Minimizing the electricity cost of coordinating houses on microgrids

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***Abstract*—This manuscript presents a comprehensive mathematical model for multi-objective optimization problem of the microgrid. The microgrid consists of houses and local plants, each seen as independent agents with their specific goals. We, also, propose a heuristic algorithm for optimizing the electricity cost by using the concept of load shifting and renewable power sharing among houses in the microgrid for a particular price. Also, the algorithm minimizes the loss of energy by prioritizing power exchange between close houses and minimize discomfort factor. The findings have shown that houses and micro plants working in microgrid setting can make a significant saving. The results have illustrated that our algorithm guarantee nobody will lose in the microgrid.**

***Index Terms*—Microgrid, home automation, energy management, optimization algorithm, and resource allocation.**

I. Introduction

The inhabitants of the globe have risen dramatically in last 20 years [1], [2]. Consequently, World's energy needs are ever increasing, and the investment in new power plants (traditional or/and renewable) is not going to cover the future demand [3]. Furthermore, energy conservation, energy-efficient appliances, smart household appliances usability and load management have been used to tackle the increase electricity demand [4].

Power grid consists of generation plants, substations, transformers, transmission lines and end users [3]. Nowadays, the electricity grid has become more complicated as new entities (such as distributed micro plants, renewable plants, and distributors) have been introduced [5]. The Smart Grid is an enhanced grid in which information and communication technology is used to improve the power system and increase the profit of consumers, distributors and generation companies. The key features of such infrastructure are reliability, efficiency, sustainability, manageability of resources, and market-enabling [5], [6].

The microgrid provides electricity to islands, rural areas, and remote operations that have limited or no access to the National Electricity Grid (NEG). A microgrid is a set of houses and domestic resources working as a single controllable system [6]. Microgrid uses diesel generators (DG) and diesel pickup system for generating electricity. Also, integration of renewable power plants such as wind turbine and PV array; storage system is used to accommodate the surplus renewable power [5]–[7].

Many studies investigate methods for optimizing the electricity cost in houses, based on price, availability of renewable power, or user preferences. For example, studies [8]–[10] use algorithms that find the optimal cost of electricity, whereas [11]–[13] use heuristic methods which only guarantee suboptimal value.

Authors in [14] propose a predictive control approach for minimizing the electricity cost of the microgrid. They have used MILP to formulated their model of the microgrid. To solve this single objective optimization problem, authors have used LP solver to find optimal plan. However, their time resolution is low which may not scale up well with such NP-hard problem. Optimizing the electricity cost of a grid-tied microgrid is the main goal of study [15]. Authors uses MILP-based model for the microgrid. Nevertheless, computational time problem has not been tackled seriously in this study. Authors have proposed empirical evaluation and the used low time resolution is in [14].

In study [16], Authors have used a static optimization tool (Mixed Integer Linear Programming (MILP)) for optimizing a load of a microgrid. The study compares the cost of 20 houses working individually and the same houses working in a microgrid setup. However, it does not tackle the important issue of computation time. In studies of this type, the time complexity of the particular algorithm increases with the system's granularity or the number of available appliances.The authors are only able to present examples that allocate resources over relatively large time slots for a couple of appliances only.Studies [17]–[19] have proposed similar MILP-based models for the microgrid.

manuscript [1] investigates the sharing of local renewable energy in a micro-grid. A greedy energy search algorithm is used to match the predicted renewable power with the predicted house consumption. The proposed approach also minimizes the power loss incurred while transferring electricity power along power lines by choosing the nearest house to share renewable power with. Unfortunately the proposed algorithm does not scale well with the length of the time slots.

In this work, we investigate the effectiveness of an MILP-based strategy that can be used to solve a particular energy allocation problem within a given microgrid. Our contributions are as follows: We have designed a comprehensive model for a microgrid that allows renewable power sharing. Also, a heuristic energy algorithm is proposed for the multi-objective optimization problem. Additionally, we have designed a set of rules that control the power exchange among agents in the microgrid. Design unique version of $\epsilon-$constraint method to convert the multi-objective optimization problem to a single objective optimization problem. Finally, the framework gives preliminary results.

