A Robust Optimization Approach for Demand Side Scheduling Considering Uncertainty of Manually Operated Appliances

Y. F. Du, L. Jiang, *Member, IEEE*, Y. Z. Li, and Q. H. Wu, *Fellow, IEEE*

***Abstract*—Manually operated appliances (MOAs) are manually operated based on users' real-time demands and their energy consumption is uncertain to other schedulable appliances (SAs). This paper represents energy consumption scheduling of home appliances under the uncertainty of the MOAs as a robust optimization problem, as uncertainty distribution of MOAs is usually unknown and not easily estimated. Among all possible energy consumption cases of the MOAs, the robust approach takes into account the worst case to reduce electricity payment of all home appliances, based on the real-time electricity pricing scheme combined with inclining block rate. Intergeneration projection evolutionary algorithm, which is a nested heuristic algorithm with inner genetic algorithm and outer particle swarm optimization algorithm, is adopted to solve the robust optimization problem. Case studies are based on one day case, and one month case with various combinations of SAs and MOAs. Simulation results illustrate the effectiveness of the proposed approach in reduction of electricity payment compared with approach without considering the uncertainty of MOAs, and approach considering MOAs with fixed pattern.**

***Index Terms*—Energy consumption scheduling, robust optimization approach, manually operated appliances, schedulable appliances, demand response.**

NOMENCLATURE

Abbreviations

MOA	Manually operated appliance.
SA	Schedulable appliance.
LOT	Length of operation time.
OTI	Operation time interval.
DR	Demand response.
PSO	Particle swarm optimization.
GA	Genetic algorithm.
RTP	Real-time pricing.
TOUP	Time-of-use pricing.
CPP	Critical-peak pricing.
IBR	Inclining block rate.
IP-GA	Intergeneration projection genetic algorithm.
IP-EA	Intergeneration projection evolutionary algorithm.
EMC	Energy management controller.
ESS	Energy scheduling system.
A1	Approach without considering MOAs.
A2	Approach considering the fixed pattern of MOAs.
A3	Approach considering the worst impact of MOAs.

Variables

l_t	Total energy consumption of home appliances at time slot t.
prc_t	The electricity price based on RTP-IBR at time slot t.
e_t	Real-time electricity price at time slot t.
c	Threshold of energy consumption.
P	Power vector of an appliance.
p_θ	Power consumption of an appliance at time slot θ.
α	Earliest start time of appliance's operation.
β	Deadline of appliance's operation.
γ	Length of operation time.
ϵ	Multiplier of electricity price when l_t exceeds c.
s	Scenario index.
ρ_s	The probability of scenario s.
NS	The number of scenarios.
t, θ	The index of time slot.
i, j	The index of SA/MOA.
m, n	The number of MOAs/SAs.
b, d	The index of non-interruptible/interruptible SA.
f, g	The index of non-interruptible/interruptible MOA.
x_i^t	Energy consumption of SA i at time slot t.
u_j^t	Energy consumption of MOA j at time slot t.
x_b^t	Energy consumption of non-interruptible SA b at time slot t.
x_d^t	Energy consumption of interruptible SA d at time slot t.
u_f^t	Energy consumption of non-interruptible MOA f at time slot t.
u_g^t	Energy consumption of interruptible MOA g at time slot t.
t_b	Start time of the operation of non-interruptible SA b.
t_f	Start time of the operation of non-interruptible MOA f.
γ_f^{min}	Minimum LOT of non-interruptible MOA f.
γ_f^{max}	Maximum LOT of non-interruptible MOA f.
γ_g^{min}	Minimum LOT of interruptible MOA g.
γ_g^{max}	Maximum LOT of interruptible MOA g.
T	Horizon of energy consumption scheduling.
X_b	Energy consumption schedule of non-interruptible SA b.

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X_d	Energy consumption schedule of interruptible SA d.
U_f	Energy consumption case of non-interruptible MOA f.
U_g	Energy consumption case of interruptible MOA g.
$\mathbf{X}$	Energy consumption schedule of SAs.
$\mathbf{U}$	Energy consumption case of MOAs.
$\mathbf{U}_s$	Energy consumption case of MOAs in scenario s.
c_1, c_2	The acceleration constants in PSO.
r_1, r_2	The randomly generated numbers in range of [0,1] in PSO.
w	The inertia weight factor in PSO.
k	The iteration index in PSO.
v_ζ^k, p_ζ^k	The velocity/position of the particle ζ at the kth iteration in PSO.
$pbest_\zeta^k$	The best position of the particle ζ among k iterations in PSO.
$gbest^k$	The best position of the particle swarm among k iterations in PSO.

Sets

H	Set of time slots in a day.
$\mathbf{\Omega}$	Set of energy consumption schedules of SAs.
$\mathbf{\Gamma}$	Set of energy consumption cases of MOAs.
$\mathbf{\Gamma}_s$	Set of energy consumption cases of MOAs in scenario s.

I. Introduction

SMART grid tends to apply the advances from the information and communication technology, control and optimization methodologies to improving the efficiency and reliability of conventional power system [1]. Nowadays smart grid is undergoing profound transformation due to the increasing penetration of renewable generation, the fast development and deployment of new equipments such as energy storage apparatus and power electronics, and the active participation of demand response (DR) from the user side [2]. DR changes electricity usage of end users from their normal consumption patterns in response to the real-time electricity price, or to incentive payments, or when system reliability is jeopardized [3].

Smart home technology tends to achieve energy saving and provide users with maximal comfort and convenience, such as, the thermal comfort brought by the network of temperature sensors [4], the convenience coming along with the wireless area network [5], and the comfort and convenience brought by the human-computer interaction through the adaption of services according to the context of the environment and the involved users [6]. All these technologies will motivate more users to participate in DR and thus improve the efficiency and the reliability of power system [7] [8], reduce the overall peak energy demand [9], and lower the risk of outages [10].

Several pricing schemes have been proposed in DR, such as real-time pricing (RTP), time-of-use pricing (TOUP), critical-peak pricing (CPP), and RTP combined with inclining block rate (RTP-IBR) [11] [12]. TOUP and hourly-based pricing tariff have been applied in practice, such as the Economy 7 tariff in UK with higher electricity price for daytime and lower price for night time [13] and hourly based real-time electricity price announced a day head by the Illinois Power Company in US [14]. Compared with other pricing schemes, the RTP carries on more real-time information of power system and would bring more economic benefits to power system [15], but it may aggregate energy consumption in periods with the lower price to cause peak demand [12]. RTP-IBR can reserve the economic benefits of the RTP, meanwhile avoid the possible aggregation effect caused by the RTP as the electricity price will increase to a higher value when the user's total energy consumption exceeds a threshold [11].

Many optimization methods have been proposed to schedule the energy consumption of home appliances for reducing users' electricity payment, such as linear programming [11], mixed integer linear programming [16], genetic algorithm (GA) [12], branch and bound method [17], and game theory [18]. Home appliances can be classified as schedulable appliances (SAs) and manually operated appliances (MOAs) whose energy consumption is manually controlled by the the real-time demands of user and cannot be scheduled ahead like SAs. In [11], [16]–[18], fixed operation pattern is assumed for MOAs, which sacrifices users' comfort and convenience as the usage of MOAs is dependent upon users' real-time demands and is affected by many random external factors. Moreover, the scheduling optimal results will be degraded when there is uncertainty of the MOAs' energy consumption. In [12], the MOAs have not been considered when energy consumption is scheduled based on the RTP-IBR, which would make users confront the risk of high electricity payment due to the excess of the energy consumption threshold set by IBR when the energy consumption of MOAs is accidentally involved. Based on the best knowledge of authors, the uncertainty of energy consumption of the MOAs has not been considered in the energy consumption scheduling of home appliances. As MOAs usually consume around 30% to 40% of the total energy consumption of home appliances [11] [12] [16]–[18], it is necessary and worth to consider the uncertainty of MOAs so as to further improve the overall efficiency of the optimal scheduling scheme of home appliances.

