Decentralized supply chain decisions on Lead time quote and pricing with a risk-averse supplier

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Abstract: We consider a decentralized supply chain containing a risk-averse supplier and a risk-neutral retailer with lead time- and price-sensitive demands. A Stackelberg game is employed to model the lead time quote and pricing decision process between the two members under the conditional value-at-risk (CVaR) criterion. A unique equilibrium is obtained. Using the corresponding centralized mode as a benchmark, we find that a less risk-averse supplier is better to cooperate and share risk with the retailer to improve the entire supply chain's efficiency. With a uniformly distributed realized lead time, the impact of the supplier's risk aversion on the decisions can be characterized by a few threshold values of the late delivery penalty cost. In particular, when the unit delay penalty cost exceeds a certain level, a more risk-averse supplier will counter-intuitively quote a shorter lead time by risking a higher delay penalty cost.

Keywords: decentralized supply chains; price; quoted lead time; risk aversion; Stackelberg game; conditional value-at-risk (CVaR)

1. Introduction

In addition to price, short and consistent delivery lead time has become an important factor in gaining competitive advantage in supply chain management (Li and Lee 1994; Lederer et al. 1997; Welborn 2008), especially when the market demand is both lead time-sensitive and price-sensitive (Boyaci et al. 2003; Ray et al. 2004; Zhao et al. 2012). However, the supply chain is often subject to uncertainty due to uncontrolled operational situations, such as machine breakdowns, material inconsistencies, quality errors, and process bottlenecks (Das et al. 2003). Thus, the realized delivery lead time of a customer's order may deviate from the promised one. Such deviation may incur costs to suppliers or service providers. Some transportation carriers and make-to-order suppliers of critical components, such as FedEx and Beta LAYOUT, a custom printed-circuit-board supplier, offer delay refunds (Afèche et al. 2013). The tardiness risk is common in other manufacturing industries as well. The initial public offering (IPO) prospectus of Zhejiang Canaan Technology Limited, a professional pharmaceutical equipment supplier, shows that the delayed delivery of orders leads to risks of default and customer churn (http://cybpdf.stcn.com/site2/20140825/3582694337990670222.PDF). From the supplier's viewpoint, on the one hand, a shorter quoted lead time may attract more customers and increase the market share; on the other hand, it may impose higher operational pressures and decrease on-time delivery performance, which will damage the supplier's reputation and incur tardiness costs. Therefore, the uncertain realized lead time may trigger a supplier's risk consciousness (Li, Lin et al. 2014), and the supplier should quote an appropriate lead time that carefully balances the two aspects above.

In reality, managers often show different risk preferences in their decision-making (Roussanov and Savor, 2014; Brenner, 2014), and their decisions under uncertainties are not always aimed at optimizing expected profit or cost (Wong, 2012, 2014; Shavit et al., 2014). A risk-averse manager and a risk-seeking manager can make quite different decisions in the same situation. Moreover, a supply chain member's risk preference has a great impact on the decisions of other members and the revenue of the whole supply chain (Tsay, 2002).

Because in some manufacturing (especially in the durable goods industry and equipment manufacturing), lead time is quoted by the supplier, the supplier should be more responsible for the tardiness than the retailer for long-term cooperation. In this case, the supplier may be more risk-averse than the retailer. Besides, globalization and standardization increases competition between suppliers with the same or similar products. Then, the retailer can shift to other suppliers due to high wholesale price (Shen et al. 2013), long lead time or bad timeliness. In this era of information explosion, bad reputation can spread quickly and make it difficult for the supplier to attract more retailers. Thus, the supplier will be more aware of risks. Moreover, if the delay badly causes losses to the retailer or the customers, the supplier should undertake the due obligations. For example, a retailer purchases frozen turkeys from a foreign supplier and the quoted lead time is December 20th. However, the supplier delivers the order on December 26th. Because the demand of turkey after Christmas decreases sharply and the price falls, the retailer suffers heavy losses. Then, the retailer has the right to require cancellation of the order and even indemnity from the supplier, according to Contract Law. For another example, in the furniture customization industry, late deliveries do occur due to bad weather, material shortages and other uncertain operational factors. In China, consumers also can cancel the order or claim for compensation from the retailer for severely late-delivered furniture. Accordingly, the retailer will share the tardiness cost with the supplier on the basis of their contract or agreement. Both the manufacturer and the retailer are responsible for the late delivery. Inversely, when the product is produced and delivered ahead of time, the customer can accept or refuse the delivery if the time is inappropriate for him. Then the product especially customized product has to be stored in the warehouse, which increases the supplier's holding costs (Li 2007), especially for drop-shipping suppliers.

Motivated by the supply chain lead-time uncertainty setting shown above with supply chain managers' risk attitude towards uncertainty, this paper aims to investigate lead-time quote and pricing decisions in a decentralized supply chain with a risk-averse supplier and a risk-neutral retailer. In previous research, decisions of risk-averse suppliers with risk-neutral retailers have been studied on other supply chain management issues (Shen 2013a, Shen 2013b, Yoo 2014). In practice, the operation timeline is as follows: (1) Notified of the market information (i.e., customer sensitivity about lead time and price) by the retailer, the supplier quotes the lead time and the wholesale price. (2) The retailer decides on the retail price according to the supplier's decisions. (3) The quoted lead time and the retail price are offered to the customers. (4) When a customer places an order with the retailer, it will be passed to the supplier. (5) The supplier ships the completed product to the retailer's warehouse or directly to the customer's home. Then, the lead time is realized, which may differ from the quoted lead time. Because the market demand is sensitive to the supplier's quoted lead time and the retailer's selling price, the decision making of both parties becomes a Stackelberg game with the supplier as the leader and the retailer as the follower.

A substantial amount of research has discussed lead time management and lead time uncertainty issues. However, to date, only a few studies on lead time issues have taken the managers' risk preferences into account. Afèche et al. (2013) investigate revenue-maximizing tariffs that depend on realized lead times for a provider serving multiple time-sensitive customer types. They assume that customers are risk-averse and do not consider the risk preferences of the supply sides. Li (2007) investigates the effects of lead time, lead time uncertainty and the risk aversion of the supplier on the optimal production policy. Li et al. (2014b) examine how the risk aversion of the decision maker influences pricing and lead time strategies. However, both these two papers only create a single-firm framework. To the best of our knowledge, there is no existing research that has addressed the influence of managers' risk aversion on the optimal lead-time and pricing decisions in the supply chain context. This paper attempts to fill this research gap.

This paper examines how the supplier's risk aversion influences the optimal price selection, the promised delivery lead time, and the utility of both members in the supply chain. Moreover, we introduce a centralized supply chain as a benchmark to investigate the performance of the decentralized supply chain. We also study the interaction of the supplier's risk aversion and other parameters, including marketing sensitivity and operational costs. The main findings are summarized as follows:

(i) The more risk-averse the supplier, the lower the demand rate, the lower the supplier's utility, and the lower the retailer's profits.

(ii) The decision efficiency of the decentralized supply chain improves when the supplier becomes more risk-averse.

(iii) Numerical examples show that if the supplier can improve the timeliness of the delivery, then the utility of the entire supply chain (both decentralized and centralized supply chains) may increase, whereas the efficiency of the supply chain may decrease. Here, delivery timeliness is defined as the maximum possible value of the supplier's inherent lead time, which is independent of the external demand rate.

(iv) In special cases, such as the realized lead time following uniform distribution, the impact of the supplier's risk aversion on the optimal quoted lead time, the optimal wholesale price and the optimal retail price decisions can be characterized by a few threshold values associated with the unit penalty cost of late delivery. A counter-intuitive finding is observed, that is, when the unit delay penalty cost reaches a certain level, a more risk-averse supplier will quote a shorter lead time, which essentially indicates that the supplier is willing to risk paying higher delay penalty cost.

The managerial insights above, together with the analytical models and results established in this paper, make new contributions to the literature and could help the managers with risk aversion make appropriate decisions on lead time quotes and pricing in supply chain management.

2. Literature Review

There are three streams of literature related to our paper. The first involves pricing and lead-time decisions in the supply chain. The second focuses on lead time uncertainty. The last stream examines the impacts of the members' risk aversion behaviors on supply chain decisions.

First, research on pricing and lead-time decisions in the supply chain has achieved fruitful results. For example, Boyaci and Ray (2003) investigate the lead-time and pricing decisions of a firm selling two substitutable products with capacity costs. Pekgün et al. (2008) consider the coordination of marketing and production departments in a firm that serves customers who are sensitive to quoted price and lead time. Zhao et al. (2012) study whether a firm will offer a single lead time and price quote or a menu of lead times and prices for customers to choose from. All the research above just examines a single firm's decisions. The competition and cooperation among firms in the supply chain have also been studied. Hua et al. (2010) use a two-stage optimization technique and a Stackelberg game to determine the optimal decisions on delivery lead time and prices in a centralized and a decentralized dual-channel supply chain. Hong et al. (2012) focus on pricing and lead time competition in a duopoly industry consisting of two large firms and several small firms with Nash Equilibrium. Boute et al. (2014) study the coordination of the retailer's inventory decisions and the supplier's lead times on a make-to-order basis.

With regard to lead time uncertainty, previous research has provided deep insights into this problem. Liu et al. (2007) examine the optimal wholesale price, retail price and planned lead-time decisions in a decentralized supply chain with uncertain realized lead time. Kouvelis et al. (2008) consider replenishment decisions for a supplier with uncertain lead times in a constant demand-rate environment. Maiti et al. (2009) study inventory problem with advance payment incorporating price dependent demand and stochastic lead time. Wu et al. (2012) extend the newsvendor problem with price and lead-time decisions under lead-time and demand uncertainty. Xu and Rong (2012) address the lead time uncertainty problem in two-echelon supply chain systems with a framework, which inherits the methodology of the minimum variance control theory. Fang et al. (2013) analyze the marginal effect on the cost of reductions in (i) lead time mean, (ii) lead time variance, and (iii) demand variance. All of the aforementioned literature implicitly assumes that the members' risk preferences are risk-neutral, which ignores the influences of the agents' risk-averse behaviors on decisions in the supply chain. In this paper, we extend the research of Liu et al. (2007) and suppose the supplier is risk-averse under lead time uncertainty.

