Outage Probability and Average Error Performance of Modulation Schemes under Nakagami-q (Hoyt) and Nakagami-n (Rice) Fading Channels

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***Abstract*—This work provides exact and approximated (more tractable) expressions for the evaluation of the outage probability and average error performance of a wide range of modulation schemes under Nakagami-q (Hoyt) and Nakagami-n (Rice) fading channels. While the exact theoretical results (obtained by direct integration over the fading statistics) are given in infinite series form, a convergence analysis indicates that truncated versions of the series with very few terms are able to provide accurate estimates, thus leading to several analytically tractable yet accurate approximations. The comparison with simulation results corroborates the correctness of the exact theoretic expressions as well as the accuracy of their approximated forms.**

***Keywords*—Nakagami-q (Hoyt) fading, Nakagami-n (Rice) fading, outage probability, average error performance.**

I. Introduction

The outage probability and average bit/symbol error rate have extensively been used as performance metrics for digital communication systems. The development of accurate methods and formulas for the evaluation of these metrics over fading channels has been an area of long-time interest. While expressions are known for simpler fading models such as Rayleigh and Nakagami-m [1], the results known in the literature for the average error performance of modulation schemes under Nakagami-q (Hoyt) and Nakagami-n (Rice) fading channels are given in integral forms obtained by means of the moment generating function (MGF) method [1]. Although the average error performance under Nakagami-q (Hoyt) fading has been analysed individually for some M-ary modulation schemes in [2], general expressions for both fading environments are, to the best of the author's knowledge, not available in the existing literature. Moreover, the expressions known for the outage probability under both fading scenarios are based on functions defined as integrals that are difficult to manipulate in analytical studies. By direct integration over the probability density function (PDF) of the signal-to-noise ratio (SNR), this work derives general expressions that can readily be employed in the error performance evaluation of a wide range of modulation schemes under both Nakagami-q (Hoyt) and Nakagami-n (Rice) fading channels as well as new expressions for the outage probabilities based on power and exponential functions, which are well suited for analytical manipulations and efficient numerical evaluations. Analytical results are given in infinite series form, however a convergence analysis shows that truncated versions of the series with very few terms can attain the desired levels of accuracy. Approximations to the exact expressions are also provided in order to enable a simpler performance evaluation. In summary, compared to previous work, this paper provides new analytical results based on simpler expressions that are analytically tractable and computationally efficient.

The rest of the paper is organised as follows. First, Section II provides novel analytical results for the outage probability under Nakagami-q (Hoyt) and Nakagami-n (Rice) fading channels. The counterpart results for the average probability of error are provided in Section III. The validity of the exact analytical results along with the accuracy of the proposed approximated forms is demonstrated with some numerical results in Section IV. Finally, Section V summarises and concludes the paper.

II. Outage Probability

The probability of outage, $P_{\rm out}$, defined as the probability that the instantaneous SNR per symbol, γ, falls below a specified SNR threshold, γ_{th}, i.e., $P_{\rm out} = P(\gamma \leq \gamma_{th}) = F_\gamma(\gamma_{th})$, can be evaluated based on the cumulative distribution function (CDF) of the SNR, $F_\gamma(\gamma)$, which for the particular case of Nakagami-q (Hoyt) fading can be expressed in terms of the Rice Ie-function [3, eq. (3)] or the Marcum Q-function [4, eq. (8)] given their equivalence [5, eq. (1)]. These functions are defined as integrals and can also be expressed as infinite series of power, exponential and modified Bessel functions of the first kind, which is not ideal for analytical manipulations. Analytically tractable expressions in terms of power and exponential functions only are derived in this section for Nakagami-q (Hoyt) and Nakagami-n (Rice) fading channels.

A. Outage Probability under Nakagami-q (Hoyt) Fading

Under Nakagami-q (Hoyt) fading the instantaneous SNR per symbol is distributed according to [1, eq. (2.11)]:

$$f_\gamma^{\rm H}(\gamma) = \frac{1+q^2}{2q\overline{\gamma}} \exp\left(-\frac{(1+q^2)^2}{4q^2}\frac{\gamma}{\overline{\gamma}}\right) I_0\left(\frac{1-q^4}{4q^2}\frac{\gamma}{\overline{\gamma}}\right) \quad (1)$$

where $\overline{\gamma}$ is the average SNR, $q \in [0, 1]$ is the Nakagami-q fading parameter and $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind [6, eq. (8.431)]. Introducing [6, eq.

