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Efficient budget balancing cartel equilibria with imperfect monitoring*

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Summary. We modify the infinitely repeated Cournot game with imperfect monitoring of Green and Porter (1984) and Abreu, Pearce and Stacchetti (1986) to include heterogenous products and the possibility of balanced budget side payments (Holmström 1982). It is shown that a transfer mechanism which induces the efficient outcome exists under a reasonable technical assumption in contrast to the preceding authors. Intuitively, the existence of an observable random price vector rather than a single price makes it possible to identify "likely" defectors, eliminating the need for collective punishments.

1. Introduction

Historically, the analysis of oligopoly has played an important role in the economic literature. The best known solution to the oligopoly problem goes back to Cournot (1897) who analyzed a single period game in the absence of any binding agreement between firms. The Cournot solution specifies an industry output which is less than the competitive one but exceeds that of the monopoly. This last fact has led to criticism of the Cournot solution on the grounds that it unrealistically predicts excessively competitive behaviour (Stigler 1964). Several authors have appealed to dynamic considerations to explain how an oligopoly might be able to behave approximately like a monopoly even in the absence of any ability to make binding agreements.

One of the first rigorous formulations of the oligopoly problem as an infinitely repeated game was presented by Friedman (1971). He showed how a cooperative level of output may be sustained as a non-cooperative Nash equilibrium of the infinitely repeated game: Following unilateral deviation from the cooperative quantity, each firm responds by producing the Cournot quantity

* This paper is based on the last chapter of Demougin's dissertation at the University of Western Ontario.

forever. If future profits are discounted sufficiently slowly, the anticipated reversion to Cournot behaviour effectively outweighs any possible single period gains from defection. Abreu (1986) has recently generalized this result and characterized the punishment strategies which support the maximum possible level of collusive profit. These strategies typically specify punishments in excess of Cournot reversion.

The success of the preceding approach requires that cheating be detectable. This is possible only if either outputs are common knowledge or price is deterministic. Neither of these possibilities is very realistic. Several authors have therefore reformulated the oligopoly problem as a game of incomplete information: each firm is privately informed of its own output and the market price contains a stochastic component. The literature includes Green and Porter (1984), Porter (1983) and more recently Abreu, Pearce and Stachetti (1986). The latter demonstrate that subject to very mild regularity requirements, the symmetric equilibrium which offers the oligopolist the greatest possible expected profits is fully described by 2 quantities which correspond to the best and worst possible symmetric sequential equilibrium. Firms decide which of the two quantities to produce simply by remembering the state and the price of the preceding period. For either state, the equilibrium describes two complementary sets of prices, which leads to either the cooperative or the "punishment" quantities. The alternation between the two states is shown to follow a Markov process.

Because it is not in general possible to perfectly distinguish between random fluctuations in demand and evidence of defection, the optimal equilibrium includes periods in which output exceeds the cooperative level. The message of the preceding literature is therefore that the fully cooperative outcome is unattainable under imperfect monitoring.[1]

The purpose of this paper is to demonstrate that the validity of the preceding result hangs crucially on 2 assumptions:

(i) the oligopolist' outputs are perfect substitutes
(ii) side payments are not feasible.

Although the literature on price competition has long recognized the importance of product differentiation, the study of quantity competition has traditionally been dominated by the assumption that outputs are perfect substitutes. This is because under complete information, the analysis of Cournot competition is essentially unaffected by the degree of substitutability between outputs. This is not the case when firms are privately informed. If outputs are homogeneous, only one market price is observed. An abnormally low price may indicate the presence of defection but *not* the identity of the defector. When outputs are heterogeneous, a *vector* of market prices, one for each firm, is

[1] However, even in this case the Cournot solution predicts excessively competitive behaviour. The equilibrium described by Abreu, Pearce and Stachetti (1986) makes a cartel better off than producing the Cournot quantity in each period.

publicly observed and it may be possible to design a mechanism which penalizes likely defectors.

