On Stock Market Illiquidity and Real-Time GDP Growth

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Abstract

This study examines whether stock market illiquidity forecasts real UK GDP growth using data over the period 1989q1-2012q2. Apart from standard linear model specifications, we also utilize non-linear models, which allow for regime switching behavior in terms of a liquid versus an illiquid market regime and over the phases of the business cycle. Our findings support a statistically significant negative relationship between stock market illiquidity and future UK GDP growth over and above the usual control variables. This relationship is found to be stronger during periods of highly illiquid market conditions and weak economic growth. Our out-of-sample forecasting analysis indicates that a regime-switching model of illiquid versus liquid market conditions predicts UK growth better than any other model. Actually, this model is the only one to significantly outperform the GDP growth forecasts published in the *Bank of England's Inflation Report*.

JEL Codes: G12, C32, C51, C52.
Keywords: Stock market illiquidity, Divisia money, GDP growth, Non-linear model, Real-time data.

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1. Introduction

The President of the Federal Reserve Bank of Boston, Eric Rosengren (2010) pointed out that the seriousness of the recent financial crisis was underestimated by economic forecasters because financial links, such as provision of liquidity, to the real economy were "*only crudely incorporated into most macroeconomic modeling*" (p. 221). Adding to this, Borio (2013) noted that for most of the postwar period "*financial factors in general progressively disappeared from macroeconomists' radar screen*" (p.1). However, provision of liquidity has become a central issue in the literature since the recent financial crisis (see Bridges and Thomas, 2012; Angelini et al., 2011; Naes et al., 2011; Acharya et al., 2011; Joyce et al., 2011; Blanchard et al., 2010; Hameed et al., 2010; Brunnermeir and Pedersen, 2009; Borio, 2008; Adrian and Shin, 2008). Additionally, in response to the crisis, UK (and global) monetary policy followed an unprecedented path of interest rate cuts. UK interest rate cuts came to a halt in March 2009 and since then the Bank of England (BoE) base rate stands at a record low of 0.5%. BoE also decided to support the economy further by boosting liquidity. The above operation, known as Quantitative Easing (QE), consisted of large purchases of mainly longer-term government bonds and related assets. Between March 2009 and July 2012, the Monetary Policy Committee (MPC) authorized a total of £375bn of QE. The impact of QE on the economy works via three main channels: the macro/policy news channel, the signaling channel and the portfolio rebalancing channel (see Martin and Milas, 2012, for a critical analysis).

In this study, we examine an additional channel through which economic growth may be affected: the prevailing stock market liquidity conditions. There are various reasons why stock market liquidity can be an informative leading indicator for future economic conditions. Firstly, market liquidity can act as a signaling mechanism, revealing the

information set of investors. During periods of high uncertainty or negative outlook regarding the future state of the economy, investors move their capital away from high-risk investments, reducing their exposure or fleeing the stock market altogether, investing in short-term fixed income securities, preferably government debt (*flight to quality or flight to safety*). If these shifts in investors' portfolio composition are related to fears that stock market liquidity may dry up, then a "*flight to liquidity*" is observed (Longstaff, 2004). These effects become more pronounced during periods of financial distress, where the actions of market participants, and in particular institutional investors, tend to be correlated. Brunnermeier and Pedersen (2009) show that a reinforcing mechanism between market liquidity and funding liquidity (the interaction between securities' market liquidity and financial intermediaries' availability of funds) leads to liquidity spirals and institutional investors are forced to shift their holdings towards stocks with low margins. Stock market liquidity can alternatively affect the real economy through an investment channel. In particular, a liquid secondary market can facilitate the financing of long-run projects in the real economy (Levine and Zervos, 1998). It is also well-established that liquidity has a first-order effect on the premium that investors demand to withhold risky assets (see, for example, Amihud, 2002, and Acharya and Pedersen, 2005). As a result, a liquid stock market may lower the cost of capital for firms, and hence boost high return projects that stimulate earnings and productivity growth (Levine, 1991).

The main contribution of this study is that it examines whether stock market illiquidity forecasts real UK GDP growth once other financial variables, such as the term spread (see e.g. Chinn and Kucko, 2010, Estrella, 2005, and Estrella and Hardouvelis, 1991), asset prices (see e.g. Zaher, 2007) and stock market uncertainty (see e.g. Fornari and Mele, 2009), have been accounted for. Stock and Watson (2003) provide an extensive review of

the literature on forecasting macroeconomic variables, namely inflation and real output growth, by using asset prices in addition to monetary aggregates. Their work concludes that most assets (short-term interest rates, term spreads and stock returns) do not provide stable and strong predictive power. We build upon this strand of research by suggesting stock market illiquidity as an additional leading indicator of economic growth using data for the period 1989q1-2012q2

In doing so, we pay attention to a particular dimension of stock market illiquidity, namely the price impact, which measures the resilience of stock prices to changes in trading activity. Blume and Keim (2012) show that illiquidity measures that attempt to estimate the price impact of trades do a better job at capturing liquidity, and are robust to regime changes such as the change in minimum tick size to decimals. Following Naes et al. (2011), we use the illiquidity measure of Amihud (2002), which is defined as the average ratio of daily absolute returns to daily trading volume (hereafter RtoV). This measure is appealing because it is easy to compute for long time periods given the wide availability of returns and trading volume data.[1] RtoV is also considered to be a good proxy for trading costs and the depth of the market without requiring intraday data, as we need for bid-ask spreads to be meaningful (see Acharya and Pedersen, 2005). Additionally, we use a modified price impact ratio, which is defined as the average ratio of daily absolute stock returns to daily turnover ratio (hereafter RtoTR), essentially replacing the trading volume of a stock with its turnover ratio in the denominator of Amihud's ratio (see Florackis at al., 2011, for a detailed analysis). [2]

[1] In addition, it is intuitively attractive because the average daily price response associated with a dollar of trading volume renders it a good proxy for the theoretically founded Kyle's price impact coefficient (see Goyenko et al., 2009).
[2] See also Florackis et al. (2011) for the advantages of RtoV over traditional proxies of liquidity.

From a methodological perspective, an important contribution of our study is that it allows for an asymmetric relationship between liquidity and economic growth and ultimately links stock market liquidity to macro-liquidity provision. As Joyce et al. (2011) observe, central banks purchases of assets can improve market functioning when financial markets are dysfunctional or, in other words, when liquidity has dried up.[3] In other words, one might expect injections of macro-liquidity to be more pronounced in periods where market liquidity is too low and when the economy is underperforming. This suggests a regime-switching model which assesses the impact of stock market liquidity on economic growth depending on the existence of a liquid/illiquid market regime as well as during different phases of the business cycle. In this study, we opt for a Smooth Transition model, which is able to capture regime switching behaviour in a flexible way and facilitates its economic interpretation (see van Dijk et al., 2002, for a discussion of the benefits of this class of models. To this end, our study substantially differs from Naes et al. (2011), who use solely linear model specifications to examine the impact of stock market liquidity on GDP growth in the US and Norway after controlling for the effects of other financial variables.

Furthermore, we use data available to policymakers at the time of their forecasts and policy decisions adding realism to our models. In terms of monetary variables, we include either M4 or divisia money. The latter has been argued to have a closer relationship to expenditure, as it weighs the components of the money supply in proportion to their usefulness in making transactions (see Darrat et al., 2005; Hancock, 2005).

[3] Acknowledging the considerable uncertainty of the impact of QE on the economy, Bank of England Deputy Governor (Monetary Policy) Charlie Bean (2012), points out that "it is possible that the effectiveness of policy depends on the state of the economy". Prominent economic commentators appear mindful of this very issue. For instance, David Smith (Economics Editor of *The Sunday Times*) referred to QE as an emergency tool and noted that its implementation depends on whether one thinks that "this is an emergency or merely a period of soft growth" (Smith, 2011).

Our results document a statistically significant negative relationship between stock market illiquidity and subsequent real economic growth over and above the usual control variables, such as real money, term spread and global economic activity. The results also provide evidence that divisia money, which has a close relationship to aggregate spending, is a better predictor of UK growth than the routinely used M4 measure of money supply.

Furthermore, we provide evidence that the impact of both market illiquidity and divisia money becomes stronger during periods of illiquid market conditions and during periods of (very) weak economic growth. Using a counterfactual experiment, our findings suggest that had market liquidity not dried up so dramatically since 2007, the depth in UK recession would have been less severe by some 2.3 percentage points. Finally, our out-of-sample forecasting analysis provides evidence in favor of a regime-switching model of illiquid versus liquid market conditions in predicting real UK GDP growth better than any other linear or non-linear model. Furthermore, using formal statistical tests, we find that this is the only one out of a wide range of utilized models that provides significantly more accurate UK GDP growth forecasts than those published in the BoE's *Inflation Report*.

The remainder of the paper proceeds as follows. Section 2 outlines our modeling strategy. Section 3 provides details of the dataset used. Section 4 reports our empirical findings. Finally, Section 5 summarizes our findings, discusses policy implications and offers some suggestions for future research.

2. Linear and non-linear models

The starting point of our analysis is a linear model of the form:

$$y_t = \beta_0 + \beta_{illiq} illiq_{t-l} + \beta_{\mathbf{X}}' \mathbf{X}_{t-l} + v_t \,, \tag{1}$$

where y_t is the annual GDP growth rate, $illiq_{t-l}$ is a measure of stock market illiquidity, $\mathbf{X}_{t-l}$ is a vector of control variables and v_t is an error term.[4] A large number of potential candidates exist for the $\mathbf{X}_{t-l}$ vector of control variables. We proceed by including in the vector $\mathbf{X}_{t-l}$ the following control variables: lagged GDP growth, the slope of the term structure of interest rates (term spread), annual real money growth and a measure of global economic activity. The slope of the term structure is approximated by the spread between the yield on the 10-year UK government bond and the 3-month T-bill rate. Annual real money growth (i.e. nominal money growth minus the Retail Price Index (RPI) inflation rate) is approximated by two measures of money: broad money (M4) and divisia money. Global economic activity is proxied by the annual real GDP growth rate in the US.[5] In preliminary analysis, we also included a measure of oil prices. Oil prices have been shown to significantly affect real economic activity in the US (see e.g. Ravazollo and Rothman, 2013, Hamilton, 2003, Hamilton, 1996, and references therein). We experimented with the annual growth of the real price of oil (i.e. the annual growth in the price of oil in £ minus the RPI inflation rate). This variable entered all our models with a negative sign (higher oil prices depress economic activity) but it was statistically insignificant.[6]

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[4] The $illiq_t$ and $\mathbf{X}_t$ regressors do not need to share the same lag length l. For our empirical analysis, we considered lagged values up to lag 5 for both regressors. Our empirical models favor a choice of l =1 for both $illiq_t$ and $\mathbf{X}_t$ based on the Akaike Information Criterion (AIC).

[5] We proxy global economic activity by US real GDP since US output is ranked 1st based on World Bank's database and it accounted for approximately 23% of World's GDP in 2011. In a VAR model of UK economic growth, Garratt et al. (2003) proxy global economic activity by OECD's GDP. In preliminary analysis, we also considered annual real GDP growth for the OECD countries; this is highly correlated with US growth based on revised data (the correlation coefficient is equal to 0.89). However, real-time OECD GDP data are only available from 2002 onwards. Therefore, we opted for the US real-time dataset.