Optimization problems considering uncertain variables can be tackled by stochastic approach, robust approach and the hybrid robust/stochastic approach [19] [20]. Most of such optimization problems are dealt with by stochastic approach which usually requires a known probability distribution of the uncertain variable. For example, [21]–[23] schedule house load or unit commitment under uncertain renewable energy source with a known probability distribution, and the hybrid approach requires the probability distribution of the uncertain variable as well [19] [20]. As an effective method for addressing optimization problems with unknown probability distribution of the uncertainties, robust approach has been applied in power system recently [24]–[26]. Since users' real-time demands and behaviors are usually affected by many random factors and external disturbances, the probability distribution of MOAs' energy consumption cannot be accurately estimated. Therefore, the robust approach is adopted to solve the energy scheduling problem with the consideration of the uncertainty of MOAs' energy consumption, and the problem is formulated with two-level framework.

Taking into account the uncertainty of MOAs' energy consumption, the energy scheduling of home appliances is formulated as a robust optimization problem in this paper. The robust optimization approach treats the energy consumption case of MOAs as disturbance to the scheduling scheme of SAs, and the maximal disturbance of MOAs, i.e., the MOAs' energy consumption case with the worst impact, is the energy consumption case of MOAs that causes maximum electricity payment of all home appliances. The robust approach will minimize this maximum electricity payment and the problem of energy scheduling is formulated as a min-max, two-level optimization problem. The proposed robust approach is compared with other two approaches, one without considering the impact of MOAs and one considering the MOAs with fixed pattern.

This paper employs the intergeneration projection evolutionary algorithm (IP-EA) to solve the proposed robust optimization problem [27]. The IP-EA, which is a two-level evolutionary algorithm with inner genetic algorithm (GA) and outer particle swarm optimization (PSO) algorithm, is effective in solving the proposed nonlinear problem with two-level framework [27].

The rest of the paper is organised as follows. The energy scheduling system is introduced in Section II. Section III presents the impact of the uncertainty of MOAs' energy consumption. The robust optimization approach is introduced in Section IV and case study is conducted in Section V. Finally, the paper is concluded in Section VI.

II. Energy Scheduling System

The structure of the energy scheduling system is shown in Fig. 1. Based on the electricity price and user's demands, the energy management controller (EMC) schedules energy consumption of home appliances a day ahead. The system structure is introduced at first and then the model formulation is presented.

A. System Structure

EMC is the main part of the energy scheduling system (ESS) and it is connected to a smart power distribution system with digital communication capability through computer networking [11]. Electricity price is transmitted to the EMC from the smart power distribution system and users' demands for operations of appliances are input together by in-home display device which is a part of the EMC. After EMC works out the optimal scheduling scheme of home appliances, it transmits the control signals to appliances through the home area network and several communication protocols have been proposed [28] [29].

Home appliances are categorized into MOAs and SAs. MOAs are manually operated based on users' real-time demands and users must be available to operate them. SAs are appliances whose energy consumption can be scheduled ahead and automatically controlled by the EMC and does not require the users' participation during their operations, though users are required to prepare for the operations and preset the operation time intervals and operation lengths. For example, after users put food in the oven and set the operation interval and the operation length, EMC will start the oven at the optimal time slot automatically, so is the case for the washing machine. Based on this definition, oven and washing machine are classified as SAs, and they are scheduled automatically no matter whether users are at home or not.

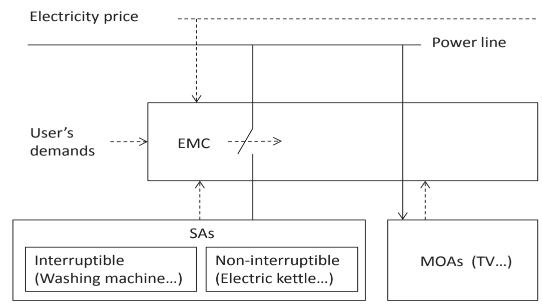


Fig. 1. Energy scheduling system

SAs include interruptible and non-interruptible appliances [11]. Interruptible appliances can suspend their operations during the operation processes, and then restart again to continue their operations. For example, washing machine is interruptible as it can be paused during the process of washing. Once non-interruptible appliances are started, they cannot be stopped until they finish their tasks, such as the electric kettle. The common MOAs include hair drier, lights, laptop and TV. For SAs, clothes dryer, oven, water heater and electric kettle are non-interruptible, washing machine and humidifier are interruptible.

The EMC only schedules energy consumption for the SAs, not for the MOAs since the operations of MOAs are manually controlled based on the real-time demands of users. However, it does not mean that the MOAs can be ignored when the energy consumption of the SAs is scheduled. The EMC schedules energy consumption for the SAs taking into account the uncertainty and the impact of MOAs' energy consumption.

B. Model of User's Demands

Firstly, the model of the electricity pricing is given. As stated in Section I, the pricing scheme RTP-IBR keeps the benefits of RTP and avoids the possible aggregation of energy consumption caused by the RTP as well. We schedule the energy consumption based on this pricing scheme. In the pricing scheme RTP-IBR, users are charged at a higher electricity price than RTP at the corresponding time when the total energy consumption within a certain period exceeds a threshold. The pricing scheme RTP-IBR is given as:

$$prc_t(l_t) = \begin{cases} e_t, & \text{if } 0 \leq l_t < c \\ \varepsilon \cdot e_t, & \text{if } l_t \geq c \end{cases} \tag{1}$$

where l_t denotes the total energy consumption at time slot t, e_t and prc_t denote RTP and the electricity price based on

RTP-IBR at time slot t, respectively, c denotes the threshold and ε is a coefficient greater than 1 [11] [12].

Model of user's demands is formulated. User's demands include the length of operation time (LOT) and the operation time interval (OTI) for home appliances [12]. Let γ denote the LOT and $[\alpha, \beta]$ denote the OTI of an appliance, where α is the earliest start time of operation and β is the deadline that the operation must be finished. Considering the general operation time of appliances, 1 hour is divided into 5 time slots and the energy consumption is scheduled with 12-minute time resolution. One day is mapped to 120 time slots and the LOT and the OTI are represented via time slots with one time slot representing 12 minutes. For example, when it takes an hour for washing machine to finish the washing task and the operation is pre-specified between 12 am and 12 pm, the LOT of washing machine is 5 and the OTI is from 1 to 60, i.e., $\gamma = 5, \alpha = 1, \beta = 60$ for washing machine. If the operation length of an appliance is not an integer multiple of 12 minutes and the mapped LOT is not an integer, the LOT of the appliance is rounded up to the nearest integer [12]. It is noted that though the energy consumption of MOAs cannot be scheduled in advance, users' demands for MOAs are also modelled via the possible OTIs and the ranges of LOTs which are pre-specified by users. For example, the watching time of TV is pre-specified between 6 pm and 12 am and the watching length is pre-specified between 3 hours and 4 hours while when and how long users watch TV are still dependent upon users' real-time demand.

III. Impact of the Uncertainty of MOAs' Energy Consumption

Based on the pricing scheme RTP-IBR, the electricity payment is

$$f(\mathbf{X}, \mathbf{U}) = \sum_{t=1}^{T} prc_t(l_t(\mathbf{X}, \mathbf{U})) \cdot l_t(\mathbf{X}, \mathbf{U}) \tag{2}$$

where $\mathbf{X}$ denotes the energy consumption schedule of SAs, $\mathbf{U}$ denotes the energy consumption case of MOAs, $\mathbf{X}$ and $\mathbf{U}$ are matrixes in which each row stands for the energy consumption schedule of a certain appliance, T is the scheduling horizon that indicates the number of time slots ahead which the energy consumption schedule is made for SAs, and $T = 120$ since the energy consumption of appliances is scheduled a day ahead with one hour divided into 5 time slots and the time slot duration is 12 minutes, and prc_t is the electricity price based on RTP-IBR at time slot t as shown in (1).

