Optimal strategies for pay-as-you-go pension finance: A sustainability framework

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Abstract

The aim of this paper is to design an automatic balancing mechanism to restore the sustainability of a pay-as-you-go (PAYG) pension system based on changes in its main variables, such as the contribution rate, normal retirement age and indexation of pensions. Using nonlinear optimisation, this mechanism, identifies and applies an optimal path of these variables to a PAYG system in the long run and absorbs fluctuations in longevity, fertility rates, salary growth or any other events in a pension system.

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1 Introduction

Public pension systems are usually financed on a pay-as-you-go (PAYG) basis where pensions for retirees are paid by the contributions of the working-age population. A successful PAYG system requires a balance between the expenditure on pensions and the income from contributions made by the active workers over time, usually termed inter-generational solidarity.[1]

Birth rates have dramatically decreased and with continuous improvements in life expectancy, due to improved health care and medical innovations[32], pensions are paid over a longer time horizon. This causes great difficulties for pension finances and raises serious concerns about the sustainability of the pay-as-you-go pension systems. In the U.S., the Old-Age, Survivors, and Disability Insurance programme (OASDI, 2015 [33]), forecasts that the net present value of expenditure on pensions exceeds the net present value from contributions through the period 2015-2089 and that there is a 50% probability of trust fund depletion by the end of 2034 under intermediate assumptions. In Europe, *The European Commission* (White Paper (2012) [15], Green Paper 2010 [14]) shows a clear increase in life expectancy at birth for males and females of 7.9 and 6.5 years respectively from 2010 to 2060 and that this will cause an increase of the pension expenditure from 10% to 12.5% of GDP in 2060[2]. Furthermore, the increase in unemployment rates, resulting from the recent global crisis, have exerted additional stress on pension systems.

The common trend resulting from these events, that negatively affect the financial health of the systems, is a wave of parametric, or even structural reforms, by changing the formula to calculate the initial pension from a Defined Benefit (DB) PAYG to a Notional Defined Contribution (NDC), with the aim of reducing the expenditure on pensions.[3]

In this respect, Boado-Penas et al. (2008) [9] warn that some politicians, researchers and public opinion mistakenly consider the annual cash-flow deficit or surplus to be an indicator of the pay-as-you-go system's solvency or sustainability[4]; i.e. they confuse an annual liquidity indicator with a sustainability indicator. An actuarial balance

[1] Haberman and Zimbidis (2002) [19] define inter-generational solidarity as the willingness of different groups of people, in this case young and old generations, to participate in a common pool sharing actual experience, including any losses emerging.

[2] In both reports the age of retirement, the contribution rate and the indexation of pensions are fixed according to current legislation in 2010 and 2012.

[3] See Whitehouse et al. (2009) [39], Whitehouse (2009b) [40], OECD (2012) [25]. See Holzmann et al. (2012) [20] for more details on NDCs.

[4] According to Knell et al. (2006) [22], the term sustainability has many definitions, though it almost always refers to the fiscal policies of a government, the public sector or the pension system. On the other hand, the concept of solvency, mostly applicable in the insurance sector, refers to the ability of a pension scheme's assets to meet the scheme's liabilities indicator. Henceforth we will use the term sustainability.

needs to be compiled in order to assess whether or not a system is sustainable in the long run. The actuarial balance, according to Barr and Diamond (2009) [4], is necessary to have the "big picture" of the whole system looking for explicit or implicit assets and not only focusing on the future liabilities of the pay-as-you-go system. The actuarial balance, Boado-Penas and Vidal-Meliá (2013) [10], also supplies a positive incentive to improve financial management by eliminating or at least minimising the traditional mismatch between relatively short-vision of both politician and voters -often only four years- and the indefinite time horizon of the system itself.

Even though the information provided by the actuarial balance can help to take better and more informed decisions, some countries have decided not to use it, i.e., U.S. Other countries, without compiling any balance, have adopted some parametric reforms via emergency modifications in legislation. In this respect, Vidal-Meliá et al. (2009) and (2010) [36] [37] define *Automatic Balance Mechanisms* (ABMs) as a set of pre-determined measures established by law to be applied immediately as required according to a sustainability indicator or any other indicator that reflects the financial health of the system. Its purpose, through successive application, is to re-establish the financial equilibrium of pay-as-you-go pension systems with the aim of making those systems viable without the repeated intervention of the legislators. According to D'Addio and Whitehouse (2012) [12] three main automatic mechanisms can be considered for changing pension values. First, adjustments can be made in benefit levels to reflect changes in life expectancy; second, adjustments can be made through revaluation of earlier years contribution bases and third, adjustments may occur through the indexation of pension payments. In practice, OECD (2012) [25], Turner (2007, 2009) [34] [35] state that at least 12 countries link the indexation of pensions to life expectancy or some other type of indicator.

It is important to highlight that Haberman and Zimbidis (2002) [19] and Godínez-Olivares et al. (2015) [17], propose parametric reforms for restoring the liquidity in the PAYG pension schemes using a buffer fund. In particular, Haberman and Zimbidis (2002) [19], using standard linearisation procedures, obtain a closed solution for the optimal paths of the contribution rate and retirement age while trying to keep these variables at the same level over time. Godínez-Olivares et al. (2015) [17] propose an Automatic Balance Mechanism to restore the liquidity of the system at the end of a 20-year horizon. In this case, the authors use a logarithmic function to minimise the growth rate of the contribution rate, retirement age and indexation on pensions.

In this paper, for the very first time according to the authors' knowledge, two different automatic balancing mechanisms are developed using nonlinear optimisation techniques to restore the sustainability over a 75-year time horizon into a DB-pay-as-you-go pension system while keeping the system liquid at all times. The functional objective function is set to guarantee that the net present value of the income from

contributions is sufficient to cover the pension expenditure in the long run.
Following this introduction, the next section of the paper describes the methodology to compile the actuarial balance with the aim of assessing the sustainability of the system. The third section describes the optimisation techniques in a pay-as-you-go pension system and the mathematical preliminaries together with the main notation and definitions. Section 4 describes the two Automatic Balance Mechanisms proposed to guarantee the sustainability into a PAYG pension scheme. Section 5 shows a representative application given a population structure and suggests how an ABM should be designed for both symmetric and asymmetric cases. Section 6 conclude and make suggestions for further research.