(8.445)] into (1) and integrating between zero and γ_{th} with the aid of [6, eq. (3.351.1)] yields:

$$P_{\text{out}}^{\text{H}} = \frac{2q}{1+q^2}\sum_{k=0}^{\infty}\frac{(2k)!}{(k!)^2}\left(\frac{1}{2}\frac{1-q^4}{(1+q^2)^2}\right)^{2k} \times\left[1-e^{-\frac{(1+q^2)^2}{4q^2}\frac{\gamma_{th}}{\overline{\gamma}}}\sum_{j=0}^{2k}\frac{1}{j!}\left(\frac{(1+q^2)^2}{4q^2}\frac{\gamma_{th}}{\overline{\gamma}}\right)^j\right] \quad (2)$$

which only depends on power and exponential terms of $\gamma_{th}/\overline{\gamma}$. Notice that (2) also represents the SNR CDF.

The truncation of the infinite series in (2) to the first term (i.e., $k=0$) yields:

$$P_{\text{out}}^{\text{H}} \approx \frac{2q}{1+q^2}\left[1-e^{-\frac{(1+q^2)^2}{4q^2}\frac{\gamma_{th}}{\overline{\gamma}}}\right] \quad (3)$$

An asymptotic approximation for high SNR can be obtained by noting that:

$$f_{\gamma}^{\text{H}}(\gamma) \approx \frac{1+q^2}{2q\overline{\gamma}} \quad (4)$$

when $\overline{\gamma}\to\infty$, which integrated between zero and γ_{th} yields:

$$P_{\text{out}}^{\text{H}} \approx \frac{1+q^2}{2q}\frac{\gamma_{th}}{\overline{\gamma}} \quad (5)$$

These results not only provide simple and tractable expressions but also elegant and straightforward relations between the involved parameters and the resulting outage probability.

B. Outage Probability under Nakagami-n (Rice) Fading

Under Nakagami-n (Rice) fading the instantaneous SNR per symbol is distributed according to [1, eq. (2.16)]:

$$f_{\gamma}^{\text{R}}(\gamma) = \frac{(1+n^2)e^{-n^2}}{\overline{\gamma}}e^{-(1+n^2)\frac{\gamma}{\overline{\gamma}}}I_0\left(2n\sqrt{(1+n^2)\frac{\gamma}{\overline{\gamma}}}\right) \quad (6)$$

where $n\geq 0$ is the Nakagami-n fading parameter. Introducing [6, eq. (8.445)] into (6) and integrating between zero and γ_{th} with the aid of [6, eq. (3.351.1)] yields the following result:

$$P_{\text{out}}^{\text{R}} = e^{-n^2}\sum_{k=0}^{\infty}\frac{n^{2k}}{k!} \times\left[1-e^{-(1+n^2)\frac{\gamma_{th}}{\overline{\gamma}}}\sum_{j=0}^{k}\frac{1}{j!}\left((1+n^2)\frac{\gamma_{th}}{\overline{\gamma}}\right)^j\right] \quad (7)$$

which only depends on power and exponential terms of $\gamma_{th}/\overline{\gamma}$. Notice that (7) also represents the SNR CDF.

Following the same procedures employed in Section II-A, the following one-term approximation is obtained:

$$P_{\text{out}}^{\text{R}} \approx e^{-n^2}\left[1-e^{-(1+n^2)\frac{\gamma_{th}}{\overline{\gamma}}}\right] \quad (8)$$

along with the following high SNR approximation:

$$P_{\text{out}}^{\text{R}} \approx (1+n^2)e^{-n^2}\frac{\gamma_{th}}{\overline{\gamma}} \quad (9)$$

which not only provide analytically tractable algebraic forms but also enable an efficient numerical evaluation.

