Distribution Forecast Targeting in an Open-economy, Macroeconomic Volatility and Financial Implications[1]

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# Distribution Forecast Targeting in an Open-economy, Macroeconomic Volatility and Financial Implications

**Abstract**

In an open-economy faced with parameter uncertainty, this paper uses distribution forecasts to investigate the impact of alternative inflation targeting policies on macroeconomic volatility and their potential implications on financial stability. Theoretically, Domestic Inflation Targeting (DIT) leads to less volatility than Consumer Price index Inflation Targeting (CPIIT) for several macroeconomic variables and, in particular, for the interest rate. Empirically, a positive relationship between interest rate volatility and financial instability emerges for the US, UK and Sweden since the early 1990s. Bridging theory and empirical evidence, we conclude that the choice of the inflation targeting regime has an important impact on macroeconomic volatility and potential implications for financial stability.

JEL Classification: E52, E58, F41.

Key Words: Macroeconomic volatility; financial stability; interest rate volatility; multiplicative uncertainty; Markov jump linear quadratic systems; open-economy; optimal monetary policy; inflation index.

# 1 Introduction

Central banks are called to take monetary policy decisions in an uncertain contest. As a result, policy practice and research have always been challenged to experiment more efficient ways to tackle uncertainty. In this respect, during the last decade, distribution forecasts of the main macroeconomic variables, commonly known as fan charts, have become an important instrument for both monetary policy decisions and communication with the public[1]. A key feature of a distribution forecast is its volatility at each future point in time. This information matters in that lower volatility implies more forecast accuracy and, in general, less expected uncertainty surrounding the path of the variable at issue.

Motivated by the pervasive role played by uncertainty in the decision making process of any economic agent, this paper first theoretically investigates to what extent, if any, alternative inflation targeting policies impact on the expected volatility of the macroeconomic variables in presence of parameter uncertainty. We choose an open-economy framework as in this case the presence of the exchange rate even more separates alternative inflation targeting policies. In this framework, we compare the performance of different inflation targeting policies in terms of the expected volatility of the macroeconomic variables. Our motivation, in doing so, stems from the fact that central banks continuously face various types of uncertainty in setting the monetary policy, an important one being parameter uncertainty. Furthermore, this matters for the private sector, which has to constantly take decisions subject to the expected distribution forecast of inflation, output gap and the interest rate.

In line with this motivation, our interest on "raw" expected volatilities rather than a function of these volatilities, as a utility based welfare measure, is due to the fact that the former bears the advantage to be operational for policy decisions. Specifically, investigating expected volatilities of the macroeconomic variables is consistent with the *inflation forecast targeting* operating procedure in use at various central banks as the Bank of England, Sweden's Riksbank, Norway's Norges Bank, and the Reserve Bank of New Zealand. In contrast, a utility based welfare measure it is not (Holmsen, Qvigstad, Røisland and Solberg-Johansen 2008, Svensson 2010, Adolfson, Laséen, Lindé, and Svensson 2011). As to the policies, the focus is on Domestic Inflation Targeting (DIT), where the central bank aims to stabilize inflation related to the goods domestically produced, and CPI Inflation Targeting (CPIIT), which also considers the goods

[1] Indeed, a large number of central bank during this period have released to the general public the distribution forecasts of inflation and real activity by publishing these forecasts on their website. Some central banks have also published the distribution forecast of the interest rate.

imported from the rest of the world.

These alternative policies can be respectively referred to the stabilization of core inflation and headline inflation. Core inflation, which excludes international food and energy prices from the consumer basket, tends to be used as a proxy for domestic inflation[2]. Generally, central banks target headline inflation. Yet there is one central bank, the Bank of Thailand, that explicitly targets core inflation. Furthermore, in most central banks core inflation is constantly monitored and plays an important role in decision-making and communication. The Norges Bank of Norway, for example, reports on its home page both current core inflation and CPI inflation stating that uses the former as an operational guide since it can better indicate the underlying trend of inflation.

With respect to the Fed, however, it has been found that core inflation is not necessarily the best predictor of total inflation (Crone, Khettry, Mester and Novak, 2013). Nevertheless, as argued by Mishkin (2007), both for the purposes of internal deliberations and for communications with the public, central bankers are truly concerned with the underlying rate of inflation, for which core inflation can be a useful proxy.

Thus, in our opinion, comparing the performance of the alternative targeting policies in terms of the expected volatility of the main macrovariables should not simply be used to judge the superiority of one policy over the other in this specific respect. Rather, more broadly, volatilities comparison is useful to see how both policies can complement each other in decision-making and communication with the public. We think this especially matters when it is acknowledged that policymakers continuously face, among the others, parameter uncertainty. With this caveat in mind, the main result of our analysis is that, considering parameter uncertainty, DIT implies less volatility of the main macroeconomic variables than CPIIT, in particular for the interest rate. This finding is relevant for real-world monetary policy effectiveness. Indeed, best practice monetary policy is largely implemented via forward guidance by steering short-term interest rates and shaping the expected path of these short rates. By making this task harder, interest rate volatility plays against here. Empirically, Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez, Uribe (2011) show that interest rate volatility matters as it affects output, consumption and investment in emerging small open economies. Thus, this result is important *per se* as it suggests that giving more attention to the stabilization of domestic inflation can reduce macroeconomic volatility.

[2]It is worth noticing that the correlation between domestic price inflation and CPI inflation for the countries that we empirically study in this paper, i.e. US, UK and Sweden is, respectively, 0.85, 0.87, and 0.92 .

In addition, beyond being a key variable in the real sector, the interest rate is a key variable also in the financial sector. Sharp increases in the official interest rate strain financial markets, as it occurred for example in 1994 in the US. In general, changes in the current and expected future official rates are transmitted to market rates and asset prices. We thus conjecture that excessive short-term interest rate volatility can be associated with financial instability. In empirically testing this hypothesis, we find a significant positive relation between interest rate volatility and financial instability in all US, UK and Swedish economies since the early 1990s. This second result of the paper thus suggests that concentrating on DIT rather than CPIIT can also assist in fostering financial stability.

The intuition for these findings is based on the combined action of three factors. The *level of policy activism* implied by the choice of the inflation targeting policies, the *consideration of parameter uncertainty* on the part of the central bank, and the transmission mechanism of monetary policy to the real and financial sector. Our findings show that under CPIIT there is more policy activism than under DIT. Thus, when the central bank decides the optimal policy and takes into account model parameter uncertainty, a more active policy results in more volatility for most of the macroeconomic variables, market rates and asset prices.

Arguably, the findings that we have obtained have important policy implications for inflation-targeting economies like the US, UK and Sweden. According to our theoretical results, more emphasis on DIT (or to a targeting policy closer to DIT than CPIIT) would lower interest rate volatility. The beneficial impact of the latter potentially extends to financial stability. In March 2013, Ben Bernanke noted that in order to address financial stability concerns the Federal Open Market Committee (FOMC) amongst other things now provides greater clarity concerning the likely course of the federal funds rate. Indeed, the empirical part of our paper shows that lower interest rate volatility (which we proxy by the 2-year moving standard deviation of the interest rate and a GARCH representation) reduces financial instability in all three economies.

The literature on the choice of the inflation measure to stabilize has identified various important factors to consider. Mankiw and Reis (2003) in a static and closed economy set-up show that monetary policy should target inflation in the sticky-price sector. The same result, in a dynamic set-up, is found by Aoki (2001) and Benigno (2004), respectively in a closed economy and a monetary union, and by Gali and Monacelli (2005) in an open economy. This finding suggests one should target domestic inflation as it tends to be stickier than CPI inflation. Regarding this literature, we also find that domestic inflation should be targeted, although this finding depends on different factors. Specifically, longer transmission lags necessary to affect domestic inflation versus CPI inflation, larger exposure of CPI inflation to foreign shocks, and structural parameters uncertainty that makes CPI inflation more volatile than domestic inflation.

CPI inflation has been also questioned as the inflation measure to target considering economic indeterminacy (Batini, Levine and Pearlman 2005), and external price shocks (Eckstein and Segal 2010). Contrasting results, instead, emerge considering alternative producers price setting behaviors (Corsetti, Dedola and Leduc, 2010), the elasticity of substitution between domestic and foreign goods (Sutherland, 2006), and the intertemporal elasticity of substitution in consumption (De Paoli, 2009a, Kirsanova, Leith and Wren-Lewis, 2006). CPI inflation seems, finally, preferable to domestic inflation in presence of complete and immediate exchange rate pass-through (Svensson 2000), sticky wages (Campolmi 2014), or if imports are production inputs and not only used in final consumption (Jakab and Karvalits 2009).

With respect to the previous litterature, the novelty of this work is twofold. It frames the comparison between alternative inflation measures within the *inflation forecast targeting* operating procedure in use at many central banks, and accounts for parameter uncertainty.

This innovation is carried out by comparing distribution forecasts associated with alternative inflation targeting policies. We do so in three steps. First, we obtain distribution forecasts considering parameter uncertainty along with exogenous shocks. Then, we associate these distribution forecasts to alternative inflation measure to stabilize by varying the weights of domestic and CPI inflation in a standard loss function; this is consistent with the procedure indicated for example by Holmsen, Qvigstad, Røisland and Solberg-Johansen (2008) for the Bank of Norway. Finally, we compare the impact of alternative inflation measure to stabilize on the distribution forecasts of the main macroeconomic variables using appropriate statistics. In this way, the paper contributes to the literature offering a new standpoint, based on inflation forecast targeting and parameter uncertainty -which is a formidable challenge to real-world monetary policy- for assessing the ability of DIT to help reducing macroeconomic volatility. It also departs from the previous literature suggesting an indirect link between the choice of the inflation targeting policies and financial stability, where interest rate volatility acts as a drive belt.

The paper is organized as follows. Section 2 presents the model and its calibration. Model simulations under the alternative inflation targeting policies are reported and discussed in Section 3 where the role played by model parameter uncertainty in the policy assessment is also analyzed. Section 4 relies on US, UK and Swedish data to show empirically that lower interest rate uncertainty has a beneficial impact also on financial stability. Section 5 concludes.

# 2 The model

The model adopts a New Keynesian framework drawing on Flamini (2007) and the methodology developed by Svensson and Williams (2007) to compute the optimal monetary policy when the central bank have limited information on the behaviour of the private sector[3].

Regarding the optimal policy, a standard approach employed in the literature consists of modeling central bank and private sector behavior with a quadratic loss function and linear aggregate demand and supply, respectively. This approach, in presence of additive exogenous shocks, leads to the well known Certainty Equivalence result: the same optimal policy with or without shocks. Thus, the model would generate *mean forecasts* for each variable in response to a shock, i.e. impulse response functions, rather than the much more useful *distribution forecasts*, i.e. impulse response distribution forecasts. The limitation is therefore clear: with mean forecasts, important information used in policy decisions consisting of the uncertainty associated with the forecast is lost. To avoid Certainty Equivalence and therefore obtain useful distribution forecasts, we relax the strong assumption usually held in the literature that central banks know with certainty the model of the economy. Thus, when an exogenous shock hits, several possible expected paths of the economy are possible, which result in a distribution forecast for each macroeconomic variables. To consider model parameter uncertainty, which has nature of *multiplicative* uncertainty, along with exogenous shocks, which instead have nature of *additive* uncertainty, the modeling strategy follows the Svensson and Williams (2007) approach based on Markov jump-linear-quadratic systems[4].

## 2.1 The household

The economy is populated by a continuum of consumers/producers indexed by $j \in [0, 1]$ sharing the same preferences and living forever. The representative household seeks to maximize the expected value of an intertemporal utility of the form

[3] This section reports a concise description of the model in order to allow a clear presentation of how *model uncertainty* affects the expected dynamics of the economy. Details on the derivation of the structural relations can be found in Flamini (2007).

