Outage Probability and Average Error Performance of Modulation Schemes under η-μ and κ-μ Fading Channels in Terms of Elementary Functions

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***Abstract*—This work presents exact and approximated expressions for the outage probability and average error performance of modulation schemes under η-μ and κ-μ fading channels. Under physical fading models the analytical results can be expressed in terms of elementary functions of the relevant parameters, which not only leads to more tractable expressions but also more efficient numerical evaluations. Simple asymptotic approximations are derived as well. While exact results are given in infinite series forms, a convergence analysis demonstrates that truncated versions of the series with very few terms can achieve the desired level of accuracy. The comparison with simulation results demonstrates the correctness of the exact theoretic expressions as well as the accuracy of their approximated forms.**

***Keywords*—η-μ fading, κ-μ fading, outage probability, average error performance.**

I. Introduction

The η-μ and κ-μ distributions [1] have become popular models for small-scale fading under various non-line-of-sight and line-of-sight conditions, respectively. These models have been proven to fit experimental data more accurately than other fading models such as one-sided Gaussian, Rayleigh, Nakagami-m, Nakagami-q (Hoyt) and Nakagami-n (Rice) [2], which can be obtained as particular cases of these distributions. While some expressions are known in the literature for the outage probability and average error rate of modulation schemes under η-μ and κ-μ fading channels, the existing theoretical results are valid for particular classes of modulation formats and/or based on complicated functions that are difficult to manipulate in analytical studies and lead to inefficient numerical evaluations.

In [3], expressions are provided for the moment generating function (MGF) and average error rate of some binary coherent modulations, which are given in the form of infinite series of Meijer's G-functions [4, eq. (9.301)]. While a simpler form for the MGF based on elementary functions is provided in [5], it cannot be easily manipulated analytically owing to its algebraic form, thus requiring the numerical evaluation of the average error performance [5, eq. (10)]. Exact expressions are provided in [6] for the average of the Gaussian Q-function and the average of the product of two Gaussian Q-functions over η-μ and κ-μ distributions, which are frequently found in the evaluation of the average error performance of modulation schemes under fading channels. The results are given in terms of Appell's hypergeometric [6, eq. (12)] and confluent hypergeometric [6, eq. (14)] functions of two variables and their Lauricella's counterparts of three variables [6, eqs. (19) and (21)]. A general expression for the outage probability under η-μ fading channels is provided in [7] in terms of a Lauricella's confluent hypergeometric function of two variables [4, eq. (9.261.2)], which under physical fading models can be expressed as a series of first-order Marcum Q-functions [2, eq. (4.33)], modified Bessel functions of the first kind [4, eq. (8.431)] and Jacobi polynomials [4, eq. (8.960.1)]. Some other related studies have evaluated analytically the outage probability under η-μ and/or κ-μ fading channels considering more specific scenarios with diversity combining techniques, background noise and interference with different combination of fading models in the interfered and interfering links [8–11]. However, the results given are typically expressed in terms of equally complex functions.

As it can be observed from the discussion above, the theoretical results known in the literature are based on complicated functions that are difficult to manipulate in analytical studies and usually lead to inefficient numerical evaluations.

In this context, this work derives new expressions that can readily be employed in the evaluation of the outage probability and error performance of a broad range of modulation formats under η-μ and κ-μ fading channels. Moreover, it is shown that under physical models the exact results can be expressed in terms of elementary functions, which are well suited for both analytical manipulations and efficient numerical evaluations.

The rest of the paper is organised as follows. First, Section II provides novel analytical results for the outage probability under η-μ and κ-μ fading channels. The counterpart results for the average probability of error are provided in Section III. The validity of the exact analytical results along with the accuracy of the proposed approximated forms is demonstrated with some numerical results in Section IV. Finally, Section V summarises and concludes the paper.

II. Outage Probability

The probability of outage, P_{out}, defined as the probability that the instantaneous signal-to-noise ratio (SNR) per symbol, γ, falls below a specified SNR threshold, γ_{th}, is here obtained by evaluating:

$$P_{\text{out}} = P(\gamma \le \gamma_{th}) = \int_0^{\gamma_{th}} f_\gamma(\gamma) d\gamma \tag{1}$$

where $f_\gamma(\gamma)$ is the probability density function (PDF) of the instantaneous SNR per symbol. Analytically tractable expressions in terms of elementary (power and exponential) functions are derived in this section for η-μ and κ-μ fading channels.