III. Average Error Probability

The conditional bit and symbol error probabilities of a large number of modulation schemes is given by expressions of the form $\mathcal{Q}(\alpha\sqrt{\gamma})$, where:

$$\mathcal{Q}(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty}e^{-t^2/2}dt \quad (10)$$

is the Gaussian Q-function and α is a constant that depends on the specific combination of modulation and detection [1]. The corresponding average error probability under fading therefore involves the evaluation of an integral of the form [1, eq. (5.1)]:

$$\mathcal{I} = \int_0^{\infty}\mathcal{Q}(\alpha\sqrt{\gamma})f_{\gamma}(\gamma)d\gamma = \int_0^{\infty}\text{erfc}\left(\frac{\alpha z}{\sqrt{2}}\right)f_{\gamma}\left(z^2\right)zdz \quad (11)$$

where the right-hand side of (11) is obtained for convenience by applying the change of variable $\gamma = z^2$ and employing the relation:

$$\mathcal{Q}(x) = \frac{1}{2}\text{erfc}\left(\frac{x}{\sqrt{2}}\right) \quad (12)$$

where

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty}e^{-t^2}dt \quad (13)$$

is the complementary Gaussian error function.

For some modulation schemes the conditional error probability is given by integer powers of the Gaussian Q-function and the average performance under fading channels is obtained by evaluating integrals of the form [1, eq. (5.88)]:

$$\mathcal{I}_m = \int_0^{\infty}\mathcal{Q}^m(\alpha\sqrt{\gamma})f_{\gamma}(\gamma)d\gamma = \int_0^{\infty}\mathcal{Q}^m(\alpha z)f_{\gamma}(z^2)\,2z\,dz \quad (14)$$

This section derives exact solutions to (11) and approximated closed-form solutions to (14) under Nakagami-q (Hoyt) and Nakagami-n (Rice) fading environments.

A. Average Error Under Nakagami-q (Hoyt) Fading

Introducing [6, eq. (8.445)] and (1) into (11), the resulting integral can be solved with the aid of [7, eq. (4.3.9)] to yield:

$$\mathcal{I}^{\mathrm{H}} = \frac{1+q^2}{4\sqrt{\pi}q\overline{\gamma}} \sum_{k=0}^{\infty} \frac{\Gamma\left(2k+\frac{3}{2}\right)}{(k!)^2(2k+1)\left(\frac{\alpha^2}{2}\right)^{2k+1}} \left(\frac{1-q^4}{8q^2\overline{\gamma}}\right)^{2k} \times {}_2F_1\left(2k+1, 2k+\frac{3}{2}; 2k+2; -\frac{(1+q^2)^2}{2q^2\alpha^2\overline{\gamma}}\right) \quad (15)$$

where $\Gamma(\cdot)$ denotes the gamma function [6, eq. (8.310.1)] and ${}_2F_1(\cdot)$ is the Gauss hypergeometric function [6, eq. (9.100)]. By taking the first term (i.e., $k=0$) of the sum in (15), and noting that:

$${}_2F_1\left(1, \frac{3}{2}; 2; x\right) = \frac{2\left(1-\sqrt{1-x}\right)}{x\sqrt{1-x}} \quad (16)$$

and

$$\Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2} \quad (17)$$

the following approximation is obtained:

$$\mathcal{I}^{\mathrm{H}} \approx \frac{q}{1+q^2}\left(1-\sqrt{\frac{2q^2\alpha^2\overline{\gamma}}{(1+q^2)^2+2q^2\alpha^2\overline{\gamma}}}\right) \quad (18)$$

Under high SNR conditions ($\overline{\gamma} \to \infty$), the expression in (15) can be approximated asymptotically by:

$$\mathcal{I}^{\mathrm{H}} \approx \frac{1+q^2}{4q\alpha^2\overline{\gamma}} \quad (19)$$

which provide tractable forms and simple relations between the involved parameters and the resulting error performance.

Introducing [6, eq. (8.445)] and (1) into (14) along with the approximation $\mathcal{Q}(x) \approx e^{-ax^2-bx-c}$, where (a,b,c) are fitting coefficients (see [8] and Table I therein), yields an integral of the form in [6, eq. (3.462.1)] [7, eq. (A6)], which leads to the following approximated result:

$$\mathcal{I}_m^{\mathrm{H}} \approx \frac{1+q^2}{\Upsilon q\overline{\gamma}} e^{\frac{(bm\alpha)^2}{8\Upsilon}-cm} \times \sum_{k=0}^{\infty} \frac{(4k+1)!}{2^{2k+1}(k!)^2}\left(\frac{1-q^4}{8\Upsilon q^2\overline{\gamma}}\right)^{2k} D_{-4k-2}\left(\frac{bm\alpha}{\sqrt{2\Upsilon}}\right) \quad (20)$$

