# Financial engineering in pricing agricultural derivatives based on demand and volatility


**Purpose**

The purpose of this paper is twofold. First, we propose a financial engineering framework to model commodity prices based on market fundamentals such as market demand processes and demand functions. This framework explains the relation between demand, volatility and the leverage effect of commodities. It is also shown how the proposed framework can be used to price derivatives on commodity prices. Second, we estimate the model parameters for agricultural commodities and discuss the implications of the results on derivative prices. In particular, we see how leverage effect (or inverse leverage effect) is related to market demand.

**Design/methodology/approach**

This paper uses a power demand function along with the Cox, Ingersoll and Ross (CIR) mean-reverting process to find the price process of commodities. Then by using the Ito theorem the constant elastic volatility (CEV) model is derived for the market prices. The partial differential equation (PDE) that the dynamics of derivative prices satisfy is found and, by the Feynman-Kac theorem, the market derivative prices are provided within a Monte-Carlo simulation framework. Finally, by using a maximum likelihood estimator (MLE), the parameters of the CEV model for the agricultural commodity prices are found.

**Findings**

The results of this paper show that derivative prices on commodities are heavily affected by the elasticity of volatility and, consequently, by market demand elasticity. The empirical results show that different groups of agricultural commodities have different values of demand and volatility elasticity.

**Practical implications**

The results of this paper can be used by practitioners to price derivatives on commodity prices and by insurance companies to better price insurance contracts. As in many countries agricultural insurances are subsidised by the government, the results of this paper are useful for setting more efficient policies.

**Originality/value**

Approaches that use the methodology of financial engineering to model agricultural prices and compute the derivative prices are rather new within the literature and still need to be developed for further applications.

**Keywords:** CEV model, demand elasticity, CIR model

# Introduction

Commodities go through boom and bust cycles. The upward movements and the increased volatility of commodity prices have been largely attributed to emerging markets and massive capital flows into the commodity markets by institutional investors, portfolio managers and speculators. While sometimes demand or supply is unexpectedly higher or lower than usual, and inventories can usually stabilize volatility, there are sometimes near-stockouts, and inventories can approach their maximum storage capacity. However, as commodities are goods, their market prices are strongly affected by market demand. Not only prices, but also price fluctuations can thus be affected by demand. More precisely, at market equilibrium, once market demand functions and demand processes are known, market price processes can be determined. Since price fluctuations are closely related to market prices, there is a natural linkage between the market demand function, its elasticity, and price fluctuations (Keynes 1936, Kaldor 1939, Deaton and Laroque 1992, 1995, 1996, Assa 2015). Therefore, market demand affects derivative prices in two ways, first by changing the price dynamics and second by changing volatility. Apparently, these effects are different when an option is in the money, at the money or out of the money. For instance, if volatility has a reverse relation with prices, volatility will offset the rise of a put option price when it is out of the money. While commodity derivative pricing typically begins with Black (1976), which is itself a variation of Black and Scholes (1973), as we explained above, since both prices and volatility are being affected by market demand, a richer model for prices is needed – one that provides an explicit link with market demand.

In this paper, we explore the linkages between commodity demand, price dynamics and volatility by setting up the Cox, Ingersoll and Ross (1985) model (CIR) for market demand process and the constant elastic volatility (CEV) model for market prices. A simple application of the Ito theorem shows that, if market demand process follows a CIR mean-reverting model and if market demand function is a power function, then price dynamics will follow a CEV process, whose elasticity parameter is in close relation with demand elasticity. The CEV model is an extension of the Black-Scholes-Merton (BSM) model that was primarily developed in 1973 to estimate derivative prices. Over the years, several researchers have noticed the shortcomings of the BSM model and proposed new variations of it. Among them, the Cox (1975) CEV model, for which volatility is a power function of market prices, quickly drew a lot of attention within academia and industry, due to its simplistic approach to model stochastic volatility (for further reading on CEV models see Beckers (1980)).