A. Outage Probability under η-μ Fading

Under η-μ fading the instantaneous SNR per symbol, γ, is distributed according to [1, eq. (26)]:

$$\begin{aligned} f_\gamma^{\eta\mu}(\gamma) = & \frac{2\sqrt{\pi}\mu^{\mu+\frac{1}{2}} h^\mu}{\Gamma(\mu) H^{\mu-\frac{1}{2}}} \frac{\gamma^{\mu-\frac{1}{2}}}{\overline{\gamma}^{\mu+\frac{1}{2}}} \\ & \times \exp\left(-2\mu h \frac{\gamma}{\overline{\gamma}}\right) I_{\mu-\frac{1}{2}}\left(2\mu H \frac{\gamma}{\overline{\gamma}}\right) \end{aligned} \tag{2}$$

where $\overline{\gamma}$ is the average SNR per symbol, η and μ are the fading parameters, h and H are functions of η (see [1] for details), $\Gamma(\cdot)$ is the gamma function [4, eq. (8.310.1)], and $I_\nu(\cdot)$ is the νth-order modified Bessel function of the first kind [4, eq. (8.431)]. Since μ represents half the number of multipath clusters, 2μ takes integer values in physical fading models. Taking this into account, introducing [4, eq. (8.445)] into (2) and integrating between zero and γ_{th} with the aid of [4, eq. (3.351.1)] leads to the following results:

$$P_{\text{out}}^{\eta\mu} = \sqrt{\pi} \sum_{k=0}^{\infty} \frac{H^{2k}}{k!\, 2^{2\mu+2k-1} h^{\mu+2k}} \frac{\gamma\left(2\mu+2k, 2\mu h \frac{\gamma_{th}}{\overline{\gamma}}\right)}{\Gamma(\mu)\Gamma(\mu+k+\frac{1}{2})} \tag{3}$$

$$\begin{aligned} = & \sqrt{\pi} \sum_{k=0}^{\infty} \frac{H^{2k}}{k!\, 2^{2\mu+2k-1} h^{\mu+2k}} \frac{\Gamma(2\mu+2k)}{\Gamma(\mu)\Gamma(\mu+k+\frac{1}{2})} \\ & \times \left[1 - e^{-2\mu h \frac{\gamma_{th}}{\overline{\gamma}}} \sum_{j=0}^{2\mu+2k-1} \frac{1}{j!} \left(2\mu h \frac{\gamma_{th}}{\overline{\gamma}}\right)^j\right] \end{aligned} \tag{4}$$

The more compact expression of (3), which depends on the lower incomplete gamma function $\gamma(\cdot,\cdot)$ [4, eq. (8.350.1)], is more suitable for numerical evaluations while (4), which only depends on elementary (power and exponential) terms of $\gamma_{th}/\overline{\gamma}$, provides a more tractable expression suitable for analytical manipulations. Interestingly, the result in (3) is valid for any arbitrary value of μ (including non-integer values of 2μ). Notice that both results also represent the SNR CDF.

As mentioned in Section I, the result given in [7] for the outage probability under η-μ physical fading models is a series of first-order Marcum Q-functions [2, eq. (4.33)], modified Bessel functions of the first kind [4, eq. (8.431)] and Jacobi polynomials [4, eq. (8.960.1)]. The expressions in (3)-(4) also provide exact results with much simpler forms.

B. Outage Probability under κ-μ Fading

Under κ-μ fading the instantaneous SNR per symbol, γ, is distributed according to [1, eq. (10)]:

$$\begin{aligned} f_\gamma^{\kappa\mu}(\gamma) = & \frac{\mu(1+\kappa)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}} \exp(\mu\kappa)} \frac{\gamma^{\frac{\mu-1}{2}}}{\overline{\gamma}^{\frac{\mu+1}{2}}} \\ & \times \exp\left(-\mu(1+\kappa)\frac{\gamma}{\overline{\gamma}}\right) I_{\mu-1}\left(2\mu\sqrt{\kappa(1+\kappa)\frac{\gamma}{\overline{\gamma}}}\right) \end{aligned} \tag{5}$$

where κ and μ are the fading parameters (see [1] for details). Taking into account that μ represents the number of multipath clusters (i.e., it takes integer values in physical fading models) and following the same procedure as in Section II-A, the following results for the outage probability are obtained:

