mathcal{H}_R + \overline{\mathcal{H}}_R = \left(\mathbf{1}, \mathbf{1}, \mathbf{2}, +\frac{3}{2}, -\frac{1}{2}\right) + \left(\mathbf{1}, \mathbf{1}, \mathbf{2}, -\frac{3}{2}, +\frac{1}{2}\right), \tag{4.8}$$

i.e.

$$Q_{Z'} = \frac{1}{5}Q_C - \frac{2}{5}T_{3_R} + Q_\zeta \tag{4.9}$$

but include a brief discussion on the alternative case and its phenomenological implications in the next section. The electromagnetic $U(1)$ charge is then given by the combination,

$$Q_{\text{EM}} = T_{3_L} + Y. \tag{4.10}$$

The augmentation of the states in Table 4.1 with the states given by (4.4), guarantees that the low–energy EFT above and below the intermediate breaking scale is completely free of gauge and gravitational anomalies. These states satisfy the string charge assignments and would originate from a different fixed point to that of the **16** containing the MSSM. We note here that we envisage problems with gauge coupling unification due to the presence of these extra doublets and so we include triplets in order to slow down the running of $SU(3)$, hopefully reinstating one–loop unification. The number of triplets is treated as a free parameter, n_D. Specifying a string model, however, would restrict n_D as alluded to in Section 2.5.1. This issue will be discussed in further detail in Chapter 5.

The spectrum of our model above the left–right symmetry breaking scale is summarised in Table 4.2. The spectrum below the intermediate symmetry breaking scale is shown in Table 4.3. The anomaly freedom of our model allows the $U(1)_{Z'}$ combination generated by (4.9) to be viable to low energies, limited only by experimental constraints, which include gauge coupling unification and proton decay suppression. These will be discussed further in the following chapter.

4.2 Superpotentials

First we briefly discuss the superpotentials above and below the $SU(2)_R$ gauge symmetry breaking. We may be able to impose phenomenological constraints on the allowed couplings which may indicate our low–energy $U(1)_{Z'}$ symmetry breaking scale. The most general superpotentials, up to trilinear couplings, at both the high scale, invariant under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_C \times U(1)_\zeta$, shown in (4.11), and at the low scale, invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Z'}$, shown in (4.12), are

$$W = h_Q Q_L Q_R H_0 + h_L L_L L_R H_0 \tag{4.11a}$$

$$+ \lambda_H H_L H_R H_0 + \lambda_{H'} H'_L H'_R H_0 \tag{4.11b}$$

$$+ h_{H_L} H_L H'_L S + h_{H_R} H_R H'_R \overline{S} \tag{4.11c}$$

$$+ \mu H_0 H_0 \phi + \mu_1 L_L H_L \phi + \mu_2 L_R H_R \phi + \mu_D D\overline{D}\phi + \mu_S S\overline{S}\phi + \mu_3 \phi\phi\phi \tag{4.11d}$$

$$+ \eta_1 L_R \overline{\mathcal{H}}_R S + \eta_2 H_R \mathcal{H}_R \overline{S} + \eta_3 H'_L \mathcal{H}_R H_0 + \eta_4 H'_R \overline{\mathcal{H}}_R \phi + \eta_5 \mathcal{H}_R \overline{\mathcal{H}}_R, \tag{4.11e}$$

and

$$W = h_u Q_L u^c_L H^u + h_d Q_L d^c_L H^d + h_e L_L e^c_L H^d + h_\nu L_L \nu^c_L H^u \tag{4.12a}$$

$$+ \lambda_H H_L N_R H^d + \lambda'_H H_L E_R H^u + \lambda_{H'} H'_L E'_R H^d + \lambda'_{H'} H'_L N'_R H^u \tag{4.12b}$$

$$+ h_L H_L H'_L S + h_R E_R E'_R \bar{S} + h'_R N_R N'_R \bar{S} \tag{4.12c}$$

$$+ \mu_D D\overline{D}\phi + \mu_S S\overline{S}\phi + \mu_\phi \phi\phi\phi \tag{4.12d}$$

$$+ \mu H^u H^d \phi + \mu_1 L_L H_L \phi + \mu_2 e^c_L E_R \phi + \mu_{2'} \nu^c_L N_R \phi \tag{4.12e}$$

$$+ \eta_1 \nu^c_L S + \eta_2 N_R \overline{S} + \eta_3 H'_L H^u + \eta_4 N'_R \phi, \tag{4.12f}$$

where all indices have been suppressed. Terms in (4.11d) may also be written as bilinear terms, as ϕ is a gauge–singlet. However, bilinear terms do not appear in the string–scale superpotential due to conformal invariance. We presume that states not present here, but present in the full string–derived model, acquire mass at the string scale and, as we are only considering the low–energy aspects of this model, do not include them here. In the same way, we do not include terms in the superpotential that would not appear in a string–derived superpotential, *i.e.* bilinear terms. We emphasise that this analysis is string–inspired; we must satisfy the string constraints but have more freedom to choose states that allow our model to satisfy phenomenological constraints also.

It is interesting to note here that the extra quarks, $D + \overline{D}$, exhibit a global $U(1)$ symmetry. This global $U(1)$ results in the lightest extra quark being stable. In order to suppress the abundances of nuclear isotopes which contain such stable relics, the lightest exotic quarks should be heavier than 10 TeV [102]. This scale, M_D, will become important when we discuss gauge coupling unification in the next chapter.

The $\mu_{2'}$ term may induce an unstable lightest neutralino, due to mixing between neutralinos and leptons. Thus, the LSP would not be a dark matter candidate, unlike in the MSSM. The couplings labelled by η_j are those that involve the heavy Higgs fields that break the left–right symmetry, acquiring a vev at high energies. The terms (4.11a, 4.12a) produce the MSSM Dirac masses for the quarks and leptons, including a Dirac mass term for the neutrino, admitted due to the left–right symmetry.

The couplings in (4.11b, 4.11c, 4.11d) involve couplings of the additional doublets, triplets and singlets in the model. There is also the μ parameter; the usual supersymmetric Higgs parameter akin to that in E_6 inspired models or the NMSSM. The couplings in (4.11e) are those that involve the couplings to the heavy Higgs fields. We note that the choice given in (4.9) forbids the Higgsino–neutrino mixing term $L_L\overline{\mathcal{H}}_R H_0$ at the expense of the ν_L^c fields becoming charged under $U(1)_{Z'}$, whereas the alternative choice in (4.7) allows the neutrino–Higgsino mixing term, but keeps the ν_L^c fields neutral under the respective $U(1)_{Z'}$ combination. This issue again relates to the scale of $SU(2)_R$ breaking and the consequent suppression of the left–handed neutrino masses. We will examine this question in more detail in future work. We note here that some couplings in (4.12) may still need to be suppressed to avoid conflict with the data. We defer this discussion to future analyses.

There are also gauge invariant terms that may appear in (4.12) that do not originate from renormalizable terms at the high scale, *i.e.* not present in (4.11). These terms are suppressed by at least $\frac{M_R}{M_S}$ and so, dependent on how low the $SU(2)_R$ breaking scale is, are not expected to provide large contributions. Nevertheless, these terms may have interesting phenomenological effects and so we defer these investigations to future work as this is beyond the scope of this thesis. These additional terms are shown below in (4.13). Included are bilinear terms that may originate from bilinear terms at the high scale. As discussed above, these may not be generated directly in a string–derived model as renormalizable terms and so we do not include these in (4.11) and thus (4.12).

$$\begin{aligned}
W_{\text{NR}} =& \mu'_D D\bar{D} + \mu'_S S\bar{S} + \mu'_\phi \phi\phi \\
& + \mu' H^u H^d + \mu'_1 L_L H_L + \mu'_2 e^c_L E_R + \mu'_{2'} \nu^c_L N_R \\
& + \lambda_1 Q_L d^c_L H'_L + \lambda_2 L_L e^c_L H'_L + \lambda_3 H'_L H'_L E'_R \\
& + \lambda_4 \nu^c_L E_R E'_R + \lambda_5 e^c_L E_R N'_R + \lambda_6 \nu^c_L N_R N'_R + \lambda_7 L_L H_L N'_R + H_L H'_L N_R \\
& + \lambda_8 H^u H^d N'_R + \lambda_9 D\overline{D} N'_R + \lambda_{10} N'_R S\overline{S} \\
& + \lambda_{11} H_L H^d S + \lambda_{12} L_L H^u \overline{S} \\
& + \lambda_{13} H'_L H^u \phi \\
& + \lambda_{14} N_R \overline{S}\phi + \lambda_{15} \nu^c_L S\phi + \lambda_{16} \nu^c_L N'_R S \\
& + \lambda_{17} N'_R N'_R + \lambda_{18} N'_R N'_R N'_R + \lambda_{19} N'_R N'_R \phi + \lambda_{20} N'_R \phi\phi.
\end{aligned} \tag{4.13}$$

As ν^c_L is charged under $U(1)_{Z'}$ its mass is protected and thus cannot acquire a vev greater than $M_{Z'}$ which is expected to be low. In order to have sufficiently low left–handed neutrino masses, the model admits an extended seesaw mechanism. Also, due to the additional $U(1)_\zeta$ combination in (2.31), resulting in the low–scale $U(1)_{Z'}$ combination in (4.9), we find that the Majorana mass term,

$$\nu^c_L \nu^c_L SS, \tag{4.14}$$

that violates lepton number, is allowed in our low–energy superpotential. This originates from the non–renormalizable operator

$$L_R L_R SS \mathcal{H}_R \mathcal{H}_R \tag{4.15}$$

also resulting in too light a right–handed neutrino mass due to the M_S^3 suppression. However, a larger mass may also be generated via the extended seesaw mechanism discussed below.

Extended Seesaw Mechanism

The seesaw mechanism used in this model generates a small neutrino mass, $m_\nu \sim 1$ eV, by way of a 9×9 matrix in the (ν_L, ν_L^c, S) flavour basis:

$$M = \begin{pmatrix} 0 & M_Z & 0 \\ M_Z & 0 & M_R \\ 0 & M_R & A \end{pmatrix}. \tag{4.16}$$

We note that A is dependent on the mass matrix of the singlet, S. On first look, this will be proportional to the scale at which ϕ gains mass, M_ϕ. Therefore the analysis of the neutrino mass would follow along the lines of [10, 103], where the relevant couplings are

$$h_\nu L_L \nu_L^c H^u + \mu_S S\overline{S}\phi + \eta_1 \nu_L^c S \langle \mathcal{N}_R \rangle.$$

However, further analysis would need to be conducted to specify the phenomenology of the neutrino sector. Other examples of non–GUT $U(1)$s, with origins in the heterotic string and similar neutrino sectors, are detailed in [59] including a string–inspired toy–model.

We note that the intermediate scale breaking is a free parameter in this model as it is not constrained by a doublet–triplet splitting mechanism, which is induced at the string–scale [95]. Hence, the masses of the left–handed neutrinos can be used to constrain the intermediate scale. The ν_L masses can be sufficiently suppressed by the extended seesaw mechanism, and by rendering the left–handed neutrinos unstable by the coupling to light sterile neutrinos, ϕ_m. Such states may arise, for example, from hidden sector condensates in the string models. The renormalization group evolution of the gauge and superpotential couplings, together with hidden sector dynamics are also expected to fix all scales in the string models.

Proton Decay Mediating Operators

Turning to the PDMOs, we note that with both choices in (4.7) the dimension-4 baryon number violating operator that arises from

$$Q_R Q_R Q_R \mathcal{H}_R \to u^c d^c d^c \langle \mathcal{N}_R \rangle \tag{4.17a}$$

as well as the dimension-5 baryon number violating operators

$$Q_L Q_L Q_L L_L \to QQQL \tag{4.17b}$$

$$Q_R Q_R Q_R L_R \to \begin{cases} u^c d^c d^c \nu^c \\ u^c d^c d^c e^c \end{cases} \tag{4.17c}$$

are forbidden by $U(1)_{Z'}$. The lepton number violating operators that arise from

$$Q_L Q_R L_L \mathcal{H}_R \to QLd^c \langle \mathcal{N}_R \rangle \tag{4.18}$$

$$L_L L_L L_R \mathcal{H}_R \to LLe^c \langle \mathcal{N}_R \rangle \tag{4.19}$$

are also forbidden for the model of (4.9). For the other model of (4.7), the lepton number violating operators are allowed. Hence, the PDMOs are suppressed by $M_{Z'}/M_S$, which yields adequate suppression provided that the $U(1)_{Z'}$ breaking scale is sufficiently low as discussed in [9, 10].