For the case of heterogeneous outputs, when firms are risk-neutral, side payments are feasible and the capital market is perfectly efficient we demonstrate the existence of a transfer mechanism which sustains an outcome inducing each firm to produce the optimal (cooperative) quantity in *each* period. Specifically, for each possible realization of the random price vector, the mechanism specifies the amount of money each firm must either transfer to or receive from other firms such that the sum of payments equals the sum of receipts in every period. Observe that in contrast to the collective punishment schemes of Green and Porter and Abreu et al, the transfer mechanism enforces cooperative behaviour by selectively punishing firms "suspected" of defection. In equilibrium, each firm attains the collusive level of profit.

If outputs are perfectly homogeneous, no such mechanism exists in general. Since only one price is observed, any mechanism which specifies the amount of money which a subset of firms is required to transfer to remaining firms must be essentially arbitrary. A scheme under which a defector is as likely to receive a payment as she is of being required to pay it cannot police cooperative behaviour. The latter *can* be accomplished through "dissipative" transfer schemes which require *all* firms to transfer money outside of the oligopoly but these, like the collective punishment schemes of Green and Porter and Abreu et al, are incapable of implementing the efficient (cooperative) outcome.

Our result is related to Holmström's (1982). Holmström shows that a team of agents producing a joint output with unobservable effort cannot attain pareto optimal production via balanced budgeting transfer schemes. In the Holmström model, the single joint output corresponds to the homogeneous product case in our model. In either case balanced budget pareto optimal schemes fail to exist because only one "signal" is observed: the joint monetary outcome in Holmström's case and the joint market price in ours. By contrast, when multiple signals, one for each agent are observable, our analysis shows that Pareto optimal balanced budget transfer mechanisms exist subject to reasonable restrictions.

Our paper is also related to the literature on surplus extraction in agency problems with asymmetric information. In this literature a mechanism designer observes ex-post a noisy signal correlated to the private information of the agent. Examples of this literature are Cremer and McLean (1985, 1988) and McAfee and Reny (1989) in the context of auctions and Demougin (1988) and Riordan and Sappington (1989) in the context of regulation. In all of these papers, as in the present one, the risk neutrality and the feasibility of side payments compensate for the noisy signal.

The remainder of the paper is organized as follows. In the next section, we present the one period game. In Section 3 we discuss the cooperative solution. In Section 4 we prove that the cooperative solution can be induced by an *enforceable* balanced transfer scheme. Finally, in the last section, we prove that in the context of an infinitely repeated game the cooperative solution is attainable in the absence of enforceable contracts. Concluding remarks end the paper.

2. The model

We consider a quantity setting oligopoly in which N firms produce a heterogeneous product. The cost function of each firm is common knowledge; $c_i(q_i)$, $i=1,\dots N$. The vector of market prices $\mathbf{p}=(p_1,\dots,p_N)$ is a random vector whose distribution $f(\mathbf{p}|\mathbf{q})$ depends on the entire vector of output $\mathbf{q}=(q_1,\dots,q_N)$. The realization of $\mathbf{p}$ is assumed to be common knowledge.

We assume that:

(A1) Firms are risk neutral and maximize expected profit.
(A2) $\forall\, \mathbf{q} \geqq \mathbf{0}$, $\forall i \varepsilon \{1,\dots,N\}$ define:

$$\psi_i(\mathbf{q}) := \operatorname{Sup}\{p_i \varepsilon \mathbf{R} \,|\, \exists\, \mathbf{p}_{-i} \varepsilon \mathbf{R}^{N-1} \text{ s.t. } f(p_i, \mathbf{p}_{-i}|\mathbf{q}) > 0\}$$

$\forall \mathbf{q} \geqq \mathbf{0}$, the mapping $\boldsymbol{\Psi}(\mathbf{q}) = (\boldsymbol{\psi}_1(\mathbf{q}),\dots,\boldsymbol{\psi}_N(\mathbf{q}))$ is bounded, bijective and $f(\boldsymbol{\Psi}(\mathbf{q})|\mathbf{q}) > 0$.
(A3) The probability density function $f(\mathbf{p}|\mathbf{q})$ and the mapping $\boldsymbol{\Psi}(\mathbf{q})$ are N times differentiable in the output vector.