[4] An interesting application with respect to monetary policy under financial uncertainty is provided by Williams (2012).

$$E_t \sum_{\tau=0}^{\infty} \delta^{\tau} U\left(C_{t+\tau}, \check{C}_{t+\tau-1}\right), \tag{1}$$

where $\delta$ is the intertemporal discount factor, $C_t$ is total consumption of household $j$, and $\check{C}_t$ is the total aggregate consumption. Preferences over total consumption feature habit formation *a la'* Abel (1990) captured by the following instantaneous utility function

$$U\left(C_{t+\tau}, \check{C}_{t+\tau-1}\right) = \frac{\left(C_{t+\tau}/\check{C}^{\iota}_{t+\tau-1}\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \tag{2}$$

where $\sigma > 0$ is the intertemporal elasticity of substitution and $\iota \geq 0$ captures habit persistence. Habit persistence determines the degree of backward and forward lookingness of the household, and therefore the degree of persistence in the aggregate demand. The previous literature offered a wide range of estimations for habit persistence to which a wide range of aggregate demands corresponds. This range spans from a purely backward looking aggregate demand, where a change in the previous period output gap leads to the same change in the current period output gap, to completely forward looking aggregate demand, where the previous period output gap does not affect the current period output gap[5]. Given the variety of proposed values for habit persistence, this work assumes that the central bank does not choose a specific value for this parameter but a range. In other words, the central bank is uncertain on the amount of persistence in the aggregate demand.

Back to the model, total consumption, $C_t$, is a Cobb-Douglas function of domestic good consumption, $C_t^d$, and import good consumption, $C_t^i$,

$$C_t \equiv C_t^{d^{(1-w)}} C_t^{i^{w}}, \tag{3}$$

where $w$ determines the steady state share of imported goods in total consumption and $C_t^d$, $C_t^i$ are Dixit-Stiglitz aggregates of continuum of differentiated domestic goods and import goods (henceforth indexed with $d$ and $i$ respectively),

$$C_t^h = \left[\int \left(C_t^h(j)\right)^{1-\frac{1}{\vartheta}} dj\right]^{\frac{1}{1-\vartheta}}, \quad h = d,\ i,$$

where $\vartheta > 1$ is the elasticity of substitution between any two differentiated goods and, for the

[5] For a review of the previous literature on the calibration of habits formation see Leith, Moldovan and Rossi (2009).

sake of simplicity, is the same in both sectors[6]. Finally, $P^c$ is the overall Dixit-Stiglitz price index for the minimum cost of a unit of $C_t$ and is given by

$$P_t^c = \frac{P_t^{i^w} P_t^{d^{(1-w)}}}{w^w (1-w)^{(1-w)}}, \tag{4}$$

with $P^d$, $P^i$ denoting, respectively, the Dixit-Stiglitz price index for goods produced in the domestic and import sector.

Assuming a no-Ponzi schemes condition, utility maximization subject to the budget constraint and the limit on borrowing gives the Euler equation and the Uncovered Interest Parity, which in terms of log deviations from steady state values are, respectively

$$c_t = \beta c_{t-1} + (1-\beta) c_{t+1|t} - (1-\beta) \sigma \left( i_t - \pi_{t+1|t}^c \right), \qquad \beta \equiv \frac{\iota_t (1-\sigma)}{1 + \iota_t (1-\sigma)} < 1, \tag{5}$$

$$i_t - i_t^* = s_{t+1|t} - s_t + \upsilon_t, \tag{6}$$

where for any variable $x$, the expression $x_{t+\tau|t}$ stands for the rational expectation of that variable in period $t+\tau$ conditional on the information available in period $t$ and, by means of a log-linearization, the variables $c_t$, $\pi_t^c$, $i_t$, $i_t^*$, $\left(s_{t+1|t} - s_t\right)$ and $\upsilon_t$ are log-deviations from their respective constant steady state values; finally, $c_t$ denotes total aggregate consumption, obtained considering that in equilibrium total consumption for agent $j$ is equal to total aggregate consumption, i.e. $C_t = \check{C}_t$, $\pi_t^c$ denotes CPI inflation (measured as the log deviation of gross CPI inflation from the constant CPI inflation target), and $\upsilon_t$ is a risk premium shock added to capture financial market volatility and it is modeled with a stationary univariate AR(1) process

$$\upsilon_{t+1} = \gamma_\upsilon \upsilon_t + \xi_{t+1}^\upsilon.$$

[6] Following Corsetti and Pesenti (2004), the intratemporal elasticity of substitution between domestic and import goods is set equal to one. This assumption ensures the stationarity of the model.

### 2.1.1 Domestic consumption of goods produced in the domestic sector

Preferences captured by equation (3) imply that the (log deviation of the) domestic demand for goods produced in the domestic sector, $c_t^d$, is given by

$$c_t^d = c_t - \left(p_t^d - p_t^c\right),$$

which, considering the (log-linearized version of the) price index equation (4), can be rewritten as

$$c_t^d = c_t + wq_t, \tag{7}$$

where $q_t \equiv p_t^i - p_t^d$ is the (log-deviation of the) real exchange rate.

Then, solving equation (5) for $c_t$ and combining it with equation (7) we obtain

$$c_t^d = -\sigma\left(1 - F_1 L\right)^{-1} \rho_t - \sigma\left(1 - F_1 L\right)^{-1} wq_t + wq_t, \tag{8}$$

where $F_1 < 1$ is the smaller root of the characteristic polynomial of equation (5) and

$$\rho_t \equiv \sum_{\tau=0}^{\infty} \left(i_{t+\tau|t} - \pi_{t+\tau+1|t}^d\right) \tag{9}$$

can be interpreted as the long real interest rate.

### 2.1.2 Aggregate demand for goods produced in the domestic sector

Total aggregate demand for the good produced in the domestic sector is

$$\widehat{Y}_t^d = C_t^d + Y_t^{d,d} + Y_t^{d,i} + C_t^{*d}, \tag{10}$$

where $Y_t^{d,d}$, $Y_t^{d,i}$ and $C_t^{*d}$ denote the quantity of the (composite) domestic good which is used as an input in the domestic sector, as an input in the import sector and which is demanded by the foreign sector, respectively.

While both sectors feature a continuum of unit mass of firms, indexed by $j$, that produce differentiated goods $Y_t^d(j)$ and $Y_t^i(j)$ in the domestic and import sector respectively, the two sectors differ for the input used: the domestic sector uses a composite input consisting of the domestic (composite) good itself and the (composite) import good provided by the import sector; the import sector uses a composite input consisting of the foreign good $Y_t^*$ and the domestic (composite good). Furthermore, to capture the real-world feature that production inputs tend to be rigid at business cycle frequency, sectors are assumed to use a Leontief technology. Thus, the production functions in the domestic and import sector are given respectively by

$$Y_t^d(j) = f\left[A_t^d \min\left\{\frac{Y_t^{d,d}}{1-\mu}, \frac{Y_t^{i,d}}{\mu}\right\}\right], \; Y_t^i(j) = f\left[A_t^i \min\left\{\frac{Y_t^*}{1-\mu^i}, \frac{Y_t^{d,i}}{\mu^i}\right\}\right], \; \mu, \mu^i \in [0,1], \quad (11)$$

where $f$ is an increasing, concave, isoelastic function, $A_t$ is an exogenous (sector specific) economy-wide productivity parameter, $(1-\mu)$ and $\mu$ denote, respectively, the shares of the domestic good and import good in the composite input required to produce the differentiated domestic good $j$, and $\left(1-\mu^i\right)$ and $\mu^i$ denote, respectively, the shares of the foreign good and domestic good in the composite input required to provide the differentiated import good $j$. Focusing on $\mu^i$, it is worth of note that when this parameter is positive a change of the exchange rate does not fully reflect in a change of the import goods price as the composite input consists also of the domestic good. In this case the exchange rate pass-through turns out to be *incomplete*. It is well known that the exchange rate pass-through can be quite variable over time due to numerous factors playing a role in its determination. To model pass-through uncertainty, the parameter $\mu^i$ is assumed to be uncertain[7]. Returning to the description of the technology, equation (11) implies that the quantities of the (composite) domestic good used as an input in the domestic and import sector are

$$Y_t^{d,d} = \frac{1}{A_t^d}(1-\mu) f^{-1}\left(\widehat{Y}_t^d\right), \qquad Y_t^{d,i} = \frac{1}{A_t^i}\mu^i f^{-1}\left(\widehat{Y}_t^i\right), \quad (12)$$

where $\widehat{Y}_t^i$ denotes the demand of the import good. Finally, log-linearizing equation (10) around the steady state values yields

$$\widehat{y}_t^d = \kappa_1\left(\mu^i\right) c_t^d + \kappa_2\left(\mu^i\right)\widehat{y}_t^i + \kappa_3\left(\mu^i\right) c_t^{*d}, \quad (13)$$

where $\kappa_1'\left(\mu^i\right), \; \kappa_3'\left(\mu^i\right) < 0$ and $\kappa_2'\left(\mu^i\right) > 0$.

[7] Campa and Goldberg (2006 and 2005) argue that changes in pass-through can be driven by changes in the use of imported inputs or in the composition of a country's import basket when the component products have distinct pass-through elasticities. Futhermore, various authors (Devereux and Engel 2001, Devereux, Engel and Storgaard 2004) link the pass-through variability to changes in monetary stability and the persistence of exogenous shocks, and Bacchetta and van Wincoop (2005) to changes in the market share and in the degree of differentiation of the exporting country goods.

Next, the output-gap in sector $h = d, i$ is defined as

$$y_t^h \equiv \widehat{y}_t^h - y_t^{h,n},$$

where $y_t^{h,n}$ denotes the log deviation of the natural output in sector $h$ from its steady state value. As in Svensson (2000), both $y_t^{h,n}$ and $c_t^{*d}$ are exogenous and follow, respectively

$$y_{t+1}^{h,n} = \gamma_y^{h,n} y_t^{h,n} + \eta_{t+1}^{h,n}, \qquad 0 \leq \gamma_y^{h,n} < 1, \qquad h = d, i, \tag{14}$$

where $\eta_{t+1}^{h,n}$ is a serially uncorrelated zero-mean shock to the natural output level (a productivity shock), and

$$c_t^{*d} = \overline{\beta}_y^* y_t^* + \theta^* w^* q_t, \tag{15}$$

where $\overline{\beta}_y^*$ is the income elasticity of foreign real consumption and $\theta^*$ and $w^*$ denote, respectively, the foreign atemporal elasticity of substitution between domestic and foreign goods and the share of domestic goods in foreign consumption. Finally, in line with the central banks' view of the approximate one-period lag necessary to affect aggregate demand, consumption decisions are assumed to be predetermined one period in advance. Accordingly, repeating the same derivation with preferences maximized on the basis of one period ahead information results in the aggregate demand in the domestic sector. This relation, expressed in terms of the output-gap, is given by

$$y_{t+1}^d = \beta_y y_t^d - \beta_\rho \rho_{t+1|t} + \beta_q q_{t+1|t} - \beta_{q_{-1}} q_t + \beta_{y^*} y_t^* + \beta_{y^n} y_t^{d,n} + \eta_{t+1}^d - \eta_{t+1}^{d,n}, \tag{16}$$

where $\eta_{t+1}^d$ is a serially uncorrelated zero-mean demand shock. In (16) all the coefficients are positive and functions of the structural parameters of the model. It is worth noting that, due to the uncertainty on habit persistence, it turns out that, for any period $t$, the coefficients for the previous period output gap, real exchange rate, foreign output, and natural output in the domestic sector, $\beta_y$, $\beta_{q_{-1}}$, $\beta_{y^*}$, $\beta_{y^n}$ respectively, are uncertain.

### 2.1.3 Aggregate demand of goods produced in the import sector

Aggregate demand for import goods is given by

$$\widehat{Y}_t^i = C_t^i + Y_t^{i,d} \tag{17}$$

where $Y_t^{i,d}$ denotes the amount of the import good used as an input in the domestic sector. Log-linearizing (17) around the steady state results in

$$\widehat{y}_t^i = (1 - \widetilde{\kappa})\, c_t^i + \widetilde{\kappa} \widehat{y}_t^d. \tag{18}$$

Finally, the same assumptions used to derive the aggregate demand for the domestic sector goods yield

$$y_{t+1}^i = \beta_y y_t^i - \beta_\rho^i \rho_{t+1|t} - \beta_q^i q_{t+1|t} + \beta_{q-1}^i q_t + \beta_{y^*}^i y_t^* + \beta_{y^n}^i y_t^{i,n} + \eta_{t+1}^i - \eta_{t+1}^{i,n}, \tag{19}$$

where all the coefficients are positive and depend on the structural parameters of the model, $\eta_{t+1}^i$ is a serially uncorrelated zero-mean demand shock, and the coefficients $\beta_y$, $\beta_{q-1}^i$, $\beta_{y^*}^i$, $\beta_{y^n}^i$ are uncertain.