Field	$SU(3)_C$	$\times SU(2)_L$	$\times SU(2)_R$	$U(1)_C$	$U(1)_\zeta$	β_3	β_{2L}	β_Y
Q_L^i	3	2	1	$+\frac{1}{2}$	$-\frac{1}{2}$	1	$\frac{3}{2}$	$\frac{1}{6}$
Q_R^i	$\overline{3}$	1	2	$-\frac{1}{2}$	$+\frac{1}{2}$	1	0	$\frac{5}{3}$
L_L^i	1	2	1	$-\frac{3}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$
L_R^i	1	1	2	$+\frac{3}{2}$	$+\frac{1}{2}$	0	0	1
H_0	1	2	2	0	0	0	1	1
H_L^{ij}	1	2	1	$+\frac{3}{2}$	$+\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$
$H_L^{\prime\, ij}$	1	2	1	$-\frac{3}{2}$	$+\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$
H_R^{ij}	1	1	2	$-\frac{3}{2}$	$-\frac{1}{2}$	0	0	1
$H_R^{\prime\, ij}$	1	1	2	$+\frac{3}{2}$	$-\frac{1}{2}$	0	0	1
D^n	3	1	1	$+1$	0	$\frac{1}{2}$	0	$\frac{1}{3}$
$\overline{D}^n$	$\overline{3}$	1	1	-1	0	$\frac{1}{2}$	0	$\frac{1}{3}$
$\mathcal{H}_R$	1	1	2	$+\frac{3}{2}$	$-\frac{1}{2}$	0	$\frac{3}{5}$	1
$\overline{\mathcal{H}}_R$	1	1	2	$-\frac{3}{2}$	$+\frac{1}{2}$	0	$\frac{3}{5}$	1
S^i	1	1	1	0	-1	0	0	0
$\overline{S}^i$	1	1	1	0	$+1$	0	0	0
ϕ^a	1	1	1	0	0	0	0	0

Table 4.2: High scale spectrum of Model I and $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_C \times U(1)_\zeta$ quantum numbers, with $i = 1, 2, 3$ for the three light generations, $j = 1, 2$ for the number of doublets required by anomaly cancellation, $n = 1, ..., k$, and $a = 1, ..., p$. The β_i show the contributions for each state, relevant for the RGE analysis later.

Field	$SU(3)_C$	$\times SU(2)_L$	T_{3R}	$U(1)_Y$	$U(1)_{Z'}$	β_3	β_{2L}	β_Y
Q_L^i	3	2	0	$+\frac{1}{6}$	$-\frac{2}{5}$	1	$\frac{3}{2}$	$\frac{1}{6}$
$u_L^{c\,i}$	$\overline{3}$	1	$-\frac{1}{2}$	$-\frac{2}{3}$	$+\frac{3}{5}$	$\frac{1}{2}$	0	$\frac{4}{3}$
$d_L^{c\,i}$	$\overline{3}$	1	$+\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{1}{5}$	$\frac{1}{2}$	0	$\frac{1}{3}$
L_L^i	1	2	0	$-\frac{1}{2}$	$-\frac{4}{5}$	0	$\frac{1}{2}$	$\frac{1}{2}$
$e_L^{c\,i}$	1	1	$-\frac{1}{2}$	$+1$	$+\frac{3}{5}$	0	0	1
$\nu_L^{c\,i}$	1	1	$+\frac{1}{2}$	0	$+1$	0	0	0
H^u	1	2	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{5}$	0	$\frac{1}{2}$	$\frac{1}{2}$
H^d	1	2	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{5}$	0	$\frac{1}{2}$	$\frac{1}{2}$
H_L^i	1	2	0	$+\frac{1}{2}$	$+\frac{4}{5}$	0	$\frac{3}{2}$	$\frac{3}{2}$
$H_L'^{\,i}$	1	2	0	$-\frac{1}{2}$	$+\frac{1}{5}$	0	$\frac{3}{2}$	$\frac{3}{2}$
E_R^i	1	1	$-\frac{1}{2}$	-1	$-\frac{3}{5}$	0	0	1
N_R^i	1	1	$+\frac{1}{2}$	0	-1	0	0	0
$E_R'^{\,i}$	1	1	$+\frac{1}{2}$	$+1$	$-\frac{2}{5}$	0	0	1
$N_R'^{\,i}$	1	1	$-\frac{1}{2}$	0	0	0	0	0
D^n	3	1	0	$+\frac{1}{3}$	$+\frac{1}{5}$	$\frac{1}{2}$	0	$\frac{1}{3}$
$\overline{D}^n$	$\overline{3}$	1	0	$-\frac{1}{3}$	$-\frac{1}{5}$	$\frac{1}{2}$	0	$\frac{1}{3}$
S^i	1	1	0	0	-1	0	0	0
$\overline{S}^i$	1	1	0	0	$+1$	0	0	0
ϕ^a	1	1	0	0	0	0	0	0

Table 4.3: Low scale matter spectrum of Model I and $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Z'}$ quantum numbers with β_i contributions.

Chapter 5

Phenomenological Analysis

In the previous chapter we discussed the construction of a string–inspired left–right symmetric model that accommodated an additional $U(1)$ forbidding proton decay mediating operators yet allowing for light neutrinos via an extended seesaw mechanism. It is expected that, in a full string–derived model, the gauge and matter structure is fully defined, along with intermediate symmetry breaking and matter scales. In our string–inspired approach, we may use limits on experimental data to apply stringent constraints on these intermediate scales. Here we will discuss the phenomenological implications that arise, focussing on the constraints brought about due to proton stability and gauge coupling unification.

We first analyze restrictions on intermediate gauge symmetry breaking scales coming from the suppression of proton decay mediating operators. These are forbidden up to dimension-6 but are induced once $U(1)_{Z'}$ is broken. We discuss the relevant couplings in the superpotential, from where these PDMOs originate, and look at what limits this analysis provides for our intermediate gauge breaking scales, M_R and $M_{Z'}$.

For the gauge coupling unification analysis, we discuss two classes of models mentioned previously: $SO(10) \times U(1)_\zeta \not\subset E_6$ and $SO(10) \times U(1)_\zeta \subset E_6$, specifying a toy–model with $U(1)_\zeta$ embedded in E_6. We look at how their respective spectra affect the renormalization group equations. These are run for the SM gauge couplings and we focus on how each class may accommodate the low–energy limits of $\alpha_3(M_Z)$ and $\sin^2\theta_W(M_Z)$ in a comparative analysis.

5.1 Proton Stability Constraints

Here we analyze constraints coming from the suppression of PDMOs. As we saw in Chapter 4, the $U(1)_\zeta$ combination forbids PDMOs up to dimension-6, due to the charge assignments of $SU(2)_L$ and $SU(2)_R$ doublets having Q_ζ of opposite sign. Once the low scale combination $U(1)_{Z'}$, a linear combination of $U(1)_\zeta, U(1)_{B-L}$ and T_{3_R}, is broken, operators that cause rapid proton decay will be induced. Thus, we may constrain the $U(1)_{Z'}$ breaking scale by requiring sufficient suppression of PDMOs. Depending on which high scale superpotential operators the PDMOs originate from, requiring sufficient suppression may also constrain the $SU(2)_R$ breaking scale.

The strongest constraints will come from dimension-4 and -5 operators. It is expected that dimension-6 operators will be sufficiently suppressed, once limits on lower dimensional operators are satisfied. Dimension-5 PDMOs originate from the term

$$\lambda_{\text{III}} QQQL, \tag{5.1a}$$

which in the SM is the dimension-6 PDMO that is not forbidden. Introducing SUSY allows a dimension-5 PDMO to originate from it, thus carrying a $\frac{1}{M_S}$ suppression. In our construction, this model is gauge invariant when coupled to two $U(1)_{Z'}$ breaking singlets,

QQQL$\overline{SS}$

The dimension-4 PDMOs are

$$\lambda'_{\rm I} u^c_L d^c_L d^c_L, \tag{5.2a}$$

$$\lambda'_{\rm II} Q_L L_L d^c_L, \tag{5.2b}$$

and, in LRS models with additional $U(1)$s, are induced from nonrenormalizable terms,

$$\lambda_{\rm I} u^c_L d^c_L d^c_L \langle \mathcal{N}_R \rangle \Phi_1, \tag{5.3a}$$

$$\lambda_{\rm II} Q_L L_L d^c_L \langle \mathcal{N}_R \rangle \Phi_2, \tag{5.3b}$$

where Φ_1 and Φ_2 are generic combinations of $U(1)_{Z'}$ breaking singlets ensuring these terms are gauge invariant and satisfy the string selection rules. $\langle \mathcal{N}_R \rangle$ is the vev that breaks $SU(2)_R$ at M_R. In general, the vevs of both sets of intermediate symmetry breaking states, $\left(\mathcal{N}_R + \overline{\mathcal{N}}_R\right)$ and $\left(S + \overline{S}\right)$ determine the suppression of these operators via

$$\lambda'_{\rm I} \sim \frac{\langle \mathcal{N}_R \rangle}{M_S} \left(\frac{\langle \Phi \rangle}{M_S} \right)^n, \tag{5.4a}$$

$$\lambda'_{\rm II} \sim \frac{\langle \mathcal{N}_R \rangle}{M_S} \left(\frac{\langle \Phi' \rangle}{M_S} \right)^{n'}. \tag{5.4b}$$

In our model, the dimension-4 PDMOs originate in the higher dimensional operators

$$\lambda_{\rm I} u_L^c d_L^c d_L^c \langle \mathcal{N}_R \rangle S, \tag{5.5a}$$

$$\lambda_{\rm II} Q_L L_L d_L^c \langle \mathcal{N}_R \rangle \overline{S}. \tag{5.5b}$$

Thus, in our model, this equates to $n = n' = 1$, *i.e.* these operators are induced at quintic order. The suppression of the dimension-5 operator, in (5.1), is

$$\frac{\lambda'_{\rm III}}{M_S} \sim \left(\frac{\langle \overline{S} \rangle}{M_S} \right)^2 . \tag{5.6}$$

Experimental limits [3] impose constraints on the product, $\langle \lambda'_{\rm I} \cdot \lambda'_{\rm II} \rangle$ such that

$$\langle \lambda'_{\rm I} \cdot \lambda'_{\rm II} \rangle \lesssim 10^{-29}, \tag{5.7}$$

where we have assumed the

$$p \to e^+ \pi^0$$

channel dominates. We have also used the current bound [33],

$$m_{\tilde{d}^c} \gtrsim 1.11\,\text{TeV},$$

assuming SUSY masses are degenerate at $M_{\rm SUSY}$. Given these assumptions we also find that

$$\frac{\lambda_{\rm III}}{M_S} \lesssim 10^{-25},$$

which indicates that

$$\lambda_{\rm III} \lesssim 7 \cdot 10^{-8}, \tag{5.8}$$

with the heterotic string scale taken to be $\sim 5 \cdot 10^{17}$GeV. As both our M_R and $M_{Z'}$ scales are not constrained by other phenomenological data (constraints from GCU are explored in the next section), we may treat them as free parameters here. Taking $SU(2)_R$ to be broken at the string scale, results in

$$M_{Z'} \sim M_S \left(\frac{1}{\tau_p} \frac{M^4_{\text{SUSY}}}{m^5_p} \right)^{\frac{1}{2}} \sim 1200\,\text{GeV}, \tag{5.9}$$

which is just beyond the current limits on Z'_{LR} [33]. Allowing $\frac{M_R}{M_S} \lesssim 1$ would increase $M_{Z'}$, should future experiments raise the limits. This value of $M_{Z'}$ sufficiently suppresses the dimension-5 coupling which indicates an upper limit to be

$$M_{Z'} \lesssim 2 \cdot 10^5\,\text{GeV}, \tag{5.10}$$

should proton decay be detected soon. Constraints from the unification analysis are expected to more accurately determine the behaviour of these intermediate scales. It is interesting to note that as the lifetime of the proton is pushed higher, the mass limit on a proton–protecting Z' decreases. This indicates that current limits on proton lifetime experiments are close to our theoretical prediction, should a Z' be discovered at LHC energies. Further analysis of the couplings of such a Z' would be required in order to identify any distinguishing signatures that could be exhibited. This is beyond the scope of this current work and is left for future exploration.

