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Integrated inventory management and supplier base reduction in a supply chain with multiple uncertainties

Dong-Ping Song [a,*], Jing-Xin Dong [b], Jingjing Xu [a,1]

[a] School of Management, Plymouth University, Drake Circus, Plymouth PL4 8AA, UK
[b] Business School, Newcastle University, Newcastle NE1 4SE, UK

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ABSTRACT

This paper considers a manufacturing supply chain with multiple suppliers in the presence of multiple uncertainties such as uncertain material supplies, stochastic production times, and random customer demands. The system is subject to supply and production capacity constraints. We formulate the integrated inventory management policy for raw material procurement and production control using the stochastic dynamic programming approach. We then investigate the supplier base reduction strategies and the supplier differentiation issue under the integrated inventory management policy. The qualitative relationships between the supplier base size, the supplier capabilities and the total expected cost are established. Insights into differentiating the procurement decisions to different suppliers are provided. The model further enables us to quantitatively achieve the trade-off between the supplier base reduction and the supplier capability improvement, and quantify the supplier differentiation in terms of procurement decisions. Numerical examples are given to illustrate the results.

1. Introduction

Manufacturer-oriented supply chain systems are concerned with the effective management of the flow and storage of goods in the process from the procurement of raw materials from the suppliers to the delivery of finished goods to the customers. Here the flow refers to transporting materials and the storage refers to holding inventory. One of the key challenges in supply chain management is how to appropriately tackle and respond to a variety of uncertain factors such as supply uncertainty and disruption (Lu, Huang, & Shen, 2011; Snyder et al., 2012), imperfect production or defective items (Pal, Sana, & Chaudhuri, 2013; Sana, 2010), unreliable machines (Pal et al., 2013; Song & Sun, 1998); stochastic processing times (Buzacott & Shanthikumar, 1993; Song, 2013), and random demands (Masih-Tehrani, Xu, Kumara, & Li, 2011).

Traditionally, procurement policy about lot sizing decisions for raw materials was often separated from the production control systems so that the complexity and interplay of the functional areas like procurement, inventory, production and scheduling are decomposed and reduced. Such treatment is useful to simplify the management problem, and may be appropriate in situations with loose connections between functional areas. However, from a systemic viewpoint, it may lead to sub-optimal solutions and the system may perform far away from the optimum. In the last two decades, much attention has been paid to the coordination between procurement management and production management along the development of the supply chain management concept (e.g. Arshinder, Kanda, & Deshmukh, 2008; Goyal & Deshmukh, 1992; and the references therein). This paper will consider the optimal integrated procurement and production problem for a manufacturing supply chain with multiple suppliers in the presence of multiple uncertainties such as uncertain material supplies, stochastic production times, and random customer demands. In the following, we review and classify the relevant literature into two groups. The first group focuses on the sourcing (procurement) problems among multiple suppliers under supply uncertainty; and the second group focuses on the integrated inventory management for production systems subject to two or multiple types of uncertainties.

With respect to the first group, Snyder et al. (2012) described several forms of supply uncertainty including: disruptions, yield uncertainty, capacity uncertainty, lead-time uncertainty, and input cost uncertainty; however, the boundaries among these forms are often blurry. Sourcing from multiple suppliers is an important strategy to deal with the supply uncertainty. The general sourcing (ordering) problem is to determine from how many and which suppliers to source the commodity (or raw material) and in what quantities in order to minimise total expected cost. A significant number of studies have been conducted in this area in the last

* Corresponding author. Tel.: +44 1752 585630; fax: +44 1752 585633.
E-mail addresses: Dongping.song@plymouth.ac.uk (D.-P. Song), jingxin.dong@ncl.ac.uk (J.-X. Dong), Jingjing.xu@plymouth.ac.uk (J. Xu).
[1] Tel.: +44 1752 585630; fax: +44 1752 585633.

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two decades (cf. the review papers: Ho, Xu, & Dey, 2010; Minner, 2003; Qi, 2013; Thomas & Tyworth, 2006; Snyder et al., 2012). More specifically, Lau and Zhao (1993) considered an inventory system with two suppliers subject to stochastic lead times and demands. They presented a procedure to determine the optimal order-splitting policy (i.e. the total order quantity, reorder point and proportion of split between two suppliers). Anupindi and Akella (1993) studied a dual sourcing problem with stochastic demand and supply uncertainty (e.g. random disruption and yield uncertainty). They proved that the optimal ordering policy has three action regions depending on the on-hand inventory level. Agrawal and Nahmias (1997) developed a mathematical model to optimise the number of suppliers with yield uncertainty. They assumed that the yield from an order is the placed order size multiplied by a normal random variable, which implies that larger orders have higher yield variance. Such effect favours smaller orders from many suppliers. On the other hand, more suppliers incur additional fixed costs associated with each supplier such as qualifying new suppliers, supplier development, and more logistics problems. Their model is able to find the optimal number of suppliers that balances these two competing objectives. Berger, Gerstenfeld, and Zeng (2004) examined the single versus multiple sourcing problem from the risk management viewpoint and presented a decision-tree based optimisation model to evaluate the performance of the two procurement approaches. Here the risks refer to catastrophic events that affect many/all suppliers, and unique events that affect only a single supplier. Berger and Zeng (2006) extended the above decision-tree approach to considering unpredictable operations interruptions caused by all suppliers failing to satisfy the buyer's demand so that the optimal size of the supply base can be determined. Ruiz-Torres and Mahmoodi (2007) also utilised the decision tree approach to determine the optimal number of suppliers taking into account various levels of supplier failure probability and possible procurement cost savings gained from using less reliable suppliers. Dada, Petruzzi, and Schwarz (2007) formulated a newsvendor model for the procurement decisions from multiple unreliable suppliers in which demand is stochastic and supply uncertainty can reflect disruptions, yield uncertainty, and capacity uncertainty. They showed that if a given supplier is not used, then no more expensive suppliers than this supplier should be used. Federgruen and Yang (2008, 2009) examined the supplier selection and diversification issues in the similar inventory system to that of Dada et al. (2007), but with different cost structures.

Burke, Carrillo, and Vakharia (2007) contrasted the preference of single versus multiple supplier sourcing strategies in a single period, single product sourcing decisions under demand uncertainty. They showed that single sourcing strategy is preferred only when supplier capacities are large relative to the product demand and when the manufacturer does not obtain diversification benefits. In other cases, the multiple sourcing strategy is preferred. Jokar and Sajadieh (2008) considered a multiple sourcing inventory system with stochastic lead-times and constant demand under the reorder point-order quantity inventory control policy on a continuous-review scheme. They presented a mathematical model that is able to determine the optimal number of identical suppliers and quantify the difference between multiple-sourcing and sole-sourcing strategies. Sarkar and Mohapatra (2009) formulated a model with a decision tree-like structure to determine the optimal size of supply base by considering the risks of supply disruption caused by different types of events. Masih-Tehrani et al. (2011) showed that risk diversification is preferred in a multi-manufacturer-one-retailer system with stochastically dependent supply capacities, and indicated that if the retailer ignores the effect of dependent disruptions it would overestimate the fill rate and tend to order more than the optimum. Lu et al. (2011) considered the optimal sourcing policy in a supply chain with product substitution and dual sourcing under random supply failures. Mirahmadi, Saberi, and Teimoury (2012) used a decision tree approach to determine the optimal number of suppliers taking into account the supply risk and the associated costs (e.g. cost of supplier development, missing discount in volume, loss due to supply postponement). Silbermayr and Minner (2012) presented a semi-Markov decision process for the optimal sourcing problem with multiple suppliers in which demands, lead times and supplier availability are all stochastic. They showed that the optimal sourcing strategy (depending on the on-hand inventory, the outstanding orders and the supplier availability statuses) is rather complex. Pal, Sana, and Chaudhuri (2012a) addressed a multi-echelon suppler chain with two suppliers in which the main supplier may face supply disruption and the secondary supplier is reliable but more expensive, and the manufacturer may produce defective items. Arts and Kiesmuller (2013) studied a serial two-echelon periodic-review inventory system with two supply modes, and showed that dual-sourcing can lead to significant cost savings in cases with high demand uncertainty, high backlogging cost or long lead times.

With regard to the second group that addresses production-inventory control in the presence of uncertainties, a rich literature existed. The earliest relevant research could date back to early 1960s, e.g. Clark and Scarf (1960) studied the multi-stage or multi-echelon inventory systems with random demand and deterministic lead-time. When two or more types of uncertainties are modelled, the optimal production and inventory policies are often addressed within a single-stage, two-stage, or three-stage context. In the following, we mainly select the relevant literature considering two or multiple uncertainties with an emphasis on stochastic lead times.

Hadley and Whitin (1963) addressed a single-stage inventory management problem and identified the optimal inventory control policies for some special cases with restrictive assumptions, e.g. orders do not cross each other and they are independent. Zipkin (1986) characterized the distributions of inventory level and inventory position in continuous-time single-stage models with stochastic demand and lead times. Bassok and Akella (1991) investigated the optimal production level and order quantity with supply quality and demand uncertainty. Song and Zipkin (1996) studied a single-stage system with random demand and Markov modulated lead-times, and were able to characterise the optimal inventory control policy. Berman and Kim (2001) examined the optimal dynamic ordering problem in a two-stage supply chain with Erlang distributed lead-times, exponential service times and Poisson customer arrivals. They showed the optimal ordering policy has a monotonic threshold structure. Berman and Kim (2004) extended the above model to including revenue generated upon the service considering both exponential and Erlang lead times. He, Jewkes, and Buzacott (2002) considered a two-echelon make-to-order system with Poisson demand, exponential processing times, and zero lead times for ordering raw materials, and explored the structure of the optimal replenishment policy. Yang (2004) studies a periodic-review production control problem where both the raw material supply and product demand are exogenous and random. He was able to establish the partial characterisation of the optimal policies under both strict convex and linear raw material purchasing/selling costs. Simchi-Levi and Zhao (2005) investigated the safety stock positioning problem in multistage supply chains with tree network structure with stochastic demands and lead times, in which a continuous-time base-stock policy is used in each stage to control its inventory. Mukhopadhyay and Ma (2009) considered the optimal procurement and production quantity for a remanufacturing company with uncertain market demand. Song (2009) investigated the optimal integrated ordering and production control in a supply chain with a single supplier and multiple uncertainties. Muharremoglu and Yang (2010) applied the base-stock policy to single and multistage inventory systems with stochastic lead times and provided a method to determine base-stock levels and to compute the costs of a given base-stock policy. Pal, Sana, and Chaudhuri (2012b) presented an analytical method to optimise the production rate and raw material order size in a three-layer supply chain subject to imperfect quality raw materials, unreliable machine and defective product reworking. Sana (2012) presented a collaborative inventory model for a three-layer supply chain subject to defective items in production and transportation, and determined the optimal production rate, order quantity, and number of shipments. Song (2013) examined the optimal and sub-optimal integrated ordering and production policies in several stochastic supply chain systems.

From the above literature review, it can be observed that the first group of studies mainly focused on the sourcing strategies from multiple suppliers with supply uncertainty and did not explicitly consider the production decisions and processing uncertainty; whereas the second group mainly focused on production and inventory management in supply chain systems with two or more types of uncertainties (e.g. demand and lead time) but with a single supplier. From the supply chain integration perspective, manufacturers intend to reduce supplier base to a manageable size so that a closer relationship with suppliers can be established (Goffin, Szwejczewski, & New, 1997). The manufacturer may gain benefits of cheaper unit costs and more reliable delivery performance from suppliers, whereas the suppliers can gain benefits of larger and more stable demands. On the other hand, from the supply uncertainty perspective, manufacturers intend to source from multiple suppliers to buffer against the supply uncertainty. It is clear that there is a trade-off between reducing the impact of uncertainties (by having a larger number of suppliers) and reducing the procurement costs associated with each supplier (by having a smaller size of supplier base and closer relationship). However, it is believed that the supplier base reduction strategy is related to the procurement policy, the production policy and other stochastic factors in the manufacturing supply chain. Therefore, there is a need to investigate the integrated material procurement and production control policy and supplier base reduction strategies in manufacturer-oriented supply chains with multiple types of uncertainties.