2 The actuarial balance (AB)

The paper by Plamondon et al. (2002) [29] is a first attempt to conceptualise the actuarial balance of the PAYG system defined as the difference between an income rate and a cost rate computed over various periods, in a currency unit and as a percentage of insured earnings. After that, Boado-Penas and Vidal-Meliá (2013)[5] [10] and Billig and Ménard (2013) [8] describe the different types of actuarial balance for the pay-as-you-go pension systems.
According to Boado-Penas and Vidal-Meliá (2013) [10], the main methodology used to compile the actuarial balance in nonfinancial Defined Benefit systems could be described as an aggregate accounting projection model that compares the net present value (NPV) of the expenditure on pensions and the income from contributions, in a long time horizon.
This kind of actuarial balance uses a forecast demographic scenario to determine the future evolution of the number of contributors and pensioners according to the rules of the pension system. The macroeconomic scenario that determines the amounts of future contributions and pensions is exogenous.
In practice, the actuarial balance (AB) at time zero is defined as the difference between the NPV of future contributions valued at zero and the NPV of future pension benefits valued at zero.
In practice, allowing for particular differences between countries, actuarial balances are compiled, on a regular basis, in countries such as U.S, Japan or Canada, amongst others. The actuarial balance of the U.S. social security programs, (OASDI 2015) [33], is aimed at measuring the system's financial solvency over a 75 year time horizon. In Japan, the Actuarial Affairs Division of the Ministry of Health, Labour and Welfare

[5] This paper also includes the methodology of the actuarial balance sheet, in the accounting sense of the term, that is used in the Swedish notional pension system.

(AAD 2009 [2]) compiles an actuarial balance at least every five years with a 95-year time[6] that includes an automatic balancing mechanism to make the system sustainable in such horizon of time.[7] In Canada, actuarial valuation reports on the Canada Pension Plan (CPP) are prepared by the Office of the Chief Actuary (OCA, 2012 [27]) every three years. These reports determine a minimum contribution rate and show projections of the Plan's contributions, expenditures and assets for the next 75 years.[8] After compiling an actuarial balance, the next section presents the nonlinear optimisation model for two different types of Automatic Balance Mechanisms, to restore the sustainability into the system, involving changes in the contribution rate, retirement age and indexation on pensions.

3 Optimisation techniques in a PAYG pension system

This section presents the optimisation[9] techniques used to calculate the optimal and smooth paths of the variables involved in the PAYG system such as the contribution rate, the age of normal retirement and the indexation of pensions. We extend the optimisation model developed by Godínez-Olivares et al. (2015) [17] which focused only on the liquidity of the system in a 20-year time horizon. In this paper, we are particularly interested in finding the optimal paths of the key variables that guarantee not only the short run equilibrium (liquidity in the system) but also the long-run sustainability of the pension finance.

In a general nonlinear optimisation problem (NLO)[10] the functions and parameters involved are:

(a) The decision variables that are represented by $\{d\}_n = \{d_n^0, d_n^1, ..., d_n^v\} \in D$; where $D \in \Re^n$ is the decision space , n is the step of the process with $n \in \mathbb{N}$ and v is the number of variables included in d. Specifically in this model $d_n^j = (c_n^j, x_n^{(r)j}, \lambda_n^j)$ where $\{c\}_{n\in\mathbb{N}}$ is the contribution rate, $\{x^{(r)}\}_{n\in\mathbb{N}}$ is the age of retirement and $\{\lambda\}_{n\in\mathbb{N}}$ is the indexation of pensions; and n is the year.

[6] The U.S and Japanese actuarial balances include the financial assets (buffer fund) at the beginning of the valuation period and also the level of the fund that reaches a magnitude of one-year's expenditure at the end of the time horizon.

[7] A modified indexation is applied to both the revaluation of the contribution bases and pensions in payment. It takes into account both improvements in life expectancy and population increases (or decreases).

[8] Each actuarial valuation report is based on a number of best-estimate assumptions that reflect the judgement of the Chief Actuary of the CPP as to future demographic and economic conditions.

[9] Optimisation techniques are extensively used in economics in the intertemporal consumption context (see for example, Obstfeld and Rogoff (1996) [24], Wilcoxen (1989) [41], Walde (2010) [38]). However, there are not many application on pensions systems (see for example, Devolder et al. (2002) [13], Feldstein and Liebman (2002) [16], Haberman and Vigna (2002) [18], Roberts (2002) [30] and Sayan and Kiraci (2001) [31]).

[10] For more details see Bertsekas (1999) and (2005) [6] [7].

(b) A function $f_n(d_n^j, n)$, that is called, the objective function of the NLO. In our case, $f_n(d_n^j, n)$ is expressed by a minimisation function;

(c) The set of feasible solutions $\mathcal{F} = \{d_n^j \in D | h_k(d_n^j) = 0, k = 1, ...l$ and $g_j(D) \leq 0, j = 1, 2, ...m\}$; where h_k for $k = 1, ...l$ represent l linear constraints and g_j for $j = 1, ..., m$ represent m nonlinear constraints. In our model, following the ideas introduced by Godínez-Olivares et al. (2015) [17], the constraints are defined by the upper and lower bounds and change of rates of the key variables at time n and also by the liquidity restrictions, and;

(d) If d^* minimise (or maximise) f, then d^* is called an optimal solution of the NLO. If $\mathcal{F} = \emptyset$ then the NLO is not feasible, i.e., there are no d^* that minimise (or maximise) the objective function.

Following Godínez-Olivares et al. (2015) [17], we applied the Generalized Reduced Gradient (GRG) algorithm to find the optimal path of the variables. This iterative method allows us to find the optimal solution of the NLO. The GRG is the extension of the gradient method to constrained and bounded optimisation problems. It was first developed by Abadie and Carpenter (1969) [1] and is still today one of the most powerful nonlinear optimisation algorithms.[11]

In practice, building the model (modelling phase) is as important and sometimes more difficult than the solving phase (Nemirovski (1999) [23]). The modelling phase, i.e. the formulation of the Automatic Balance Mechanism, is discussed in the following section.

4 Automatic Balancing Mechanism (ABMs) to restore sustainability

This section presents two different designs of Automatic Balance Mechanisms to restore both the liquidity and sustainability of a pay-as-you-go pension system to find the optimal path for the decision variables -contribution rate, age of normal retirement and indexation of pensions- in a PAYG pension scheme.

The first Automatic Balance Mechanism (SA -Sustainability ABM-) restores the system's sustainability measured via the actuarial balance as the difference between the net present value of the future income from contributions and the expenditure on pensions in the long run. The second Automatic Balance Mechanism (SAF -Sustainability ABM including the buffer fund-) takes into account the accumulated value of the financial assets (buffer fund) emerging from the difference between the income from contributions and expenditure on pensions in each year.

[11] See, for example, Kallrath (1999) [21], Bazaraa et al. (1993) [5] and Wright (1996) [42].