[6] Alquist et al. (2011) discuss the issue of using real versus nominal oil prices in predicting real economic activity and Hamilton (1996) considers nonlinear transformations of the oil price. We also considered the nominal growth in oil prices as well as the (real) price of oil relative to its 1-year and 2-year Moving Average. All these measures were found to be insignificant.

While including in the $\mathbf{X}_{t-l}$ vector these commonly used predictors of economic growth, our study pays particular attention to the role of stock market illiquidity as a leading indicator of the UK business cycle. This modeling choice has been motivated by the channels through which stock market liquidity can affect the real economy, as discussed in the Introduction. To proxy stock market illiquidity ($illiq_{t-l}$), we rely on the price impact ratios suggested by Amihud (2002) and Florackis et al. (2011), i.e. the Return-to-Volume (RtoV) and Return-to-Turnover Ratio (RtoTR), respectively, calculated for the FTSE100 index, which contains the biggest capitalization stocks listed on London Stock Exchange. In particular, RtoV is defined as:

$$RtoV_{i,Y} = \frac{1}{N_Y} \sum_{d=1}^{Y} \frac{|R_{i,d}|}{VOL_{i,d}} \tag{2}$$

where $|R_{i,d}|$ is the absolute return of asset *i* on day *d*, $VOL_{i,d}$ is the trading volume in monetary terms of asset *i* on day *d* and N_Y is equal to the number of days in the examined window *Y*, while RtoTR is defined as:

$$RtoTR_{i,Y} = \frac{1}{N_Y} \sum_{d=1}^{Y} \frac{|R_{i,d}|}{TR_{i,d}} \tag{3}$$

where $|R_{i,d}|$ is the absolute return of asset *i* on day *d*, $TR_{i,d}$ is the turnover ratio of asset *i* on day *d* and N_Y is equal to the number of days in the examined window *Y*. An increase in both measures is equivalent to a drop in liquidity; that is, our variables are measures of *stock market illiquidity*.

Assessing the direct impact of stock market illiquidity on economic growth is not straightforward because stock market illiquidity is correlated with changes in monetary

policy. For instance, Adrian and Shin (2008), Brunnermeier and Pedersen (2009), Gromb and Vayanos (2010) and Hameed et al. (2010), show that the relationship between changes in monetary policy and stock market liquidity appears to be "unevenly" or "asymmetrically" pro-cyclical with monetary policy, that is, expansionary (contractionary) monetary policy leads to smaller (larger) increases (decreases) in liquidity. We also note that stock market liquidity may be correlated with stock returns and stock volatility; liquidity tends to be lower and volatility tends to be higher during bear markets. Fornari and Mele (2009) pay close attention to the impact of stock market volatility on the US business cycle. They argue in favor of stock market volatility measures derived from absolute returns on the grounds that these measures are more robust to the presence of outliers than volatility measures derived from squared returns. We took notice of this issue in preliminary analysis by allowing stock market volatility to enter as a separate regressor in our empirical models. In particular, following Fornari and Mele (2009) we constructed stock market volatility measures based on the 1-year and 2-year moving average of past annualized absolute FTSE100 returns. Inclusion of these stock market volatility measures did not affect the estimates of our illiquidity proxies reported below and we failed to find any statistical significance for these volatility measures.

To allow for possible asymmetries in the behavior of illiquidity, a non-linear version of (1) is given by:

$$y_t = \beta_0 + (\beta_{illiq,1} illiq_{t-l} + \beta_{\mathbf{X},1}{}' \mathbf{X}_{t-l})\theta^s{}_{t-l} + (\beta_{illiq,2} illiq_{t-l} + \beta_{\mathbf{X},2}{}' \mathbf{X}_{t-l})(1-\theta^s{}_{t-l}) + u_t\,, \quad (4)$$

where

$$\theta^s{}_{t-l} = 1 - \frac{1}{1+e^{-\gamma^s(s_{t-l}-\tau^s)/\sigma(s_{t-l})}} \quad (5)$$

is the logistic transition function discussed in van Dijk et al. (2002) and s_{t-l} is the transition variable. According to (4)-(5), GDP growth y_t exhibits regime-switching behavior depending on whether s_{t-l} is below or above an endogenously estimated threshold, τ^s, with regime weights $\theta^s{}_{t-l}$ and $(1-\theta^s{}_{t-l})$, respectively. When s_{t-l} is below the threshold τ^s, then $\theta^s{}_{t-l} \to 1$. In this case, the impact of $illiq_{t-l}$ and $\mathbf{X}_{t-l}$ is given by $\beta_{illiq,1}$ and $\beta_{\mathbf{X},1}{}'$, respectively. When s_{t-l} is above the threshold τ^s, then $\theta^s{}_{t-l} \to 0$. In this case, the impact of $illiq_{t-l}$ and $\mathbf{X}_{t-l}$ is given by $\beta_{illiq,2}$ and $\beta_{\mathbf{X},2}{}'$, respectively. In (4)-(5), we assume a common intercept β_0, which is testable. The parameter γ^s ($\gamma^s > 0$) determines the smoothness of the transition between regimes. We make γ^s dimension-free by dividing it by the standard deviation of s_{t-l} (Granger and Teräsvirta, 1993).

Furthermore, we choose illiquidity ($illiq_{t-l}$) and lagged GDP growth (y_{t-l}) as possible alternative transition variable candidates. This allows us to assess the impact of illiquidity during a liquid regime (when $illiq_{t-l} < \tau^{illiq}$) as opposed to an illiquid regime (when $illiq_{t-l} > \tau^{illiq}$) and during periods of low growth (when $y_{t-l} < \tau^y$) as opposed to periods of normal or high growth (when $y_{t-l} > \tau^y$).

3. Data description

Both RtoV and RtoTR are calculated for the FTSE100 index and they are expressed in percentage deviations from their 2-year Moving Average (MA) starting from 1989q1.[7]

[7] In preliminary analysis, we also considered the level of illiquidity as well as illiquidity relative to its 1-year Moving Average. Empirical results using these alternative measures produced similar coefficient

Thomson Reuters Datastream is the source for FTSE 100 daily returns, trading volumes, and market values. Data on M4, money divisia, the yield on the 10-year UK government bond, the 3-month T-bill rate and real-time vintages of GDP are available from the BoE database. The Retail Price Index (RPI) is available from the Office for National Statistics (ONS) database, whereas real-time vintages of US GDP are available from the website of the Federal Reserve Bank of Philadelphia. GDP and money data are seasonally adjusted.[8]

Figure 1 plots the deviations of RtoTR and RtoV measures from their 2-year MA (left axis) together with annual GDP growth rates based on the last available vintage of data produced by the ONS in 2012q2 (right axis). From Figure 1, the two measures of illiquidity strongly co-move. We note that stock market illiquidity, as measured by both RtoTR and RtoV, rises up to 20% above its 2-year MA around the 1990-91 recession. The market turns also illiquid during the Asian financial crisis and the Russian default in 1997-1998 and following the burst of the dot-com bubble, the adverse impact of which reached its height in 2002q3 (see Berger and Bouwman, 2008). During the recent financial crisis, the stock market became highly illiquid. Illiquidity rises sharply in 2007, prior to the economic slowdown, and reaches its peak in 2008q4. It then eases between 2009 and early 2012 (when £325bn of QE was implemented). Finally, Figure 1 superimposes a threshold value of -16.141% for the deviation of RtoV from its 2-year MA; we return to this issue below.

Figure 2 plots real money growth rates based on the M4 and divisia measures of money, the annual US GDP growth rate based on the last available vintage of data in 2012q2 and the term spread. We note that the recent US recession has been less deep than

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estimates but inferior statistical fit. For this reason, the main results presented in this study are based on percentage deviations from their 2-year Moving Average.

[8] We have access to revised M4 and divisia money data and use these in our estimations. However, we note that revisions of UK monetary aggregates occur mainly as a result of changes to the seasonal adjustment by the ONS (Garratt et al., 2009). The US real-time GDP dataset is available from: http://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/data-files/ROUTPUT/

in the UK and that the US economy has somewhat recovered since 2010. Moreover, if money divisia represents money movements in the economy more accurately than M4, one would expect QE injections to show up more in divisia money and less so in M4. We note from Figure 2 that real M4 growth reached its peak in the beginning of 2009 and has been falling rapidly since then. On the other hand, real divisia money increased during the first round of QE in 2009. It then fell between 2010 and mid 2011 and somewhat recovered since then. The slope of the term structure has been decreasing over the 2010-2012 period, which suggests that the post crisis recovery has been, at best, anemic.

4. Empirical results

4.1. In-sample analysis

We begin by estimating, over expanding windows of data, versions of the linear model (1) based on the different measures of money and liquidity. The first data window runs from 1989q2 to 2002q4 and uses the first release of the 2003q1 real-time data vintage. Each successive data window is extended by one observation, and hence the last data window runs from 1989q2 to 2012q1 and uses the 2012q2 real-time data vintage. This setup delivers 38 expanding windows.

Table 1 reports estimates of 4 versions of the linear model (1) using combinations of the two different measures of liquidity and money over the last vintage of data which covers the whole sample period 1989q2-2012q1. In all models, the impact of illiquidity is highly significant; an increase in stock market illiquidity is associated with lower subsequent economic growth; a 100 percentage point increase in the illiquidity gap depresses GDP growth by around 0.6 percentage points.[9] The slope of the term structure is

[9] Figure 1 shows that illiquidity movements of this magnitude and beyond have occurred since 2007.

also statistically significant. Divisia money is statistically significant too in columns (i) (iii). On the other hand, the statistical significance of M4 is much weaker; this finding arguably confirms the superiority of divisia money over M4 in predicting GDP growth.[10] There is some weak evidence that global economic activity, as proxied by US GDP growth, affects UK growth only for the model that includes RtoTR and divisia money. Amongst all estimated models, the model with the RtoV measure of illiquidity and divisia money in column (iii) delivers the best fit, since it yields the lowest AIC value.[11]

To get an idea of how the in-sample performance of the estimated linear models evolves over succesive real-time vintages, Figure 3 plots their AIC values. From Figure 3, the model with the RtoV measure of illiquidity and the divisia measure of money has the lowest AIC value; this is more evident from 2009 onwards. In what follows, we restrict our attention to the RtoV measure of illiquidity and money divisia and proceed by presenting the results for non-linear models using these measures. In addition to the AIC values of the linear models of Table 1, Figure 3 also reports the AIC value of an autoregressive (AR) process, which we use in the following section for forecasting analysis, and two non-linear versions of equations (4)-(5) using RtoV and divisia money and allowing for regime switching behavior to depend on RtoV and y_{t-1}, respectively.[12] We note that as we enter the financial crisis period, all models deteriorate in performance; nevertheless, the non-linear models continue to perform better than the remaining linear ones.

[10] Using a recursive linear VAR, Garratt et al (2009) conclude that money is a weak predictor of real UK growth. Their analysis considers the M0, M3 and M4 measures of money but not divisia money.

[11] Using recursive estimates, unreported plots (available on request) of the residuals +/-2*standard errors suggest reasonable parameter constancy with the notable exception of most of the 2007-2010 period.

[12] We also estimated the non-linear model (4)-(5) using RtoTR and divisia money or M4. These produced an inferior statistical fit to the ones reported in Table 2. In the interest of space, we abstract from reporting these estimates. Full details are readily available upon request.