This paper considers a manufacturing supply chain with multiple suppliers in the presence of multiple uncertainties with an emphasis on integrated inventory management, supplier base reduction and supplier differentiation. The objective function is the expected total cost consisting of raw material inventory cost, raw material ordering processing costs, finished goods inventory cost, and customer demand backlogging cost. The integrated inventory management problem concerns the joint decision-making including when and in what quantities to procure raw materials from which suppliers, and when to produce finished goods. The supplier base reduction strategy concerns the trade-off between reducing the number of suppliers and requiring the higher capabilities of suppliers (such as higher shipping capacity, shorter lead-time, more reliable delivery, and lower order processing cost). The supplier differentiation concerns the difference and relationship of the procurement decisions between different suppliers. Here the supplier differentiation is slightly different from the concept of order splitting in the literature, which refers to dividing a large order into smaller orders among multiple suppliers in order to reduce the effective replenishment lead-time (Thomas & Tyworth, 2006). Specific research objectives of this study include:

- formulate the integrated inventory management problem for raw material procurement and production control in a manufacturing supply chain with multiple suppliers and multiple types of uncertainties, and seek the optimal policy;
- establish the qualitative relationship between the expected total cost and the supplier base size, and the supplier capabilities (such as the suppliers' delivery capacity, the suppliers' delivery reliability); and establish the qualitative relationship of the optimal procurement decisions between different suppliers;
- evaluate the quantitative impacts of the supplier based reduction strategies, i.e. supplier base reduction combined with supplier capability improvement, on the system performance so that a trade-off can be achieved; and also quantify the supplier differentiation.

The rest of this paper is organised as follows. In the next section, the system under consideration is described and formulated mathematically. The optimal integrated policy for raw material procurement and production control is presented using the stochastic dynamic programming approach. In Section 3, the qualitative relationships between the expected total cost and the supplier base size, and the supplier capabilities are established. In Section 4, several supplier base reduction strategies are presented and the trade-off effect is discussed. In Section 5, the supplier differentiation issue is addressed and insights are provided. In Section 6, we extend the model by relaxing a key assumption. In Section 7, a range of numerical scenarios are analysed to verify and illustrate the analytical results. The best trade-off between the supplier base reduction and the supplier capability improvement is achieved. The structural characteristics of the optimal procurement and production policies are explored and discussed. Finally, the main contributions and the managerial insights are concluded in Section 8.

2. Optimal integrated procurement and production policy

The supply chain under consideration consists of three levels of entities, i.e. suppliers, manufacturer and customers. It is assumed there is sufficient warehouse capacity to store raw material (RM) and finished goods (FG). There are N suppliers that are contracted with the manufacturer to supply the raw materials. The manufacturer may place different sizes of orders to different suppliers to buffer against the uncertainty in RM supply. The quantity of an order to supplier i, denoted as q_i, is a decision variable that is constrained by the maximum order quantity Q_i, which represents the delivery capacity of supplier i. The RM replenishment lead-time from supplier i to the manufacturer is a random variable following an exponential distribution with the mean $1/\lambda_i$. The lead-time is assumed to be independent on actual order quantity, which may be justified by the fact that the dispatching equipment usually can deliver up to the quantity Q_i in a single trip. It is assumed that one unit of RM is required to produce one unit of product. The manufacturer produces one product at a time and the processing time is exponentially distributed with the mean $1/u$. Physically, u represents the production (or service) rate (e.g. Veatch & Wein, 1994). It is a control variable that takes 0 or U, which represents an action "not produce" or "produce at a speed U", respectively (the model can be extended easily to the case of allowing u taking more values between 0 and U, but the results remain the same). Customer demands arrive one at a time following a Poisson process with arriving rate ξ. A demand is satisfied immediately if there are FGs stored in the warehouse; otherwise unmet demands are backlogged.

It is assumed that the supply chain is integrated in the sense that the suppliers and the manufacturer have an agreement that the manufacturer can adjust the order quantity at any future decision point before it arrives (Song, 2009). This may be regarded as a type of partnership between the suppliers and the manufacturer, in which the supply chain aims to meet the final customers as closely as possible and reduces the downstream inventory level. Note that downstream inventory usually incurs higher inventory holding costs since more time and effort has been committed. However, two types of costs associated with RM procurement and delivery processes will be incurred to the manufacturer. The first is a fixed cost which is charged whenever there exists a non-zero outstanding order regardless of the order size. The second is a variable cost which is proportional to the order size of the outstanding order. The assumption that the outstanding order can be modified at any time before arrival together with the exponential lead-time assumption implies that there is no more than one outstanding order at any time for each supplier. The assumption of at most one outstanding order at any time was first introduced in Hadley and Whitin (1963), and often used in other literature, e.g. Berman and Kim (2001, 2004) and Kim (2005). The exponential order replenishment lead-time was also assumed in Berman and Kim (2004), Kim (2005), and Silbermayr and Minner (2012). The exponential manufacturing time has been adopted in more literature, e.g. Ching, Chan, and Zhou (1997), Feng and Yan (2000), Feng and Xiao (2002), He et al. (2002), Song and Sun (1998) and Veatch and Wein (1992, 1994). The Poisson demand arrival is one of the most common assumptions in the related literature (e.g. Buzacott & Shanthikumar, 1993). Note that the assumption of adjustable outstanding orders is rather restrictive from the practical perspective. We will relax this assumption in the late sections and investigate the impact of such assumption on the main results.

The decisions of order quantities for RMs are constrained by the maximum order quantity Q_i; while the production rate is constrained by the capacity U and the availability of raw materials. Let $x_1(t)$ denote the on-hand inventory level of RMs at time t and $x_2(t)$ denote the on-hand inventory level of FGs at time t. Here $x_2(t)$ could be negative, which represents the number of backlogged demands. The manufacturer needs to make two types of decisions: the production rate $u \in \{0,U\}$ and the RM order quantities q_i for $i \in [1,N]$ subject to $0 \leqslant q_i \leqslant Q_i$. When $q_i = 0$, it implies that supplier i is not selected to supply raw materials at the current decision-making epoch. Define the control decision vector $\mathbf{u} := (u, q_1, q_2, \ldots, q_N)$. From the assumption that the outstanding orders are adjustable before reaching the manufacturer, the outstanding orders can be treated as control variables. Therefore, the system state can be described by a vector $\mathbf{x} = (x_1,x_2)$, which represents the inventory levels of RMs and FGs. It should be pointed out that when the adjustable outstanding order assumption is relaxed, the system state must include the status of the outstanding orders to all suppliers (see Section 6).

The system state space is denoted by $X = \{\mathbf{x} = (x_1,x_2)|x_1 \in Z^+$ and $x_2 \in Z\}$. The evolution of the system state is driven by three types of events: the arrival of raw materials from one of the suppliers, the completion of production of a finished product, and the arrival of a customer demand. In other words, the system state will not change unless one of the above events occurs. It should be pointed out that due to the memoryless properties of the Poisson process and the exponential distribution, the remaining time for a shipment, a production and for a demand whose arrival completion was interrupted by an event still follows the same exponential distribution.

We focus on state-feedback policies, in which the decisions are triggered by the system state changes. Therefore, $\mathbf{u}(t)$ should be understood as $\mathbf{u}(\mathbf{x}(t))$, where $\mathbf{x}(t)$ represents the current system state at time t. Define an admissible control set $\Omega = \{\mathbf{u} = (u(\mathbf{x}), q_1(\mathbf{x}), q_2(\mathbf{x}), \ldots, q_N(\mathbf{x}))|u(\mathbf{x}) \in \{0,U\}$ if $x_1 > 0$, $u = 0$ if $x_1 \leqslant 0$; $0 \leqslant q_i(\mathbf{x}) \leqslant Q_i$ for $i = 1, 2, \ldots, N\}$. To simplify the narrative, we often simplify $u(\mathbf{x})$ and $q_i(\mathbf{x})$ as u and q_i by omitting the system state in the rest of the paper. The integrated inventory management problem is to find the optimal joint policy $\mathbf{u} \in \Omega$ by minimising the infinite horizon expected discounted cost.

$$J(\mathbf{x}_0) = \min_{\mathbf{u}} \left[E\left(\int_0^\infty e^{-\beta t} G(\mathbf{x}(t), \mathbf{u}(t)) dt | \mathbf{x}(0) = \mathbf{x}_0 \right) \right] \tag{1}$$

where $0 < \beta < 1$ is a discount factor, $\mathbf{x}_0$ is the initial system state, and $G(\mathbf{x}(t),\mathbf{u}(t))$ represents the raw material holding costs, finished goods inventory costs, customer demand backlog costs, production costs, raw material fixed ordering costs, raw material variable ordering costs, which may be defined as

$$G(\mathbf{x}(t), \mathbf{u}(t)) = g(\mathbf{x}(t)) + c_p u(t) + \sum_{i=1}^{N} \left(c_i^f \cdot I\{q_i(t) > 0\} + c_i^v q_i(t) \right) \tag{2}$$

where $I\{.\}$ is an indicator function, which takes 1 if the condition is true, takes 0 otherwise; $c_i^f \geqslant 0$ and $c_i^v \geqslant 0$ are cost coefficients representing fixed ordering costs and variable ordering costs respectively; c_p represents the production cost coefficient. Here the ordering costs (fixed and variable) are charged as long as the orders have been placed but have not reached the manufacturer. This can be interpreted as the aggregated costs including order handling, shipping and in-transition inventory costs. In (2), $g(\mathbf{x}(t))$ represents the raw material inventory holding costs, finished goods holding costs and demand backlogging cost, defined by

$$g(\mathbf{x}(t)) = c_1 x_1(t) + c_2^+ x_2^+(t) + c_2^- x_2^-(t) \tag{3}$$

where c_1 and c_2^+ are holding cost for raw material and finished goods respectively; c_2^- is the backlog cost; and $x_2^+(t) := \max\{0, x_2(t)\}$, and $x_2^-(t) := \max\{0, -x_2(t)\}$.

The problem in (1) is a continuous time Markov decision problem, which can be transformed into an equivalent discrete-time Markov chain problem by using the uniformisation technique (Puterman, 1994; Song, 2013), e.g. let $v = \xi + U + \sum_1^N \lambda_i$ be the uniform transition rate. The details of the transforming process are given in the Appendix and the similar formulation process can be referred to Song (2013). From the stochastic dynamic programming theory, we have the following results.

Proposition 1. *The optimal integrated procurement and production policy* $(u^*(\mathbf{x}), q_1^*(\mathbf{x}), q_2^*(\mathbf{x}), \ldots, q_N^*(\mathbf{x}))$ *is given by:*

$$q_i^*(\mathbf{x}) = \arg\min \left\{ c_i^v q_i + I\{q_i > 0\} \cdot c_i^f + \lambda_i J(x_1 + q_i, x_2) | 0 \leqslant q_i \leqslant Q_i \right\}, \quad \text{for } i = 1,2,\ldots,N \tag{4}$$

$$u^*(\mathbf{x}) = \begin{cases} U & c_p + J(x_1 - 1, x_2 + 1) < J(x_1, x_2), x_1 > 0 \\ 0 & \text{othewise} \end{cases} \tag{5}$$

Physically, the quantity $c_i^v q_i + c_i^f + \lambda_i (J(x_1 + q_i, x_2) - J(x_1, x_2))$ is the additional cost incurred when an order with non-zero size q_i is placed to supplier i compared to no order is placed to supplier i. On the other hand, the quantity $U \cdot (c_p + J(x_1 - 1, x_2 + 1) - J(x_1,x_2))$ is the additional cost incurred when the manufacturer is producing at the speed U compared to producing nothing. Therefore, the manufacturer should produce nothing if the additional cost is non-negative.

The optimal policy given in Proposition 1 is implicit. To implement it in reality, we need to know the explicit optimal cost function $J(\mathbf{x})$ or the additional cost incurred for the procurement decisions and the production decisions at any system state. From the Appendix, we have the following result.

