ON THE APPLICATION OF LOSS FUNCTIONS FOR DETERMINING HAZARDOUS CONCENTRATIONS

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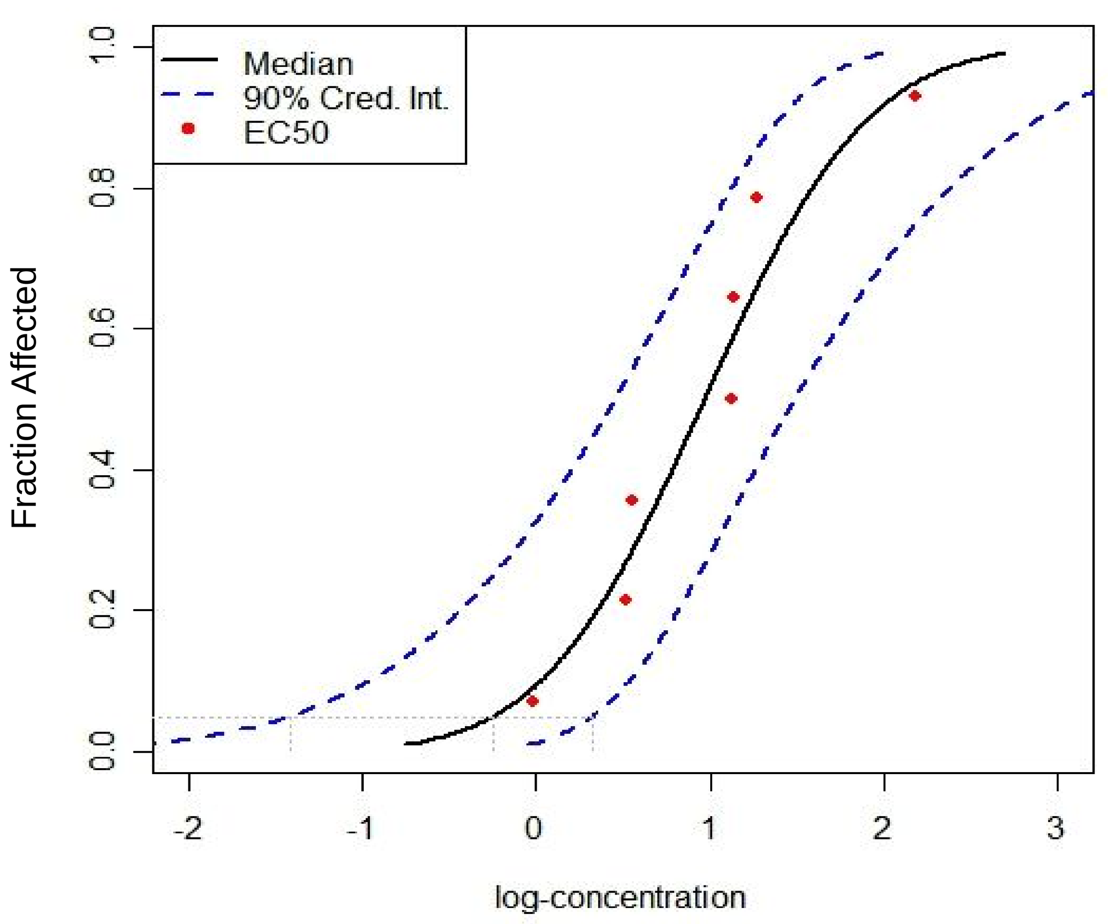
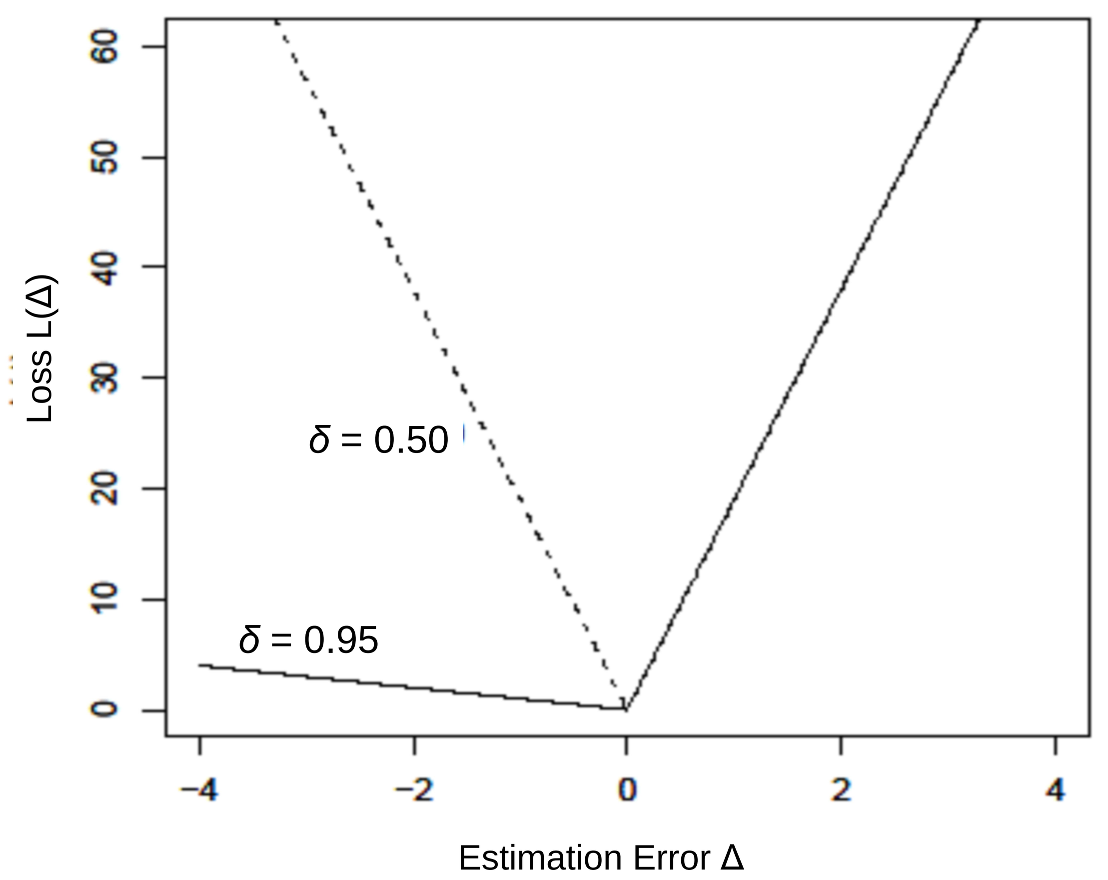
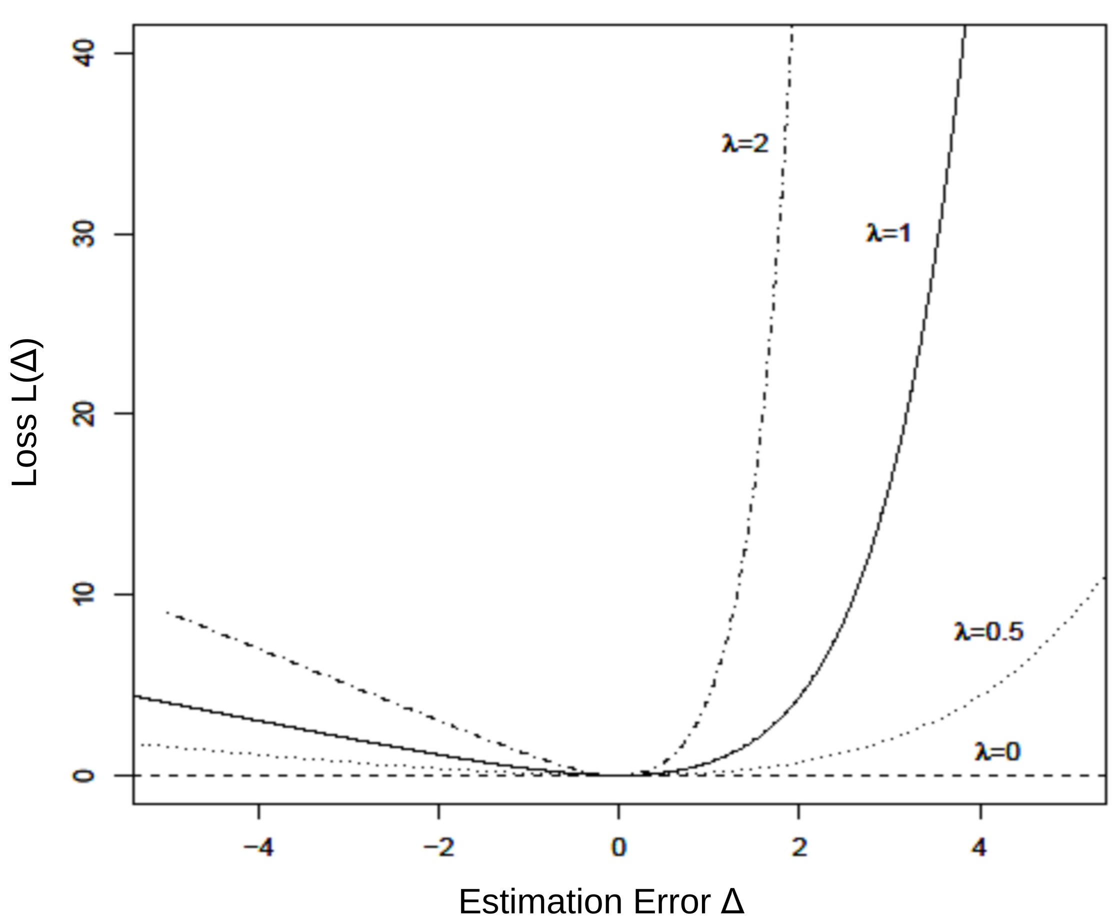