Estimates of these non-linear models based on the last available vintage of data are reported in Table 2. Specification (i) uses the RtoV measure of illiquidity as the regime-switching variable, whereas specification (ii) uses lagged GDP growth (y_{t-1}) as the regime-switching variable. Consider first the non-linear model in Table 2(i). The estimate of the smoothness parameter γ^{illiq} suggests a very sharp switch from one regime to the other.[13] During the liquid regime, i.e. when illiquidity drops 16% below its 2 year MA (this threshold value is statistically significant), real divisia money growth has a significant impact on GDP growth. On the other hand, an increase in stock market liquidity does not have any statistical effect in this regime. However, during the illiquid regime both an increase in stock market liquidity and real divisia money growth are strong predictors of GDP growth. Market illiquidity exerts a highly significant effect; a 100 percentage point increase in the illiquidity gap is associated with a decline in GDP growth by almost a full percentage point. At the same time, the impact of real divisia money growth is twice as high as its impact during the liquid regime. Initially, we also allowed for regime-switching effects from the remaining regressors but failed to find convincing evidence. Imposing regime-independent effects from lagged growth, the slope of the term structure, global economic activity and the intercept facilitated robust convergence of the non-linear model and improved its statistical fit. Lagged economic growth and the slope of the term structure have a statistically significant impact. Global economic activity exerts a positive but statistically weak effect (with a *t*-ratio of 1.60).

Next, we turn our attention to the non-linear model in Table 2(ii). The estimate of the smoothness parameter γ^{y} suggests a sharp switch from one regime to the other, whereas

[13] van Dijk et al. (2002) note the difficulty in getting an accurate estimate of γ. The likelihood function is very insensitive to γ and therefore, precise estimation of this parameter is unlikely.

the statistically significant estimated threshold value of 1.22% distinguishes between a regime of weak economic growth, since the UK economy has witnessed an average growth of 2.27% over our sample period, and a regime of relatively normal growth or better. Illiquidity effects are insignificant when lagged annual GDP growth rate exceeds the 1.22% threshold. On the other hand, real divisia money growth exerts a statistically significant impact. However, during periods of weak economic growth (i.e. below 1.22%), stock market illiquidity exerts a strong impact on economic growth (i.e. 10 times bigger) and, at the same time, the effect of real divisia money growth doubles in magnitude. Lagged economic growth and the slope of the term structure have a significant impact, which is invariant to the state of the economy, whereas global economic activity has a statistically weak effect (with a *t*-ratio of 1.35). We also note that the non-linear model using RtoV as the transition variable (Table 2(i)) delivers a lower AIC compared to the non-linear model in Table 2(ii) and the linear models in Table 1 (see Figure 3). Furthermore, both models presented in Table 2 exhibit parameter constancy. The related test is reported in the last row of this table.

Our use of expanding windows of data and successive real-time vintages allows us to examine how policy perceptions about the drivers of GDP growth evolve over time. This is because the release of additional data together with data revisions trigger re-estimation of empirical models. We demonstrate this in Figure 4, which gives an idea of how the use of expanding windows of data and successive real-time vintages affect the estimated coefficients and the confidence intervals of market illiquidity, divisia money, global economic activity and the slope of the term structure, derived from the linear model in Table 1 (iii), which has the best in-sample fit amongst all linear models. As the financial crisis kicks in, the impact of real divisia money growth increases. Stock market illiquidity

has a statistically significant and rising impact from 2009 onwards. Quite strikingly, the coefficient on global economic activity is statistically significant only until 2011q3. Between 2009 and 2011q3, global economic activity has a rising impact which then drops in size and significance at the same time when divisia money and illiquidity effects become stronger. Finally, the coefficient of the slope of the term structure is fairly stable and always significant. From Figure 4, re-estimation of the empirical model based on the release of additional data delivers the message that monetary, liquidity and global economic activity developments are all important drivers during the 2008-2009 UK recession and the subsequent short-lived recovery, whereas global economic conditions appear to weigh less towards the end of the examined period.

To save space, we abstract from providing plots of the parameter estimates of the non-linear models over expanding windows and successive real-time vintages. With the exception of the parameter estimate on the global economic activity, which remains as in the case of the linear models statistically significant only until 2011q3, these plots corroborate to a large extent the results of Table 2 and they are available on request.

To assess the regime-switching impact of stock market illiquidity on UK GDP growth, we restrict our attention to the non-linear model in Table 2(i), which delivers a better statistical fit in terms of AIC. Using the estimates in Table 2(i), Figure 5 plots together the annual UK GDP growth rate and the regime-switching impact of illiquidity calculated as $\beta_{illiq,1}\theta^{illiq}{}_{t-1} + \beta_{illiq,2}(1-\theta^{illiq}{}_{t-1})$. Compared with the estimates of the linear models reported in Table 1, our non-linear model reveals a more subtle response of GDP growth to stock market illiquidity. The impact switches from zero during liquid market conditions (i.e. when illiquidity fluctuates below the estimated threshold; see Figure 1) to -0.009 during illiquid market conditions. In the latter case, the lack of liquidity takes its toll on the

economy dragging GDP growth down; this is indeed notable during the 1990-1991, 2008-2009 and 2011q4-2012q1 recessions and to a much lesser extent during the 1997-1998 Asian and Russian crises and the burst of the dot-com bubble in the early 2000s.

To further assess the implications of our non-linear model estimates in Table 2(i) since the beginning of the financial crisis in 2007q3, Figure 6 compares actual GDP growth rates with the counterfactual GDP growth rates implied by the liquid (when $illiq_{t-1} < \tau^{illiq}$) and the illiquid (when $illiq_{t-1} > \tau^{illiq}$) regimes.[14] Returning to Figure 1, we note that, since 2007q3, GDP growth was largely determined by the illiquid regime. In fact, illiquidity conditions became much more severe compared with the 1990-1991 recession period or any other period. From Figure 6, it is apparent that the GDP growth rate implied by the illiquid regime is much closer to the actual growth rate than the implied growth rate from the liquid regime in this period. Estimates from the illiquid regime imply a recession depth of 7.3% in 2009q1, closely matching the 6.9% figure based on the actual data. By contrast, estimates of the liquid regime would suggest a much smoother fall in UK GDP, predicting a less severe recession depth of 4.6% with a delay of one quarter.

What is the implication of these estimates? Had market liquidity not dried up so dramatically from 2007q3 onwards, the UK economy would have witnessed a substantially less severe recession of 2.3 (i.e. 6.9 minus 4.6) percentage points and delayed by one quarter. But as QE is implemented and liquidity conditions improve (from Figure 1 illiquidity eases up between 2009-2012), the GDP growth implied by the illiquid and liquid regimes draw closer to each other. That said, in 2012q1, the GDP growth implied by the

[14] Counterfactual GDP growth rates are given by $y_t^{liquid} = \beta_0 + \beta_{illiq,1} illiq_{t-1} + \beta_{\mathbf{X},1}{}' \mathbf{X}_{t-1}$ for the liquid regime and by $y_t^{illiquid} = \beta_0 + \beta_{illiq,2} illiq_{t-1} + \beta_{\mathbf{X},2}{}' \mathbf{X}_{t-1}$ for the illiquid regime.

illiquid regime was still tracking "better" actual GDP growth which was, nonetheless, flat. This, arguably, provided a valid justification for additional QE, which was eventually authorised by the MPC in July 2012.

4.2. Forecasting evaluation

Our expanding windows setup delivers 37 one-step-ahead forecasts of GDP growth in real time for all linear and non-linear models. These are compared with (i) one-step-ahead forecasts derived from an AR(1) model, where the AR (1) order has been selected on the basis of the AIC, (ii) the real-time mean forecasts published in BoE's *Inflation Report*, and (iii) the median value of forecasts from all of the estimated models.[15] Table 3 reports the Root Mean Squared Error (RMSE) and the Mean Absolute Error (MAE) for these forecasts.

According to both criteria, the non-linear model with RtoV as the transition variable and divisia money (Model 5 in Table 3) is ranked first. The linear version of the above model with RtoV and divisia money (Model 3), is ranked second. The non-linear model with lagged growth as the transition variable (Model 6) is ranked third and the median of all forecasts (Model 9) is ranked fourth. According to the RMSE (MAE), BoE forecasts (Model 7) are ranked seventh (eighth), whereas the AR model (Model 8) is ranked last.

We provide an out-of-sample forecast comparison of our models using the modified Diebold and Mariano test DM* (for more details, see Harvey et al., 1997, Diebold and Mariano, 1995, and the Appendix). Before turning into a detailed discussion of our results, we note that the literature has challenged the DM* test in two aspects. First, West (1996,

[15] The BoE forecasts refer to the MPC's best collective judgement of the outlook for GDP growth and they are available from http://www.bankofengland.co.uk/publications/Pages/inflationreport/irprobab.aspx. These forecasts are based on the assumption that the BoE base rate follows market expectations (see Britton et al., 1998). BoE also reports one-step-ahead predictions that assume constant interest rates over the forecast period. Their correlation with the one-step-ahead GDP growth forecasts based on market interest rate expectations is 0.99.

2001) and West and McCracken (1998) analyzed modification of forecast comparison tests in light of the use of estimated model parameters in the computation of such tests. However, van Dijk and Franses (2003) pointed out, that for the DM* test under quadratic loss, such parameter estimation uncertainty is asymptotically irrelevant. van Dijk and Franses (2003) went on to argue that corrections of the type suggested by West (1996, 2001) and West and McCracken (1998) are not necessary when examining the statistical significance of RMSE reductions (which is precisely what we are doing in the current paper). Second, under the assumption that the estimation sample size and the number of out-of-sample forecasts tend to infinity, McCracken (2000) and Clark and McCracken (2001) showed that, if the underlying forecasting models are nested, the asymptotic distribution of the DM*-statistic is not standard normal. van Dijk and Franses (2003) noted that these conditions on the sample size and the number of out-of-sample forecasts effectively mean that expanding windows of data are used for estimation.[16]

Table 4 reports the pair-wise out-of-sample forecast comparison using the modified Diebold-Mariano test (DM*). The first entry in cell (i,j) (for i=1,…,9 and j=1,…,9) contains the p-values of the DM* statistic under the null hypothesis of equal forecast accuracy of models i and j against the one-sided alternative that the RMSE of model j is lower. The second entry in (i,j) contains the p-values of the DM* statistic under the null hypothesis of equal forecast accuracy of models i and j against the one-sided alternative that the MAE of model j is lower.

[16] On the other hand, when forecasting is based on fixed-length rolling estimation windows (something we also do below), Giacomini and White (2006) proved that the asymptotic distribution of the DM*-statistic is standard normal when comparing forecasts generated by nested models. Nevertheless, it does not necessarily follow that this asymptotic approximation is a good one (see the discussion in Clark and West, 2007).