$$P_{\text{out}}^{\kappa\mu} = \sum_{k=0}^{\infty} \frac{(\mu\kappa)^k e^{-\mu\kappa}}{k!} \frac{\gamma\left(\mu+k, \mu(1+\kappa)\frac{\gamma_{th}}{\overline{\gamma}}\right)}{\Gamma(\mu+k)} \tag{6}$$

$$\begin{aligned} = & \sum_{k=0}^{\infty} \frac{(\mu\kappa)^k e^{-\mu\kappa}}{k!} \\ & \times \left[1 - e^{-\mu(1+\kappa)\frac{\gamma_{th}}{\overline{\gamma}}} \sum_{j=0}^{\mu+k-1} \frac{1}{j!}\left(\mu(1+\kappa)\frac{\gamma_{th}}{\overline{\gamma}}\right)^j\right] \end{aligned} \tag{7}$$

The comments made for (3)-(4) are also applicable to (6)-(7). To the best of the author's knowledge, no comparable results are known in the literature.

III. Average Error Probability

The conditional bit and symbol error probability of modulation schemes frequently involves terms of the form $\mathcal{Q}(\alpha\sqrt{\gamma})$, where:

$$\mathcal{Q}(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt \tag{8}$$

is the Gaussian Q-function and α is a constant that depends on the particular combination of modulation format and detection method [2]. As a result, obtaining the average error probability under fading requires the evaluation of integrals of the form [2, eq. (5.1)]:

$$\mathcal{I} = \int_0^\infty \mathcal{Q}(\alpha\sqrt{\gamma}) f_\gamma(\gamma) d\gamma = \int_0^\infty \text{erfc}\left(\frac{\alpha z}{\sqrt{2}}\right) f_\gamma\left(z^2\right) z dz \tag{9}$$

where the right-hand side of (9) is obtained for convenience by applying the change of variable $\gamma = z^2$ and employing the relation:

$$\mathcal{Q}(x) = \frac{1}{2}\text{erfc}\left(\frac{x}{\sqrt{2}}\right) \tag{10}$$

where

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt \qquad (11)$$

is the complementary Gaussian error function. Exact solutions to (9) and asymptotic approximations in terms of elementary functions are derived in the following sections.

A. Average Error under η-μ Fading

Introducing [4, eq. (8.445)] and (2) into (9), the resulting integral can be solved with the aid of [12, eq. (4.3.9)] to yield:

$$\mathcal{I}^{\eta\mu} = \sum_{k=0}^{\infty} \frac{h^\mu H^{2k}}{k!(2\mu+2k)} \frac{\Gamma(2\mu+2k+\frac{1}{2})}{\Gamma(\mu)\Gamma(\mu+k+\frac{1}{2})} \left(\frac{2\mu}{\alpha^2\overline{\gamma}}\right)^{2\mu+2k} \times {}_2F_1\left(2\mu+2k, 2\mu+2k+\tfrac{1}{2}; 2\mu+2k+1; -\tfrac{4\mu h}{\alpha^2\overline{\gamma}}\right) \qquad (12)$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gauss hypergeometric function [4, eqs. (9.11)]. Taking into account that 2μ takes integer values in physical fading models, a more tractable expression in terms of elementary (power) functions of $\overline{\gamma}$ can be obtained by introducing (21) from the Appendix into (12), which yields:

$$\begin{aligned}\mathcal{I}^{\eta\mu} = \sum_{k=0}^{\infty} \frac{h^\mu H^{2k}}{k!} \frac{\Gamma(2\mu+2k+\frac{1}{2})}{\Gamma(\mu)\Gamma(\mu+k+\frac{1}{2})} \Bigg[\frac{(2\mu+2k-1)!}{(2h)^{2\mu+2k}\left(\frac{1}{2}\right)_{2\mu+2k}} \\ - 2\sqrt{1+\frac{4\mu h}{\alpha^2\overline{\gamma}}} \left(\frac{2\mu}{4\mu h+\alpha^2\overline{\gamma}}\right)^{2\mu+2k} \\ \times \sum_{j=0}^{2\mu+2k-1} \frac{(2\mu+2k-j)_j}{\left(\frac{3}{2}\right)_j} \left(\frac{\alpha^2\overline{\gamma}}{4\mu h}\right)^{j+1} \Bigg]\end{aligned} \qquad (13)$$

where $(x)_n = \Gamma(x+n)/\Gamma(x)$ is the Pochhammer symbol. Notice that (13) is given in terms of elementary functions of $\overline{\gamma}$ and therefore not only provides a more tractable algebraic form but also a significantly more efficient numerical evaluation than the known results in terms of hypergeometric functions.

