Notional defined contribution pension schemes: Why does only Sweden distribute the survivor dividend?

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Abstract

The aim of this paper is to analyse the role of the survivor dividend in notional defined contribution (NDC) pension schemes. At present this feature can only be found in the Swedish defined contribution (DC) scheme. We develop a model that endorses the idea that the survivor dividend has a strong basis for enabling the NDC scheme to achieve financial equilibrium and that not including the dividend is a non-transparent way of compensating for increases in longevity and/or legacy costs from old pension systems. We also find that the average effect of the dividend remains unchanged for any constant annual rate of population growth, that contributors who reach retirement age always get a higher return than the scheme does, and that population growth enables cohorts with more years of contributions to benefit to a greater extent from the dividend effect.

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1. Introduction.

The introduction of what are known as notional (or non-financial) defined contribution pension accounts (NDCs) as a component of modern multi-pillar pension systems in some countries has been one of the main innovations of the last two decades as regards pension reform. They can be found in Italy (1995), Kyrgyzstan (1997), Latvia (1996), Poland (1999), Sweden (1999), Brazil[1] (1999) and Mongolia (2000). Other countries such as Germany, Austria, France, Finland, Portugal and Norway have incorporated some sort of adjustment mechanism that can also be found in NDCs to help calculate or index the initial retirement pension. According to Holzmann et al (2012), more countries including Egypt, China and Greece are seriously considering the introduction of NDCs[2].

A notional defined contribution scheme is a pay-as-you-go scheme (PAYG) that deliberately mimics a financial defined contribution scheme (FDC) by paying an income stream whose present value over a person's expected remaining lifetime equals the accumulated capital at retirement.

The practical application of NDCs first came about in the early 1990s, and since the mid-1990s they have been introduced in a number of countries. For Holzmann & Palmer (2012), NDC schemes work well from a practical point of view, as shown by the experiences of Italy, Latvia, Poland and Sweden, but they could be made to work even better.

This paper deals specifically with the so-called survivor dividend (also known as inheritance gains), which is distributed in the saving phase on a birth cohort basis from the account balances of participants who do not survive to retirement in NDC pension schemes[3]. This feature can currently only be found in the Swedish DC scheme.

According to Pensionsmyndigheten (2013), the Swedish pension scheme aims to redistribute resources from individuals with shorter-than-average life spans to those who live longer during the pay-out (or decumulation) phase. This arrangement works in exactly the same way as in DC-funded pillars when the beneficiary chooses the pay-out option of receiving the retirement benefit as a life annuity. NDC schemes do not allow phased withdrawals[4], where the retirement savings of deceased beneficiaries are distributed to their inheritors. Unlike with the DC-funded pillar, in the pay-in phase the pension balances of deceased persons are redistributed each year to surviving insured persons in the same birth cohort. This entitlement is completely different from the inheritance options in DC-funded pillars, where the accumulated capital in the individual account is distributed to inheritors and/or transformed into a survivor benefit.

Very little attention has been paid to the subject of the survivor dividend in the economic literature. Not even the report published by the Swedish authorities, Pensionsmyndigheten (2013), explains in depth its actuarial foundation and the effect it has on the scheme's financial equilibrium. As far as we know, the paper by Boado-Penas & Vidal-Meliá (2014) is the only one that has shed any light on whether it would be justified to include the survivor dividend when calculating affiliate pension balances in an NDC framework with no population changes. It concludes that including the survivor dividend in the calculation of the initial pension is by no means irrelevant because the pension could rise by up to 21.84% depending on the mortality scenario used.

The aim of this paper is to carry out an in-depth analysis of the role of the survivor dividend in NDC schemes. With this aim in mind, we first extend the model developed by Boado-Penas & Vidal-Meliá (2014). Second, we show that contributors who reach retirement age

always get a higher return than the scheme does, and that population growth enables cohorts with more years of contributions to benefit to a greater extent from the dividend effect. The results reached in the numerical example we present endorse the fact that the model really works.

Following this brief introduction, in Section 2 we present the model. Its main novelties are the introduction of changes in the growth of the active population and the possibility of exploring the effect of the survivor dividend on the relationship between the individual's internal rate of return (IRR) for contributors and the scheme's IRR. In Section 3 we present a complex example representative of a generic NDC scheme. Specifically, we provide a numerical illustration of the effects of the survivor dividend on the scheme's financial equilibrium when the economically active population is not constant, plus the impact of the survivor dividend on the individual's IRR for contributors who reach retirement age. The paper ends with the conclusions, possible directions for future research and an appendix with some proofs of the formulas used in Section 2.

2. The Model.

In this section we extend the actuarial overlapping generations model (OLG) developed by Boado-Penas & Vidal-Meliá (2014) based on those first put forward by Settergren & Mikula (2005), Boado-Penas et al. (2008) and Vidal-Meliá & Boado-Penas (2013). These papers were to some extent inspired by the accounting framework for organizing, summarizing and interpreting data on transfer systems and the life cycle developed in Lee (1994), Willis (1988) and Arthur & McNicoll (1978).

As we will see later, the main extensions to this model are the introduction of changes in the economically active population, the possibility of exploring the effect of the survivor dividend on the relationship between the individual's IRR for contributors who reach retirement age and the scheme's IRR.

The model's main features and assumptions are:

- The defined contribution rate credited to the individual, θ_a, is fixed over time.

- The initial pension depends on the value of the accumulated notional account, the expected mortality of the cohort in the year the contributor reaches retirement, and a future indexation rate, λ, i.e. pensions in payment increase or decrease at an annual rate of λ.

- The accumulated capital in the notional account reflects each participant's individual contributions and the fictitious returns these contributions generate over the course of the participant's working life, plus the inheritance capital.

- The account balances of participants who do not survive to retirement are distributed as inheritance capital to the accounts of surviving participants on a birth cohort basis.

- The scheme does not provide a minimum pension.

- Contributions and benefits are payable yearly in advance.

- Participants' lives last $(w-1-x_e)$ periods, where $(w-1)$ is the highest age to which it is possible to survive and x_e is the earliest age of entry into the pension scheme.

- The age giving entitlement to retirement pension, x_e+A, is fixed.

- The individual contribution base grows at an annual rate of g.

- The economically active population increases or decreases over time at an annual rate of γ, affecting all groups of contributors equally.

- The scheme's income from contributions (wage bill) also grows (decreases) at rate G = $(1+g)(1+\gamma)-1$.

- When the pension scheme reaches a mature state $t = w\text{-}1\text{-}x_e\text{-}A$ years from inception, A generations of contributors and $(w\text{-}(x_e+A))$ generations of pensioners coexist at each moment in time.

The demographic-financial structure at any moment t is given by:

1. Age:

$$\overbrace{x_e,\ x_e+1,\ x_e+2,\ ....x_e+A-1,}^{\text{Contributors' ages}}\ \underbrace{x_e+A,\ x_e+A+1,........... w\text{-}1}_{\text{Pensioners' ages}} \qquad \text{[1.]}$$

2. Number of contributors by age at time t:

$$N_{(x_e,\,t)} = N_{(x_e,\,1)}\cdot(1+\gamma)^{t\text{-}1},\ \ N_{(x_e+1,\,t)} = N_{(x_e+1,\,1)}\cdot(1+\gamma)^{t\text{-}1},\ ....., N_{(x_e+A-1,\,t)} = N_{(x_e+A-1,\,1)}\cdot(1+\gamma)^{t\text{-}1} \quad \text{[2.]}$$

where $N_{(x_e+k,\,1)} = N_{(x_e,\,1)\cdot k}P_{x_e}$, with $_kP_{x_e}$ being the probability that an individual aged x_e will be alive at age x_e+k.