- The hazardous concentration to p% of species (HC_p), defined up to a specific ecological community, is the concentration such that the probability a randomly selected species from the assemblage will have its toxicological endpoint violated is p%.
- The HC_5 has become a standard benchmark safety limit which is presumed to have little adverse effect to species at the community level.
- **Figure 1 (left).** An example Gaussian Species Sensitivity Distribution (SSD) with estimated median and 90% credible limits. Dashed-grey lines = HC_5 (and 90% credible interval).
- Having adequately modelled the 'true' SSD, one needs to estimate the HC_5.
- A popular estimator is the Aldenberg & Jaworska (2000; *Ecotox. Environ. Saf.* 46: 1-18) δ-estimator, where δ is a measure of certainty. The data in Fig. 1 is the cadmium exposed soil organism data from Aldenberg & Jaworska.
- Proposals have emerged for one to use the lower one-sided δ = 95% underestimate confidence limit in order to have conservativeness.



- A loss function measures the 'cost' (not necessarily financial) of making an error in estimation.
- The precautionary principle would imply that over-estimating the HC_p would be worse than under-estimating it. Therefore we would place a higher loss on overestimation.
- In the Bayesian paradigm we can use available toxicity data (e.g. EC_{50}s) to update prior knowledge for the distribution of the HC_p — known as the *posterior* distribution.
- A popular choice of estimator in decision theoretic statistics is the Bayes rule — the decision which minimises the statistically expected loss with respect to the posterior distribution of the true HC_p.
- **Figure 2 (left).** The Generalised Absolute Loss function. Aldenberg & Jaworska's class of estimators corresponds to Bayes rules under this loss function class. The symmetric 'V' shape corresponds to the median (δ = 0.50) estimator, and the union of the solid lines corresponds to the lower one-sided (δ = 0.95) underestimate confidence limit. Δ = the estimation error ($\Delta > 0$: overestimation, $\Delta < 0$: underestimation). L(Δ) = cost of estimation error (arbitrary scale). GAL is parameterised by C_2 / C_1 — the cost of overestimation relative to underestimation.
- Clearly the asymmetrical sub-class (δ > 0.50) is appealing from a conservative perspective. However, is the linearity reflective of a risk managers true cost-benefit portfolio? Furthermore, is punishing overestimation 19 times more than underestimation reasonable?



- A risk manager can choose any suitable loss function which represents their requirements.
- An alternative loss function is the modified– LINear Exponential (LINEX) function. This is a non-linear asymmetrical loss function parameterised by conservatism control parameter λ. As $\lambda > 0$ increases, so does the level of conservatism.
- **Figure 3 (left).** Example of standard-LINEX loss functions for some different values of λ.
- A modified version of LINEX allows for the risk manager to specify loss on a scale which is independent of the SSD variability.
- Reducing estimators to decision-theoretic interpretations *potentially* allows for more transparency in estimation methods.
- An algorithm for obtaining a suitable value of λ is suggested in Hickey *et al.* (2009; *Ecotox. Environ. Saf*. 72: 293-300).

Loss Function	Parameterisation	HC_5 Estimate (µg Cd / mg)
GAL (A&J δ = 0.95)	$C_2 = 19C_1$	0.038
GAL (A&J δ = 0.50)	$C_2 = C_1$	0.568
GAL (A&J δ = 0.05)	$19C_2 = C_1$	2.112
Modified-LINEX	$\lambda = 0.5$	0.633
Modified-LINEX	$\lambda = 1$	0.542
Modified-LINEX	$\lambda = 3$	0.235
Modified-LINEX	$\lambda = 5$	0.055

- It is important for a risk manager to understand the level of conservatism in their estimates, otherwise they might reject or allow a substance inappropriately.
- **Table 1 (left).** HC_5 estimates for the classical cadmium toxicity dataset (discussed in Aldenberg and Jaworska 2000) for two different loss function classes: GAL and modified-LINEX, and different parameterisations.
- There is a wide range in estimates — reinforcing the need to suitably estimate the HC_5.
- An introduction to loss functions for estimating HC_ps is discussed in Hickey *et al.* (2009).
- **Overall conclusion:** some current HC_p estimation methods are arbitrary, but loss functions can potentially help risk managers to control the degree of conservatism in more appropriate ways.