5.2 Gauge Coupling Analysis

We now turn to our analysis of gauge coupling unification. Here we discuss specific examples of our $U(1)_\zeta$ external to $SO(10)$ and how their respective spectra affect the unification of the SM gauge couplings: α_3, α_{2L} and α_Y. We run the RGEs from the high scale to the low scale via intermediate gauge symmetries breaking and matter states decoupling, in order to check agreement with experimental data.

Model I

This model is an example of a three generation, free fermionic model that yields an unbroken, anomaly free $U(1)$ symmetry. Heterotic string models with this property break the $SO(10)$ symmetry to the left–right symmetric subgroup [51] and are therefore supersymmetric and completely free of gauge and gravitational anomalies. The $U(1)_\zeta$ symmetry in the string models is an anomaly free, family universal symmetry that forbids the dimension-4, -5 and -6 PDMOs, while allowing for the SM fermion mass terms. A combination of $U(1)_\zeta$, $U(1)_{B-L}$ and $U(1)_{T_{3_R}}$, the $U(1)_{Z'}$ remains unbroken down to low energies and forbids baryon number violation while allowing for lepton number violation. Hence, it allows for the generation of small left–handed neutrino masses via a seesaw mechanism, specifically an extended seesaw with the singlets, S and $\overline{S}$ [73]. Proton decay mediating operators are only generated when the $U(1)_{Z'}$ is broken. Thus, the scale of the $U(1)_{Z'}$ breaking, $M_{Z'}$, is constrained by proton lifetime limits and can be within reach of the contemporary experiments. The model is summarised by the spectra in Table 4.2, above the intermediate $SU(2)_R$ breaking scale M_R, and Table 4.3, below M_R, and the superpotentials of (4.11) and (4.12); above and below M_R respectively. As seen previously, the combination forming our proton-protecting $U(1)_\zeta$, in (2.31), does not admit an E_6 embedding, *i.e.* our matter cannot originate from a single **27** of E_6. Rather than accommodate an E_6 charge assignment, we have taken advantage of the intermediate LRS gauge structure, with $SU(2)_L$ and $SU(2)_R$ doublets acquiring $U(1)_\zeta$ charges of opposite sign. However, as we saw in the previous chapter, the effective $U(1)_\zeta$ became anomalous requiring additional anomaly cancelling doublets. The large β–function contribution of the ACDs increases the likelihood of approaching a Landau pole before the unification scale. Extra triplets are also included in an attempt to negate any adverse gauge coupling unification effects of the doublets *i.e.* the triplets serve to slow down the running of α_3 such that unification is reached before the Landau pole. As our $U(1) \not\subset E_6$, the number of triplets is not restricted by the number of complete representations. This will be investigated further when we discuss the GCU analysis of this model.

Model II

The second class of models preserves the E_6 embedding of the $U(1)_\zeta$ and is akin to Z' models arising in string–inspired E_6 models [38, 39, 104–106]. For these models, the spectrum consists of three generations of **27** representations that decompose under $SO(10) \times U(1)_\zeta$ as:

$$\mathbf{27}^i \rightarrow \mathbf{16}^i_{\frac{1}{2}} + \mathbf{10}^i_{-1} + \mathbf{1}^i_{2} \tag{5.11}$$

where $i = 1, 2, 3$. Under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_C \times U(1)_\zeta$, this results in a similar spectrum to the LRS model. The main difference between the spectra of these models is that Model II includes full **27** representations of E_6 where Model I does not.

The states of the MSSM and the right–handed neutrino lie in the **16** of $SO(10)$ and so decomposes exactly as for Model I,

$$\begin{aligned} Q^i_L &\sim \left(\mathbf{3}, \mathbf{2}, \mathbf{1}, +\frac{1}{2}, +\frac{1}{2}\right); \qquad & L^i_L &\sim \left(\mathbf{1}, \mathbf{2}, \mathbf{1}, -\frac{3}{2}, +\frac{1}{2}\right); \\ Q^i_R &\sim \left(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{2}, -\frac{1}{2}, +\frac{1}{2}\right); \qquad & L^i_R &\sim \left(\mathbf{1}, \mathbf{1}, \mathbf{2}, +\frac{3}{2}, +\frac{1}{2}\right), \end{aligned} \tag{5.12a}$$

with the proviso that the charges under $U(1)_\zeta$ all take the same sign. The **10** decomposes as

$$H^i \sim (\mathbf{1}, \mathbf{2}, \mathbf{2}, 0, -1)\,; \qquad D^i \sim (\mathbf{3}, \mathbf{1}, \mathbf{1}, +1, -1)\,; \quad \overline{D}^i \sim \left(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1}, -1, -1\right). \tag{5.12b}$$

The remaining singlets are neutral under the SM gauge group and are used to break the $U(1)_{Z'}$. In addition to the complete $SO(10)$ representations above, the E_6 spectrum includes a bidoublet,

$$H_0 \sim (\mathbf{1}, \mathbf{2}, \mathbf{2}, 0, -1)\,, \tag{5.12c}$$

that facilitates gauge coupling unification. Upon breaking of the intermediate gauge symmetry, this bidoublet will form the electroweak Higgs doublets. The model also contains the pair of heavy Higgs right–handed doublets,

$$\mathcal{H}_R + \overline{\mathcal{H}}_R = \left(\mathbf{1}, \mathbf{1}, \mathbf{2}, \frac{3}{2}, \frac{1}{2}\right) + \left(\mathbf{1}, \mathbf{1}, \mathbf{2}, -\frac{3}{2}, -\frac{1}{2}\right). \tag{5.12d}$$

These would be used to break the $SU(2)_R$ symmetry as outlined in Model I. Symmetry breaking mechanisms and a full spectrum analysis of this model, including construction of the superpotential, are beyond the scope of this demonstrative analysis.

Intermediate Gauge and Matter Scales

To conduct a direct comparison of the two model classes, we require the intermediate gauge structure of both models to be equivalent and so it is instructive to detail the symmetry breaking patterns at this stage. The SM gauge group will be embedded in $SO(10)$. As previously mentioned, this is broken to the LRS gauge group via the addition of basis vectors, $\alpha, \beta,$ and γ at the string scale, M_S. The $SU(2)_R$ is then broken at some intermediate scale, M_R. Suppression of PDMOs requires that this be around the string scale for phenomenologically accessible values of $M_{Z'}$. In keeping the scale of M_R high, $M_{Z'}$ is required to be sufficiently low for adequate suppression of dimension-4, -5 and -6 operators that induce rapid proton decay.

We contrast the analysis in the LRS heterotic string models with the models that admit the E_6 embedding of the $U(1)_{Z'}$ charges. In both models there are six intermediate gauge symmetry and matter scales between M_S and M_Z, corresponding to:

M_R: *$SU(2)_R$ breaking scale.* In Model I, the neutral components of $\mathcal{H}_R + \overline{\mathcal{H}}_R$ acquire a vev to break the $SU(2)_R$ symmetry and leave the hypercharge and the orthogonal combination, $U(1)_{Z'}$, unbroken. Thus, the pair of heavy Higgs', $\left(\mathcal{H}_R + \overline{\mathcal{H}}_R\right)$, and $SU(2)_R$ gauge bosons, W_R, decouple from the spectrum at this scale. This is assumed to be the case in Model II also, although the symmetry breaking mechanism has not been analyzed for this toy–model.

M_D: *Colour triplet scale.* The additional colour triplets in Model I acquire a mass at this scale. The presence of this scale may also help resolve the discrepancy between the MSSM unification scale and string–scale unification [30]. For Model II, due to the embedding of the bidoublets and triplets in a fundamental **10** representation of $SO(10)$, this scale is equivalent to $M_{Z'}$. This is because the triplets and doublets acquire their mass from the $U(1)_{Z'}$ breaking singlets. For Model I, this is not necessarily the case.

$M_{Z'}$: *$U(1)_{Z'}$ breaking scale.* The $U(1)_{Z'}$ is broken at this scale by SM gauge singlets acquiring vevs. The anomaly cancelling doublets also acquire mass at this scale in Model I and, in both models, only the MSSM spectrum survives to lower scales. From the analysis of our PDMOs, this should be within an accessible energy range, with our previous results indicating $M_{Z'} \sim \mathcal{O}\,(\text{TeV})$.

M_{SUSY}: *SUSY breaking scale.* The current bounds from the LHC will be included here to allow for a phenomenologically viable supersymmetry scale. We take the

SUSY masses to be degenerate and only the SM states remain down to the lower scales. The additional Higgs doublet of the MSSM is also decoupled at this scale. Further analysis of the SUSY spectrum is left to future work.

M_t, M_h: *Top quark and Higgs boson mass scale.* At M_t, the top quark is decoupled and at M_h, the remaining Higgs doublet is also decoupled, leaving the remaining spectrum to run down to the Z–scale; at which the gauge data is extracted.

By starting from the string scale and evolving the couplings down to M_Z, our analysis may test whether the predictions of these models are in accordance with low–energy experimental data.

Low–energy inputs

For our analysis, we take the following values for the masses and couplings [33]:

$$\begin{aligned} M_Z &= 91.1876 \pm 0.0021 \text{ GeV} & \sin^2\theta_W(M_Z)\big|_{\overline{\text{MS}}} &= 0.23116 \pm 0.00012 \\ \alpha^{-1} \equiv \alpha^{-1}_{\text{EM}}(M_Z) &= 127.944 \pm 0.014 & \alpha_3(M_Z) &= 0.1184 \pm 0.0007. \end{aligned} \tag{5.13}$$

We also include the top quark mass of $M_t \sim 173.5$ GeV [33] and the Higgs boson mass of $M_h \sim 125$ GeV [1, 2] in our analysis but these are not expected to have a large effect.

5.2.1 Renormalization Group Equations

For the RGE running of both models, we follow the methods outlined in [30]. String unification implies that the SM gauge couplings are unified at the heterotic string scale, M_S. As discussed previously, taking into consideration nonperturbative effects and intermediate scale threshold contributions, one may resolve the discrepancy between the MSSM GUT scale, M_{GUT}, and the heterotic string scale, M_S. The one–loop RGEs for the couplings are given by

$$\frac{4\pi}{\alpha_i(\mu)} = k_i \frac{4\pi}{\alpha_{\text{string}}} + \beta_i \log \frac{M^2_{\text{string}}}{\mu^2} + \Delta_i^{(\text{total})}, \tag{5.14}$$

where β_i are the one–loop β–function coefficients, and $\Delta_i^{(\text{total})}$ represents possible corrections from the additional gauge or matter states *i.e.* in each model, we initially assume the MSSM spectrum between the string scale, M_S, and the Z–scale, M_Z, and treat all perturbations as effective correction terms. Explicitly, in our case, the gauge coupling RGEs are,

$$\begin{aligned}\frac{1}{\alpha_i(M_S)} =& \frac{1}{\alpha_i(M_Z)} - \frac{b_i^{\text{MSSM}}}{2\pi} \log \frac{M_S}{M_Z} - \frac{1}{2\pi} \sum_j b_{ij}^{\text{IM}} \log \frac{M_S}{M_j} + \frac{b_i^{\text{SP}}}{2\pi} \log \frac{M_{\text{SUSY}}}{M_Z} \\ &+ \frac{b_i^{\text{t}}}{2\pi} \log \frac{M_t}{M_Z} + \frac{b_i^{\text{h}}}{2\pi} \log \frac{M_h}{M_Z},\end{aligned} \tag{5.15}$$

where $i = Y, 2L$ and 3, and j represents the intermediate scales M_R, M_D and $M_{Z'}$. At the string unification scale we have

$$\alpha_S \equiv \alpha_3(M_S) = \alpha_2(M_S) = k_1 \alpha_Y(M_S), \tag{5.16}$$