where:

$$\Upsilon = am\alpha^2 + \frac{(1+q^2)^2}{4q^2\overline{\gamma}} \quad (21)$$

and $D_\nu(\cdot)$ is the parabolic cylinder function [6, eq. (9.240)]. Truncating the infinite series to its first term (i.e., $k=0$), and noting that:

$$D_{-2}(x) = e^{-x^2/4} - \sqrt{2\pi}xe^{x^2/4}\mathcal{Q}(x) \quad (22)$$

yields the simpler form:

$$\mathcal{I}_m^{\mathrm{H}} \approx \frac{1+q^2}{2\Upsilon q\overline{\gamma}} e^{-cm}\left[1-bm\alpha\sqrt{\frac{\pi}{\Upsilon}}e^{\frac{(bm\alpha)^2}{4\Upsilon}}\mathcal{Q}\left(\frac{bm\alpha}{\sqrt{2\Upsilon}}\right)\right] \quad (23)$$

which under high SNR conditions ($\overline{\gamma} \to \infty$) simplifies to:

$$\mathcal{I}_m^{\mathrm{H}} \approx \frac{(1+q^2)e^{-cm}}{2amq\alpha^2\overline{\gamma}}\left[1-b\sqrt{\frac{\pi m}{a}}e^{\frac{b^2m}{4a}}\mathcal{Q}\left(b\sqrt{\frac{m}{2a}}\right)\right] \quad (24)$$

An exact solution to (14) under Nakagami-q (Hoyt) fading is provided in [9, eq. (14)] for the particular case $m=2$. While the results in (20)-(24) are approximated, they present more tractable forms and are valid for arbitrary m, which allows their use in a wider range of scenarios (e.g., [1, eq. (8.39)]).

B. Average Error Under Nakagami-n (Rice) Fading

Introducing [6, eq. (8.445)] and (6) into (11), the resulting integral can be solved with the aid of [7, eq. (4.3.9)] to yield:

$$\mathcal{I}^{\mathrm{R}} = \frac{(1+n^2)e^{-n^2}}{2\sqrt{\pi}\overline{\gamma}} \sum_{k=0}^{\infty} \frac{\Gamma\left(k+\frac{3}{2}\right)}{(k!)^2(k+1)\left(\frac{\alpha^2}{2}\right)^{k+1}} \left(\frac{n^2(1+n^2)}{\overline{\gamma}}\right)^{k} \times {}_2F_1\left(k+1, k+\frac{3}{2}; k+2; -\frac{2(1+n^2)}{\alpha^2\overline{\gamma}}\right) \quad (25)$$

The counterpart of (18) for (25) is given by:

$$\mathcal{I}^{\mathrm{R}} \approx \frac{e^{-n^2}}{2}\left(1-\sqrt{\frac{\alpha^2\overline{\gamma}}{2(1+n^2)+\alpha^2\overline{\gamma}}}\right) \quad (26)$$

Under high SNR conditions ($\overline{\gamma} \to \infty$), (25) can also be approximated asymptotically by the more tractable form:

$$\mathcal{I}^{\mathrm{R}} \approx \frac{(1+n^2)e^{-n^2}}{2\alpha^2\overline{\gamma}} \quad (27)$$

Following the same procedures as in Section III-A, the following approximated solution to (14) is obtained:

$$\mathcal{I}_m^{\mathrm{R}} \approx \frac{1+n^2}{\Omega\overline{\gamma}} e^{\frac{(bm\alpha)^2}{8\Omega}-cm-n^2} \times \sum_{k=0}^{\infty} \frac{n^{2k}(2k+1)!}{2^k(k!)^2}\left(\frac{1+n^2}{\Omega\overline{\gamma}}\right)^{k} D_{-2k-2}\left(\frac{bm\alpha}{\sqrt{2\Omega}}\right) \quad (28)$$

where:

$$\Omega = am\alpha^2 + \frac{1+n^2}{\overline{\gamma}} \tag{29}$$

Truncating the infinite series to its first term (i.e., $k = 0$) yields the simpler form:

$$\mathcal{I}_m^{\mathrm{R}} \approx \frac{1+n^2}{\Omega\overline{\gamma}} e^{-cm-n^2} \left[1 - bm\alpha\sqrt{\frac{\pi}{\Omega}} e^{\frac{(bm\alpha)^2}{4\Omega}} \mathcal{Q}\left(\frac{bm\alpha}{\sqrt{2\Omega}}\right)\right] \tag{30}$$

which under high SNR conditions ($\overline{\gamma} \to \infty$) simplifies to:

$$\mathcal{I}_m^{\mathrm{R}} \approx \frac{1+n^2}{am\alpha^2\overline{\gamma}} e^{-cm-n^2} \left[1 - b\sqrt{\frac{\pi m}{a}} e^{\frac{b^2 m}{4a}} \mathcal{Q}\left(b\sqrt{\frac{m}{2a}}\right)\right] \tag{31}$$

Notice that the approximated expressions are well suited for analytical manipulations owing to their algebraic simplicity. The correctness of the exact results along with the accuracy of their approximated forms are assessed in the next section.

IV. Numerical Results

The theoretical expressions derived analytically in this paper were evaluated numerically and compared (for validation purposes) with results obtained from Monte Carlo simulations. To obtain the simulation results, a sufficiently large number of random SNR values were generated according to the SNR distribution of the considered fading channel. Then, for each SNR value: 1) an outage decision was made by comparing the SNR value with the SNR threshold γ_{th}, and 2) an error decision was made by comparing a random number uniformly distributed in the interval $[0, 1]$ with the instantaneous probability of error given by $\mathcal{Q}^m(\alpha\sqrt{\gamma})$. The resulting vectors of outage and error binary decisions were used to compute the average outage and error probabilities, respectively.

Figs. 1 and 2 compare the exact expressions for the outage [(2) and (7)] and average error [(15) and (25)] probabilities (considering a sufficiently large number of terms in the truncated series) with numerical results obtained from Monte Carlo simulations. As it can be appreciated, there are no differences between theoretical and simulated curves, which corroborates the correctness of the obtained theoretical results. It is worth mentioning that for M-ary modulations the α parameter is inversely proportional to the modulation order M, which explains the higher average error rate observed in Fig. 2 for lower values of α (i.e., higher modulation orders). This results in a certain SNR penalty (e.g., ≈20 dB in the particular numerical example shown in Fig. 2).

Figs. 3 and 4 illustrate the convergence rate of the same exact theoretical expressions in terms of the maximum relative error as a function of the value of k (i.e., number of terms considered in the truncated versions of the infinite series minus one). The results are shown for the outage probability P_{out} (Fig. 3) and the error performance integral $\mathcal{I}$ (Fig. 4) only. Note that the accuracy of $\mathcal{I}_m$ in (20) and (28) is constrained by that of the employed approximation to the Gaussian Q-function and cannot be improved by increasing the number of terms. As it can be appreciated, the relative error quickly decreases as the number of terms employed in the truncated series increases, indicating that the evaluation of just a few terms is sufficient to provide accurate numerical results. It is interesting to note the existence of a lower bound in the relative error, which is due to the finite precision of the employed computer. It is also worth mentioning that the relative errors shown in Figs. 3 and 4 correspond to the maximum relative error observed over the whole range of SNR values. This maximum error usually occurs in the region of low SNR. At higher SNR values, which is the typical region of operation targeted by wireless communication systems, the actual relative error is lower than the values shown in Figs. 3 and 4. These relative errors represent the worst-case (i.e., low SNR) relative errors.

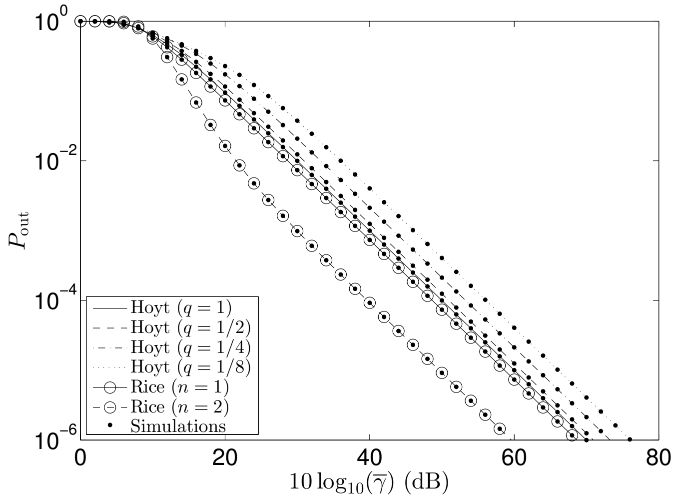

Fig. 1. Outage probability as a function of the average SNR ($\gamma_{th} \sim 10$ dB).