Proposition 2. *Let* $J_0(\mathbf{x}) = 0$ *for any* $\mathbf{x} \in X$, $J_k(\mathbf{x}) = +\infty$ *for* $\mathbf{x} \notin X$ *and* $k \geqslant 0$, *and*

$$J_{k+1}(\mathbf{x}) = (\beta + v)^{-1} \Big[g(\mathbf{x}) + \xi J_k(x_1, x_2 - 1) + U \cdot \min\{c_p + J_k(x_1 - 1, x_2 + 1), J_k(\mathbf{x})\} + \sum_{i=1}^{N} \min\left\{ c_i^v q_i + I\{q_i > 0\} \cdot c_i^f + \lambda_i J_k(x_1 + q_i, x_2) | 0 \leqslant q_i \leqslant Q_i \right\} \Big] \tag{6}$$

for $k \geqslant 0$ and $\mathbf{x} \in X$, where $J_k(\mathbf{x})$ is the k-stage cost function for state $\mathbf{x}$, then

$$\lim_{k \to +\infty} J_k(\mathbf{x}) = J(\mathbf{x}), \text{ for } \mathbf{x} \in X \tag{7}$$

where $J(\mathbf{x})$ is defined in (1).

The convergence of the k-stage policy and cost function to the infinite-horizon optimal policy and cost again follows from the fact that a finite number of controls are taken at each state (Bertsekas, 1976, Chapter 6, Propositions 8–12). Based on Proposition 2, the value iteration algorithm below can be used to approximate the optimal cost function.

The value iteration algorithm

Specify the maximum iteration number K and the error allowance ε which is a small positive number. Let k denote the iteration number,

Step 0: Set $k = 0$ and $J_0(\mathbf{x}) \equiv 0$ for any $\mathbf{x} \in X$; and define $J_k(\mathbf{x}) := +\infty$ for $\mathbf{x} \notin X$ and $k \geqslant 0$.
Step 1: Compute $J_{k+1}(\mathbf{x})$ using Eq. (6).
Step 2: Calculate $\delta = \max\{|J_{k+1}(\mathbf{x}) - J_k(\mathbf{x})| \text{ for } \mathbf{x} \in X\}$.
Step 3: If $\delta < \varepsilon$ or $k > K$ go to Step 4; otherwise replace k by $k + 1$ and go to Step 1.
Step 4: Output $J_{k+1}(\mathbf{x})$ and the resulting policy $(u(\mathbf{x}), q_1(\mathbf{x}), q_2(\mathbf{x}), \ldots, q_N(\mathbf{x}))$ realising the minimisation of the right-hand-side of (6). Terminate the algorithm.

3. Impact of supplier base reduction and supplier capability improvement

We define the supplier capability as its ability to provide higher service level (e.g. higher delivery capacity, faster delivery, more reliable delivery) or lower ordering costs (e.g. cheaper fixed or variable order processing costs). This section investigates the impacts of the supplier base reduction and the supplier capability improvement on the total expected cost.

Proposition 3. *(i) $J(\mathbf{x})$ is decreasing as λ_i increases; (ii) $J(\mathbf{x})$ is decreasing as U increases.*

Proof. For assertion (i), let $J'(\mathbf{x})$ denote the expected discounted cost with λ'_i, which is greater than λ_i. Define the uniform transition rate $\upsilon = \xi + U + \sum_1^n \lambda'_i$ for both cases λ_i and λ'_i. We want to prove $J(\mathbf{x}) \geqslant J'(\mathbf{x})$ by the induction approach. For any $\mathbf{x}$, it is obvious that $J_0(\mathbf{x}) \equiv 0 \geqslant J'_0(\mathbf{x}) \equiv 0$. Suppose $J_k(\mathbf{x}) \geqslant J'_k(\mathbf{x})$. We want to show $J_{k+1}(\mathbf{x}) \geqslant J'_{k+1}(\mathbf{x})$. From Proposition 2,

$$\begin{aligned} J_{k+1}(\mathbf{x}) = (\beta+\upsilon)^{-1}\Big[&g(\mathbf{x}) + \xi J_k(x_1, x_2 - 1) + U \cdot \min\{c_p + J_k(x_1 - 1, \\ &x_2 + 1), J_k(\mathbf{x})\} + \sum_1^n (\lambda'_i - \lambda_i) J_k(\mathbf{x}) + \sum_1^n \min\Big\{c_i^v q_i + I\{q_i > 0\} \\ &\cdot c_i^f + \lambda_i J_k(x_1 + q_i, x_2) | 0 \leqslant q_i \leqslant Q_i\Big\}\Big] \end{aligned} \tag{8}$$

and

$$\begin{aligned} J'_{k+1}(\mathbf{x}) = (\beta+\upsilon)^{-1}\Big[&g(\mathbf{x}) + \xi J'_k(x_1, x_2 - 1) + U \cdot \min\{c_p \\ &+ J'_k(x_1 - 1, x_2 + 1), J'_k(\mathbf{x})\} + \sum_1^n \min\{c_i^v q_i + I\{q_i > 0\} \\ &\cdot c_i^f + \lambda'_i J'_k(x_1 + q_i, x_2) | 0 \leqslant q_i \leqslant Q_i\Big\}\Big] \end{aligned} \tag{9}$$

Suppose that $\arg\min\left\{c_i^v q_i + I\{q_i > 0\} \cdot c_i^f + \lambda_i J_k(x_1 + q_i, x_2) | 0 \leqslant q_i \leqslant Q_i\right\} = q_i^* > 0$; otherwise it is straightforward from the induction hypotheses. This implies that

$$c_i^v q_i^* + \left(c_i^f + \lambda_i J_k(x_1 + q_i^*, x_2)\right) < \lambda_i J_k(\mathbf{x})$$

Namely,

$$(\lambda'_i - \lambda_i) J_k(\mathbf{x}) > (\lambda'_i - \lambda_i) \cdot \left(c_i^v q_i^* + \left(c_i^f + \lambda_i J_k(x_1 + q_i^*, x_2)\right)\right) \Big/ \lambda_i; \tag{10}$$

It follows,

$$\begin{aligned} &\min\left\{c_i^v q_i + I\{q_i > 0\} \cdot c_i^f + \lambda_i J_k(x_1 + q_i, x_2) | 0 \leqslant q_i \leqslant Q_i\right\} + (\lambda'_i - \lambda_i) J_k(\mathbf{x}) \\ &= \left(c_i^v q_i^* + \left(c_i^f + \lambda_i J_k(x_1 + q_i^*, x_2)\right)\right) + (\lambda'_i - \lambda_i) J_k(\mathbf{x}) \\ &> \left(c_i^v q_i^* + \left(c_i^f + \lambda_i J_k(x_1 + q_i^*, x_2)\right)\right) + (\lambda'_i - \lambda_i) \cdot \left(c_i^v q_i^* + \left(c_i^f + \lambda_i J_k(x_1 + q_i^*, x_2)\right)\right) / \lambda_i; \\ &= \lambda'_i \cdot \left(c_i^v q_i^* + \left(c_i^f + \lambda_i J_k(x_1 + q_i^*, x_2)\right)\right) / \lambda_i; \\ &= \lambda'_i \cdot \left(c_i^v q_i^* + c_i^f\right) / \lambda_i + \lambda'_i \cdot J_k(x_1 + q_i^*, x_2); \\ &> c_i^v q_i^* + c_i^f + \lambda'_i \cdot J_k(x_1 + q_i^*, x_2); \\ &> c_i^v q_i^* + c_i^f + \lambda'_i \cdot J'_k(x_1 + q_i^*, x_2); \\ &> \min\left\{c_i^v q_i + I\{q_i > 0\} \cdot c_i^f + \lambda_i J'_k(x_1 + q_i, x_2) | 0 \leqslant q_i \leqslant Q_i\right\} \end{aligned}$$

By the induction hypotheses, we have $J_{k+1}(\mathbf{x}) \geqslant J'_{k+1}(\mathbf{x})$. By Proposition 2, we have $J(\mathbf{x}) \geqslant J'(\mathbf{x})$. Hence, assertion (i) is true. Assertion (ii) can be proved similarly. This completes the proof. □

Note that the material lead-time from supplier i follows an exponential distribution with average $1/\lambda_i$ and variance $1/\lambda_i^2$. This implies that, as λ_i increases, the average delivery time and its variability are reduced. Therefore, Proposition 3(i) is in agreement with the intuition that shorter expected material lead time and more reliable delivery is beneficial to the manufacturer since the manufacturer could maintain lower level of raw material inventories to buffer against the uncertainty in material supply. Proposition 3(ii) can be similarly interpreted.

Remark 1. (i) $J(\mathbf{x})$ is decreasing as N increases; (ii) $J(\mathbf{x})$ is decreasing as Q_i increases; (iii) $J(\mathbf{x})$ is increasing as c_i^f or c_i^v increases.

The first two assertions in Remark 1 are intuitively true from the relaxation argument that an optimal solution of a relaxed problem cannot be worse. The third assertion is intuitive and can be shown from Proposition 2 by the induction approach. Physically, assertion (i) indicates that the manufacturer can reduce the cost if the supplier base increases; while assertions (ii) and (iii) imply that having a supplier with higher delivery capacities or lower order processing costs is more beneficial to the manufacturer.

4. Supplier base reduction strategies

From the supply chain management perspective, the manufacturer may want to establish a closer relationship with suppliers through supplier management strategies, e.g. the supplier base reduction. Reducing supplier base implies that the selected suppliers will have more sales. In return, the manufacturer often requires or expects that those selected suppliers can provide higher service level or lower ordering costs.

According to the supplier's capability improvement, it gives rise to three types of supplier base reduction strategies:

- the manufacturer reduces its supplier base size, whereas the selected suppliers provide higher delivery capacity. This strategy is called supplier base reduction with higher delivery capacity (SBR-HDC);
- the manufacturer reduces its supplier base size, whereas the selected suppliers offer shorter and more reliable delivery service. This strategy is called supplier base reduction with shorter and more reliable delivery (SBR-SRD);

- the manufacturer reduces its supplier base size, whereas the selected suppliers offer more efficient order processing service, which leads to lower fixed ordering costs. This strategy is called supplier base reduction with lower fixed ordering cost (SBR-LFC).

Proposition 3(i) and Remark 1 indicate that the supplier base reduction and the supplier capability improvement have opposite impacts on the total cost. A trade-off exists. However, it is not intuitive to identify the trade-off point because of their interaction and the dependency on the integrated procurement and production policy. Note that the supplier base size and the supplier capability are changing simultaneously under each of the supplier base reduction strategies, identifying the trade-off point is essentially a one-dimensional optimisation problem. This is not difficult to solve, e.g. using the value iteration method iteratively. In terms of the location of the trade-off point, we have the following conjecture.

Conjecture 1. Under one of the above supplier base reduction strategies, the system performance has a U-shape with respect to the degree of each supplier base reduction strategy. Here the degree of supplier base reduction strategy measures how far the supplier base has been reduced.

In Conjecture 1, U-shape is used in a broad sense. It includes the cases of monotonic increasing or monotonic decreasing situations. If the system performance is monotonically increasing as the size of supplier base decreases, then the best supplier base reduction strategy is to use a single supplier. On the other hand, if the system performance is monotonically decreasing as the size of supplier base decreases, then the best supplier base reduction strategy is to have the maximum number of available suppliers. We will verify this conjecture in the numerical experiment section.

It should be pointed out that in real cases extra costs may be incurred or intangible benefit may be generated in the processes of supply base reduction and supply capability improvement, which may affect the trade-off point in the supplier base reduction strategies. However, this is a complicated and case-dependent issue, which deserves further research.

5. Supplier differentiation

Another important supplier management strategy is to differentiate suppliers when they have different supply capabilities. In other words, when suppliers have different delivery parameters (represented by $Q_i,\ c_i^f,\ c_i^v$, and λ_i in our model), the manufacturer should differentiate its procurement decisions in order to achieve the best performance.