(A2) is our central technical assumption. It requires that for every feasible output level there exists a bounded square which covers the entire support of $\mathbf{p}$ and for which it is true that the upper right corner $\boldsymbol{\Psi}(\mathbf{q})$ has a positive density. (A2) also requires that the mapping $\boldsymbol{\Psi}(\mathbf{q})$ be invertible. There are many economically meaningful environments which satisfy the conditions of (A2), as the following example shows.

Example. Assume $N=2$ and $q_i = d_i(\mathbf{p}, \varepsilon_i)$ with

(i) $\partial d_i/\partial p_i < 0$, $\partial d_i/\partial p_j > 0$, $\partial d_i/\partial \varepsilon_i > 0$,
$(\partial d_1/\partial p_1)(\partial d_2/\partial p_2) - (\partial d_1/\partial p_2)(\partial d_2/\partial p_1) > 0$
(ii) $(\varepsilon_1, \varepsilon_2) \varepsilon [0,1] \times [0,1]$ is a random vector,
(iii) $(1,1)$ has a positive density.

To see that $\mathrm{d}(\cdot,\cdot)$ satisfies (A2) note that for any output vector the requirements in (i) imply that $\partial \psi_i/\varepsilon_j > 0$, $j=1,2$.

3. The cooperative solution

The payoff of firm i, given (p_i, q_i), is:

$$\pi_i(p_i, q_i) = p_i q_i - c_i(q_i). \tag{1}$$

The cooperative solution is the vector of output $\mathbf{q}^c = (q_1^c,\dots,q_N^c)$ which solves the following control problem:

$$\operatorname*{Max}_{q_1,\dots,q_n} \int_0^{\Psi(q)} \left\{ \sum_{i=0}^{n} \pi_i(p_i, q_i) \right\} f(\mathbf{p}|\mathbf{q}) d\mathbf{p}. \tag{2}$$

Since our objective is not to derive the conditions, which guarantee the existence of the cooperative solution, we assume that the following requirement is satisfied:

(A4) Problem (2) has at least one solution with the profit of each firm being strictly positive.

From the work of Abreu [1986] and Friedman [1971], it is well known that under some regularity conditions, the production vector $\mathbf{q}^c$ can be sustained by implicit collusion as a solution of a *non-cooperative equilibrium* of a super game when either outputs are common knowledge or prices are deterministic.

4. Unobservable outputs

In this section, we consider the single period game under the additional requirements:

(A5) Production is the private information of each firm.
(A6) Firms can write *enforceable* contracts and have "infinitely deep pockets".

Assumption (A6) is essential for the solution of the single period game[2]. When we consider the super game this assumption is dropped. Since the vector of prices depends on the state of nature, and output is not observable, a firm cannot monitor the production of its competitors. A contract between the firms is a vector of transfers $\mathbf{T}=(T_1,\ldots,T_N)$. These transfers can only be conditioned by the variables that are commonly observed, which in the present case is the vector of prices. The feasibility of the contract requires that we do not introduce money from outside of the system. That is, we require that for any price vector:

$$\sum_{i=1}^{N} T_i(\mathbf{p}) \leqq 0. \tag{3}$$

We will say that contract satisfies the production incentive criteria if $\mathbf{q}^c$ is a Nash equilibrium of the production game faced by the firms. A contract will be said to be "efficient" if it satisfies the production incentive criteria and if no money leaves the system; that is if eq. (3) is binding in every feasible state of nature.