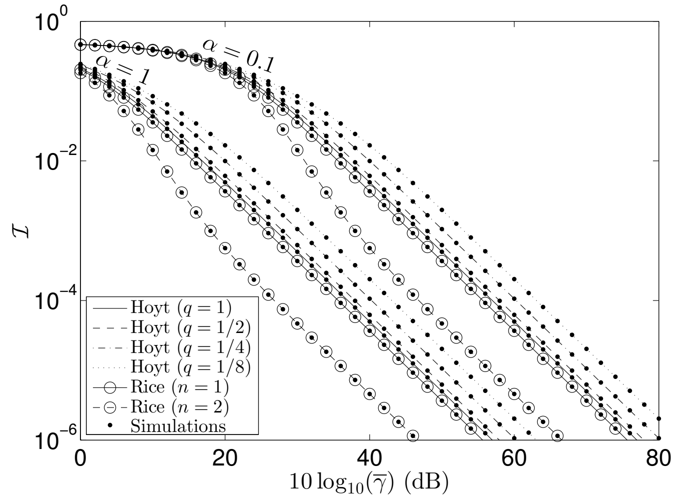

Fig. 2. Error performance integral $\mathcal{I}$ as a function of the average SNR.

Tables I and II show the minimum number of terms required in the truncated series for the outage probability and the error performance integral $\mathcal{I}$ in order to guarantee a relative error no greater than 1% (the accuracy of $\mathcal{I}_m$ in (20) and (28) is constrained by that of the employed approximation to the Gaussian Q-function and cannot be improved by increasing the number of terms; however the values shown in Table II

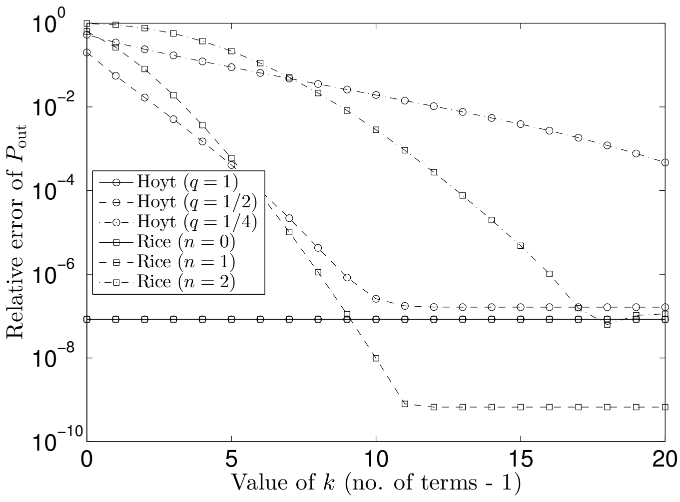

Fig. 3. Relative error of P_{out} as a function of k ($\gamma_{th} \sim 10$ dB).

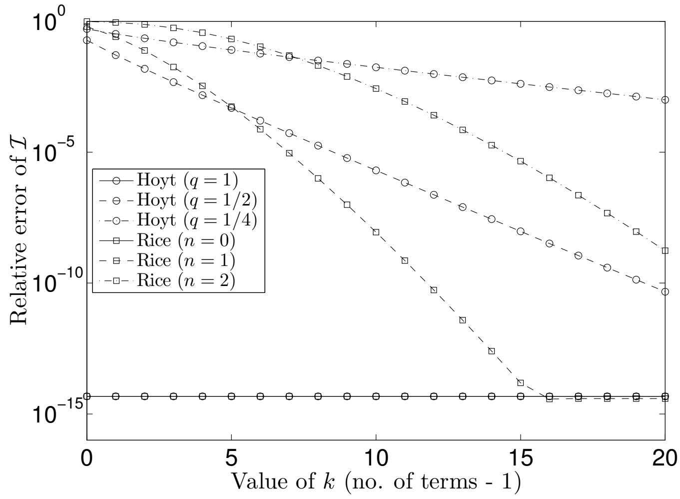

Fig. 4. Relative error of $\mathcal{I}$ as a function of k ($\alpha = 0.1$).