The non-linear model with RtoV as the transition variable and divisia money (Model 5), which according to Table 3 is ranked first, delivers a statistically significantly lower MAE relative to all models. It also delivers a significantly lower RMSE relative to all models with the exception of Models 3, 6, 7 and 9, where we only marginally fail to reject the null hypothesis of equal RMSE. The non-linear model with lagged growth as the transition variable (Model 6), which is ranked third, delivers a significantly lower MAE relative to two models only (Model 2 and Model 8). With the exception of the non-linear model with RtoV as the transition variable and money divisia (Model 5), no other model delivers a significantly lower MAE relative to the BoE forecasts (Model 7). All models, with the exception of the BoE forecasts, deliver a significantly lower RMSE relative to the AR model (Model 8); when the MAE criterion is used, Model 2 and the BoE forecasts fail to outperform the AR model. The pooled forecasts (Model 9) constructed by taking the median value across the point forecasts generated by all models deliver significantly lower RMSE and MAE relative to Models 1, 2 and 8.[17]

To visualize how well the models forecast out-of-sample GDP growth, Figure 7a plots the real-time GDP growth rate together with its forecasts from the two non-linear models (where RtoV and lagged growth as used as transition variables) and the forecasts from the BoE published in its *Inflation Report*. Figure 7b plots the real-time GDP growth rate together with its forecasts from of our preferred model (i.e. non-linear with RtoV as the transition variable-model5) and from two linear models (i.e. models 3 and 4 from Table 3). The forecasting performance of the estimated models appears rather similar during liquid

[17] In a forecasting exercise based on final-time data, Kapetanios et al. (2008) consider a wide range of linear and non-linear models of UK growth and conclude that although individual models hardly outperform autoregressive (AR) forecasts, combining forecasts does improve out-of-sample performance. Moreover, Teräsvirta et al. (2005) find that a dynamic STAR model specification generally outperforms linear AR models in forecasting a series of macroeconomic variables in the G7. However, neither study utilizes proxies of aggregate stock market liquidity.

conditions and up until 2007. When growth turns negative in 2008q4, a negative outcome is predicted only by the two non-linear models. The non-linear model with lagged growth as the transition variable predicts a deeper recession compared to the other models. It also predicts the trough of the recession one quarter earlier than it occurred (in 2009q1 instead of 2009q2), whereas the remaining models forecast the trough of the recession with a delay of one quarter. We also note that the two non-linear models come closer than any other model in predicting the 0.01% real-time (flat) growth in 2012q1.

For robustness reasons, our forecasting exercise also reports results based on a sequence of fixed-length rolling windows where each successive window is constructed by shifting the preceding window ahead by one observation. Again, our setup delivers 37 one-step-ahead forecasts of GDP growth in real time for all linear and non-linear models. Table 5 reports the Root Mean Squared Error (RMSE) and the Mean Absolute Error (MAE) for these forecasts and Table 6 reports the pair-wise out-of-sample forecast comparison using the modified Diebold-Mariano test (DM*). Results based on the rolling forecasts are broadly consistent with those based on expanding windows. The non-linear model with RtoV as the transition variable and divisia money (Model 5) is ranked first (see Table 5) and delivers a statistically significantly lower RMSE and MAE relative to all remaining models (see Table 6). As with expanding windows, the linear version of the above model with RtoV and divisia money (Model 3) is ranked second (see Table 5) according to rolling windows, whereas the non-linear model with lagged growth as the transition variable (Model 6) is now ranked fifth according to the MAE criterion and sixth according to the RMSE (see Table 5). From Table 6, with the exception of the non-linear model with RtoV as the transition variable and money divisia (Model 5), no other model delivers a significantly lower MAE or RMSE relative to the BoE forecasts (Model 7). For the sake of

brevity, we do not present here the figures that visualize how well the models forecast out-of-sample GDP growth based on rolling windows. These figures are available upon request.

5. Conclusions

This study examines the predictive ability of stock market illiquidity with respect to real UK GDP growth. We focus on two measures of stock market illiquidity, namely the price impact ratios introduced by Amihud (2002) and Florackis et al. (2011), respectively, and we use data available to policymakers in real time to document a statistically significant relationship between stock market illiquidity and the subsequent real UK economic activity over and above the usual control variables, such as real money, term spread and global economic activity. Furthermore, our findings support the use of divisia money, which has a close relationship to aggregate spending, as a better predictor of UK growth than the routinely used M4 measure of money supply.

We also find that the predictive ability of both stock market illiquidity and divisia money is regime-switching; it becomes stronger during periods of illiquid market conditions and during periods of weak economic growth. We provide a counterfactual experiment which suggests that had liquidity not dried up so dramatically since 2007, the depth of UK recession would have been substantially less severe by some 2.3 percentage points. The previous findings offer indirect support to the implementation of QE by the BoE. Indeed, QE, which boosts liquidity and supports monetary growth, is bound to be more effective in the current context of illiquid conditions and weak economic growth where both liquidity and monetary growth are strong leading indicators of economic growth. An out-of-sample forecasting exercise confirms the superiority of a regime-switching model of illiquid versus liquid stock market conditions in predicting UK growth

better than any other model. In fact, this is the only model that significantly outperforms the forecasts published in the BoE's *Inflation Report*.

To further assess the importance of liquid versus illiquid stock market conditions for the macro-economy, our work can be extended to allow for regime-switching liquidity effects in joint estimates of output growth, inflation and the policy interest rate within a structural Vector Autoregressive framework. We also view the construction of global measures of stock market liquidity by pooling information from the US, UK and Eurozone stock markets as a very promising avenue for future research towards identifying a successful predictor of the world business cycle.

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References

Acharya, V.A. and Pedersen L.H. (2005) "Asset Pricing with Liquidity Risk." *Journal of Financial Economics*, 77, 375-410.

Acharya, V.A., Shin, H.S. and Yorulmazer, T. (2011) "Crisis Resolution and Bank Liquidity." *Review of Financial Studies*, 24, 2166-2205.

Adrian T. and Shin, H. (2008) "Liquidity, Monetary Policy and Financial Cycles" Federal Reserve Bank of New York, *Current issues in Economics and Finance*, 14, 1-7.

Alquist, R., L. Kilian, and Vigfusson, R.J. (2011) "Forecasting the Price of Oil". In Handbook of Economic Forecasting, edited by Graham Elliott and Allan Timmermann, forthcoming. Amsterdam: North Holland.

Amihud, Y. (2002). "Illiquidity and Stock Returns: Cross-section and Time-series Effects." *Journal of Financial Markets*, 5, 31–56.

Angelini, P., Nobili, A. and Picillo, C. (2011) "The Interbank Market after August 2007: What Has Changed, and Why?" *Journal of Money, Credit and Banking*, 43, 924-958.

Bean, C. (2012) "What we Know: Reflections on 15 years at the BoE", The Financial Times, Friday May 4 2012.

Berger, A.N. and Bouwman, C.H.S. (2008) "Financial Crises and Bank Liquidity Creation", University of Pennsylvania Working Paper No. 37.

Blanchard, O., Dell'Ariccia, G. and Mauro, P. (2010) "Rethinking Macroeconomic Policy." *Journal of Money, Credit and Banking*, 42, 199-215.

Blume, M.E. and Keim, D.B. (2012) "Institutional Investors and Stock Market Liquidity: Trends and Relationships", University of Pennsylvania - Wharton School. Available at SSRN: http://ssrn.com/abstract=2147757.

Borio, C. (2009). "Ten Propositions about Liquidity Crises." Bank of International Settlements Working Paper No. 293.

Borio, C. (2013). "The Financial Cycle and Macroeconomics: What Have we Learnt?" *Journal of Banking and Finance*, forthcoming.

Borio, C. (2008). "The Financial Turmoil of 2007-? A Preliminary Assessment and Some Policy Considerations." Bank of International Settlements Working Paper No. 251.

Bridges, J. and Thomas, R. (2012) "The Impact of QE on the UK Economy - Some Supportive Monetarist Arithmetic." Bank of England Working Paper No. 442.

Britton, E., P. Fisher and Whitley, J. (1998) "The Inflation Report Projections: Understanding the Fan Chart." *Bank of England Quarterly Bulletin*, 38, 30-37.

Brunnermeier, M. and Pedersen, L. (2009) “Market Liquidity and Funding Liquidity.” *Review of Financial Studies*, 22, 2201-2238.

Clark, T.E. and M.W. McCracken (2001). “Tests of equal forecast accuracy and encompassing for nested models.” *Journal of Econometrics*, 105, 85-110.

Clark, T.E. and K.D. West (2007). “Approximately Normal Tests for Equal Predictive Accuracy in Nested Models.” *Journal of Econometrics*, 138, 291−311.

Chinn, M.D. and Kucko, J. (2010) “The Predictive Power of the Yield Curve across Countries and Time.” NBER Working Paper No. 16398.

Darrat, A.F., Chopin, M.C. and Lobo, B.J. (2005) “Money and Macroeconomic Performance: Revisiting Divisia Money.” *Review of Financial Economics*, 14, 93-101.

Diebold, F.X. and Mariano, R.S. (1995) “Comparing Predictive Accuracy.” *Journal of Business and Economic Statistics*, 13, 253-263.

Estrella, A. (2005). “Why Does the Yield Curve Predict Output and Inflation.” *Economic Journal*, 115, 722-744.

Estrella, A. and Hardouvelis, G. (1991) “The Term Structure as a Predictor of Real Economic Activity.” *Journal of Finance*, 46, 555-576.

Florackis, C. Gregoriou, A. and Kostakis, A. (2011) “Trading Frequency and Asset Pricing on the London Stock Exchange: Evidence from a New Price Impact Ratio.” *Journal of Banking and Finance*, 35, 3335-3350.

Fornari, F. and Mele, A. (2009) “Financial Volatility and Economic Activity.” London School of Economics Discussion Paper No. 642.

Garratt, A., Lee, K., Pesaran, M.H. and Shin, Y. (2003) “A Long Run Structural Macroeconomic Model of the UK.” *Economic Journal*, 113, 412-455.

Garratt, A., G. Koop, E. Mise and Vahey, S.P. (2009) “Real-time Prediction with U.K. Monetary Aggregates in the Presence of Model Uncertainty.” *Journal of Business and Economic Statistics*, 27, 480-491.

Giacomini, R., and H. White (2006). “Tests of Conditional Predictive Ability.” *Econometrica*, 74, 1545-1578.

Goyenko, R.Y., C.W. Holden and Trzcinka, C.A. (2009) “Do Liquidity Measures Measure Liquidity?” *Journal of Financial Economics*, 92, 153-181.

Granger, C.W.J., and Teräsvirta, T. (1993) “Modeling Nonlinear Economic Relationships.” Oxford: Oxford University Press.

Gromb, D. and Vayanos, D. (2010) “A Model of Financial Market Liquidity based on Intermediary Capital.” *Journal of European Economic Association*, 8, 456-466.

Hameed A., Kang, W. and Viswanathan, S. (2010) “Stock Market Declines and Liquidity.” *Journal of Finance*, 65, 257-293.

Hamilton, J.D. (1996). “This is what Happened to the Oil Price-Macroeconomy Relationship.” *Journal of Monetary Economics*, 38, 225-230.

Hamilton, J.D. (2003). “What is an Oil Shock?” *Journal of Econometrics*, 113, 363-398.

Hancock, M. (2005) “Divisia Money.” *Bank of England Quarterly Bulletin*, 45, 39-46.

Harvey, D., S. Leybourne and Newbold, P. (1997) “Testing the Equality of Prediction Mean Squared Errors.” *International Journal of Forecasting*, 13, 281-291.

Joyce, M., Tong, M, and Woods, R. (2011). “The United Kingdom’s Quantitative Easing Policy: Design, Operation, and Impact.” *Bank of England Quarterly Bulletin*, 51, 200-212.

Kapetanios, G., Labhard, V. and Price, S. (2008) “Forecast Combination and the Bank of England's Suite of Statistical Forecasting Models.” *Economic Modelling*, 25, 772-792.

Levine, R. (1991). “Stock Markets, Growth, and Tax Policy.” *Journal of Finance*, 46, 1445-1465.

Levine, R. and Zervos, S. (1998) “Stock Market, Banks and Economic Growth.” *American Economic Review*, 88, 537-558.