### 2.1.4 Aggregate supply in the domestic sector

We now assume that firm $j$ takes

$$Y_t^d(j) = \widehat{Y}_t^d \left( \frac{P_t^d(j)}{P_t^d} \right)^{-\vartheta}$$

as the demand for its own variety, where $P_t^d(j)$ is the nominal price for variety $j$. Since the composite input is a convex combination of both aggregates of domestic and import goods, as shown by equation (11), it follows that the input price is $W_t \equiv (1-\mu)\, P_t^d + \mu P_t^i$. Furthermore, adopting the Calvo (1983) staggered price scheme, the firm chooses in any period the new price with probability $(1-\alpha)$ or keeps the previous period price indexed to past inflation with probability $\alpha$. The parameter $\alpha$ determines the degree of price stickiness and exerts a major impact on the slope of the Phillips curve, that is the response of inflation to fluctuations in resource utilization. This relation seemed to have varied in the last two decades possibly due to an anchoring of inflation expectations via better monetary policy (Mishkin 2007, Boivin and Giannoni 2006, and Roberts 2006), or due to changes in the price-setting behaviour dependent on the level and variability of inflation (among the others, Cogley and Sbordone 2005 and Fernandez-Villaverde and Rubio-Ramirez 2007). To account for this uncertainty on the slope of the Phillips curve, the parameter $\alpha$ is assumed to be uncertain. Finally, we assume that when the firm can choose the optimal price, it chooses it two periods in advance. This assumption is motivated by the fact that domestic sector firms take both production and retailing decisions. The implication is that monetary policy needs a two-period lag to affect domestic inflation. This is in line with the central banks' experience of an approximate two-period lag for monetary policy to have the highest impact on inflation. Recalling that all the varieties are produced with the same technology, there is a unique input requirement function for each $j$ given by $\frac{1}{A_t^d} f^{-1}\left[Y_t^d(j)\right]$ and the variable cost of producing the quantity $Y_t^d(j)$ is $W_t \frac{1}{A_t^d} f^{-1}\left[Y_t^d(j)\right]$. It follows that the decision problem for firm $j$ at time $t$ is

$$\max_{\widetilde{P}_{t+2}^d} E_t \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \widetilde{\lambda}_{t+\tau+2}^d \left\{ \frac{\widetilde{P}_{t+2}^d \left(\frac{P_{t+\tau+1}^d}{P_{t+1}^d}\right)^\zeta}{P_{t+2+\tau}^d} \widehat{Y}_{t+\tau+2}^d \left[\frac{\widetilde{P}_{t+2}^d \left(\frac{P_{t+\tau+1}^d}{P_{t+1}^d}\right)^\zeta}{P_{t+2+\tau}^d}\right]^{-\vartheta} - \frac{W_{t+\tau+2}}{P_{t+\tau+2}^d} \frac{f^{-1}\left[\widehat{Y}_{t+\tau+2}^d \left(\frac{\widetilde{P}_{t+2}^d \left(\frac{P_{t+\tau+1}^d}{P_{t+1}^d}\right)^\zeta}{P_{t+2+\tau}^d}\right)^{-\vartheta}\right]}{A_{t+\tau+2}^d} \right\}, \tag{20}$$

where $\widetilde{\lambda}_t^d$, $\widetilde{P}_{t+2}^d$ and $\zeta$ denote, respectively, the marginal utility of domestic goods, the new price chosen in period $t$ for period $t+2$ and the degree of indexation to the previous period inflation rate[8]. Following Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003), the parameter $\zeta$ introduces inflation inertia in the Calvo model of pricesetting. Empirical evidence on $\zeta$ is characterized by contrasting results as reported by Kimura and Kurozumi (2007). It is therefore difficult to pin down a value for $\zeta$ and the paper proceeds by assuming that this parameter belongs to the set of the uncertain parameters.

Finally, following Svensson (2000), we set $\delta = 1$ to ensure the natural-rate hypothesis and assuming that the purchasing power parity holds in the long run, the log-linearized version of

[8] Recalling that consumption decisions are predetermined one period in advance, the marginal utility of domestic goods $\widetilde{\lambda}_t^d$ is obtained by the following first-order condition with respect to $C_{t+1}^d$

$$E_t U_d\left(C_{t+1}^d, C_{t+1}^i\right) = E_t\left[\lambda_{t+1} P_{t+1}^d\right] \equiv E_t \widetilde{\lambda}_{t+1}^d,$$

where $\lambda_t$ is the marginal utility of nominal income in period $t$.

the Phillips curve for the domestic sector turns out to be

$$\pi_{t+2}^{d} = \frac{1}{1+\zeta}\left[\zeta\pi_{t+1}^{d} + \pi_{t+3|t}^{d} + \frac{(1-\alpha)^{2}}{\alpha(1+\omega\vartheta)}\left(\omega y_{t+2|t}^{d} + \mu q_{t+2|t}\right)\right] + \varepsilon_{t+2} \tag{21}$$

$$= \phi_{\pi}\pi_{t+1}^{d} + (1-\phi_{\pi})\,\pi_{t+3|t}^{d} + \phi_{y}^{d} y_{t+2|t}^{d} + \phi_{q}^{d} q_{t+2|t} + \varepsilon_{t+2}, \tag{22}$$

where $\omega$ in (21) is the output elasticity of the marginal input requirement function and $\varepsilon_{t+2}$ is a zero-mean i.i.d. cost-push shock. In (22) all the implicitly defined coefficients are positive and $\phi_y^d$ and $\phi_q^d$ are uncertain due to the uncertainty on $\alpha$ and $\zeta$.

### 2.1.5 Aggregate supply in the import sector

In the import sector, the input is a convex combination of the aggregate of domestic goods and of the foreign good, with price $P_t^* S_t$, where $P_t^*$ is the price in foreign currency of the foreign good. It follows that the price of the composite input is $F_t \equiv \mu^i P_t^d + \left(1-\mu^i\right) P_t^* S_t$.

Now, relaxing the assumption that pricing decisions are predetermined and keeping all the remaining assumptions used to derive the Phillips curve in the domestic sector results in

$$\pi_{t}^{i} = \frac{1}{1+\zeta}\left[\zeta\pi_{t-1}^{i} + \pi_{t+1|t}^{i} + \frac{\left(1-\alpha^{i}\right)^{2}}{\alpha^{i}(1+\omega\vartheta)}\left(\omega y_{t}^{i} + q_{t}^{i}\right)\right] \tag{23}$$

$$= \phi_{\pi}\pi_{t-1}^{i} + (1-\phi_{\pi})\,\pi_{t+1|t}^{i} + \phi_{y}^{i} y_{t}^{i} + \phi_{q}^{i} q_{t}^{i}, \tag{24}$$

where $\alpha^i$ is the probability of not updating optimally the price in the import sector and is assumed to be uncertain, $q_t^i$ denotes (the log deviation of) the price of the composite input in the import sector expressed in terms of the import goods price, $p_t^i$, and is defined as

$$q_{t}^{i} \equiv \left(1-\mu^{i}\right)(s_t + p_t^{*}) + \mu^{i} p_t^{d} - p_t^{i}, \tag{25}$$

where $p_t^*$ is the (log) foreign price level. Relaxing the assumption of predetermined pricing decisions is motivated by the fact that the import sector only acts as a retailer for the foreign goods and, in practice, retailers do not set their price before they take effect as much as producers do. It is worthy of note that while $\mu^i$ determines the degree of completeness of the pass-through as discussed before, $\alpha^i$ determines the speed of the pass-through. Hence, uncertainty on $\alpha^i$ and $\mu^i$ captures two dimensions of the uncertainty on the exchange rate pass-through.

## 2.2 CPI inflation and the uncovered interest parity

CPI-inflation, $\pi_t^c$, is given by

$$\pi_t^c = (1-w)\,\pi_t^d + w\pi_t^i, \tag{26}$$

where $w$ is the steady state share of imported goods in total consumption and determines the degree of openness of the economy. In order to eliminate the non-stationary nominal exchange rate, it is convenient to express the Uncovered Interest Parity in terms of $q_t^i$ obtaining

$$q_{t+1|t}^i - q_t^i = \left(1-\mu^i\right) r_t - \left(1-\mu^i\right)\left(i_t^* - \pi_{t+1|t}^*\right) - \left(\pi_{t+1|t}^i - \pi_{t+1|t}^d\right) - \left(1-\mu^i\right)\upsilon_t, \tag{27}$$

where $r_t$ is the short term real interest rate defined as $r_t \equiv i_t - \pi_{t+1|t}^d$.

## 2.3 Central bank, rest of the world, and deep parameter uncertainty

The behavior of the central bank consists of minimizing the following loss function:

$$E_t \sum_{\tau=0}^{\infty} \beta^\tau \left[\mu^c \pi_{t+\tau}^{c2} + \mu^d \pi_{t+\tau}^{d2} + \lambda y_{t+\tau}^{d2} + \nu \left(i_{t+\tau} - i_{t+\tau-1}\right)^2\right], \tag{28}$$

where $\mu^c$, $\mu^d$, $\lambda$ and $\nu$ are weights that express the preferences of the central bank for alternative CPI and domestic inflation targets along with the output stabilization target, and the instrument smoothing target, respectively[9].

It is worth noticing that in the New Keynesian literature on optimal monetary policy the central bank preferences are modeled either directly in terms of volatility for inflation, output gap and first difference of the interest rate, or in terms of a quadratic approximation of the utility function of the household[10].

The first way bears the advantage to be operational. Indeed, in contrast with a loss function that approximates the utility of the representative consumer, it does not depend on the specific assumptions of the model (e.g. household preferences, inflation inertia, habit persistence, predetermined pricing decisions) which would imply fixed weights in the loss functions.

For this reason it is consistent with the *inflation forecast targeting* operating procedure adopted in several central banks as, for example, the Bank of England, Sweden's Riksbank, Norway's Norges Bank, and the Reserve Bank of New Zealand. Describing this procedure, first

[9] Regarding the motivation for an interest rate smoothing preferences in the Central Bank loss function see, for example, Svensson (2010), Holmsen et al. (2008), and Flamini and Fracasso (2011).

[10] See for example Svensson (2000, 2010) for the former and Corsetti et al. (2010) for the latter.

the staff computes alternative distribution forecasts associated with different interest rate paths minimizing a standard loss function of the type of expression (28). These optimal distribution forecasts are constructed by varying the weights and/or the discount factor in the central bank loss function. Then the Board selects the policy associated with the specific distribution forecast that suits best its preferences. Holmsen, Qvigstad, Røisland and Solberg-Johansen (2008) describe accurately this operating procedure and add at p. 22 that "From the point of view of the staff, the loss function and its relative weights are meant to represent the preferences of the Board. This is in contrast to much of the recent monetary policy literature, where the loss function approximates the utility loss of the representative consumer".

The rest of the world is exogenous and described by stationary univariate AR(1) processes for foreign inflation and income, and a Taylor rule for monetary policy, respectively

$$\pi^*_{t+1} = \gamma^*_\pi \pi^*_t + \varepsilon^*_{t+1}, \tag{29}$$

$$y^*_{t+1} = \gamma^*_y y^*_t + \eta^*_{t+1}, \tag{30}$$

$$i^*_t = f^*_\pi \pi^*_t + f^*_y y^*_t + \xi^*_t, \tag{31}$$

where the shocks are white noises.