where $k_1 = 5/3$ is the canonical $SO(10)$ normalization in both cases due to our hypercharge embedding discussed previously. Using the relations

$$\alpha_2(M_Z) = \frac{\alpha(M_Z)}{\sin^2\theta_W(M_Z)}; \qquad \alpha_Y(M_Z) = \frac{\alpha(M_Z)}{1 - \sin^2\theta_W(M_Z)}, \tag{5.17}$$

where $\alpha(M_Z) \equiv \alpha_{\text{EM}}(M_Z)$, and the unification condition, (5.16), we may obtain expressions for $\sin^2\theta_W(M_Z)$ and $\alpha_3(M_Z)$ [30],

$$\begin{aligned}\sin^2\theta_W(M_Z)\big|_{\overline{MS}} =& \Delta^{\sin^2\theta_W}_{\text{MSSM}} + \Delta^{\sin^2\theta_W}_{\text{IM}} + \Delta^{\sin^2\theta_W}_{\text{SP}} + \Delta^{\sin^2\theta_W}_{\text{IG}} \\ &+ \Delta^{\sin^2\theta_W}_{\text{Top}} + \Delta^{\sin^2\theta_W}_{\text{Higgs}} + \Delta^{\sin^2\theta_W}_{\text{TC}}\end{aligned} \tag{5.18}$$

and

$$\alpha_3(M_Z)|_{\overline{MS}} = \Delta^{\alpha_3}_{\text{MSSM}} + \Delta^{\alpha_3}_{\text{IM}} + \Delta^{\alpha_3}_{\text{SP}} + \Delta^{\alpha_3}_{\text{IG}} + \Delta^{\alpha_3}_{\text{Top}} + \Delta^{\alpha_3}_{\text{Higgs}} + \Delta^{\alpha_3}_{\text{TC}}. \tag{5.19}$$

As mentioned, Δ_{MSSM} represents the one–loop contributions from the spectrum of the MSSM between the unification scale and the Z–scale. The following five Δ terms correspond to corrections from the intermediate matter thresholds, the light SUSY partner thresholds, the intermediate vector bosons corresponding to the $SU(2)_R$ symmetry breaking, the top quark contribution and the electroweak Higgs contribution respectively. The last term,

$$\Delta^{\sin^2\theta_W}_{\text{TC}} = \Delta^{\sin^2\theta_W}_{\text{HS}} + \Delta^{\sin^2\theta_W}_{\text{Yuk.}} + \Delta^{\sin^2\theta_W}_{\text{2-loop}} + \Delta^{\sin^2\theta_W}_{\text{Conv.}}, \tag{5.20}$$

includes the corrections due to heavy string thresholds, those arising from Yukawa couplings, two–loops and scheme conversion. These corrections were shown to be small for certain classes of heterotic string models constructed within the free fermionic framework [30] and are neglected for this demonstrative analysis.

Rearranging the RGEs for $\sin^2\theta_W(M_Z)$, we obtain

$$\Delta^{\sin^2\theta_W}_{\text{MSSM}} = \frac{1}{1+k_1}\left[1 + \frac{\alpha}{2\pi}\left(k_1 b_2^{\text{MSSM}} - b_Y^{\text{MSSM}}\right)\log\frac{M_S}{M_Z}\right]; \tag{5.21a}$$

$$\Delta^{\sin^2\theta_W}_{\text{IM+IG}} = \frac{1}{2\pi}\frac{k_1\alpha}{(1+k_1)}\sum_j\left(b_{2j}^{\text{IM+IG}} - b_{1j}^{\text{IM+IG}}\right)\log\frac{M_S}{M_j}; \tag{5.21b}$$

$$\Delta^{\sin^2\theta_W}_{\text{SP}} = -\frac{1}{2\pi}\frac{k_1\alpha}{(1+k_1)}\left(b_1^{\text{SP}} - b_2^{\text{SP}}\right)\log\frac{M_{\text{SUSY}}}{M_Z}, \tag{5.21c}$$

where $\alpha = \alpha_{\text{EM}}(M_Z)$, M_j are the intermediate gauge and matter scales discussed earlier and we have used $b_1 \equiv \frac{b_Y}{k_1}$. Similarly for $\alpha_3(M_Z)$, we have:

$$\Delta^{\alpha_3}_{\text{MSSM}} = \frac{1}{1+k_1}\left\{\frac{1}{\alpha} + \frac{1}{2\pi}\left[(1+k_1)\, b_3^{\text{MSSM}} - (b_2^{\text{MSSM}} + k_1 b_1^{\text{MSSM}})\right]\log\frac{M_S}{M_Z}\right\}; \quad (5.22a)$$

$$\Delta^{\alpha_3}_{\text{IM+IG}} = \frac{1}{2\pi}\frac{1}{(1+k_1)}\sum_j\left[(1+k_1)\, b_{3j}^{\text{IM+IG}} - \left(b_{2j}^{\text{IM+IG}} + k_1 b_{1j}^{\text{IM+IG}}\right)\right]\log\frac{M_S}{M_j}; \quad (5.22b)$$

$$\Delta^{\alpha_3}_{\text{SP}} = -\frac{1}{2\pi}\frac{1}{(1+k_1)}\left[(1+k_1)\, b_3^{\text{SP}} - (b_2^{\text{SP}} + k_1 b_1^{\text{SP}})\right]\log\frac{M_{\text{SUSY}}}{M_Z}. \quad (5.22c)$$

Δ_{Higgs} and Δ_{Top} take the same form as Δ_{SP} for both $\alpha_3(M_Z)$ and $\sin^2\theta_W(M_Z)$.

Numerical Analysis – Model I

The β–functions for Model I correspond to,

$$b_i^{\text{MSSM}} = \begin{pmatrix} 11 \\ 1 \\ -3 \end{pmatrix}; \quad (5.23a)$$

$$b_{ij}^{\text{IM+IG}} = \begin{pmatrix} -4 & \frac{2}{3}n_D & 18 \\ 0 & 0 & 6 \\ 0 & n_D & 0 \end{pmatrix}; \quad (5.23b)$$

$$b_i^{\text{SP}} = \begin{pmatrix} \frac{25}{6} \\ \frac{25}{6} \\ 4 \end{pmatrix}; \qquad b_i^{\text{t}} = \begin{pmatrix} \frac{17}{18} \\ \frac{1}{2} \\ \frac{2}{3} \end{pmatrix}; \qquad b_i^{\text{h}} = \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \\ 0 \end{pmatrix}. \quad (5.23c)$$

where $i = Y, 2L$ and $3C$ and $j = M_R, M_D$ and $M_{Z'}$. Both (5.23a) and (5.23c) are generic for models based on the MSSM. The matrix, (5.23b), is specific to our model but its form is generic for models that have the same symmetry breaking pattern. For calculating the one–loop β–functions below M_{SUSY} we used [107]

$$b_i = \frac{g^3}{16\pi^2}\left[-\frac{11}{3}t_2(V) + \frac{2}{3}t_2(F) + \frac{1}{3}t_2(S)\right] \quad (5.24)$$

where for $SU(N), t_2(V) = N$ and $t_2(F) = t_2(S) = \frac{1}{2}$. Incorporating supersymmetry sets $t_2(F) = t_2(S)$ for scalar multiplets and $t_2(V) = t_2(F)$ for vector multiplets. Thus,

$$b_i^{\text{SUSY}} = \frac{g^3}{16\pi^2}\left[-3t_2(V) + t_2(S)\right]. \tag{5.25}$$

The β–function contributions for Model I are shown in Tables 4.2 and 4.3, and are also summarised in (5.23). The numerical output of equations (5.21) and (5.22) is generated subject to the variation of the scales. The hierarchy of the scales was constrained to be

$$M_S \gtrsim M_R > M_D \gtrsim M_{Z'} \gtrsim M_{SUSY} > M_Z. \tag{5.26}$$

The relevant mass ranges for the intermediate scales were scanned to find phenomenologically viable areas of the parameter space. When first running the RGEs, we restricted the allowed range of $\sin^2\theta_W(M_Z)$ and $\alpha_3(M_Z)$ to 5σ from the central values shown in (5.13). The RGEs were run in Mathematica. Restricting the output to the experimentally constrained interval produced no phenomenologically viable results. Allowing the values of $\sin^2\theta_W(M_Z)$ and $\alpha_3(M_Z)$ to run freely and restricting the relevant mass scales to (in GeV)

$$\begin{aligned} 2\cdot 10^{16} \leq \ M_S \ &\leq 5\cdot 10^{17}; & 10^5 \leq \ M_D \ &\leq 10^{12}; \\ 10^9 \leq \ M_R \ &\leq 5\cdot 10^{17}; & 10^3 \leq \ M_{Z'}, M_{\text{SUSY}} \ &\leq 10^{10}, \end{aligned} \tag{5.27}$$

also produced no phenomenologically viable results, as shown in Figure 5.1 (see (5.13) for current experimental limits).

Numerical analysis – Model II

To further elucidate the constraints on the LRS heterotic string models arising from coupling unification, we contrast the outcome with the corresponding results

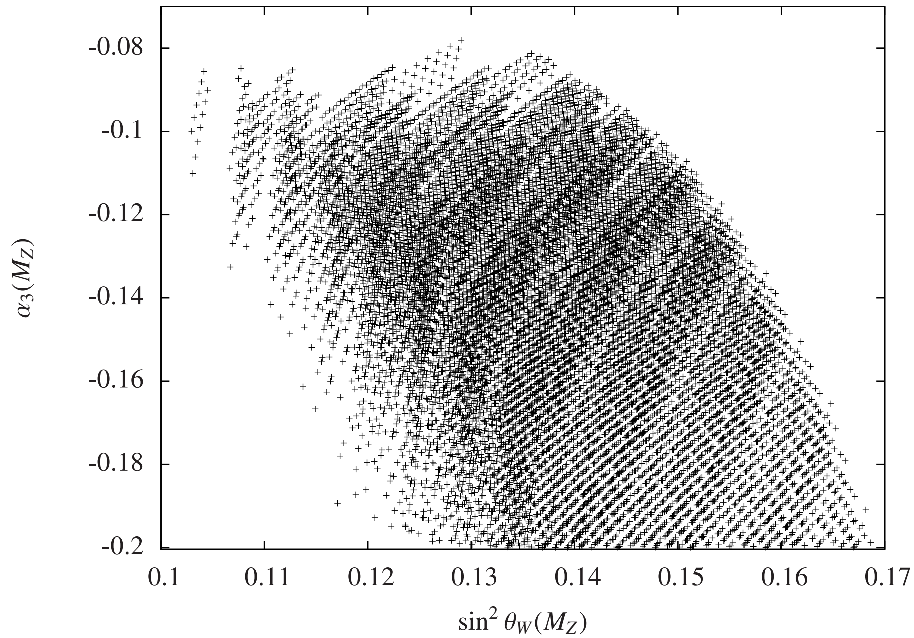

Figure 5.1: Freely running $\sin^2\theta_W(M_Z)$ and $\alpha_3(M_Z)$: $\sin^2\theta_W(M_Z)$ vs. $\alpha_3(M_Z)$ with $0 < \alpha_{\text{string}} \lesssim 0.1$.

when the $U(1)_\zeta$ charges are embedded in E_6 representations. We run the RGEs in exactly the same way as shown for the LRS model, restricting the mass scales to the hierarchy

$$M_S \gtrsim M_R \gtrsim M_D = M_{Z'} \gtrsim M_{SUSY} \gg M_Z. \tag{5.28}$$

As discussed earlier, $M_D \stackrel{!}{=} M_{Z'}$ due to the states originating in the **10** representation acquiring mass through couplings with the singlet that breaks $U(1)_{Z'}$. A string–derived model would afford more flexibility that we do not make use of in our analysis here.

In this case, the β–functions for $b_i^{\text{MSSM}}, b_i^{\text{SP}}, b_i^{\text{t}}$ and b_i^{h} are identical to (5.23a) and (5.23c) respectively, and so their contributions are identical to those in Model I. For

$b_{ij}^{\text{IM+IG}}$, we have

$$b_{ij}^{\text{IM+IG}} = \begin{pmatrix} -4 & 2 & 3 \\ 0 & 0 & 3 \\ 0 & 3 & 0 \end{pmatrix}. \tag{5.29}$$

At this stage, for comparative clarity, we keep separate the β–functions for the scales M_D and $M_{Z'}$. Reasons for doing so will be elaborated on later.