Theorem 1. *There exists at least one efficient contract.*

In order to prove this result, we will implicitly define a contract by a system of $(N-1)$ Volterra equations of the first kind with multiple integral. We will show that the contract is efficient and that the system of Volterra equations is solvable.

Proof. For $i=1,\ldots,(N-1)$ we define $T_i(\cdot)$ implicitly by the following integral equation:

$$\int_0^{\psi(q)} T_i(\mathbf{p}) f(\mathbf{p}|\mathbf{q}) d\mathbf{p} = \tag{4i}$$

$$\int_0^{\psi(q)} \frac{1}{N} \left\{ \sum_{\substack{j=1 \\ j\neq i}}^{N} \pi_j(p_j, q_j) - (N-1)\pi_i(p_i, q_i) \right\} f(\mathbf{p}|\mathbf{q}) d\mathbf{p}.$$

[2] Some implications of (A6) are discussed at the end of this section.

Finally, define the N-th transfer function:

$$T_N(\mathbf{p}) = -\sum_{i=1}^{N-1} T_i(\mathbf{p}). \tag{5}$$

Using this definition it is easy to see $T_N(\cdot)$ satisfies an equation of the form (4i) for $i = N$; simply take the expected value of (5) and substitute using (4i). From the definition, it is also obvious that the sum of transfers will always add up to zero. To prove that the contract is efficient, we only need to show that the production vector $\mathbf{q}^c$ is a Nash equilibrium of the production game given $\mathbf{T}$. Suppose, that all the firms, except i, decide to produce the cooperative quantity. What is the best response of firm i? Firm i maximizes its expected profit with respect to its own output q_i. The profit of a firm is the sum of its payoff and its transfer, $\pi_i(\mathbf{p}) + T_i(\mathbf{p})$. Using (4i), the firm's expected profit is:

$$\int_0^{\psi(q)} \{\pi_i(p_i, q_i) + T_i(\mathbf{p})\} f(\mathbf{p}|\mathbf{q}) d\mathbf{p} = \int_0^{\psi(q)} \frac{1}{N} \left\{ \sum_{j=1}^{N} \pi_j(p_j, q_j) \right\} f(\mathbf{p}, \mathbf{q}) d\mathbf{p}. \tag{6}$$

When each of its competitors produce the cooperative quantity, firm i's problem is:

$$\text{Max} \int_0^{\psi(q^c_{-i}, q_i)} \frac{1}{N} \left\{ \sum_{j=1, j\neq i}^{N} \pi_j(p_j, q_j^c) + \pi_i(p_i, q_i) \right\} f(\mathbf{p}|\mathbf{q}^c_{-i}\, q_i) d\mathbf{p}. \tag{7}$$

By construction the cooperative output solves this problem. To conclude the proof we must show that the integral eqs. (4i) $i = 1, \ldots, (N-1)$ are all solvable. For this purpose let us rewrite equation (4i) in the standard form of a Volterra equation of the first kind with multiple integrals and let $v_i(\mathbf{q})$ denote the right-hand side of the eq. (4i):

$$\begin{aligned} \int_0^{\psi} T_i(\mathbf{p}) g(\mathbf{p}|\psi) d\mathbf{p} &= t_i(\psi) \\ g(\mathbf{p}|\psi) &= f(\mathbf{p}|\psi^{-1}(\psi)) \\ t_i(\psi) &= v_i(\psi^{-1}(\psi)). \end{aligned} \tag{8i}$$

We need to show that each of the above equations satisfy the necessary requirements for solvability. These requirements were initially formulated by Volterra (see e.g. Volterra (1897) or Volterra and Peres (1936)), and for convenience are reproduced in the appendix. It is useful to note the following equality:

$$\begin{aligned} t_i(\psi) = \int_0^{\psi} \frac{1}{N} \Bigg\{ & \sum_{j=1, j\neq i}^{N} \pi_j(p_j, \psi_j^{-1}(\psi)) \\ & - (N-1)\pi_i(p_i, \psi_i^{-1}(\psi)) \Bigg\} f(\mathbf{p}, \psi^{-1}(\psi)) d\mathbf{p}. \end{aligned} \tag{9}$$

From this equality, it becomes immediately apparent that the regularity requirements, R.2 and R.3 as formulated in the appendix, are satisfied for each of

the functions $t_i(\cdot)$, $i=1,\ldots,N-1$. Furthermore, the assumptions (A2) and (A3) guarantee that $t_i(\cdot)$ is N times differentiable in Ψ for all i's. From (A2) we also conclude that R.4 holds and from (A3) that R.5 and R.1 are satisfied.