A few methods exist to evaluate the risk degree of supply chain members, such as the mean-variance analysis method (Markowitz, 1959), the value-at-risk (VaR) method and the conditional value-at-risk (CVaR) method (Artzner, 1997). Among these methods, CVaR has recently attracted much attention, partially because it is a coherent risk measure, has a clear and suitable economic interpretation and can be written as a linear stochastic programming model (Rudloff, 2014). Ozgun et al. (2011) investigate the potential impact of the retailer's risk aversion on both the manufacturer's and the retailer's rebate promotions under the CVaR criterion. Li, Chen et al. (2014) consider a Nash bargaining problem in a dual-channel supply chain with stochastic demand under the CVaR criterion. Koenig and Meissner (2015) model a dynamic capacity control decision of a risk-averse monopolistic firm with the CVaR criterion. In this paper, CVaR is adopted as the decision criterion to measure the risk-averse supplier's performance.

In fact, the research of Liu et al. (2007) and Li et al. (2014b) is the most related to this paper. The decentralized supply chain model in Liu et al. (2007) can be regarded as a special case of our model by letting the supplier's risk indicator take a value of 1, i.e., become risk-neutral. Liu et al. (2007) focus on the comparison of the centralized and the decentralized supply chain, while this paper concentrates on the impacts of the supplier's risk aversion on the lead time decisions and the utilities of supply chain members. The differences between the research of Li et al. (2014b) and this paper are: (1) Li et al. (2014b) study a single firm while this paper considers a supply chain environment and the members' game process. (2) Referring to Liu et al. (2007) and Wu et al. (2012), we define the distribution of the realized lead time as a multiplicative form consisting of a function of the external demand rate and a random variable representing the system's inherent realized lead time. Different from Li et al. (2014b), this formulation simplifies the calculation and is general to cover different scenarios. (3) Li et al. (2014b) consider two decision schemes: quoted lead time and price are independent variables; price dependents on quoted lead time. The game process in this paper obtains the optimal price given any quoted lead time first, and then the optimal lead time and the optimal demand rate. (4) Due to different settings, different calculation methods and different decision processes, the optimal decisions and the effects of the parameters we find are also different from those in Li et al. (2014b). For example, our numerical study illustrates that when the consumers are more price sensitive, the supplier tends to quote a shorter lead time, which is the opposite in Li et al. (2014b).

The remainder of the paper is organized as follows. In Section 3, we formulate the decision models and provide the equilibrium solution. In Section 4, we carry out model discussions to examine the performance of the decentralized supply chain and the effects of various parameters. Finally, section 5 concludes and indicates future research opportunities.

3. Decision Models

3.1 Basic Model

We consider a supply chain consisting of a risk-averse supplier and a risk-neutral retailer selling a single type of product in a competitive market. The supplier decides on the quoted lead time and the wholesale price, while the retailer decides on the retail price. Let λ denote the demand rate from the market, which is sensitive to the quoted lead time (l) and the retail price (p). We assume a linear demand model as follows (Zhao et al. 2012):

$$\lambda(p,l) = \lambda_0 - \alpha p - \beta l, \tag{1}$$

where λ_0 represents the market size, and α β represent the market's sensitivity factors to the retail price and the quoted lead time, respectively.

The supplier's cost in relation to the lead time includes the holding cost of the finished goods when an order is delivered earlier than the quoted lead time and the penalty cost when an order is tardy. Let T denote the realized delivery lead time, which is a random variable with the cumulative distribution function (cdf), $R_\lambda(t)$ and the probability density function (pdf), $r_\lambda(t)$, depending on the demand rate, λ. Then, the supplier's cost associated with the delivery lead time is given by

$$C(l,\lambda) = h(l-T)^+ + b(T-l)^+, \tag{2}$$

where h denotes the holding cost per unit item per unit time; b denotes the penalty cost per unit item per unit time; $b > h \geq 0$ represents the practical scenarios; and $(l-T)^+ = \max(0, l-T)$.

Thus, the supplier's profit rate is

$$\pi_s(p,w,l) = [w - c_s - C(l,\lambda)]\lambda(p,l), \tag{3}$$

where w and c_s denote the supplier's wholesale price and unit operating cost, respectively.

If the supplier is risk-averse, we adopt the CVaR criterion to evaluate his performance. According to Rockafellar and Uryasev (2000), the supplier's objective is to maximize the following utility function:

$$CVaR_{\eta}\left(\pi_s\left(p,w,l\right)\right)=\max_{v\in R}\left\{v+\frac{1}{\eta}E\left[\min\left(\pi_s\left(p,w,l\right)-v,0\right)\right]\right\}, \tag{4}$$

where E is the expectation operator, and $\eta\in(0,1]$ is a parameter to represent the degree of the supplier's risk aversion, which is called a supplier's risk indicator. Here, $\eta=1$ means that the supplier is risk neutral; otherwise, the smaller η is, the more risk-averse the supplier will be. In addition, v denotes the probable upper limit of the supplier's profit under a certain risk level of η, and R denotes the real set.

Meanwhile, the retailer's profit rate is

$$\pi_r(p,w,l)=(p-w-c_r)\lambda(p,l), \tag{5}$$

where c_r denotes the retailer's variable cost per unit. For any given $\lambda_0,\alpha,\beta,c_r$ and w,l, because the demand rate must be positive to ensure that product sales are profitable, the retail price should satisfy the following condition:

$$w+c_r<p<p^{\max}-c_w l\,, \tag{6}$$

where $p^{\max}=\lambda_0/\alpha$ is the maximum feasible retail price, and $c_w=\beta/\alpha$ can be interpreted as the customer's waiting cost per unit for the quoted delivery lead time, l (Wu et al. 2012).

Therefore, the objective of the retailer who is considered to be risk-neutral is to maximize the following utility function:

$$\max_p \pi_r(p,w,l)=\max_p(p-w-c_r)\lambda(p,l)\,. \tag{7}$$

3.2 The Decisions through the Stackelberg Game

Consider the supplier as a Stackelberg game leader who determines the wholesale price, w, and the quoted lead time, l, first; the retailer is then a follower who decides the retail price, p, based on the supplier's decision. We give the retailer's best response function first and then propose a model to decide the Stackelberg equilibrium strategies of the two players.

From Eq.(7), the best response of the retailer for the given l and w can be obtained as follows:

$$p^*=\frac{1}{2}(p^{\max}-c_w l+w+c_r). \tag{8}$$

It can be proved that p^* satisfies condition (6). Hence, the retailer's optimal profit can be written as a function of λ, that is, $\pi_r^*=\lambda^2/\alpha$.

Next, we consider the supplier's lead time strategy. Plugging Eq. (8) into Eq. (1), we get $\lambda=[\lambda_0-\beta l-\alpha(w+c_r)]/2$. Rewrite it, and we then have

$$w=p^{\max}-c_w l-2\lambda/\alpha-c_r. \tag{9}$$

Plug Eq. (9) into Eq. (3). Then, the supplier's profit rate can be written as

$$\pi_s(\lambda,l)=[p^{\max}-c_w l-2\lambda/\alpha-c_r-c_s-C(l,\lambda)]\lambda. \tag{10}$$

Thus, the supplier's utility can be expressed as

$$CVaR_\eta(\pi_s(\lambda,l))=\max_{v\in R}\left\{v+\frac{1}{\eta}E[\min(\pi_s(\lambda,l)-v,0)]\right\}. \tag{11}$$

Plug Eq. (10) into Eq. (11), and let $A=p^{\max}-c_r-c_s$. Then, we have

$$\begin{aligned}CVaR_\eta(\pi_s(\lambda,l))=\max_{v\in R}\Bigg\{&v-\frac{1}{\eta}\int_0^l\{v-[A-c_w l-2\lambda/\alpha-h(l-t)]\lambda\}^+dR_\lambda(t)\\&-\frac{1}{\eta}\int_l^{+\infty}\{v-[A-c_w l-2\lambda/\alpha-b(t-l)]\lambda\}^+dR_\lambda(t)\Bigg\}.\end{aligned} \tag{12}$$

Following the definition of CVaR and the method in Li, Lin et al. (2014), the optimal v^* can be obtained from Eq. (12), and the optimal lead-time decision, l^*, and the corresponding demand rate, λ^*, can then be calculated. We use a sequential procedure to find the optimal lead time, l^*, with a given λ first. Then, we plug $l^*(\lambda)$ into the supplier's utility function to solve a single-variable problem to obtain the optimal λ^*. The results are described in the following Lemmas 1 and 2 and Proposition 1.

Lemma 1. *For a fixed λ, the supplier's optimal quoted lead time is given by*

$$l^*=(1-D)R_\lambda^{-1}(C\eta)+DR_\lambda^{-1}(1-(1-C)\eta), \tag{13}$$

where $C=\dfrac{b-c_w}{h+b}, D=\dfrac{b}{h+b}, b>c_w$, *and* $R_\lambda(t)$ *is the* cdf *of the realized lead time.*

It should be noted that it is usually very difficult to specify the distribution of the realized lead time from a real supply chain. Most literature on similar problems often assumes the lead time as the steady-state customer sojourn time in an M/M/1 queue system (Boyaci and Ray 2003; Zhao et al. 2012). However, the M/M/1 lead time distribution has technical restrictions, and the practical operation environment can actually hardly match its strict assumptions (Wu et al. 2012). Hence, a more general lead time model is desirable. According to Liu et al. (2007), the realized lead time may be determined by the external demand rate, λ, and the system's internal factors, which are independent of λ. Thus, we assume that the random lead time, T, has the multiplicative form: $T=g(\lambda)U$, where U represents the system's inherent realized lead time independent of the external demand rate, and it is a random variable that follows the cdf, $\Phi(u)$ and the pdf, $\phi(u)$; $g(\lambda)$ is a nonnegative, twice differentiable and increasing convex functions of λ. Through our knowledge of statistics, we have $R_\lambda(t)=\Phi(t/g(\lambda))$ and $r_\lambda(t)=\phi(t/g(\lambda))/g(\lambda)$.

Note that $E\left(g\left(\lambda\right)U\right)=g\left(\lambda\right)E\left(U\right)$ and $Var\left(g\left(\lambda\right)U\right)=g^2\left(\lambda\right)Var\left(U\right)$. Therefore, $g(\lambda)$ can be interpreted as an external factor that affects the scale and variability of the random realized lead time. The rationale behind the lead time model above is that the delivery lead time may increase (e.g., proportionally) as the external demand rate increases because of possible longer production time and/or more required transportation resources. Moreover, this lead time model is general enough to cover a range of different settings, e.g., it can represent the GI/M/1 queue system (Liu et al. 2007). Technically, this model can also overcome computation difficulties to find the optimal decisions.