Let $\mathbf{\Omega}$ represent all the possible energy consumption schedules of SAs and $\mathbf{\Gamma}$ represent all the possible energy consumption cases of MOAs. For a certain $\mathbf{X}$, $f(\mathbf{X}, \mathbf{U})$ is uncertain due to the uncertainty of $\mathbf{U}$. Among all the possible energy consumption cases for MOAs, there exists an energy consumption case of MOAs that makes the smallest impact to electricity payment, i.e., with this energy consumption case of MOAs, the electricity payment is smallest. The electricity payment with the smallest impact of MOAs is formulated as

$$f^{\mathrm{L}}(\mathbf{X}) = \min_{\mathbf{U} \in \mathbf{\Gamma}} f(\mathbf{X}, \mathbf{U}). \tag{3}$$

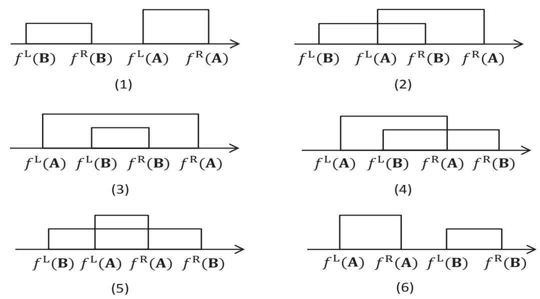


Fig. 2. Six possible relationships between electricity payment intervals

Similarly, there exists an energy consumption case of MOAs that makes the worst impact to electricity payment, corresponding to the highest electricity payment

$$f^{\mathrm{R}}(\mathbf{X}) = \max_{\mathbf{U} \in \mathbf{\Gamma}} f(\mathbf{X}, \mathbf{U}). \tag{4}$$

Therefore, for a certain energy consumption schedule of SAs $\mathbf{X}$, the electricity payment $f(\mathbf{X}, \mathbf{U}) \in [f^{\mathrm{L}}(\mathbf{X}), f^{\mathrm{R}}(\mathbf{X})]$.

For any two different energy consumption schedules of SAs $\mathbf{A}$ and $\mathbf{B}$, the electricity payments are within $[f^{\mathrm{L}}(\mathbf{A}), f^{\mathrm{R}}(\mathbf{A})]$ and $[f^{\mathrm{L}}(\mathbf{B}), f^{\mathrm{R}}(\mathbf{B})]$, respectively. Fig. 2 shows all the possible relationships between these two electricity payment intervals [27]. For relationships shown in Fig. 2-(1) and Fig. 2-(6), the performance comparison between schedules $\mathbf{A}$ and $\mathbf{B}$ is deterministic, since the electricity payment intervals of the two schedules are not interacted and overlapped and the electricity payment with one energy consumption schedule is always less than the payment with the another schedule regardless of the uncertainty of the MOAs. However, the uncertainty of MOAs' energy consumption should be taken into account for relationships shown in Fig. 2-(2) - Fig. 2-(5) when schedules $\mathbf{A}$ and $\mathbf{B}$ are compared, since the electricity payment intervals of two schedules are overlapped and thus the comparison of electricity payment is uncertain due to the uncertainty of MOAs' energy consumption.

IV. Robust Optimization Approach

As shown in Fig. 2, the uncertainty of MOAs' energy consumption has impact on the electricity payment whose interval is between Eqn. (3) and Eqn. (4), it is necessary to take into account the uncertainty of MOAs' energy consumption for energy consumption scheduling. In this section, a robust approach is proposed to deal with the uncertainty of MOAs' energy consumption. A complete optimization model is given at first, then a heuristic algorithm, the intergeneration projection evolutionary algorithm (IP-EA), is adopted for solving the robust optimization problem.

A. Complete Optimization Model

Among all the possible cases of MOAs' energy consumption, the robust optimization approach takes into account the case that causes $f^{\mathrm{R}}(\mathbf{X})$, i.e., with the worst impact to electricity payment of all home appliances, and the problem of scheduling energy consumption of SAs with the consideration of the worst impact of MOAs is represented as

$$\begin{aligned} &\min_{\mathbf{X}\in\mathbf{\Omega}} f^{\mathrm{R}}(\mathbf{X}) \\ &f^{\mathrm{R}}(\mathbf{X}) = \max_{\mathbf{U}\in\mathbf{\Gamma}} f(\mathbf{X},\mathbf{U}) \\ &f(\mathbf{X},\mathbf{U}) = \sum_{t=1}^{T} prc_t(l_t(\mathbf{X},\mathbf{U}))\cdot l_t(\mathbf{X},\mathbf{U}) \end{aligned} \tag{5}$$

which is equivalent to

$$\begin{aligned} &\min_{\mathbf{X}\in\mathbf{\Omega}} \max_{\mathbf{U}\in\mathbf{\Gamma}} f(\mathbf{X},\mathbf{U}) \\ &f(\mathbf{X},\mathbf{U}) = \sum_{t=1}^{T} prc_t(l_t(\mathbf{X},\mathbf{U}))\cdot l_t(\mathbf{X},\mathbf{U}) \end{aligned} \tag{6}$$

with the electricity price prc_t presented in (1). For a home with n SAs including the interruptible and non-interruptible SAs and m MOAs, the energy consumption schedule of SAs is

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} x_1^1 & x_1^2 & \dots & x_1^t & \dots & x_1^T \\ x_2^1 & x_2^2 & \dots & x_2^t & \dots & x_2^T \\ \vdots & \vdots & & \ddots & & \vdots \\ x_n^1 & x_n^2 & \dots & x_n^t & \dots & x_n^T \end{bmatrix} \tag{7}$$

where each row of the matrix $\mathbf{X}$ represents the energy consumption schedule of a SA within T time slots. The energy consumption case of MOAs is

$$\mathbf{U} = \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_m \end{bmatrix} = \begin{bmatrix} u_1^1 & u_1^2 & \dots & u_1^t & \dots & u_1^T \\ u_2^1 & u_2^2 & \dots & u_2^t & \dots & u_2^T \\ \vdots & \vdots & & \ddots & & \vdots \\ u_m^1 & u_m^2 & \dots & u_m^t & \dots & u_m^T \end{bmatrix} \tag{8}$$

where each row of the matrix $\mathbf{U}$ represents the energy consumption case of a MOA. Therefore, the total energy consumption at time slot t is

$$l_t(\mathbf{X},\mathbf{U}) = \sum_{i=1}^{n} x_i^t + \sum_{j=1}^{m} u_j^t, t\in\{1,2,\cdots,T\}. \tag{9}$$

1) Constraints: Based on the classifications of appliances, the constraints of the energy consumption of appliances are presented in this section and the illustrative examples for the constraints of appliances are shown in Fig. 3. Let $P=[p_1,p_2,\cdots,p_\gamma]$ denote the power vector of an appliance, which represents the appliance's power consumption during the whole operation process. For example, the power vector of the clothes dryer is [1.2 1.2 1] kW, which shows that the power consumption of the clothes dryer in the first, second and third time slot are 1.2, 1.2 and 1 kW, respectively.

When the appliance b belongs to the non-interruptible SAs, the energy consumption schedule X_b is

$$X_b = \Big\{ x_b^t \,|\quad \begin{aligned} &x_b^{t_b+\theta} = \tfrac{p_{\theta+1}}{5}, \text{for all } \theta = 0,1,\cdots,\gamma_b - 1 \\ &t_b\in[\alpha_b,\beta_b-\gamma_b+1], \\ &x_b^t = 0,\ t\in H\backslash[t_b,t_b+\gamma_b-1], \\ &H=\{1,2,\cdots,T\} \Big\} \end{aligned} \tag{10}$$

where t_b is the start time slot for the appliance's operation and $t_b\in[\alpha_b,\beta_b-\gamma_b+1]$ since the operation should start ahead the deadline by at least the length of operation time, the expression $t\in H\backslash[t_b,t_b+\gamma_b-1]$ indicates that t belongs to H excluding the range $[t_b,t_b+\gamma_b-1]$, and since one hour is divided into 5 time slots with 12 minutes in each time slot, the energy consumption in each time slot is $1/5$ of the power which is the energy consumption in an hour. (10) shows the constraints of non-interruptible SAs: the operation is within the OTI $[\alpha_b,\beta_b]$, the energy consumption is continuous and reaches LOT γ_b. When the appliance d belongs to the interruptible SAs, the energy consumption schedule X_d is

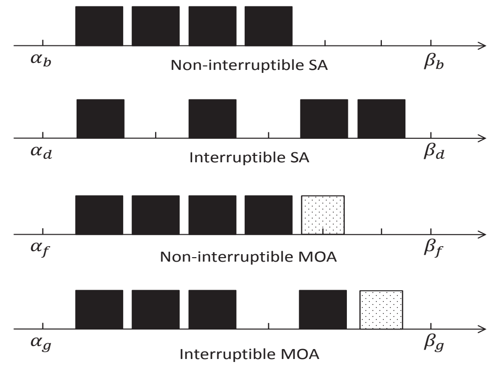


Fig. 3. Illustrative examples for the constraints of appliances

$$X_d = \Big\{ x_d^t \,|\quad \begin{aligned} &x_d^{t_\theta} = \tfrac{p_\theta}{5}, \text{for all } \theta = 1,\cdots,\gamma_d \\ &\alpha_d \le t_1 < t_2 < t_3 < \cdots < t_{\gamma_d} \le \beta_d, \\ &x_d^t = 0,\ t\in H\backslash\{t_1,t_2,t_3,\cdots,t_{\gamma_d}\}, \\ &H=\{1,2,\cdots,T\} \Big\} \end{aligned} \tag{11}$$

which shows the constraints of interruptible SAs: the operation is within the OTI $[\alpha_d,\beta_d]$, the energy consumption is interruptible and reaches LOT γ_d. The LOT is 4 for both the interruptible and non-interruptible SAs in Fig. 3.