The next subsections describe how the two different Automatic Balance Mechanisms are built.

4.1 Sustainability ABM (SA)

The optimisation function, $f_n(d_n^j, n)$, to minimise for the SA for a time horizon N, subject to some constraints, is as follows:

$$min_{c_n x_n, \lambda_n} \sum_{n=0}^{N} \frac{c_n W_n(g_n, x_n^{(r)})}{(1+\delta)^n} - \sum_{n=0}^{N} \frac{B_n(g_n, x_n^{(r)}, \lambda_n)}{(1+\delta)^n}$$

$$s.t. = \begin{cases} c_{min} \le c_n \le c_{max}; x_{min}^{(r)} \le x_n^{(r)} \le x_{max}^{(r)}; \\ \lambda_{min} \le \lambda_n \le \lambda_{max}; \\ c_{1\Delta} \le \dfrac{c_{n+1}}{c_n} \le c_{2\Delta}; x_{1\Delta}^{(r)} \le \dfrac{x_{n+1}^{(r)}}{x_n^{(r)}} \le x_{2\Delta}^{(r)}; \\ \lambda_{1\Delta} \le \dfrac{\lambda_{n+1}}{\lambda_n} \le \lambda_{2\Delta}; \\ \dfrac{c_n W_n(g_n, x_n^{(r)})}{B_n(g_n, x_n^{(r)}, \lambda_n)} \ge 1; \end{cases} \tag{4.1}$$

where:

c_n is the contribution rate during year n; $W_n(g_n, x_n^{(r)})$ is the total contribution base[12] paid at n that depends on the growth of salaries g_n and the retirement age at n, $x_n^{(r)}$; B_n is the total expenditure on pensions at n that depends on g_n, $x_n^{(r)}$, and the indexation of pensions at n, λ_n; $\delta > 0$ is the discount rate.

$c_{min}, x_{min}^{(r)}, \lambda_{min} \in \Re$ and $c_{max}, x_{max}^{(r)}, \lambda_{max} \in \Re$ are lower and upper bounds of the decision variables respectively. These bounds are set in order to avoid possible unrealistic changes in the key variables of the pension system.

Smooth constraints are also necessary to prevent jumps in path of the contribution rate, age of retirement and indexation of pensions. Mathematically, smooth constraints are set as: $c_{1\Delta} \le \frac{c_{n+1}}{c_n} \le c_{2\Delta}; x_{1\Delta}^{(r)} \le \frac{x_{n+1}^{(r)}}{x_n^{(r)}} \le x_{2\Delta}^{(r)}; \lambda_{1\Delta} \le \frac{\lambda_{n+1}}{\lambda_n} \le \lambda_{2\Delta}$

where $c_{1\Delta}, c_{2\Delta}, x_{1\Delta}^{(r)}, x_{2\Delta}^{(r)}, \lambda_{1\Delta}, \lambda_{2\Delta} \in \Re$.

Finally, the liquidity restriction $c_n W_n(g_n, x_n^{(r)}) \ge B_n(g_n, x_n^{(r)}, \lambda_n)$ for all n is imposed to ensure that the income from contributions at n covers the annual expenditure on pensions. Consequently, the liquidity indicator that measures the liquidity of the system, taking only into account the income from contributions and the expenditure of pensions is defined as:

[12] The contribution base of the system is defined as the total wages of the working population.

$$L_n = \frac{c_n W_n(g_n, x_n^{(r)})}{B_n(g_n, x_n^{(r)}, \lambda_n)} \tag{4.2}$$

Since $\delta > 0$ and thanks to the liquidity restriction, the optimal solution for this Automatic Balance Mechanism built by equation 4.1 should be zero. However, it is well-known that in practice it is not possible to have an exact equilibrium for the valuation problem, and that is the reason for minimising the difference between the income from contributions and the expenditure on pensions.

4.2 An alternative design for the Sustainability ABM (SAF)

The amount of the buffer fund, F_n, or even the returns that it generates, might have a positive effect on a pension system facing unexpected changes in the demographic or economic projections. For example, the returns generated by the buffer fund in Spain covered the shortfall in contributions during 2010. With this in mind, the Sustainability ABM including the buffer fund (SAF), that includes the accumulation of the fund over time, can be expressed as:

$$F_n = (1 + J_n)F_{n-1} + c_n W_n(g_n, x_n^{(r)}) - B_n(g_n, x_n^{(r)}, \lambda_n), \tag{4.3}$$

where J_n is the return[13] of the fund during year n. The variable F_n fluctuates deliberately to absorb changes in fertility, mortality projections and any other events that might affect the liquidity and sustainability indicator in the pension system.
In contrast to the Automatic Balance Mechanism built in 4.1, the SAF is focused on the minimisation of the present value of the buffer fund. The objective function to minimise, subject to some constraints, is as follows:

$$min_{c_n x_n, \lambda_n} \sum_{n=0}^{N} \frac{F_n(c_n, g_n, x_n^{(r)}, \lambda_n, J_n)}{(1+\delta)^n}$$

$$s.t. = \begin{cases} c_{min} \leq c_n \leq c_{max}; x_{min}^{(r)} \leq x_n^{(r)} \leq x_{max}^{(r)}; \\ \lambda_{min} \leq \lambda_n \leq \lambda_{max}; \\ c_{1\Delta} \leq \dfrac{c_{n+1}}{c_n} \leq c_{2\Delta}; x_{1\Delta}^{(r)} \leq \dfrac{x_{n+1}^{(r)}}{x_n^{(r)}} \leq x_{2\Delta}^{(r)}; \\ \lambda_{1\Delta} \leq \dfrac{\lambda_{n+1}}{\lambda_n} \leq \lambda_{2\Delta}; \\ F_n \geq 0 \end{cases} \tag{4.4}$$

[13] According to Yermo (2008) [43] one of the most remarkable aspects of the regulatory environment of the reserve funds surveyed is that with the exception of Ireland, Japan, Korea, and Sweden, there are no major investment limitations. The study of the different investment strategies is out of the scope of this paper, therefore our analysis assumes a risk-free investment rate as in the case of the United States, Belgium and Spain.

The same constraints for the contribution rate, the age of retirement and the indexation of pensions as in the Sustainability ABM (eq. 4.1) are imposed. However, the liquidity constraint is now set as $F_n \geq 0$, for all n, to ensure liquidity in the system.
Under the Sustainability ABM including the buffer fund, the natural liquidity indicator that emerges from the objective function and takes into account the accumulated value of the buffer fund is expressed as follows:

$$Lf_n = \frac{(1 + J_n)F_{n-1} + c_n W_n(g_n, x_n^{(r)})}{B_n(g_n, x_n^{(r)}, \lambda_n)} \tag{4.5}$$

The same liquidity indicator as for the Sustainability ABM (without the inclusion of the buffer fund eq. 4.2), can be calculated but, under the Sustainability ABM including the buffer fund model, we cannot guarantee that its value would be higher than or equal to one.