A simple asymptotic approximation under high SNR conditions ($\overline{\gamma} \to \infty$) can be obtained by taking the first term (i.e., $k = 0$) of the sum in (12) and noting that ${}_2F_1(a, b; c; 0) = 1$:

$$\mathcal{I}^{\eta\mu} \approx \frac{h^\mu}{2\mu} \frac{\Gamma(2\mu+\frac{1}{2})}{\Gamma(\mu)\Gamma(\mu+\frac{1}{2})} \left(\frac{2\mu}{\alpha^2\overline{\gamma}}\right)^{2\mu} \qquad (14)$$

which provides a tractable form and a simple relation between the involved parameters and the resulting error performance.

B. Average Error under κ-μ Fading

Introducing [4, eq. (8.445)] and (5) into (9), the resulting integral can be solved with the aid of [12, eq. (4.3.9)] to yield:

$$\begin{aligned}\mathcal{I}^{\kappa\mu} = \frac{1}{2\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{\mu^k \kappa^{-\mu} e^{-\mu\kappa}}{k!\,(\mu+k)} \frac{\Gamma(\mu+k+\frac{1}{2})}{\Gamma(\mu+k)} \\ \times \left(\frac{2\mu\kappa(1+\kappa)}{\alpha^2\overline{\gamma}}\right)^{\mu+k} \\ \times\ {}_2F_1\left(\mu+k, \mu+k+\frac{1}{2}; \mu+k+1; -\tfrac{2\mu(1+\kappa)}{\alpha^2\overline{\gamma}}\right)\end{aligned} \qquad (15)$$

A more convenient expression in terms of elementary functions of $\overline{\gamma}$ can be obtained based on (21) as follows:

$$\begin{aligned}\mathcal{I}^{\kappa\mu} = \frac{1}{2\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{\mu^k \kappa^{-\mu} e^{-\mu\kappa}}{k!} \frac{\Gamma(\mu+k+\frac{1}{2})}{\Gamma(\mu+k)} \\ \times \Bigg[\frac{(\mu+k-1)!\,\kappa^{\mu+k}}{\left(\frac{1}{2}\right)_{\mu+k}} \\ - 2\sqrt{1+\frac{2\mu(1+\kappa)}{\alpha^2\overline{\gamma}}} \left(\frac{2\mu\kappa(1+\kappa)}{2\mu(1+\kappa)+\alpha^2\overline{\gamma}}\right)^{\mu+k} \\ \times \sum_{j=0}^{\mu+k-1} \frac{(\mu+k-j)_j}{\left(\frac{3}{2}\right)_j} \left(\frac{\alpha^2\overline{\gamma}}{2\mu(1+\kappa)}\right)^{j+1} \Bigg]\end{aligned} \qquad (16)$$

The asymptotic approximation for high SNR is given by:

$$\mathcal{I}^{\kappa\mu} \approx \frac{\kappa^{-\mu} e^{-\mu\kappa}}{2\sqrt{\pi}\mu} \frac{\Gamma(\mu+\frac{1}{2})}{\Gamma(\mu)} \left(\frac{2\mu\kappa(1+\kappa)}{\alpha^2\overline{\gamma}}\right)^{\mu} \qquad (17)$$

which is a simple power function of $\overline{\gamma}$.

IV. Numerical Results

The theoretical expressions derived analytically in this paper were evaluated numerically and compared (for validation purposes) with results obtained from Monte Carlo simulations. To obtain the simulation results, a sufficiently large number of random SNR values were generated according to the SNR distribution of the considered fading channel. Then, for each SNR value: 1) an outage decision was made by comparing the SNR value with the SNR threshold γ_{th}, and 2) an error decision was made by comparing a random number uniformly distributed in the interval $[0, 1]$ with the instantaneous probability of error given by $\mathcal{Q}(\alpha\sqrt{\gamma})$. The resulting vectors of outage and error binary decisions were used to compute the average outage and error probabilities, respectively.