Corollary 1. *(1) If* $g\left(\lambda\right)=1$, *i.e.,* $T=U$, *where the random variable* U *follows the cdf* $\Phi(u)$, *then the optimal quoted lead time is given by*

$$\bar{l}=(1-D)\Phi^{-1}\left(C\eta\right)+D\Phi^{-1}\left(1-(1-C)\eta\right). \tag{14}$$

(2) If $g\left(\lambda\right)$ *takes a general form, then*

$$l^*(\lambda)=g(\lambda)\bar{l}. \tag{15}$$

Eq. (14) is the direct result of Lemma 1. Here, $\bar{l}$ can be regarded as the system's configuration lead time, i.e., the lead time quoted to an unfilled order under the assumption that the realized delivery lead time would be independent of the demand rate. From $R_\lambda(t)=\Phi(t/g(\lambda))$, it is easy to derive that $R_\lambda^{-1}(k)=\Phi^{-1}(k)\cdot g(\lambda)$. Comparing Eq. (14) with Eq. (13), we obtain Eq. (15).

Further, plugging Eq. (15) into the supplier's objective function (12) to solve a single-variable problem for the optimal demand rate λ, we can obtain the following Lemma 2.

Lemma 2. *The optimal demand rate,* λ^*, *is the unique solution to the following equation:*

$$\left(A-\frac{4\lambda^*}{\alpha}\right)-\left[g\left(\lambda^*\right)+\lambda^* g'\left(\lambda^*\right)\right]\cdot\left(c_w\bar{l}+\varphi\right)=0, \tag{16}$$

where $\varphi=\frac{h}{\eta}\int_0^{\Phi^{-1}(C\eta)}(\bar{l}-u)d\Phi(u)+\frac{b}{\eta}\int_{\Phi^{-1}(1-(1-C)\eta)}^{+\infty}(u-\bar{l})d\Phi(u)$, *which stands for the supplier's average lead time-related costs, including the holding cost and the tardiness (penalty) cost.*

By computation, we find that $c_w\bar{l}+\varphi=-\frac{h}{\eta}\int_0^{\Phi^{-1}(C\eta)}ud\Phi(u)+\frac{b}{\eta}\int_{\Phi^{-1}(1-(1-C)\eta)}^{+\infty}ud\Phi(u)$ does not contain $\bar{l}$ any more, that is, λ^* is independent of $\bar{l}$.

Hence, combining Lemma 1 and Lemma 2, we have Proposition 1 as follows.

Proposition 1. *The unique Stackelberg equilibrium for the decentralized supply chain with a* *risk-averse supplier under the CVaR criterion is given by*

$$p^* = \frac{1}{2}(p^{\max} - c_w l^* + w^* + c_r),$$

$$w^* = p^{\max} - c_w l^* - 2\lambda^* / \alpha - c_r,$$

$$l^* = (1-D)R_{\lambda^*}^{-1}\left(C\eta\right) + DR_{\lambda^*}^{-1}\left(1-(1-C)\eta\right).$$

In addition, if $T = g(\lambda)U$, *then the optimal demand rate,* λ^*, *satisfies the following equation:*

$$\left(A - \frac{4\lambda^*}{\alpha}\right) - [g(\lambda^*) + \lambda^* g'(\lambda^*)] \cdot (c_w \overline{l} + \varphi) = 0.$$

The retailer's optimal profit and the supplier's optimal utility are

$$\pi_r^* = (\lambda^*)^2 / \alpha, \tag{17}$$

$$CVaR_\eta(\pi_s)^* = \frac{2(\lambda^*)^2}{\alpha} + (\lambda^*)^2 \cdot (c_w \overline{l} + \varphi) \cdot g'(\lambda^*). \tag{18}$$

It should be noted that the optimal demand rate, λ^*, is given implicitly in Proposition 1. An explicit form of λ^* is possible only if we know the specific distribution of the delivery lead time.

For better understanding, we consider a special case as an example by assuming that the realized lead time follows a uniform distribution, i.e., $T \sim \mathrm{U}(0, l_{\max})$. Similarly, Wanke (2008) and Sazvar et al. (2013) also consider the supply lead time as a random variable with uniform distribution. Therefore, the system's inherent realized lead time U follows $U \sim \mathrm{U}(0, u_{\max})$ and $l_{\max} = g\left(\lambda\right) u_{\max}$. Here, $u_{\max}$ is the maximum possible value of the system's inherent realized lead time independent of the external demand rate, and it can represent the supplier's delivery timeliness. We also assume that the demand factor $g\left(\lambda\right) = \lambda$. Based on these assumptions, we can obtain the optimal decisions for the decentralized supply chain using Proposition 1.

Corollary 2. *If the realized lead time,* T, *follows a uniform distribution with support over* $(0, l_{\max})$ *and the demand factor* $g\left(\lambda\right) = \lambda$, *then the optimal demand rate is*

$$\lambda^* = \frac{A\alpha\left(b+h\right)}{4\left(b+h\right) + \alpha u_{\max}\left[2b\left(h+c_w\right) - \eta\left(bh + c_w^2\right)\right]}, \tag{19}$$

and the optimal quoted lead time is

$$l^{*}=\frac{A\alpha u_{\max}\left(b-c_{w}\eta\right)}{4\left(b+h\right)+\alpha u_{\max}\left[2b\left(h+c_{w}\right)-\eta\left(bh+c_{w}^{2}\right)\right]}. \tag{20}$$

From Corollary 2, it is easy to show that the optimal quoted lead time l^{*} increases in the unit delay cost b.

4. Model Discussion

In this section, we use the corresponding centralized supply chain as a benchmark and define the decision efficiency to investigate the performance of the decentralized supply chain in relation to the supplier's risk indicator. We then perform a sensitivity analysis to examine the impacts of some key parameters on both members' decisions. Further, we explore the management implications of these impacts.

4.1 Analysis of the utilities and supply chain efficiency

First, we explore the impacts of the supplier's risk aversion and other parameters on the utilities of the supplier and the retailer. Moreover, we also investigate the relationship between the supplier's risk aversion and the decentralized supply chain's efficiency. From Eq. (17) and Eq. (18), the following propositions can be obtained.

Proposition 2. *The relationships between* $\left(\lambda^{*}, CVaR_{\eta}(\pi_{s})^{*}, \pi_{r}^{*}\right)$ *and different parameters are found as follows:*

(1) When the supplier's risk indicator η *increases,* λ^{*}*,* $CVaR_{\eta}(\pi_{s})^{*}$ *and* π_{r}^{*} *also increase.*

(2) When the unit holding cost, h*, or the unit penalty cost,* b*, increases,* λ^{*}*,* $CVaR_{\eta}(\pi_{s})^{*}$ *and* π_{r}^{*} *decrease.*

(3) When the market price sensitivity factor, α*, or the lead time sensitivity factor,* β*, increases,* λ^{*}*,* $CVaR_{\eta}(\pi_{s})^{*}$ *and* π_{r}^{*} *decrease.*

Proposition 2(1) provides a generic result of the structural relationship between $\left(\lambda^{*}, CVaR_{\eta}(\pi_{s})^{*}, \pi_{r}^{*}\right)$ and η. In other words, under a general distribution of the system's inherent realized lead time, *U*, the optimal market demand rate will decrease as the supplier becomes more risk averse, and the utilities of both the supplier and the retailer will also decrease accordingly.

Proposition 2(2) states that when the unit holding cost or the unit penalty cost increases, the demand rate will decrease, as will the utilities of the supplier and the retailer. Therefore, the higher the lead time-related unit costs are, the more customers and profits the supply chain will lose. The implication is that the decision makers should pay attention to lead-time cost management because the reduction of the unit holding or tardy cost would benefit both the supplier and the retailer.

Proposition 2(3) suggests that if the market becomes more price-sensitive or more lead time-sensitive, the optimal market demand rate and the utilities of the supplier and the retailer will all decrease. This relationship may be explained by the fact that more price-sensitive customers may shift orders to lower-price retailers, whereas more lead time-sensitive customers may shift orders to other stores with quicker services, which results in lower demand rates and consequently reduces the utilities of the supplier and the retailer.

To examine the decentralized supply chain's efficiency, we use the corresponding centralized supply chain as a benchmark for comparative purposes. We will further investigate the impacts of the decision maker's risk aversion on the decision efficiency of the decentralized supply chain.

For the centralized supply chain, the utility of the total supply chain is defined as follows:

$$\pi_c(p_c, l_c) = CVaR_\eta(\pi_s^c) + \pi_r^c = \max_{v_c \in R}\left\{v_c + \frac{1}{\eta}E[\min(\pi_s^c - v_c, 0)]\right\} + \pi_r^c, \tag{21}$$

where π_s^c is given by (3) and π_r^c is given by (5). Here the superscript c is used to denote the corresponding variables in the centralized supply chain.

We use the same method applied in the decentralized mode to solve for the optimal $p_c^*, l_c^*, \lambda_c^*$. Hence, Proposition 3 can be obtained.

Proposition 3. *If the optimal decisions for the centralized supply chain with a risk-averse supplier under the CVaR criterion is given by* (p_c^*, l_c^*)*, then*

$$p_c^* = p_{\max} - c_w l_c^* - \lambda_c^* / \alpha, \tag{22}$$

$$l_c^* = (1-D)R_{\lambda_c^*}^{-1}(C\eta) + DR_{\lambda_c^*}^{-1}(1-(1-C)\eta). \tag{23}$$

If $T = g(\lambda_c)U$*, then the optimal demand rate,* λ_c^**, satisfies the following equation:*

$$\left(A - \frac{2\lambda_c^*}{\alpha}\right) - [g(\lambda_c^*) + \lambda_c^* g'(\lambda_c^*)] \cdot (\varphi + c_w \bar{l}) = 0. \tag{24}$$

The optimal utility of the centralized supply chain is

$$\pi_c^* = \frac{(\lambda_c^*)^2}{\alpha} + (\lambda_c^*)^2 \cdot (c_w \bar{l} + \varphi) \cdot g'(\lambda_c^*). \tag{25}$$

Define $e_\pi = \left(\pi_r^* + CVaR_\eta(\pi_s)^*\right) / \pi_c^*$ as the decision efficiency of the decentralized supply chain. To establish the relationship between the supplier's risk aversion and the decision efficiency of the decentralized supply chain, we introduce the following Lemma 3.