3. Average wage by age at time t:

$$y_{(x_e,\,t)} =\ y_{(x_e,\,1)}\cdot(1+g)^{t\text{-}1},\ y_{(x_e+1,\,t)} =\ y_{(x_e+1,\,1)}\cdot(1+g)^{t\text{-}1},......., y_{(x_e+A-1,\,t)} = y_{(x_e+A-1,\,1)}\cdot(1+g)^{t\text{-}1}$$
$$\text{[3.]}$$

This demographic structure means that the age-wage structure (contribution bases) only undergoes proportional changes. The slope of the age-wage structure is constant.

After the main assumptions have been detailed, for the sake of clarity this section will be divided into three subsections: a description of the pension scheme in the mature state, the definition and calculation of the survivor dividend, and the effect of the survivor dividend on the scheme's financial equilibrium.

2.1. Description of the pension scheme in the mature state.

The main implications of the NDC scheme's being in a mature state are (1) it pays full benefits to all generations of retirement pensioners, (2) the dependency ratio[5], dr_t, stabilizes, and (3) the financial ratio, fr_t, is constant due to the fact that the average pension and the average contribution base evolve at the rate of variation in wages. Hence the total contribution rate (θ_t) that ensures equality between contribution revenue and pension expenditure is constant over time, and the scheme's income from contributions is equivalent to the present actuarial value of the pensions awarded in that year.

The initial average pension in year t for those individuals who reach retirement age x_e+A, $\overline{P}_{(x_e+A,\,t)}$, can be calculated as:

$$\overline{P}_{(x_e+A,\ t)} = \underbrace{\frac{K^{acT}_{(x_e+A,\ t)}}{N_{(x_e+A,t)}}}_{\overline{K}^{ac}_{(x_e+A,\ t)}} \cdot \frac{1}{\ddot{a}^{\lambda}_{x_e+A}} = \frac{\sum\limits_{c=1}^{A} P_{(x_e+A,\ c,\ t)} \cdot N_{(x_e+A,\ c,\ t)}}{N_{(x_e+A,\ t)}}$$ [4.]

where:

$N_{(x_e+A,\ t)}$: Number of contributors aged x_e+A, whereas $N_{(x_e+A,c,t)}$ is the number of contributors aged x_e+A who have been contributing for the last c years. Therefore

$$N_{(x_e+A,\ t)} = \sum_{c=1}^{A} N_{(x_e+A,\ c,\ t)}$$

$\ddot{a}^{\lambda}_{x_e+A} = \sum\limits_{k=0}^{w-x_e-A-1} \left[\dfrac{1+\lambda}{1+G}\right]^{k} \cdot {}_{k}p_{x_e+A}$: Present value at age x_e+A of 1 monetary unit of a lifetime pension payable in advance and indexed at rate λ, with a technical interest rate equal to G. This is also called the annuity factor or actuarial divisor, with ${}_{k}p_{x_e+A}$ being the probability that an individual aged x_e+A will be alive at age x_e+A+k.

$K^{acT}_{(x_e+A,t)}$: Total accumulated notional capital at time t for all individuals who reach age x_e+A. Therefore $K^{acT}_{(x_e+A,t)} = \sum\limits_{c=1}^{A} K^{ac}_{(x_e+A,\ c,\ t)} \cdot N_{(x_e+A,\ c,\ t)}$, where $K^{ac}_{(x_e+A,\ c,t)}$ represents the accumulated notional capital at age x_e+A for one individual who has been contributing for the last c years.

$K^{ac}_{(x_e+A,t)} = \dfrac{K^{acT}_{(x_e+A,t)}}{N_{(x_e+A,\ t)}}$: Average accumulated notional capital at time t for one individual aged x_e+A.

Henceforth F will be used to denote $\left[\dfrac{1+\lambda}{1+G}\right]$ throughout this paper.

With population growth of $\gamma > 0$, once the individual joins the labour market they will continue working nonstop until retirement age. The only exit from the labour market in this model is early death. Therefore there are A different contribution pathways that will determine A different pensions, as contributors might be working for 1 year, 2 years…, A-1 years.

Contributions of individuals who die before reaching retirement age are included in the notional capital. Consequently, in the case of $\gamma > 0$, $P_{(x_e+A,\ c,t)}$ is the initial pension at ordinary retirement age for individuals who entered the labour market at age x_e+A-c, i.e. for one individual who has been contributing for the last c years, whereas $\overline{P}_{(x_e+A,\ t)}$ is the average pension for individuals who retire at the ordinary retirement age, this being a weighted average pension of the A different pensions once settled.

As formula [4.] shows, in line with the definition of an NDC scheme being one that deliberately mimics an FDC scheme, the process for calculating the initial amount of the retirement pension is similar to the one used for calculating the future instalments of a deferred lifetime annuity, a type of annuity contract where payments are not scheduled to begin until a future date, in this case the age of retirement.

The balanced contribution rate, θ_t, makes spending on pensions equal to the revenue from contributions, and hence:

$$\overbrace{\theta_t \cdot \sum_{k=0}^{A-1} y_{(x_e+k,\ t)} \cdot N_{(x_e+k,\ t)}}^{\textit{Aggregate contributions}} = \underbrace{\overline{P}_{(x_e+A,\ t)} \cdot \overbrace{\sum_{k=0}^{t-1} N_{(x_e+A+k,\ t)}}^{N_{(x_e+A,t)} \cdot \ddot{a}^{\lambda}_{x_e+A}} \cdot F^k}_{\textit{Expenditure on pensions}} \qquad \textbf{[5.]}$$

Therefore once the mature state is reached, the macro contribution rate, θ_t, is constant from an actuarial point of view and can be expressed as the product of the dependency ratio and the financial ratio:

$$\theta_t = dr_t \cdot fr_t = \cfrac{\overbrace{\overline{P}_{(x_e+A,\ t)} \cdot \sum_{k=0}^{w\text{-}x_e-A-1} N_{(x_e+A+k,\ t)} F^k}^{\textit{Expenditure on pensions}}}{\underbrace{\sum_{k=0}^{A-1} y_{(x_e+k,\ t)} \cdot N_{(x_e+k,\ t)}}_{\textit{Contribution base}}} = \theta_{t+1} = ... = \theta \qquad \textbf{[6.]}$$

2.2. Definition and determination of the survivor dividend when the economically active population increases.

As in the Swedish NDC scheme, we follow the principle that each monetary unit contributed is paid out in the form of a retirement or old-age benefit, but not necessarily to the individual who made the contributions. An accumulated survivor dividend is included for any individual who reaches retirement age. The account balances of participants who do not survive to retirement are distributed as inheritance capital to the accounts of survivors on a birth cohort basis[6].

In this model, the amount of the survivor dividend can mainly be quantified using formulas **[7.]** and **[8.]**

Formula **[7.]** shows the mathematical expression of the accumulated survivor dividend at retirement age at time t for an individual who belongs to the initial group (that entered the scheme at age x_e) and has therefore contributed since entering the pension scheme:

$$D^{ac}_{(x_e+A,\ A,\ t)} = \underbrace{K^{ac}_{(x_e+A,\ A,\ t)} - \theta_a \cdot \sum_{k=0}^{A-1} y_{(x_e+k,\ -A+k+t)} \cdot (1+G)^{A-k}}_{K^i_{(x_e+A,\ A,t)}} = \sum_{k=1}^{A-1} D_{(x_e+k,\ -A+k+t)} \cdot (1+G)^{A-k} \qquad \textbf{[7.]}$$

where $D_{(x_e+k,\ -A+k+t)}$ is the dividend distributed at time $t\text{-}A+k$ for individuals aged x_e+k.

As mentioned above, the accumulated survivor dividend at a specific age is the portion of the credited account balances of participants resulting from the distribution, on a birth cohort basis, of the account balances of participants who do not survive to retirement. In other words it is the difference between credited capital $K^{ac}_{(x_e+A,\ A,\ t)}$, which includes contributions and indexation on contributions for members from the same cohort who died, and individual credited notional capital $K^i_{(x_e+A,\ A,\ t)}$.