Proposition 4. *Under the optimal integrated procurement and production policy $(u^*(\mathbf{x}), q_1^*(\mathbf{x}), q_2^*(\mathbf{x}), \ldots, q_N^*(\mathbf{x}))$ given in Proposition 1, we have the following relationships of the procurement decisions between different suppliers:*

(i) *If $Q_l < Q_j$ and all other parameters are the same for suppliers l and j, then $q_l^*(\mathbf{x}) \leqslant q_j^*(\mathbf{x})$.*
(ii) *If $c_l^f > c_j^f$ and all other parameters are the same for suppliers l and j, then $q_l^*(\mathbf{x}) \leqslant q_j^*(\mathbf{x})$, and if $q_l^*(\mathbf{x}) > 0$, then $q_j^*(\mathbf{x}) = q_l^*(\mathbf{x})$.*
(iii) *If $c_l^v > c_j^v$ and all other parameters are the same for suppliers l and j, then $q_l^*(\mathbf{x}) \leqslant q_j^*(\mathbf{x})$.*
(iv) *If $\lambda_l < \lambda_j$ and all other parameters are the same for suppliers l and j, then $q_l^*(\mathbf{x}) \leqslant q_j^*(\mathbf{x})$.*

Proof. From Proposition 1, we know that the optimal order quantity is determined by

$$q_i^*(\mathbf{x}) = \arg\min\left\{c_i^v q_i + I\{q_i > 0\}\cdot c_i^f + \lambda_i J(x_1 + q_i, x_2) | 0 \leqslant q_i \leqslant Q_i\right\},$$
for $i = 1, 2, \ldots, N$

Assertion (i) is obvious from the above equation. To show the rest of assertions, we introduce the following two types of discrete functions to simplify the narrative,

$$f_i(\mathbf{x}, n) := c_i^v \cdot n + I\{n > 0\}\cdot c_i^f \tag{11}$$

$$h_i(\mathbf{x}, n) := \lambda_i J(x_1 + n, x_2) \tag{12}$$

It implies that

$$q_i^*(\mathbf{x}) = \arg\min\{f_i(\mathbf{x}, n) + h_i(\mathbf{x}, n) | 0 \leqslant n \leqslant Q_i\} \tag{13}$$

For assertion (ii): because $q_i^*(\mathbf{x})$ are non-negative integers, we only need to show if $q_l^*(\mathbf{x}) > 0$, then $q_j^*(\mathbf{x}) = q_l^*(\mathbf{x})$. Suppose $q_l^*(\mathbf{x}) = m > 0$. It follows, $f_l(\mathbf{x}, m) + h_l(\mathbf{x}, m) \leqslant f_l(\mathbf{x}, n) + h_l(\mathbf{x}, n)$ for $0 \leqslant n \leqslant Q_l$. That is,

$$c_l^v \cdot m + c_l^f + h_l(\mathbf{x}, m) \leqslant h_l(\mathbf{x}, 0) \tag{14}$$

$$c_l^v \cdot m + c_l^f + h_l(\mathbf{x}, m) \leqslant c_l^v \cdot n + c_l^f + h_l(\mathbf{x}, n) \quad \text{for } 0 < n \leqslant Q_l \tag{15}$$

Note that $c_l^f > c_j^f,\ c_l^v = c_j^v,\ Q_l = Q_j$, and $h_l(\mathbf{x}, n) = h_j(\mathbf{x}, n)$. The above two equations lead to,

$$f_j(\mathbf{x}, m) + h_j(\mathbf{x}, m) \leqslant f_j(\mathbf{x}, n) + h_j(\mathbf{x}, n) \quad \text{for } 0 \leqslant n \leqslant Q_j \tag{16}$$

It follows, $q_j^*(\mathbf{x}) = m = q_l^*(\mathbf{x})$. Thus, assertion (ii) is true.

For assertion (iii): we want to show that if $q_l^*(\mathbf{x}) > 0$, then $q_j^*(\mathbf{x}) \geqslant q_l^*(\mathbf{x})$. Suppose $q_l^*(\mathbf{x}) = m > 0$. We have the same Eqs. (14) and (15). Note that $c_l^v > c_j^v,\ c_l^f = c_j^f,\ Q_l = Q_j$, and $h_l(\mathbf{x}, n) = h_j(\mathbf{x}, n)$. It follows,

$$c_j^v \cdot m + c_j^f + h_j(\mathbf{x}, m) \leqslant h_j(\mathbf{x}, 0) \tag{17}$$

$$c_j^v \cdot m + c_j^f + h_j(\mathbf{x}, m) \leqslant c_j^v \cdot n + c_j^f + h_j(\mathbf{x}, n) \quad \text{for } 0 < n \leqslant m \tag{18}$$

That yields, $q_j^*(\mathbf{x}) \geqslant m = q_l^*(\mathbf{x})$. Therefore, assertion (iii) is true. For assertion (iv), it can be similarly proved. This completes the proof. □

Physically, Proposition 4 states that larger orders should be placed to the suppliers with higher supply capabilities, which is in agreement in intuition. In particular, Proposition 4(ii) provides a further interesting insight that if two suppliers only differ in the fixed ordering cost, the optimal non-zero procurement decisions to the supplier with higher fixed ordering cost should be the same as that to the supplier with lower fixed ordering cost. The interpretation is that if the optimal decision at a system state **x** is to place a non-zero order to the supplier with higher fixed ordering cost (which implies that this procurement decision can offset the incurred fixed ordering costs to both suppliers), then the optimal order size to these two suppliers at the system state **x** will not be affected by the difference of their fixed ordering costs. The results in Proposition 4 are qualitative. However, our model is able to quantify the differences of the optimal procurement decisions for different suppliers, which will be illustrated in the numerical example section.

6. Extension to the case with non-adjustable outstanding orders

A key assumption in our model is that the outstanding orders are adjustable before they reach the manufacturer. This assumption is rather restrictive and only represents a very special type of supply chain relationship. We will relax this assumption and extend the model to the cases with non-adjustable outstanding orders in this section. However, we keep the assumption that at most one outstanding order is allowed for each supplier, which is common in the literature.

Since the outstanding orders are not allowed to change once they are issued, the manufacturer's new ordering decisions should depend on not only the inventory-on-hand of raw materials and finished goods, but also the inventory-in-transition, i.e. the outstanding orders (Silbermayr & Minner, 2012). The system state space can now be described by $X := \{(\mathbf{x},\mathbf{y})|\mathbf{x} = (x_1,x_2)$ s.t. $x_1 \in Z^+$ and $x_2 \in Z$; $\mathbf{y} = (y_1, y_2, \ldots, y_N)$ s.t. $0 \leqslant y_i \leqslant Q_i$ for $i = 1, 2, \ldots, N\}$. The manufacturer needs to make two types of decisions at any state $(\mathbf{x},\mathbf{y})$: the production rate $u \in \{0,U\}$, and the raw material order quantities $0 \leqslant q_i \leqslant Q_i \cdot I\{y_i = 0\}$ for $i = 1, 2, \ldots, N$. Clearly, if $y_i > 0$ for $i = 1, 2, \ldots, N$, which represents the situation that there are non-zero outstanding orders to every supplier, then no new orders can be placed to any supplier. The admissible control set is defined as $\Omega = \{\mathbf{u} = (u(\mathbf{x},\mathbf{y}), q_1(\mathbf{x},\mathbf{y}), q_2(\mathbf{x},\mathbf{y}), \ldots, q_N(\mathbf{x},\mathbf{y}))|u(\mathbf{x}, \mathbf{y}) \in \{0,U\}$ if $x_1 > 0$, $u = 0$ if $x_1 \leqslant 0$; $0 \leqslant q_i(\mathbf{x}) \leqslant Q_i \cdot I\{y_i = 0\}$ for $i = 1, 2, \ldots, N\}$. To simplify the narrative, we simplify $u(\mathbf{x})$ and $q_i(\mathbf{x})$ as u and q_i by omitting the system state and let $\mathbf{q} := (q_1, q_2, \ldots, q_N)$.

The cost function $J(\mathbf{x},\mathbf{y})$ can be defined similar to (1) by replacing $G(\mathbf{x},\mathbf{u})$ with $G(\mathbf{x},\mathbf{y},\mathbf{u})$, which represents the incurred unit-time cost at state $(\mathbf{x}, \mathbf{y})$ taking control action $\mathbf{u}$,

$$G(\mathbf{x},\mathbf{y},\mathbf{u}) = g(\mathbf{x}) + c_p u + \sum_{i=1}^{N}\left(c_i^f \cdot I\{y_i + q_i > 0\} + c_i^v \cdot (y_i + q_i)\right) \tag{19}$$

Following the uniformization technique (define $v = \xi + U + \sum_1^N \lambda_i$) and the stochastic dynamic programming theory, the Bellman optimality equation is given as follows

$$\begin{aligned} J(\mathbf{x},\mathbf{y}) = (\beta + v)^{-1} \min_{u\in\{0,U\},0\leqslant q_i\leqslant Q_i\cdot I\{y_i=0\}} \Big[& G(\mathbf{x},\mathbf{y},\mathbf{u}) + \xi J(x_1, x_2 \\ & - 1, \mathbf{y}+\mathbf{q}) + uJ(x_1 - 1, x_2 + 1, \mathbf{y}+\mathbf{q}) + \sum_{i=1}^{N} \lambda_i J(x_1 \\ & + y_i + q_i, x_2, y_1 + q_1, \ldots, y_{i-1} + q_{i-1}, 0, y_{i+1} \\ & + q_{i+1}, \ldots, y_N + q_N\} + (U - u)J(\mathbf{x}, \mathbf{y}+\mathbf{q})\Big] \end{aligned} \tag{20}$$

To simplify the narrative, let $(x_1 + y_i + q_i,\ x_2,\ \mathbf{y} + \mathbf{q}\backslash y_i + q_i) := (x_1 + y_i + q_i,\ x_2,\ y_1 + q_1, \ldots, y_{i-1} + q_{i-1},\ 0,\ y_{i+1} + q_{i+1}, \ldots, y_N + q_N,)$. The above equation can be simplified as,

$$\begin{aligned} J(\mathbf{x},\mathbf{y}) = (\beta + v)^{-1} \Bigg[& g(\mathbf{x}) + \min_{0\leqslant q_i\leqslant Q_i\cdot I\{y_i=0\}} \Bigg\{ \xi J(x_1, x_2 - 1, \mathbf{y}+\mathbf{q}) \\ & + U \cdot \min\{c_p + J(x_1 - 1, x_2 + 1, \mathbf{y}+\mathbf{q}), J(\mathbf{x}, \mathbf{y}+\mathbf{q})\} \\ & + \sum_{i=1}^{N}\Big(c_i^f \cdot I\{y_i + q_i > 0\} + c_i^v \\ & \cdot (y_i + q_i) + \lambda_i J(x_1 + y_i + q_i, x_2, \mathbf{y}+\mathbf{q} \setminus y_i + q_i)\Big)\Bigg\}\Bigg] \end{aligned} \tag{21}$$

From (21), the optimal integrated procurement and production policy $(u^*(\mathbf{x},\mathbf{y}), \mathbf{q}^*(\mathbf{x},\mathbf{y}))$ for the case with non-adjustable outstanding orders can be given by:

$$\begin{aligned} \mathbf{q}^*(\mathbf{x},\mathbf{y}) = (q_1^*(\mathbf{x},\mathbf{y}), q_2^*(\mathbf{x},\mathbf{y}), \ldots, q_N^*(\mathbf{x},\mathbf{y})) = \arg\min_{0\leqslant q_i\leqslant Q_i\cdot I\{y_i=0\}} \Big\{ & \xi J(x_1, x_2 - 1, \mathbf{y}+\mathbf{q}) \\ & + U \cdot \min\{c_p + J(x_1 - 1, x_2 + 1, \mathbf{y}+\mathbf{q}), J(\mathbf{x}, \mathbf{y}+\mathbf{q})\} \\ & + \sum_{i=1}^{N}\Big(c_i^f \cdot I\{y_i + q_i > 0\} + c_i^v \cdot (y_i + q_i) + \lambda_i J(x_1 + y_i + q_i, x_2, \mathbf{y}+\mathbf{q} \setminus y_i + q_i)\Big) \Big\} \end{aligned} \tag{22}$$

$$u^*(\mathbf{x},\mathbf{y}) = \begin{cases} U & c_p + J(x_1 - 1, x_2 + 1, \mathbf{y}+\mathbf{q}^*) < J(x_1, x_2, \mathbf{y}+\mathbf{q}^*), x_1 > 0 \\ 0 & \text{othewise} \end{cases} \tag{23}$$

Apart from the system state, another important difference between (22) and (23) and (4) and (5) is that the procurement decisions are coupled and they are also coupled with the production decision in (22) and (23), whereas they are de-coupled in Proposition 1. Nevertheless, from (21), it is clear that the value function approximation procedure in Proposition 2 can be similarly applied to the case with non-adjustable outstanding orders.