Lemma 3. *The optimal demand rates of the two supply chain modes satisfy the following inequalities:* $\lambda^* < \lambda_c^* < 2\lambda^*$.

The result of Lemma 3 is similar to that of the risk-neutral supply chain cases given in Liu et al. (2007). Lemma 3 indicates that the centralized supply chain can capture a higher demand rate than the decentralized supply chain, but the demand rate will not exceed $2\lambda^*$ for a given market environment and system condition.

Proposition 4. *If* $g(\lambda)=\lambda^n(n=0,1,2,3)$*, then* e_π *decreases as* η *increases.*

Proposition 4 indicates that the decentralized supply chain's efficiency can be improved if the supplier is more risk-averse, probably because the supplier's risk aversion has a stronger effect on the centralized supply chain's utility than that of the decentralized supply chain. The implication is that if the supplier is less risk-averse, it is probably more necessary to adopt the centralized supply chain because seeking cooperation between two members can share the risk of lead time uncertainty.

4.2 Analysis of the decisions

Next, we discuss the impacts of the supplier's risk-averse degree η, and other parameters on the optimal pricing and lead-time decisions in the decentralized supply chain.

Proposition 1 indicates that the decision variables (p^*, w^*, l^*) are highly related to the probability distribution of the realized lead time, and there is no closed form for the optimal demand rate λ^*, under the general lead time distribution. Therefore, it is difficult to establish the relationship between η and the decision variables for general distribution cases. For analytical tractability, we consider a special case in which the realized lead time follows a uniform distribution and the demand factor $g(\lambda)=\lambda$ to simplify the analysis in this sub-section. The optimal decisions for this case can be obtained through Corollary 2. Therefore, the following propositions can be deduced.

Proposition 5. *If the realized lead time,* T*, follows a uniform distribution with support over* $(0,l_{\max})$ *and the demand factor* $g(\lambda)=\lambda$*, then there is a set of threshold parameters* (b_1,b_2,b_3) *subject to* $b_1<b_2<b_3$ *that can characterize the following structural properties:*

(1) When $b\in(0,b_1)$, $\dfrac{dl^*}{d\eta}<0, \dfrac{dp^*}{d\eta}>0, \dfrac{dw^*}{d\eta}>0$.

(2) When $b\in(b_1,b_2)$, $\dfrac{dl^*}{d\eta}<0, \dfrac{dp^*}{d\eta}>0, \dfrac{dw^*}{d\eta}<0$.

(3) When $b\in(b_2,b_3)$, $\dfrac{dl^*}{d\eta}<0, \dfrac{dp^*}{d\eta}<0, \dfrac{dw^*}{d\eta}<0$.

(4) When $b\in(b_3,\infty)$, $\dfrac{dl^*}{d\eta}>0, \dfrac{dp^*}{d\eta}<0, \dfrac{dw^*}{d\eta}<0$.

Proposition 5 provides interesting managerial insights. It describes the relationships between the supplier's risk aversion and the lead-time and pricing decisions in relation to the supplier's unit penalty cost, b, for delayed orders. Such relationships can be characterized by three threshold levels and be divided into four cases:

Case 1: When the unit penalty cost is very low, the more risk-averse the supplier, the longer the quoted lead time the supplier will promise, and the lower the wholesale price he will set. Meanwhile, the retailer will select a lower selling price to balance the impact of the longer quoted lead time and the lower wholesale price.

Case 2: When the unit penalty cost is a bit low, the more risk-averse the supplier, the longer the lead time he will quote, and the higher the wholesale price he will set to balance the impact of the low unit penalty cost. However, as the demand rate is affected by the longer lead time, the retailer will still decrease the retail price to attract more customers.

Case 3: When the unit penalty cost is a bit high, the more risk-averse the supplier, the longer the lead time he will quote, and the higher the wholesale price he will set. Due to the higher wholesale price, the retailer will also increase the retail price to guarantee a positive unit profit rate.

Case 4: When the unit penalty cost is very high, the more risk-averse the supplier, the shorter the lead time he will quote. This case appears to be counterintuitive because the supplier will risk paying higher delay penalty by quoting a shorter lead time. According to common sense, the extremely high penalty cost will push the supplier to quote a rather long lead time, which, however, makes the supplier lose competitiveness with respect to time. Thus, such counterintuitive behavior may be explained by the fact that the risk-averse supplier tends to attract more customers and set higher wholesale prices to cancel out the higher delay penalty. In practice, the shorter quoted lead time may justify the higher wholesale price. Similar to Case 3, the retailer will increase the retail price accordingly.

It is worth noting that because the values of $\left(b_1, b_2, b_3\right)$ have been given in closed forms in Appendix, the results of Propositions 5 are easy to calculate and implement from a practical application perspective.

Because the quoted lead time is directly associated with the market lead time sensitivity factor, β, and the supplier's unit holding cost, h, we specifically discuss the impacts of these two parameters on the decisions under the general lead time distribution. The results are summarized in Proposition 6.

Proposition 6. *The relationships between* (p^*, w^*, l^*) *and* β and h are found as follows*:*

(1) *When the market lead time sensitivity factor,* β*, increases, the optimal quoted lead time,* l^**, decreases.*

(2) *When the holding cost per unit per unit time,* h*, increases,* l^**, decreases, while the optimal retail price,* p^**, and the optimal wholesale price,* w^**, increase.*

Proposition 6 shows that if the market becomes more lead time-sensitive, the supplier's optimal quoted lead time will be shortened to avoid losing too many customers. As the unit holding cost increases, it is better for the supplier to quote a shorter lead time. The optimal wholesale price and the optimal retail price will then increase to balance the impact of a shorter quoted lead time. In some situations, the customers are willing to receive shipments earlier than the quoted lead time. This situation can be represented by setting the holding cost, h, to zero.

Regarding the price sensitivity factor, α, because we cannot rigorously verify its impact, we will investigate it in the numerical experiment shown in Figure 1 with the realized lead time following uniform distribution and the assumption that $g\left(\lambda\right) = \lambda$. Based on the data of Liu et al. (2007), the parameters are set as follows: $\lambda_0 = 16$, $\beta = 0.08$, $h = 0.03$, $b = 0.3$, $c_s = 9.5$, $c_r = 0.8$,

$u_{\max} = 10$ and η ranges from 0.1 to 1. We then take $\alpha = 1.0, 1.2, 1.4$. In Figure 1 (a), we can observe that with the increase of the market price sensitivity factor, α, the supplier tends to quote a shorter lead time. This result is the opposite of that of the single stage model in Li, Lin et al. (2014) because of the complicated interactions among the lead-time decision, the wholesale pricing and the retail pricing decisions in the two-stage supply chain context. Figure 1 (b) shows that when the market price sensitivity factor increases, the wholesale price and the retail price both decrease. In addition, we can also observe that the wholesale price and the retail price are much less sensitive to the supplier's risk aversion degree than to the market price sensitivity factor.

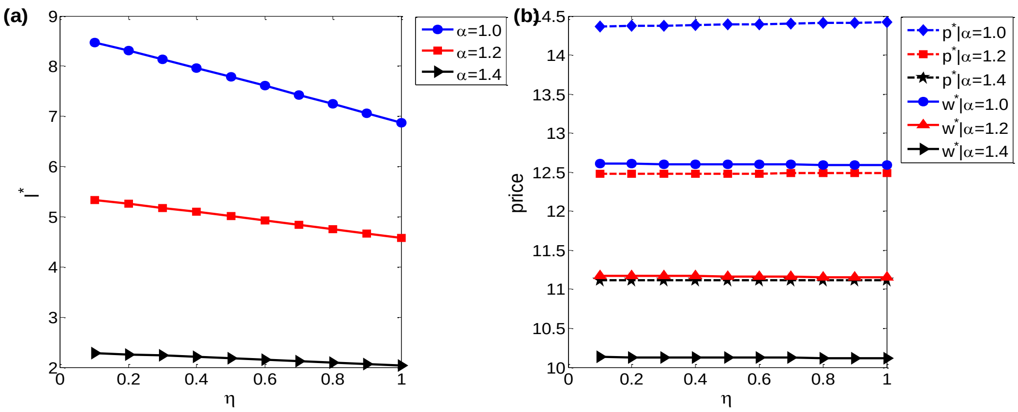

Figure 1. The impacts of η on the optimal decisions of both members under the different α

5. Conclusion

In this paper, we study lead time quote and pricing decision problem in a decentralized supply chain that consists of a risk-averse supplier and a risk-neutral retailer. The external demand rate is assumed to be sensitive to both retail price and quoted lead time, while the realized lead time is uncertain and can deviate from the quoted lead time. The supplier determines the quoted lead time and the wholesale price to maximize his own utility, taking risk aversion into account, while the retailer determines the retail price to maximize his profit rate. A Stackelberg game is employed to model the decision process with the supplier as the leader and the retailer as the follower under the CVaR criterion. A unique equilibrium is obtained under the reasonable assumption that the realized lead time has a multiplicative form consisting of two components (one is a function of the external demand rate, and the other is a random variable representing the system's inherent realized lead time). We then examine the impact of the supplier's risk aversion on the decision efficiency of the decentralized supply chain by using the corresponding centralized supply chain as a benchmark. Furthermore, we analyze how the supplier's risk aversion and other parameters affect the decision variables and the utilities of both the supplier and the retailer and illustrate managerial insights.

The main contributions of this paper include the following:

(i) We present a model for the optimal lead time quote and pricing decision problem in a decentralized supply chain that consists of a risk-averse supplier and a risk-neutral retailer, with the demand rate being sensitive to both retail price and quoted lead time;

(ii) We establish a number of structural relationships, e.g., (a) between the optimal demand rate, the retailers' utilities and the supplier's risk-averse indicator; (b) between the optimal demand rate, the retailers' utilities and the system parameters (including the unit holding cost and the unit delay penalty cost), and the environmental parameters (including the market price sensitivity factor and the lead time sensitivity factor); and (c) between the optimal decisions and, the lead-time sensitivity factor and the unit holding cost;

(iii) Regarding the corresponding centralized supply chain as a benchmark, we argue that the efficiency of the decentralized supply chain increases as the supplier becomes more risk-averse;

(iv) With the realized lead time following uniform distribution, we establish that the impacts of the supplier's risk aversion on the optimal decisions can be characterized by a few threshold values associated with the unit delay penalty cost. A counterintuitive behavior is observed, i.e., when the unit delay penalty cost is high to a certain level, a more risk-averse supplier will quote a shorter lead time, which indicates that the supplier is willing to risk a higher delay penalty cost.