TABLE I. NUMBER OF TERMS IN (2) & (7) FOR 1% MAXIMUM RELATIVE ERROR.

	Nakagami-q (Hoyt)			**Nakagami-n (Rice)**		
	$q = 1$	$q = \frac{1}{2}$	$q = \frac{1}{4}$	$n = 0$	$n = 1$	$n = 2$
$\gamma_{th} \sim 5$ dB	1	3	8	1	5	10
$\gamma_{th} \sim 10$ dB	1	4	14	1	5	10
$\gamma_{th} \sim 15$ dB	1	4	14	1	5	10

TABLE II. NUMBER OF TERMS IN (15) & (25) FOR 1% MAXIMUM RELATIVE ERROR.

	Nakagami-q (Hoyt)			**Nakagami-n (Rice)**		
	$q = 1$	$q = \frac{1}{2}$	$q = \frac{1}{4}$	$n = 0$	$n = 1$	$n = 2$
$\alpha = 0.01$	1	4	14	1	5	10
$\alpha = 0.1$	1	4	13	1	5	10
$\alpha = 1$	1	3	7	1	4	9

TABLE III. AVERAGE COMPUTATION TIME (MS) REQUIRED TO EVALUATE (2) & (7) TO 1% MAXIMUM RELATIVE ERROR.

	Nakagami-q (Hoyt)			**Nakagami-n (Rice)**		
	$q = 1$	$q = \frac{1}{2}$	$q = \frac{1}{4}$	$n = 0$	$n = 1$	$n = 2$
$\gamma_{th} \sim 5$ dB	2.6	5.2	12.1	1.9	5.8	10.8
$\gamma_{th} \sim 10$ dB	2.6	6.6	20.5	2.0	5.8	10.8
$\gamma_{th} \sim 15$ dB	2.6	6.6	20.7	2.0	5.9	10.9

TABLE IV. AVERAGE COMPUTATION TIME (S) REQUIRED TO EVALUATE (15) & (25) TO 1% MAXIMUM RELATIVE ERROR.

	Nakagami-q (Hoyt)			**Nakagami-n (Rice)**		
	$q = 1$	$q = \frac{1}{2}$	$q = \frac{1}{4}$	$n = 0$	$n = 1$	$n = 2$
$\alpha = 0.01$	1.15	2.25	5.85	1.16	2.60	4.37
$\alpha = 0.1$	1.15	2.24	5.44	1.16	2.60	4.37
$\alpha = 1$	1.15	1.85	3.26	1.16	2.25	3.98

were observed to be enough to approach the best attainable accuracy). As it can be appreciated, only a few terms are needed to provide accurate (virtually exact) results. It is worth noting that for Nakagami-q (Hoyt) with $q = 1$ and Nakagami-n (Rice) with $n = 0$, which are both equivalent to Rayleigh fading, only one term (i.e., $k = 0$) is required in the series in order to provide accurate (exact) results. In both cases, (18) and (26) lead to the same expression obtained in [1, eq. (5.6)] for Rayleigh fading channels based on the MGF approach.

Tables III and IV show the average time of computation required with the obtained analytical expressions to produce numerical results with a maximum relative error of 1% (based on the number of terms shown in Tables I and II). These results were obtained with an average general purpose computer (Intel Core i3-3220 CPU at 3.30 GHz with 8 GB of DDR3 SDRAM at 1600 MHz) by averaging the computation times observed in a set of 1000 evaluations of the corresponding analytical expressions. As it can be appreciated, the computation of the error performance integral $\mathcal{I}$ requires a significantly longer time than the outage probability. This is due to the presence of the Gauss hypergeometric function in the analytical results of (15) & (25), which is a complex function whose evaluation can take a significant amount of time depending on the particular value of its parameters. On the other hand, the analytical results for the outage probability obtained in (2) & (7) are based on elementary (power and exponential) functions and as a result lead to an almost instantaneous evaluation. In any case, and despite the code employed to obtain these results was not optimised for computational efficiency, the resulting computation times are within reasonable practical limits.