The rest of this paper is organized as follows: Section 2 defines our allocation problem. MILP formulation is presented in section 3, and the 4th section illustrates the results which are followed by discussions and conclusion.

II. Allocation Problem

A. The microgrid

A microgrid consists of a set of houses $\mathcal{H}$ (each house has a set of rooms, $\mathcal{M}$), and a set of micro plants $\mathcal{R}$, Fig. 6. The distance between a particular house, h, and renewable plant, r, is d_h^r distance unit, where $r \in \mathcal{R}$, and $h \in \mathcal{H}$. Some houses (like h_1, and h_4 in Fig. 6) may have a top-roof renewable power plant, Diesel Generator (DG), or Combined Heat and Power (CHP), r_1 and r_4, respectively, and therefore they are able to receive energy from it in a particularly efficient way, but in general the houses in the system may receive their power from any of the plants in the microgrid or the NEG. The energy exchange within a microgrid is controlled by a Local Microgrid Optimizer (LMO). The local plants generate electricity that can be either consumed by the owner (house), neighbors (houses), or exported to the NEG.

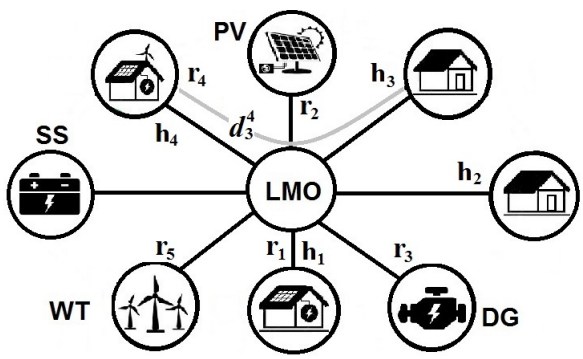


Fig. 1: Microgrid diagram

Fig. 2 explains the potential energy trades among a house, a generator, and the NEG. Houses can merely use electricity. The electricity comes in the house either from a generator (belongs to the house or neighbors) or from the NEG. The labels on the arcs describe the cost that the agent at the end of the arrow will have to pay to the agent at the other end to get electricity from it. We assume that the energy produced by generator r can be sent to house h at a unit cost $\gamma_{r,h}$ or exported to the NEG at a cost ζ_r or a house can buy energy from the NEG at a cost λ_h. All costs might change over time.

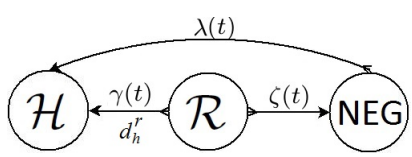
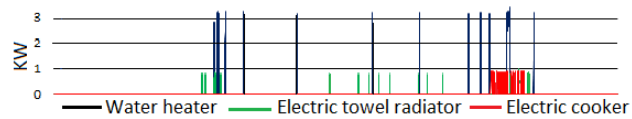


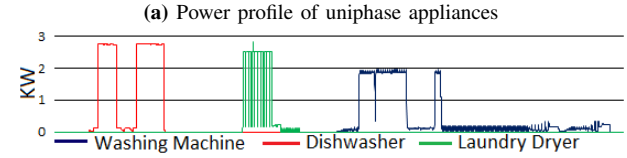
Fig. 2: Diagram shows power exchange among micro-grid components.