It is straightforward to extend the results in Remark 1 to the case with non-adjustable outstanding orders. However, it is challenging to establish the property in Proposition 3 mathematically due to the coupled relationships of the procurement and production decisions. However, we will use numerical examples to illustrate that the results in Proposition 3 and Proposition 4 can carry over to the case with non-adjustable outstanding orders. In addition, a full factorial experiment will be conducted to investigate the impact of various factors (including the parameters representing three different types of uncertainties) and their interactions on the system performance.

7. Numerical examples

This section consists of four sub-sections. In Section 7.1, we numerically illustrate the analytical results about the impacts of the supplier base size and the supplier capabilities on the system performance. In Section 7.2, the trade-off relationships under the supplier base reduction strategies (in Conjecture 1) are verified using a range of scenarios. The best trade-off point under each supplier base reduction strategy is identified. In Section 7.3, we illustrate the results about supplier differentiation in Proposition 4, and examine the control structure of the optimal integrated procurement and production policies. In Section 7.4, the case of non-adjustable outstanding orders is discussed. We focus on the impacts of the supplier capabilities on the system performance, the supplier differentiation, and the control structure of the optimal policies, in comparison with the case of adjustable outstanding orders. We also conducted a full factorial experiment to evaluate the impact of various factors and their interactions on the total cost. To simplify the computation effort, the system state space is limited into a finite area with $x_1 \in [0,20]$ and $x_2 \in [-50,20]$, which is large enough for the scenarios in our experiments because of the small scale of the parameter setting. The value iteration algorithm in Section 2 will be terminated when the cost difference is less than 10^{-3}.

7.1. Impacts of supplier base size and supplier capabilities on system performance

The common system parameters are set as follows: $\beta = 0.1$; $c_i^v = 0.5$; $c_1 = 1.0$; $c_2^+ = 2.0$; $c_2^- = 8.0$; $c_p = 1.0$; $U = 1.0$; $\xi = 0.8$. Here the inventory holding cost for the finished goods (c_2^+) is twice of that for the raw materials (c_1), and the penalty for backlog (c_2^-) is much higher than the inventory costs. The other cost parameters such as ordering variable costs and production costs are at the comparable levels to the raw material inventory cost. The production utilisation on average is at 80% if the aggregated supply rate is greater than the demand rate 0.8. Although the above parameter setting is for illustrative purpose, it is generally reasonable in terms of the overall operations. Assume that all suppliers have the same capability. We vary one parameter at a time to examine its impact on the cost function. The rationale for selecting the range of the parameters in the following cases is to ensure that the maximum supply capacity exceeds the customer demand rate on average.

- *Case 1:* The number of suppliers (N) varies from 3 to 4, 5, 6, 7, 8, and 9 (eight scenarios); while $Q_i = 5$, $\lambda_i = 0.1$, $c_i^f = 0.5$.
- *Case 2:* The suppliers' delivery capacity (Q_i) varies from 3 to 4, 5, 6, 7, 8, and 9 (eight scenarios); while $N = 5$, $\lambda_i = 0.1$, $c_i^f = 0.5$.
- *Case 3:* The suppliers' delivery rate (λ_i) varies from 0.04 to 0.06, 0.08, 0.10, 0.12, 0.14, and 0.16 (eight scenarios); while $N = 5$, $Q_i = 5$, $c_i^f = 0.5$.

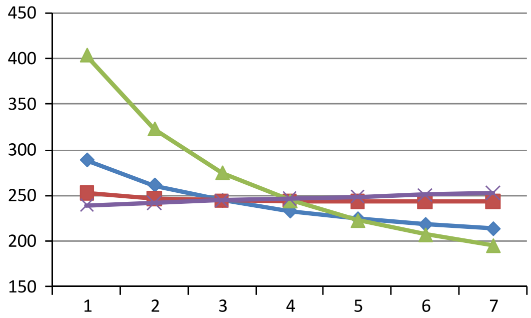

Fig. 1. Impact of supplier capabilities on the system performance. (Diamond line – Case 1; square line – Case 2; triangle line – Case 3; cross line – Case 4).

- *Case 4:* The suppliers' fixed ordering cost $\left(c_i^f\right)$ varies from 0.3 to 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9 (eight scenarios); while $N = 5$, $Q_i = 5$, $\lambda_i = 0.1$.

The results of the above cases are shown in Fig. 1, in which the vertical-axis represents the cost, and the horizontal-axis represents eight scenarios in each case. In cases 1, 2 and 3, the cost functions are decreasing as the number of suppliers increases (Case 1), or the suppliers' delivery capacity increases (Case 2), or the suppliers' delivery rate increases (Case 3). On the other hand, the cost functions are increasing as the suppliers' fixed ordering cost increases. This quantifies the analytical results in Proposition 3 and Remark 1, e.g. Fig. 1 shows the relative impacts of these parameters on the system performance within the given ranges of the parameters.

If we regard the scenario with $N = 5,\ Q_i = 5,\ \lambda_i = 0.1,\ c_i^f = 0.5$ as the reference point, the parameter in each case is increasing by 20% for the varying scenarios. This reveals that with the same percentage of parameter changes, their impacts on the cost are quite different, e.g. λ_i has the most significant impact on the cost, followed by $N,\ c_i^f$, and Q_i. The implication is that raw material delivery time and its reliability appear to be more important compared to the other three aspects. Moreover, it can be observed that the cost is more sensitive when λ_i or N is smaller. This may be explained by the fact that when λ_i or N is smaller, the system has lower capability to meet customer demands and may incur heavy backlog penalty costs.

7.2. Evaluate the supplier base reduction strategies

In this sub-section, three supplier base reduction strategies are evaluated and their trade-off points will be identified. The common parameters are set as follows: $\beta = 0.1$; $c_i^v = 0.5$; $c_1 = 1.0$; $c_2^+ = 2.0; c_2^- = 8.0$; $c_p = 1.0$. Other system parameters take different values to represent different scenarios. The scenarios are designed generally to ensure that the suppliers are able to supply adequate raw materials and the manufacturer is able to meet customer demands in long term and operates with a reasonable production utilisation. Assume that all suppliers have the same capability.

For the SBR-HDC strategy, we examine different combinations of N and Q_i as shown in Table 1, which represents eight levels of the strategy. From level A1 to A8, the number of suppliers is decreasing from 9 to 2, while the suppliers' delivery capacity Q_i is increasing from 1 to 8. At each level, a number of scenarios are created to evaluate the performance of the given strategy, in which λ_i takes three levels at (0.08, 0.10, 0.12), U takes three levels at (1.0, 1.2, 1.4), ξ takes three levels at (0.7, 0.8, 0.9), and c_i^f takes three levels at (0.4, 0.5, 0.6). Therefore, in total there are 81 different scenarios for each level of the strategy. The last column in Table 1 gives the average costs over 81 scenarios at state (0, 0). It can be seen from Table 1, the average total cost has a U-shape for the different levels of SBR-HDC strategy, and the best trade-off point is at $(N, Q_i) = (7, 3)$ with the total expected cost 207.97.

Table 1
Performance of SBR-HDC strategy.

Level	(N, Q_i)	Scenarios of $\left(\lambda_i, U, \xi, c_i^f\right)$	$J(0,0)$
A1	(9,1)	81	259.48
A2	(8,2)	81	211.88
A3	(7,3)	81	207.97
A4	(6,4)	81	212.83
A5	(5,5)	81	222.63
A6	(4,6)	81	238.09
A7	(3,7)	81	262.81
A8	(2,8)	81	306.04

For the SBR-SRD strategy, we examine eight different combinations of N and λ_i as shown in Table 2. From level B1 to B8, the number of suppliers is decreasing from 9 to 2, while the suppliers' delivery rate (or speed) λ_i is increasing from 0.03 to 0.17. At each level, total 81 different scenarios are created to evaluate the performance of the given strategy, in which Q_i takes three levels at (4, 5, 6), U takes three levels at (1.0, 1.2, 1.4), ξ takes three levels at (0.7, 0.8, 0.9), and c_i^f takes three levels at (0.4, 0.5, 0.6). The last column in Table 2 gives the average costs over 81 scenarios at state (0, 0). Table 2 shows that the average total cost is of a U-shape with respect to the level of SBR-SRD strategy, and the best trade-off point is at $(N, \lambda_i) = (4, 0.13)$ with the total expected cost 203.53.

For the SBR-LFC strategy, we examine eight different combinations of N and c_i^f as shown in Table 3. From level C1 to C8, the number of suppliers is decreasing from 9 to 2, while the suppliers' fixed ordering cost c_i^f is decreasing from 0.9 to 0.2. At each level, total 81 different scenarios are created to evaluate the performance of the given strategy, in which Q_i takes three levels at (4, 5, 6), λ_i takes three levels at (0.08, 0.10, 0.12), U takes three levels at (1.0, 1.2, 1.4), and ξ takes three levels at (0.7, 0.8, 0.9). The forth column in Table 3 gives the average costs over 81 scenarios at state (0, 0). Table 3 shows that the average total cost appears monotonic increasing as the level of SBR-LFC increases, and the best trade-off point is at $\left(N, c_i^f\right) = (9, 0.9)$ with the total expected cost 202.71.

Table 2
Performance of SBR-SRD strategy.

Level	(N, λ_i)	Scenarios of $\left(Q_i, U, \xi, c_i^f\right)$	$J(0,0)$
B1	(9,0.03)	81	391.60
B2	(8,0.05)	81	292.43
B3	(7,0.07)	81	246.05
B4	(6,0.09)	81	221.34
B5	(5,0.11)	81	208.24
B6	(4,0.13)	81	203.53
B7	(3,0.15)	81	207.99
B8	(2,0.17)	81	229.17

Table 3
Performance of SBR-LFC strategy.

Level	$\left(N, c_i^f\right)$	Scenarios of (Q_i, λ_i, U, ξ)	$J(0,0)$	$\left(N, c_i^f\right)$	$J(0,0)$
C1	(9,0.9)	81	202.71	(9,3.0)	248.24
C2	(8,0.8)	81	204.77	(8,2.6)	243.96
C3	(7,0.7)	81	208.28	(7,2.2)	241.10
C4	(6,0.6)	81	214.01	(6,1.8)	240.32
C5	(5,0.5)	81	223.26	(5,1.4)	242.93
C6	(4,0.4)	81	238.53	(4,1.0)	251.51
C7	(3,0.3)	81	265.43	(3,0.6)	271.62
C8	(2,0.2)	81	317.32	(2,0.2)	317.32

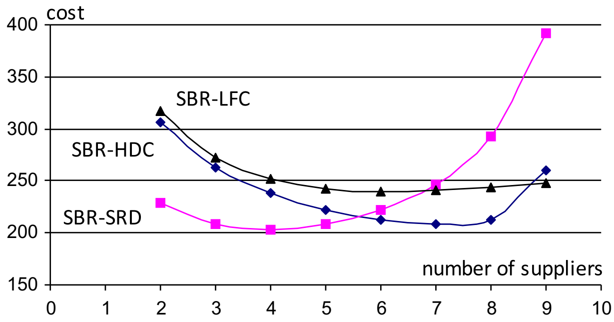

Fig. 2. Performance of three types of supplier base reduction strategies at eight levels.

The reason that we did not see the U-shape of the cost function is the setting of the cases, e.g. the cases have not covered the sufficiently large range, or the fixed ordering cost is not decreasing quickly enough to cancel out the effect of the supplier base reduction. For example, if we re-design level C1 to C8 to be those in the fifth column in Table 3, we would have the average costs over 81 scenarios at state (0,0) in the sixth column. Clearly, we can now see the U-shape of the average cost with the best trade-off at $\left(N,\ c_i^f\right) = (6, 1.8)$.