Further research may be conducted in the following directions. First, the model may be extended to a dual-channel supply chain with a direct channel and a traditional channel. Second, because a more risk-neutral supplier or a timelier supplier is better for the centralized supply chain, how to achieve supply coordination between the supplier and the retailer deserves more research. Third, other types of uncertainties, such as random demands, may be included in the model.

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Appendix

Proof for Lemma 1, Lemma 2 and Proposition 1

The supplier's objective is to maximize

$$\begin{aligned} CVaR_\eta(\pi_s(\lambda,l)) &\equiv \max_{v\in R} G(v,\lambda,l) \\ &= \max_{v\in R}\left\{ v - \frac{1}{\eta}\int_0^l \left\{v-[A-c_w l-2\lambda/\alpha-h(l-t)]\lambda\right\}^+ dR_\lambda(t) \right. \\ &\quad \left. -\frac{1}{\eta}\int_l^{+\infty}\left\{v-[A-c_w l-2\lambda/\alpha-b(t-l)]\lambda\right\}^+ dR_\lambda(t)\right\}, \end{aligned}$$

where $[x]^+ = \max(x,0)$.

For any given λ, l, solve the problem $\max_{v\in R} G(v,\lambda,l)$ first. Consider three cases below.

Case (1): If $v \le [A-(c_w+h)l-2\lambda/\alpha]\lambda$, then $v-[A-c_w l-2\lambda/\alpha-h(l-t)]\lambda \le 0$ and $\frac{[A-(c_w-b)l-2\lambda/\alpha]\lambda - v}{b\lambda} > l$. Therefore,

$$G(v,\lambda,l)=v-\frac{1}{\eta}\int_{\frac{[A-(c_w-b)l-2\lambda/\alpha]\lambda-v}{b\lambda}}^{+\infty}\left\{v-[A-c_w l-2\lambda/\alpha-b(t-l)]\lambda\right\}dR_\lambda(t).$$

Taking the first-order partial derivative of $G(v,\lambda,l)$ with respect to v, we have

$$\frac{dG(v,\lambda,l)}{dv}=1-\frac{1}{\eta}\left\{1-R_\lambda\left(\frac{[A-(c_w-b)l-2\lambda/\alpha]\lambda-v}{b\lambda}\right)\right\}.$$

Because the range of v is $v\le[A-(c_w+h)l-2\lambda/\alpha]\lambda$, noticing that

$\lim\limits_{v\to-\infty}\dfrac{dG(v,\lambda,l)}{dv}=1>0$ --and that $\dfrac{dG(v,\lambda,l)}{dv}$ decreases with the increase of v, we need to discuss the value of $\left.\dfrac{dG(v,\lambda,l)}{dv}\right|_{v=[A-(c_w+h)l-2\lambda/\alpha]\lambda}=1-\dfrac{1}{\eta}\left\{1-R_\lambda\left(\dfrac{l\cdot(b+h)}{b}\right)\right\}.$

(1-i) If $1-\dfrac{1}{\eta}\left\{1-R_\lambda\left(\dfrac{l\cdot(b+h)}{b}\right)\right\}\le 0$, there exits $\bar{v}$ that satisfies $\left.\dfrac{dG(v,\lambda,l)}{dv}\right|_{v=\bar{v}}=0$ to maximize $G(v,\lambda,l)$. Therefore, we have

$$\bar{v}=[A-(c_w-b)l-2\lambda/\alpha-bR_\lambda^{-1}(1-\eta)]\cdot\lambda,$$

where R_λ^{-1} is the inverse of the realized lead time distribution function, R_λ.

(1-ii) If $1-\dfrac{1}{\eta}\left\{1-R_\lambda\left(\dfrac{l\cdot(b+h)}{b}\right)\right\}>0$, $\dfrac{dG(v,\lambda,l)}{dv}>0$, and the maximum value of $G(v,\lambda,l)$ is obtained at the upper boundary of v, i.e., $\bar{v}'=[A-(c_w+h)l-2\lambda/\alpha]\lambda$.

Case (2): If $[A-(c_w+h)l-2\lambda/\alpha]\lambda<v\le[A-c_w l-2\lambda/\alpha]\lambda$, then

$$\begin{aligned}G(v,\lambda,l)=v&-\frac{1}{\eta}\int_0^{\frac{v-[A-(c_w+h)l-2\lambda/\alpha]\lambda}{h\lambda}}\left\{v-[A-(c_w+h)l-2\lambda/\alpha+ht]\lambda\right\}dR_\lambda(t)\\&-\frac{1}{\eta}\int_{\frac{[A-(c_w-b)l-2\lambda/\alpha]\lambda-v}{b\lambda}}^{+\infty}\left\{v-[A-(c_w-b)l-2\lambda/\alpha-bt]\lambda\right\}dR_\lambda(t).\end{aligned}$$

Taking the first-order partial derivative of $G(v,\lambda,l)$ with respect to v, we have

$$\begin{aligned}\frac{dG(v,\lambda,l)}{dv}=1-\frac{1}{\eta}\Bigg\{&R_\lambda\left(\frac{v-[A-(c_w+h)l-2\lambda/\alpha]\lambda}{h\lambda}\right)+1\\&-R_\lambda\left(\frac{[A-(c_w-b)l-2\lambda/\alpha]\lambda-v}{b\lambda}\right)\Bigg\}.\end{aligned}$$

Similarly, it can also be proved that $G(v,\lambda,l)$ is concave in v.

Because the range of v is $[A-(c_w+h)l-2\lambda/\alpha]\lambda < v \le [A-c_w l-2\lambda/\alpha]\lambda$,

$$\left.\frac{dG(v,\lambda,l)}{dv}\right|_{v=[A-(c_w+h)l-2\lambda/\alpha]\lambda} = 1-\frac{1}{\eta}\left\{1-R_\lambda\left(\frac{l\cdot(b+h)}{b}\right)\right\} \text{ and}$$

$$\left.\frac{dG(v,\lambda,l)}{dv}\right|_{v=[A-c_w l-2\lambda/\alpha]\lambda} = 1-\frac{1}{\eta} < 0,$$

the value of $\left.\frac{dG(v,\lambda,l)}{dv}\right|_{v=[A-(c_w+h)l-2\lambda/\alpha]\lambda} = 1-\frac{1}{\eta}\left\{1-R_\lambda\left(\frac{l\cdot(b+h)}{b}\right)\right\}$ needs to be discussed.

(2-i) If $1-\frac{1}{\eta}\left\{1-R_\lambda\left(\frac{l\cdot(b+h)}{b}\right)\right\} \le 0$, $\frac{dG(v,\lambda,l)}{dv} \le 0$, and the maximum value of $G(v,\lambda,l)$ is obtained at the lower boundary of v, i.e., $\bar{\bar{v}} = [A-(c_w+h)l-2\lambda/\alpha]\lambda$.

(2-ii) If $1-\frac{1}{\eta}\left\{1-R_\lambda\left(\frac{l\cdot(b+h)}{b}\right)\right\} > 0$, there exits $\bar{\bar{v}}'$ that satisfies $\left.\frac{dG(v,\lambda,l)}{dv}\right|_{v=\bar{\bar{v}}'} = 0$ to maximize $G(v,\lambda,l)$.

Case (3): If $v > [A-c_w l-2\lambda/\alpha]\lambda$, then

$$\begin{aligned} G(v,\lambda,l) = v &- \frac{1}{\eta}\int_0^l \left\{v-[A-(c_w+h)l-2\lambda/\alpha+ht]\lambda\right\} dR_\lambda(t) \\ &- \frac{1}{\eta}\int_l^{+\infty} \left\{v-[A-(c_w-b)l-2\lambda/\alpha-bt]\lambda\right\} dR_\lambda(t). \end{aligned}$$

Taking the first-order partial derivative of $G(v,\lambda,l)$ with respect to v, we have

$\frac{dG(v,\lambda,l)}{dv} = 1-\frac{1}{\eta} < 0.$ Because the range of v is $v > [A-c_w l-2\lambda/\alpha]\lambda$, the maximum value of $G(v,\lambda,l)$ is obtained at the lower boundary of v. It should be noted that the situation $v = [A-c_w l-2\lambda/\alpha]\lambda$ has been considered in Case (2) when $[A-(c_w+h)l-2\lambda/\alpha]\lambda < v \le [A-c_w l-2\lambda/\alpha]\lambda$.

For $G(v,\lambda,l)$ is a continuous function of v, thus as $1-\frac{1}{\eta}\left\{1-R_\lambda\left(\frac{l\cdot(b+h)}{b}\right)\right\} \le 0$,

$G(\overline{v},\lambda,l) > G(\overline{\overline{v}},\lambda,l)$; as $1-\frac{1}{\eta}\left\{1-R_{\lambda}\left(\frac{l\cdot(b+h)}{b}\right)\right\} > 0$, $G(\overline{v}',\lambda,l) < G(\overline{\overline{v}}',\lambda,l)$.

Hence, the solution of $\max_{v\in R} G(v,\lambda,l)$ can be obtained as follows:

$$v^{*}=\begin{cases}\overline{v},\ 1-\frac{1}{\eta}\left\{1-R_{\lambda}\left(\frac{l\cdot(b+h)}{b}\right)\right\}\le 0\\ \overline{\overline{v}}',\ else\end{cases}.$$

Next, we solve $\max_{l,\lambda} G(v^{*},\lambda,l)$ for optimal l with a given λ, i.e., to obtain $l^{*}(\lambda)$ as a function of λ. Then we solve $\max_{\lambda} G(v^{*},\lambda,l^{*}(\lambda))$ for optimal λ. Consider the two cases below.