B. Electrical Appliances

Each house h is equipped with a set of appliances $\mathcal{A}_h = \{A_1, \ldots, A_{m_h}\}$, Appliances in a micro-grid are the main energy outlets. The appliances in the system can be switched ON/OFF without disrupting their functionalities. Air-conditioning (AC) units and heaters are examples of suitable appliances, whereas TV sets and Computers do not fit into such framework. Appliances in a microgrid can either be interruptible or uninterruptible, uniphase or multiphase. Interruptible appliances are designed to be switched ON/OFF at any time such as heaters, Fig. 3a. Uninterruptible appliances are not designed to be switched OFF once they have been switched ON until they finish a particular task, such as Dishwasher. Heaters are, also, examples of uniphase appliance. Any such appliance can either be OFF or ON, and when it is ON, it uses approximately a constant amount of power (nominal power). The restriction to the use of uniphase appliances is that only have a single ON state, without loss of generality, appliances that can run at one of several power levels can be simulated by a combination of several uniphase appliances. Multiphase appliances work in different phases, each using a certain amount of power. Fig. 3b shows the typical power profile of three distinct multiphase appliances. Within a given phase, multiphase appliances cannot be switched off. We also assume that some appliances may have constraints on how often they are run while others might be controlled by environmental factors such as the level of charge of a battery, or particular desired values of room temperatures [20].



(a) Power profile of uniphase appliances



(b) Power profile of multiphase appliances

Fig. 3: Uniphase vs Multiphase appliances

The total energy consumed by appliance A (P_T) can be modeled by the following equation:

$$P_A = \int P(t)dt \tag{1}$$

Finding continuous function that represent the power profile of each appliance is not easy. Therefore, we assume that each appliance A operates in $\Delta_A > 0$ phases and for each appliance, the power profile vector is modeled by $(\alpha_1, \ldots, \alpha_{\Delta_A})$, $\forall j\, \alpha_j \geq 0$. We assume that $\Delta_{\min}$ is length of the shortest phase. When switched on, appliance A progresses through each of its phases, starting from phase 1 up until phase Δ_A at which point the appliance is switched OFF. We also assume that for each appliance we know whether it is interruptible or not, and the number of times it must be used, n_A.

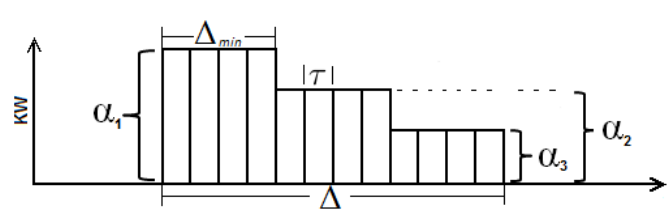


Fig. 4: Multiphase appliance modeling

C. Optimization Problem

The microgrid consists of distinct agents each with their goals and priorities; houses need the energy to run their appliances

according to pre-defined plans, generators sell energy to the homes in the microgrid or the NEG, houses want to purchase cheap energy whereas generators want to maximize their profit. We can associate a cost function Ψ_h to each house $h \in \mathcal{H}$:

$$\Psi_h = \int \lambda_h L_g^h dt + \sum_r \int \gamma_r^h G_r^h dt, \tag{2}$$

where L_g^h describes the amount of energy from the NEG used by house h over time, and G_r^h the amount of energy generated from plant r used by h. The profit function Ξ_r to each $r \in \mathcal{R}$:

$$\Xi_r = \int \zeta_r E_g^r dt + \sum_h \int \gamma_r^h G_r^h dt, \tag{3}$$

E_g^r shows the amount of energy produced by r that is sold to the NEG. The discomfort function of house, h, is defined as follows:

$$\omega_h = \sum_{m \in \mathcal{M}} \int |T_{in}^{h,m}(t) - T_{opt}^{h,m}| dt, \tag{4}$$

Microgrid discomfort is calculated by $\Omega = \sum_{h \in \mathcal{H}} \omega_h$. The energy lose, Λ, in power line between r and h is defined by:

$$\Lambda = \sum_{\forall r \in \mathcal{R}} \sum_{h \in \mathcal{H}} \int \varrho_{r,h} d_h^r G_h^r dt, \tag{5}$$

where $\varrho_{r,h}$ is lose rate between r and h in kWh/m. The problem of allocating energy to houses in a micro-grid in a way that is cost effective for the grid entities can then be cast as a multi-objective optimization problem [21].

$$\min(\Psi_h : h \in \mathcal{H}; -\Xi_r : r \in \mathcal{R}; \Lambda; \Omega) \tag{6}$$

III. MILP Formulation

This Section gives a (multi-objective) mathematical programming formulation of the problem. It, also, discusses the hardness of the problem and suggests a heuristic algorithm to tackle it.