Figs. 1 and 2 compare the exact expressions for the outage [(4) and (7)] and average error [(13) and (16)] probabilities (considering a sufficiently large number of terms in the truncated series) with results obtained from Monte Carlo simulations. As it can be appreciated, there are no differences between analytical and simulation results, which corroborates the correctness of the obtained theoretical expressions. Note the higher error rates in Fig. 2 for lower values of α, which are due to the inversely proportional relation between α and the modulation order of M-ary modulations (i.e., higher-order modulations, which are less robust against radio propagation errors, are characterised by lower values of α [2]). This results in a certain SNR penalty (e.g., ≈13 dB in the case of Fig. 2).

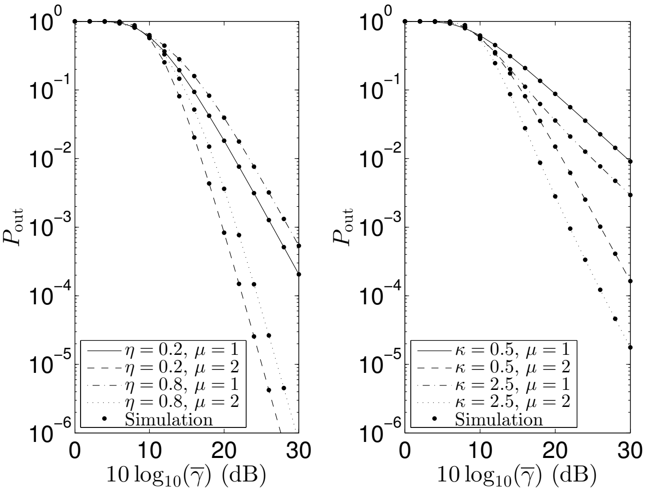

Fig. 1. Outage probability as a function of the average SNR ($\gamma_{th} \sim 10$ dB).

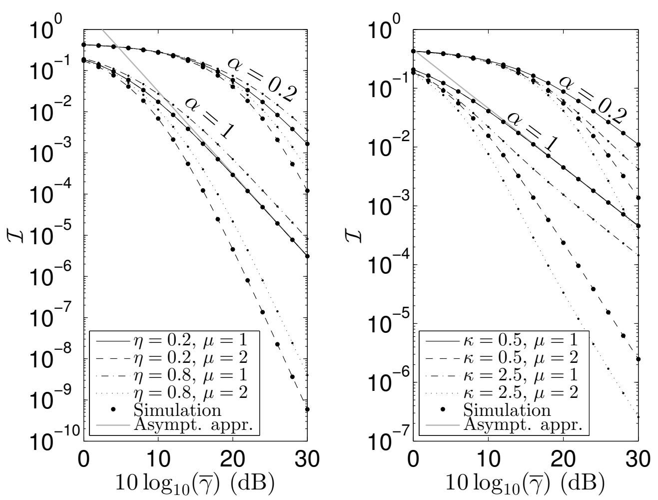

Fig. 2. Error performance integral $\mathcal{I}$ as a function of the average SNR.

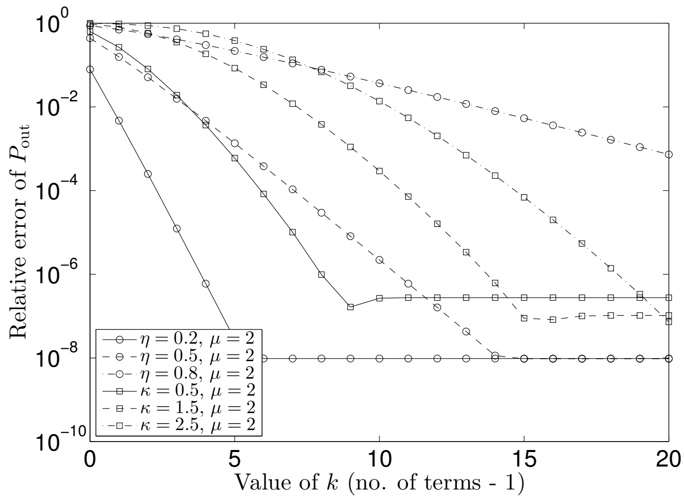

Fig. 3. Relative error of $P_{\rm out}$ as a function of k ($\gamma_{th} \sim 10$ dB).