Longstaff, F.A. (2004) “The Flight-to-Quality Premium in U.S. Treasury Bond Prices.” *Journal of Business*, 77, 511-525.

Martin, C. and Milas, C. (2012) “Quantitative Easing: A Sceptical Survey.” *Oxford Review of Economic Policy*, 28, 750-764.

McCracken, M.W. (2000). “Robust out-of-sample inference.” *Journal of Econometrics*, 99, 195-223.

Naes, R., Skjeltorp, J.A. and Ødegaard B.A. (2011). “Stock Market Liquidity and the Business Cycle.” *Journal of Finance*, 66, 139-176.

Ravazollo, F. and Rothman, P. (2013) “Oil and US GDP: A Real-time Out-of-Sample Examination.” *Journal of Money, Credit and Banking*, 45, 449-463.

Rosengren, E. (2010) “The Impact of Liquidity, Securitization, and Banks on the Real Economy.” *Journal of Money, Credit and Banking*, 42, 221-228.

Smith, D. (2011) “Scrambling for Ways to Rev up the Recovery.” *The Sunday Times*, 18 September 2011.

Stock, J.H. and Watson, M.W. (2003) “Forecasting Output and Inflation: The Role of Asset Prices.” *Journal of Economic Literature*, 41, 788-829.

Teräsvirta, T., D. van Dijk, and Medeiros, M.C. (2005) “Linear Models, Smooth Transition Autoregressions, and Neural Networks for Forecasting Macroeconomic Time Series: A re-examination.” *International Journal of Forecasting*, 21, 755-774.

van Dijk, D. and P.H. Franses (2003). “Selecting a nonlinear time series model using weighted tests of equal forecast accuracy.” *Oxford Bulletin of Economics and Statistics*, 65, 727-744.

van Dijk, D., T. Teräsvirta, and Franses, P.H. (2002) “Smooth Transition Autoregressive Models – A Survey of Recent Developments.” *Econometric Reviews*, 21, 1-47.

West, K.D. (1996). Asymptotic inference about predictive ability, *Econometrica*, 64, 1067-1084.

West, K.D. (2001). “Tests for forecast encompassing when forecasts depend on estimated regression parameters.” *Journal of Business & Economic Statistics*, 19, 29-33.

West, K.D. and M.W. McCracken (1998). “Regression based tests of predictive ability.” *International Economic Review*, 39, 817-840.

Zaher, F. (2007). “Evaluating Factor Forecasts for the UK: The Role of Asset Prices.” *International Journal of Forecasting*, 23, 679-693.

Appendix

Diebold-Mariano (DM) and modified DM test statistic

At forecast horizon *h*, this is computed by weighting the forecast loss differentials between two competing models *i* and *j* equally, where the loss differential for observation *t* is given by $d_t \equiv \left[g\left(e_{it|t-h}\right) - g\left(e_{jt|t-h}\right)\right]$, where $g(.)$ is a general function of forecast errors (e.g. RMSE or MAE). The null hypothesis of equal accuracy of the forecasts of two competing models, can be expressed in terms of their corresponding loss functions, $\mathrm{E}\left[g\left(e_{it|t-h}\right)\right] = \mathrm{E}\left[g\left(e_{jt|t-h}\right)\right]$, or equivalently, in terms of their loss differential, $\mathrm{E}\left[d_t\right] = 0$. Let $\bar{d} = \frac{1}{P}\sum_{t=R+h}^{R+P+h-1} d_t$ denote the sample mean loss differential over *t* observations, such that there are *P* out-of-sample point forecasts and *R* observations have been used for estimation. The Diebold-Mariano test statistic follows asymptotically the standard normal distribution: $DM = \frac{\bar{d}}{\sqrt{\frac{2\pi \hat{f}_d(0)}{P}}} \xrightarrow{d} N(0,1)$, where $N(.)$ is the normal distribution and $\hat{f}_d(0)$ is a consistent estimate of the spectral density of the loss differential at frequency 0. To counteract the tendency of the *DM* test statistic to reject the null too often when it is true in cases where the forecast errors are not bivariate normal, Harvey et al. (1997) propose a modified Diebold-Mariano test statistic:

$DM^* = \left[\frac{P + 1 - 2h + P^{-1}h(h-1)}{P}\right]^{1/2} DM \xrightarrow{d} t_{(P-1)}$, where *DM* is the original Diebold and Mariano (1995) test statistic for *h*-step-ahead forecasts and $t_{(P-1)}$ refers to the Student's *t* distribution with $P - 1$ degrees of freedom.

List of Tables

Table 1
Linear estimates of Real UK GDP growth

	Dependent Variable: *Real GDP Growth*			
	(i) Illiquidity measured by RtoTR	*(ii)* Illiquidity measured by RtoTR	*(iii)* Illiquidity measured by RtoV	*(iv)* Illiquidity measured by RtoV
Explanatory Variables				
Intercept	-0.141 (-0.84)	-0.064 (-0.30)	-0.077 (-0.48)	-0.025 (-0.12)
GDP growth$_{t-1}$	0.784 (10.72)	0.882 (12.62)	0.786 (11.22)	0.886 (13.20)
Illiquidity$_{t-1}$	-0.006 (-3.06)	-0.007 (-3.42)	-0.006 (-4.22)	-0.007 (-4.43)
Real divisia money growth$_{t-1}$	0.104 (3.49)	-	0.106 (3.72)	-
Real M4 growth$_{t-1}$	-	0.031 (1.49)	-	0.035 (1.74)
Term spread$_{t-1}$	0.130 (2.86)	0.134 (2.69)	0.134 (3.10)	0.143 (2.99)
US GDP growth$_{t-1}$	0.084 (1.27)	0.032 (0.52)	0.040 (0.66)	-0.014 (-0.17)
Adjusted R^2	0.88	0.87	0.89	0.88
Regression standard error	0.77	0.81	0.74	0.78
AIC	2.38	2.49	2.30	2.41

Notes: This Table shows the estimates of linear models of Real UK GDP growth over the period 1989q2-2012q1 using the last vintage or real GDP (2012q2). *t*-ratios are given in parentheses. AIC stands for the Akaike Information Criterion.

Table 2
Non-linear estimates of Real UK GDP growth

	Dependent Variable: *Real GDP Growth*	
	(i)	***(ii)***
Intercept	0.046 (0.28)	0.036 (0.21)
GDP growth$_{t\text{-}1}$	0.746 (11.06)	0.760 (12.47)
term spread$_{t\text{-}1}$	0.151 (3.63)	0.134 (3.32)
US GDP growth$_{t\text{-}1}$	0.090 (1.60)	0.084 (1.35)
	Illiquid regime of:	***Growth regime of:***
	$illiq_{t-1} < \tau^{illiq}$	$y_{t-1} < \tau^{y}$
Illiquidity$_{t\text{-}1}$	0.001 (0.01)	-0.010 (-3.83)
Real divisia money growth$_{t\text{-}1}$	0.083 (2.15)	0.178 (4.23)
	Illiquid regime of:	***Growth regime of:***
	$illiq_{t-1} > \tau^{illiq}$	$y_{t-1} > \tau^{y}$
Illiquidity$_{t\text{-}1}$	-0.009 (-5.48)	-0.001 (-0.80)
Real divisia money growth$_{t\text{-}1}$	0.160 (4.62)	0.084 (2.35)
τ^{illiq}	-16.141 (-6.71)	-
γ^{illiq}	99.29 (0.37)	-
τ^{y}	-	1.224 (4.66)
γ^{y}	-	46.24 (0.18)
Adjusted R^2	0.90	0.90
Regression standard error	0.70	0.71
AIC	2.21	2.23
Parameter constancy F-test [p-value]	0.67 [0.74]	0.62 [0.78]

Notes: This Table shows estimates of non-linear models of Real UK GDP growth over the period 1989q2-2012q1 using the last vintage or real GDP (2012q2). *t*-ratios are given in parentheses. Parameter constancy is an *F*-test of parameter constancy of the non-linear model which involves testing the statistical significance of the cross-product of all regressors in the non-linear model and time trend (see van Dijk et al., 2002).

Table 3
Ranking of forecasts by RMSE and MAE criteria. Expanding windows

Model *i*	*RMSE (Ranking in parenthesis)*	*MAE (Ranking in parenthesis)*
i=1	0.872 (6)	0.633 (5)
Linear model with RtoTR and Divisia money		
i=2	0.924 (8)	0.675 (7)
Linear model with RtoTR and M4		
i=3	0.805 (2)	0.588 (2)
Linear model with RtoV and Divisia money		
i=4	0.859 (5)	0.635 (6)
Linear model with RtoV and M4		
i=5	0.704 (1)*	0.505 (1)**
Non-linear model with RtoV and Divisia money		
(RtoV is the transition variable)		
i=6	0.820 (3)	0.592 (3)
Non-linear model with RtoV and Divisia money		
(lagged GDP growth is the transition variable)		
i=7	0.885 (7)	0.689 (8)
BoE forecasts published in the BoE's *Inflation Report*		
i= 8	1.027 (9)	0.732 (9)
AR model		
i=9	0.828 (4)	0.604 (4)
Median of all forecasts		

Notes: This Table reports the Root Mean Squared Error (RMSE) and the Mean Absolute Error (MAE) criteria of the forecasts associated with each model (from *i*=1 to *i*=9). RMSE and MAE criteria are based on the expanding windows one-step-ahead forecasts over the period 2003q1-2012q1. The ranking of each model, in terms of its forecasting accuracy, is reported in parentheses. * (**) indicates the models with the highest ranking based on RMSE (MAE).

Table 4
Pair-wise out-of-sample forecast comparison using the modified Diebold-Mariano test (DM*). Expanding windows

	Against Model j								
Model i	1	2	3	4	5	6	7	8	9
1	-	0.854 0.866	**0.075*** **0.020***	0.411 0.525	**0.095*** **0.050***	0.210 0.148	0.565 0.746	0.955 0.938	**0.074*** **0.053***
2	0.146 0.134	-	**0.041*** **0.029***	**0.061*** **0.031***	**0.063*** **0.025***	0.139 **0.073***	0.353 0.558	0.932 0.855	**0.023*** **0.016***
3	0.925 0.980	0.959 0.971	-	0.876 0.904	0.123 **0.089***	0.601 0.551	0.809 0.877	0.977 0.974	0.811 0.817
4	0.589 0.475	0.939 0.969	0.124 **0.096***	-	**0.078*** **0.032***	0.331 0.197	0.599 0.724	0.965 0.934	0.216 0.121
5	0.905 0.950	0.937 0.975	0.877 0.911	0.922 0.968	-	0.891 0.918	0.892 0.962	0.966 0.980	0.870 0.920
6	0.790 0.852	0.861 0.927	0.399 0.449	0.669 0.803	0.109 **0.082***	-	0.757 0.875	0.935 0.941	0.547 0.618
7	0.435 0.254	0.647 0.442	0.191 0.123	0.401 0.276	0.108 **0.038***	0.243 0.125	-	0.834 0.651	0.239 0.147
8	**0.045*** **0.062***	**0.068*** 0.145	**0.023*** **0.026***	**0.035*** **0.066***	**0.034*** **0.020***	**0.065*** **0.059***	0.166 0.349	-	**0.022*** **0.025***
9	0.926 0.947	0.977 0.984	0.189 0.183	0.784 0.879	0.130 **0.080***	0.453 0.382	0.761 0.853	0.978 0.985	-

Notes: This Table presents pair-wise out-of-sample forecast comparisons for the 9 forecasting models and expanding windows, at the $h = 1$ forecast horizon using the modified (*DM**) Diebold-Mariano test statistic of Harvey et al. (1997). The first entry in cell (*i*,*j*) contains the *p*-values of the modified DM* statistic of Harvey et al. (1997) for testing the null hypothesis of equal forecast accuracy of models *i* and *j* against the one-sided alternative that the RMSE of model *j* is lower. The second entry in (*i*,*j*) contains the *p*-values of the modified DM* statistic of Harvey et al. (1997) for testing the null hypothesis of equal forecast accuracy of models *i* and *j* against the one-sided alternative that the MAE of model *j* is lower. An asterisk (*) denotes statistical significance at the 10% level. *i*=1 refers to the linear model with RtoTR and Divisia money. *i*=2 refers to the linear model with RtoTR and M4. *i*=3 refers to the linear model with RtoV and Divisia money. *i*=4 refers to the linear model with RtoV and M4. *i*=5 refers to the non-linear model with RtoV and Divisia money where RtoV is the transition variable. *i*=6 refers to the non-linear model with RtoV and Divisia money where lagged GDP growth is the transition variable. *i*=7 refers to the BoE forecasts published in the BoE's *Inflation Report*. *i*=8 refers to the AR model. *i*=9 refers to the median of all forecasts.