A. Appliances modeling and linear constraints

We assume that each instance of the problem is solved over a fixed time horizon and that time is divided into a finite set of time slots, $\mathcal{T} = \{1, \ldots, T\}$, all of length τ with $0 < \tau < \Delta_{\min}$. We assume that τ divides the length of each phase within the system. We identify the m_h appliances in house h with the numbers $1, 2, \ldots, m_h$. Without loss of generality, we also assume that each appliance i runs through Δ_i^h phases, of length τ. We use a dedicated binary variable $x_{i,j}^h(t)$ for appliance i in phase j. The variable holds the appliance ON/OFF state at time t.

$$P_{i,j}^h(t) = \alpha_{i,j}^h \cdot x_{i,j}^h(t) \in \left\{0, \ldots, \alpha_{\Delta_i^h}^h\right\}. \tag{7}$$

We also assume that appliance i in h can only be run between time slot $t_s^{h,i}$ and $t_f^{h,i}$ (with $t_s^{h,i} \leq t_f^{h,i}$), in a so called comfort interval specified by the user. We model this using Eq. (8),

$$\sum_{t=0}^{t_s^{h,i}-1} x_j^{h,i}(t) + \sum_{t=t_{f1}^{h,i}+1}^{t_T} x_j^{h,i}(t) = 0, \tag{8}$$

where either sums may be empty if $t_s^{h,i} = 1$ or $t_f^{h,i} = T$. If both equalities hold (say if the user does not specify a comfort interval) the constraints vanish. To enforce appliance i in h runs n_i^h times in $\{t_s^{h,i}, \ldots, t_f^{h,i}\}$, we need the following constraints

$$\sum_{t \in \{t_s^{h,i}, \ldots, t_f^{h,i}\}} x_j^{h,i}(t) = n_i^h. \tag{9}$$

Phases can be kept in order by imposing the following constraint,

$$\sum_{t \in \mathcal{T}} \left[t \cdot x_{j+1}^{h,i}(t) - t \cdot x_j^{h,i}(t)\right] \geq 1. \tag{10}$$

and to prevent interruption between any two consecutive phases, we use constraint (10) with "=" replacing "≥".

The operation of some appliances depends on external conditions rather than initial user demands. For instance charging a battery depends on the battery charging state $\Theta_i^h(t)$ and its charging rate, α_i^h, whereas the operations of an Air Conditioning (AC) unit depends on the room temperature, $T_{in}^{h,i}(t)$, the outside temperature and the device heating or cooling power [20]. Appropriate constraints in such cases replace those in (9). In the case of batteries, we need to use Eqs.(11) and (12).

$$\Theta_i^h(t) = \Theta_i^h(t-1) + \frac{1}{4} \cdot \pi \cdot P_{i,1}^h(t) \quad \forall t : t \in \left\{t_s^{h,i}, \ldots, t_f^{h,i}\right\} \tag{11}$$

$$\Theta_i^h(t_s^{h,i}) = \underline{\beta}_i^h, \quad \Theta_i^h(t_f^{h,i}) = \overline{\beta}_i^h \tag{12}$$

where $\underline{\beta}_i^h$ is the initial state of charge of the battery, $\overline{\beta}_i^h$ is the desired final state of charge of the battery (usually full), and π is the battery charging efficiency.

In the case of heating/cooling units, the main task of the given unit is to keep the room temperature within the comfort level $[T_{min}^{h,i}, T_{max}^{h,i}]$ during b_i^h specified time intervals $I_1^h, \ldots, I_{b_i}^h$. The relationship between room temperature and the power allocated to the appliance is shown in Eq. (13).

$$T_{in}^{h,i}(t) = \epsilon \cdot T_{in}^{h,i}(t-1) + (1-\epsilon)\left[T_{out}(t) - \frac{\eta}{\kappa} P_{i,1}^h(t)\right] \tag{13}$$
$$T_{min}^{h,i} \leq T_{in}^{h,i}(t) \leq T_{max}^{h,i} \ \forall t : t \in I_1^h \cup \ldots I_{b_i}^h$$

where ϵ is the appliance inertia, η is efficiency of the system (with $\eta > 0$ for a heating appliance and $\eta < 0$ in the case of cooling), κ is the thermal conductivity, $T_{out}(t)$ is outside temperature at time t.