When the economically active population grows over time (see Section 2.1.), there are A different contribution pathways that will determine A different contribution profiles.

Formula **[7.]** therefore needs to be modified to account for the different profiles. Formula **[8.]** quantifies the average accumulated dividend at age x_e+A, taking into account the different A contribution profiles:

$$\overline{D}^{ac}_{(x_e+A,\ t)} = \frac{\sum\limits_{c=1}^{A} D^{ac}_{(x_e+A,\ c,\ t)} \cdot N_{(x_e+A,\ c,\ t)}}{N_{(x_e+A,\ t)}} = \overline{K}^{ac}_{(x_e+A,\ t)} - \overline{K}^{i}_{(x_e+A,\ t)}$$

$$= \qquad\qquad\qquad\qquad\qquad \text{[8.]}$$

$$\frac{1}{N_{(x_e+A,\ t)}} \left[\theta_a \sum\limits_{k=0}^{A-1} N_{(x_e+k,-A+k+t)} \cdot y_{(x_e+k,-A+k+t)} \cdot (1+G)^{A-k} - \sum\limits_{c=1}^{A} K^{i}_{(x_e+A,\ c,\ t)} \cdot N_{(x_e+A,c,\ t)} \right]$$

In short, when the active population increases, the accumulated dividend for the individual who reaches retirement age depends on the mortality rates by age, the rate of population growth and the number of years contributed.

2.3. The effect of the survivor dividend on the scheme's financial equilibrium.

The survivor dividend plays a crucial role in the scheme's financial equilibrium and - as shown in Appendix 1 - it is easy to prove the equivalence between the macro (balanced) contribution rate, θ_t, and the credited individual contribution rate, θ_a, in the model.

If the amount of the pension is determined from the individual notional capital without considering the survivor dividend, then the new balanced contribution rate, θ_t^*, and the credited individual contribution rate, θ_a, are different because the retirement benefits are strictly lower than they should be (as the survivor dividend is not distributed among the survivors).

The relation between both contribution rates can be expressed using the so-called dividend effect, as shown in formula **[9.]**:

$$\theta_a = \theta_t^* \cdot \frac{K^{ac}_{(x_e+A,t)}}{K^{i}_{(x_e+A,t)}} = \theta_t^* \cdot \left(1 + \overbrace{\frac{D^{ac}_{(x_e+A,t)}}{K^{i}_{(x_e+A,t)}}}^{\text{Dividend effect} > 0} \right) = \theta_t^* \cdot \frac{\overline{P}_{(x_e+A,\ t)}}{\overline{P}^{i}_{(x_e+A,\ t)}} \qquad \text{[9.]}$$

where $\overline{K}^{i}_{(x_e+A,t)}$ is the average individual accumulated notional capital at time t for individuals aged x_e+A, without taking into account the survivor dividend, and $\overline{P}^{i}_{(x_e+A,\ t)}$ is the average pension of an individual who retires at the ordinary age at t, again without considering the dividend.

The dividend effect measures either the increase in the initial retirement pension after inclusion of the survivor dividend or the decrease in the balanced contribution rate if the dividend is not included in the pension calculation. Therefore $\theta_a > \theta_t^*$ because the scheme saved money by not including the survivor dividend.

If $\theta_t = \theta_a$ were contributed instead of θ_t^*, the pension scheme would continuously accumulate financial reserves because ignoring the survivor dividend produces savings when longevity is constant over time. In practice these reserves could finance the increase in spending on pensions resulting from increases in longevity and/or legacy costs from old pension systems. Indeed, in Poland and Latvia these revenues provide funds for other social insurance commitments that have no specific source of funding. Both countries decided to introduce funded components and, as a result, the revenue for the pay-as-you-go pillars was

reduced due to contributions being transferred to funded accounts. Hence the inheritance gains help to cover the double payment burden.

In year t, assuming constant longevity, the amount of the scheme's surplus, S_t, is easy to quantify because it is connected to the survivor dividend, specifically to the average accumulated dividend:

$$S_t = (\theta_a - \theta_t^*) \cdot \sum_{k=0}^{A-1} y_{(x_e+k,\ t)} \cdot N_{(x_e+k,\ t)} = N_{(x_e+A,\ t)} \cdot \overbrace{(\overline{K}_{(x_e+A,\ t)}^{ac} - \overline{K}_{(x_e+A,\ t)}^{i})}^{\overline{D}_{(x_e+A,t)}^{ac}} \qquad [10.]$$

The cost of longevity measured as the increase in life expectancy at retirement age, Δe_{x_e+A}, that could be financed if the dividend were not distributed, in the simplest case where $\lambda = G$, can be quantified following the reasoning presented in Appendix 1.

3. Numerical example.

This section presents a numerical example to illustrate the important role played by the survivor dividend in the NDC framework. To do this we basically use the closed-form expressions developed in Section 2.

Individuals are assumed to join the labour market at age 16 (x_e) onwards and contribute 16% (θ_a) of their contribution base until they reach 65 (x_e+A). The individual contribution bases grow at an annual accumulative rate (g) of 1.6%, while the retirement pension, once settled, is constant in real terms ($\lambda = 0$).

For the purposes of comparison, the mortality tables[7] used are those for Poland in 2009 (PL), Latvia in 2010 (LT) and Sweden (SW) in 2011[8].

3.1. Baseline case: zero population growth

In the case of zero population growth, all individuals enter the labour market at age 16 and work continuously until retiring at age 65. The amount of pension payable to individuals aged 65 is determined, after 49 contribution years, according to the formulas shown in the previous section. Under this scenario the balanced contribution rate (θ_t) is 16% and coincides with the credited contribution rate (θ_a).

The first vertical axis of Figure 1 (see after Table 1 below) shows the numbers of contributors and pensioners by age depending on the mortality scenario (SW, PL or LT) in a mature state from a cross-sectional point of view. The second vertical axis represents the contribution base and pension structure under the three different mortality scenarios (SW-w&p, PL-w&p and LT-w&p).

The main values making up the scheme's financial equilibrium under the three mortality tables are shown in Table 1.

[Insert Table 1]

[Insert Figure 1]

It can be seen that the mortality pattern has a significant effect on the amount of the retirement pensions provided by the scheme. Under the mortality scenario with the highest life expectancy (Figure 1, SW, in blue), the actuarially fair pensions are much lower than those

under the other mortality scenarios (PL, in black, and LT, in grey). Consequently under the SW mortality scenario, the annuity divisor, $\ddot{a}^{\lambda}_{x_e+A}$, used to calculate the amount of the initial retirement pension is higher (17.23) than those under the other mortality scenarios (15.05 and 14.39 for PL and LT respectively).

Given that the initial assumptions are analogous for all three scenarios, mortality rates play a crucial role in achieving the scheme's financial equilibrium. As Table 1 shows for the SW scenario, the scheme's dependency rate (dr_t) is higher (38.86%) than for the other two scenarios (30.20% and 27.57% for PL and LT respectively), whereas the scheme's financial rate (fr_t) is lower (41.18% as against 52.98% and 58.04% for PL and LT respectively). These results come naturally from formula **[6.].** If there is a defined contribution rate, θ_t, fixed over time, and if the dependency ratio is determined by the mortality scenario, then the financial ratio has to be adapted accordingly.

For the SW scenario, the amount of the pension once settled at retirement age, $\overline{\beta}_{(x_e+A,\,t)}$, amounts to 68.77% of the average salary. The impact of the dividend effect, De_t, on the initial pension is not very significant as it would only rise by 7.39% after the balances of participants who do not survive to retirement are distributed. For the same scenario, if the survivor dividend were not included in the calculation of the initial retirement pension, a discrepancy would arise between the credited contribution rate equal to 16%, θ_t, and the rate necessary to finance the pension, θ_t^*, in this case 14.90%. As can be seen in formula **[9.]**, the direct link between both rates is the dividend effect $De_t = \left(\theta_t / \theta_t^*\right) - 1$.