Since all the requirements in the appendix are satisfied, each of the eqs. (8i) are solvable. Note that the equation might not be defined for all $\Psi\varepsilon[0,\psi_1]X \ldots X[0,\psi_N]$. If this is the case, we can simply extend the functions such that the requirements R.1 to R.5 remain satisfied. By Volterra's theorem the extended problem has a solution which by construction must satisfy the original problem. In this case the solution is obviously not unique since the set of feasible extensions is generally large.

This concludes the proof. Q.E.D.

It is worthwhile to examine why this solution concept breaks down for a homogeneous oligopoly. For simplicity's sake, we only consider the case of a duopoly. We can argue from either a technical or a heuristic perspective. Technically, for a homogeneous duopoly, the analogue of eq. (4i) becomes:

$$\int_0^{\psi(q_1+q_2)} T_i(\mathbf{p})f(\mathbf{p}|q_1+q_2)d\mathbf{p} = \int_0^{\psi(q_1+q_2)} \tfrac{1}{2}\{\pi_j(p_j,q_j)-\pi_i(p_i,q_i)\}f(\mathbf{p}|q_1+q_2)d\mathbf{p}.$$

Without extreme restriction on the inverse demand and the cost function, this equation is not solvable. To prove this, consider a change in the production vector which leaves the total output constant. The left-hand side of (4i) cannot be affected by any such changes, since the output enters only as a sum into this integral. For the right-hand side this argument fails, because output enters non-linearly into the payoffs of the firms.

The essential difference between the homogeneous and the heterogeneous oligopoly is that in the former case only one market price is observed, whereas in the latter case there exists a *vector* of prices upon which the transfers can be conditioned. For the homogeneous case, one can easily find a contract which satisfies the production incentive criteria. However, as we have just proved, this contract would have to induce "dissipative" transfers (perhaps donations to hospitals or the purchase of an art collection!). Intuitively, a production incentive contract needs to identify and penalize a firm which overproduces. In the case of a homogeneous oligopoly, there is no meaningful way to distinguish between firms. When punishment is called for, it must be imposed collectively so that the sum of transfers is negative and the efficient solution cannot be achieved.

To see why it is intuitively reasonable that an efficient contract exists for a heterogenous oligopoly, consider a duopoly for the sake of simplicity. For each firm, consider Δ_i, the difference between its expected price and its realized price. Suppose it is agreed that the firm with the larger Δ_i must make a positive transfer to its competitor. Finally, let the transfer satisfy the requirement that, if neither firm cheats, the expected transfer of a firm is just zero. Consider the production decision of firm i which believes that j produces q_j^c. If i produces more than q_i^c, the probability that $(\Delta_i\text{-}\Delta_j)>0$ increases. This means that i expects to make a payment to J. Theorem 1 simply states that one can find a transfer function for which it is true that (i) any change from q_i^c increases the expected

transfer to be paid by firm i by more than it expects to gain from cheating, and (ii) the negative of this function has a symmetric effect for firm j.

The main point of this section is that product differentiation, by providing more information, allows for more effective collusion then the homogeneous case. The technique chosen to prove the theorem appears to make the result excessively strong because we imposed very weak restrictions on the distribution of prices[3]. A important consequence is that even "coarse information" seems to be sufficient to eliminate the distortionary consequences of the information asymmetries between the firms.