B. Objective Function and Additional Constraints

For the purpose of our experiments we simplify the general model presented in Section II-C. The cost function in Eq.(2) is replaced by the linear function

$$\Psi_h = \sum_{t \in \mathcal{T}} \left\{\lambda(t) L_g^h(t) + \sum_{r \in \mathcal{R}} \left[\gamma_r^h(t) G_r^h(t)\right]\right\} \forall h : h \in \mathcal{H}, \tag{14}$$

and similarly, the profit function in Eq.(3) is replaced by

$$\Xi_r = \sum_{t \in \mathcal{T}} \left\{\zeta(t) E_g^r(t) + \sum_{h \in \mathcal{H}} \left[\gamma_r^h(t) G_r^h(t)\right]\right\} \forall r : r \in \mathcal{R}. \tag{15}$$

Note that we are assuming that the cost of the energy from the NEG, λ, and the profit obtained selling energy to the grid, ζ, may vary over time but are otherwise identical for all houses and generators in the system. Also if r belongs to h then $\gamma_r^h(t)$= 0 $\forall t$, and Ψ_h is the right-hand side of (14) minus Ξ_r.

The discomfort function in Eq.(4) is replaced by Eq.(16)

$$\Omega = \sum_{h\in\mathcal{H}} \sum_{r\in\mathcal{R}} \sum_{t\in I_j^r} |T_{in}^{h,r}(t) - T_{opt}^{h,r}|. \tag{16}$$

The lose power function in Eq.(4) is replaced by Eq.(17),

$$\Lambda = \sum_{h\in\mathcal{H}} \sum_{r\in\mathcal{R}} \sum_{t\in\mathcal{T}} \varrho_{r,h} d_h^r G_h^r(t) \tag{17}$$

Few constraints need to be added to the system. There are the renewable power constraints

$$E_g^r(t) + \sum_{h\in\mathcal{H}} G_r^h(t) = P_r(t) \quad \forall t : t \in \mathcal{T}, \forall r : r \in \mathcal{R}, \tag{18}$$

where $P_r(t)$ is the renewable power generated by r, and power balance equations, enforcing that the allocated power at any time slot, t, must equal power demand at that time

$$L_g^h(t) + \sum_{r\in\mathcal{R}} G_r^h(t) = \sum_{i\in\mathcal{A}_h} \sum_{j=0}^{\Delta_i^h} P_j^{h,i}(t), \quad \forall t : t \in \mathcal{T}, \tag{19}$$

All cost functions and constraint are linear. Therefore, we will use MILP formulation for our model.

C. MILP Formulation Issues

There are two issues with the objective function in Eq.(6). The first issue is that we can not use the absolute value of a variable directly in linear programming. Therefore, we need to represent Eq. (16) to eliminate absolute sign. Due to page limit, we can not explain it here, readers are refereed to our previous paper [22] for more details. The second issue is that, to the best of our knowledge, there is no LP solver can tackle MILP-based multi-objective optimization problem (MOOP). There are many ways used to convert MOOP to single objective optimization problem (SOOP). The most suitable method for our problem is ϵ-constraint [23]. However,the main issue with this approach is that the solution is biased to one objective which will cause fairness issues. Scalarizing method is another way to convert MOOP to SOOP and it is better than ϵ-constraint concerning fair profit. Therefore, we will use a combination of both method; we called this method "Scalarizing ϵ-constraint".

$$\text{Min} \left(\sum_{h\in\mathcal{H}} w_h \Psi_h + \sum_{r\in\mathcal{R}} w_r \Xi_r + w''\Omega + w'\Lambda \right), \tag{20}$$

and extra constraints

$$\Psi_h \leq \tilde{\Psi}_h \qquad \forall h : h \in \mathcal{H} \tag{21}$$

$$\Xi_r \geq \tilde{\Xi}_r \qquad \forall r : r \in \mathcal{R} \tag{22}$$

where w_h, w_r, w' and w'' are weights to bias the optimization toward cost, comfort, or power lose. $\tilde{\Psi}_h$, and $\tilde{\Xi}_r$ are the optimal costs of the energy allocation problem for house h and renewable plant r, considered as isolated units connected solely to the NEG.