MOAs are categorized into two groups: non-interruptible and interruptible ones. The energy consumption of a non-interruptible MOA f is

$$U_f = \Big\{ u_f^t \,|\quad \begin{aligned} &u_f^{t_f+\theta} = \tfrac{p_{\theta+1}}{5}, \text{for all } \theta = 0,1,\cdots,\gamma_f - 1 \\ &t_f\in[\alpha_f,\beta_f-\gamma_f+1], \gamma_f\in[\gamma_f^{\min},\gamma_f^{\max}], \\ &u_f^t = 0,\ t\in H\backslash[t_f,t_f+\gamma_f-1], \\ &H=\{1,2,\cdots,T\} \Big\} \end{aligned} \tag{12}$$

where t_f is the start time slot for the appliance's operation, $\gamma_f^{\min}$ and $\gamma_f^{\max}$ denote the minimum and maximum LOT of MOA f, respectively. (12) shows the constraints of non-interruptible MOAs: the operation is within the OTI $[\alpha_f,\beta_f]$, the LOT γ_f is within the range $[\gamma_f^{\min},\gamma_f^{\max}]$ and is flexible based on users' real-time demands. The energy consumption of an interruptible MOA g is

$$U_g = \Big\{ u_g^t \,|\quad \begin{aligned} &u_g^{t_\theta} = \tfrac{p_\theta}{5}, \text{for all } \theta = 1,\cdots,\gamma_g \\ &\alpha_g \le t_1 < t_2 < t_3 < \cdots < t_{\gamma_g} \le \beta_g, \\ &u_g^t = 0,\ t\in H\backslash\{t_1,t_2,t_3,\cdots,t_{\gamma_g}\}, \\ &\gamma_g\in[\gamma_g^{\min},\gamma_g^{\max}], H=\{1,2,\cdots,T\} \Big\} \end{aligned} \tag{13}$$

which shows the constraints of interruptible MOAs: the operation is within the OTI $[\alpha_g, \beta_g]$, the energy consumption is interruptible and reaches LOT γ_g. The range of LOT is $[4, 5]$ for the illustrative MOAs in Fig. 3. Note that the constraints of MOAs are only to define the possible cases of MOAs' energy consumption and energy consumption of MOAs is still controlled by users in real time.

2) Complete Model Formulation: Finally, the objective of the robust optimization approach with the consideration of the constraints of appliances is represented as

$$
\begin{aligned}
&\min_{\mathbf{X}\in\mathbf{\Omega}} \max_{\mathbf{U}\in\mathbf{\Gamma}} f(\mathbf{X},\mathbf{U}) \\
&f(\mathbf{X},\mathbf{U}) = \sum_{t=1}^{T} prc_t(l_t(\mathbf{X},\mathbf{U}))\cdot l_t(\mathbf{X},\mathbf{U}) \\
&\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} x_1^1 & x_1^2 & \dots & x_1^t & \dots & x_1^T \\ x_2^1 & x_2^2 & \dots & x_2^t & \dots & x_2^T \\ \vdots & \vdots & & \ddots & & \vdots \\ x_n^1 & x_n^2 & \dots & x_n^t & \dots & x_n^T \end{bmatrix} \\
&\mathbf{U} = \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_m \end{bmatrix} = \begin{bmatrix} u_1^1 & u_1^2 & \dots & u_1^t & \dots & u_1^T \\ u_2^1 & u_2^2 & \dots & u_2^t & \dots & u_2^T \\ \vdots & \vdots & & \ddots & & \vdots \\ u_m^1 & u_m^2 & \dots & u_m^t & \dots & u_m^T \end{bmatrix} \\
&l_t(\mathbf{X},\mathbf{U}) = \sum_{i=1}^{n} x_i^t + \sum_{j=1}^{m} u_j^t, t\in\{1,2,\cdots,T\} \\
&prc_t(l_t(\mathbf{X},\mathbf{U})) = \begin{cases} e_t, & \text{if } 0 \le l_t(\mathbf{X},\mathbf{U}) < c \\ \varepsilon\cdot e_t, & \text{if } l_t(\mathbf{X},\mathbf{U}) \ge c \end{cases} \\
&\mathbf{\Omega} = \left\{ \mathbf{X} | \mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}, \begin{array}{l} X_i \text{ subject to } (10) \\ \text{if } i \text{ is a non-interruptible SA} \\ X_i \text{ subject to } (11) \\ \text{if } i \text{ is an interruptible SA} \\ i = \{1,2,\cdots,n\} \end{array} \right\} \\
&\mathbf{\Gamma} = \left\{ \mathbf{U} | \mathbf{U} = \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_m \end{bmatrix}, \begin{array}{l} U_j \text{ subject to } (12) \\ \text{if } j \text{ is a non-interruptible MOA} \\ U_j \text{ subject to } (13) \\ \text{if } j \text{ is an interruptible MOA} \\ j = \{1,2,\cdots,m\} \end{array} \right\}
\end{aligned}
\tag{14}
$$

The min-max problem represented in (14) is a two-level, robust optimization problem. Each $\mathbf{X}$ corresponds to a $\mathbf{U}$ that causes the highest electricity payment and the value of $\mathbf{X}$ (the variable of the outer level optimization) has direct impact on the value of $\mathbf{U}$ (the variable of the inner level optimization). All the variables are discrete since the energy consumption of appliances is determined by the on/off states of appliances. When the appliance is operated (on state), the value of variable is the energy consumption of the appliance; and when the appliance is not operated (off state), the value of variable is zero.

B. Solution Algorithm

Since the model for the optimization problem of energy consumption scheduling is the same as the optimization model in [27], the optimal solution to energy consumption scheduling can be obtained through IP-GA as well. In this paper, we adopt the intergeneration projection evolutionary algorithm (IP-EA) to solve the robust optimization problem of energy consumption scheduling. IP-GA is a nested heuristic algorithm with inner GA and outer GA while IP-EA is a nested heuristic algorithm with inner GA and outer PSO algorithm. It is noted that IP-GA and IP-EA are essentially the same in finding the optimal solution except that the IP-EA is more computationally efficient, since the computational effort required by PSO is less than that of the GA to obtain same quality of the final solution [30].

GA mimics the process of natural selection. After evaluating the fitness of each individual in generation, selecting individuals with high fitness, crossover and mutation, the new generation with better fitness is obtained, and the above process cycles until the individual with the satisfactory fitness is found [12]. PSO algorithm is based on the behavior of particles of a swarm. Particles in a swarm approach to the optimum by tracking the best location of individual particle ($pbest$) and the best location of particle swarm ($gbest$), which is formulated as

$$
\begin{aligned}
v_\zeta^{k+1} &= wv_\zeta^k + c_1 r_1(pbest_\zeta^k - p_\zeta^k) + c_2 r_2(gbest^k - p_\zeta^k) \\
p_\zeta^{k+1} &= p_\zeta^k + v_\zeta^{k+1}
\end{aligned}
\tag{15}
$$

where v_ζ and p_ζ are the velocity and the position of the particle ζ, respectively, k is the iteration index, w is the inertia weight factor, c_1 and c_2 are the acceleration constants, and r_1 and r_2 are randomly generated numbers in range of $[0, 1]$ [31].

The flowchart of IP-EA for solving the problem of energy consumption scheduling is shown in Fig. 4. The outer PSO algorithm is for searching the optimal schedule of SAs' energy consumption and the inner GA is for searching the energy consumption case of MOAs with the worst impact to electricity payment. More specifically, for a trial schedule of SAs' energy consumption, the worst case of MOAs' energy consumption is obtained through GA. Then we can get the electricity payment with the trial schedule of SAs' energy consumption and the worst case of MOAs' energy consumption, and this electricity payment is used for tracking the optimal energy consumption schedule among the swarm of energy consumption schedules of SAs. The optimal schedule of SAs' energy consumption is obtained after the convergency of the electricity payment with the worst impact of MOAs.