4.3 Symmetric and asymmetric designs for the Sustainability ABM (SA) and the alternative design (SAF)

Most of parametric reforms are based on the general view that expenditure on pensions is increasing, based on an increase in life expectancy, and/or on the fact that the amount of the buffer fund is decreasing. In this respect, Automatic Balance Mechanisms are designed to face adverse demographic and economic changes (asymmetric ABM), but these could also be designed to benefit the system's participants in good times.
Palmer [28] states that under a symmetric Automatic Balance Mechanism, any surplus (defined as the difference between the income from contributions and the expenditure on pensions at n) that might arise would be automatically distributed. In the absence of a symmetric ABM, an undistributed surplus is maintained in the system. Furthermore, Alho et al. [3] state that the balancing mechanism can be symmetric, adjusting for both positive and negative deviations from the financial health indicator. Analytically, for the asymmetric case the change in the contribution rate and age of normal retirement are enforced to be strictly greater or equal to zero (strictly lower or equal to zero for the indexation of pensions).
In our model, under the *symmetric* design, our ABMs determine whether the contribution rate, the age of normal retirement (and indexation of pensions) are reduced (increased) when the system has a surplus and increased (decreased) in periods of deficit. Analytically, relative changes in the contribution rate and age of normal retirement are enforced to be strictly greater or equal to one (strictly lower or equal to one for the indexation of pensions) under the asymmetric design. In contrast, under the *symmetric* case, the rate of change in the variables could be positive or negative.

5 Numerical Example

This section presents a numerical example using the ABMs defined by the optimisation problems 4.1 and 4.4 developed in Section 4. First, the main data and assumptions are presented, secondly the results are discussed and finally this section provides a sensitivity analysis of the results.

5.1 Data

- We use the demographic structure of the European population (Figure 1a) from 2013 to 2087 obtained from Eurostat[14]. It can be seen that the peak of population in 2013 (Figure 1a, grey pyramid) is at ages 40-50 in 2013, corresponding to the demographic boom in the 1960s and early 1970s. By 2087, there are no clear peaks in the population. Also, the age-dependency ratio[15], decreases over time, with 3.63 contributor financing one pensioner in 2013 to 1.94 in 2087 as shown in Figure 1b. For comparison purposes the demographic structure has been normalised.[16]
- The European salary and pension structures over time have been used.[17]
- The salaries (g_n) are assumed to increase at an annual constant rate of 2.5%[18] while the buffer fund is assumed to increase at an annual rate (J_n) of 3%[19].
- The initial pension is set at 55% of final salary[20] and $P_{x,n}$ can be written as:

$$P_{x,n} = P_{x-1,n-1} * (1 + \lambda_{n-1}),$$

where $P_{x_n^{(r)},n} = P_{x_n^{(r)},n} \mathbb{1}_{\{x_n^{(r)},n=1\}} + P_{x_n^{(r)},n} * (1 + g_n)^n \mathbb{1}_{\{x_n^{(r)},n>1\}}$.

- The lower bounds for the contribution rate, age of normal retirement and indexation of pensions are given respectively by 15%, 65 and -5%; the upper bounds

[14] Data obtained from Eurostat Database, http://ec.europa.eu/eurostat, accessed 1 March 2015.

[15] This ratio measures the number of elderly people relative to those of working age. It is calculated as the number of contributors divided by the number of pensioners.

[16] The normalisation rescales data values of the population per age to a new range of values between zero and one.

[17] Data obtained from Eurostat Database, http://ec.europa.eu/eurostat accessed 1 March 2015. The salary structure, shown in Figure 1c, has been normalised to allow comparisons.

[18] Average growth of the hourly labour costs in Europe from 2004 to 2014. Data obtained from Eurostat Database, http://ec.europa.eu/eurostat, accessed 1 March 2015.

[19] This value, which is used for the SAF, is in line with the average value of the Euribor rates during the last 15 years.

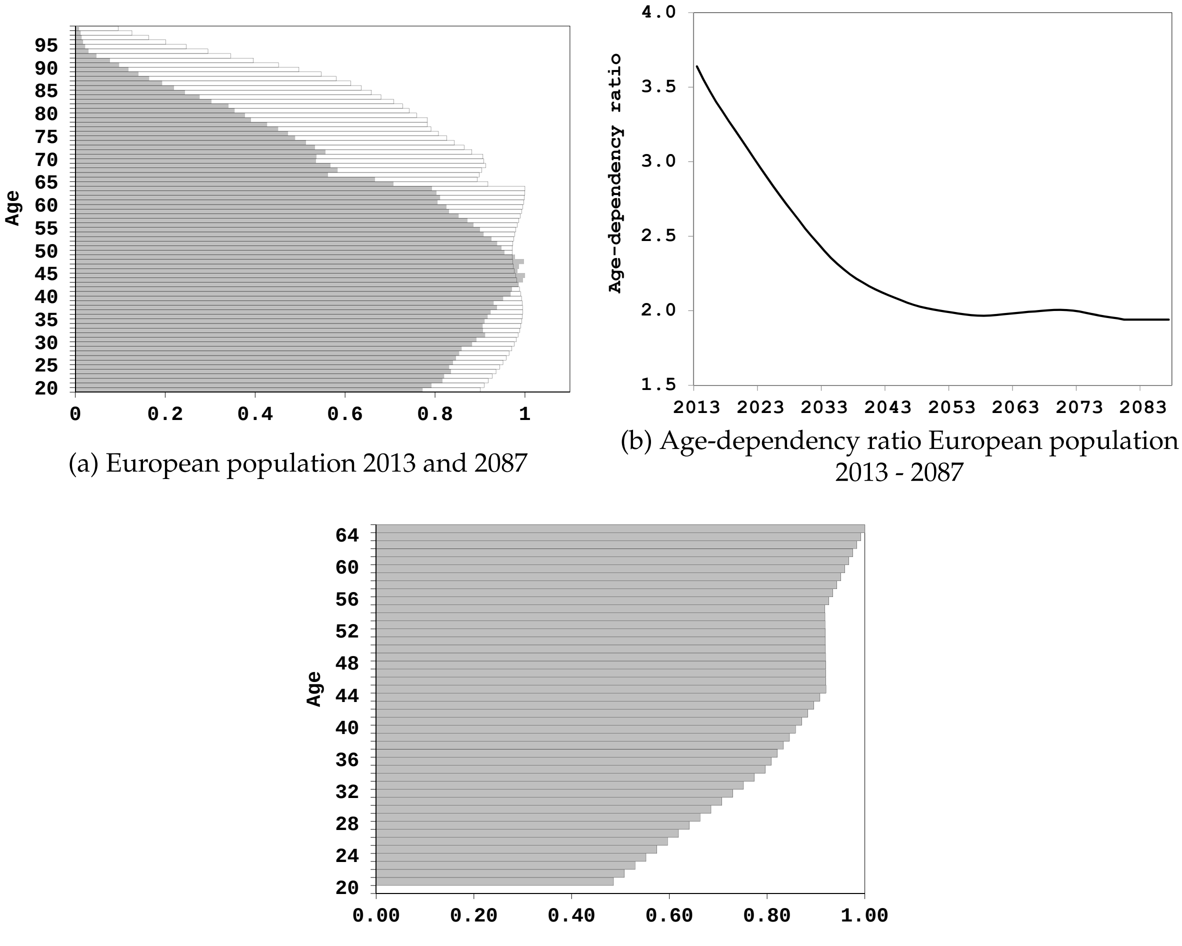
[20] According to Creighton (2014) [11], this level is in line with the average replacement rate which measures the pension as a percentage of a worker's pre-retirement income.