Running the RGEs for Model II, we find that unification does occur, as found in previous literature. We note that the phenomenologically viable results (see Figure 5.2) required $M_S \sim M_{\text{GUT}} \sim 2 \cdot 10^{16}$ GeV as expected. The other intermediate scales were found to be (in GeV)

$$10^{13} \leq M_R \leq 10^{16}; \qquad 10^3 \leq \; M_D \; \leq 10^8; \quad 10^3 \leq \; M_{\text{SUSY}} \; \leq 1 \cdot 10^6, \tag{5.30}$$

with $M_{Z'}$ between $1 - 10^5$ TeV. Fine–tuning the M_{SUSY} allows for $M_{Z'}$ to be in agreement with current experimental bounds.

Model I vs Model II

From our numerical analysis above, we found that, for unification to occur, the $U(1)_\zeta$ charges must have an E_6 embedding. We can, more precisely, elucidate the contrast between the two cases by examining the contributions of the intermediate gauge and matter thresholds, the light SUSY thresholds and the Higgs and top quark thresholds, to $\sin^2\theta_W(M_Z)$ and $\alpha_3(M_Z)$. If we take M_S to coincide with the MSSM unification scale and with M_R as well, then (5.21a) and (5.22a), which only contain the MSSM contributions, are in good agreement with the observable data, with

$$\Delta_{\text{MSSM}}^{\sin^2\theta_W} = 0.231 \qquad \text{and} \qquad \Delta_{\text{MSSM}}^{\alpha_3} = 0.117. \tag{5.31}$$

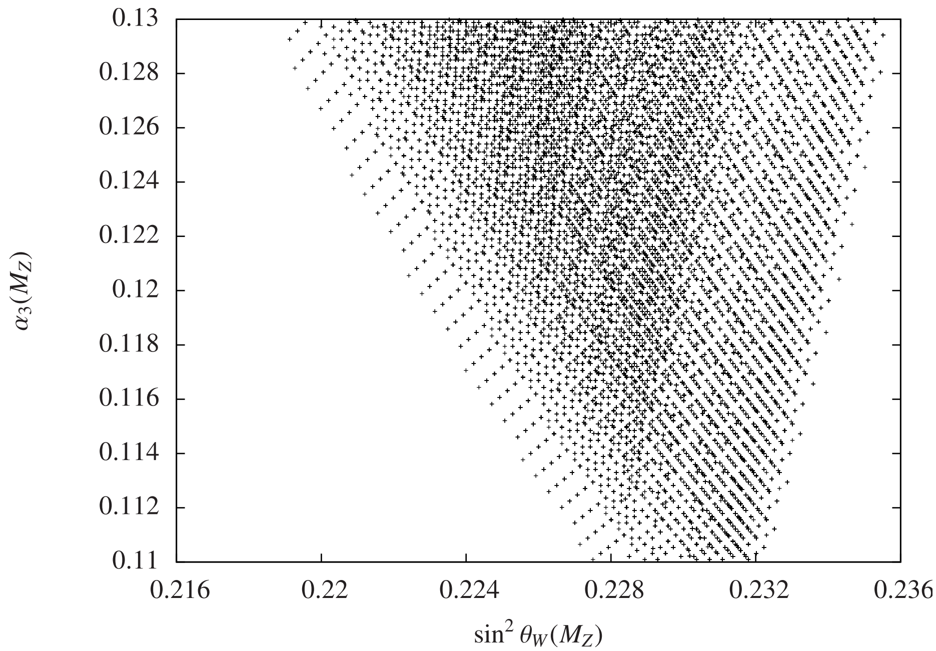

Figure 5.2: Freely running $\sin^2\theta_W(M_Z)$ and $\alpha_3(M_Z)$: $\sin^2\theta_W(M_Z)$ vs. $\alpha_3(M_Z)$ with $0 < \alpha_{\text{string}} \lesssim 0.1$ for Model II.

For both models, as M_t and M_h are fixed, the top and Higgs contributions equate to

$$\Delta^{\sin^2\theta_W}_{\text{Higgs}} = 1.63 \cdot 10^{-5}; \qquad \Delta^{\alpha_3}_{\text{Higgs}} = 6.28 \cdot 10^{-3}; \tag{5.32a}$$

$$\Delta^{\sin^2\theta_W}_{\text{Top}} = 3.33 \cdot 10^{-5}; \qquad \Delta^{\alpha_3}_{\text{Top}} = -1.71 \cdot 10^{-2}. \tag{5.32b}$$

The lower bound on M_{SUSY} is also equivalent for both models, as we have taken the superpartner masses to be degenerate at $M_{\text{SUSY}} \sim 1\,\text{TeV}$, and contributes as

$$\Delta^{\sin^2\theta_W}_{\text{SP}} = -3.10 \cdot 10^{-3}; \qquad \Delta^{\alpha_3}_{\text{SP}} = 4.73 \cdot 10^{-3}. \tag{5.32c}$$

As M_{SUSY} increases it is clear that the SUSY spectrum contributions will increase, however, these are not expected to be significant given the mass hierarchy of (5.26). Therefore, the corrections arising from the intermediate gauge and matter thresholds in (5.21b) and (5.22b) must cancel to reaffirm unification. Using the expressions in (5.21b) and (5.22b) we find that, in the case of the spectrum and charge assignments in the LRS heterotic string model, shown in Tables 4.2 and 4.3, the threshold corrections from intermediate gauge and matter scales are given by

$$\Delta^{\sin^2\theta_W}_{\mathrm{IM+IG}} = \frac{1}{2\pi}\frac{k_1\alpha}{1+k_1}\left(\frac{12}{5}\log\frac{M_S}{M_R} - \frac{2n_D}{5}\log\frac{M_S}{M_D} - \frac{24}{5}\log\frac{M_S}{M_{Z'}}\right), \tag{5.33a}$$

$$\Delta^{\alpha_3}_{\mathrm{IM+IG}} = \frac{1}{2\pi\,(k_1+1)}\left(4\log\frac{M_S}{M_R} + 2n_D\log\frac{M_S}{M_D} - 9\log\frac{M_S}{M_{Z'}}\right). \tag{5.33b}$$

In the case of models that admit an E_6 embedding of the $U(1)_\zeta$ charges, the same threshold corrections are given by

$$\Delta^{\sin^2\theta_W}_{\mathrm{IM+IG}} = \frac{1}{2\pi}\frac{k_1\alpha}{1+k_1}\left(\frac{12}{5}\log\frac{M_S}{M_R} - \frac{6}{5}\log\frac{M_S}{M_D} + \frac{6}{5}\log\frac{M_S}{M_H}\right), \tag{5.34a}$$

$$\Delta^{\alpha_3}_{\mathrm{IM+IG}} = \frac{1}{2\pi\,(k_1+1)}\left(4\log\frac{M_S}{M_R} + 6\log\frac{M_S}{M_D} - 6\log\frac{M_S}{M_H}\right). \tag{5.34b}$$

We see from (5.33a) that the corrections from the intermediate doublet and triplet thresholds contribute with equivalent sign. Treating the number of triplets, n_D as a free parameter allows us to solve the relations

$$\Delta^{\sin^2\theta_W}_{\mathrm{IM+IG}} = \Delta^{\alpha_3}_{\mathrm{IM+IG}} = 0. \tag{5.35}$$

However, we find that, due to the requirement that $M_R \gg M_D$, (5.35) cannot be satisfied. We therefore conclude that a low scale Z' in the LRS heterotic string models is incompatible with the gauge data at the Z–boson scale. In contrast, from (5.34) we see that the corresponding corrections cancel each other, provided that $M_H = M_{Z'} = M_D$. This is expected to be the case as both the triplet and doublet masses are generated by the Z' breaking vev. This cancellation is, of course, the well known cancellation that occurs when the representations fall into $SU(5)$ multiplets. Allowing $M_R \simeq \frac{M_S}{2}$, *i.e.* $M_R \sim 10^{16}$ GeV, then compensates for the SUSY threshold at $\sim 1\,\mathrm{TeV}$, enabling accommodation of the low–energy data, as illustrated in Figure 5.2.

Chapter 6

Accommodating $U(1)_\zeta \subset E_6$ in heterotic string models

Here we present the potential route one may take to build a model that incorporates the proton–protecting $U(1)_\zeta$ described above but with the E_6 embedding required for gauge coupling unification. As we saw in Section 2.3, at the level of the NAHE set, the visible GUT gauge group is $SO(10)$. We then went on to see the various breaking patterns following the further addition of basis vectors. However, we may also add basis vectors that enhance the gauge symmetry. For example, we saw previously, employing

$$\mathbf{x} \equiv \left\{ \overline{\psi}^{1,\dots,5} \overline{\eta}^{1,2,3} \right\}$$

resulted in gauge bosons, additional to the ones in the NS–sector, that enhance $SO(10) \times U(1)_\zeta \rightarrow E_6$. In this example we find that these bosons will enhance $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_C \times U(1)_\zeta$ to the Pati–Salam gauge group with an additonal $U(1)$. This $U(1)$ is, in fact, a rotation of our original $U(1)_\zeta$ and remains anomaly free due to the presence of all of the states in the **27** representation of E_6 in our low–energy effective theory. Here we discuss how viable heterotic string models with E_6 embedding of the $U(1)_{Z'}$ charges may be obtained, extending our initial discussion in [74].

6.1 Enhancement to E_6 in string models

From the analysis of Section 5.2, we found the $SO(10) \times U(1)_\zeta \not\subset E_6$ model, Model I, gave results inconsistent with gauge coupling constraints. On the other hand, with Model II, we found that the gauge coupling data seems to indicate the $U(1)_\zeta$ charges must admit an E_6 embedding. We emphasize that the indication is that the charges must admit an E_6 embedding and *not* that the E_6 symmetry is necessarily realised.

In our previous model, we found that the gauge bosons generating our visible group, $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_C \times U(1)_\zeta$, originated in the NS sector only. In general, the vector bosons that generate the visible four dimensional gauge group in the string models arise from two principal sectors: the untwisted sector and the sector $\mathbf{x} = \left\{\overline{\psi}^{1,\cdots,5}, \overline{\eta}^{1,2,3}\right\}$. We have not included the basis vector $\mathbf{x}$ in this analysis thus far. As mentioned earlier, in Section 2.4.3 we found that including the $\mathbf{x}$ vector at the level of the NAHE set, along with the relevant GGSO projection coefficients, enhanced $SO(10) \times U(1)_\zeta \rightarrow E_6$. This is due to the gauge bosons that transform in the $\mathbf{16}$ and $\overline{\mathbf{16}}$ of $SO(10)$.

Before adding the $\mathbf{b}_j$ vectors to complete the NAHE set, the gauge group is $SO(44)$, with $\mathcal{N} = 4$ spacetime SUSY, generated by the basis vectors

$$\mathbf{b} = \mathbb{1}, \mathbf{S}.$$

At this stage the gauge bosons originate only in the NS sector.