The results shown in Fig. 5 (for the outage probability and error performance integral $\mathcal{I}$) and Fig. 6 (for the integral $\mathcal{I}_m$) demonstrate that the proposed approximations provide an accurate characterisation of the real performance over a wide range of average SNR values, which is a consequence of the rapid convergence of the infinite series. For the particular case selected for Nakagami-n (Rice) fading (i.e., $n = 2$), which exhibits a slightly different shape compared to the rest of cases in Fig. 2, the accuracy degrades in the region of low SNR values. However, for lower values of the Nakagami-n

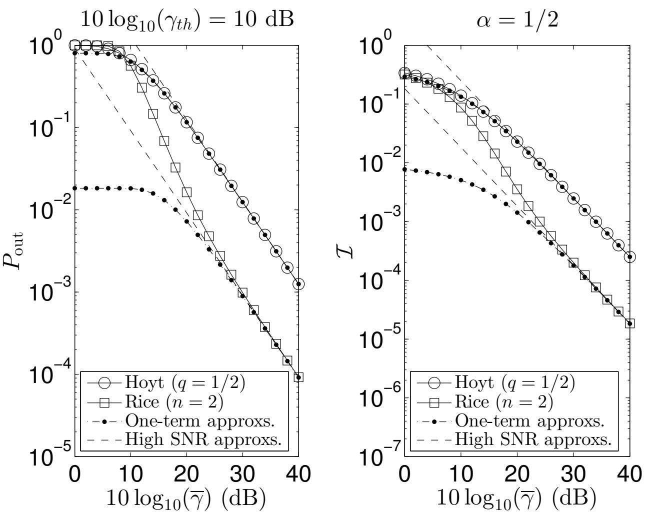

Fig. 5. Comparison of exact and approximated theoretical expressions.

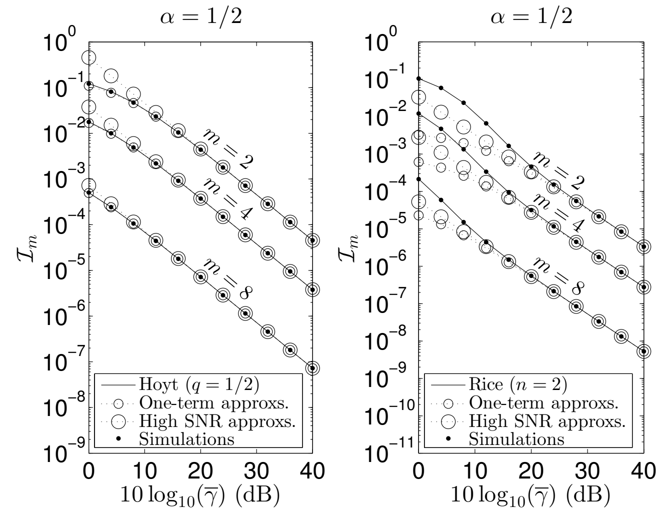

Fig. 6. Error performance integral $\mathcal{I}_m$ as a function of the average SNR.

fading parameter, and for arbitrary values of the Nakagami-q fading parameter, the approximated expressions provide more accurate results. Interestingly, the accuracy of all the proposed approximations improves as the average SNR increases, providing (virtually) exact results in the region of high SNR values (i.e., the region of interest in the design of wireless communication systems). Owing to this appealing feature, along with their tractable algebraic forms, the obtained approximations constitute suitable tools not only for the performance evaluation of modulation schemes under Nakagami-q (Hoyt) and Nakagami-n (Rice) fading channels but also for analytical manipulations in the context of these fading environments.

V. Conclusions

This paper has presented novel results for the evaluation of the outage probability and average error performance of modulation schemes under Nakagami-q (Hoyt) and Nakagami-n (Rice) fading channels. Exact expressions have been given in infinite series form, but a convergence analysis has demonstrated that truncation to very few terms can provide accurate results. Several approximations based on elementary functions have been derived as well. The presented expressions not only provide analytically tractable forms and high levels of accuracy but are also useful in a wide range of scenarios, including cases not covered by previous known results.

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