are 20%, 70 and 5% respectively.

- For smooth changes, it is also assumed that the change in the contribution rate varies between 0.3% and 0.7%, the age of normal retirement between 1.5 and 4 months and the indexation of pensions between -0.5% and 0.5%. These values are in line with the most important reforms in the 34 OECD member countries[21] between January 2009 and September 2013.

- No unemployment is considered in our analysis.

Figure 1: Normalised European population structure in 2013 (grey) and 2087 (transparent), evolution of the age-dependency ratio, and salary structure in 2013.



(a) European population 2013 and 2087

(b) Age-dependency ratio European population 2013 - 2087

(c) Salary structure European population 2013

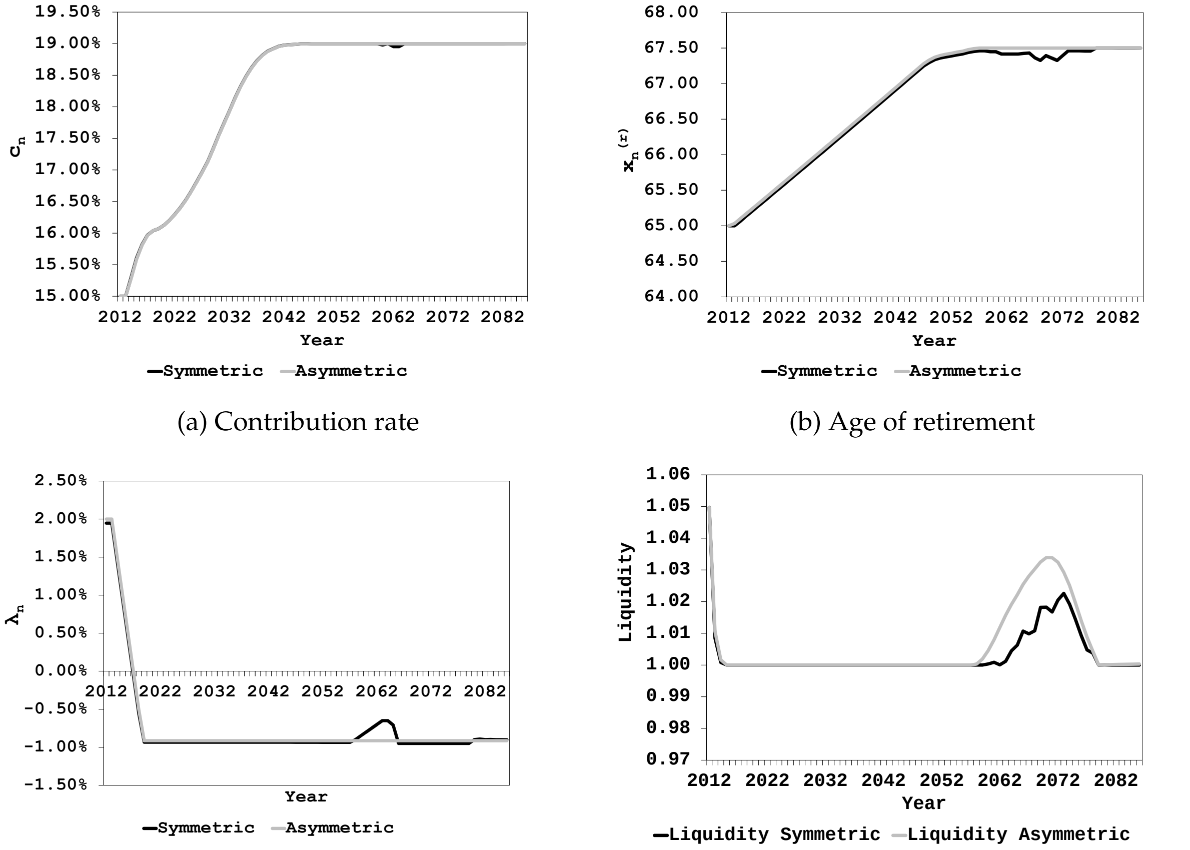
5.2 Results

Figure 2 shows the optimal paths[22] for the Sustainability ABM when the three decision variables (contribution rate, retirement age and indexation of pensions) are modified

[21]See OECD (2013) [26].

[22]For both ABMs the optimal solutions found are local minimums, however, as the initial values for the decision variables are close to the real values, the local minimums are the best solutions.

Figure 2: Results of SA when the three variables are projected simultaneously for the symmetric (black line) and asymmetric (grey line) with European population structure.