D. MILP-based Heuristic

Let MINCOST denote the version of our problem restricted to a single house, with m uniphase appliances, to be allocated in one of two possible time slots. Also assume that the available renewable power is always $\frac{1}{2}\sum_{i=1}^{m} \alpha_i$, and the NEG electricity price is $\lambda > 0$. A straightforward reduction from the PARTITION problem [24] shows that MINCOST is NP-hard. In our experiments, we resort to an MILP-based heuristic algorithm to get a feasible solution in acceptable time. We have used an off-the-shelf LP-solver to generate a feasible solution but without running the optimization process to completion. The LP-solver uses dual relaxation to find a lower bound on the optimum and stops as soon as the difference between the cost of the best feasible solution so far, and the lower bound on the optimum becomes smaller than a predefined threshold.

IV. EMPIRICAL EVALUATION

All the experiments in this paper have been done on a PC with an Intel(R) core(TM) i7-2600 CPU @ 3.4 GHZ, RAM is 16 GB, 64-bit Operating System (windows 7). Besides, Gurobi has been used to solve MILP problem, whereas the Java was the tool to build our model.

1) Communal Input Setting: We will use the following input for all case studies: τ = 5 minutes, T= 288 slots, ζ = 4.5 P/KWH, ξ = 0.0 P/KWH, $\gamma(t)$ = 8.5 P/KWH, π = 0.8, ϵ = 0.96, η= 30 $KW/°$C, κ = 0.98, T_{min}= 18.0 °C, ϱ = 0.02 kWh/m, and T_{max}=22.0 °C. Also, all weights, w, are equal to 1. The first chart in Fig. (5) shows two pricing schemes, a "Fixed" and "Dynamic" pricing; the second chart illustrates predicted renewable power in two days, partly cloudy, and sunny day; last chart in the same figure demonstrates the outside temperature.

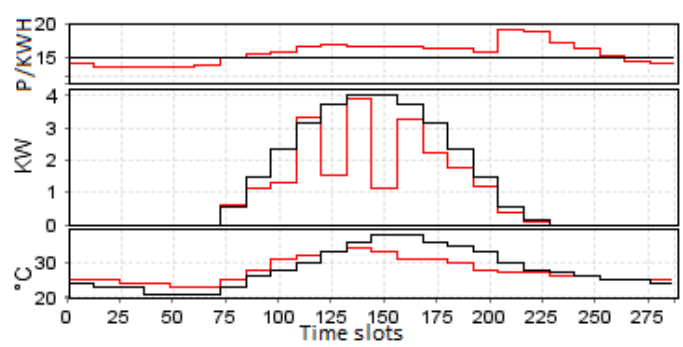


Fig. 5: Electricity Price, one fixed pricing and two dynamic schemes.

A. First case study

The primary purpose of this case study is to compare between ϵ-constraint method and our modified version "Scalarizing ϵ-constraint" method regarding fair profit issue.

1) Input setting : Let us assume that we have 5 identical houses working in a microgrid; each house has a PV array (2.5 kWh) and 8 appliances, see Table I and Table II. Additionally, t_s and t_f for all appliances are 1 and 288, respectively.

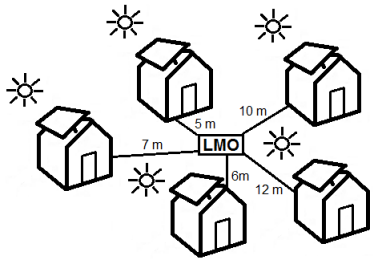


Fig. 6: Diagram of microgrid with 5 coordinating houses and 5 PV arrays.