The impact of the dividend effect, De_t, on the initial pension is not insignificant for the other two scenarios, with the pension rising by 18.32% using the Polish mortality tables (PL) and 23.12% using the Latvian tables (LT). As a result, the replacement rates reached are also higher due to inclusion of the dividend. There is a direct relation between the dividend effect and the replacement rates. Indeed, with the variables provided in Table 1, it is easy to see that $De_t = \left(\overline{\beta}_{(x_e+A,\,t)} / \overline{\beta}^i_{(x_e+A,\,t)}\right) - 1$.

As seen in Table 1, if the survivor dividend were not included in the calculation of the initial retirement pension, the pension scheme could handle an unexpected increase in life expectancy at retirement age (Δe_{x_e+A}) of between 1.48 and 3.76 years, depending on the scenario, before exhausting the accumulated surplus. Again, the dividend effect can be used to explain the increase in life expectancy at retirement needed to neutralize the lack of dividend. Given this, it is not difficult to check in Table 1 that $De_t = \Delta e_{x_e+A} / e_{x_e+A}$.

3.2. Population changes

The growth in population means that the retirees' generation can be split into A different cohorts, whose common factor is the number of years contributed since joining the labour market. This section explores two additional assumptions about the rate of population growth: (1) that the number of contributors of all ages grows at an annual rate of $\gamma = 0.01$ over time (henceforth PL+), and (2) that the number of contributors of all ages decreases by an annual rate of $\gamma = -0.01$ (henceforth PL-). The Polish mortality scenario (referred to in the previous section as PL) is taken as a reference when analysing the effect of population changes, whether increases or decreases, because its survivor dividend has an average effect on the results, falling between the other two scenarios.

The results presented in the previous sub-section are recalculated taking into account the effect of population changes under the two additional assumptions described above. Figure 2 is included later in this sub-section for a better understanding of what happens when the economically active population grows. In addition, the relationship between the IRR for contributors by years of contributions and the IRR for the scheme itself is studied under the population growth scenario. The main results are shown in Figures 3 and 4.

The main values making up the scheme's financial equilibrium under the three population scenarios are shown in Table 2.

[Insert Table 2]

What really draws the attention is that the ratio between the numbers of contributors and pensioners, the ratio between the average salary and pension, and the effect of the survivor dividend remain unchanged when the economically active population is not constant over time. However, the explanation is obvious. Given that the contribution rate is the same under the three population scenarios, the ratio between the number of pensioners and the number of contributors (dr_t) must also be the same because it depends on the mortality scenario and, according to equation **[11.]** in Appendix 1, the ratio stabilizes because both groups evolve (increase or decrease) at exactly the same rate as population growth (γ). Therefore, according to equation **[6.]**, the scheme's average replacement rate (fr_t) has to be the same for all three population scenarios.

The rate of population growth has a direct effect on the sustainable scheme's rate of return (G), which in the NDC framework largely determines the amount of the initial retirement benefit provided by the schemes. Consequently, the higher the population growth, the higher the amount of accumulated notional capital, which implies a higher amount of initial pension.

[Insert Figure 2]

The scheme's sustainability derives from an adjustment to the average initial pension, directly linked to the annuity factor, $\ddot{a}^{\lambda}_{x_e+A}$, and the accumulated notional capital reached at retirement age. With an annual increase in population of γ =0.01 and for a given amount of credited notional capital at retirement age, the initial amount of the pension awarded will be higher than with a decrease in population of 1% (γ =-0.01) or constant population growth of 0% (γ =0). This is because the annuity divisor used is lower (13.73 as against 16.59 for the decrease in population and 15.05 for constant population respectively). The higher the value of *G*, the lower the value of the annuity divisor.

In our example, generation members who retire at age 65 could come from any of 49 different cohorts depending on the number of years contributed. This determines 49 (*A*) different amounts of pension that set the generation's average initial pension, linked to the average number of years contributed by those who reach retirement age. Therefore growth in the economically active population brings about changes in the average years contributed (AYC), as shown in Table 2. Under the assumption that the number of contributors of all ages grows at an annual rate of γ =0.01 over time, the AYC is 38.97 as opposed to 49 for γ =0 or γ =-0.01, where all the contributors who reach retirement age started working at the

entry age of 16, i.e. A years ago. It can be said that the value assigned to γ has an inverse influence on the AYC for the generation that retires at time t.

In spite of the growth in population that brings about a reduction in the AYC (see Table 2), the average replacement rate, $\overline{\beta}_{(x_e+A,\ t)}$, reached for PL$^+$ is even higher (95.07%) than in the other two cases (78.68% and 86.75% for PL$^-$ and PL respectively).

The data provided in Table 2 clearly show us that the amount of the pension with an equal number of years of contributions for the case of positive population growth is much higher than for the case of stable population and higher than for a decrease in population. This is to be expected given that the scheme's sustainable return with population growth is higher.

Another underlying issue is whether or not the variation in population has an influence on the dividend effect. According to the data shown in Table 2, the dividend effect remains constant for any value of γ, but we need to study what happens when the population increases. In the case of population growth there is a vector of pensions - A different pensions - so it is important to find out whether the impact of the dividend remains constant for cohorts belonging to the same generation of retirees when there are changes in the rate of population growth.

The answer can be seen in Figure 2, which shows the dividend effect for each of the A cohorts that make up the retirees' generation under the assumption that the population grows annually at a constant rate of 0.01, 0.02 or 0.04. For the values of γ assumed, the AYCs are 38.97, 31.67 and 22.20 respectively, given that γ has an inverse influence on the AYC for the generation that retires at time t.

Figure 2 shows that, for example, under the assumption that the population grows annually at a constant rate of 0.01 (represented by the solid black line), those contributors who reach retirement age having started working at the earliest age possible, i.e. age 16, benefit from a dividend effect of 19.07%. This is higher than the dividend effect with zero population growth (18.32%) but lower than for the scenarios in which the population grows annually at a constant rate of 2% or 4%, represented by the solid grey and blue lines respectively. Under the scenario with the highest population growth, the dividend effect for contributors who started working at the earliest age reaches 20.99%, whereas for the intermediate assumption it is 19.77%. And it is not only contributors with 49 contributed years that obtain a higher than average dividend effect; many other contributor cohorts with fewer years of contributions also obtain a higher survivor dividend. For example, as shown in Figure 2 under the scenario with the highest population growth, the dividend effect for those cohorts with more than 33 years of contributions is higher than the average, whereas for those cohorts with fewer than 33 contributed years it is lower.

Under the other two population scenarios, the number of years contributed are 38 and 42 for γ=0.02 and γ=0.01 respectively.

Population growth therefore enables cohorts with more years of contributions to benefit to a greater extent from the dividend effect. The more the number of contributors grows, the larger the pension for cohorts with more years of contributions compared to what it would have been without including the survivor dividend. Nevertheless, it must be stressed that the average effect of the dividend remains constant for any value of γ.

-The contributors' IRR and its relation to the scheme's IRR (G).

To provide a numerical illustration, we will look at the relationship between the contributors' IRR and that for the scheme itself under the population growth scenario.

Generally speaking, the contributors' IRR - with or without the survivor dividend - depends on age of entry to the labour market and the time the calculation is done (at age of entry, at retirement age, s years after entering the labour market, etc.). As we will see later, this is because the age reached by the contributor is very important when computing the IRR. The impact of these two conditioning factors underlying the results of the contributors' IRR are explored in Figures 3 and 4.

For the three mortality scenarios (SW, solid blue line, PL, solid black line, and LT, solid grey line), Figure 3 presents the expected $IRR_{(xe+A,\,A-K,\,t)}$ for contributors who reach retirement age taking into account the survivor dividend and distinguishing by age of entry to the labour market, i.e. according to the number of years contributed.