To have a more intuitive view of the performances at different levels of the supplier base reduction strategies, the average costs are shown in Fig. 2, in which the data in the last column of Table 3 are used for SBR-LFC. From Tables 1–3 and Fig. 2, it verifies the results in Conjecture 1, i.e. the total cost is of U-shape under each of three supplier base reduction strategies. It should be pointed out that we have varied two parameters on discrete basis to represent different levels of supplier base reduction strategies, this may not cover all real settings and effects. In addition, in practice there may be extra costs associated with supplier capability improvement or maintaining the supplier base, which may impact on the best trade-off point under the supplier base reduction strategies.

7.3. Supplier differentiation

This sub-section aims to illustrate the results about supplier differentiation in Proposition 4, and examine the control structure of the optimal integrated procurement and production policies. The common system parameters are set as follows: $\beta = 0.1$; $c_1 = 1.0$; $c_2^+ = 2.0$; $c_2^- = 8.0$; $c_p = 1.0$; $U = 1.0$; $\xi = 0.8$. To simplify the discussion, we consider the cases with two suppliers, i.e. $N = 2$, but they have different supply capabilities. We vary one parameter at a time to examine four cases below.

- *Case D1:* Supplier 1 has a delivery capacity $Q_1 = 5$, and supplier 2 has $Q_2 = 6$; other parameters are the same for both suppliers, i.e. $c_i^f = 0.5,\ c_i^v = 0.2,\ \lambda_i = 0.1$, for $i = 1, 2$.
- *Case D2:* Supplier 1 has ordering fixed cost $c_1^f = 0.5$, and supplier 2 has $c_2^f = 1.0$; other parameters are the same for both suppliers, i.e. $Q_i = 5,\ c_i^v = 0.2,\ \lambda_i = 0.1$, for $i = 1, 2$.
- *Case D3:* Supplier 1 has ordering variable cost $c_1^v = 0.2$, and supplier 2 has $c_2^v = 0.4$; other parameters are the same for both suppliers, i.e. $Q_i = 5,\ c_i^f = 0.5,\ \lambda_i = 0.1$, for $i = 1, 2$.
- *Case D4:* Supplier 1 has delivery rate $\lambda_1 = 0.1$, and supplier 2 has $\lambda_2 = 0.2$; other parameters are the same for both suppliers, i.e. $Q_i = 5,\ c_i^f = 0.5,\ c_i^v = 0.2$, for $i = 1,2$.

The optimal procurement and production decisions in the above four cases are partially shown in the (x_1,x_2) plane in Figs. 3–6, respectively, in which the numbers indicate the optimal order sizes (for the procurement decisions) or whether the manufacturer should produce products (1 represents producing with rate U, and 0 represents producing nothing), and their positions correspond to the system state $\mathbf{x} = (x_1,x_2)$. For example, at the state $\mathbf{x} = (3,0)$ in Fig. 3, the optimal procurement decisions to both suppliers are the same with an order size 5, whereas the optimal production decision is producing with the rate U. The results in Figs. 3–6 illustrate the qualitative results in Proposition 4. Namely, the order sizes to the suppliers with higher supply capability are not less than that to the supplies with lower supply capability. Fig. 4 confirms the results in Proposition 4(ii), i.e. non-zero procurement decisions to the suppliers with higher fixed ordering cost are indeed the same as that to the suppliers with lower fixed ordering cost. More importantly, through numerical examples we are able to quantify the differences between the procurement decisions to different suppliers given the different supplier capabilities.

In addition, the results in Figs. 3–6 also illustrate the structural characteristics of the optimal procurement and production decisions. For example, the optimal procurement decisions to each supplier are characterised by two switching regions (one with no order and the other with non-zero orders), and the order size shows the monotonic property with respect to the raw material and finished goods inventory levels. The optimal production decisions can also be characterised by two switching regions (one with producing nothing and the other with producing the rate U). The boundary curves between two switching regions are monotonically increasing or decreasing. More specifically, the procurement decisions may be approximated by a set of (s,S)-type policies constrained by the delivery capacity, e.g. in Fig. 3(a) and (b), the optimal procurement decisions are determined by the (s,S) policy with $s = 5$ and $S = 8$ constrained by the delivery capacity $Q_1 = 5$ and $Q_2 = 6$ when $x_2 = 0$ (i.e. the order size is given by: $\max\{S - x_1, Q_i\}$ for $x_1 \leqslant s$; and 0 for $x_1 > s$); and with $s = 4$ and $S = 8$ constrained by the delivery capacity $Q_1 = 5$ and $Q_2 = 6$ when $x_2 = 1$. As for the production decisions, it appears to be fairly robust to system parameters (e.g. exactly the same in Figs. 3–5, and slightly different from Fig. 6). The implication is that based on the characteristics of the optimal policy, we are able to construct near-to-optimal but much simpler parameterised policies to determine the procurement and production decisions (Song, 2013).

7.4. The case with non-adjustable outstanding orders

This sub-section considers the cases with non-adjustable outstanding orders. It consists of three parts. The first part examines the impact of supplier capabilities on the system performance, in which the results are compared to the cases with adjustable outstanding orders; the second part examines the supplier differentiation and the decision structure of the optimal policies; and the third part presents a full factorial experiment to evaluate the impact of various factors and their interactions on the system performance. We limit our experiments within the situations of two suppliers to avoid the high dimension of the state space and the computational difficulty.

7.4.1. Impact of supplier capabilities on the system performance

The common system parameters are set as follows: $\beta = 0.1$; $c_1 = 1.0$; $c_2^+ = 2.0$; $c_2^- = 8.0$; $c_p = 1.0$; $U = 1.0$; $\xi = 0.8$. Assume two suppliers have the same supply capabilities. We vary one parameter at a time to examine three cases: (i) the suppliers' delivery capacity (Q_i) varies from 3 to 4, 5, 6, 7, 8, and 9; while $N = 5,\ \lambda_i = 0.1,\ c_i^f = 0.5$. (ii) the suppliers' delivery rate (λ_i) varies from 0.04 to 0.06, 0.08, 0.10, 0.12, 0.14, and 0.16; while $N = 5,\ Q_i = 5,\ c_i^f = 0.5$. (iii) the fixed ordering cost (c_i^f) varies from 0.3 to 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9; while $N = 5$, $Q_i = 5$, $\lambda_i = 0.1$. The results of three cases are given in Tables 4–6 respectively, in which the second column is the optimal cost under the adjustable

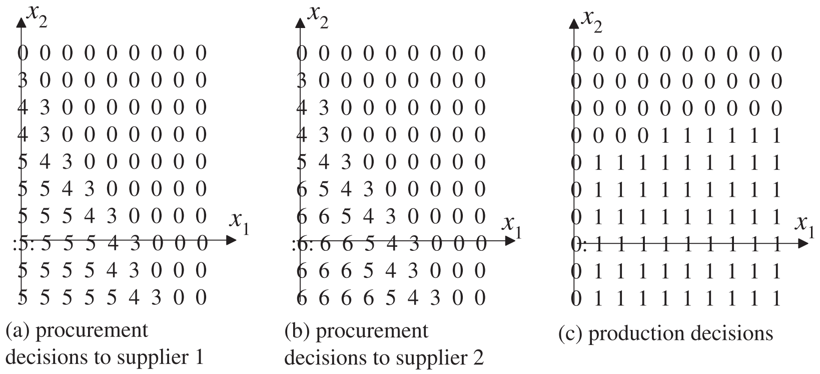

Fig. 3. The optimal procurement and production decisions in case D1.

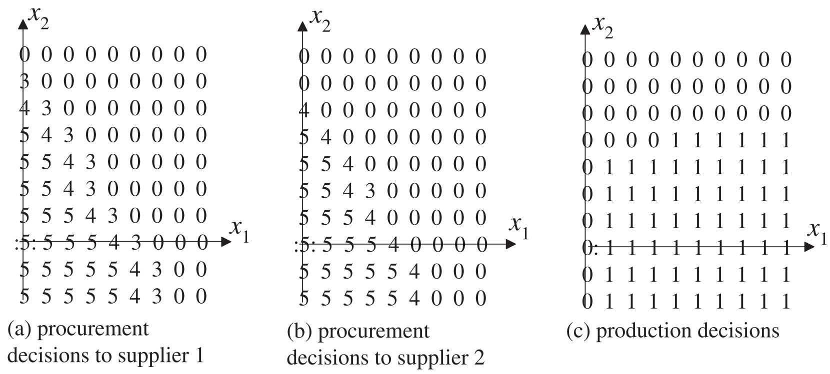

Fig. 4. The optimal procurement and production decisions in case D2.

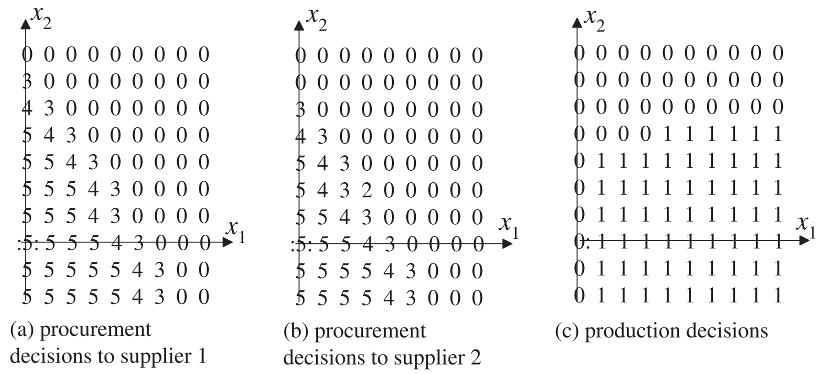

Fig. 5. The optimal procurement and production decisions in case D3.

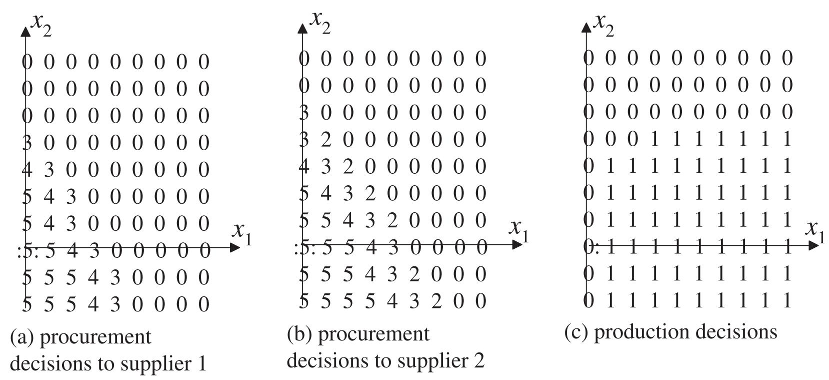

Fig. 6. The optimal procurement and production decisions in case D4.

Table 4
Impact of suppliers' delivery capacity on the system cost.

Q_i	Cost with adjustable	% of change	Cost with non-adjustable	% of change
3	359.18	14.01	359.29	12.88
4	329.61	4.63	330.25	4.07
5	315.03	0.00	316.85	0.00
6	307.62	−2.35	311.16	−1.73
7	303.73	−3.59	309.23	−2.31
8	301.85	−4.18	309.23	−2.31
9	301.01	−4.45	309.23	−2.31

Table 6
Impact of fixed ordering cost on the system cost.

c_i^f	Cost with adjustable	% of change	Cost with non-adjustable	% of change
0.3	311.66	−1.07	313.61	−1.02
0.4	313.36	−0.53	315.23	−0.51
0.5	315.03	0.00	316.85	0.00
0.6	316.68	0.52	318.47	0.51
0.7	318.32	1.04	320.09	1.02
0.8	319.95	1.56	321.70	1.53
0.9	321.56	2.07	323.30	2.04

Table 5
Impact of suppliers' delivery rate on the system cost.