Table I: Multiphase uninterruptible appliances

Laundry Dryer	α in KW	3.2	0.28	0	3.2	0.28
	ϕ in minutes	15	10	5	20	10
Dishwasher	α in KW	0.2	2.7	0.2	2.7	0.2
	ϕ in minutes	5	15	15	20	5
Washing Machine	α in KW	2.2	0.28	2.2	0.28	-
	ϕ in minutes	10	20	10	20	-

Table II: Interruptible appliances

Interruptible appliances	α	Depend on
Water heater	3.1 KW/t	-
Electric Towel Radiator	1.5 KW/t	-
Electric cooker	2.5 KW/t	-
Plug-in Hybrid Electric Vehicle	0.35 KW /t	$\Theta(t_s)$=2.0, $\Theta(t_f)$=16.0
Air conditioner	2.3 KW/t	T_{min}=18, T_{max}=22

2) Finding: Table IV shows a comparison between using ϵ-constraint method and our modified version "Scalarizing ϵ-constraint". We have used four scenarios with each method. Furthermore, the standard deviation (last column in the table) shows how good our method regarding profit fairness. By contrast, ϵ-constraint method achieved the same overall saving but just h^* gets almost all the profit. Moreover, we have repeated these four scenarios with ω'= 0, see last column in Table IV (Λ^*) to see how much renewable power this model has saved.

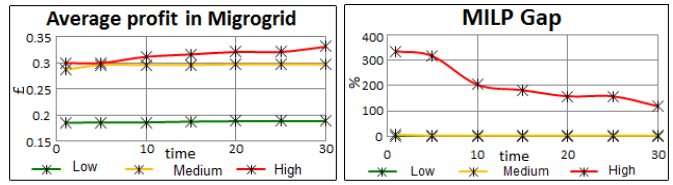
B. Second case study

1) Input setting : 20 houses with variant renewable generation capacities, Table (III), and three independent renewable plants (PV array with maximum generation capacity = 5KW/H, two wind turbines, with 1KW/H, 10KW/H generation capacity, respectively) will be used to investigate the performance of our model. We have used Dynamic pricing in Fig. 5. We will use three scenarios (Low-demand, Medium-demand, and High-demand) to examine the effect of electricity demand on saving. Due to the page limit, we can not fit all the input data for 20 houses. So, we had to put all details in a technical report [25].

Table III: PV array generation capacity of houses

House No	5,10,15	1,6,11,16,19	2,7,12,17,20	3,8,13,18	4,9,14
Capacity	0.0 KW	1.0 KW	1.5 KW	2.0 KW	2.5 KW

2) Findings: Fig. 7a displays the average profit of three scenarios, Low-demand, Medium-demand, and High-demand. The houses with high demand, in general, can make more profit because the relationship between saving and renewable power consumption is positive. In contrast, Fig. 7b shows the relative MILP Gap (duality gap) of the three scenarios. MILP Gap of High-demand scenario is still above 100% after 30 minutes of running time that means the solution found could be far from optimality, it could be so close to optimality though. Besides, the first and second scenarios are so close to optimality because MILP gap is less than 1 %. Fig. 8 illustrates the profit made by each component in the microgrid in three scenarios.



(a) Average profit in microgrid **(b)** the relative MILP Gap

Fig. 7: The result of low demand, medium demand, and high demand.

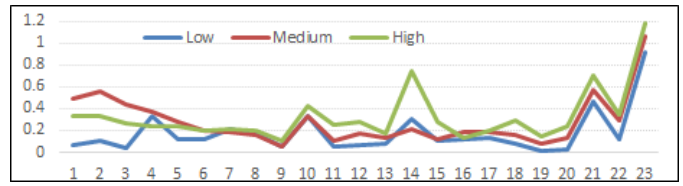


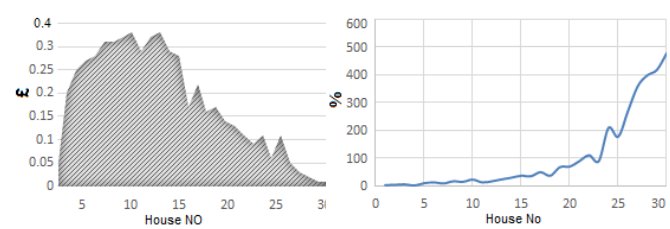
Fig. 8: The profit of entity in micro-grid in three scenarios.