Turning to the presence of uncertainty on some structural parameters, it is worth noting that it introduces multiplicative uncertainty in the model. This implies that the certainty-equivalence principle does not hold anymore and the optimal policy in presence of uncertainty differs from the one in presence of certainty. To model multiplicative uncertainty and compute the equilibrium we follow the Markov Jump-Linear-Quadratic approach developed by Svensson and Williams (2007). Leaving to the Appendix the description of this method, we specify here the assumptions on the parameter uncertainty faced by the central bank. First, the central bank only knows a band for each uncertain deep parameter and considers any realization as equally likely. For example, if there is only one uncertain parameter, say $\varphi$, a benchmark value is chosen, $\overline{\varphi}$, and the lower and upper bound of the support of the distribution are set equal to $\overline{\varphi} - x\overline{\varphi}$ and $\overline{\varphi} + x\overline{\varphi}$ respectively, where the coefficient $x$ modules the variance of the distribution and therefore the amount of uncertainty. Second, model parameter uncertainty and shocks to the economy are assumed to be independent. Third, the central bank is assumed not to know how the structural parameters co-move together, should they be dependent.

### 2.4 Calibration

Solving the model requires the calibration of two groups of parameters. The first consists of the parameters that are assumed to be known with certainty, while the second one consists of the benchmark values for the uncertain parameters[11].

The choice of the parameters assumed to be known with certainty follows Svensson (2000) as the current model is similar in structure to the Svensson's one. These parameters, with respect to the domestic economy, are the output elasticity of the marginal input requirement function, $\omega = 0.8$; the elasticity of substitution between varieties of the same type of good $\vartheta = 1.25$; the intertemporal elasticity of substitution, $\sigma = 0.5$; the share of import good in the composite input to produce the domestic good, $\mu = 0.1$; the share of import goods in domestic consumption, $w = 0.3$. With respect to the foreign sector, the elasticity of substitution between domestic and import goods for foreign consumers is $\theta^* = 2$; the share of the domestic good in foreign consumption is $w^* = 0.15$; the income elasticity of foreign real consumption is $\overline{\beta}_y^* = 0.9$; and the coefficients for the foreign Taylor rule are $f_{\pi^*} = 1.5$, and $f_{y^*} = 0.5$. Finally, the exogenous cost push and demand shocks have variances $\sigma_\pi^2 = \sigma_y^2 = 1$; the natural output shocks have variances $\sigma_{y^{d,n}}^2 = \sigma_{y^{i,n}}^2 = 0.5$ and AR(1)-parameter $\gamma_y^{d,n} = \gamma_y^{i,n} = 0.96$, and finally the risk premium, foreign inflation and output have AR(1) process-parameter $\gamma_{y^*} = \gamma_{\pi^*} = \gamma_\upsilon = 0.8$ and variances $\sigma_\nu^2 = \sigma_{\pi^*}^2 = \sigma_{y^*}^2 = 0.5$. As to the central bank preferences, the weights in the loss function under DIT and CPIIT are, respectively, $\mu^d = 1,\ \mu^c = 0$ and $\lambda = 0.5$, and $\mu^d = 0,\ \mu^c = 1$ and $\lambda = 0.5$.

The benchmark values of the uncertain parameters follow Banerjee and Batini (2003) as to the measure of habit formation in the utility function, $\overline{\iota} = 0.8$ and Smets and Wouters (2005) as to the degree of indexation to the previous period inflation rate, $\overline{\zeta} = 0.66$. The probability on not optimally updating the price in the current period in the domestic and import sector, $\overline{\alpha}$, and $\overline{\alpha^i}$, are set equal to 0.5 following Svensson (2000) and Flamini (2007), respectively. Finally, the value of the share of domestic good in the composite input to supply the import good, $\overline{\mu^i}$, is set to 0.35 consistently with Flamini (2007) and such that the lower and upper bound of the support of the $\mu^i$ distribution are realistic for the uncertainty level considered in the analysis; specifically the lower and upper bounds are 0.245 and 0.405.

[11] In this paper we assume that the central bank is uncertain on some key parameters. This does not mean that the true value of the remaining parameters is known in the real world and corresponds to the value specified in the calibration suggested by the previous literature. Nevertheless, this is a problem of the literature at large and is beyond the scope of the current paper.

### 2.4.1 Robustness check

The current model is also similar in spirit to the Leitemo and Söderström (2005) model. Although the latter is not microfounded, its parametrization for the exogenous disturbances provides a valid alternative to check for the robustness of the results. In the Leitemo and Söderström model, the cost-push shock and the demand shock are AR(1) processes and their AR(1)-coefficients, $\gamma_\pi$ and $\gamma_y$, are set equal to 0.3 (this is a difference with the previous calibration where the AR(1)-coefficients for these two shocks are implicitly set equal to zero). The variances for these shocks are $\sigma_y^2 = 0.656$ and $\sigma_\pi^2 = 0.389$, while the variance for the shocks to the risk premium, foreign inflation, and foreign output gap are $\sigma_\nu^2 = 0.844$, $\sigma_{\pi^*}^2 = 0.022$, and $\sigma_{y^*}^2 = 0.083$, respectively[12]. For the risk premium AR(1)-coefficient $\gamma_\upsilon$, Leitemo and Söderström considers the interval $[0,1]$. In the current analysis, having to choose one value, $\gamma_\upsilon$ is set equal to 0.5.

To recap, all the parameters known with certainty and associated with the Svensson (2000) and the Leitemo and Söderström (2005) calibrations are reported, respectively in Panels a and b of Table 1, while the benchmark values of the uncertain parameters are reported in Table 2.

# 3 Macroeconomic volatility under DIT and CPIIT

Parameter uncertainty poses a major challenge to real world monetary policy. In this work, the consideration of model parameter uncertainty is what allows moving from *mean forecast targeting* to *distribution forecast targeting.* The latter means that, given a specific policy, e.g. DIT or CPIIT, and given an exogenous disturbance, the solution of the optimization problem implies a *correspondence* that associates any point in time with a distribution forecast for each variable. This information richness is lost with mean forecasts targeting. In this case, due to the certainty equivalence principle, the optimal policy response to an exogenous shock implies a *function* that associates any point in time with, exactly, one value for each variable. Thus, the relevance of accounting for model parameter uncertainty lies in shedding light on the expected volatility of the variables at any current and future point in time. This, for policymakers, is a key aspect of the economic outlook and is normally assessed in policy decisions via the inflation forecast targeting operating procedure.

[12] Leitemo and Roisland (2002) find these variances with a structural VAR on the Norwegian economy.

### 3.1 Distribution forecasts to a cost-push shock in presence of general uncertainty

The analysis starts with the unconditional distribution forecasts of the impulse responses to a (one standard deviation) cost-push shock reported in Figures 1-2. The distribution forecasts are generated assuming *general* uncertainty, which encompasses uncertainty on the pass-through, $\left(\mu_j^i, \alpha_j^i\right)$, on the persistence in the private sector's behaviour, $(\iota_j, \zeta_j)$, and on the slope of the domestic AS, $(\alpha_j)$. In each figure, the first and second column report the distribution forecasts of the main macroeconomic variables under the optimal policies of domestic and CPIIT respectively[13]. Assuming an uncertainty level of 30% on all the uncertain parameters, Figures 1-2 have been generated by drawing an initial mode of the Markov chain from its stationary distribution, simulating the chain for a sequence of periods forward, and then repeating this procedure for 1000 simulations runs[14]. Thus these figures display mean (dashed line), and quantiles (grey bands), of the empirical distribution. In particular, the dark, medium and light grey band show the 30%, 60%, and 90% probability bands, respectively. Figures 1-2 consider, respectively, high and low central bank preferences for smoothing the interest rate path[15]. Strong attention on smoothing the interest rate implies a mild monetary policy where there is almost no attempt to buffer the shock. This case is interesting as starts to reveal the impact of model parameter uncertainty and alternative inflation indexes on the distribution forecasts; it thus provides a benchmark. In the latter case, low preferences for interest rate smoothing, the monetary policy is more realistic and the different impact of model parameter uncertainty on the distribution forecasts linked to alternative target inflation indexes is fully revealed.

Figure 1 features a high preference for interest rate smoothing. Here, visual inspection shows that the volatility of the macroeconomic variables distribution tends to be higher under CPIIT. In Figure 2, switching to a low preference for interest rate smoothing, and therefore to a more active policy, the previous result is strongly amplified: DIT implies much less volatility of the projections of the economy, in particular of the interest rates, and a surprisingly better ability to absorb the cost-push shock. Focusing on the interest rate in the more realistic case portrayed by Figure 2, monetary policy with DIT is expected to be tighter than neutral in the initial five periods to get back to neutral afterwards. In contrast, with CPIIT, the distribution forecast

[13] Although this paper focus on the expected interest rate volatility associated with alternative inflation targeting policies, it is informative to investigate also the volatility of the other macroeconomic variables.

[14] The results presented in this and the next sections are robust to smaller and larger uncertainty levels.

[15] Specifically, the interest rate smoothing preferences parameter, $\nu$, in the loss function (28), is 0.05 in Figure 1 and 0.002 in Figure 2.

allows anticipating the type of policy only in the first two periods leaving policymakers in the darkness in the subsequent periods. Indeed, with respect to the distribution forecast associated with DIT, the one associated with CPIIT signals a policy expected to be even tighter than neutral in the first two periods, but then provides no guidance of anticipation in terms of whether it will tighten or ease afterwards.

Furthermore, it is worth noting a sharp increase in the interest rate in the first two periods under CPIIT which will be discussed in Section 4 with reference to the possible implications of interest rate uncertainty on financial instability.

Summing up, these findings suggest the following: first, buffering a cost-push shock under DIT leads to less volatility in the distribution forecasts than under CPIIT, in particular for the interest rate. Second, with CPIIT it is much more difficult to forecast the interest rate path after the initial periods. Third, if the central bank is called to set a less smooth interest rate path, that is, a more active policy, then CPIIT leads to much more expected volatility in the economic outlook than DIT.

These findings also suggest important potential implications for financial stability because market rates and asset prices are related to the behavior of the official rate analyzed here. Before discussing these implications and investigating empirically the relation between interest rate uncertainty and financial stability, which will be the subject matter of section 4, we introduce some statistics to deepen the analysis of the interest rate volatility in presence of a cost-push shock. Then we extend the analysis to other macroeconomic variables and shocks in order to gain a general outlook associated with the alternative targeting policies.

### 3.2 Measuring the volatility of $i$ in presence of a cost-push shock

On the basis of the previous analysis with high and low interest smoothing preferences, a natural question to ask is whether the volatility of the macroeconomic variables is monotonous in the preferences for smoothing. This is relevant given the uncertainty on the smoothing preferences of the central bank and, more in general, the time varying degree of activism in monetary policy possibly related to central bank judgment. To address this question, Figure 3 focuses on the cost-push shock case and presents the *standard deviation* of the distribution forecasts of the nominal interest rate for the periods considered above and for interest rate smoothing values in the set $V = \{0.002,\ 0.005,\ ...,0.04\}$[16]. Explaining this figure, each sub plot reports two

[16] Section 3.3 will extend the analysis to other macroeconomic variables and shocks.

surfaces that describe the standard deviation of the distribution forecasts under CPI and DIT. The uncertainty cases considered are uncertainty (i) on the pass-through, (ii) on the persistence of the behaviour of households and firms, (iii) on the degree of price flexibility in the domestic sector (AS slope uncertainty), and (iv) on all the previous sources, i.e. general uncertainty.

A first result is that either the CPIIT surface is always above the DIT surface (in the uncertainty on the pass-through, on the persistence in the behaviour of households and firms, and general uncertainty cases, first, second, and forth column respectively), or the two surfaces tend to overlap with the DIT one slightly above the CPI one for small preferences on interest rate smoothing (in the cases of uncertainty on the slope of the Phillips curve in the domestic sector, third column). This shows that under the pass-through, persistence, and general uncertainty cases the CPIIT policy results systematically in a larger standard deviation for the interest rate distribution forecast than DIT. Instead, when we consider the case of uncertainty on the degree of price flexibility in the domestic sector, the standard deviation associated with DIT tends to be higher than the one associated with CPIIT.

Second, the volatility of the distribution forecasts of the interest rate tend to be monotonically increasing in the preference for not smoothing the interest rate. Yet, it is interesting to note that, decreasing interest rate smoothing, the volatility under CPIIT tends to increase more than under DIT.

These findings are relevant as they generalize to a broad set of interest rate smoothing preferences the previous findings reported in Figures 1-2: DIT leads to less variability of the distribution forecasts of the interest rate in the presence of a cost-push shock, and it is less sensitive to interest rate smoothing.