Case (1): If $1-\frac{1}{\eta}\left\{1-R_{\lambda}\left(\frac{l\cdot(b+h)}{b}\right)\right\}\le 0$, then

$$\max_{v\in R} G(v,\lambda,l)=\overline{v}-\frac{1}{\eta}\int_{\frac{[A-(c_w-b)l-2\lambda/\alpha]\lambda-\overline{v}}{b\lambda}}^{+\infty}\left\{\overline{v}-[A-c_w l-2\lambda/\alpha-b(t-l)]\lambda\right\}dR_{\lambda}(t).$$

Substituting $\overline{v}$ into the equation above, we have

$$G(v^{*},\lambda,l)=G(\overline{v},\lambda,l)=\left[A-(c_w-b)l-\frac{2\lambda}{\alpha}-\frac{b}{\eta}\int_{R_{\lambda}^{-1}(1-\eta)}^{+\infty}tdR_{\lambda}(t)\right]\lambda.$$

For a fixed λ, taking the first-order derivative of $G(\overline{v},\lambda,l)$ with respect to l, we can get $\frac{dG(\overline{v},\lambda,l)}{dl}=-(c_w-b)\lambda.$ From the condition $1-\frac{1}{\eta}\left\{1-R_{\lambda}\left(\frac{l\cdot(b+h)}{b}\right)\right\}\le 0$, the range of l is $0\le l\le l_0$, where $l_0=\left(\frac{b}{b+h}\right)\cdot R_{\lambda}^{-1}(1-\eta)$.

If $b\le c_w$, then $\frac{dG(\overline{v},\lambda,l)}{dl}\le 0$. It is optimal for the supplier to quote a zero lead time. Thus, we assume that $b>c_w$ to avoid this trivial solution. Therefore, $\frac{dG(\overline{v},\lambda,l)}{dl}>0$, and the supplier's best quoted lead time is given by $l_1^{*}=l_0=\left(\frac{b}{b+h}\right)\cdot R_{\lambda}^{-1}(1-\eta)$.

Case (2): If $1-\frac{1}{\eta}\left\{1-R_{\lambda}\left(\frac{l\cdot(b+h)}{b}\right)\right\}>0$, then $\max_{v\in R} G(v,\lambda,l)=G(\overline{\overline{v}}',\lambda,l)$, where

$\overline{\overline{v}}'$ satisfies $1-\frac{1}{\eta}\left[R_\lambda\left(\mathrm{M}\right)+1-R_\lambda\left(N\right)\right]=0$,

and $M=\dfrac{\overline{\overline{v}}'-[A-(c_w+h)l-2\lambda/\alpha]\lambda}{h\lambda}, N=\dfrac{[A-(c_w-b)l-2\lambda/\alpha]\lambda-\overline{\overline{v}}'}{b\lambda}$.

Therefore,

$$\max_{v\in R} G(v,\lambda,l)=G(\overline{\overline{v}}',\lambda,l)$$
$$=\left\{(A-c_w l-\frac{2\lambda}{\alpha})-\frac{hl}{\eta}R_\lambda(M)+\frac{h}{\eta}\int_0^M tdR_\lambda(t)+\frac{bl}{\eta}\left[1-R_\lambda(N)\right]-\frac{b}{\eta}\int_N^{+\infty}tdR_\lambda(t)\right\}\lambda.$$

From the condition $1-\frac{1}{\eta}\left\{1-R_\lambda\left(\frac{l\cdot(b+h)}{b}\right)\right\}>0$, we have $l>l_0$.

For a fixed λ, taking the first-order derivative of $G(\overline{\overline{v}}',\lambda,l)$ with respect to l and combining it with $1-\frac{1}{\eta}\left[R_\lambda\left(\mathrm{M}\right)+1-R_\lambda\left(N\right)\right]=0$, we can get

$$\frac{dG(\overline{\overline{v}}',\lambda,l)}{dl}=-\lambda\left[(c_w-b)+\frac{1}{\eta}(h+b)R_\lambda(M)\right]$$
$$=-\lambda\left\{(c_w+h)-\frac{1}{\eta}(h+b)[1-R_\lambda(N)]\right\}.$$

If $\dfrac{dG(\overline{\overline{v}}',\lambda,l)}{dl}$ =0, then we have

$$M^*=R_\lambda^{-1}\left(\frac{b-c_w}{h+b}\eta\right),\ \ N^*=R_\lambda^{-1}\left(1-\left(1-\frac{b-c_w}{h+b}\right)\eta\right).$$

Note that $l=\dfrac{hM}{h+b}+\dfrac{bN}{h+b}$. We can get $l_2^*=(1-D)R_\lambda^{-1}\left(C\eta\right)+DR_\lambda^{-1}\left(1-(1-C)\eta\right)$,

where $C=\dfrac{b-c_w}{h+b}, D=\dfrac{b}{h+b}$. As assumed above, $b>c_w$.

Because $DR_\lambda^{-1}\left(1-(1-C)\eta\right)>l_0$, $l_2^*>l_0$, l_2^* is feasible. Taking the second-order derivative of $G(\overline{\overline{v}}',\lambda,l)$ with respect to l, we get

$$\frac{d^2G(\overline{\overline{v}}',\lambda,l)}{dl^2}=-\lambda\left[\frac{1}{\eta}(h+b)r_\lambda(M)\frac{dM}{dl}\right]$$
$$=-\lambda\left[\frac{1}{\eta}(h+b)^2\cdot\frac{r_\lambda(M)\cdot r_\lambda(N)}{br_\lambda(M)+hr_\lambda(N)}\right]\leq 0.$$

Therefore, $G(\bar{\bar{v}}', \lambda, l)$ is concave in l. Thus, l_2^* is the optimal lead-time decision.

Because $\frac{dG(\bar{v}, \lambda, l)}{dl} > 0$, together with $0 < l_1^* \le l_0 \le l_2^*$, we have

$G(\bar{v}, \lambda, l_1^*) < G(\bar{v}, \lambda, l_2^*)$. Note that $G(\bar{v}, \lambda, l_2^*) < G(\bar{\bar{v}}', \lambda, l_2^*)$. It follows that $G(\bar{v}, \lambda, l_1^*) < G(\bar{\bar{v}}', \lambda, l_2^*)$. Hence $l^* = l_2^*$, and we only need to take $G(\bar{\bar{v}}', \lambda, l_2^*)$ into consideration. Thus, Lemma 1 is proved.

Note that $T = g(\lambda)U$ and $l^* = g(\lambda)\bar{l}$, where

$\bar{l} = (1-D)\Phi^{-1}\left(C\eta\right) + D\Phi^{-1}\left(1-(1-C)\eta\right)$.

We know that $R_\lambda(t) = \Phi(t / g(\lambda))$ and $R_\lambda^{-1}(k) = \Phi^{-1}(k)\cdot g(\lambda)$.

Substitute l^* into $G(\bar{\bar{v}}', \lambda, l)$. Using the results $1-\frac{1}{\eta}[R_\lambda(M)+1-R_\lambda(N)] = 0$ and

$M^* = R_\lambda^{-1}\left(C\eta\right), N^* = R_\lambda^{-1}\left(1-(1-C)\eta\right)$, the supplier's optimal profit rate can be transformed into

$$G(\bar{\bar{v}}', \lambda, l^*(\lambda)) = \left\{(A-\frac{2\lambda}{\alpha}) + \frac{hg(\lambda)}{\eta}\int_0^{\Phi^{-1}(C\eta)} u d\Phi(u) - \frac{bg(\lambda)}{\eta}\int_{\Phi^{-1}(1-(1-C)\eta)}^{+\infty} u d\Phi(u)\right\}\lambda.$$

Taking the first-order derivative of $G(\bar{\bar{v}}', \lambda, l^*(\lambda))$ with respect to λ, we have

$$\frac{dG(\bar{\bar{v}}', \lambda, l^*(\lambda))}{d\lambda} = \left(A - \frac{4\lambda}{\alpha}\right)$$
$$-\left[g(\lambda)+\lambda g'(\lambda)\right]\left[-\frac{h}{\eta}\int_0^{\Phi^{-1}(C\eta)} u d\Phi(u) + \frac{b}{\eta}\int_{\Phi^{-1}(1-(1-C)\eta)}^{+\infty} u d\Phi(u)\right].$$

Taking the second-order derivative of $G(\bar{\bar{v}}', \lambda, l^*(\lambda))$ with respect to λ, we get

$$\frac{d^2G(\bar{\bar{v}}', \lambda, l^*(\lambda))}{d\lambda^2} = -\frac{4}{\alpha}$$
$$-\left[2g'(\lambda)+\lambda g''(\lambda)\right]\cdot\left[-\frac{h}{\eta}\int_0^{\Phi^{-1}(C\eta)} u d\Phi(u) + \frac{b}{\eta}\int_{\Phi^{-1}(1-(1-C)\eta)}^{+\infty} u d\Phi(u)\right].$$

As $g(\lambda)$ is a nonnegative and increasing convex function of λ, $2g'(\lambda)+\lambda g''(\lambda) \ge 0$.

In addition, because

$$-\frac{h}{\eta}\int_0^{\Phi^{-1}(C\eta)} ud\Phi(u)+\frac{b}{\eta}\int_{\Phi^{-1}(1-(1-C)\eta)}^{+\infty} ud\Phi(u)$$
$$=c_w\overline{l}+\frac{h}{\eta}\int_0^{\Phi^{-1}(C\eta)}(\overline{l}-u)d\Phi(u)+\frac{b}{\eta}\int_{\Phi^{-1}(1-(1-C)\eta)}^{+\infty}(u-\overline{l})d\Phi(u)>0$$

(note that $C=\frac{b-c_w}{h+b}$), $\frac{d^2G(\overline{\overline{v}}',\lambda,l^*(\lambda))}{d\lambda^2}$ is negative. Thus, $G(\overline{\overline{v}}',\lambda,l^*(\lambda))$ is concave in λ.