Remarks Since values of outer-level variables affect the values of inner-level variables, this kind of problem with two-level framework is equivalent to a problem with product term [24]–[26], which makes the problem in general NP-hard to solve [25], and it cannot be solved by linear programming. The outer approximation (OA) algorithm is adopted in [24]-[26] to solve the similar max-min problems after the inner minimization problem has been converted to an equivalent dual maximization problem via the duality theory. However, this method cannot be applied in the proposed problem as all the

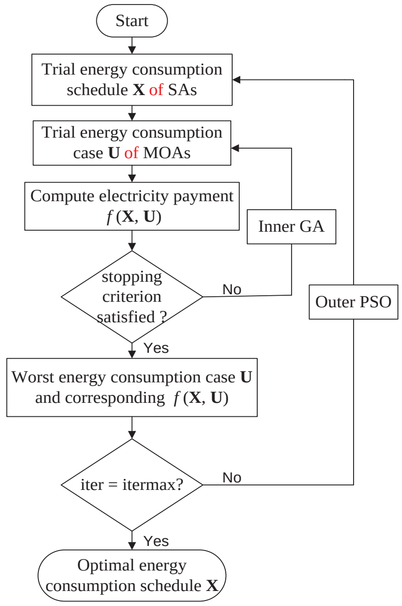


Fig. 4. Flowchart of intergeneration projection evolutionary algorithm

variables are discrete and the equivalent dualization requires the continuousness of variables [32]. Future studies will be focused on converting the proposed problem to an equivalent continuous one. Evolutionary algorithm is effective in solving nonlinear problems and problems with discrete variables [33]–[38], and the IP-EA with two-level structure is effective in solving the proposed two-level problem with discrete variables [27]. It is noted that with the two-level structure of the algorithm, the IP-EA solves the proposed problem directly without the conversion of the original problem. Though the IP-EA cannot guarantee the global optimality, the effectiveness of the usage of the IP-EA for solving the problem has been verified via case studies.

V. Case Study

In this section, simulation studies are carried out to verify the effectiveness of the proposed approach. Eight typical SAs and six typical MOAs are considered and the parameters of SAs and MOAs are given in Table I and Table II based on their operation characteristics. The first six SAs are non-interruptible and the last two SAs are interruptible, and the first three MOAs are interruptible and the last three MOAs are non-interruptible. For appliances whose powers are not represented as vectors in Table I and Table II, their powers are constant during the operation processes. The RTP data in August 2012 is adopted from the Ameren Illinois Power Company [14], which is the electricity price used for energy scheduling. Taking into account the uncertainty and the prediction error of the RTP, the electricity price varying between 90% and 110% of the RTP data is used for back test of the energy scheduling [11]. We assume that the coefficient $\varepsilon = 1.4423$ and the energy consumption threshold $c = 0.45$ kWh [11] [12]. Note that the threshold is for the energy consumption within 12 minutes. Two cases, one day operation of all SAs

TABLE I
PARAMETERS OF SAS

SA	OTI	LOT	Power (kW)
Electric kettle [12]	1-25	1	1.5
Clothes dryer [12] [39]	61-90	3	[1.2 1.2 1]
Oven [40] [41]	71-85	3	[2.1 1.9 1.9]
Water heater [12] [42]	86-105	3	[1.7 1.7 1.4]
Electric radiator [12] [43]	96-110	5	[2.2 1.8 1.8 1.8 1.8]
Dishwasher [12]	101-120	2	0.6
Washing machine [12]	1-60	5	0.38
Humidifier [12]	1-30	8	0.05

TABLE II
PARAMETERS OF MOAS

MOA	OTI	LOT	Power (kW)
Electric iron [40] [41]	61-70	3	[1.7 1.5 1.5]
Vacuum cleaner [12]	71-80	3	1.5
Hair drier [12]	101-110	1	1
Lights [12]	81-120	30-35	0.2
Laptop [40]	86-115	15-20	0.1
TV [12]	91-120	15-20	0.1

and MOAs on August 3rd 2012, and one month operation of different combinations of home appliances in August 2012, are presented. All simulations are implemented in MATLAB on Intel Core-i3 3.3-GHz personal computer with 8 GB RAM.

A. *Performance of the IP-EA*

For the inner GA, the population size of each generation is 200, the probability of crossover is 0.8 and the max generation number is 100 [12] [44]. For the outer PSO, the number of particles is 20, the inertial weight factor w decreases linearly from 0.9 to 0.4 with the increase of the iteration index, both the acceleration constants c_1 and c_2 are 2, and the max iteration number is 300 [45] [46]. The computational time of the IP-EA is 113 minutes within the scheduling horizon of a day. Fig. 5-(a) shows the evolution process of GA for searching the worst case of MOAs' energy consumption for a trial schedule of SAs' energy consumption. Fig. 5-(b) presents the convergency performance of the electricity payment with the worst impact of MOAs in the evolution process of PSO for searching the optimal energy consumption schedule of SAs, and the electricity payment with the worst impact of MOAs decreases by 8.54% from 57.17 cents to 52.29 cents.

Remarks The objective of the proposed approach is to improve an installed ESS via upgrading scheduling algorithm only, and thus no extra hardware cost is needed. Though the computational time of the optimization in the simulation test on a current PC is around 2 hours, it is within the scheduling horizon of a day and the proposed approach only runs optimization once per day. Thus the implementation of the proposed optimization in house level does not require a very powerful processor for the EMC. With the further development of computer technology and reduction of hardware cost, the implementation cost of the ESS will be further reduced.

B. *One Day Case*

To verify the advantage of the proposed approach, the energy consumption schedule of one day period August 3rd 2012 obtained by the proposed approach is compared with other two approaches without considering the uncertainty of

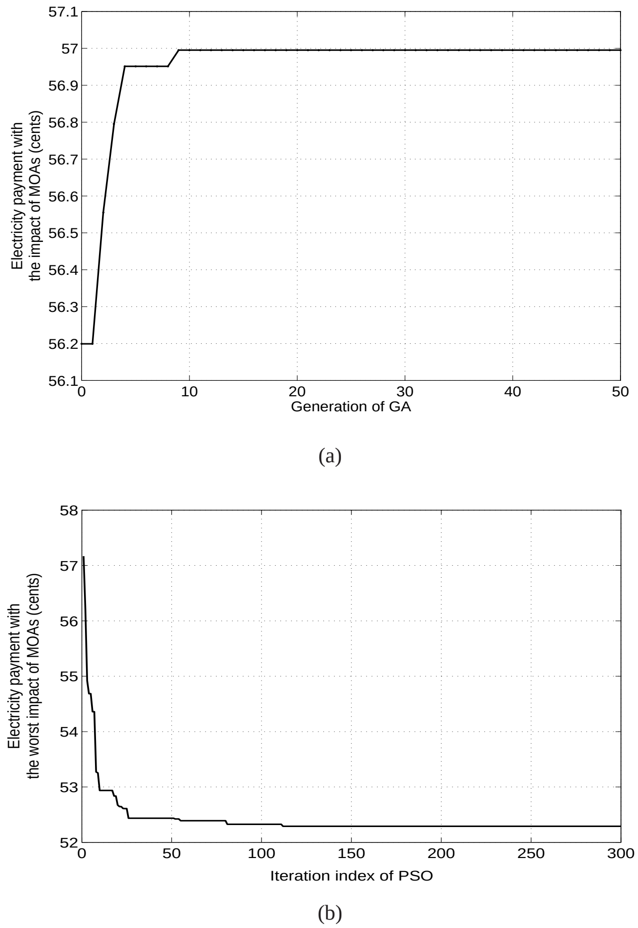


Fig. 5. (a) Evolution process of GA for searching the worst case of MOAs' energy consumption (b) Evolution process of PSO for searching the optimal energy consumption schedule of SAs

TABLE III
ASSUMED ENERGY CONSUMPTION PERIODS OF MOAS

MOA	Energy consumption period
Electric iron	61-63
Vacuum cleaner	73-75
Hair drier	110
Lights	85-117
Laptop	101-115
TV	94-113

TABLE IV
COMPARISON OF THE WORST ELECTRICITY PAYMENT

Approach	Worst electricity payment (cents)
Without considering MOAs	65.02
Considering the fixed pattern of MOAs	61.83
Considering the worst impact of MOAs	57.89

MOAs' energy consumption, including the approach without considering MOAs which minimizes the electricity payment of SAs and the approach considering the fixed pattern of MOAs which minimizes the electricity payment of all home appliances with an assumed energy consumption of MOAs. For convenience, the approach without considering MOAs, the approach considering the fixed pattern of MOAs and the proposed approach considering the worst impact of MOAs are referred to as A1, A2 and A3, respectively. Both A1 and A2 are implemented through the IP-EA. In the case study, the energy consumption periods of MOAs are assumed as shown in Table III within the OTIs and ranges of LOTs of MOAs in Table II for A2. It is noted that the fixed energy consumption of MOAs is assumed for A2 in the decision making process of SAs' energy consumption, the energy consumption of MOAs is still controlled by users in real time and can be consumed at anytime in OTIs with any possible LOTs. The following three aspects are compared.