In order to quantitatively compare the volatility of the distribution forecasts associated with the two policies it is informative to compute the ratio of the means (along all the smoothing preferences values and the periods considered) of the standard deviations in the two policy cases, i.e.

$$R^{\sigma} \equiv \frac{mean\,|_{\nu,t} std^{c}_{\nu,t}\,(variable)}{mean\,|_{\nu,t} std^{d}_{\nu,t}\,(variable)},$$

where $std^{h}_{\nu,t}\,(variable)$, $h = c, d$, denote the standard deviation of the distribution forecast of the considered variable for period $t$, and smoothing preferences value $\nu$, and $c$ and $d$ denote CPI and DIT, respectively. Table 3 presents the statistic $R^{\sigma}$ for various uncertainty types.

INSERT TABLE 3 HERE

This analysis shows that in almost all uncertainty cases, DIT dominates CPIIT. Furthermore, when we focus on the more representative case of general uncertainty, which includes all the previous cases, the mean of the standard deviation under CPIIT is 2.79 times larger than under DIT.

### 3.3 Targeting policies and macroeconomic volatility: the overall economic outlook

Do the earlier results associated with the $R^\sigma$ statistic hold for the other variables and external disturbances? It is worth asking this question as the willingness to follow a targeting policy favoring interest rate predictability might be related to other shocks and considerations on the predictability of other macroeconomic variables. Interestingly, this section shows that earlier findings tend to hold to a remarkable extent in a more general setting. Considering CPI and domestic inflation, $\pi^c$ and $\pi^d$ respectively, the short term real interest rate, $r$, and the real exchange rate, $q$, along with the additional (one standard deviation) shocks to the aggregate demand, the foreign interest rate, the natural output, the risk premium, and the foreign output, Tables 4-5 report the $R^\sigma$ ratio for the *general uncertainty* case.

INSERT TABLES 4-5 HERE

To discuss the results associated with the ratio $R^\sigma$ it is useful to define alternative dominance intervals around the no-dominance point, i.e. $R^\sigma = 1$. We thus select intervals endpoints starting from the case in which one policy performs outstandingly better than the other. We let this case be the one in which a policy leads to a volatility *at most half* as large as the other policy volatility and call it "Strong Dominance". As a result, Strong Dominance cutoff values are 0.5 and 2 and the related intervals are $(0, 0.5]$ and $[2, \infty)$. Next, we consider the opposite case, i.e. when policies do not perform in a significantly different way. This case is useful to identify and filter out close calls, i.e. similar performances potentially difficult to make a decision about. We let this case be the one in which a policy leads to a volatility *at least nine tenth* as large as but *smaller* than the other and call it “Weak Dominance”[17]. It follows that the Weak Dominance cutoff values are 0.9 and $1.\bar{1}$, and the related intervals are $[0.9, 1)$ and $(1, 1.\bar{1}]$.

[17] Although the choice of the 9/10 cutoff is a priori not unreasonable, less conservative cutoff values would not change the line of the results.

This definition of Strong and Weak Dominance implicitly delimits an in-between space where one policy performs significantly, but not outstandingly, better than the other. This is the case in which one policy leads to a volatility *more than half and less than nine tenth* as large as the other. We call this the "Dominance" case and it consists of the intervals $(0.5,\ 0.9)$ and $(1.\bar{1},\ 2)$. To recap, the alternative dominance intervals are

$$\begin{array}{ccc} \text{Strong Dominance} & \Longleftrightarrow & 0 < R^{\sigma} \leq 0.5 \text{ or } R^{\sigma} \geq 2 \\ \text{Dominance} & \Longleftrightarrow & 0.5 < R^{\sigma} < 0.9 \text{ or } 1.\bar{1} < R^{\sigma} < 2 \\ \text{Weak Dominance} & \Longleftrightarrow & 0.9 \leq R^{\sigma} < 1 \text{ or } 1 < R^{\sigma} \leq 1.\bar{1} \end{array}$$

Turning to the results, Tables 4 describes the performance of the two policies under the Svensson (2000) calibration. Abstracting from the *weak dominance* cases, DIT is *strongly dominant* or *dominant* in 44.4% of the cases, while it is dominated in 27.7% of the cases[18]. Interestingly, DIT strongly dominates in approximately one fifth of the cases, yet it is never strongly dominated. Checking for the robustness of these results, the analysis based on the Leitemo and Söderström (2005) calibration corroborates the previous findings. Indeed, results in Table 5 show that DIT is strongly dominant or dominant in the 63.8% while it is dominated in the 16.6% of the cases.

It is worth noting that the cases in which DIT is dominated tend to pertain to CPI inflation, as we would expect, and also to the real exchange rate. As to the former, except for the cost-push shock, both the distribution forecasts of domestic and CPI inflation are not very sensitive to exogenous disturbances. Thus the two policies tend to be similar in their ability to stabilize inflation even if each one is better at stabilizing its own measure of inflation[19]. As to the latter, the real exchange rate, with a demand, natural output, risk premium, and foreign output shock, CPIIT performs better as is shown in Table 4-5. This is due to the fact that it aims to stabilize both domestic and import inflation, which determine the real exchange rate.

Shocks to the risk premium, foreign interest rate and foreign output gap deserve a final comment. In these cases the shocks impact on the nominal exchange rate via the uncovered interest parity. Then, if the central bank does not react, the shock propagates to CPI inflation. Thus, with CPIIT the central bank has to respond to these shocks. Yet, the central bank may not be willing to react to shocks that affect the nominal exchange rate. Leitemo and Söderström (2005) maintain that it should not. Their argument is that there is uncertainty about how the

[18] DIT is strongly dominant in 8 cases, dominant in 8 cases, weakly dominant in 4 cases, weakly dominated in 6 cases, dominated in 10 cases, and strongly dominated in 0 cases.

[19] The impulse response distribution forecasts for the complete set of shocks are available upon request.

exchange rate is determined and the effect of exchange rate movements on the economy. This implies that rules with the exchange rate are more sensitive to model uncertainty. Thus, a monetary policy developed in the context of an exchange rate model could perform poorly if that model is incorrect. Empirical evidence in this respect seems to favor no policy reaction to the nominal exchange rate. Lubik and Schorfheide (2007) find that Australia and New Zealand did not react to movements in the exchange rate while Canada and the UK did. Also considering optimal policy and parameter uncertainty, Justiniano and Preston (2010) find that Australia, Canada and New Zealand do not respond to the exchange rate.

Describing the mechanism that generates the paper's results, two factors stand out: more policy activism under CPIIT than under DIT and the presence of model parameter uncertainty. The first factor is shown in Figures 4-5 computed assuming no model parameter uncertainty. These figures displays the impulse response function of the nominal interest rate to a cost-push shock under the two alternative policies for high and low smoothing preferences, Figure 4 and 5 respectively. Measuring monetary policy activism by the volatility (in terms of std) of the impulse response function around its long run value, under CPIIT this volatility is 1.3 times larger than under DIT when $\nu = 0.05$, and 4.53 times larger when $\nu = 0.002$.

More policy activism under CPIIT than under DIT is due to i. different lags in the transmission of the policy action to CPI and domestic inflation, and ii. to a larger exposure of CPI inflation to foreign shocks. Different lags arise as the pricing decisions for domestic firms embed not only retailing decisions but production decisions too, and therefore are more subject to information delays. It follows a longer lag for policy action to affect domestic inflation than CPI inflation via the output gap. This is the policy transmission that occurs through the aggregate demand channel and the switching demand exchange rate channel. It follows also that shocks to the exchange rate and the price of the foreign goods in foreign currency affect domestic inflation with a lag via $q_t$ in the AS for the domestic sector, while they affect directly import inflation via $q_t^i$ in the AS for the import sector[20].

Furthermore, more policy activism depends on a larger exposure of CPI inflation to foreign shocks. Indeed, via the uncovered interest parity, the latter causes exchange rate volatility exerting a stronger impact on CPI inflation than on domestic inflation because import sector inputs are more intensive in foreign goods than domestic sector inputs. As a result, under CPIIT the central bank is more solicited to intervene in order to prevent exchange rate volatility from

[20] The impact of the exchange rate on the domestic price of the foreign good is amply documented in the literature and usually referred to as the Direct Exchange Rate channel.

leading to too much CPI inflation volatility. Hence, CPIIT implies a more pronounced trade-off between CPI inflation and interest rate volatility.

What happens when more policy activism is associated with the consideration of model parameter uncertainty in the design of the optimal monetary policy? When parameter uncertainty is taken into account we move from one expected path for the interest rate (Figures 4-5) to a set of expected paths, which form the distribution forecast for the interest rate (third row in Figures 1-2). At this point, the degree of policy activism expands the width of the distribution forecast. Indeed, the larger the initial monetary policy stimulus, the more the uncertainty on the private sector behavior can lead to future changes in the policy.

Finally, a wider distribution forecast for the interest rate results in wider distribution forecasts for most of the other macroeconomic variables, which is the result shown in Figure 1-2 and reported, more generally, in Tables 4-5.

# 4 Interest rate volatility and financial stability

Our theoretical results suggest that the choice of the inflation targeting policy, specifically DIT, can reduce interest rate uncertainty. Since the interest rate is a key variable both for the real and the financial sector of the economy, we argue that interest rate volatility can favour financial instability. Interestingly, financial instability, in turn, can feedback to the transmission mechanism of monetary policy (Baum et al., 2013).

To start, we note that sharp increases in interest rates strain financial markets, as it occurred for example in 1994 in the US. In this respect, the spike in the official rate reported in Figure 2 under CPIIT in the first two periods suggests that this reaction to a cost-push shock is likely to add to financial instability.

We then draw on macroeconomic theory, in particular on the transmission mechanism of monetary policy. Indeed, changes in the official rate set by the central bank, along with changes in the expectations concerning future official rates, directly impact on market rates and asset prices. Short-term market rates follow the current and expected official rates, although neither automatically nor exactly of the same amount. With respect to securities, other things equal, higher short-term interest rates lower equities prices. Further, expected short-term interest rates determine the long-term interest rate, which is inversely related to the price of bonds. Hence, the larger the volatility featured by the interest rate distribution forecast, the larger the volatility on market rates and asset prices. The volatility of these variables, in turn, affects financial instability. Regarding the US for example, Nelson and Perli (2005) develop a financial fragility index based on the volatility of several assets including options on Eurodollar. The inclusion of this variable is interesting in that the implied volatility calculated from these options provide a measure of the expected volatility of very short-term rates, which are strictly related to the official rate.

Although we have not introduced a formal theoretical model linking financial instability to interest rate uncertainty along the lines discussed in the theoretical section, we now provide some preliminary empirical evidence using US, UK and Swedish monthly data since the early 1990s (our sample choice is dictated by the availability of data; data ends in 2013:M3 for the US, in 2013:M1 for Sweden and in 2011:M12 for the UK). Our empirical results reported in this section are by no means definitive; what we do is provide some initial evidence that such an impact does exist.

Thus, we do not claim, at this stage, any methodological advances. Certainly it is possible to build a model that theoretically explores the conjectured relation between financial instability and interest rate uncertainty. However, we believe that the model we use, along with the empirical evidence provided in this section, can serve as a useful baseline for policymakers to consider, in a broader perspective, the targeting policy choices in an era characterized by increasing financial instability.

To fix ideas, Figure 6 plots the Federal Reserve Bank of Kansas Financial Stress Index (FSI) together with the effective federal funds rate. The index, provided by the website of the Federal Reserve Bank of Kansas, pools information from 11 financial variables (see Hakkio and Keeton, 2009) and is available from 1990 onwards[21]. An increase in the index denotes more financial stress/instability. Figure 7 plots the Bank of England's base rate together with the UK FSI compiled by the International Monetary Fund (see Balakrishnan et al, 2009); this measure provides a broad spectrum measure of stress across money, foreign exchange and equity markets in the UK (we have data for the index until the end of 2011). We note that both measures of financial stress follow a similar pattern. They rise during the Russian debt default of 1998 and the dot-com crash of 2000; they also rise sharply in 2007-2009. We also note that UK's FSI index is high in late 1992 following the exit from the European Exchange Rate Mechanism

[21] The Kansas index is a composite index of the 3-month LIBOR/T-Bill spread, the 2-year swap spread, the Aaa/10-year Treasury spread, the Baa/Aaa spread, the off-the-run/on-the-run 10-year Treasury spread, the high-yield bond/Baa spread, the consumer Asset-Backed Securities/5-year Treasury spread, the correlation between returns on stocks and Treasury bonds, the implied volatility of overall stock prices (VIX), the idiosyncratic volatility of bank stock prices and the cross-section dispersion of bank stock returns.