Table 5
Ranking of forecasts by RMSE and MAE criteria. Rolling windows

Model *i*	*RMSE (Ranking in parenthesis)*	*MAE (Ranking in parenthesis)*
i=1	0.853 (4)	0.632 (4)
Linear model with RtoTR and Divisia money		
i=2	0.984 (8)	0.712 (8)
Linear model with RtoTR and M4		
i=3	0.806 (2)	0.592 (2)
Linear model with RtoV and Divisia money		
i=4	0.930 (7)	0.670 (6)
Linear model with RtoV and M4		
i=5	0.696 (1)*	0.516 (1)**
Non-linear model with RtoV and Divisia money		
(RtoV is the transition variable)		
i=6	0.903 (6)	0.669 (5)
Non-linear model with RtoV and Divisia money		
(lagged GDP growth is the transition variable)		
i=7	0.886 (5)	0.689 (7)
BoE forecasts published in the BoE's *Inflation Report*		
i= 8	1.101 (9)	0.751 (9)
AR model		
i=9	0.833 (3)	0.600 (3)
Median of all forecasts		

Notes: This Table reports the Root Mean Squared Error (RMSE) and the Mean Absolute Error (MAE) criteria of the forecasts associated with each model (from *i*=1 to *i*=9). RMSE and MAE criteria are based on the rolling windows one-step-ahead forecasts over the period 2003q1-2012q1. The ranking of each model, in terms of its forecasting accuracy, is reported in parentheses. * (**) indicates the models with the highest ranking based on RMSE (MAE).

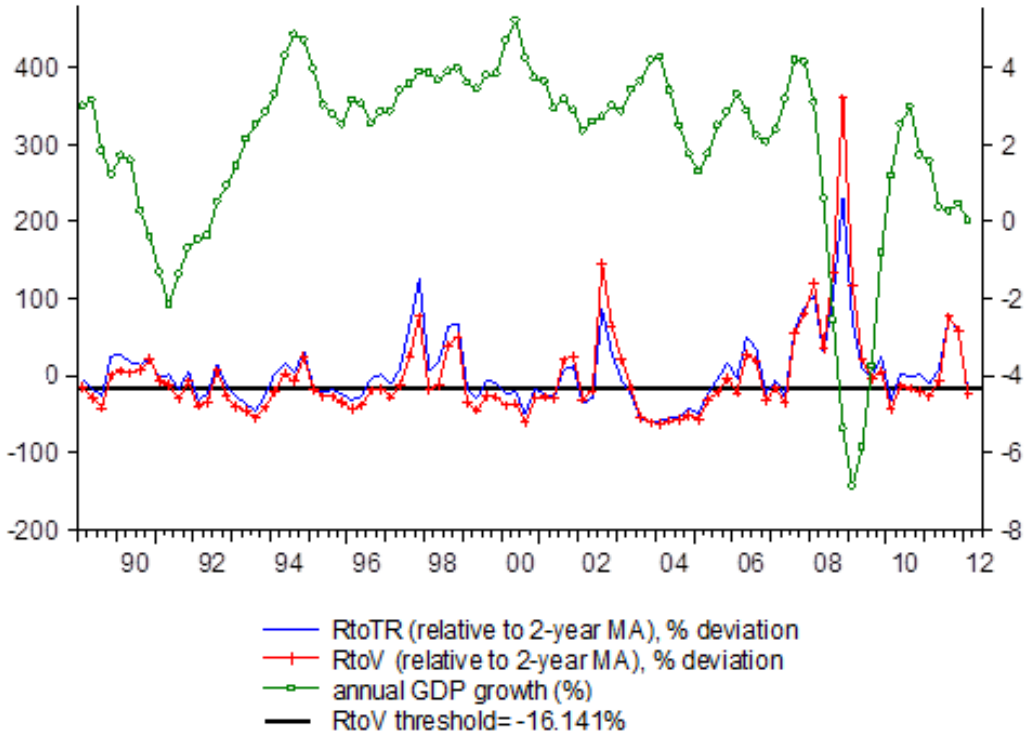
Table 6
Pair-wise out-of-sample forecast comparison using the modified Diebold-Mariano test (DM*). Rolling windows

	Against Model j								
Model i	1	2	3	4	5	6	7	8	9
1	-	0.905 0.871	**0.029*** **0.011***	0.772 0.701	**0.034*** **0.023***	0.648 0.664	0.582 0.688	0.983 0.948	0.341 0.177
2	**0.095*** 0.129	-	**0.032*** **0.035***	**0.040*** **0.016***	**0.009*** **0.006***	0.262 0.319	0.242 0.419	0.875 0.690	**0.012*** **0.009***
3	0.971 0.989	0.968 0.965	-	0.908 0.887	**0.047*** **0.054***	0.780 0.815	0.697 0.799	0.987 0.976	0.767 0.622
4	0.228 0.299	0.960 0.984	**0.092*** 0.113	-	**0.015*** **0.015***	0.413 0.479	0.373 0.568	0.908 0.815	**0.066*** **0.070***
5	0.966 0.977	0.991 0.994	0.953 0.964	0.985 0.985	-	0.971 0.990	0.903 0.954	0.985 0.984	0.970 0.957
6	0.352 0.336	0.738 0.681	0.220 0.185	0.587 0.503	**0.029*** **0.010***	-	0.430 0.592	0.864 0.759	0.273 0.195
7	0.418 0.312	0.758 0.581	0.303 0.201	0.627 0.432	**0.097*** **0.046***	0.570 0.408	-	0.867 0.676	0.348 0.206
8	**0.017*** **0.052***	0.125 0.310	**0.013*** **0.024***	**0.092*** 0.180	**0.015*** **0.016***	0.136 0.241	0.133 0.324	-	**0.014*** **0.019***
9	0.659 0.823	0.988 0.991	0.233 0.378	0.934 0.930	**0.030*** **0.043***	0.727 0.805	0.652 0.794	0.986 0.981	-

Notes: This Table presents pair-wise out-of-sample forecast comparisons for the 9 forecasting models and rolling windows, at the h = 1 forecast horizon using the modified (*DM**) Diebold-Mariano test statistic of Harvey et al. (1997). The first entry in cell (i,j) contains the p-values of the modified DM* statistic of Harvey et al. (1997) for testing the null hypothesis of equal forecast accuracy of models i and j against the one-sided alternative that the RMSE of model j is lower. The second entry in (i,j) contains the p-values of the modified DM* statistic of Harvey et al. (1997) for testing the null hypothesis of equal forecast accuracy of models i and j against the one-sided alternative that the MAE of model j is lower. An asterisk (*) denotes statistical significance at the 10% level. i=1 refers to the linear model with RtoTR and Divisia money. i=2 refers to the linear model with RtoTR and M4. i=3 refers to the linear model with RtoV and Divisia money. i=4 refers to the linear model with RtoV and M4. i=5 refers to the non-linear model with RtoV and Divisia money where RtoV is the transition variable. i=6 refers to the non-linear model with RtoV and Divisia money where lagged GDP growth is the transition variable. i=7 refers to the BoE forecasts published in the BoE's *Inflation Report*. i=8 refers to the AR model. i=9 refers to the median of all forecasts.