1) Energy Consumption Schedule: Fig. 6 and Fig. 7 show the comparison of energy consumption schedule of SAs between A1 and A3, and A1 and A2, respectively. The gray area in Fig. 6 is the possible energy consumption period of the electric iron and the vacuum cleaner and Fig. 7 is the assumed energy consumption of the electric iron and the vacuum cleaner. We can see from Fig. 6 and Fig. 7 that, compared with A1, some SAs are shifted from low electricity price periods to periods with a little higher electricity price to avoid the risk of much higher electricity price charged in low price periods, which is caused by the excess of threshold set by IBR when operations of MOAs happen to be in these low price periods. More specifically, the energy consumption of the cloth dryer and the oven is shifted from 61-63, 72-74 time slots to 88-90, 83-85 time slots, respectively, to avoid all the possible consumption periods of the electric iron and the vacuum cleaner for A3. By comparison, only the energy consumption of the oven is shifted and the energy consumption of the cloth dryer still remains in low price period for A2. That is to say, A3 will consider all the possible cases of energy consumption of MOAs and A2 will only consider a certain pattern of MOAs' energy consumotion.

With the energy consumption schedules of SAs and the electricity price shown in Fig. 6 and Fig. 7, the electricity payments of A1, A2 and A3 are 24.48 cents, 25.07 cents and 25.13 cents, respectively, when all the MOAs are not used, and the worst electricity payments are 59.06 cents, 56.21 cents and 52.29 cents, respectively, when all the MOAs are used with the consideration of the worst impact of MOAs' energy consumption. Though compared with A1 and A2, the electricity payment of A3 increases by 2.66% and 0.24%, respectively, when all MOAs are not used, it is noted that the worst electricity payment of A3 drops by 11.46% and 6.97% when all the MOAs are used. The little sacrifice of SAs will help greatly to reduce the risk of much higher payment caused by the uncertainty of MOAs' energy consumption.

2) Electricity Payment with the Worst Impact of MOAs and RTP: Based on the energy consumption schedules of SAs obtained from A1, A2 and A3, the worst impact of uncertainty on RTP, i.e., 110% of the RTP used for energy scheduling, and the worst impact of MOAs' energy consumption are considered. As shown in Table IV, the electricity payment obtained by A3 is 57.89 cents, reduced 10.97% from A1 (65.02 cents) and 6.37% from A2 (61.83 cents). Note that MOAs are included to calculate the total electricity payment, together with SAs, even when the MOAs' impact is not considered in the decision making process of SAs' energy consumption schedule.

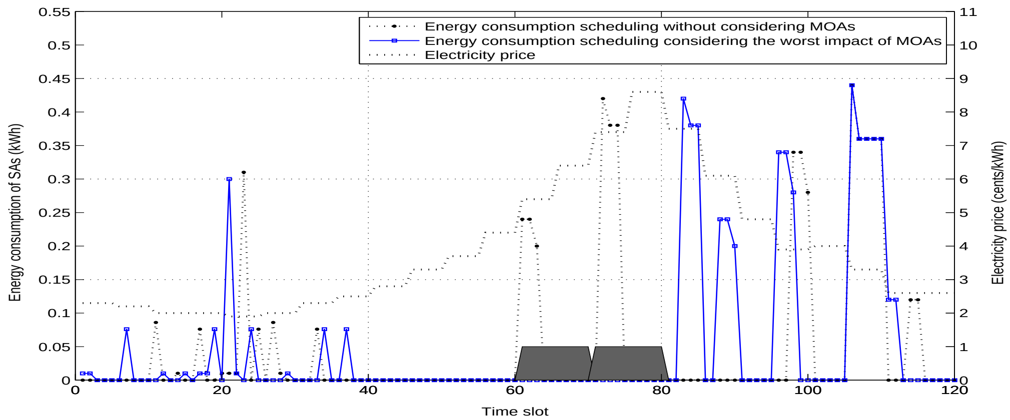


Fig. 6. Energy consumption comparison between the approach considering the worst impact of MOAs and the approach without considering MOAs

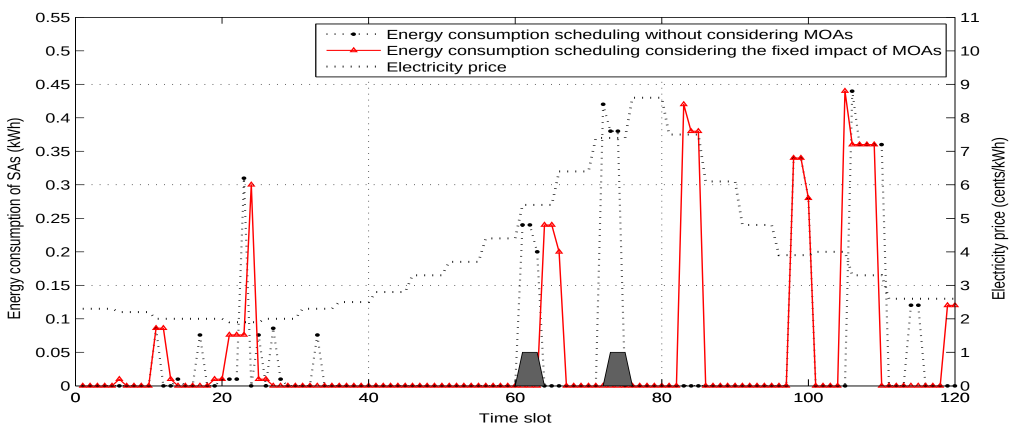


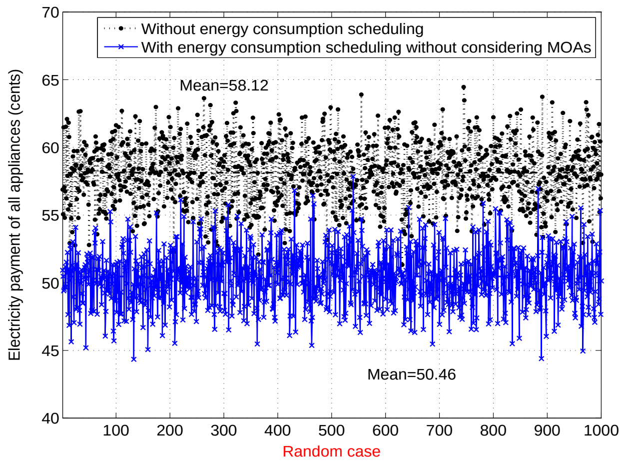
Fig. 7. Energy consumption comparison between the approach considering the fixed pattern of MOAs and the approach without considering MOAs

3) Electricity Payment with Random Impact of MOAs and RTP: The effectiveness of A3 is tested under 1000 random cases where the MOAs' energy consumption is evenly distributed among all the possible cases, and $\pm 10\%$ random noises are added into the RTP. Fig. 8 shows the comparison of electricity payment under random MOAs and uncertain RTP among the case without energy consumption scheduling of SAs (which the energy consumption of SAs is randomly distributed), and scheduling obtained by A1, A2 and A3 (as shown in Fig. 6 and Fig. 7), respectively. The average electricity payments of 1000 random cases are 48.42 cents (A3), 50.46 cents (A1), 49.76 cents (A2), and 58.12 cents for the situation without energy consumption scheduling, respectively. Through A3, the average electricity payment decreases by 16.69%, 4.04% and 2.69% compared with the situation without energy consumption scheduling, A1 and A2, respectively. The comparison of average electricity payment under random cases is tested through the hypothesis testing. The hypothesis that the average electricity payment of A3 is smaller than that of other situations is accepted with the probability of 99.95%. The procedure of the hypothesis testing is presented in Appendix [47] [48].