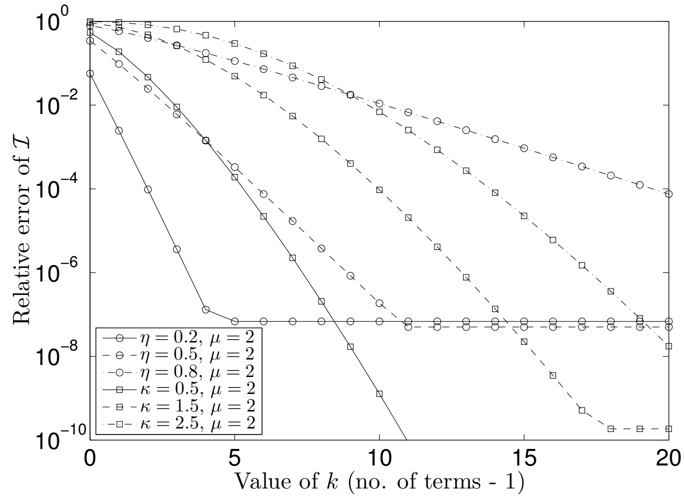

Fig. 4. Relative error of $\mathcal{I}$ as a function of k ($\alpha = 1$).

Fig. 2 also illustrates the asymptotic approximations in (14) and (17). Notice that only selected cases are shown for the sake of clarity (the same trend was observed in all cases). As it can be observed, these asymptotic approximations provide very accurate results in the region of high SNR (i.e., the SNR region of interest in the design of wireless communication systems). The level of accuracy provided by these approximations along with their analytical tractability make of these expressions useful tools that can be employed to obtain simple yet accurate estimations of the performance of a broad range of modulation formats under practical operation conditions.

Figs. 3 and 4 illustrate the convergence rate of the exact theoretical expressions in terms of the maximum relative error as a function of the value of k (i.e., number of terms considered in the truncated versions of the infinite series minus one). As appreciated, the relative error quickly decreases as the number of terms employed in the truncated series increases, indicating that the evaluation of just a few terms is sufficient to provide accurate numerical results. It is interesting to note the existence of a lower bound in the relative error, which is due to the finite precision of the employed computer. It is also worth mentioning that the relative errors shown in Figs. 3 and 4 correspond to the maximum relative error observed over the whole range of SNR values. This maximum error usually occurs in the region of low SNR. At higher SNR values, which is the typical region of operation targeted by wireless communication systems, the actual relative error is lower than the values shown in Figs. 3 and 4. These relative errors represent the worst-case (i.e., low SNR) relative errors.

Tables I and II show the minimum number of terms required in the truncated series for the outage probability and the error performance integral $\mathcal{I}$, respectively, to guarantee a relative error no greater than 1%. As observed, only a few terms are sufficient to provide accurate (virtually exact) results.

Tables III and IV show the average time of computation required with the obtained analytical expressions to produce numerical results with a maximum relative error of 1% (based on the number of terms shown in Tables I and II). These results were obtained with an average general purpose computer (Intel Core i3-3220 CPU at 3.30 GHz with 8 GB of DDR3 SDRAM

TABLE I. NUMBER OF TERMS IN (4) & (7) FOR 1% MAXIMUM RELATIVE ERROR.

	η			κ		
	0.2	0.5	0.8	0.5	1.5	2.5
$\mu = 1$	2	4	11	4	6	8
$\mu = 2$	2	5	15	5	9	12
$\mu = 3$	2	6	18	6	11	16

TABLE II. NUMBER OF TERMS IN (13) & (16) FOR 1% MAXIMUM RELATIVE ERROR.

	η			κ		
	0.2	0.5	0.8	0.5	1.5	2.5
$\mu = 1$	2	3	9	3	6	7
$\mu = 2$	2	5	14	5	8	12
$\mu = 3$	2	5	17	6	11	15

TABLE III. AVERAGE COMPUTATION TIME (MS) REQUIRED TO EVALUATE (4) & (7) TO 1% MAXIMUM RELATIVE ERROR.