The figure also compares the cohort IRRs with the scheme's IRR (G, broken black line), which is the same as the return that contributors would get if the survivor dividend were not included when calculating the initial retirement pension. As expected, the IRR decreases as the age of entry to the labour market increases because the contributors benefit less from the distribution of deceased persons' contributions. It can also be observed that, depending on the mortality scenario, the IRR varies between 3.41% for SW (the scenario with the lowest mortality rates) and 5.41% for LT (the scenario with the highest mortality rates) at a labour market entry age of 16. This difference between mortality scenarios tends to decrease as the age of entry to the labour market increases. If the individual joins the labour market at age 56, the IRR varies between 2.97% for SW and 3.74% for LT.

[Insert Figure 3]

Hence the individuals' IRR varies significantly depending on the number of contribution years for each cohort within the same generation, and contributors who reach retirement age always get a higher return than the scheme because their notional capital also includes contributions from affiliates who die before reaching retirement age.

Figure 4 shows the results of the study set for cohorts of contributors who joined the labour market at the earliest age possible, i.e. 16 in our example, differentiating by age attained.

[Insert Figure 4]

The outcomes when the survivor dividend is taken into account, $IRR(sd)_{(xe+s,\,t)}$, are represented in Figure 4 as SW (solid blue line), PL (solid black line) and LT (solid grey line). It can be seen that the IRR increases with age, due to the fact that the survivors' accumulated notional capital increases yearly as a result of the distribution of the dividend.

The results when the survivor dividend is not taken into account are represented in Figure 4 as SW* (broken blue line), PL* (broken black line) and LT* (broken grey line). A maximum return equal to G is only achieved by survivors who reach retirement age, and therefore the scheme permanently accumulates reserves if the dividend is not included when calculating the pension.

The results shown in Figure 4 do not exactly match one of the stated properties for NDCs according to Palmer (2006): *Property 1. At any time the present value of an individual's lifetime benefit equals the individual's account balance. For each participant and at all times, the amount in the account, K, is the present or expected value of his or her benefit. The value of the account is determined by the individual's*

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own contributions and the system's internal rate of return; and Diamond (2006): *An NDC is supposed to provide benefits for different cohorts that have a present discounted value that equals the value of the account, using the internal rate of return (IRR) (of the system) for a discount rate.*

When they say the *system's internal rate of return* or the *internal rate of return (IRR) (of the system)*, both authors seem to be referring to what in this paper has been defined as G. Hence the property stated by Palmer (2006), IRR=G, is only fulfilled in two specific cases for the contribution cohort that joins the labour market at the earliest possible age.

Case 1: When the IRR is computed at age of entry into the labour market under the assumption that the survivor dividend is taken into account to calculate the retirement benefit.

Case 2: When the IRR is valued at retirement age under the assumption that the survivor dividend is not taken into account to calculate the retirement benefit.

Both cases are identified in Figure 4 for contributors who entered the labour market at the earliest possible age. For all other cases, depending on whether or not the scheme takes the survivor dividend into account when calculating the retirement pension, the expected IRR for different contributors is (very) different from G, and this difference depends on the number of years expected to be contributed and the survival probabilities attributed to affiliates.

4. Concluding comments and directions for future research.

Among those countries in which NDC schemes have been introduced, only Sweden applies what is known as the survivor dividend. Surprisingly little attention has been given to this subject in the economic literature and not even the report published by the Swedish authorities explains in any depth why this survivor dividend is applied.

As far as we are aware, the paper by Boado-Penas & Vidal-Meliá (2014) is the only one that has shed any light on whether it would be justified to include the survivor dividend when calculating affiliate pension balances in an NDC framework. In the present paper we have extended their model to account for changes in the economically active population, and the effect of the survivor dividend on the relationship between the individual's IRR for contributors who reach retirement age and the scheme's IRR.

We find that when the active population changes, the model endorses the idea that the survivor dividend has a sound basis which enables the NDC scheme to achieve financial equilibrium. To put it another way, the paper demonstrates that the survivor dividend enables the balanced contribution rate applied to be the same as the individual credited rate. A similar outcome was reached by Palmer (2012) regarding the equivalence between these two contribution rates in an NDC framework, but without explicitly considering the effect of the survivor dividend.

We also find that the average effect of the survivor dividend remains unchanged for any constant annual rate of population growth, that contributors who reach retirement age always get a higher return than the scheme does, and that population growth enables cohorts with more years of contributions to benefit to a greater extent from the dividend effect. The higher the number of contributors, the higher the pension for those cohorts with more years of contributions, compared to what they would have received without inclusion of the survivor dividend.

On the practical side, it can be said that the numerical example developed in the paper is close to reality. It confirms that our model really works because the results make sense and provide us with some useful values regarding the magnitude of the dividend effect, the unexpected increase in life expectancy at retirement age that the NDC scheme could handle

if the survivor dividend were not included in the calculation of the initial retirement pension, and the contributors' IRR computed according to a set of different scenarios.

In short, this topic is particularly important for the design of pension reforms and therefore this research could have a considerable impact for those countries that are currently rethinking the structure of their public pension systems. Taking the survivor dividend explicitly into account increases the political attractiveness of the reform by providing higher initial retirement benefits. The issue of transparency is also important, because not including the dividend means that systems tend to accumulate financial resources (as in countries such as Poland, Italy and Latvia) as a non-transparent way to protect their systems against the longevity risk and/or to finance legacy costs from former pension arrangements.

Finally, based on the model presented in this paper, a number of important directions for future research can be identified.

First, the coverage of any unexpected increase in longevity in cases where the SD is not distributed could be explored further in order to evaluate whether the SD is a potential solution for covering longevity risk in NDCs. Increases in longevity can be reflected not only by an increase in life expectancy at retirement age but also by a decrease in mortality rates or an increase in survival probabilities. Probabilities, meanwhile, can be evaluated as being either constant over time or age-specific depending on the mortality model.

The second direction would consist of evaluating the impact of introducing a minimum pension on the scheme's financial equilibrium. According to Holzmann & Palmer (2006), NDC schemes should be supplemented with a minimum income (pension) guarantee. For Barr & Diamond (2009), the purpose of pensions is to provide an adequate income stream when the individual is unable to work due to disability or retirement, so it would be a good idea to introduce a minimum pension in order to maintain a minimum standard of living.

Third, it would be interesting to design a fully integrated NDC model with retirement and permanent disability. An NDC scheme is widely defined as a PAYG scheme that deliberately mimics an FDC scheme. In most countries with mandatory individual capitalization accounts (Reyes (2010)), disability insurance is fully integrated into the FDC scheme. At the same time, according to Autor & Duggan (2006), OECD (2010) and Burkhauser et al. (2013), disability insurance is a big challenge for policy makers today. Hence, given that NDC schemes have positive features that could help to improve the efficiency of disability insurance, it would be useful to develop a theoretical model that fully integrated the disability contingency into an NDC framework. The methodology developed by Ventura-Marco & Vidal-Meliá (2014) could be a reference for designing this integrated model.

Finally, insurance innovation could be incorporated into the model, as proposed by Murtaugh et al (2001) and Brown & Warshawsky (2013) for funded systems, by integrating retirement and long-term care (LTC) annuities. The NDC framework could be useful for this purpose. This suggestion stems from the fact that LTC as a contributory contingency has been provided in the German contributory pension system (Rothgang (2010)) since the mid-1990s. Barr (2010) also gives sound reasons for extending social security to provide mandatory cover for LTC.

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Appendix 1: Proofs of some formulas/equations included in Section 2.