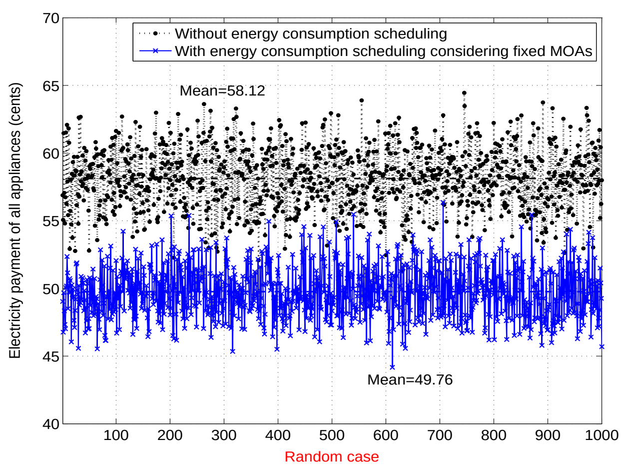
C. Impact of Electricity Price on MOAs' OTIs

Taking into account the possible impact of electricity price on users' preference, the OTIs of some MOAs would be more likely to be set in periods with low electricity price. To research on the impact of MOAs' OTIs being set in low price periods, the OTIs of electric iron, vacuum cleaner and hair drier are reset as follows, $[61-65]$, $[71-75]$, $[106-110]$, in periods with lower electricity price within the previous OTIs in Table II, and the OTIs of lights, laptop and TV remain the same since users' preference for the operations of these appliances seems not to be affected by the electricity price. With the new MOAs' OTIs, the electricity payment with random impact of MOAs and RTP is compared between A1 and A3. Under 1000 random cases of MOAs' energy consumption and RTP, the average electricity payment is 48.12 cents for A3 and it is 52.99 cents for A1. Through the proposed A3, the average electricity payment decreases by 9.19% compared with A1. The hypothesis that the average electricity payment of A3 is smaller than that of A1 is accepted with the probability of 99.95% [47] [48]. The procedure of the hypothesis testing is presented in Appendix.

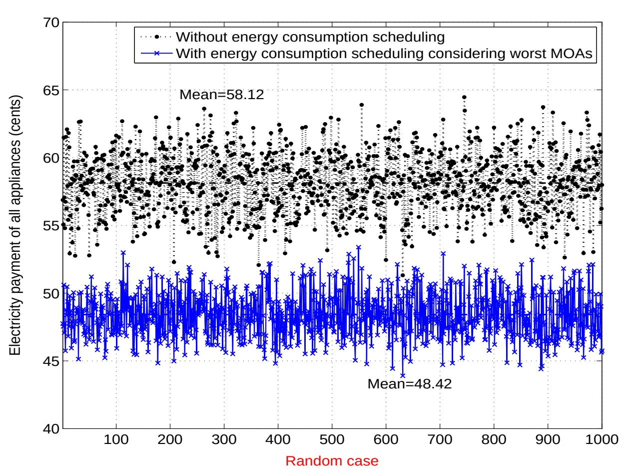
Table V shows the comparison of electricity payment with random impact of MOAs and RTP between A3 and A1.



(a)



(b)



(c)

Fig. 8. Comparison of electricity payment between (a) no scheduling and scheduling without considering MOAs (b) no scheduling and scheduling considering the fixed pattern of MOAs (c) no scheduling and scheduling considering the worst impact of MOAs

TABLE V
COMPARISON BETWEEN THE PROPOSED APPROACH AND THE APPROACH WITHOUT CONSIDERING MOAS

Electricity payment (cents)	Without price impact	With price impact
Approach without considering MOAs	50.46	52.99
Approach considering worst impact of MOAs	48.42	48.12
Percentage	-4.04%	-9.19%

Without the consideration of the impact of electricity price on MOAs' OTIs, the electricity payment decreases by 4.04% comparing A3 with A1, and it is 9.19% when the price impact on MOAs' OTIs is considered, i.e., users will benefit more from the proposed A3 compared with A1 when the possible impact of electricity price on MOAs' OTIs is taken into account. Since users will be more likely to operate MOAs in periods with low electricity price when the price impact on MOAs' OTIs is considered, and the energy consumption of SAs is scheduled in low price periods as well for A1, the risk of exceeding the threshold of energy consumption set by IBR increases for A1, and the proposed A3 helps reduce the risk effectively.

D. One Month Case with Different Combinations of Home Appliances

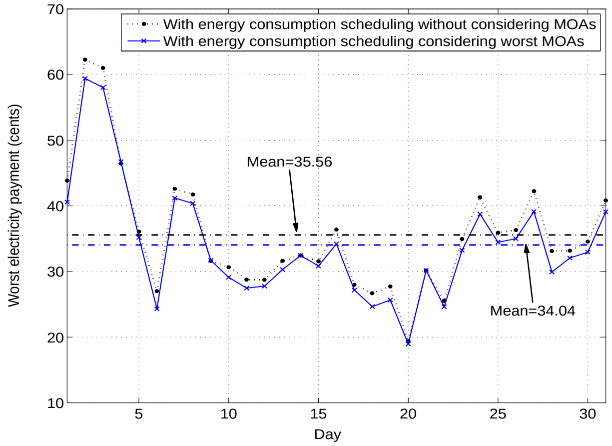
To further verify the effectiveness of the proposed A3, it is tested in one month period, August 2012, with different combinations of home appliances. Choosing $7-8$ SAs in Table I and $5-6$ MOAs in Table II, the electricity payment with the worst impact of MOAs and RTP, and the payment with random impact of MOAs and RTP are compared between A3 and A1, A3 and A2, respectively. As shown in Fig. 9, the average value of worst electricity payments in 31 days obtained by A3 is 34.04 cents, reduced 4.27% from A1 (35.56 cents) and 3.76% from A2 (35.37 cents). Fig. 10 shows the comparison of electricity payment with random impact of MOAs and RTP. The average values of payments with random impact of MOAs and RTP in 31 days are 29.42 cents (A3), 29.93 cents (A1) and 29.87 cents (A2), respectively. Through A3, the electricity payment with random impact of MOAs and RTP deceases by 1.70% and 1.51% compared with A1 and A2, respectively.

E. Case of MOAs' Usage Probability

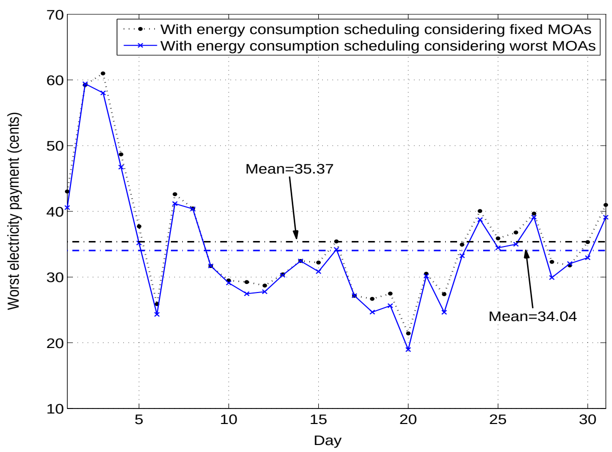
The proposed approach assumes that all MOAs will be used during the planning. Under this assumption, to avoid violating the IBR threshold and accommodate all MOAs, energy consumption of some SAs may be shifted to periods with a little higher electricity price, which may cause the increase of electricity payment if some MOAs are not actually used. Taking into account the usage probability of MOAs, a new approach of energy scheduling is formulated as

$$\min_{\mathbf{X}\in\mathbf{\Omega}} \sum_{s=1}^{NS} \rho_s \cdot \max_{\mathbf{U_s}\in\mathbf{\Gamma_s}} f(\mathbf{X},\mathbf{U_s}) \tag{16}$$

where s is the index of scenario and each scenario represents one combination of MOAs. Considering each MOA may be