(a) Contribution rate

(b) Age of retirement

(c) Indexation of pensions

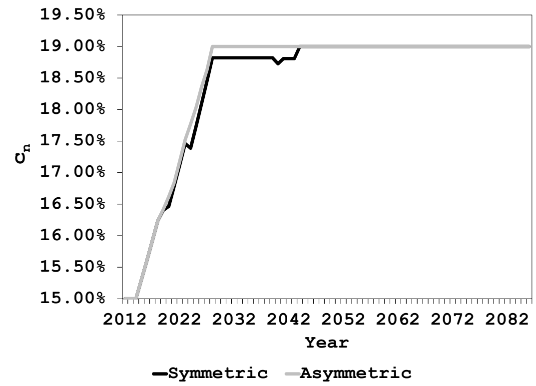
(d) Liquidity indicator (L_n)

simultaneously under both the asymmetric (base case) and symmetric cases. Under both cases, the contribution rate (Figure 2a) stabilises at 19%, and only from 2060 to 2064 is the contribution rate lower under the symmetric design. At the same time, the age of normal retirement (Figure 2b) stabilises at 67.5 under both scenarios, although for the symmetric case the retirement age is lower than under the asymmetric design from 2055 to 2077. The indexation of pensions (Figure 2c) stabilises at -1%[23] at the end of the period of analysis. The liquidity indicator, defined as the ratio of the income from contributions to the expenditure on pensions at every year, stabilises around one. The difference between the symmetric and asymmetric cases is explained by the population structure: the age-dependency ratio (Figure 1b) shows a small blip in the projected ratio between 2058 and 2069, i.e. there are more active workers to finance the pensioners.

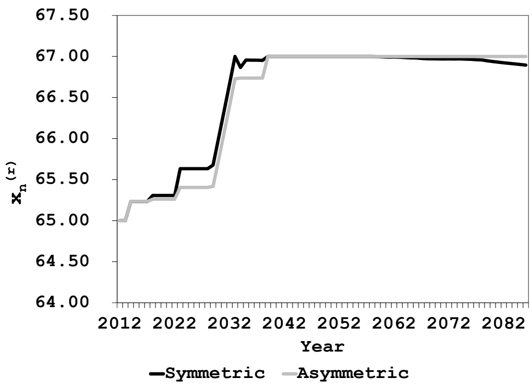
[23]Negative indexation would be a very unpopular and an unlikely measure to be taken. When lower bound of the indexation is set to zero percent, the SA ABM stabilizes at 21% for the contribution rate, 68.44 for the age of normal retirement, and 0% for the indexation of pensions; the SAF ABM, stabilizes at 20.45%, 68 and 0% respectively. However some PAYG pension systems like Sweden has already adopted a negative indexation for some years to make its pension system solvent. For this reason, we would keep a more general setting in our model allowing for negative indexation with a minimum value of -5%.

Figure 3 shows the results of the SAF, which is the modified Automatic Balance Mechanism that takes into account the amount of financial assets of the system. Under this design, the optimal paths of the contribution rate, age of normal retirement (and indexation of pensions) are expected to take lower (higher) values than for the Sustainability ABM design. Figure 3 shows that, under the symmetric design (black line), the contribution rate (Figure 3a) needs to increase to 19%, as in the SA. However, the age of normal retirement increases to 66.8 at the end of the study, 0.7 less than under the Sustainability ABM design. The indexation of pensions takes the same path as for the SA design.

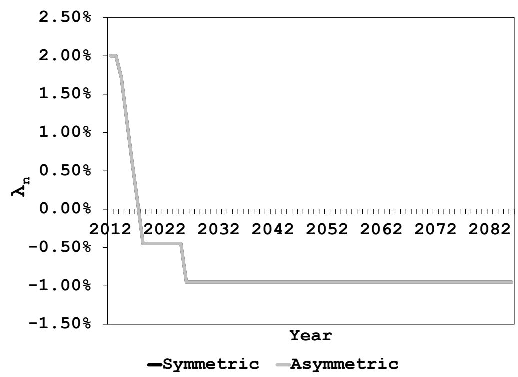
Figure 3: Results of the SAF when the three variables are projected simultaneously - for the symmetric (black line) and asymmetric scenario (grey line) with European population structure.



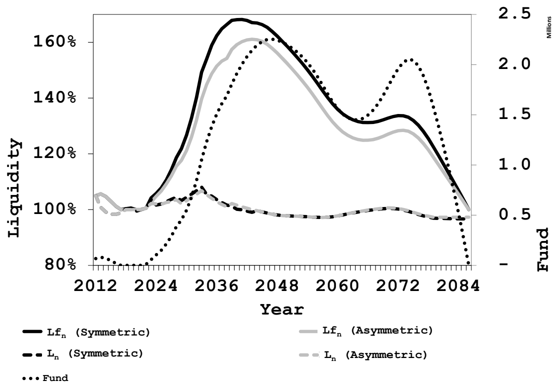
(a) Contribution rate



(b) Age of retirement



(c) Indexation of pensions



(d) Liquidity indicator

Figure 3d shows the liquidity indicator without buffer fund defined by eq. (4.2) (dashed line) and including the amount of the buffer fund defined by eq. (4.5) (solid line) under both the symmetric (black line) and asymmetric (grey line) designs. The liquidity indicator including buffer fund remains always greater than 1, that is, the system always has enough money to cover the expenditure on pensions in every year. However, if we do not take into account the buffer fund (dashed line), the system shows a permanent deficit from year 2040 onwards.

The Automatic Balance Mechanism could also be designed around having only one modified variable rather than having three variables that are modified simultaneously. For the Sustainability ABM, if the contribution rate is the only decision variable, it would need to increase to 31.43%, whereas if the age of retirement is the only variable it would increase to 78.04 years. If the indexation of pensions is set as the only decision variable, the benefits would decrease at an annual rate of 5.15%.

5.3 Sensitivity analysis

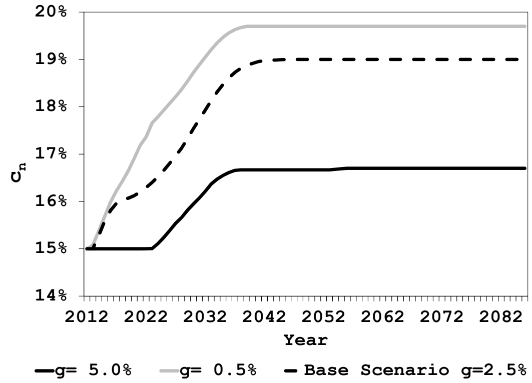
This sub-section performs a sensitivity analysis for different levels of salaries growth (g_n), percentages of the contribution base to calculate the initial pension, and age-dependency ratios in order to see how the main variables of the system react to these changes. From now on, only the Sustainability ABM under an asymmetric design is analysed.[24]

Different levels of growth of salaries (g_n)