This is a stark result, because we would think that in making side-payments, contingent on information available after the agreement has been reached, the level of efficiency attainable by the agreement should depend on the quality of the information. The same difficulty has been encountered in the literature mentioned in the introduction on surplus extraction in principal-agent models with asymmetric information[4]. Demougin and Garvie (1990) have reviewed this literature. In particular, they discuss the difficulty associated with simultaneously assuming that firms are risk-neutral and have "infinitely deep pockets". Specifically, the solutions which are derived could not be applied in an actual market, because on some sample paths some of the players would be driven into bankruptcy. In other words, applying this observation to the above problem seem to suggest that the economic environment used to derive Theorem 1 is not well specified.

First, for the problem at hand an important message follows from the work by Demougin and Garvie (1990). Applying their result to the present difficulty suggests that if the elasticity of the probability density function of the signals (in our case the prices) with respect to the private information (in our case is the quantity produced by a firm) is large enough a cartel should be able to sustain the solution derived in Theorem 1 even if the firm faces limited liability. The actual size of the elasticity required to sustain Theorem 1 would naturally depend on the financial constraints of the firms. In general, this is a difficult problem. More importantly, though, we will see in the next section that this entire predicament can be avoided in the case on an infinitely repeated game if the capital market is perfectly efficient.

5. The infinitely repeated game

The preceding section assumed that the firms can write binding contracts to enforce the transfer mechanism and that the firms never face any binding financial constraints. In the one shot game and in the absence of such a contract a firm required to make a positive transfer would renege on its agreement.

[3] We are thankful to an anonymous referee for pointing the problem out and its relation to the existing literature.

[4] See Cremer and McLean (1985, 1988), Demougin (1988), McAfee and Reny (1989) and Riordan and Sappington (1989).

The purpose of this section is to show that when the capital market is perfectly efficient, and when the oligopoly game is infinitely repeated if future profits are discounted sufficiently slowly, there exists an equilibrium in which **T** is voluntarily implemented in each period even in the absence of enforceable contracts. The argument will be based on the paper by Friedman (1971). At the expense of using more complex mathematics the results could probably be generalized using optimal punishment scheme similar to Abreu (1986).

Suppose the firms agree on a contract Φ. Given any such agreement, a strategy σ_i^t for firm i at time t will specify how much i will produce at the beginning of period t and, once the price vector Ψ^t is observed, how much i will transfer. The strategy σ_i^t will depend on the history known to firm i at time t. We will denote this history as H_i^t. In general, H_i^t will include all of the past price vectors, all of the past transfer vectors and all of the past production of firm i. Given the contract Φ, denote the strategy of firm i by $\sigma_i = \{\Phi; \sigma_i^1\}$. Define the strategy σ_i^c by (10) where $\mathbf{q}^N = (q_1^N, \ldots, q_N^N)$ is the single period Cournot equilibrium output and T^c is the contract defined by Theorem 1.

$$\sigma_i^{c^1} = \begin{cases} q_i^1 = q_i^c \\ T_i^1 = T_i^c(\mathbf{p}^1) \end{cases}$$

$$\begin{cases} q_i^t = \begin{cases} q_i^c & \text{if } T^s = T^c(\mathbf{p}^s) \quad \forall s,\ s < t \\ q_i^N & \text{otherwise} \end{cases} \\ T_i^t = \begin{cases} T_i^c(\mathbf{p}^t) & \text{if } T^s = T^c(\mathbf{p}^s) \quad \forall s,\ s < t \\ \mathbf{0} & \text{otherwise} \end{cases} \end{cases} \tag{10}$$