The optimal demand rate, λ^*, is the unique solution to

$$\left.\frac{dG(\overline{\overline{v}}',\lambda,l^*(\lambda))}{d\lambda}\right|_{\lambda=\lambda_m^*}=\left(A-\frac{4\lambda^*}{\alpha}\right)$$
$$-\left[g(\lambda^*)+\lambda^*g'(\lambda^*)\right]\left[-\frac{h}{\eta}\int_0^{\Phi^{-1}(C\eta)}ud\Phi(u)+\frac{b}{\eta}\int_{\Phi^{-1}(1-(1-C)\eta)}^{+\infty}ud\Phi(u)\right]=0.$$

In addition, the supplier's optimal profit rate can be written as

$$G(\overline{\overline{v}}',\lambda^*,l^*(\lambda^*))=\frac{2(\lambda^*)^2}{\alpha}+(\lambda^*)^2g'(\lambda^*)\cdot\left\{-\frac{h}{\eta}\int_0^{\Phi^{-1}(C\eta)}ud\Phi(u)+\frac{b}{\eta}\int_{\Phi^{-1}(1-(1-C)\eta)}^{+\infty}ud\Phi(u)\right\}.$$

Hence, Lemma 2 is proved. This completes the proof of Proposition 1.

Proof for Proposition 2

(1) Taking the first-order derivative of $\varphi+c_w\overline{l}$ with respect to η, we get

$$\frac{d(\varphi+c_w\overline{l})}{d\eta}=-\frac{1}{\eta^2}\left\{h\int_0^{\Phi^{-1}(C\eta)}[\Phi^{-1}(C\eta)-u]d\Phi(u)+\right.$$
$$\left.b\int_{\Phi^{-1}(1-(1-C)\eta)}^{+\infty}[u-\Phi^{-1}(1-(1-C)\eta)]d\Phi(u)\right\}<0.$$

Differentiating Eq. (16), Eq. (18) and Eq. (17) with respect to η respectively, we have

$$\frac{d\lambda^*}{d\eta}=\frac{-[g(\lambda^*)+\lambda^*g'(\lambda^*)]\cdot\dfrac{d(\varphi+c_w\overline{l})}{d\eta}}{\dfrac{4}{\alpha}+[2g'(\lambda^*)+\lambda^*g''(\lambda^*)]\cdot(\varphi+c_w\overline{l})}>0,$$

$$\frac{dCVaR_\eta(\pi_s)^*}{d\eta}=-\lambda^*g(\lambda^*)\cdot\frac{d(\varphi+c_w\overline{l})}{d\eta}>0 \text{ and } \frac{d\pi_r^*}{d\eta}=\frac{2\lambda^*}{\alpha}\cdot\frac{d\lambda^*}{d\eta}>0.$$

(2) Differentiating Eq. (16) with respect to α, we have

$$\frac{d\lambda^*}{d\alpha}=\frac{-\lambda_0/\alpha+4\lambda^*/\alpha+\left[g(\lambda^*)+\lambda^*g'(\lambda^*)\right]c_w\overline{l}}{4+\alpha\left[2g'(\lambda^*)+\lambda^*g''(\lambda^*)\right]\left(c_w\overline{l}+\varphi\right)}.$$

Using Eq. (16), we have $-\lambda_0/\alpha + 4\lambda^*/\alpha + \left[g(\lambda^*) + \lambda^* g'(\lambda^*)\right] c_w \bar{l} < 0$.

Therefore, $\dfrac{d\lambda^*}{d\alpha} < 0$.

Differentiating Eq. (18) with respect to α, we get (using Eq. (16) and Eq. (15))

$$\frac{dCVaR_\eta(\pi_s)^*}{d\alpha} = -\frac{\lambda^*}{\alpha}\left(p^{\max} - 2\lambda^*/\alpha - c_w l^*\right).$$

Using Eq. (9), because the wholesale price should not be negative, we know that

$p^{\max} - 2\lambda/\alpha - c_w l^*$ must be positive. Thus, $\dfrac{dCVaR_\eta(\pi_s)^*}{d\alpha} < 0$ can be obtained.

Differentiating Eq. (17) with respect to α, we have $\dfrac{d\pi_r^*}{d\alpha} = \dfrac{2\lambda^*}{\alpha} \cdot \dfrac{d\lambda^*}{d\alpha} - \dfrac{(\lambda^*)^2}{\alpha^2} < 0.$

Note that $c_w = \beta/\alpha$ and $C = \dfrac{b - c_w}{h + b}$. Taking the first-order derivative of $\varphi + c_w \bar{l}$ with respect to β, we get $\dfrac{d(\varphi + c_w \bar{l})}{d\beta} = \dfrac{1}{\alpha}\bar{l} > 0.$

Differentiating Eq. (16), Eq. (18) and Eq. (17) with respect to β, respectively, we get

$$\frac{d\lambda^*}{d\beta} = \frac{-[g(\lambda^*) + \lambda^* g'(\lambda^*)] \cdot \dfrac{d(\varphi + c_w \bar{l})}{d\beta}}{\dfrac{4}{\alpha} + [2g'(\lambda^*) + \lambda^* g''(\lambda^*)] \cdot (\varphi + c_w \bar{l})} < 0,$$

$$\frac{dCVaR_\eta(\pi_s)^*}{d\beta} = -\lambda^* g(\lambda^*) \cdot \frac{d(\varphi + c_w \bar{l})}{d\beta} < 0 \quad \text{and} \quad \frac{d\pi_r^*}{d\beta} = \frac{2\lambda^*}{\alpha} \cdot \frac{d\lambda^*}{d\beta} < 0.$$

(3) Taking the first-order derivative of $\varphi + c_w \bar{l}$ with respect to b and h, respectively, we get

$$\begin{aligned}\frac{d(\varphi + c_w \bar{l})}{db} &= \frac{1}{\eta}\int_{\Phi^{-1}(1-(1-C)\eta)}^{+\infty} \left(u - \Phi^{-1}\left(1-(1-C)\eta\right)\right) d\Phi(u) \\ &+ (1-C)(1-D)\left[\Phi^{-1}(1-(1-C)\eta) - \Phi^{-1}(C\eta)\right] > 0\end{aligned} \quad \text{and}$$

$$\begin{aligned}\frac{d(\varphi + c_w \bar{l})}{dh} &= \frac{1}{\eta}\int_0^{\Phi^{-1}(C\eta)} [\Phi^{-1}(C\eta) - u] d\Phi(u) \\ &+ CD\left[\Phi^{-1}\left(1-(1-C)\eta\right) - \Phi^{-1}(C\eta)\right] > 0.\end{aligned}$$

Differentiating Eq. (16) with respect to h and b, respectively, we get

$$\frac{d\lambda^*}{dh}=\frac{-[g(\lambda^*)+\lambda^* g'(\lambda^*)]\cdot\dfrac{d(\varphi+c_w\overline{l}\,)}{dh}}{\dfrac{4}{\alpha}+[2g'(\lambda^*)+\lambda^* g''(\lambda^*)]\cdot(\varphi+c_w\overline{l}\,)}<0 \text{ and}$$

$$\frac{d\lambda^*}{db}=\frac{-[g(\lambda^*)+\lambda^* g'(\lambda^*)]\cdot\dfrac{d(\varphi+c_w\overline{l}\,)}{db}}{\dfrac{4}{\alpha}+[2g'(\lambda^*)+\lambda^* g''(\lambda^*)]\cdot(\varphi+c_w\overline{l}\,)}<0.$$

Differentiating (18) with respect to h and b, respectively, we can get

$$\frac{dCVaR_\eta(\pi_s)^*}{dh}=-\lambda^* g(\lambda^*)\cdot\frac{d(\varphi+c_w\overline{l}\,)}{dh}<0 \text{ and}$$

$$\frac{dCVaR_\eta(\pi_s)^*}{db}=-\lambda^* g(\lambda^*)\cdot\frac{d(\varphi+c_w\overline{l}\,)}{db}<0.$$

Differentiating (17) with respect to h and b, respectively, we have

$$\frac{d\pi_r^*}{dh}=\frac{2\lambda^*}{\alpha}\cdot\frac{d\lambda^*}{dh}<0 \text{ and } \frac{d\pi_r^*}{db}=\frac{2\lambda^*}{\alpha}\cdot\frac{d\lambda^*}{db}<0.$$

Proof for Proposition 3

Substituting Eq. (3) into Eq. (21), we have

$$\pi_c(p_c,l_c)=\max_{v_c\in R}\left\{v_c-\frac{1}{\eta}\int_0^l\{v_c-[w_c-c_s-h(l_c-t)]\lambda_c\}^+ dR_{\lambda_c}(t)\right.$$
$$\left.-\frac{1}{\eta}\int_l^{+\infty}\{v_c-[w_c-c_s-b(t-l_c)]\lambda_c\}^+ dR_{\lambda_c}(t)\right\}+\pi_r^c.$$

With a similar transformation on the demand Eq. (1), we can obtain Eq. (22), i.e.,

$p_c=p_{\max}-c_w l_c-\lambda_c/\alpha$. Then, from Eq. (5), we get

$$\pi_r^c=\pi_r^c(\lambda_c,l_c)=(p_{\max}-c_w l_c-\lambda_c/\alpha-w_c-c_r)\lambda_c.$$

Additionally, the objective function is transformed into $\pi_c(\lambda_c,l_c)$. Let

$$G(v_c,\lambda_c,l_c)=\max_{v_c\in R}\left\{v_c-\frac{1}{\eta}\int_0^l\{v_c-[w_c-c_s-h(l_c-t)]\lambda_c\}^+ dR_{\lambda_c}(t)\right.$$
$$\left.-\frac{1}{\eta}\int_l^{+\infty}\{v_c-[w_c-c_s-b(t-l_c)]\lambda_c\}^+ dR_{\lambda_c}(t)\right\}$$

For any given λ_c,l_c, solve the problem $\max_{v_c\in R}G(v_c,\lambda_c,l_c)$ first, using the same method in the decentralized supply chain (see Proof for Lemma 1). Similarly, the solution can be obtained as the form of:

$$v_c^* = \begin{cases} \bar{v}_c, \ 1 - \dfrac{1}{\eta}\left\{1 - R_{\lambda_c}\left(\dfrac{l_c \cdot (b+h)}{b}\right)\right\} \le 0 \\ \bar{\bar{v}}'_c, \ else \end{cases}.$$

Then, $\pi_c(\lambda_c, l_c) = \begin{cases} G(\bar{v}_c, \lambda_c, l_c) + \pi_r^c(\lambda_c, l_c), \ 1 - \dfrac{1}{\eta}\left\{1 - R_{\lambda_c}\left(\dfrac{l_c \cdot (b+h)}{b}\right)\right\} \le 0 \\ G(\bar{\bar{v}}'_c, \lambda_c, l_c) + \pi_r^c(\lambda_c, l_c), \ else. \end{cases}$

Let $F(v_c^*, \lambda_c, l_c) = \pi_c(\lambda_c, l_c)$ and solve $\max\limits_{l_c, \lambda_c} F(v_c^*, \lambda_c, l_c)$ for l_c^*, and λ_c^* in a sequential procedure. The process is similar to that of Proposition 1 and is omitted to save space.