In this paper, we argue that the CEV model from the finance literature can explain the effects of market demand on prices and volatility. Like Assa (2015), where the author presents market demand process based on market fundamentals, in this paper it is argued that the CVE model can be based on market price fundamentals such as market demand function. To the best of the author's knowledge, the only important work using the CEV model for modelling commodity prices is Geman and Shih (2009), where the authors use real commodity data to describe the differences between seven diffusion processes including the geometric Brownian motion (GBM) and the CEV model, but with little explanation of the effects of market fundamentals on commodity market prices. In this paper, we consider the CEV model for the first time for agricultural commodities, and observe that the CIR model for demand process along with power demand function can support the CEV model for the price process of a large group of agricultural commodities.

The relationship between stock prices and volatility has been discussed in several theories. In general, this relation is important because of financial leverage on the variance of returns, which depends on the firm's portfolio. For instance, French, Schwert and Stambaugh (1987) examine the relation between stock returns and stock market volatility and find evidence that the expected market risk premium is positively related to the predictable volatility of stock returns. In commodity markets, the leverage effect can vary within different groups of commodities. For example, Kristoufek (2014) shows that, while oil prices have a leverage effect, gas prices have an inverse leverage effect.

In this paper, by setting up the CIR model for demand and the CEV model for price processes we provide a framework to discuss the leverage (or inverse leverage) effect of prices. Interestingly, we observe that agricultural market leverage effect can be categorised by the type of commodities. While livestock commodities and orange juice have a leverage effect, the non-livestock commodities (except orange juice) have an inverse leverage effect. For the non-livestock commodities, except orange juice, one can interpret the inverse leverage effect as panic caused by food inflation break out. It is necessary to recall that the Black model (or in principle the BSM model) does not have either a leverage or an inverse leverage effect. The CEV model incorporates prices in modelling volatility by using a new elasticity parameter. As mentioned earlier, because of a strong linkage between this notion of the elasticity and the market demand function, our model can reproduce leverage (or inverse leverage) effects similar to the actual agricultural data.

There are different studies in the literature for estimating the CEV parameters, for instance, see a three-step estimation method in MacBeth and Merville (1980) (also Emanuel and MacBeth (1982)), Schroder (1989) for using estimation based on non-central chi-square distribution and a generalised method of moment (GMM) used in Geman and Shih (2009).

In this paper, by using the maximum likelihood estimator (MLE) the parameters of a CEV model are estimated in a discrete time framework for 13 kinds of daily future agricultural commodity prices, within two groups: livestock and non-livestock commodities. Using a likelihood-ratio-test we test the goodness of fit of the CEV model against the BSM model and we observe that, in most cases, the likelihood-ratio-test rejects the BSM model in favour of the CEV model.

Before closing this section, it is worth mentioning that modelling commodity prices has been developed in different strands of the literature. For instance, storage models and factor models are among the most well-known models. Storage models were introduced in the mid-1930s by Keynes (1936) and Kaldor (1939). Gustafson (1958), for policy applications, defined a set of optimal storage rules and Deaton and Laroque (1992, 1995 and 1996) introduced speculators to the storage model for the first time. For further reading on storage models, see Muth (1961), Beck (1993) and Williams and Wright (1991). On the other hand, the so-called factor models for commodity prices were first introduced in Brennan and Schwartz (1985) and further developed in Gibson and Schwartz (1990) and Schwartz (1997).

The rest of the paper is organised as follows. First, we introduce the CEV model and discuss the functionality of the parameters of this model. Then we show that, if the market demand follows a mean-reverting CIR model and if the demand function is a power function, then the resulting price process is a CEV model. In the next section, we will discuss the effect of the elasticity parameter on derivative prices. Finally, using a maximum likelihood estimator (MLE) we estimate the parameters of CEV models on the prices of agricultural goods and discuss the results.

## The CEV Model

In 1975, Cox extended the Black-Scholes-Merton model to the Constant Elasticity of Volatility (CEV) model which uses stochastic volatility to model equities. Cox considered a security market consisting of two assets: bonds and stock. Let $W_t (0 \leq t \leq T)$ be a standard Brownian motion (or Wiener process) on the probability space. According to Cox, the price of a bond, $b_t$, is given by

(1) $$db_t = rb_t dt,$$

where $r > 0$ and is the constant riskless return and the initial price is $b_0 = 1$.