The complex right–moving worldsheet fermions that generate the Cartan subalgebra of the observable gauge group are all periodic in the $\mathbf{x}$–sector. Thus, adding

$\mathbf{x}$ at this stage, breaks

$$SO(44) \to SO(16) \times SO(28) \tag{6.1}$$

by projecting out some of the gauge bosons from the NS sector. However, in the $\mathbf{x}$–sector, further gauge bosons appear transforming in the spinorial representation of $SO(16)$. These combine with the adjoint of $SO(16)$ and form the adjoint representation of E_8, *i.e.*

$$\mathbf{120} + \mathbf{128} \to \mathbf{248},$$

just as we saw the ξ–sector enhance the hidden $SO(16)$ in Chapter 2. The gauge group at this stage now reads

$$E_8 \times SO(28). \tag{6.2}$$

The basis vector $\mathbf{S}$ generates $\mathcal{N} = 4$ spacetime SUSY which remains unbroken. The $\mathbf{b}_j$ vectors are then included, breaking the visible gauge group to

$$E_6 \times U(1)^2 \times SO(4)^3. \tag{6.3}$$

With the inclusion of the $\mathbf{b}_j$ vectors, SUSY is now broken to $\mathcal{N} = 1$. This gauge group, with $\mathcal{N} = 1$ spacetime SUSY, may also be generated by the set of basis vectors

$$\mathbf{b} = \{\mathbb{1}, \mathbf{S}, \mathbf{x}, \zeta, \mathbf{b}_1, \mathbf{b}_2\}\,. \tag{6.4}$$

At the level of the $E_8 \times E_8$ heterotic string in ten dimensions, the vector bosons of the observable E_8 are obtained from the untwisted sector and from the $\mathbf{x}$–sector. The set $\{\mathbb{1}, \mathbf{S}, \mathbf{x}, \xi\}$ produces a model with gauge symmetry, at a generic point in the compactified space, of either $E_8 \times E_8$ or $SO(16) \times SO(16)$ depending on the GGSO phase $c\begin{pmatrix} \mathbf{x} \\ \xi \end{pmatrix} = \pm 1$.

Adding the basis vectors $\mathbf{b}_1$ and $\mathbf{b}_2$ reduces the spacetime supersymmetry to $\mathcal{N} = 1$ just as before. The observable gauge symmetry then reduces from either $E_8 \to E_6 \times U(1)^2$ or $SO(16) \to SO(10) \times U(1)^3$. The additional vectors, $\{\alpha, \beta, \gamma\}$, reduce the gauge symmetry further. Aside from the model of [108], all the quasi–realistic free fermionic models follow the second symmetry breaking pattern. That is, in all these models, the vector bosons arising from the $\mathbf{x}$–sector are projected out.

6.2 Embedding $U(1)_\zeta$ in E_6

We consider, then, the SL symmetry breaking pattern induced by the following boundary condition assignments in two separate basis vectors

$$\alpha\left\{\overline{\psi}^{1\cdots5}\right\} = \left\{\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\right\} \Rightarrow SU(5) \times U(1), \tag{6.5a}$$

$$\beta\left\{\overline{\psi}^{1\cdots5}\right\} = \{1,\,1,\,1,\,0,\,0\} \Rightarrow SO(6) \times SO(4). \tag{6.5b}$$

The inclusion of (6.5) in two separate boundary condition basis vectors reduces the $SO(10)$ gauge symmetry to $SU(3)_C \times SU(2)_L \times U(1)_C \times U(1)_L$, where $2U(1)_C = 3U(1)_{B-L}$ and $U(1)_L = 2U(1)_{T_{3_R}}$ (this was explored in more detail in Chapter 2). For appropriate choices of the GGSO projection coefficients, the vector bosons arising from the $\mathbf{x}$–sector enhance the $SU(3) \times SU(2) \times U(1)^2 \times U(1)_\zeta$ arising from the untwisted sector to $SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_{\zeta'}$, where

$$U(1)_4 = U(1)_C + 3U(1)_L - 3U(1)_\zeta; \tag{6.6a}$$

$$U(1)_2 = U(1)_C + U(1)_L + U(1)_\zeta; \tag{6.6b}$$

$$U(1)_{\zeta'} = -3U(1)_C + 3U(1)_L + U(1)_\zeta. \tag{6.6c}$$

$U(1)_4$ and $U(1)_2$ are embedded in $SU(4)_C$ and $SU(2)_R$, respectively, and $U(1)_\zeta$ is given by (2.31). The matter representations charged under this group arise from the sectors $\mathbf{b}_j$ and are complemented by states from $\mathbf{b}_j$+$\mathbf{x}$ to form the ordinary representations of the Pati–Salam model. The difference, as compared to the Pati–Salam string models of [49, 50], is that $U(1)_{\zeta'}$ is anomaly free. The reason is that all the states of the **27** representation of E_6 are retained in the spectrum, whereas in the Pati–Salam models of [49, 50] the corresponding states are projected out. The symmetry breaking of the Pati–Salam $SU(4)_C \times SU(2)_R$ group is induced by the vev of the heavy Higgs in the $(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})_{-\frac{1}{2}} + (\mathbf{4}, \mathbf{1}, \mathbf{2})_{+\frac{1}{2}}$ representation of $SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_{\zeta'}$. In addition to the weak–hypercharge, this vev leaves the unbroken combination

$$U(1)_{Z'} = \frac{1}{2}U(1)_{B-L} - \frac{2}{3}U(1)_{T_{3_R}} + \frac{5}{3}U(1)_{\zeta'}, \tag{6.7}$$

which is anomaly free and admits the E_6 embedding of the charges. Due to the incorporation of $U(1)_\zeta$ in this model, it is expected that problematic PDMOs are forbidden, while the inclusion of the full **27** representation of E_6 allows for agreement with gauge coupling unification data. A full string–model, or a string–inspired approach along the lines of Chapter 4 should be conducted to examine these issues further.

Chapter 7

Conclusions

In this thesis we constructed an $SO(10)$ string–inspired Grand Unified model that took advantage of left–right symmetric charge assignments to accommodate an additional $U(1)$, external to the $SO(10)$ and not embedded in E_6, that forbids proton decay mediating operators up to dimension–6. We also presented the formation of a leptophobic $U(1)$, that also took advantage of the LRS breaking pattern of $SO(10)$. These models were constructed within the free fermionic formulation of the heterotic string, which allowed us to break $SO(10)$ at the string scale, M_S, and form $U(1)$ combinations free of gauge and gravitational anomalies.

Having constructed a viable $U(1)$ combination in the main bulk of this thesis, that, should it survive to sufficiently low scales, would adequately suppress proton decay mediating terms, we found, using the field content in our model, it was incompatible with low–scale gauge coupling constraints. However, our comparative analysis showed that embedding this $U(1)$ in E_6 allowed for gauge coupling unification at, or close to, $M_S \sim 2 \cdot 10^{16}\,\mathrm{GeV}$, as has been previously shown in the literature.

In this analysis, we assumed that an E_6 embedding of our $U(1)$ may be derived from the string and assigned the charges according to this prescription. In the last chapter we were able to show how such a model may be derived from the string and presented how our $U(1)$ combination may be accommodated within such an embedding.

Further exploration of the phenomenology of a model that could accommodate the $U(1)$ of Chapter 6 is required. Some work has been carried out previously in [105, 109] within the context of an orbifold E_6 model. Given the equivalence between free fermion models and orbifold constructions, the resulting phenomenology would have similar implications. However, should a full string–model be built, there may be unique signatures that could distinguish (6.7) from other additional abelian symmetries beyond the Standard Model.

Appendix A

$SO(10)$ weights and roots

The simple roots of $SO(10)$ for the fundamental weight basis are

$$\begin{aligned}
\alpha_1 &= (1,-1,0,0,0)\,;\\
\alpha_2 &= (0,1,-1,0,0)\,;\\
\alpha_3 &= (0,0,1,-1,0)\,; \qquad\qquad (A.1)\\
\alpha_4 &= (0,0,0,1,-1)\,;\\
\alpha_5 &= (0,0,0,1,1)\,.
\end{aligned}$$

In Figure A.1 we construct the **16** representation. This corresponds to the notation used in (2.38) where

$$\left[\begin{pmatrix} 5 \\ 0 \end{pmatrix}\right] = |{+\tfrac{1}{2}},+\tfrac{1}{2},+\tfrac{1}{2},+\tfrac{1}{2},+\tfrac{1}{2}\rangle_\alpha \qquad (A.2a)$$

$$\left[\begin{pmatrix} 5 \\ 2 \end{pmatrix}\right] = \overline{|{-\tfrac{1}{2}},-\tfrac{1}{2},+\tfrac{1}{2},+\tfrac{1}{2},+\tfrac{1}{2}\rangle}_\alpha \qquad (A.2b)$$

$$\left[\begin{pmatrix} 5 \\ 4 \end{pmatrix}\right] = \overline{|{-\tfrac{1}{2}},-\tfrac{1}{2},-\tfrac{1}{2},-\tfrac{1}{2},+\tfrac{1}{2}\rangle}_\alpha\,. \qquad (A.2c)$$

Here, $\overline{|\quad\rangle}_\alpha$ represents the permutations of the vacua charges in the sector α.

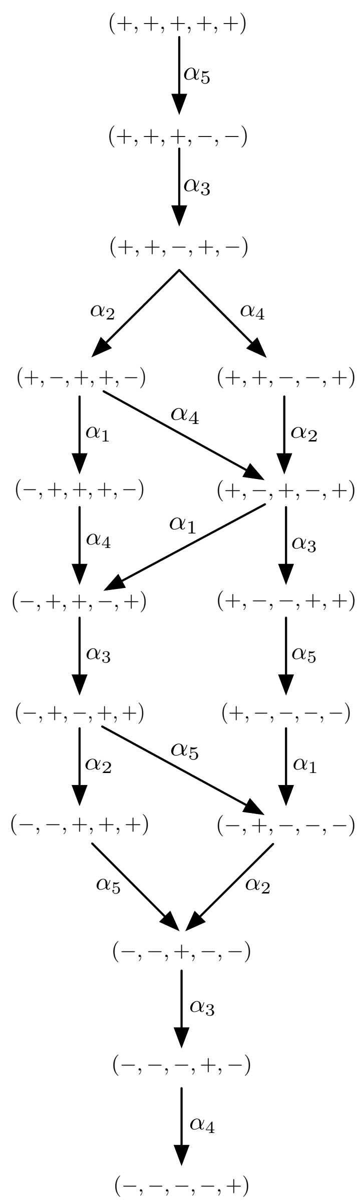

Figure A.1: Weight diagram for the **16** of $SO(10)$, where the weights are $\pm\frac{1}{2}$.

Appendix B

Rules for Model Building in the Free Fermionic Formulation

Here we present the model building rules used in constructing heterotic string models using the free fermionic formulation. These were first presented in [75, 76] and are known as the *ABK rules*.

Theorem A To any consistent fermionic string theory there corresponds a finite additive group of vectors of boundary conditions

$$\Xi \simeq \mathbb{Z}_{N_1} \oplus \cdots \oplus \mathbb{Z}_{N_k} \tag{B.1}$$

generated by a basis $\{\mathbf{b}_1, \ldots, \mathbf{b}_k\}$ which we may choose as canonical, *i.e.*

(A1) $\sum_i m_i \mathbf{b}_i = 0 \iff m_i = 0 \bmod N_i \ \forall \ i$

and which obeys the following conditions:

(A2) $\mathbb{1} \in \Xi$

(A3) $N_{ij}\, \mathbf{b}_i \cdot \mathbf{b}_j = 0 \bmod 4$,

with N_{ij} the least common multiple of N_i and N_j;

(A4) $N_i\mathbf{b}_i^2 = 0 \bmod 8$ if N_i is even;

(A5) the number of real fermions which are simultaneously periodic under any four boundary condition basis vectors $\mathbf{b}_i$, $\mathbf{b}_j$, $\mathbf{b}_l$, $\mathbf{b}_m$ is even;

(A6) the boundary condition matrix corresponding to each $\mathbf{b}_i$ is an automorphism of the Lie algebra defining the worldsheet supercharge.

Theorem B For any such group Ξ, there exist $2\Pi_{i>j}\, g_{ij}$ consistent string theories, where g_{ij} is the greatest common divisor of N_i and N_j. These correspond to the g_{ij} different choices for every coefficient $c\left[\begin{array}{c}\mathbf{b}_i\\ \mathbf{b}_j\end{array}\right]$, for $i > j$, such that:

(B1) $c\left[\begin{array}{c}\mathbf{b}_i\\ \mathbf{b}_j\end{array}\right] = \delta_{\mathbf{b}_i} e^{\frac{2\pi i n}{N_j}} = \delta_{\mathbf{b}_i} e^{\frac{\pi i(\mathbf{b}_i\cdot\mathbf{b}_j)}{2}}\, e^{\frac{2\pi i m}{N_i}}$;

with n and m (i- and j-dependent) integers, and to the two possible choices for

(B2) $c\left[\begin{array}{c}\mathbf{b}_1\\ \mathbf{b}_1\end{array}\right] = \pm e^{\frac{\pi i \mathbf{b}_1^2}{4}}$.