(ERM). Figure 8 plots the policy (repo) rate of the Swedish Central Bank (Sveriges Riksbank) together with the Financial Stress Index provided by the website of Sveriges Riksbank. The index is a composite index of the stock market, the bond market, the money market and the foreign exchange market (Johansson and Bonthron, 2013). The correlation amongst the three FSI measures is high (0.75 between the US and Swedish measures, 0.76 between the US and UK measures and 0.80 between the UK and Swedish measures). Figures 6-8 also plot our GARCH measures of interest rate uncertainty (reported in the text below). We note that uncertainty is high following the terrorist attacks of 9/11, the dot-com bubble and during the recent financial crisis.

To test the impact of interest rate uncertainty on financial instability we rely on a simple Auto Regressive (AR) model of the FSI index augmented by measures of interest rate uncertainty ($\sigma_{i_t}$); the first one is a 2-year Moving standard deviation of the interest rate, whereas the second measure derives from a simple GARCH(1,1) type of model of the interest rate[22].

Table 6 reports the empirical impact of interest rate uncertainty on financial instability using US, UK and Swedish data.

The results reveal strong persistency in the FSI. Increased interest rate uncertainty increases financial instability (for Sweden the impact is significant only based on the GARCH-type measure of interest rate uncertainty). For all countries, the GARCH type of proxy of interest rate uncertainty fits the data best as it delivers a lower regression standard error and a lower Akaike Information Criterion (AIC)[23].

The short-run impacts of interest rate uncertainty on FSI are given by the $\sigma_{i_t}$ coefficients reported in Table 6. For the US, the long-run impacts are given by 0.043/(1-0.97)=1.433, and 0.360/(1-0.971)=12.41, respectively, for the 2-year Moving standard deviation and the GARCH

[22] For the US, we estimate a GARCH(1,1) model of the form

$$i_t = \beta_0 + \beta_1 i_{t-1} + \beta_2 i_{t-2} + \beta_3 i_{t-3} + \varepsilon_t$$

where

$$\sigma^2_{\varepsilon_t} = \gamma_0 + \gamma_1 \varepsilon^2_{t-1} + \gamma_2 \sigma^2_{\varepsilon_{t-1}}$$

and $i_t$ is the interest rate. We estimate $\beta_0 = 0.020$ (0.010), $\beta_1 = 1.402(0.030)$, $\beta_2 = -0.272(0.060)$, $\beta_3 = -0.150(0.039)$, $\gamma_0 = 0.002(0.001)$, $\gamma_1 = 0.363(0.030)$ and $\gamma_2 = 0.724(0.017)$, where numbers in brackets are standard errors. For the UK, we estimate a GARCH(1,1) model of the form

$$i_t = \beta_0 + \beta_1 i_{t-1} + \beta_2 i_{t-2} + \varepsilon_t$$

where $\sigma^2_{\varepsilon_t} = \gamma_0 + \gamma_1 \varepsilon^2_{t-1} + \gamma_2 \sigma^2_{\varepsilon_{t-1}}$. We estimate $\beta_0 = 0.017(0.037)$, $\beta_1 = 1.460(0.050)$, $\beta_2 = -0.480(0.048)$, $\gamma_0 = 0.001(0.001)$, $\gamma_1 = 0.142(0.015)$ and $\gamma_2 = 0.887(0.008)$, where numbers in brackets are standard errors. For Sweden, we estimate an ARCH(1) model of the form $i_t = \beta_0 + \beta_1 i_{t-1} + \beta_2 i_{t-2} + \beta_3 i_{t-3} + \varepsilon_t$, where $\sigma^2_{\varepsilon_t} = \gamma_0 + \gamma_1 \varepsilon^2_{t-1}$. We estimate $\beta_0 = 0.025(0.010)$, $\beta_1 = 1.530(0.080)$, $\beta_2 = -0.350(0.140)$, $\beta_3 = -0.200(0.059)$, $\gamma_0 = 0.010(0.001)$ and $\gamma_1 = 0.460(0.130)$.

[23] For all countries, we also used the 1 and 3-year Moving standard deviation measures of interest rate uncertainty. Results are qualitatively similar.

type measure. For the UK, the long-run impacts are given by 0.187/(1-0.976)=7.791, and 1.156/(1-0.98)=57.8, respectively, for the 2-year Moving standard deviation and the GARCH type measure. For Sweden, the long-run impacts are given by 0.001/(1-0.934)=0.015, and, 0.167/(1-0.92)=2.087, respectively, for the 2-year Moving standard deviation and the GARCH type measure. Hence, our estimates suggest that the long-run effects are much stronger than the short-run ones.

To account for possible endogeneity issues, we used, in Table 6, lagged uncertainty ($\sigma_{it-1}$) instead of current uncertainty ($\sigma_{it}$). This made very little difference to the empirical estimates (whether the 2-year Moving standard deviation or the GARCH measure is used). Indeed, based on the 2-year Moving standard deviation measure, the coefficient on lagged uncertainty is estimated at 0.038 for the US, at 0.148 for the UK, and at 0.002 for Sweden (detailed results are available on request). Finally, to account for the fact that interest rates leveled after the financial crisis, we re-estimated our models up to 2009. Again, this made very little difference to the empirical estimates. Indeed, based on the 2-year Moving standard deviation measure, the coefficient on lagged uncertainty is estimated at 0.036 for the US, at 0.128 for the UK and at 0.001 for Sweden (full details are available on request).

# 5 Conclusions

Parameter uncertainty poses a formidable problem to central banks. This paper uses distribution forecast targeting to show that in presence of parameter uncertainty the choice of the inflation measure to stabilize remarkably affects the volatility of several macroeconomic variables, in particular of the interest rate. Specifically, we find that under DIT the volatility of the expected path for several variables turns out to be much less than under CPIIT. Consequently, under CPIIT, it is more difficult to predict the expected path of the economy, in particular with respect to the interest rate. This result matters since the less the uncertainty surrounding the expected path of the short-term interest rate, the stronger the effectiveness of the expectations channel for the transmission of monetary policy to the real side of the economy. Thus, all else equal, concentrating more on DIT would reduce macroeconomic volatility.

We also think that this result is interesting with respect to financial stability. Indeed, less uncertainty on the expected path of the official rate is transmitted to market rates and asset prices, whose volatility determines financial instability. When we take this hypothesis to US, UK and Swedish data, we find significant empirical evidence that interest rate volatility positively affects financial instability. Hence, we conclude that the choice of the inflation targeting policy can also bear important consequences on financial stability. We leave to further analysis the theoretical study of this relation via the inclusion of a financial sector.

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# Appendix

The behaviour of the private sector described by equations (16, 19, 22, 24, 27-31) is conveniently rewritten in State-space form to obtain the law of motion of the economy. Then, the central bank problem is to find the expected interest rate path that minimizes its loss given the law of motion of the economy, that is

$$Min_{\{i_{t+\tau|t}\}_{\tau=0}^{\infty}} E_t \sum_{\tau=0}^{\infty} \beta^{\tau} Y_{t+\tau}' K Y_{t+\tau}$$

subject to

$$\left[\begin{array}{c} X_{t+1} \\ x_{t+1|t} \end{array}\right] = \left[\begin{array}{cc} A_{11,t+1} & A_{12,t+1} \\ A_{21,t} & A_{22,t} \end{array}\right] \left[\begin{array}{c} X_t \\ x_t \end{array}\right] + \left[\begin{array}{c} B_{1,t+1} \\ B_{2,t} \end{array}\right] i_t + \left[\begin{array}{c} B_{1,t+1}^1 \\ B_{2,t}^1 \end{array}\right] i_{t+1|t} + \left[\begin{array}{c} \varepsilon_{t+1} \\ 0 \end{array}\right],$$

$$Y_t \equiv C_{Z,t} \left[\begin{array}{c} X_t \\ x_t \end{array}\right] + C_{i,t} i_t,$$

where the target variables, the predetermined variables, and the forward looking variables are, respectively

$$
\begin{aligned}
Y_t &= \left(\pi_t^c,\ \pi_t^d,\ y_t^d,\ i_t - i_{t-1}\right)', \\
X_t &= \left(\pi_t^d,\ \pi_{t+1|t}^d,\ \pi_{t-1}^i,\ \pi_t^*,\ y_t^d,\ y_t^i,\ y_t^*,\ i_t^*,\ y_t^{d,n},\ y_t^{i,n},\ i_{t-1},\ q_{t-1},\ q_{t-1}^i,\ \upsilon_t\right)', \\
x_t &= \left(\pi_t^i,\ q_t^i,\ \rho_t,\ \pi_{t+2|t}^d\right)',
\end{aligned}
$$

and where $K$ captures the central bank's preferences, a diagonal matrix with the diagonal $\left(\mu^c,\ \mu^d,\ \lambda,\ \nu\right)$ and off-diagonal elements equal to zero. Following the Markov Jump-Linear-Quadratic approach developed by Svensson and Williams (2007) we assume that the matrices

$$
A_{11,t},\ A_{12,t},\ B_{1,t},\ B_{1,t}^1,\ A_{21,t},\ A_{22,t},\ B_{2,t},\ B_{2,t}^1,\ C_{Z,t},\ C_{i,t}, \tag{32}
$$

are random, each free to take $n_j$ different values in period $t$ corresponding to the $n_j$ modes indexed by $j_t \in \{1, 2, ..., n\}$. This means that, for example, $A_{11,t} = A_{11,j_t}$. The mode $j_t$ is then assumed to follow a Markov process with constant and equal transition probabilities

$$
P_{jk} \equiv \Pr\{j_{t+1} = k | j_t = j\} = \frac{1}{n}, \qquad j, k \in \{1, 2, ..., n\}. \tag{33}
$$

Furthermore, modes $j_t$ and innovations $\varepsilon_t$ are assumed to be independently distributed. As to the central bank knowledge before choosing the instrument-plan $\left\{i_{t+\tau|t}\right\}_{\tau=0}^{\infty}$ at the beginning of period $t$, the information set consists of the probability distribution of $\varepsilon_t$, the transition matrix $[P_{jk}]$, the $n_j$ different values that each of the matrices can take in any mode, and finally the realizations of $X_t,\ j_t,\ \varepsilon_t,\ X_{t-1},\ j_{t-1}, \varepsilon_{t-1},\ x_{t-1}, ...$

Given (33), the unique stationary distribution of the modes associated with the Markov transition matrix $[P_{jk}]$ is a uniform distribution. This implies that the transition probabilities described by (33) capture the case of generalized modes uncertainty in which modes are serially i.i.d.. The motivation to consider this case lies in the interest of studying optimal monetary policy when the central bank only knows a band for each uncertain deep parameter and considers any realization as equally likely.

Turning to the number of modes, letting $m$ be the number of uncertain parameters and $d$ be the number of values that each parameter can take in any period, then the number of modes is $n = d^m$. In this work $d = 5$ and $m$ can be either 1 or 2 or 5 depending on the uncertainty cases described below.