Using an argument similar to Friedman [11], we can show that there exist discount rates for which the vector of strategies $\sigma = (\sigma_1^c, \ldots, \sigma_N^c)$ is a Nash equilibrium[5]. Note, though, that unlike in the standard model studied by Friedman there are now two points in time in every period where a player could potentially deviate. First, a firm could deviate by over producing at the beginning of the period and second a firm, which is required to make a negative transfer according to the equilibrium, could deviate by refusing to pay. In either case the deviation becomes suboptimal when the discount rate becomes small. The reasoning is exactly the same as in Friedman. As the discount rate goes to zero, the value of a firm's discounted profit approaches infinity, while the profitability of a single deviation remains constant. On some sample path of the equilibrium process a firm might not be able to finance the transfer from its own

[5] For $N \leqq 3$, σ is exactly a non-cooperative equilibrium. For $N > 3$, there is a slight difficulty because the firms must decide how to allocate the transfers among themselves. We could make this decision part of the equilibrium strategy, but it is easy to see that it would lead to an infinite number of (equivalent) strategies. (The reader can verify that, if in some state there are more than two firms with a positive and more than two firms with a negative transfer, there is an infinite number of combinations of transfers between the firms which leaves the total transfer of each firm constant). We assume that if $N \geqq 4$, the firms convene every period to harmonize their transfers. σ still remains a non-cooperative equilibrium, since every firm could in any single period reneg on the agreement.

cash flow. However, if the discount rate is low enough to make σ a Nash-equilibrium, we can conclude that the present value of the firm after transfer must be larger than zero. Therefore the assumption of a perfectly efficient capital market implies that in such a situation a firm will be able to borrow against future profit.

6. Conclusion

It has been shown, that under reasonable restrictions and for discount rates sufficiently close to 1 the combination of a heterogeneous oligopoly, a perfectly efficient capital market, an infinitely repeated game and the possibility of side payments guarantees that a cartel can sustain monopolistic output in every period as a non-cooperative equilibrium.

The result does not extend to homogeneous oligopoly. It is however not difficult to see that the methodology we have presented may be applied to the latter case to achieve profit levels corresponding to the optimal equilibria of Abreu et al. As we have argued, in this case the sum of transfers would have to be negative, i.e. dissipative, to achieve conformity with the desired output levels. It is precisely the dissipative nature of the transfers which prevents the cooperative profit level from being achieved in this case. The preceding observation suggests an explanation for the prevalence of apparently unprofitable donations to charities and public works. Such donations, viewed as dissipative transfers, may actually serve to police implicit collusive behaviour.

Appendix

A Volterra equation of the first kind with multiple integral is defined as follows:

$$\forall(\psi_1,..,\psi_N)\in[0,\Psi_1]..[0,\Psi_N]$$

$$\lambda(\psi_1,..,\psi_N)=\int_0^{\Psi_1}\cdot\int_0^{\Psi_N}v(x_1,.,x_N)k(x_1,.,x_N,\psi_1,.,\psi_N)dx_1\cdot dx_N$$

where $v(\cdot)$ is the unknown function.

Using the result from Volterra [1896] (alternatively, see Volterra and Peres [1936]) the above system of equations is solvable if the following conditions are satisfied:

(R.1) $\lambda(\cdot)$ and $k(\cdot)$ are n times differentiable in the variables ψ.

(R.2) $\lambda(\psi_1,..,\psi_N)=0$ if $\psi_i=0$, $i=1,..n$.

(R.3) $\forall k=1,..,n-1$

$$\frac{\partial^k\lambda}{\partial\psi_{i_1}..\partial\psi_{i_k}}(\psi_1,.,\psi_N)=0\ \forall\ \psi_j=0,\ j=1,.,N\ J\neq i_1,.,j\neq i_k$$

(R.4) $k(\psi_1,.,\psi_N,\psi_1,.,\psi_N)\neq 0$ for all possible $(\psi_1,.,\psi_N)$.

(R.5) $k(x_1,.,x_N,\psi_1,.,\psi_N)/k(\psi_1,.,\psi_N,\psi_1,.,\psi_N)$ is bounded.

The conditions R.2 and R.3 are just consistence requirements. Indeed, under R.2 and R.3 the integral on the right hand side will all be zero.

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