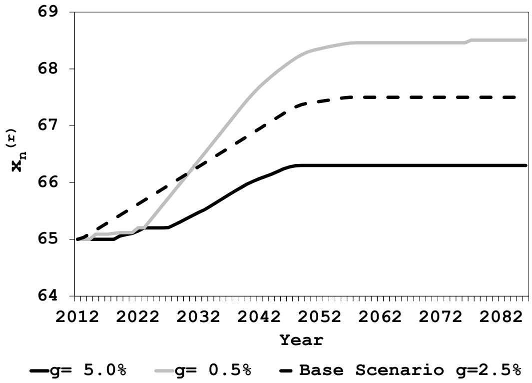
This subsection analyses two different levels of salary growth (g_n). When the growth of salaries is 0.5%, the contribution rate stabilises at 19.7% (Figure 4a), that is 0.7% more than for the base scenario, whereas the retirement age needs to increase to 68.5 (Figure 4b), one year more than the base scenario, and the indexation of pension stabilises at -2.1% (Figure 4c), that is 1.1% less than the base scenario.
If the growth of salaries is set at 5%, the contribution rate stabilises at 16.7%, 2.3% lower than the base scenario with a salary growth of 2.5%, the age of retirement stabilises at 66.3, that is 1.2 years below the base scenario, whereas the indexation of pensions stabilises at -0.4%, that is 0.6% more than the base scenario.
The liquidity of the system (Figure 4d) follows a similar path for the different growth of salary analysed. The increase from 2058 to 2070 corresponds to the increase in the age-dependency ratio shown in the demographic projections. As the liquidity of the system is always greater than 1, it is possible to conclude that the objective of the Automatic Balance Mechanism is achieved.

[24]The results for the SAF are not presented since the conclusions drawn are very similar to the SA.

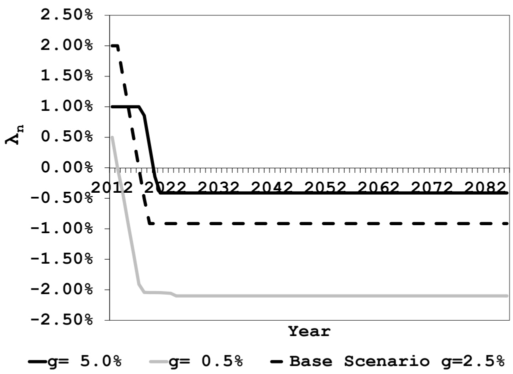
Figure 4: Results of the SA under different levels of growth of salaries: 0.5% (grey), 5% (black) and base scenario (dashed).



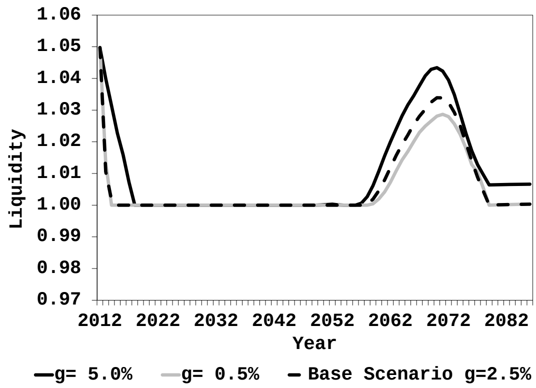
(a) Contribution rate



(b) Age of retirement



(c) Indexation of pensions



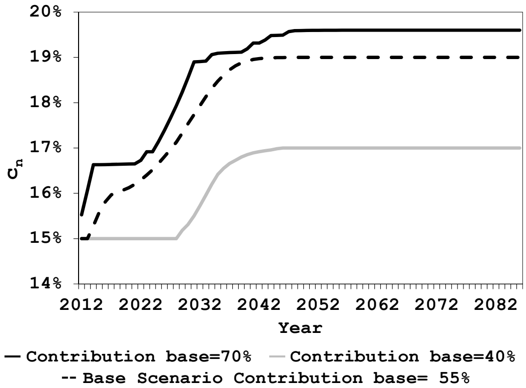
(d) Liquidity indicator (L_n)

Different percentage levels of the initial pension

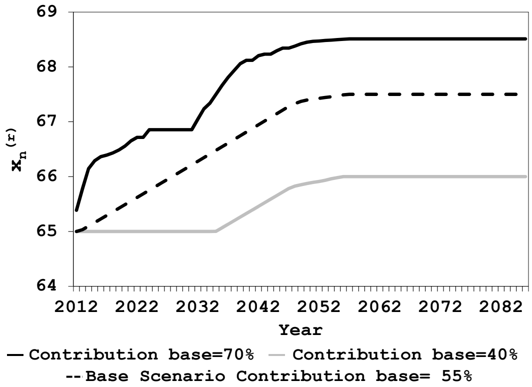
This subsection analyses different percentage levels of initial pension, specifically the initial pension is set at 40% or 70% of the contribution base. The contribution rate (Figure 5a) stabilises at 17% and 19.6% respectively, instead of 19% for the base case scenario. The age of normal retirement stabilises at age 66 and 68.5 respectively, while it stabilises at 67 under the base scenario.

The value of the indexation of pensions stabilises at -0.45% and -1.75% for a 40% and 70% of the contribution base respectively. The liquidity of the system is maintained around one for the whole of the period analysed; thus, the target of the Automatic Balance Mechanism is achieved with these levels of contribution base.

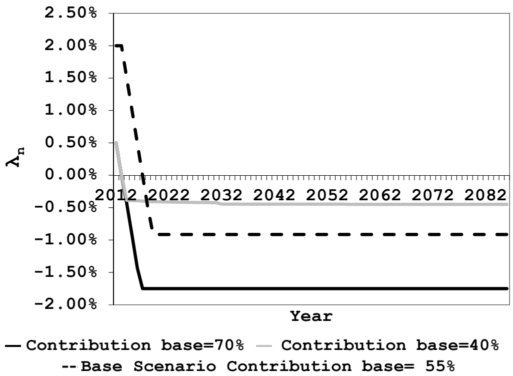
Figure 5: Results of the SA under different levels of initial pension: 40% (grey), 70% (black) and base scenario (dashed).