TABLE 1 Parameters known with certainty

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| Panel a (Svensson 2000) | | | | | | Panel b ( Leitemo and Söderström (2005)) | |
|---|---|---|---|---|---|---|---|
| $\omega$ | 0.8 | $\theta^*$ | 2 | $\sigma_\pi^2,\ \sigma_y^2$ | 1 | $\gamma_\pi,\ \gamma_y$ | 0.3 |
| $\vartheta$ | 1.25 | $w^*$ | 0.15 | $\sigma_{y^{d,n}}^2,\ \sigma_{y^{i,n}}^2$ | 0.5 | $\sigma_y^2$ | 0.656 |
| $\sigma$ | 0.5 | $\overline{\beta}_y^*$ | 0.9 | $\gamma_y^{d,n},\ \gamma_y^{i,n}$ | 0.96 | $\sigma_\pi^2$ | 0.389 |
| $\mu$ | 0.1 | $f_{\pi^*}$ | 1.5 | $\gamma_{y^*},\ \gamma_{\pi^*},\ \gamma_v$ | 0.8 | $\sigma_v^2$ | 0.844 |
| $w$ | 0.3 | $f_{y^*}$ | 0.5 | $\sigma_v^2,\ \sigma_{\pi^*}^2,\ \sigma_{y^*}^2$ | 0.5 | $\sigma_{\pi^*}^2$ | 0.022 |
| | | | | | | $\sigma_{y^*}^2$ | 0.083 |
| | | | | | | $\gamma_v$ | 0.5 |
| CPIIT | $\mu^d = 0,\ \mu^c = 1,\ \lambda = 0.5$ | | | | | | |
| DIT | $\mu^d = 1,\ \mu^c = 0,\ \lambda = 0.5$ | | | | | | |

TABLE 2 Benchmark values of the uncertain parameters

839

| | | |
|---|---|---|
| $\overline{\iota}$ | 0.8 | Banerjiee and Batini (2003) |
| $\overline{\zeta}$ | 0.66 | Smets and Wouters (2005) |
| $\overline{\alpha},\ \overline{\alpha^i}$ | 0.5 | Svensson (2000) |
| $\overline{\mu^i}$ | 0.35 | Flamini (2007) |

TABLE 3 $R^\sigma$ for various uncertainty type. Shock: cost-push. First calibration.

840

| Uncertainty type | $i$ |
|---|---|
| Pass-through | 3.68 |
| Persistence private sector behavior | 1.16 |
| Domestic AS slope | 0.91 |
| General | 2.79 |

TABLE 4 $R^\sigma$ for various shocks and variables under general uncertainty. First calibration.

841

| **Shock** | $\boldsymbol{\pi^c}$ | $\boldsymbol{\pi^d}$ | $\boldsymbol{y^d}$ | $\boldsymbol{i}$ | $\boldsymbol{r}$ | $\boldsymbol{q}$ |
|---|---|---|---|---|---|---|
| Cost-push | 1.08 | 1.15 | 1.48 | 2.79 | 2.62 | 1.44 |
| Demand | 0.89 | 1.16 | 0.95 | 1.05 | 1.05 | 0.82 |
| Foreign interest rate | 0.77 | 1.32 | 1.18 | 2.91 | 2.77 | 1.01 |
| Natural output | 0.87 | 1.11 | 0.98 | 0.97 | 0.97 | 0.75 |
| Risk premium | 0.71 | 0.90 | 0.82 | 2.09 | 2.05 | 0.77 |
| Foreign output | 0.76 | 1.16 | 0.94 | 2.22 | 2.26 | 0.88 |

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TABLE 5 $R^\sigma$ for various shocks and variables under general uncertainty. Second calibration.

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| **Shock** | $\boldsymbol{\pi^c}$ | $\boldsymbol{\pi^d}$ | $\boldsymbol{y^d}$ | $\boldsymbol{i}$ | $\boldsymbol{r}$ | $\boldsymbol{q}$ |
|---|---|---|---|---|---|---|
| Cost-push | 1.05 | 1.13 | 1.23 | 1.79 | 1.22 | 2.04 |
| Demand | 0.86 | 1.16 | 0.91 | 0.94 | 1.17 | 1.01 |
| Foreign interest rate | 0.76 | 1.31 | 1.19 | 2.23 | 1.35 | 2.91 |
| Natural output | 0.87 | 1.12 | 0.99 | 0.89 | 1.12 | 0.95 |
| Risk premium | 0.74 | 1.33 | 1.19 | 2.67 | 1.38 | 3.35 |
| Foreign output | 0.77 | 1.15 | 0.94 | 1.75 | 1.19 | 2.22 |

TABLE 6: Empirical FSI models

| | US FSI, 1990:M2-2013:M3 | | UK FSI, 1992:M10-2011:M12 | | Swedish FSI, 1995:M2-2013:M1 | |
|---|---|---|---|---|---|---|
| Intercept | -0.031 | -0.060 | -0.094 | -0.216 | 0.010 | -0.100 |
| | (0.006) | (0.008) | (0.023) | (0.097) | (0.008) | (0.009) |
| $\text{FSI}_{t-1}$ | 0.970 | 0.971 | 0.976 | 0.980 | 0.934 | 0.920 |
| | (0.004) | (0.004) | (0.005) | (0.004) | (0.025) | (0.030) |
| $\sigma_{i_t}*$ | 0.043 | | 0.187 | | 0.001 | |
| | (0.007) | | (0.080) | | (0.100) | |
| $\sigma_{i_t}**$ | | 0.360 | | 1.156 | | 0.167 |
| | | (0.120) | | (0.486) | | (0.070) |
| SER | 0.058 | 0.055 | 0.175 | 0.166 | 0.070 | 0.068 |
| AIC | -2.82 | -2.94 | -0.63 | -0.73 | -2.46 | -2.47 |

Note: Newey-West Heteroskedasticity and Autocorrelation robust standard errors in brackets. SER is the Regression Standard Error, AIC is the Akaike Information Criterion.

* Uncertainty measured by 2-year Moving standard deviation.

** Uncertainty measured by GARCH type measure.

**Figure**

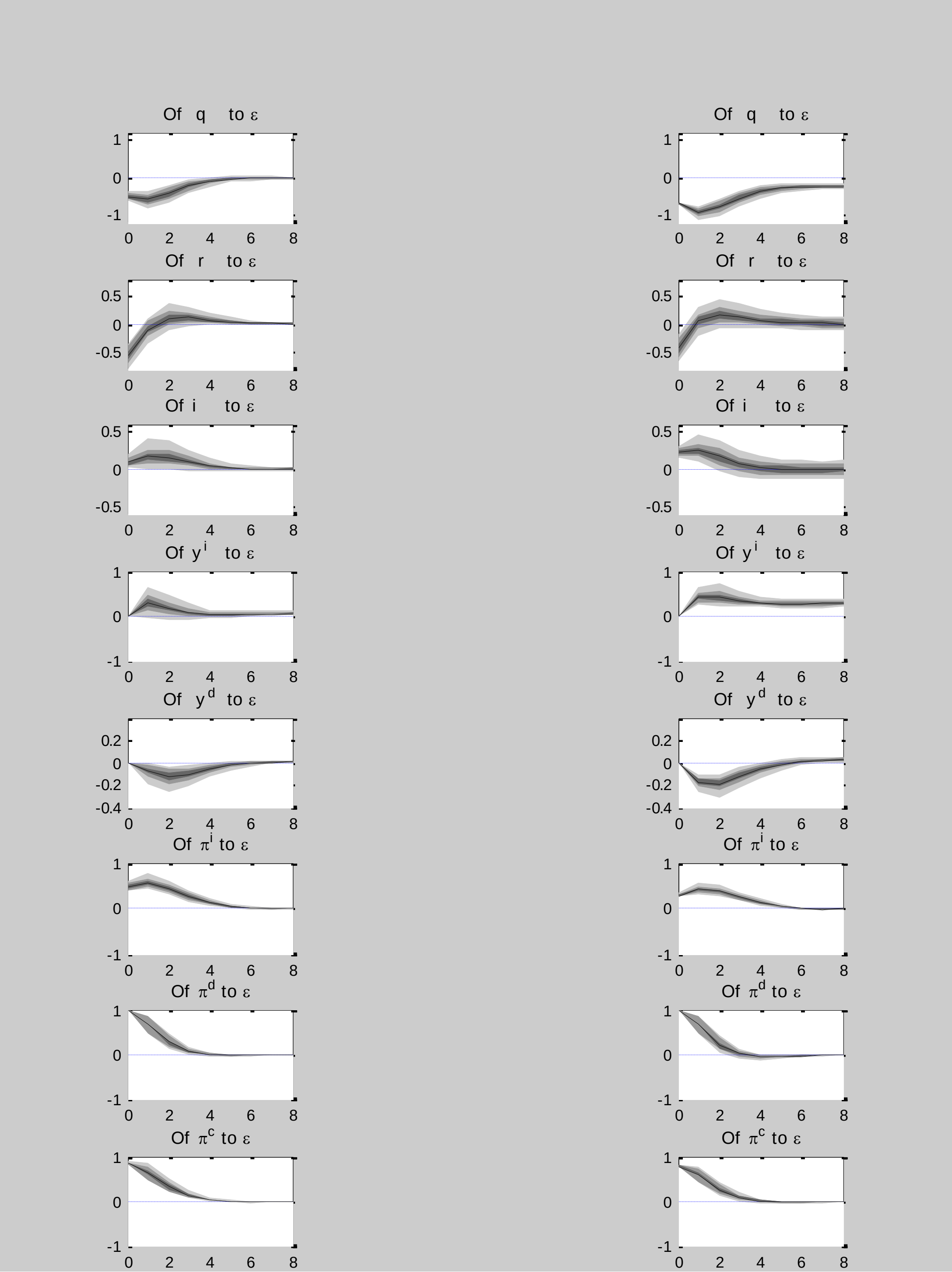



**Figure 1**: Unconditional distribution forecasts of the impulse responses to a cost-push shock in the *general uncertainty* case and for high smoothing preferences, i.e. $\nu = 0.05$. First and second column report, respectively, the distribution forecasts under the DIT and CPI IT policies. Solid lines: Mean responses. Dark/medium/light grey bands: 30/60/90% probability bands. First calibration.

Figure

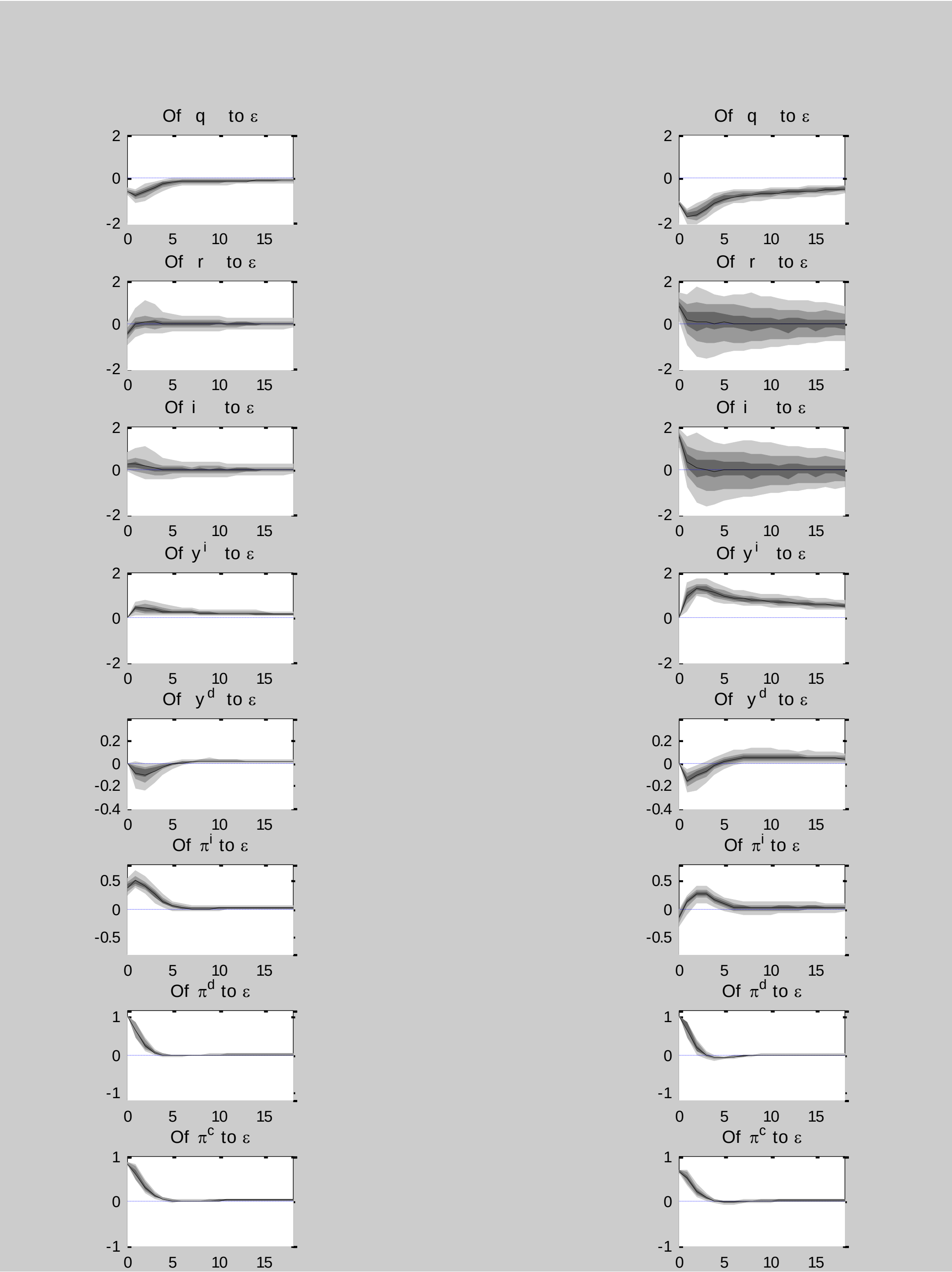



**Figure 2**: Unconditional distribution forecasts of the impulse responses to a cost-push shock in the *general* uncertainty case and for low smoothing preferences, i.e. $\nu = 0.002$. First and second column report, respectively, the distribution forecasts under the DIT and CPI IT policies. Solid lines: Mean responses. Dark/medium/light grey bands: 30/60/90% probability bands. First calibration.