Any such choice can be uniquely extended to all pairs of elements of Ξ, using the properties

(B3) $c\left[\begin{array}{c}\alpha\\ \alpha\end{array}\right] = e^{\frac{\pi i(\alpha\,\alpha+\mathbb{1}\,\mathbb{1})}{4}}\, c\left[\begin{array}{c}\alpha\\ b_1\end{array}\right]^{\frac{N_1}{2}}$,

(B4) $c\left[\begin{array}{c}\alpha\\ \beta\end{array}\right] = e^{\frac{\pi i(\alpha\,\beta)}{2}}\, c\left[\begin{array}{c}\beta\\ \alpha\end{array}\right]^{*}$,

(B5) $c\left[\begin{array}{c}\alpha\\ \beta+\gamma\end{array}\right] = \delta_\alpha\, c\left[\begin{array}{c}\alpha\\ \beta\end{array}\right] c\left[\begin{array}{c}\alpha\\ \gamma\end{array}\right]$.

The full one–loop amplitude is given by a sum over all spin–structures $\left[\begin{array}{c}\alpha\\ \beta\end{array}\right]$, corresponding to pairs of elements in Ξ, with weight $c\left[\begin{array}{c}\alpha\\ \beta\end{array}\right]$.

Bibliography

[1] CMS Collaboration, Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC, *Physics Letters B* **716** (2012), 30, arXiv:1207.7235[hep-ex].

[2] ATLAS Collaboration, Observation of a New Particle in the Search for the Standard Model Higgs Boson with the ATLAS Detector at the LHC, *Physics Letters B* **716** (2012), 1, arXiv:1207.7214[hep-ex].

[3] Super-Kamiokande Collaboration, Search for Nucleon Decay into Charged Anti-Lepton plus Meson in Super-Kamiokande I and II, *Physical Review D* **85** (2012), 25, arXiv:1203.4030[hep-ex].

[4] T. Kobayashi, H. P. Nilles, F. Plöger, S. Raby, and M. Ratz, Stringy Origin of Non-Abelian Discrete Flavor Symmetries, *Nuclear Physics B* **768** (2007), 135, arXiv: hep-ph/0611020.

[5] O. Lebedev, H. Nilles, S. Raby, S. Ramos-Sánchez, M. Ratz, P. Vaudrevange, and A. Wingerter, Heterotic Road to the MSSM with R Parity, *Physical Review D* **77** (2008), 046013, arXiv:0708.2691[hep-th].

[6] R. Kappl, B. Petersen, S. Raby, M. Ratz, R. Schieren, and P. K. Vaudrevange, String-Derived MSSM Vacua with Residual R Symmetries, *Nuclear Physics B* **847** (2011), 325, arXiv:1012.4574[hep-th].

[7] H. M. Lee, S. Raby, M. Ratz, G. G. Ross, R. Schieren, K. Schmidt-Hoberg, and P. K. Vaudrevange, Discrete R Symmetries for the MSSM and its Singlet Extensions, *Nuclear Physics B* **850** (2011), 1, arXiv:1102.3595[hep-ph].

[8] S. B. Giddings and A. Strominger, Loss of Incoherence and Determination of Coupling Constants in Quantum Gravity, *Nuclear Physics B* **307** (1988), 854.

[9] A. E. Faraggi, Proton Stability and Superstring Z', *Physics Letters B* **499** (2001), 147, arXiv: hep-ph/0011006.

[10] C. Corianò, A. E. Faraggi, and M. Guzzi, A Novel String-Derived Z' with Stable Proton, Light Neutrinos and R-Parity Violation, *The European Physical Journal C* **53** (2007), 421, `arXiv:0704.1256[hep-th]`.

[11] K. R. Dienes, New Constraints on $SO(10)$ Model-Building from String Theory, *Nuclear Physics B* **488** (1997), 141, `arXiv: hep-ph/9606467`.

[12] S. Iso, N. Okada, and Y. Orikasa, The Minimal $B-L$ Model Naturally Realized at TeV Scale, *Physical Review D* **80** (2009), 115007, `arXiv:0909.0128[hep-ph]`.

[13] L. Basso, A. Belyaev, S. Moretti, and C. Shepherd-Themistocleous, Phenomenology of the Minimal B-L Extension of the Standard Model: Z' and Neutrinos, *Physical Review D* **80** (2009), 055030, `arXiv:0812.4313[hep-ph]`.

[14] S. Khalil, TeV-Scale Gauged $B-L$ Symmetry with Inverse Seesaw Mechanism, *Physical Review D* **82** (2010), 077702, `arXiv:1004.0013[hep-ph]`.

[15] S. Iso and Y. Orikasa, TeV Scale B-L Model with a Flat Higgs Potential at the Planck Scale – in View of the Hierarchy Problem, *Progress of Theoretical and Experimental Physics* **2013** (2012), 18, `arXiv:1210.2848[hep-ph]`.

[16] T. Fukuyama, $SO(10)$ GUT in Four and Five Dimensions: A Review, *International Journal of Modern Physics A* **28** (2013), 1330008, `arXiv:1212.3407[hep-ph]`.

[17] L. Basso, Phenomenology of the Minimal B-L Extension of the Standard Model at the LHC (2011), Ph.D. Thesis, University of Southampton, `arXiv:1106.4462[hep-ph]`.

[18] A. E. Faraggi, D. Nanopoulos, and K. Yuan, A Standard-like Model in the Four-Dimensional Free Fermionic String Formulation, *Nuclear Physics B* **335** (1990), 347.

[19] A. E. Faraggi, A New Standard-like Model in the Four Dimensional Free Fermionic String Formulation, *Physics Letters B* **278** (1992), 47.

[20] A. E. Faraggi, Construction of Realistic Standard-like Models in the Free Fermionic Superstring Formulation, *Nuclear Physics B* **387** (1992), 239.

[21] M. Ambroso and B. A. Ovrut, The Mass Spectra, Hierarchy and Cosmology of B-L MSSM Heterotic Compactification, *International Journal of Modern Physics A* **26** (2011), 1569, `arXiv:1005.5392[hep-th]`.

[22] A. E. Faraggi, τ Neutrino Mass as Possible Evidence for a Superstring Inspired Standard-like Model, *Physics Letters B* **245** (1990), 435.

[23] K. R. Dienes and J. March-Russell, Realizing Higher-Level Gauge Symmetries in String Theory: New Embeddings for String GUTs, *Nuclear Physics B* **479** (1996), 69, `arXiv: hep-th/9604112`.

[24] A. E. Faraggi, Custodial Non–Abelian Gauge Symmetries in Realistic Superstring Derived Models, *Physics Letters B* **339** (1994), 223.

[25] J. C. Pati, The Essential Role of String-Derived Symmetries in Ensuring Proton-Stability and Light Neutrino Masses, *Physics Letters B* **388** (1996), 20, `arXiv: hep-ph/9607446`.

[26] S. Dimopoulos, S. Raby, and F. Wilczek, Supersymmetry and the Scale of Unification, *Physical Review D* **24** (1981), 1681.

[27] M. Einhorn and D. Jones, The Weak Mixing Angle and Unification Mass in Supersymmetric $SU(5)$, *Nuclear Physics B* **196** (1982), 475.

[28] J. Ellis, S. Kelley, and D. Nanopoulos, Precision LEP Data, Supersymmetric GUTs and String Unification, *Physics Letters B* **249** (1990), 441.

[29] V. S. Kaplunovsky, One-Loop Threshold Effects in String Unification, *Nuclear Physics B* **307** (1988), 145, `arXiv: hep-th/9205068`.

[30] K. R. Dienes and A. E. Faraggi, Gauge Coupling Unification in Realistic Free-Fermionic String Models, *Nuclear Physics B* **457** (1995), 409, `arXiv:hep-th/9505046v2`.

[31] P. Hořava and E. Witten, Heterotic and Type I String Dynamics from Eleven Dimensions, *Nuclear Physics B* **460** (1995), 27, `arXiv: hep-th/9510209`.

[32] E. Witten, Non-Perturbative Superpotentials in String Theory, *Nuclear Physics B* **474** (1996), 343, `arXiv: hep-th/9604030`.

[33] J. Beringer et al., Review of Particle Physics, *Physical Review D* **86** (2012), 010001.

[34] G. Costa, J. Ellis, G. Fogli, D. Nanopoulos, and F. Zwirner, Neutral Currents Within Beyond the Standard Model, *Nuclear Physics B* **297** (1988), 244.

[35] A. Leike, The Phenomenology of Extra Neutral Gauge Bosons, *Physics Reports* **317** (1999), 143, `arXiv:hep-ph/9805494`.

[36] P. Langacker, The Physics of Heavy Z' Gauge Bosons, *Reviews of Modern Physics* **81** (2009), 1199, `arXiv:0801.1345[hep-ph]`.

[37] D. London and J. Rosner, Extra Gauge Bosons in E_6, *Physical Review D* **34** (1986), 1530.

[38] F. Zwirner, Phenomenological Aspects of E_6 Superstring-Inspired Models, *International Journal of Modern Physics A* **03** (1988), 49.

[39] J. Hewett and T. G. Rizzo, Low-Energy Phenomenology of Superstring-Inspired E6 Models, *Physics Reports* **183** (1989), 193.

[40] P. Athron, S. F. King, D. J. Miller, S. Moretti, and R. Nevzorov, Constrained Exceptional Supersymmetric Standard Model, *Physical Review D* **80** (2009), 035009, arXiv:0904.2169[hep-ph].

[41] M. Hirsch, L. Reichert, W. Porod, and F. Staub, Phenomenology of a Supersymmetric $U(1)_{B-L} \times U(1)_R$ Extension of the Standard Model with Inverse Seesaw Mechanism, *Physical Review D* **86** (2012), 093018, arXiv:1206.3516[hep-ph].

[42] P. Pachołek, Embedding Z' models in $SO(10)$ GUTs, *The European Physical Journal C* **73** (2013), 2436, arXiv:1306.4517[hep-ph].

[43] V. Braun, Y. H. He, B. A. Ovrut, and T. Pantev, A Heterotic Standard Model, *Physics Letters B* **618** (2005), 252, arXiv: hep-th/0501070.

[44] V. Braun, Y. H. He, B. A. Ovrut, and T. Pantev, The Exact MSSM Spectrum from String Theory, *Journal of High Energy Physics* **0605** (2006), 043, arXiv: hep-th/0512177.

[45] W. Buchmüller, K. Hamaguchi, O. Lebedev, and M. Ratz, Supersymmetric Standard Model from the Heterotic–String, *Physical Review Letters* **96** (2006), 121602, arXiv: hep-ph/0511035.

[46] A. Faraggi, E. Manno, and C. Timirgaziu, Minimal Standard Heterotic–String Models, *The European Physical Journal C* **50** (2007), 701, arXiv:hep-th/0610118.

[47] I. Antoniadis, J. Ellis, J. Hagelin, and D. Nanopoulos, Supersymmetric Flipped $SU(5)$ Revitalized, *Physics Letters B* **194** (1987), 231.

[48] I. Antoniadis, J. Ellis, J. Hagelin, and D. Nanopoulos, An Improved Flipped $SU(5) \times U(1)$ Model from the Four-Dimensional String, *Physics Letters B* **208** (1988), 209.

[49] I. Antoniadis, G. Leontaris, and J. Rizos, A Three-Generation $SU(4) \times O(4)$ String Model, *Physics Letters B* **245** (1990), 161.

[50] G. K. Leontaris and J. Rizos, $\mathcal{N} = 1$ Supersymmetric $SU(4) \times SU(2)_L \times SU(2)_R$ Effective Theory from the Weakly Coupled Heterotic Superstring, *Nuclear Physics B* **554** (1999), 50, arXiv: hep-th/9901098.

[51] G. Cleaver, A. Faraggi, and C. Savage, Left-Right Symmetric Heterotic–String Derived Models, *Physical Review D* **63** (2001), 066001, arXiv: hep-ph/0006331.

[52] D. C. Dunbar, Fermionic String Models and Z_N Orbifolds, *Modern Physics Letters A* **04** (1989), 2339.