1. Financial and dependency ratios in the mature state.

Once the mature state is reached, the ratio between the number of pensioners and the number of contributors - (dr_t) - stabilizes because both groups evolve (increase or decrease) exactly equal to rate γ:

$$dr_t = \frac{\overbrace{\sum_{k=0}^{w-x_e-A-1} N_{(x_e+A+k,\,1)} \cdot (1+\gamma)^{t-1-k}}^{\overbrace{}^{Pensioners=R_t}}}{\underbrace{(1+\gamma)^{t-1} \cdot \sum_{k=0}^{A-1} N_{(x_e+k,\,1)}}_{Contributors=\,C_t}} = \frac{\sum_{k=0}^{w-x_e-A-1} N_{(x_e+A+k,\,t)} \cdot (1+\gamma)^{-k}}{\sum_{k=0}^{A-1} N_{(x_e+k,\,t)}} = dr_{t+1} = \ldots = dr = \frac{R}{C} \qquad \textbf{[11.]}$$

Also, the system's average replacement rate, expressed by the financial ratio, is already constant due to the fact that the numerator and denominator evolve equally (at the rate of variation in wages):

$$fr_t = \frac{\overbrace{\dfrac{\overbrace{P_{(x_e+A,\,t)} \cdot \sum_{k=0}^{w-1-x_e-A} N_{(x_e+A+k,\,t)} \cdot F^k}^{Expenditure\ on\ pensions}}{\underbrace{\sum_{k=0}^{w-1-x_e-A} N_{(x_e+A+k,\,t)} \cdot (1+\gamma)^{-k}}_{Pensioners=\,R_t}}}^{}}{\underbrace{\dfrac{\overbrace{\sum_{k=0}^{A-1} y_{(x_e+k,\,t)} \cdot N_{(x_e+k,\,t)}}^{Aggregate\ contribution\ base}}{\underbrace{\sum_{k=0}^{A-1} N_{(x_e+k,\,t)}}_{Contributors=\,C_t}}}} = \frac{\overline{P}_{t+1}}{\overline{W}_{t+1}} = \ldots = \frac{\overline{P}}{\overline{W}} = fr \qquad \textbf{[12.]}$$

2. Relationship between contribution rates.

The relationship between the credited contribution rate and the balanced rate according to formulas **[4] and [6]** is:

$$\underbrace{\frac{\theta_a \cdot \sum_{k=0}^{A-1} N_{(x_e+k,\,-A+k+t)} \cdot \overbrace{y_{(x_e+k,\,-A+k+t)}}^{\sum_{k=0}^{A-1} y_{(x_e+k,\,t)} \cdot N_{(x_e+k,\,t)}} \cdot (1+G)^{A-k}}{N_{(x_e+A,t)} \cdot \ddot{a}_{x_e+A}^{\lambda}} \cdot N_{(x_e+A,\,t)} \cdot \ddot{a}_{x_e+A}^{\lambda}}_{Expenditure\ on\ pensions} = \theta_t \cdot \overbrace{\sum_{k=0}^{A-1} y_{(x_e+k,\,t)} \cdot N_{(x_e+k,\,t)}}^{Aggregate\ contributions} \qquad \textbf{[13.]}$$

Therefore it is easy to see that $\theta_a = \theta_t$.

The amount of the pension ignoring the survivor dividend is calculated as follows:

$$\overline{P}^{i}_{(x_e+A,\,t)} = \frac{\overbrace{\theta_a \cdot \sum_{k=0}^{A-1} y_{(x_e+k,\,-A+k+t)} \cdot (1+G)^{A-k}}^{\overline{K}^{i}_{(x_e+A,\,t)}}}{\ddot{a}^{\lambda}_{x_e+A}} = \frac{\sum_{c=1}^{A} P^{i}_{(x_e+A,\,c,\,t)} \cdot N_{(x_e+A,\,c,\,t)}}{N_{(x_e+A,\,t)}} \qquad \textbf{[14.]}$$

where $\overline{P}^{i}_{(x_e+A,\,t)}$ is the average pension, without taking into account the survivor dividend, of an individual who retires at the ordinary age at t. It is a weighted average pension depending on the A different pensions that can be awarded.

Then,

$$\underbrace{\overline{P}^{i}_{(x_e+A,\,t)} \cdot \sum_{k=0}^{w-1-x_e-A} N_{(x_e+A+k,\,t)} \cdot F^{k}}_{Expenditure\ on\ pensions} = \overbrace{\theta^{*}_{t} \cdot \sum_{k=0}^{A-1} y_{(x_e+k,\,t)} \cdot N_{(x_e+k,\,t)}}^{Aggregate\ contributions} \qquad \textbf{[15.]}$$

and substituting the expression for $\overline{P}^{i}_{(x_e+A,\,t)}$, we get:

$$\theta_a = \theta^{*}_{t} \cdot \frac{K^{ac}_{(x_e+A,t)}}{K^{i}_{(x_e+A,t)}} = \theta^{*}_{t} \cdot \left(1 + \frac{\overbrace{\overline{D}^{ac}_{(x_e+A,t)}}^{Dividend\ effect}}{K^{i}_{(x_e+A,t)}} \right) \qquad \textbf{[16.]}$$

3.- The increase in life expectancy at retirement age, Δe_{x_e+A}, that could be financed if the dividend were not distributed.

If the survivor dividend is included:

$$\overline{P}_{(x_e+A,\,t)} = \frac{\overline{K}^{i}_{(x_e+A,\,t)} + \overline{D}^{ac}_{(x_e+A,t)}}{\ddot{a}^{\lambda}_{x_e+A}} = \frac{\overline{K}^{ac}_{(x_e+A,\,t)}}{\ddot{a}^{\lambda}_{x_e+A}} > \overline{P}^{i}_{(x_e+A,\,t)} = \frac{\overline{K}^{i}_{(x_e+A,\,t)}}{\ddot{a}^{\lambda}_{x_e+A}} \qquad \textbf{[17.]}$$

However, given the assumption that $\lambda = G$, it can be shown that:

$$\ddot{a}^{\lambda}_{x_e+A} = \sum_{k=0}^{w-x_e-A-1} F^{k}\,_{k}p_{x_e+A} = \sum_{k=0}^{w-x_e-A-1} {}_{k}p_{x_e+A} = 1 + e_{x_e+A} \qquad \textbf{[18.]}$$

where e_{x_e+A} is the curtate expectation of life for an individual aged x_e+A, i.e. the expected number of complete years remaining for an individual aged x_e+A to live.

Hence formula **[19]** should necessarily be fulfilled to neutralize the survivor dividend:

$$\overline{P}_{(x_e+A,\,t)} = \overline{P}^{i}_{(x_e+A,\,t)} \text{ if and only if } \overline{P}^{i}_{(x_e+A,\,t)} = \frac{\overline{K}^{i}_{(x_e+A,\,t)}}{1 + e'_{x_e+A}} \qquad \textbf{[19.]}$$

where:

$$e'_{x_e+A} < e_{x_e+A} \qquad \textbf{[20.]}$$

Therefore:

$$e_{x_e+A} - e'_{x_e+A} = \Delta e_{x_e+A} > 0 \qquad \textbf{[21.]}$$

with Δe_{x_e+A} being the increase in life expectancy at the ordinary retirement age, measured in years, that would neutralize the effect of the survivor dividend.