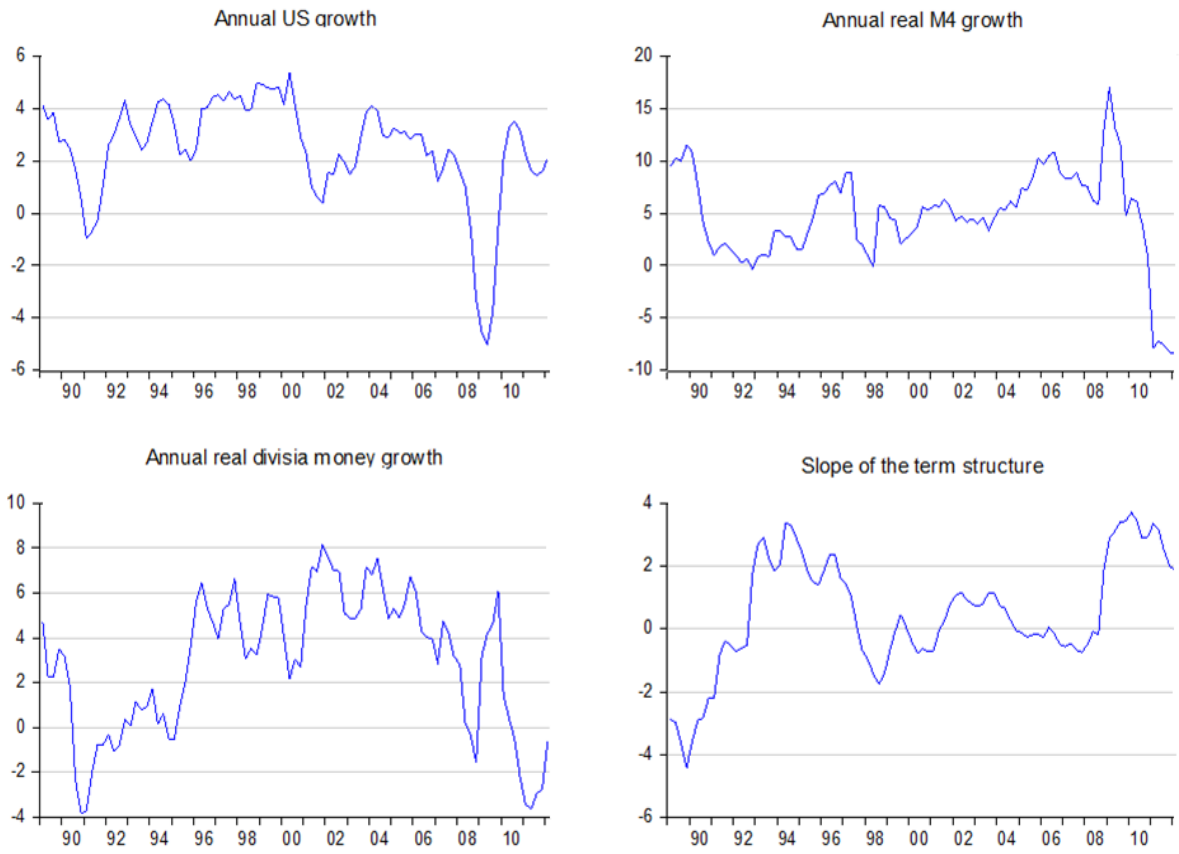
List of Figures

Figure 1
Stock market illiquidity measures and Real UK GDP growth



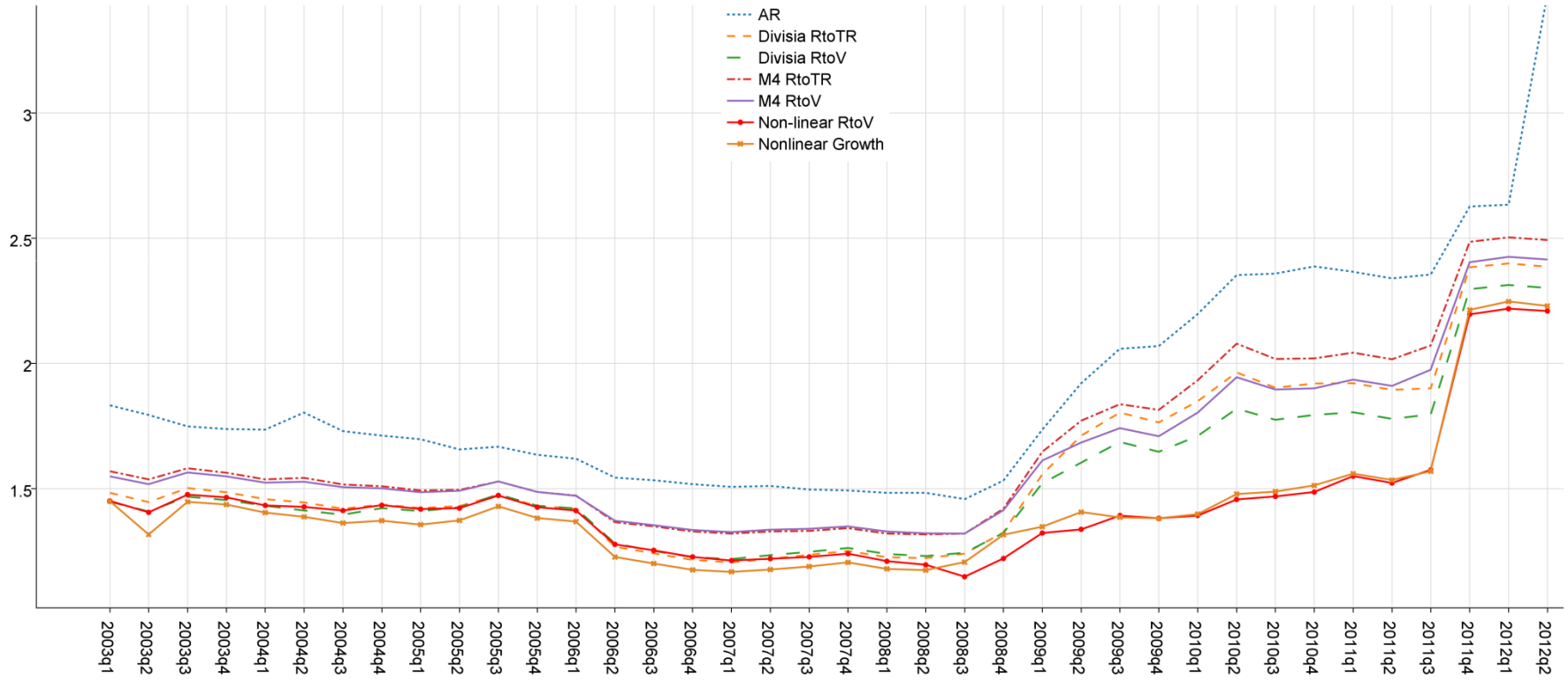
Notes: This Figure shows the time series of the two stock market illiquidity measures, Return-to-Volume (RtoV) and Return-to-Turnover Ratio (RtoTR), as percentage deviations relative to their past 2-year moving averages (LHS axis) and the real UK GDP growth rate (RHS axis) over the period 1989q2-2012q1. The Real UK GDP growth rate is calculated on the basis of the 2012q2 vintage data released by the Bank of England.

Figure 2
Real US GDP growth, real M4 growth, real divisia money growth and term spread



Notes: This Figure shows the time series of the real US GDP growth, real UK M4 growth, real UK divisia money growth rates and the slope of the term structure (term spread) in the UK market over the period 1989q2-2012q1.

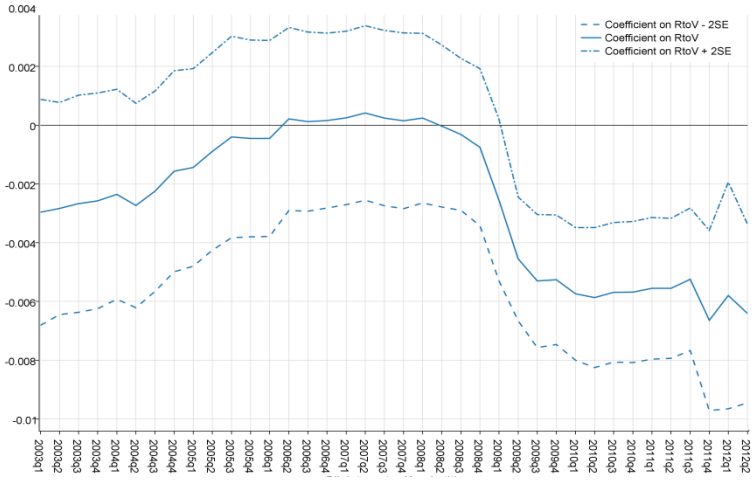
Figure 3
Akaike Information Criterion (AIC) over successive real-time vintages



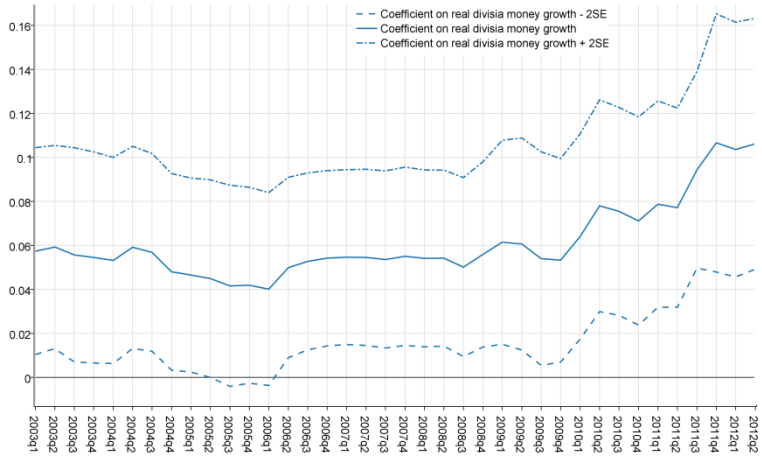
Notes: This Figure shows the value of the Akaike Information Criterion (AIC) from estimating recursively each of the models defined in Table 1 and 2 using successive real-time data vintages.

Figure 4
Parameter estimates and confidence intervals for:

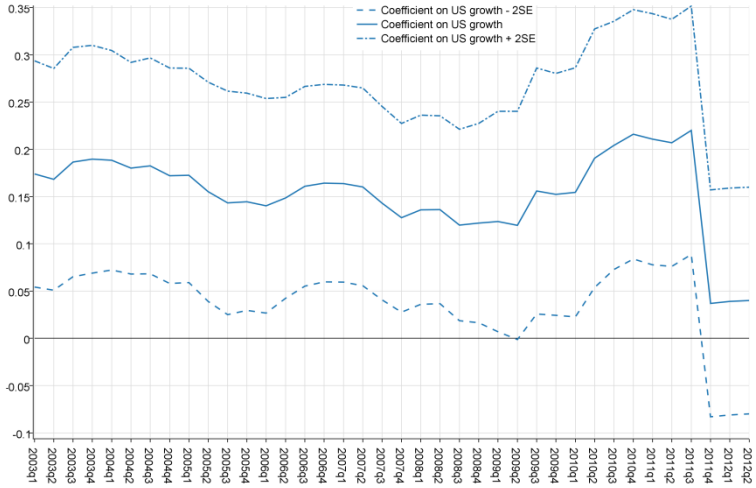
i. RtoV (stock market illiquidity)



ii. Real divisia money growth



iii. Global Economic Activity (US GDP growth)



iv. Slope of the Term Structure (term spread)