λ_i	Cost with adjustable	% of change	Cost with non-adjustable	% of change
0.04	474.29	50.55	474.42	49.73
0.06	406.67	29.09	407.12	28.49
0.08	354.43	12.51	355.48	12.19
0.1	315.03	0.00	316.85	0.00
0.12	285.26	−9.45	287.97	−9.11
0.14	262.43	−16.70	265.98	−16.05
0.16	244.55	−22.37	248.85	−21.46

outstanding order assumption, the forth column is the optimal cost under the non-adjustable outstanding order assumption, the third and fifth columns are the percentage of cost changes from the reference case (the row corresponding to 0.00%) under two assumptions respectively.

From Tables 4–6, it can be seen that the monotonic properties of the cost function with respect to the key system parameters such as supplier delivery capacity, delivery rate, and fixed ordering cost are preserved for the cases with non-adjustable outstanding orders. More interestingly, the percentages of the cost changes after varying the system parameters compared to that of the reference point are very close in two cases (under adjustable and non-adjustable assumptions). Comparing the optimal costs under two assumptions, it shows that the cost under the adjustable outstanding order assumption is slightly lower than that under the non-adjustable outstanding order assumption. This is in agreement with the intuition since the adjustable outstanding order assumption provides more flexible options for the manufacturer to manage the raw material procurement.

7.4.2. Supplier differentiation

Consider the same cases with two suppliers having different supply capabilities in Section 7.3. Note that the procurement and production decisions are now depending on not only the inventory levels of raw materials and finished goods, but also the sizes of the outstanding orders to both suppliers. As examples, the optimal procurement and production decisions for cases D1 and D4 are partially shown in Figs. 7 and 8 respectively. For example, Fig. 7(a) describes the optimal procurement decisions to supplier 1 in the (x_1,x_2) plane when there is no outstanding order to both suppliers; Fig. 7(b) and (c) describes the optimal procurement decisions to supplier 1 when there is an outstanding order to supplier 2 with the order size being 1 and 2 respectively; Fig. 7(d)–(f) describes the optimal procurement decisions to supplier 2 when the outstanding order to supplier 1 being 0, 1 and 2. Fig. 7(g) describes the optimal production decisions at any state of the outstanding orders. It should be pointed out that when there are non-zero outstanding orders to both suppliers, the procurement decisions to both suppliers are forced to be zero due to the assumption that only one outstanding order is allowed to each supplier at any time and they are not adjustable once issued.

The results in Figs. 7 and 8 reveal that the qualitative results in Proposition 4 are generally preserved for the cases with non-adjustable outstanding orders. Namely, the order sizes to the suppliers with higher supply capability are not less than that to the supplies with lower supply capability. However, it is noted that in the case D1 when the echelon inventory level (i.e. x_1+x_2) or the raw material inventory level exceeds a certain level, the optimal policy may place an order to supplier 1 (with lower delivery capacity) but place no order to supplier 2 (with higher delivery capacity). This may be interpreted as follows. When there are adequate echelon inventories or raw material inventories in the system, it is reasonable to place a relatively smaller order to the lower capacity supplier and reserve the opportunity and flexibility of placing a larger order to the higher capacity supplier to buffer against future uncertainty.

In addition, a few interesting points can be observed from Figs. 7 and 8. Firstly, the structural properties of the optimal procurement and production policies such as switching control regions and monotonicity of the boundary curves are very similar to the case with adjustable outstanding orders; however, the non-zero order size appears to be a constant that is equal or close to the maximum delivery capacity in Figs. 7 and 8. This implies that it is more appropriate to approximate the optimal procurement policy using a series of (r,Q) policies rather than the (s,S) policies. For example, in Fig. 7(a), the optimal procurement decisions can be determined by the (r,Q) policy with $r=4$ and $Q=5$ when $x_2=0$ or 1 (i.e. a fixed order size Q is placed when $x_1\leqslant r$); and in Fig. 7(d), the optimal procurement decisions can be determined by the (r,Q) policy with $r=3$ and $Q=6$ when $x_2=0$, and with $r=2$ and $Q=6$ when $x_2=1$. Secondly, comparing the procurement decisions at $\mathbf{y}$=(0,0) with that at $\mathbf{y}=(0,1)$ or $(1,0)$, the order sizes at $\mathbf{y}=(0,0)$ are generally not greater than that at $\mathbf{y}=(1,0)$ or $(0,1)$. This can be explained by the fact that at the state $\mathbf{y}=(0,0)$, the manufacturer can place two new orders simultaneously whereas at the state $\mathbf{y}=(1,0)$ or $(0,1)$ it can only place one new order. The order sizes at $\mathbf{y}=(0,1)$ are not smaller than that at $\mathbf{y}=(0,2)$, which is intuitively true considering the size of the existing outstanding order. Thirdly, the production decisions are rather insensitive to the system parameters and to the statuses of the outstanding orders. This observation is useful when designing sub-optimal and easy-to-implement production policies.

7.4.3. Impact of various factors and their interactions on the system performance

This section employs a full factorial experiment design to investigate the impact of various factors and their interactions on the system performance. This technique allows the effects of a factor to be estimated at several levels of the other factors, which can yield conclusions that are valid over a wide range of parameter settings (Montgomery, 1991). We assume two suppliers have the same supply capability and focus on the five main factors $\left(Q_i,\lambda_i,U,\xi,c_i^f\right)$. Each factor takes three levels, e.g. the suppliers' delivery capacity Q_i from (5,6,7), the raw material delivery rate

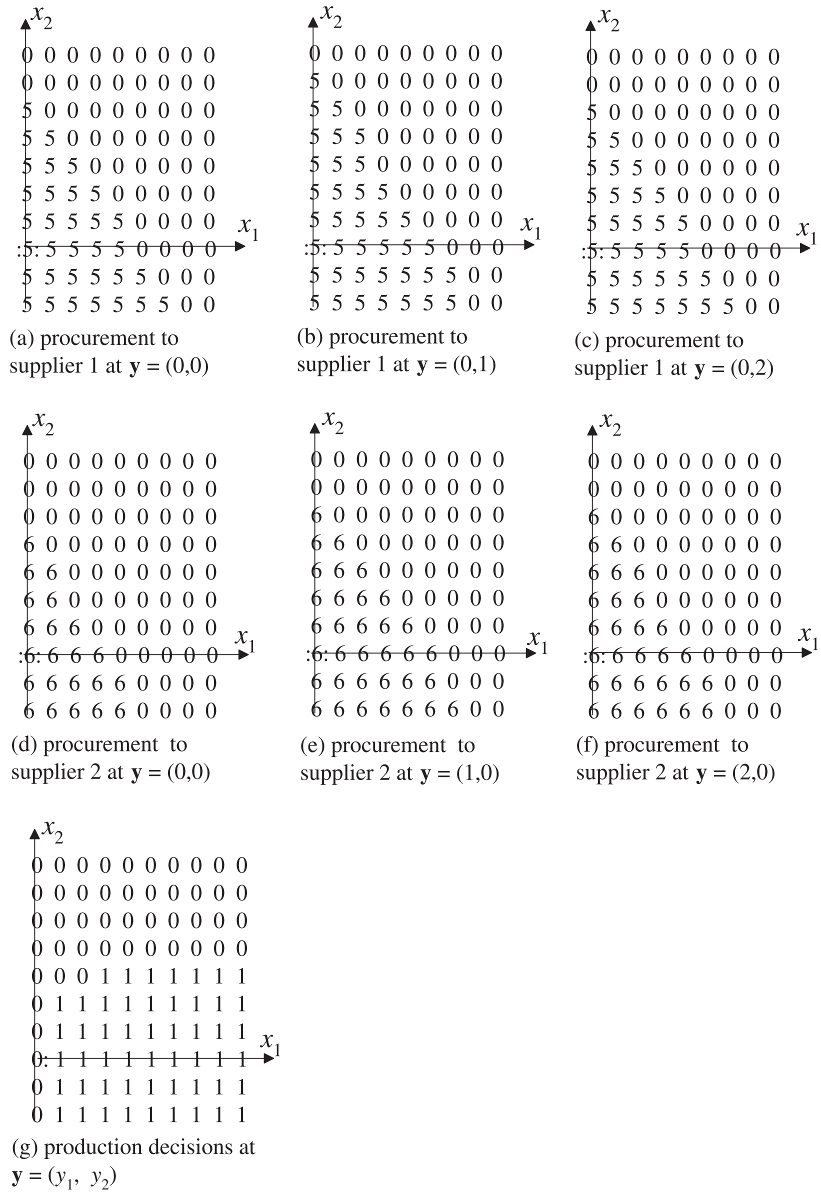

Fig. 7. The optimal procurement and production decisions in case D1 with non-adjustable outstanding orders.

λ_i from (0.10, 0.12, 0.14), the maximum production rate U from (1.0, 1.2, 1.4), the demand arrival rate ξ from (0.60, 0.72, 0.84), and the fixed ordering cost coefficient c_i^f from (0.5, 0.6, 0.7). Other system parameters are set the same as those in Section 7.1. The above ranges of the five factors are generated by increasing the value of each factor by 20% from its lowest level, which ensures that the systems have reasonable utilisations and stable in long term (i.e. maximum supply capacity exceeds the demand rate). Note that λ_i, U, and ξ represent the degree of three different types of uncertainties, the full factorial analysis can shed light on their relative importance and interactive impact on the system performance.

In total there are $3^5 = 243$ different scenarios in the full factorial experiment. We perform the analysis of variance (ANOVA) to investigate the effects of these factors and their interactions on the total cost. For each factor, the degrees of freedom (*DF*), sums of squares (*SS*), mean square (*MS*), *F* value (*F*) and probability (*P*) are given in Table 7.

For a given confidence interval 0.05, which is defined as the acceptance probability that an important factor is incorrectly rejected, all factors or interactions with a value of $P < 0.05$ are statistically significant. It can be seen from Table 7 that all factors have values $P < 0.05$ and are therefore statistically significant within the ranges considered. From the *F* value in Table 7, it shows that the forth factor (demand arrival rate) has the largest effect on the cost; the second factor (suppliers' delivery rate) and the third (manufacturer's production rate) have the second largest effect on the cost. This indicates that three factors representing three types of uncertainties have much more significant impact on the cost than the other two factors (i.e. the suppliers' delivery capacity and the suppliers' fixed ordering cost). The interactions with the factor c_i^f (i.e. the fixed ordering cost) are statistically insignificant within the ranges under consideration, whereas all other interactions have significant effect since their *P* values are less than 0.05.