Table 1: NDC scheme with survivor dividend: some selected values.				
Description variable	Notation	PL	LT	SW
Credited rate=balanced rate	$\%(\theta_a = \theta_t)$	16.00	16.00	16.00
Balanced rate without SD	$\%\,\theta_t^*$	13.52	13.00	14.90
Dividend effect	$\%\,De_t$	18.32	23.12	7.39
Demographic ratio	$\%\,dr_t$	30.20	27.57	38.86
Financial ratio	$\%\,fr_t$	52.98	58.04	41.18
Annuity divisor	$\ddot{a}_{x_e+A}^{\lambda}$	15.05	14.39	17.23
Life expectancy	e_{x_e+A} (years)	17.10	16.24	19.99
Change in life expectancy	Δe_{x_e+A} (years)	3.1323	3.7551	1.4782
Replacement rate with SD	$\%\,\overline{\beta}_{(x_e+A,\,t)}$	86.75	94.39	68.77
Replacement rate without SD	$\%\,\overline{\beta}_{(x_e+A,\,t)}^{i}$	73.32	76.66	64.04
Retirement age	x_e+A (years)	16+49=65	65	65
Base scenario with $G=(1.016)(1.00)-1=0.016$				

Table 2: NDC system with survivor dividend and population changes: some selected values.				
Description variables	Notation	PL$^+$	PL$^-$	PL
Balanced rate without SD	$\%(\theta_a = \theta_t)$	16.00		
Dividend effect	$\% \theta_t^*$	13.52		
Credited rate=balanced rate	$\% De_t$	18.32		
Demographic ratio	$\% dr_t$	30.20		
Financial ratio	$\% fr_t$	52.98		
Annuity divisor	$\ddot{a}_{x_e+A}^{\lambda}$	13.73	16.59	15.05
Replacement rate with SD	$\% \overline{\beta}_{(x_e+A,\, t)}$	95.07	78.68	86.75
Replacement rate without SD	$\% \overline{\beta}_{(x_e+A,\, t)}^{i}$	80.35	66.50	73.32
Average years contributed	AYC	38.97	49	49
Sustainable scheme's rate of return	$\% G$	2.616	0.584	1.60
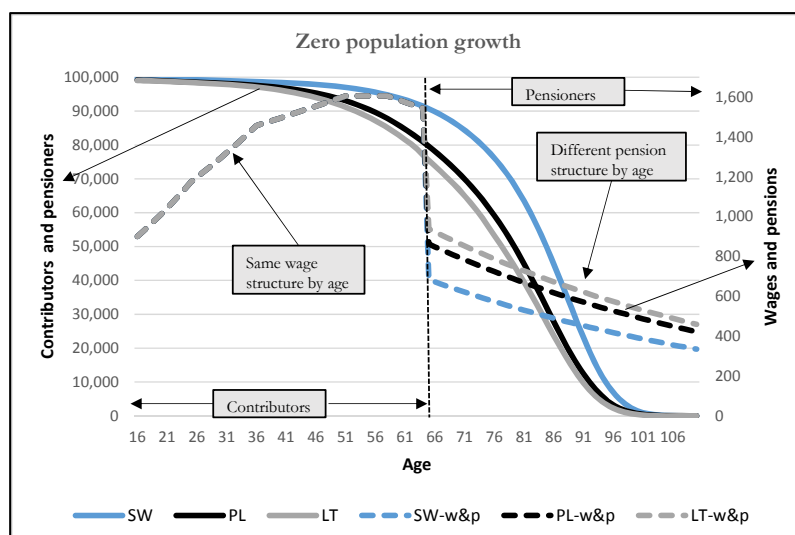
Figure 1: Structure of contributors, pensioners, wages and pensions under different mortality tables.
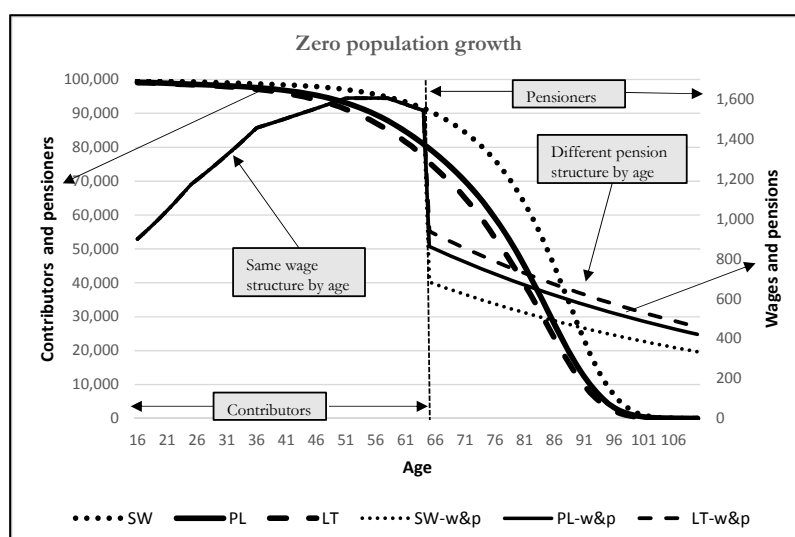
Figure 1: Structure of contributors, pensioners, wages and pensions under different mortality tables.
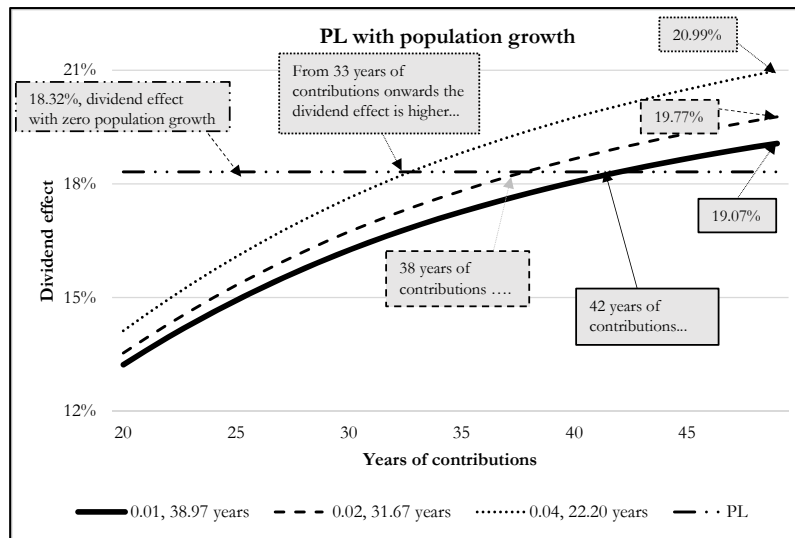
Figure 2: Dividend effect for PL with population growth by years of contributions.
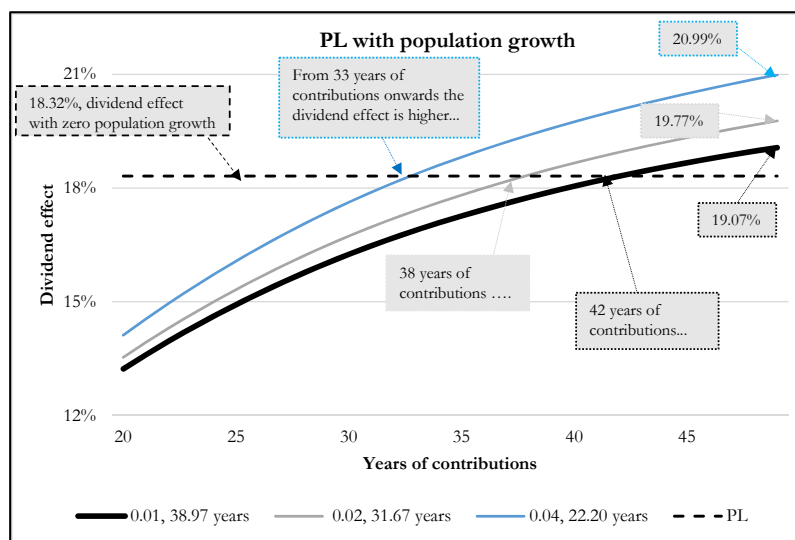