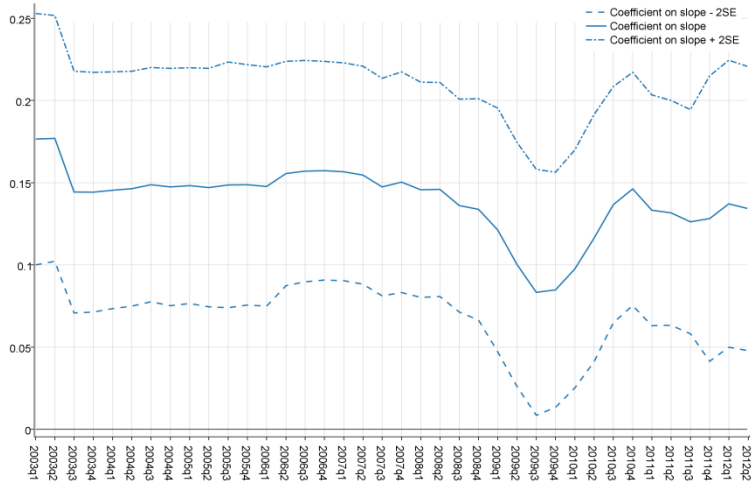
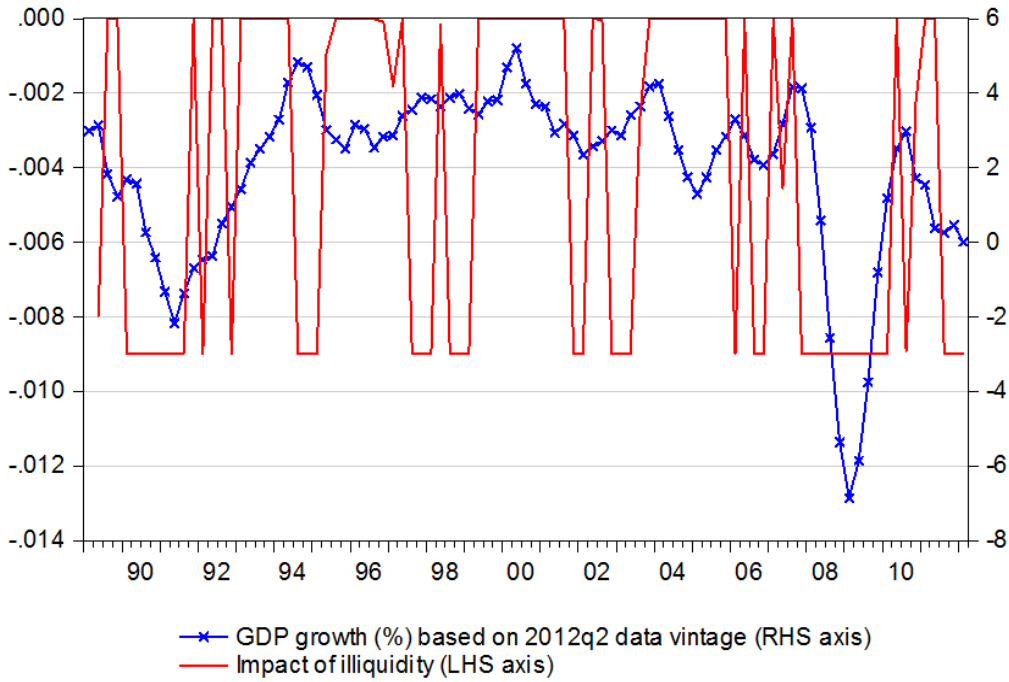
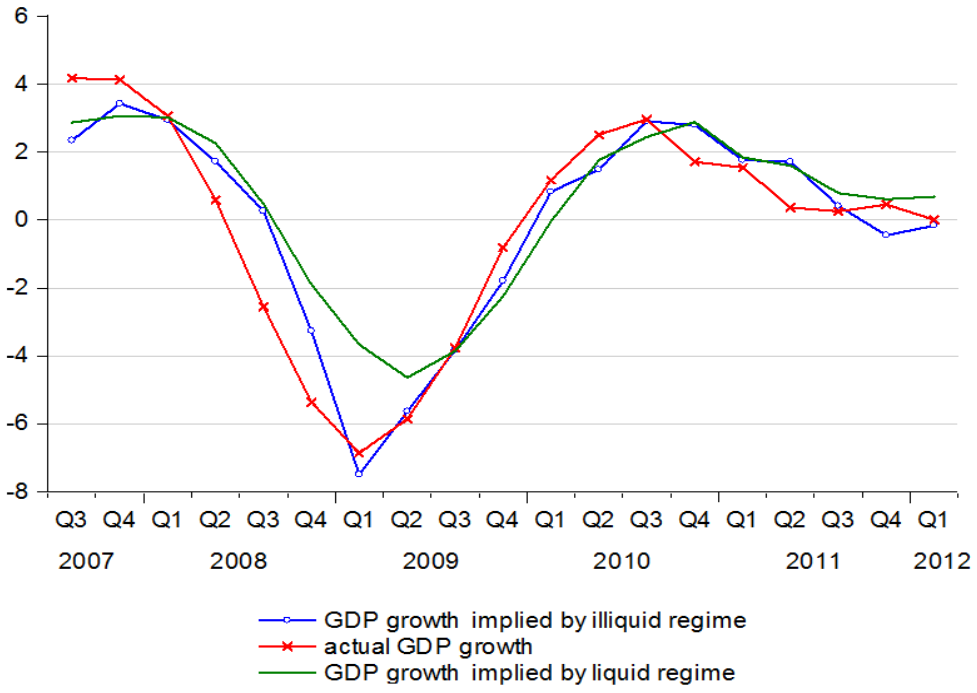


Figure 5
The regime-switching impact of illiquidity on UK economic growth, 1989q2-2012q1



***Notes*:** This Figure shows the impact of stock market illiquidity (LHS axis) and actual real UK GDP growth rate (RHS axis). The impact of illiquidity is calculated as: $\beta_{illiq,1}\theta^{illiq}{}_{t-1} + \beta_{illiq,2}(1-\theta^{illiq}{}_{t-1})$, with $\beta_{illiq,1}$=0 (imposed), $\beta_{illiq,2}$= -0.009, τ^{illiq} = -16.141%, and γ^{illiq} =99.29, based on the estimates reported in Table 2(i) for the non-linear model that includes the illiquidity measure RtoV and real money divisia growth and uses RtoV as the transition variable.

Figure 6
Actual and implied regime-specific Real UK GDP growth rates since 2007q3



Notes: This Figure shows the actual real UK GDP growth rate along with the real UK GDP growth rates since 2007q3 implied by a liquid and an illiquid market regime, respectively, based on the estimates reported in Table 2(i) for the non-linear model that includes the illiquidity measure RtoV and real money divisia growth and uses RtoV as the transition variable. The model has been estimated using the last vintage of real UK GDP data (2012q2).

Figure 7

a. One-step-ahead forecasts of Real UK GDP growth rate

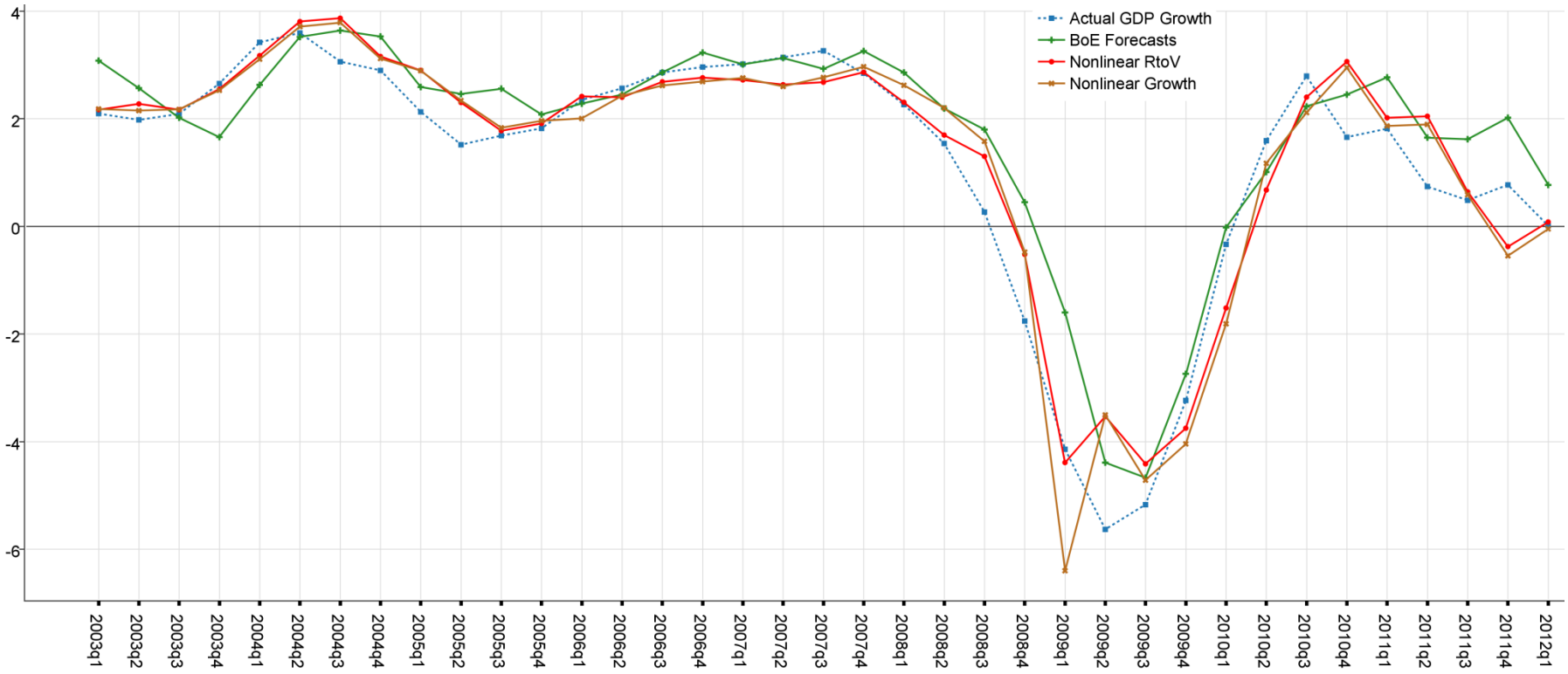
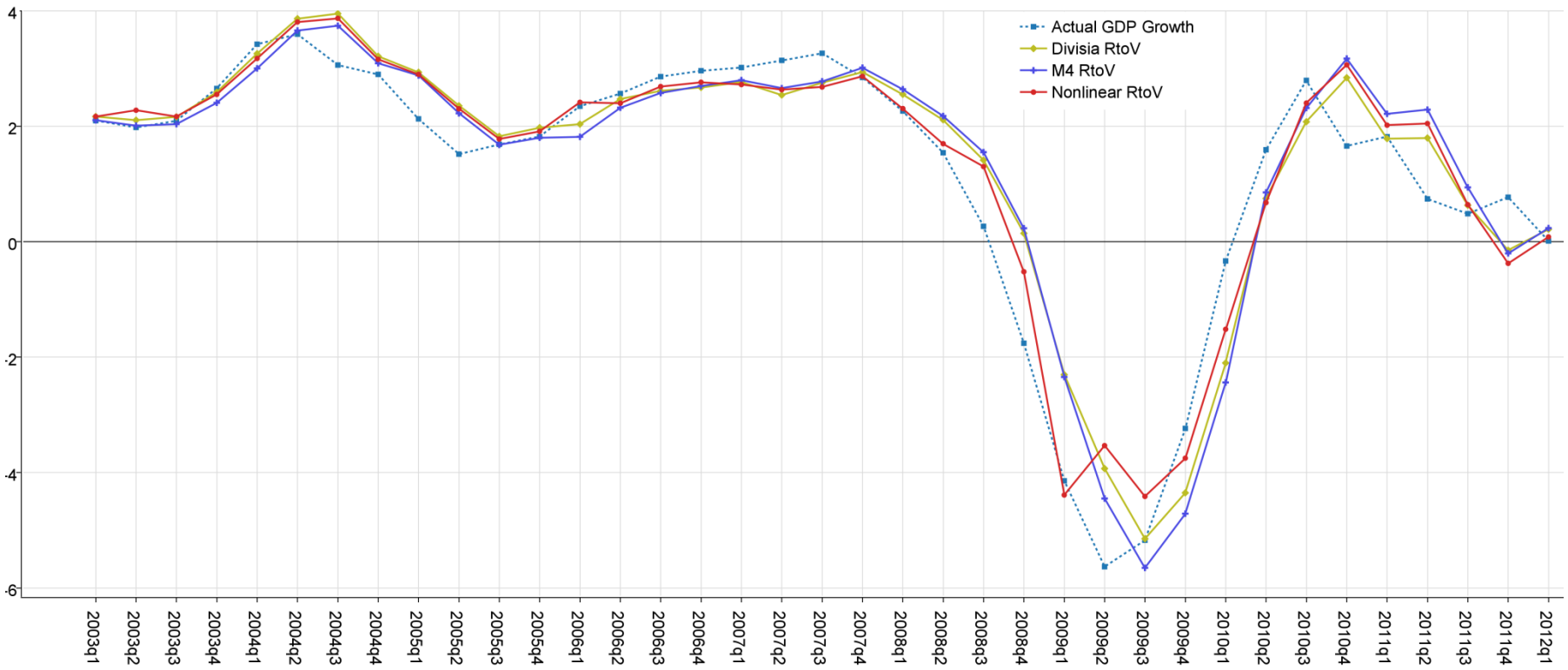


Figure 7
b. One-step-ahead forecasts of Real UK GDP growth rate (cont'd)



Notes: Figure 7a plots the real-time GDP growth rate together with its forecasts from the two non-linear models (where RtoV and lagged growth as used as transition variables) and the forecasts from the BoE as published in its *Inflation Report*. Figure 7b plots the real-time GDP growth rate together with its forecasts from of our preferred model (i.e. non-linear with RtoV as the transition variable) and from two linear models (models 3 and 4 from Table 3).