	η			κ		
	0.2	0.5	0.8	0.5	1.5	2.5
$\mu = 1$	4.2	6.9	17.0	4.8	6.8	8.8
$\mu = 2$	4.2	8.4	23.3	5.9	10.0	12.8
$\mu = 3$	4.2	9.9	27.5	7.0	11.9	16.9

TABLE IV. AVERAGE COMPUTATION TIME (MS) REQUIRED TO EVALUATE (13) & (16) TO 1% MAXIMUM RELATIVE ERROR.

	η			κ		
	0.2	0.5	0.8	0.5	1.5	2.5
$\mu = 1$	5.1	6.9	22.7	5.7	11.0	12.7
$\mu = 2$	5.5	12.8	43.9	9.6	15.8	25.3
$\mu = 3$	6.1	13.8	61.3	12.3	23.7	34.5

at 1600 MHz) by averaging the computation times observed in a set of 1000 evaluations of the corresponding analytical expressions. As it can be appreciated, the analytical results obtained in this work, which are based on elementary (power and exponential) functions, are computationally efficient and lead to an almost instantaneous evaluation in all cases. Note that the evaluation of other results in the existing literature, which are based on hypergeometric or other complex functions, would in general require significantly longer computation times (depending on the particular value of the parameters), which can be typically in the order of up to several seconds.

V. CONCLUSIONS

This work has introduced a set of novel exact and approximated expressions for the outage probability and average error performance of modulation schemes under η-μ and κ-μ fading channels, which have been given in terms of elementary functions. Owing to the algebraic simplicity of elementary functions, the presented expressions are well suited not only for analytical manipulations but also efficient numerical evaluations. The comparison with simulation results has corroborated the correctness of the obtained exact results as well as the accuracy of their approximated forms. The provided expressions can be used in a large range of scenarios, including cases not covered by previous known results.

APPENDIX
A GAUSS HYPERGEOMETRIC FUNCTION IN TERMS OF ELEMENTARY FUNCTIONS

This appendix expresses the Gauss hypergeometric function ${}_2F_1(n, n+\frac{1}{2}; n+1; x)$, where $n \in \mathbb{N}$, in terms of elementary functions. The Gauss hypergeometric function is commonly defined as an infinite series given by [4, eq. (9.14.1)]:

$$ {}_2F_1(a,b;c;x) = \sum_{j=0}^{\infty} \frac{(a)_j (b)_j}{(c)_j} \frac{x^j}{j!} \tag{18} $$

where $(x)_n = \Gamma(x+n)/\Gamma(x)$ is the Pochhammer symbol. The definition in (18) is valid for $|x| < 1$ (i.e., the convergence region of the series). Outside such interval, the function can be defined as the analytic continuation of the sum with respect to x, with the parameters a, b, c held fixed, which yields:

$$ \begin{aligned} {}_2F_1(a,b;c;x) = & \frac{\Gamma(a-b)\Gamma(c)(-x)^{-b}}{\Gamma(a)\Gamma(c-b)} \\ & \times \sum_{j=0}^{\infty} \frac{(b)_j (b-c+1)_j}{(-a+b+1)_j} \frac{x^{-j}}{j!} \\ & + \frac{\Gamma(b-a)\Gamma(c)(-x)^{-a}}{\Gamma(b)\Gamma(c-a)} \\ & \times \sum_{j=0}^{\infty} \frac{(a)_j (a-c+1)_j}{(a-b+1)_j} \frac{x^{-j}}{j!} \end{aligned} \tag{19} $$

Using [4, eq. (9.131.1.3)], the function of interest becomes:

$$ {}_2F_1(n, n+\tfrac{1}{2}; n+1; x) = (1-x)^{\frac{1}{2}-n} {}_2F_1(1, \tfrac{1}{2}; n+1; x) \tag{20} $$

Introducing (19) into (20) the following simpler expression is obtained after some algebraic manipulations:

$$ \begin{aligned} {}_2F_1(n, n+\tfrac{1}{2}; n+1; x) = & \frac{(-1)^n \, n! \, x^{-n}}{\left(\frac{1}{2}\right)_n} \\ & + 2n(1-x)^{\frac{1}{2}-n} \sum_{j=0}^{n-1} \frac{(1-n)_j}{\left(\frac{3}{2}\right)_j} x^{-j-1} \end{aligned} \tag{21} $$

Since $(1-n)_j$ is zero for $j > n-1$ the infinite sum can be truncated, thus leading to the convenient form in (21) which has a finite number of terms based on elementary functions.

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