Figure 2: Dividend effect for PL with population growth by years of contributions.
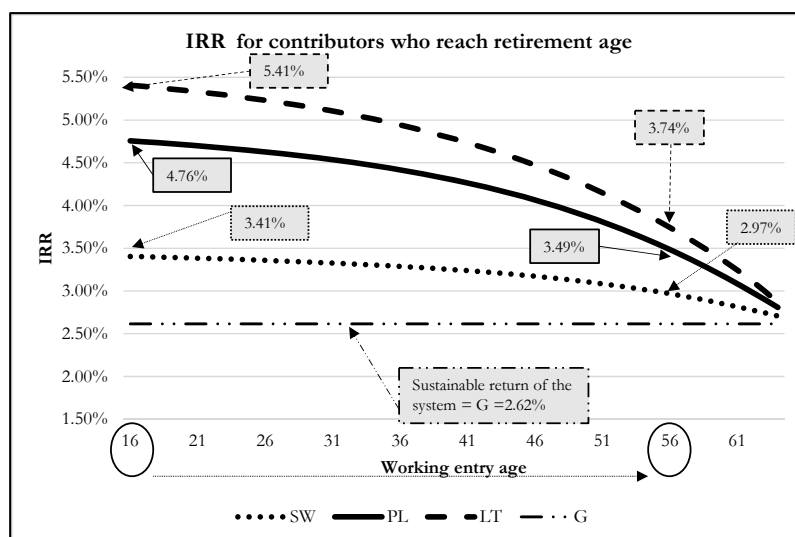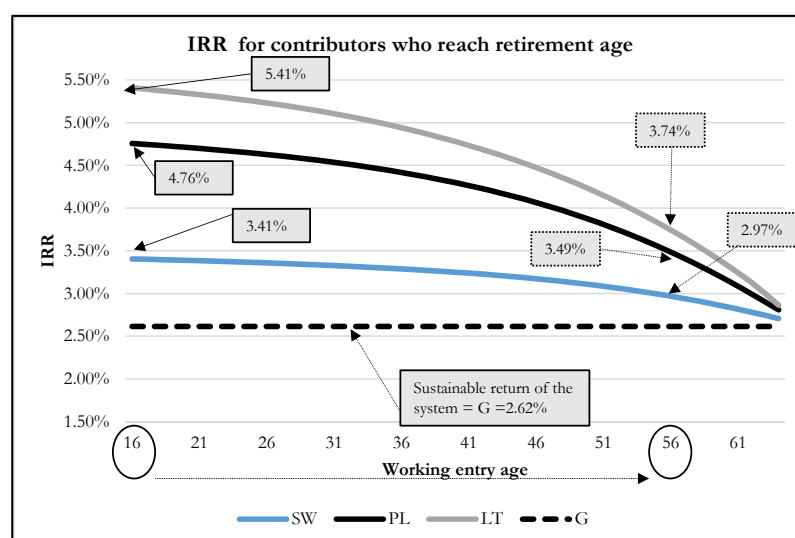
Figure 3: Expected $IRR_{(x_e+A, A-K, t)}$ taking into account the survivor dividend, with γ=0.01 and by age of entry to the labour market.

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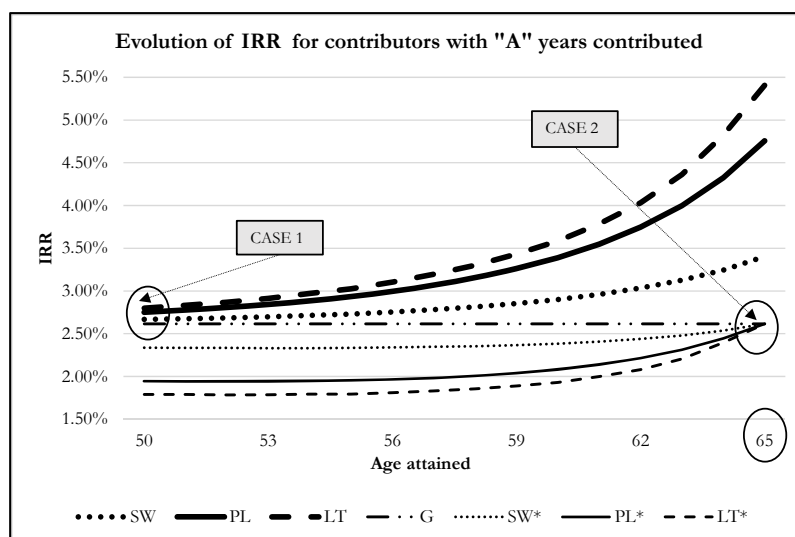
Figure 4: Evolution of $IRR_{(x_e+s,\ t)}$ **by age attained, with** γ=**0.01, for contributors who joined the labour market at age 16.**
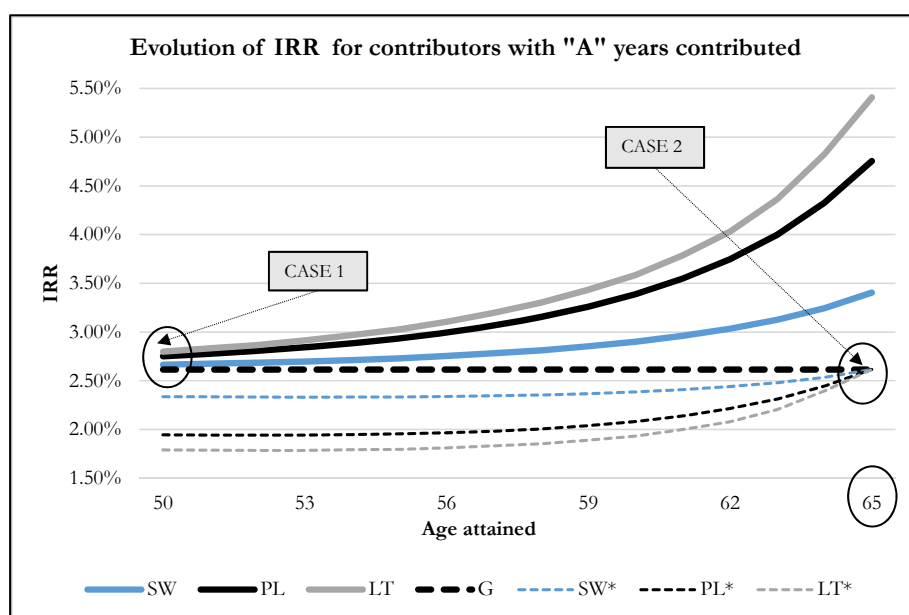

Figure 4: Evolution of $IRR_{(x_e+s,\ t)}$ **by age attained, with** γ=**0.01, for contributors who joined the labour market at age 16.**

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[1] In Brazil, the NDC reform was adopted only for the private sector.

[2] See, for example, Lindbeck & Persson (2003), Williamson (2004), Börsch-Supan (2006), Holzmann & Palmer (2006), Vidal-Meliá et al (2006), Auerbach & Lee (2009), Vidal-Meliá et al (2010), Whitehouse (2010), Auerbach & Lee (2011), Chłoń-Domińczak et al (2012) and Holzmann et al (2012).

[3] It is important not to confuse the concept of the survivor dividend (which mainly depends on mortality rates at specific ages) with the so-called "demographic dividend", Ross (2004), which is linked to fertility dynamics.

[4] See Kritzer et al (2011). One possible way of designing programmed or phased withdrawal is when the affiliate receives a pension charged on the balance of their individual capitalization account which, by remaining under the responsibility and management of the administrator, allows the retiree to benefit from the return on the fund. The pension is fixed for periods of one year and the amount is calculated by taking into account the balance of the individual account, the technical rate of interest defined by law and the life expectancy of the worker and their family according to legal mortality tables.

[5] In this model the terms dependency ratio, old-age ratio and demographic ratio are used as synonyms given that everybody participates in the labour market.

[6] A contributor can reach retirement age as active or disabled. In the case of Sweden, the current regulations on disability pension are closely linked to the old-age pension system. According to Chłoń-Domińczak et al (2012), the Swedish model for retirement pension rights for persons receiving disability benefits is to impute and pay contributions for insured periods of disability into the retirement contingency. These payments, made annually from general tax revenues, are entered on the country's accounts as a cost for the disability system and are part of the transfer from state revenues to the NDC pension fund. Disability benefits are converted into retirement benefits at age 65.

[7] Only observed mortality rates are used and not the population structure by ages.

[8] Years with the latest information available according to the Human Mortality Database (http://www.mortality.org/). Date accessed 10-05-2013.