Proof of Lemma 3

Let $k \in R^+$ be a parameter. The equation below is equivalent to Eq. (16) when $k = 1$ (corresponding to the centralized mode) and is equivalent to Eq. (24) when $k = 2$ (corresponding to the decentralized mode):

$$(A - \frac{2k\lambda^*}{\alpha}) - [g_1(\lambda^*) + \lambda^* g_1'(\lambda^*)] \cdot (c_w \bar{l} + \varphi) = 0.$$

Differentiating this equation with respect to k, we have

$$\frac{d\lambda^*}{dk} = \frac{-2\lambda^*}{\{2k + \alpha[2g_1'(\lambda^*) + \lambda^* g_1''(\lambda^*)] \cdot (c_w \bar{l} + \varphi)\}} < 0.$$ Thus, $\lambda^* < \lambda_c^*$.

Because $g(\lambda)$ is a nonnegative and monotonically increasing function, given $\lambda^* < \lambda_c^*$, we have $g(\lambda^*) < g(\lambda_c^*)$. Thus, we get

$$[g(\lambda^*) + \lambda^* g'(\lambda^*)] \cdot (\varphi + c_w \bar{l}) < [g(\lambda_c^*) + \lambda_c^* g'(\lambda_c^*)] \cdot (\varphi + c_w \bar{l}).$$

Using Eq. (16) and Eq. (24), $A - \dfrac{4\lambda^*}{\alpha} < A - \dfrac{2\lambda_c^*}{\alpha}$ or $2\lambda^* > \lambda_c^*$ can be obtained. Lemma 3 is proved.

Proof of Proposition 4

For $g(\lambda) = 1$, we have $\dfrac{de_\pi}{d\eta} = -\dfrac{\lambda^* \cdot \lambda_c^*}{\left(\pi_c^*\right)^2} \cdot \dfrac{d(\varphi + c_w \bar{l})}{d\eta} \cdot \dfrac{3}{2\alpha}\left(\lambda_c^* - 2\lambda^*\right).$

By Lemma 3, we have $\lambda_c^* - 2\lambda^* < 0$ and $\dfrac{de_\pi}{d\eta} < 0.$

For $g(\lambda)=\lambda$, we have $\frac{de_\pi}{d\eta}=\frac{\left(\lambda^*\cdot\lambda_c^*\right)^2}{\left(\pi_c{}^*\right)^2}\cdot\frac{d(\varphi+c_w\overline{l}\,)}{d\eta}\cdot\frac{1}{\frac{4}{\alpha}+2\left(\varphi+c_w\overline{l}\,\right)}\cdot\frac{4}{\alpha^2}<0.$

For $g(\lambda)=\lambda^n, n\geq 2$, we have

$$\frac{de_\pi}{d\eta}=-\frac{\left(\lambda^*\cdot\lambda_c^*\right)^2}{\left(\pi_c{}^*\right)^2}\cdot\frac{d(\varphi+c_w\overline{l}\,)}{d\eta}\cdot\frac{1}{\frac{4}{\alpha}+(2g'(\lambda^*)+\lambda^*g''(\lambda^*))\left(\varphi+c_w\overline{l}\,\right)}$$
$$\cdot\left\{\frac{n(n+1)\left(\lambda^*\right)^{n-1}}{\alpha}(\varphi+c_w\overline{l}\,)\cdot\left(\left(\lambda^*\right)^{n-1}-\left(\lambda_c^*\right)^{n-1}\right)+\frac{\left(\lambda_c^*\right)^{n-1}}{\alpha^2}\left[\left(6+2n\right)\left(\frac{\lambda^*}{\lambda_c^*}\right)^{n-1}-12\right]\right\}.$$

By Lemma 3, we know that $\left(\lambda^*\right)^{n-1}-\left(\lambda_c^*\right)^{n-1}<0$ and $\frac{\lambda^*}{\lambda_c^*}<1$. Thus, when $n=2,3$, $\frac{de_\pi}{d\eta}<0.$

Proof for Proposition 5

Differentiating Eq. (20) in Corollary 2 with respect to η, we get

$$\frac{dl^*}{d\eta}=\frac{A\alpha u_{\max}\left[\alpha u_{\max}hb^2-c_w\left(2\alpha hu_{\max}+\alpha c_wu_{\max}+4\right)b-4hc_w\right]}{\left\{4\left(b+h\right)+\alpha u_{\max}\left[2b\left(h+c_w\right)-\eta\left(bh+c_w^2\right)\right]\right\}^2}.$$

When $\alpha u_{\max}hb^2-c_w\left(2\alpha hu_{\max}+\alpha c_wu_{\max}+4\right)b-4hc_w=0$, this equation has two real roots. One root is negative while the other one is positive. Because b cannot take negative value, we let b_3 denote the positive root, i.e.,

$$b_3=\frac{c_w\left(2\alpha hu_{\max}+\alpha c_wu_{\max}+4\right)+\sqrt{c_w^2\left(2\alpha hu_{\max}+\alpha c_wu_{\max}+4\right)^2+16\alpha u_{\max}h^2c_w}}{2\alpha u_{\max}h}.$$

Then, when $b\in\left(0,b_3\right)$, $\frac{dl^*}{d\eta}<0$. Otherwise, when $b\in\left(b_3,\infty\right)$, $\frac{dl^*}{d\eta}>0$.

From Eq. (1), we know that $p^*=p^{\max}-\lambda^*/\alpha-c_wl^*$. Substitute Eq. (19) and Eq. (20) into this equation and differentiate p^* with respect to η. Similarly, we can obtain that

$$b_2=\frac{\left(3c_w^2-h^2+2\alpha c_w^2hu_{\max}+\alpha c_w^3u_{\max}\right)}{2h\left(\alpha c_wu_{\max}+1\right)}$$
$$+\frac{\sqrt{\left(3c_w^2-h^2+2\alpha c_w^2hu_{\max}+\alpha c_w^3u_{\max}\right)^2+12\left(\alpha c_wu_{\max}+1\right)h^2c_w^2}}{2h\left(\alpha c_wu_{\max}+1\right)}.$$

When $b\in\left(0,b_2\right)$, $\frac{dp^*}{d\eta}>0$. Otherwise, when $b\in\left(b_2,\infty\right)$, $\frac{dp^*}{d\eta}<0$.

From Eq. (9), we get $w^* = p^{\max} - c_w l^* - 2\lambda^* / \alpha - c_r$. Substitute Eq. (19) and Eq. (20) into this equation and differentiate w^* with respect to η. Similarly, we can obtain that

$$b_1 = \frac{\left(2c_w^2 - 2h^2 + 2\alpha c_w^2 h u_{\max} + \alpha c_w^3 u_{\max}\right)}{2h\left(\alpha c_w u_{\max} + 2\right)} + \frac{\sqrt{\left(2c_w^2 - 2h^2 + 2\alpha c_w^2 h u_{\max} + \alpha c_w^3 u_{\max}\right)^2 + 8\left(\alpha c_w u_{\max} + 2\right) h^2 c_w^2}}{2h\left(\alpha c_w u_{\max} + 2\right)}.$$

When $b \in \left(0, b_1\right)$, $\frac{dw^*}{d\eta} > 0$. Otherwise, when $b \in \left(b_1, \infty\right)$, $\frac{dw^*}{d\eta} < 0$.

Comparing b_1, b_2, b_3, it is not difficult to show that $b_1 < b_2 < b_3$. Hence, we can distinguish four cases in Proposition 5.

Proof for Proposition 6

(1) Proposition 2 has proved that $\frac{d\lambda^*}{d\beta} < 0$. Differentiating Eq. (14) with respect to β, we have

$$\frac{d\bar{l}}{d\beta} = -\frac{\eta}{(b+h)\alpha}\left[(1-D)\frac{1}{\phi(\Phi^{-1}(C\eta))} + D\frac{1}{\phi(\Phi^{-1}(1-(1-C)\eta))}\right] < 0.$$

Because $l^*(\lambda) = g(\lambda)\bar{l}$ and $g(\lambda)$ is a nonnegative and monotonically increasing function, we have $\frac{dl^*}{d\beta} = g'(\lambda^*)\bar{l} \cdot \frac{d\lambda^*}{d\beta} + g(\lambda^*) \cdot \frac{d\bar{l}}{d\beta} < 0.$

(2) Proposition 2 has proved that $\frac{d\lambda^*}{dh} < 0$. Differentiating Eq. (14) with respect to h, we have

$$\frac{d\bar{l}}{dh} = -\frac{\left(b - c_w\right)\eta}{(b+h)^2}\left[(1-D)\frac{1}{\phi(\Phi^{-1}(C\eta))} + D\frac{1}{\phi(\Phi^{-1}(1-(1-C)\eta))}\right] - \frac{b}{(b+h)^2}\left[\Phi^{-1}(1-(1-C)\eta) - \Phi^{-1}(C\eta)\right] < 0.$$

Because $l^*(\lambda) = g(\lambda)\bar{l}$ and $g(\lambda)$ is a nonnegative and monotonically increasing function, we have $\frac{dl^*}{dh} = g'(\lambda^*)\bar{l} \cdot \frac{d\lambda^*}{dh} + g(\lambda^*) \cdot \frac{d\bar{l}}{dh} < 0.$

From Eq. (1), we know that $p^* = p^{\max} - \lambda^* / \alpha - c_w l^*$. Differentiating p^* with respect to h, we get

$\frac{dp^*}{dh} = -c_w \cdot \frac{dl^*}{dh} - \frac{1}{\alpha} \cdot \frac{d\lambda^*}{dh} > 0.$ From Eq. (9), we get $w^* = p^{\max} - c_w l^* - 2\lambda^* / \alpha - c_r$.

Differentiating w^* with respect to h , we get $\frac{dw^*}{dh} = -c_w \cdot \frac{dl^*}{dh} - \frac{2}{\alpha} \cdot \frac{d\lambda^*}{dh} > 0.$

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