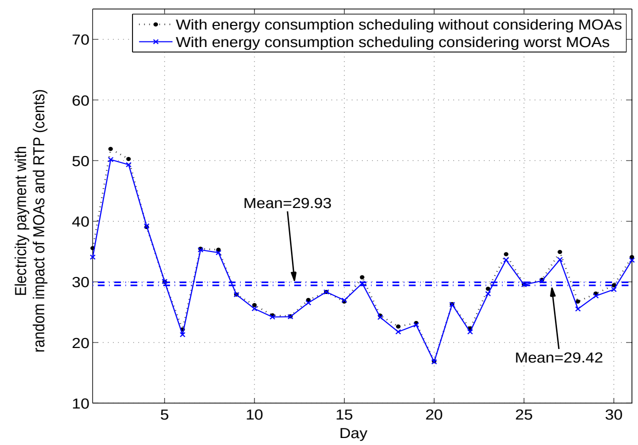


(a)

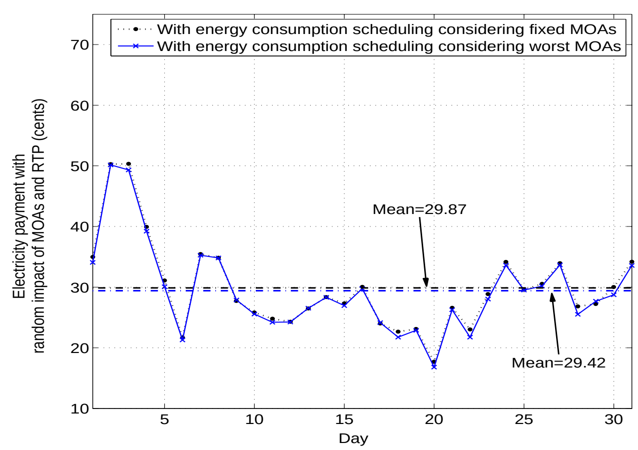


(b)

Fig. 9. Comparison of electricity payment with the worst impact of MOAs and RTP between (a) the proposed approach and the approach without considering MOAs (b) the proposed approach and the approach considering the fixed pattern of MOAs



(a)



(b)

Fig. 10. Comparison of electricity payment with random impact of MOAs and RTP between (a) the proposed approach and the approach without considering MOAs (b) the proposed approach and the approach considering the fixed pattern of MOAs

used or not, the total number of scenarios NS is 2^m for total m MOAs. ρ_s is the probability of scenario s and $\sum_{s=1}^{NS} \rho_s = 1$, $\mathbf{U_s}$ is the energy consumption case of MOAs in scenario s and $\mathbf{\Gamma_s}$ is the set of energy consumption cases of MOAs in scenario s. It is noted that it is the uncertainty of MOAs' energy consumption, i.e., when users start MOAs and how long users operate them, that is difficult to estimate rather than the probability distribution of MOAs' usage. In this section, the probability of MOAs' usage is taken into account, which is different from the probability distribution of uncertainty of MOAs' energy consumption. With the consideration of MOAs' usage probability, the energy consumption of SAs is now not scheduled considering all MOAs are used.

To test this new approach, the electric iron and the vacuum cleaner are assumed to be used with probability of 0.5 and other MOAs in Table II are assumed to be certainly used. By using the IP-EA, the energy consumption schedule of SAs on August 3rd 2012 is obtained and presented in Table VI. The electricity payment with the worst impact of MOAs and RTP, and the payment with random impact of MOAs and RTP are compared among the new approach, A1, A2 and A3, respectively. The worst electricity payments are 65.02, 61.83 and 57.89 cents for A1, A2 and A3, respectively, and remain the same as in Table IV, and it is 65.02 cents for the new approach. Electricity payments with random impact of MOAs and RTP are 42.08 cents (new approach), 42.50 cents (A1), 42.79 cents (A2) and 41.65 cents (A3), respectively. Note that the probability of MOAs' usage is now considered in the random cases of MOAs' energy consumption. Both the new approach and A3 have outperformed A1 and A2, and A3 still performs better at the electricity payment under random cases that some MOAs may not be used. It can be predicted that A3's performance may be degraded if the usage probability is reduced from 0.5, or the unused MOAs capacity increases.

VI. Conclusion

This paper has proposed a robust optimization approach for energy scheduling of home appliances, taking into account

TABLE VI
ENERGY CONSUMPTION SCHEDULE OF SAS

SA	Energy consumption period
Electric kettle	12
Clothes dryer	88-90
Oven	81-83
Water heater	96-98
Electric radiator	106-110
Dishwasher	118-119
Washing machine	17,21,22,24,32
Humidifier	1,13,15,17,21,26,27,28

the worst impact of the uncertainty of MOAs' energy consumption. The uncertainty of MOAs' energy consumption has been considered in the robust optimization approach with the consideration of the constraints of appliances, based on the electricity pricing scheme of RTP-IBR. The robust optimization approach has been implemented through the intergeneration projection evolutionary algorithm. The effectiveness of the proposed approach has been verified by case studies based on one day and one month periods. Compared with the scheduling approach without considering MOAs' uncertainty, and the scheduling approach considering MOAs' uncertainty with a fixed pattern, the proposed approach can effectively avoid the risk of high electricity payment caused by the MOAs' uncertainty and reduce the electricity payment with the worst impact of MOAs' energy consumption. The effectiveness of the proposed approach has also been verified under random cases of MOAs' energy consumption, in which the electricity payment of home appliances has also been reduced. Future research will focus on employing a global solver for the robust optimization problem and considering renewable energy generation, electric vehicle and distributed energy storage device.

APPENDIX
Process of Hypothesis Testing

Step 1. Hypothesis is defined: $\mu_* \leq \mu_{\mathrm{r}}$, where μ_* is the true value of the average electricity payment of the * approach and μ_{r} is the true value of the average electricity payment of the robust approach A3.
Step 2. Calculate t value:

$$t = \frac{\bar{x}_* - \bar{x}_{\mathrm{r}}}{\sqrt{\frac{s_*^2}{n_*} + \frac{s_{\mathrm{r}}^2}{n_{\mathrm{r}}}}}$$

where

$$\bar{x}_* = \frac{\sum_{i=1}^{n_*} x_{*i}}{n_*},\ \bar{x}_{\mathrm{r}} = \frac{\sum_{i=1}^{n_{\mathrm{r}}} x_{\mathrm{r}i}}{n_{\mathrm{r}}}$$

$$s_*^2 = \frac{\sum_{i=1}^{n_*} (x_{*i} - \bar{x}_*)^2}{n_* - 1}$$

$$s_{\mathrm{r}}^2 = \frac{\sum_{i=1}^{n_{\mathrm{r}}} (x_{\mathrm{r}i} - \bar{x}_{\mathrm{r}})^2}{n_{\mathrm{r}} - 1}$$

and x_{*i} and $x_{\mathrm{r}i}$ are the electricity payment of the * approach and the robust approach under ith random case of MOAs' energy consumption, respectively, and n_* and n_r are the number of random experiments (random cases of MOAs' energy consumption) of the * approach and the robust approach, respectively.
Step 3. Based on the desired chance of error $\alpha = 0.0005$ and the degree of freedom

$$\nu = \frac{(\frac{s_*^2}{n_*} + \frac{s_{\mathrm{r}}^2}{n_{\mathrm{r}}})^2}{\frac{s_*^4}{n_*^2(n_*-1)} + \frac{s_{\mathrm{r}}^4}{n_{\mathrm{r}}^2(n_{\mathrm{r}}-1)}},$$

find the corresponding t_{critical}, which can be obtained from the t-distribution table (Table 6.1 in [48]).
Step 4. Compare t and t_{critical}, if $t > t_{\text{critical}}$, the hypothesis is rejected.

In Section V.B, $t = 109.78$, 24.21 and 16.88 for the comparisons between A3 and the situation without energy consumption scheduling, A3 and A1, A3 and A2, respectively. In Section V.C, $t = 62.91$ for the comparison between A1 and A3. $t_{\text{critical}} = 3.291$ for all the above comparisons and since $t > t_{\text{critical}}$, all the hypothesises that the true values of the average electricity payments of other approaches are less or equal than the payment of the proposed A3 are rejected, i.e., the hypothesises that the true value of the average electricity payment of A3 is smaller than the payments of other approaches are accepted with the probability of 99.95% based on the desired chance of error $\alpha = 0.0005$.

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