C. Third case study

The main aim of this case study is to check scalability performance of our model and algorithm.

1) Input setting: In this case study, we will use up to 30 identical houses will be used in this case study, each house has 8 different appliances, nominal power of appliances and comfortable time of each house are exactly the same in first case study. Each house equipped with PV array (2.5KW). Additionally, we will repeat this case study without considering power lose to see how much we will save.

2) Findings: Fig. 9a shows that our mathematical model, and the proposed heuristic algorithm give the best saving when the number of houses is between 3 and 15 (when time resolution is 5 minutes and number of appliances around 8), because if the size of the problem becomes huge, the solution provided within particular time will be far away from optimality (large MILP gap), see Fig. 9b.



(a) Average profit of microgrid with 30 houses **(b)** MILP gap of microgrid

Fig. 9: The average profit and MILP gap of microgrid with 30 houses

Table IV: The table shows the profit in BP (£) of houses in microgrid, The profit is the saving made by the house when it works in smart grid and it equal the cost of house work in microgrid minus the cost of independent house connected to NEG only.

			$h^* = 1$	$h = 2$	$h = 3$	$h = 4$	$h = 5$	Total	Average	Standard Deviation	Λ	Λ^*
ϵ-constraint	Day 1	Fixed	1.35	0.00	0.02	0.03	0.01	1.41	0.28	0.53	0.08	0.21
		Dynamic	1.43	0.16	0.04	0.07	0.03	1.73	0.35	0.54	0.07	0.24
	Day 2	Fixed	1.02	0.04	0.15	0.18	0.43	1.82	0.36	0.35	0.11	0.39
		Dynamic	1.71	0.26	0.00	0.04	0.08	2.09	0.42	0.65	0.09	0.39
Scalarizing ϵ-constraint	Day 1	Fixed	0.23	0.27	0.31	0.21	0.39	1.41	0.28	0.01	0.08	0.18
		Dynamic	0.36	0.35	0.43	0.19	0.40	1.73	0.35	0.08	0.08	0.19
	Day 2	Fixed	0.29	0.41	0.41	0.33	0.38	1.82	0.36	0.05	0.12	0.30
		Dynamic	0.43	0.38	0.50	0.23	0.55	2.09	0.42	0.11	0.13	0.31

V. Discussion and conclusion

Regarding the first case study, we can see that the first five houses, in the Medium-demand scenario, have made a higher profit than the first five houses, in the high-demand scenario. This conflicts with the result in Fig. 7a, because, in medium-demand scenario, the first five houses have 7 to 8 appliances, and MILP gap of medium-demand, Fig. 7b, shows that the solution to the Medium-demand is so close to optimality whereas in the High-demand, it is not.

Although our method (Scalarizing ϵ-constraint) has improved the fairness issue in the microgrid, more work needs to be done, because there is still a significant difference in the profit between houses. For example, Table IV shows that $h = 5$ in Day 2 has made a £0.55 profit, whereas $h = 4$ has made a £ 0.23.

The number of entities in microgrid has a positive relationship with the profit. However, Fig. 9a shows that the relationship between the number of houses and profit is not always positive. The relationship between the number of homes and the average profit is positive up to a point, then it becomes negative, that is because we increase the number of houses whereas run time is fixed.

To conclude, this work has shown how an appropriate our mathematical model, and the proposed MILP Heuristic can be for solving the massive multi-objective optimization problem, the results indicate that the sub-optimal cost of each house in a microgrid is cheaper than the optimal value of each home working alone.

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