8. Conclusions

This paper considers the optimal integrated inventory management for raw material procurement and production control in a manufacturing supply chain with multiple suppliers in the

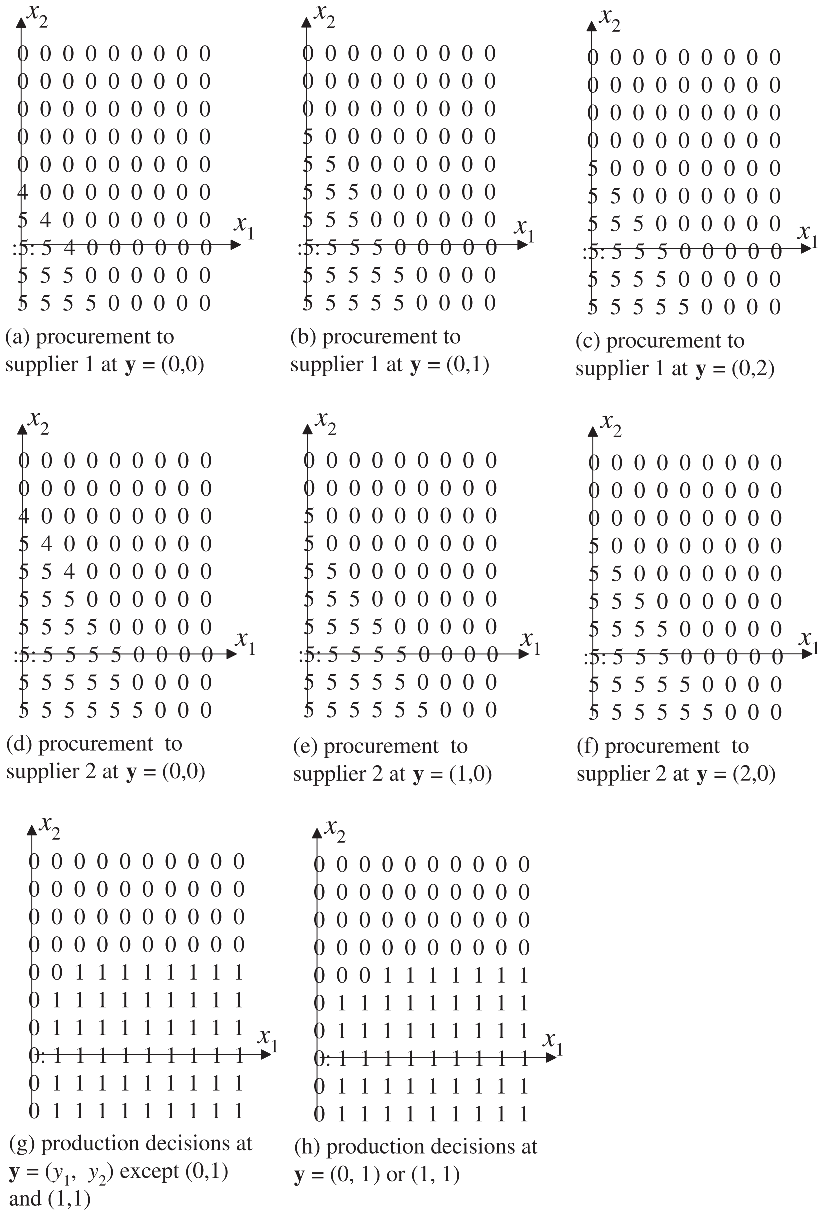

Fig. 8. The optimal procurement and production decisions in case D4 with non-adjustable outstanding orders.

presence of multiple types of uncertainties such as uncertain material supplies, stochastic production times, and random customer demands. We focus on the supplier management issues such as supplier base reduction and supplier differentiation. Our main contributions include: (i) a mathematical model and a solution method are presented for the optimal procurement and production problem with multiple suppliers and multiple uncertainties; (ii) Under the assumption that the supply chain is integrated in the sense that the suppliers and the manufacturer have an agreement that the manufacturer can adjust the order quantity at any future decision point before it arrives, we are able to analytically establish the qualitative relationships between the supplier base size and the system performance, and between the suppliers' capabilities (such as delivery capacity, delivery lead-time and reliability, and ordering cost) and the system performance. We also establish the qualitative relationship between the procurement decisions to different suppliers so that we can differentiate suppliers. The model further enables us to quantify the above relationships and achieve the best trade-off between the supplier base reduction and the supplier capability improvement. Numerical examples are provided to verify and illustrate the results; (iii) we extend the model by relaxing the assumption of adjustable outstanding orders. Numerical examples are provided to illustrate that the main results can carry over to the cases with non-adjustable outstanding orders.

The managerial insights of this study include: (i) increasing supplier base size, increasing suppliers' capacity, shortening material delivery time and improving material delivery reliability would benefit the manufacturer in the stochastic supply chain under consideration; (ii) the suppliers' delivery lead-time and its reliability has the most significant impact on the system performance compared with supplier base size, suppliers' delivery capacity and suppliers' fixed ordering cost in the range of the experimented scenarios; (iii) there exists a trade-off between the supplier base reduction and the supplier capability improvement, i.e. it appears that a U-shape relationship exists between the system performance and the level of the supplier base reduction strategies; (iv) the optimal non-zero procurement decisions to the suppliers with higher fixed ordering cost are actually the same as that to

Table 7
Analysis of variance for the total cost under multiple factors.

Source	DF	SS	MS	F	P
Q_i	2	764	382	636.85	0.000
λ_I	2	77360	38680	64486.92	0.000
U	2	67277	33639	56082.32	0.000
ξ	2	426858	213429	355828.33	0.000
c_i^f	2	288	144	240.41	0.000
$Q_i^*\lambda_I$	4	212	53	88.20	0.000
Q_i^*U	4	40	10	16.70	0.000
$Q_i^*\xi$	4	408	102	170.12	0.000
$Q_i^*c_i^f$	4	1	0	0.23	0.921
λ_i^*U	4	35	9	14.75	0.000
$\lambda_i^*\xi$	4	2409	602	1004.02	0.000
$\lambda_i^*c_i^f$	4	1	0	0.45	0.769
$U^*\xi$	4	5735	1434	2390.33	0.000
$U^*c_i^f$	4	0	0	0.00	1.000
$\xi^*c_i^f$	4	1	0	0.26	0.906
Error	192	115	1		
Total	242	581504			

Note: * Represents the interaction of two factors.

the suppliers with lower fixed ordering cost; (v) the model is able to evaluate and identify the best balance point so that the manufacturer can achieve the trade-off in managing inventory, production and supplier base by taking into account the interactions of multiple stochastic factors in the supply chain system. The optimal procurement and production decisions have good structural properties such as switching regions and monotonic boundary curves. It appears that the optimal production decisions and the optimal ordering decisions are only loosely related in the range of the experimented scenarios; (vi) under the assumption of adjustable outstanding orders, the optimal procurement policy can be closely approximated by a set of (s,S)-type policies constrained by the delivery capacities; whereas under the assumption of non-adjustable outstanding orders, it appears to be more appropriate to approximate the optimal procurement policy using a set of (r,Q)-type policies; (vii) In the situation with non-adjustable outstanding orders, the full factorial experiment via ANOVA reveals that three factors representing three types of uncertainties have much more significant impact on the cost (with the demand arrival rate has the largest effect) than other two factors (i.e. the suppliers' delivery capacity and the suppliers' fixed ordering cost) within the ranges under consideration. The interactions with the factor 'the fixed ordering cost' are statistically insignificant, whereas all other interactions have significant effect.

Some of the above results complement the findings in the literature with respect to the preference of single sourcing versus multiple sourcing strategies (Burke et al., 2007). For example, we showed that single scouring strategy is unlikely optimal in our stochastic supply chain unless the single supplier's capability is sufficiently better than multiple suppliers. Our results on the supplier differentiation in Proposition 4 confirm and complement the findings in Dada et al. (2007), e.g. if a given supplier is not used, then no more expensive suppliers than this supplier should be used. It should be pointed out that our findings are based on the adoption of the optimal integrated inventory management policy. If a non-optimal policy is applied, the results could be different. Further research could be done in the following directions: (i) extending the model to more general situations by relaxing some assumptions, e.g. allowing multiple orders to each supplier; (ii) taking into account the associated costs in supplier capability improvement and maintaining supplier base size; (iii) considering other types of uncertainties in the supply chain such as defective raw materials and imperfect production.

Acknowledgements

We thank three reviewers for providing constructive comments and pointing out relevant references, which has improved the paper significantly.

Appendix A

The problem in (1) is a continuous time Markov decision problem. Using the uniformisation technique (Puterman, 1994; Song & Sun, 1998), the continuous-time Markov chain problem can be transformed into an equivalent discrete-time problem. Let $v = \xi + U + \sum_1^N \lambda_i$ be the uniform transition rate. Under an admission control policy $\mathbf{u} \in \Omega$, the one-step transition probability $Prob(\mathbf{y}|\mathbf{x},\mathbf{u})$ is given as follows (cf. Silbermayr & Minner, 2012; Song, 2013):

$$Prob((x_1-1,x_2+1)|\mathbf{x},\mathbf{u}) = u/v,$$
$$Prob((x_1+k,x_2)|\mathbf{x},\mathbf{u}) = \sum_i \lambda_i \cdot I\{q_i = k\}/v, \text{ for } k = 1,2,\ldots,max_i\{q_i\},$$
$$Prob((x_1,x_2-1)|\mathbf{x},\mathbf{u}) = \xi/v,$$
$$Prob(\mathbf{x}|\mathbf{x},\mathbf{u}) = \left(U + \sum_1^N \lambda_i - u - \sum_1^N \lambda_i \cdot I\{q_i > 0\}\right) \Big/ v.$$

Let $0 = t_0 < t_1 < \ldots < t_k < \ldots$ be the potential state transition epochs, and $\mathbf{x}_k = \mathbf{x}(t_k)$ be the destination state of the kth transition. If $\mathbf{u}_k = \mathbf{u}(t_k)$ denotes the control decisions at time t_k, i.e. the control decision of the kth transition, it follows that $\mathbf{x}(t) = \mathbf{x}_k$ and $\mathbf{u}(t) = \mathbf{u}_k$, if $t \in [t_k, t_{k+1})$. To compute the cost function for a given initial condition $\mathbf{x}(0) = \mathbf{x}_0$ and control policy $\mathbf{u}(t)$, we have

$$E\int_0^\infty e^{-\beta t} G(\mathbf{x}(t),\mathbf{u}(t))dt = E\sum_{k=0}^\infty \int_{t_k}^{t_{k+1}} e^{-\beta t} G(\mathbf{x}_k,\mathbf{u}_k)dt$$
$$= E\sum_{k=0}^\infty \frac{1}{\beta} e^{-\beta t_k} \cdot [1 - e^{-\beta(t_{k+1}-t_k)}] \cdot G(\mathbf{x}_k,\mathbf{u}_k)$$

Note that $t_k = \sum_{j=1}^k (t_j - t_{j-1})$. Random variables $(t_j - t_{j-1})$ are independent for any $j > 0$ and follow the same exponential distribution with the uniform transition rate v. Due to the independence of the three terms on the right-hand side of the above equation and exchanging the mathematical expectation with the sum operator, the cost function can be further simplified using $Ee^{-\beta t_k} = \left(\int_0^\infty e^{-\beta\tau} \cdot ve^{-v\tau} d\tau\right)^k = \left(\frac{v}{\beta+v}\right)^k$ and $E(1 - e^{-\beta(t_{k+1}-t_k)}) = \frac{\beta}{\beta+v}$. Hence, we have

$$E\int_0^\infty e^{-\beta t} G(\mathbf{x}(t),\mathbf{u}(t))dt = \frac{1}{\beta+v}\sum_{k=0}^\infty \left(\frac{v}{\beta+v}\right)^k EG(\mathbf{x}_k,\mathbf{u}_k) \tag{A1}$$

Therefore, the problem is transformed into a discrete-time Markov chain problem with non-negative unbounded cost per step and an infinite countable state space. Following the stochastic dynamic programming theory, the Bellman optimality equation is

$$J(\mathbf{x}) = (\beta+v)^{-1} \min_{u,q_i} [G(\mathbf{x},\mathbf{u}) + \xi J(x_1,x_2-1) + uJ(x_1-1,x_2+1)$$
$$+ \sum_{i=1}^N \lambda_i I\{q_i > 0\} J(x_1+q_i,x_2\} + \left(U + \sum_{i=1}^N \lambda_i - u - \sum_{i=1}^N \lambda_i \cdot I\{q_i > 0\}\right) J(\mathbf{x})\Bigg] \tag{A2}$$

where $J(\mathbf{x})$ is defined in (1) for $\mathbf{x} \in X$. To simplify the narrative, we define $J(\mathbf{x}) := +\infty$ for $\mathbf{x} \notin X$. The derivation of (A2) can be compared to the arguments in Puterman (1994) and Song (2013). The above equation can be simplified as

$$J(\mathbf{x}) = (\beta+v)^{-1}\Big[g(\mathbf{x}) + \xi J(x_1,x_2-1)$$
$$+U \cdot \min\left\{c_p + J(x_1-1,x_2+1), J(\mathbf{x})\right\} + \sum_{i=1}^N \min\left\{c_i^v q_i + I\{q_i > 0\}\right.$$
$$\left. \cdot c_i^f + \lambda_i J(x_1+q_i,x_2) | 0 \leqslant q_i \leqslant Q_i\right\}\Big] \tag{A3}$$

The existence of a control policy to achieve the minimum in (1) follows from the fact that the one-step cost function is non-negative and only finitely many controls are considered at each state (Bertsekas, 1976). (A3) implies that the optimal policy can be described in terms of the optimal cost function.

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