**Figure**

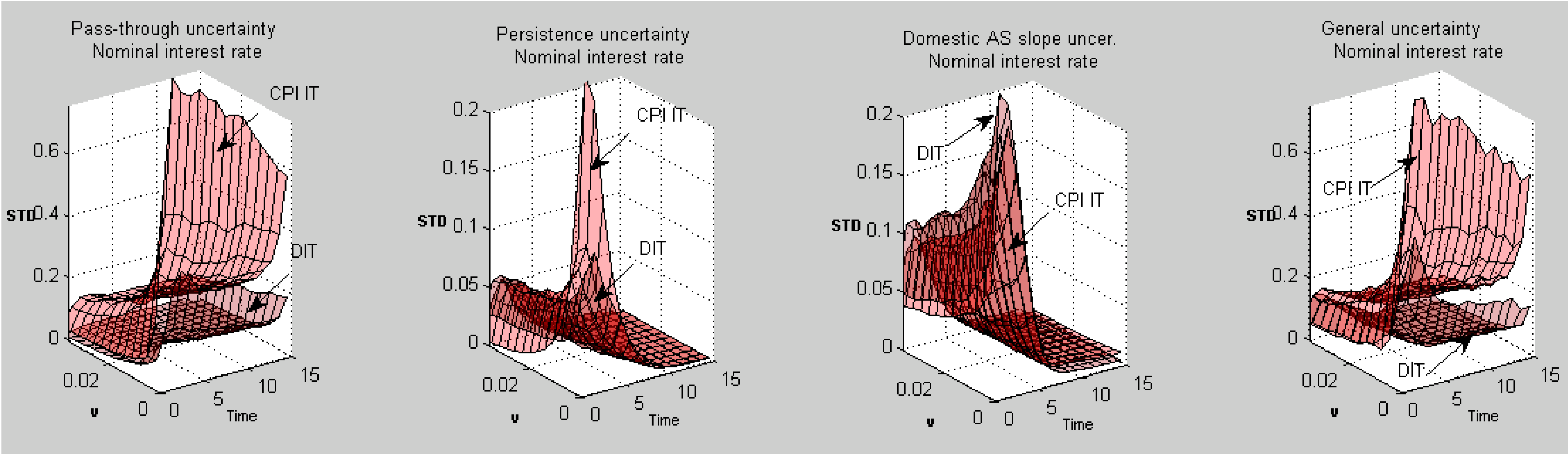



**Figure 3**: STD of the impulse response distribution to a cost-push shock under DIT and CPI IT for $\nu \in \{0.002,\ 0.005,\ ...,\ 0.04\}$ and $t \in \{0, 1, ..., 15\}$. Variables: $i$ and $y^d$, first and second row respectively. Uncertainty cases: pass-through, persistence in the behaviour of the private sector, slope of the domestic AS, and general, first, second, third and forth column respectively. First calibration.

**Figure**

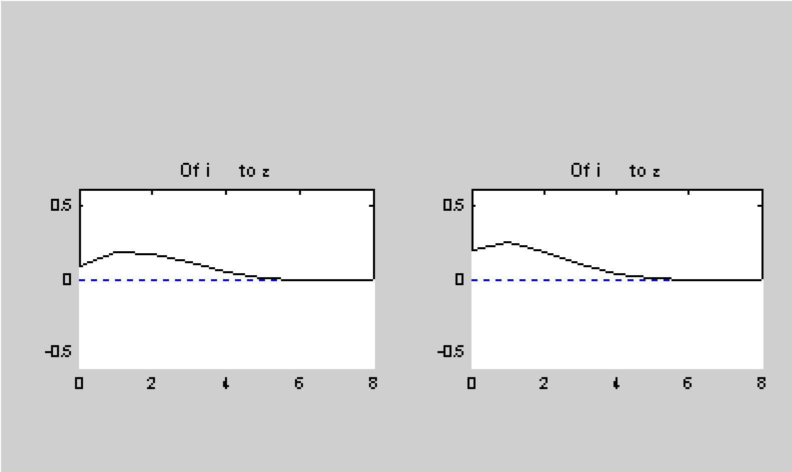



**Figure 4**: Impulse response under DIT (first) and CPI IT (second) of the nominal interest rate to a cost push-shock assuming no parameter uncertainty and for high smoothing preferences, i.e. ν = 0.05.

Figure

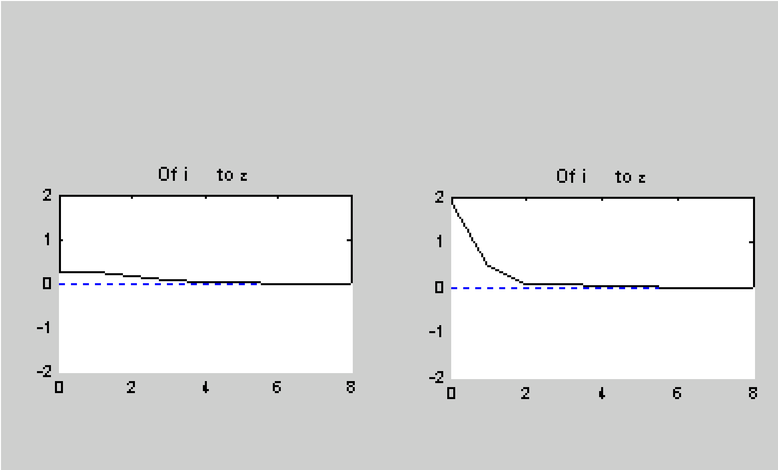
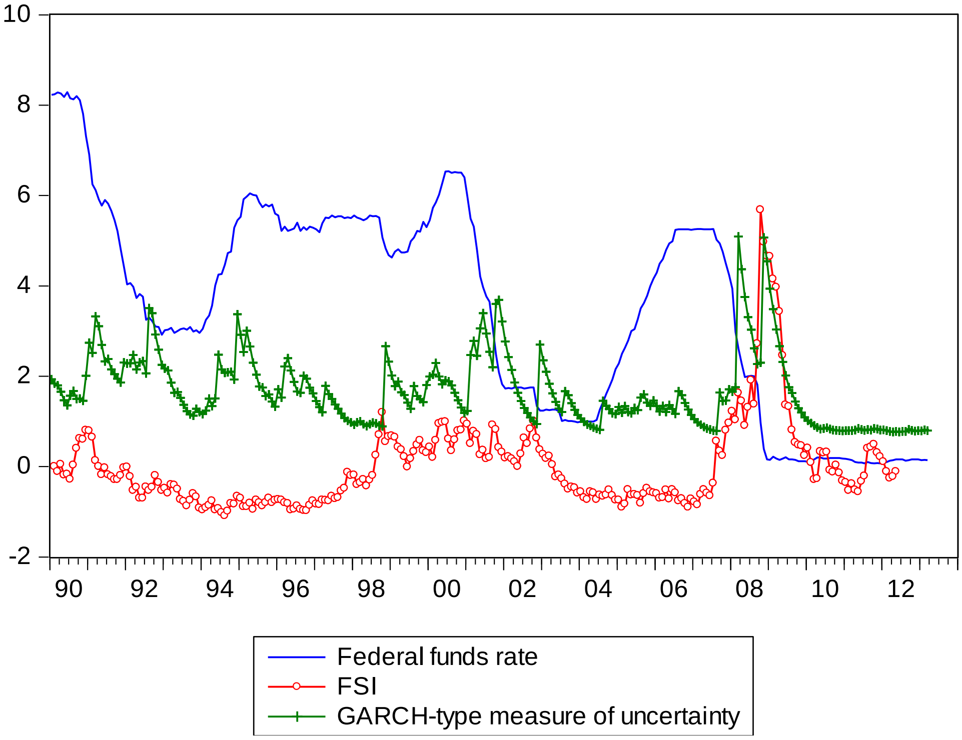



**Figure 5**: Impulse response under DIT (first) and CPI IT (second) of the nominal interest rate to a cost push-shock assuming no parameter uncertainty and for low smoothing preferences, i.e. ν = 0.002.

**Figure**

**Figure 6: Financial Stress Index, Federal funds rate and Garch-type measure of uncertainty. US data**

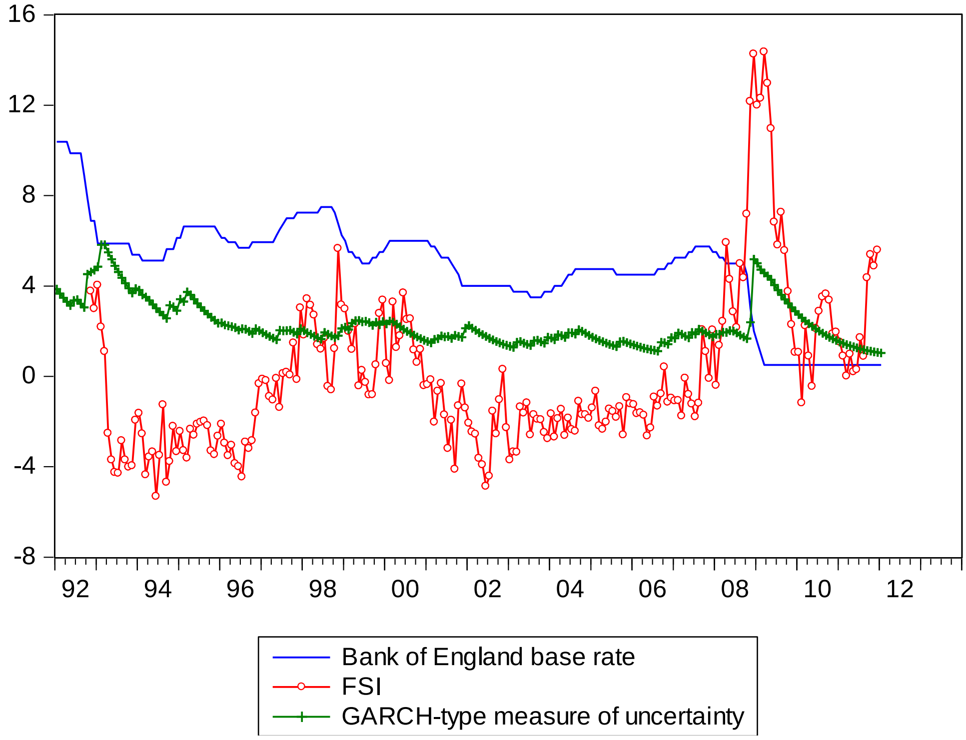



**Note:** To increase readability, the GARCH measure is multiplied by 10.

Figure

**Figure 7: Financial Stress Index, Bank of England base rate and Garch-type measure of uncertainty.**

**UK data**




**Note:** To increase readability, the GARCH measure is multiplied by 10.

**Figure**

**Figure 8: Financial Stress Index, Riksbank repo rate and Garch-type measure of uncertainty. Swedish data**