[53] D. Bailin, D. C. Dunbar, and A. Love, Four Dimensional Fermionic String Theories and Symmetric Orbifolds, *International Journal of Modern Physics A* **05** (1990), 939.

[54] D. Bailin, D. C. Dunbar, and A. Love, Bosonisation of Four Dimensional Real Fermionic String Models and Asymmetric Orbifolds, *Nuclear Physics B* **330** (1990), 124.

[55] A. E. Faraggi, Partition Functions of NAHE-Based Free Fermionic String Models, *Physics Letters B* **544** (2002), 207, `arXiv: hep-th/0206165`.

[56] R. Donagi and A. E. Faraggi, On the Number of Chiral Generations in $Z_2 \times Z_2$ Orbifolds, *Nuclear Physics B* **694** (2004), 187, `arXiv: hep-th/0403272`.

[57] A. Faraggi, C. Kounnas, S. Nooij, and J. Rizos, Classification of the Chiral $Z_2 \times Z_2$ Fermionic Models in the Heterotic Superstring, *Nuclear Physics B* **695** (2004), 41, `arXiv: hep-th/0403058`.

[58] R. Donagi and K. Wendland, On Orbifolds and Free Fermion Constructions, *Journal of Geometry and Physics* **59** (2008), 46, `arXiv:0809.0330[hep-th]`.

[59] A. E. Faraggi and M. Thormeier, String Inspired Z' Model with Stable Proton and Light Neutrino Masses, *Nuclear Physics B* **624** (2001), 20, `arXiv: hep-ph/0109162`.

[60] A. E. Faraggi and M. Masip, Leptophobic Z' from Superstring Derived Models, *Physics Letters B* **388** (1996), 15, `arXiv: hep-ph/9604302`.

[61] A. E. Faraggi and V. M. Mehta, Leptophobic Z' in Heterotic–String Derived Models, *Physics Letters B* **703** (2011), 567, `arXiv:1106.5422[hep-ph]`.

[62] A. E. Faraggi and J. C. Pati, A Family-Universal Anomalous $U(1)$ in String Models as the Origin of Supersymmetry Breaking and Squark Degeneracy, *Nuclear Physics B* **526** (1998), 21, `arXiv: hep-ph/9712516`.

[63] CDF Collaboration, Invariant Mass Distribution of Jet Pairs Produced in Association with a W Boson in $p\overline{p}$ Collisions at $\sqrt{s} = 1.96$ TeV, *Physical Review Letters* **106** (2011), 171801.

[64] L. a. Anchordoqui, H. Goldberg, X. Huang, D. Lüst, T. R. Taylor, and D. Lust, Stringy Origin of Tevatron Wjj Anomaly, *Physics Letters B* **701** (2011), 224, `arXiv:1104.2302[hep-ph]`.

[65] P. F. Perez and M. B. Wise, Baryon and Lepton Number as Local Gauge Symmetries, *Physical Review D* **82** (2010), 011901, `arXiv:1002.1754`.

[66] M. Duerr, P. F. Perez, and M. B. Wise, A Gauge Theory for Baryon and Lepton Numbers, *Physical Review Letters* **110** (2013), 1, `arXiv:1304.0576[hep-ph]`.

[67] D. J. Gross, J. A. Harvey, E. Martinec, and R. Rohm, heterotic–String Theory (I): The Free Heterotic–String, *Nuclear Physics B* **256** (1985), 253.

[68] D. J. Gross, J. A. Harvey, E. Martinec, and R. Rohm, heterotic–String Theory (II): The Interacting Heterotic–String, *Nuclear Physics B* **267** (1986), 75.

[69] D. Gross, J. Harvey, E. Martinec, and R. Rohm, heterotic–String, *Physical Review Letters* **54** (1985), 502.

[70] K. Christodoulides, Statistical Exploration of Heterotic Pati-Salam String Vacua (2012), Ph. D. Thesis, University of Liverpool.

[71] L. B. Anderson, J. Gray, A. Lukas, and E. Palti, Two Hundred Heterotic Standard Models on Smooth Calabi-Yau Threefolds, *Physical Review D* **84** (2011), 106005, `arXiv:1106.4804[hep-th]`.

[72] J. Gray, Y.-H. He, A. Ilderton, and A. Lukas, STRINGVACUA: A Mathematica Package for Studying Vacuum Configurations in String Phenomenology, *Computer Physics Communications* **180** (2009), 107, `arXiv:0801.1508[hep-th]`.

[73] A. E. Faraggi and V. M. Mehta, Proton Stability and Light Z' Inspired by String Derived Models, *Physical Review D* **84** (2011), 086006, `arXiv:1106.3082[hep-ph]`.

[74] A. E. Faraggi and V. M. Mehta, Proton Stability, Gauge Coupling Unification, and a Light Z' in Heterotic–String Models, *Physical Review D* **88** (2013), 025006, `arXiv:1304.4230[hep-ph]`.

[75] I. Antoniadis, C. Bachas, and C. Kounnas, Four-Dimensional Superstrings, *Nuclear Physics B* **289** (1987), 87.

[76] I. Antoniadis and C. Bachas, 4D Fermionic Superstrings with Arbitrary Twists, *Nuclear Physics B* **298** (1988), 586.

[77] H. Kawai, D. C. Lewellen, and S.-H. Henry Tye, Construction of Fermionic String Models in Four Dimensions, *Nuclear Physics B* **288** (1987), 1.

[78] I. Antoniadis, J. Ellis, J. Hagelin, and D. Nanopoulos, The Flipped $SU(5) \times U(1)$ String Model Revamped, *Physics Letters B* **231** (1989), 65.

[79] A. E. Faraggi, C. Kounnas, and J. Rizos, Spinor–Vector Duality in Fermionic $Z_2 \times Z_2$ Heterotic Orbifold Models, *Nuclear Physics B* **774** (2006), 29, `arXiv:0611251[hep-th]`.

[80] A. E. Faraggi, C. Kounnas, and J. Rizos, Chiral Family Classification of Fermionic Heterotic Orbifold Models, *Physics Letters B* **648** (2007), 84, eprint: hep-th/0606144.

[81] K. Christodoulides, A. E. Faraggi, and J. Rizos, Top Quark Mass in Exophobic Pati–Salam Heterotic–String Model, *Physics Letters B* **702** (2011), 81, arXiv:1104.2264[hep-ph].

[82] B. Assel, K. Christodoulides, A. E. Faraggi, C. Kounnas, and J. Rizos, Classification of Heterotic Pati–Salam Models, *Nuclear Physics B* **844** (2011), 365, arXiv:1007.2268[hep-th].

[83] B. Assel, K. Christodoulides, A. E. Faraggi, C. Kounnas, and J. Rizos, Exophobic Quasi–Realistic Heterotic–String Vacua, *Physics Letters B* **683** (2010), 306, arXiv:0910.3697[hep-th].

[84] A. E. Faraggi, Standard-like Models in the Free Fermionic String Formulation and Phenomenological Implications (1991), Ph.D. Thesis, Texas A&M University.

[85] A. E. Faraggi and D. V. Nanopoulos, Naturalness of Three Generations in Free Fermionic $Z_2^n \otimes Z_4$ String Models, *Physical Review D* **48** (1993), 3288.

[86] I. Antoniadis, J. Ellis, J. Hagelin, and D. Nanopoulos, GUT Model-Building with Fermionic Four-Dimensional Strings, *Physics Letters B* **205** (1988), 459.

[87] J. Ellis, J. Hagelin, S. Kelley, and D. Nanopoulos, Aspects of the Flipped Unification of Strong, Weak and Electromagnetic Interactions, *Nuclear Physics B* **311** (1988), 1.

[88] J. Ellis, J. S. Hagelin, S. Kelley, D. Nanopoulos, and K. a. Olive, Flipped Dark Matter, *Physics Letters B* **209** (1988), 283.

[89] G. B. Cleaver, A. E. Faraggi, and S. E. Nooij, NAHE-Based String Models with $SU(4) \times SU(2) \times U(1)$ $SO(10)$ Subgroup, *Nuclear Physics B* **672** (2003), 64, arXiv: hep-ph/0301037.

[90] G. Cleaver, D. Clements, and A. Faraggi, Flat Directions in Left-Right Symmetric String Derived Models, *Physical Review D* **65** (2002), 106003, eprint: hep-ph/0106060.

[91] A. E. Faraggi, $Z_2 \times Z_2$ Orbifold Compactification as the Origin of Realistic Free Fermionic Models, *Physics Letters B* **326** (1994), 62, arXiv: hep-ph/9311312.

[92] A. E. Faraggi, Generation Mass Hierarchy in Superstring Derived Models, *Nuclear Physics B* **407** (1993), 57, arXiv: hep-ph/9210256.

[93] T. Catelin-Jullien, A. E. Faraggi, C. Kounnas, and J. Rizos, Spinor–Vector Duality in Heterotic SUSY Vacua, *Nuclear Physics B* **812** (2009), 103, `arXiv:0807.4084[hep-th]`.

[94] G. Cleaver, A. Faraggi, and D. Nanopoulos, String Derived MSSM and M-Theory Unification, *Physics Letters B* **455** (1999), 135, `arXiv: hep-ph/9811427`.

[95] A. E. Faraggi, Doublet–Triplet Splitting in Realistic Heterotic–String Derived Models, *Physics Letters B* **520** (2001), 12, `arXiv: hep-ph/0107094`.

[96] K. R. Dienes, String Theory and the Path to Unification: A Review of Recent Developments, *Physics Reports* **287** (1996), 104, `arXiv: hep-th/9602045`.

[97] P. Ginsparg, Gauge and Gravitational Couplings in Four-Dimensional String Theories, *Physics Letters B* **197** (1987), 139.

[98] K. R. Dienes, A. E. Faraggi, and J. March-Russell, String Unification, Higher-Level Gauge Symmetries, and Exotic Hypercharge Normalizations, *Nuclear Physics B* **467** (1996), 44, eprint: `hep-th/9510223`.

[99] A. E. Faraggi, Aspects of Non-Renormalizable Terms in a Superstring Derived Standard-like Model, *Nuclear Physics B* **403** (1993), 101, `arXiv: hep-th/9208023`.

[100] A. Faraggi, Fractional Charges in a Superstring-Derived Standard–like Model, *Physical Review D* **46** (1992), 3204.

[101] G. B. Cleaver and A. E. Faraggi, On the Anomalous U(1) in Free Fermionic Superstring Models, *International Journal of Modern Physics A* **14** (1999), 2335, `arXiv: hep-ph/9711339`.

[102] J. Rich, D. Lloyd Owen, and M. Spiro, Experimental Particle Physics Without Accelerators, *Physics Reports* **151** (1987), 239.

[103] C. Corianò and A. E. Faraggi, String Inspired Neutrino Mass Textures in Light of KAMLAND and WMAP, *Physics Letters B* **581** (2004), 99, `arXiv: hep-ph/0306186`.

[104] S. King, S. Moretti, and R. Nevzorov, Theory and Phenomenology of an Exceptional Supersymmetric Standard Model, *Physical Review D* **73** (2006), 035009, `arXiv: hep-ph/0510419`.

[105] F. Braam, A. Knochel, and J. J. Reuter, An Exceptional SSM from E6 Orbifold GUTs with Intermediate LR Symmetry, *Journal of High Energy Physics* **1006** (2010), 13, `arXiv:1001.4074[hep-ph]`.

[106] J. P. Hall and S. F. King, NMSSM+, *Journal of High Energy Physics* **1301** (2013), 76, `arXiv:1209.4657[hep-ph]`.

[107] T. Cheng and L. Li, Gauge Theory of Elementary Particle Physics, *Oxford University Press* (1984), 536.

[108] L. Bernard, A. E. Faraggi, I. Glasser, J. Rizos, and H. Sonmez, String Derived Exophobic $SU(6) \times SU(2)$ GUTs, *Nuclear Physics B* **B868** (2012), 1, arXiv:1208.2145[hep-ph].

[109] F. Braam, J. Reuter, and D. Wiesler, The Pati-Salam Supersymmetric SM (PSSSM), *AIP Conference Proceedings* **1200** (2010), 458, arXiv:0909.3081[hep-ph].