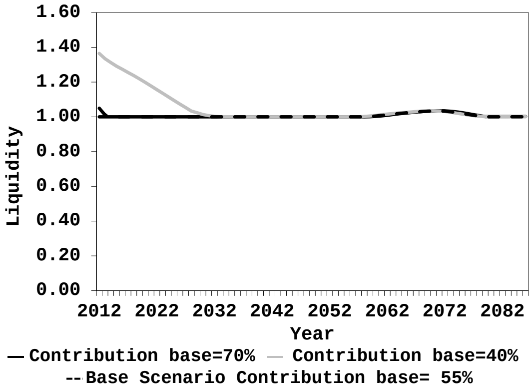
(a) Contribution rate



(b) Age of retirement



(c) Indexation of pensions



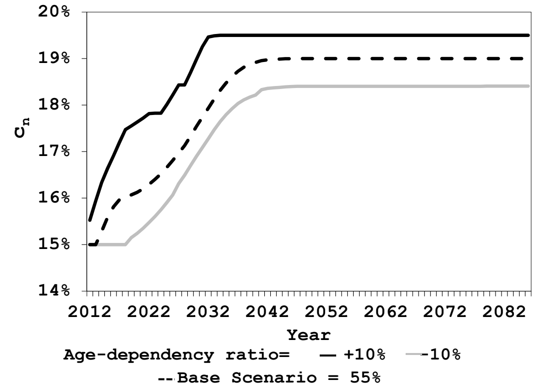
(d) Liquidity indicator (L_n)

Different levels of age-dependency ratio

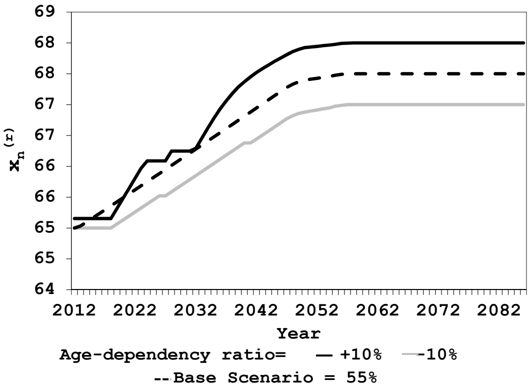
A lower (higher) level of the age-dependency ratio translates into a smaller or greater number of pensioners in our study[25]. When the age-dependency ratio (grey line) is 10% lower (Figure 6a), the contribution rate stabilises below the base case scenario by 0.6%, the age of retirement (Figure 6b) by 0.5 years and the indexation of pensions above by 0.41%. Whereas, when the age-dependency ratio increases by 10% , the contribution rate and the age of retirement are higher by 0.5% and 0.5 years respectively with respect to the base scenario. The indexation of pensions (Figure 6c) is lower by 0.20%. In this case, the liquidity of the system (Figure 6d) is obtained and, again, from 2058 to 2069 the system shows a surplus owing to the given demographic structure during these years.

[25]The constant increase or decrease in the age-dependency ratio affects all ages by the same percentage.

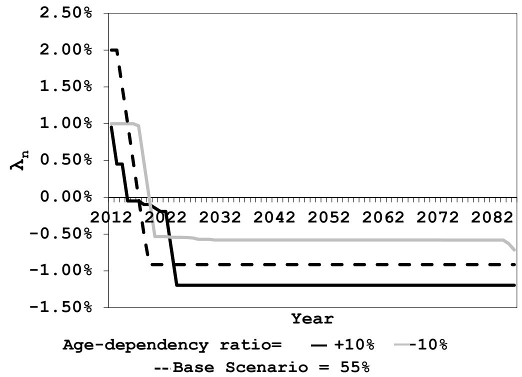
Figure 6: Results of the SA under different levels of age-dependency ratio: -10% (grey), +10% (black) and base scenario (dashed).



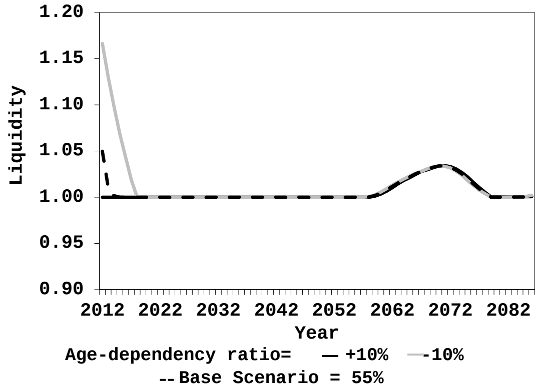
(a) Contribution rate



(b) Age of retirement



(c) Indexation of pensions



(d) Liquidity indicator (L_n)

This section has shown how the three sensitivities presented (different levels of growth of salaries (g_n), percentage of the levels of the initial pension and levels of age-dependency ratio) are all in the expected direction. It is also remarkable that, even in the most optimistic scenario, the key variables will need to be adjusted to guarantee the sustainability of the system in the long run.

6 Conclusions

Restoring the long-term sustainability of a Pay-As-You-Go pension system is on the agenda for most governments. The expenditure on pensions is increasing in line with the increase in longevity, the forecasts for the ageing of the baby-boom generation and any other random events that negatively affect to the financial health of the system. At the same time, the income from contributions does not increase at the same rate mainly due to the decline in the fertility rates.

Social security decisions to restore the sustainability usually involve political risk in the sense that the horizon of the policymakers is less than that of the system itself. This paper aims to provide a solution to restore the long-term sustainability of a pay-

as-you-go system using automatic balancing mechanisms, isolating the measures to be taken from the political arena. Using optimisation techniques, the Automatic Balance Mechanisms presented in this paper propose a theoretical framework to calculate the optimal path of the contribution rate, age of retirement and indexation of pensions.
Some politicians, researchers and sections of public opinion mistakenly consider the annual cash-flow deficit or surplus, that is the liquidity indicator, to be an indicator of the PAYG system's sustainability. Therefore, the two Automatic Balance Mechanisms presented in this paper focus on restoring the sustainability of the pension scheme while keeping the system liquid at all times. The Automatic Balance Mechanisms built are based on minimising the present value of future cash flows (income from contributions and expenditure on pensions) including or not the accumulated value of the buffer fund of the system.
The Automatic Balance Mechanisms presented in this paper could be implemented as an alternative to the parametric pension reforms of the pay-as-you-go systems around the world. Furthermore, the paper discusses two different designs that could be implemented, the symmetric or asymmetric design. Under the Sustainability ABM, no surpluses are accumulated, whereas it is possible to build an alternative Automatic Balance Mechanism which includes a buffer fund, and accumulates financial assets (SAF). For both SA and SAF mechanisms, the contribution rate and indexation of pension stabilise at the same level, 19% and -1% respectively, whereas the age of normal retirement, when the buffer fund is included (SAF), stabilises 0.7 years below the SA design, i.e. 67.5 and 66.8 respectively.
Our paper shows that the sustainability of a generic European pension system is not guaranteed in the long run, even in the most optimistic scenario with a lower dependency ratio or an increase in the total contribution base.
Finally, based on the model presented in this paper, at least two important directions for future research can be identified. First, it would be interesting to apply Automatic Balance Mechanisms to different countries which have recently carried out parametric reforms and assess whether these reforms have any mathematical basis in the long run, in terms of our optimisation framework. Another direction would be to make the model more representative of the real world by means of a periodic recalibration of the optimal path of the key variables in line with current economic and demographic projections.

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