The price of stock $S_t$, follows the following stochastic differential equation

(2) $$dS_t = \mu S_t + \sigma {S_t}^{\alpha} dW_t,$$

where $\mu$ is the percentage drift and $\sigma$ is the percentage volatility, with the restriction of $\mu \in \mathbb{R}$, $\sigma \geq 0$. Furthermore, $\alpha$ is the elasticity of variance, known as the CEV parameter, which was originally believed to be in the interval $[0,1]$. The initial price is $S_0 = s > 0$.

Under the CEV model, volatility $\sigma(S_t, t)$, is assumed to be $\sigma {S_t}^{\alpha}$ whereas under the Black-Scholes-Merton model it is assumed to be $\sigma S_t$. As a result, any changes in the volatility move randomly with $S_t$ under the CEV model. To provide meaning to the parameter $\alpha$ , it is important to realize that there are two parameters in the CEV model that affect volatility: $\sigma$ and $\alpha$. While larger $\sigma$ can increase the uncertainty, larger $\alpha$ can increase the effect of price changes on volatility. To measure this effect, a measure of relative volatility effect (RVE) is introduced as follows

(3) $$RVE = \frac{S \frac{\partial \sigma(t,S)}{\partial S}}{\sigma(t,S)}.$$

For a CEV model with $\sigma(t,S) = \sigma S^{\alpha}$, one has $RVE = \alpha$. Therefore, the relative volatility effect will increase when $\alpha$ increases. However, from the definition of *RVE,* one realises that this is nothing but the elasticity of volatility with respect to changes in prices

(4) $$Elasticity = \frac{\frac{\partial \sigma(t,S)}{\sigma(t,S)}}{\frac{\partial S}{S}} = \frac{S \frac{\partial \sigma(t,S)}{\partial S}}{\sigma(t,S)} = RVE.$$

Since price and volatility become inversely related with a negative volatility when $\alpha < 1$, it was assumed by Cox that $\alpha$ is in the interval $[0,1]$. This phenomenon is called the leverage effect. When $\alpha > 1$, price and volatility move in the same direction, so if price increases, volatility also increases. This phenomenon is called the inverse leverage effect. The assumption $\alpha > 1$ was first considered by Emanuel and MacBeth (1982).

## Derivative pricing with CEV model

In this section, the price of a derivative (option) on commodity prices is developed. But before, we need to mention that in Cox and Ross (1985), a closed form formula for call/put option prices for $0 < \alpha < 1$ is provided. However, for three reasons we adopt a Monte-Carlo simulation method here. First, we need a general arbitrage free pricing method for all $\alpha > 0$, and not only for $0 < \alpha < 1$. Second, the formula provided in Cox and Ross (1985) consists of infinite sums and integrals which make us ultimately to approximate the option prices, whereas Monte-Carlo methods seem to be more feasible and accurate. Third, the general method we introduce here can be used for pricing derivatives with any underlying asset whose prices follow a stochastic diffusion.

Let us consider a price process $S_t$ as follows

$$dS_t = \mu(t, S_t)dt + \sigma(t, S_t)dW_t. \tag{5}$$

Let us consider a derivative $D(S_T)$ on the underlying asset $S_t$. We denote the dynamic of the derivative price process with $D(t, S_t)$ where $D(T, S_T) = D(S_T)$. Using an arbitrage free argument, the dynamic of a derivative has to satisfy the following Merton Black-Scholes PDE

$$\begin{gathered} \frac{\partial D}{\partial t}(t,x) + \frac{1}{2}\sigma(t,x)^2 \frac{\partial^2 D}{\partial x^2}(t,x) + x\frac{\partial D}{\partial x}(t,x) - rD(t,x) = 0 \\ D(T,x) = D(x) \end{gathered}. \tag{6}$$

Using parameters from the CEV model, one gets

$$\begin{gathered} \frac{\partial D}{\partial t}(t,x) + \frac{1}{2}\sigma^2 x^{2\alpha} \frac{\partial^2 D}{\partial x^2}(t,x) + rx\frac{\partial D}{\partial x}(t,x) - rD(t,x) = 0 \\ D(T,x) = D(x) \end{gathered}. \tag{7}$$

To numerically solve this problem we use the Feynman-Kac theorem. For readers' benefit the Feynman-Kac theorem is repeated here. Consider the following PDE:

(8) $$\frac{\partial u(t,x)}{\partial t} + \frac{1}{2}\sigma^2(t,\ x)\left(\frac{\partial^2 u(t,\ x)}{\partial x^2}\right) + \mu(t,\ x)\left(\frac{\partial u(t,\ x)}{\partial x}\right) - V(t,\ x)u(t,\ x) + k(t,\ x) = 0,$$
$$u(T,\ x) = \psi(x).$$

The Feynman-Kac theorem asserts that the solution $u$ to this equation is given by

(9) $$u(t,\ x) = E^Q\left[\int_t^T e^{-\int_t^s V(\tau,X_\tau)d\tau}\, k(s,\ X_s)ds + e^{-\int_t^s V(\tau,X_\tau)d\tau}\, \psi(X_T)\middle| X_t = x\right]$$

where $Q$ is given as a probability measure for which the following holds

(10) $$dX_s = \mu(s,\ X_s)ds + \sigma(s,\ X_s)dW_s^Q\ s \in [0,\ T].$$

Here $W_s^Q$ is a standard Brownian motion under the probability measure $Q$. In this paper, the derivative prices are

(11) $$D(y,\ t) = E\left[e^{-r(T-t)}D(y_T)\middle|y_t = y\right],$$

where

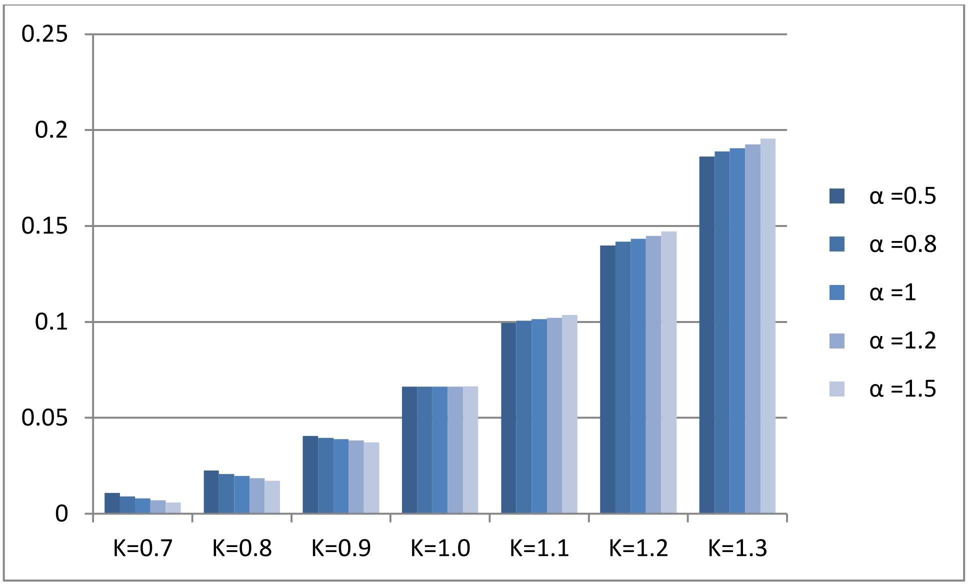
(12) $$dy_s = ry_s ds + \sigma y_t^\alpha dW_s^Q\ s \in [0,\ T].$$

In Table 1 we illustrate this process, synthetically, using numerical methods to price the put option $\max\{K - S_T, 0\}$ for different values of elasticity parameter $\alpha$. Table 1 reports the prices of a put option where $r = 0.004, \sigma = 0.1$ for different strike prices and elasticity parameters, where the strike price is considered as a measure of moneyness. For each simulation we generated 10,000 paths of 120 days.

**Table 1: Put option prices for different alphas, in the money, at the money and out of the money**

| | Moneyness | α =0.5 | α =0.8 | α =1 | α =1.2 | α =1.5 |
|---|---|---|---|---|---|---|
| K=0.7 | ITM | 0.0109 | 0.0091 | 0.0081 | 0.0071 | 0.0058 |
| K=0.8 | ITM | 0.0225 | 0.0208 | 0.0197 | 0.0186 | 0.0172 |
| K=0.9 | ITM | 0.0406 | 0.0396 | 0.0389 | 0.0383 | 0.0373 |
| K=1.0 | ATM | 0.0663 | 0.0662 | 0.0663 | 0.0663 | 0.0665 |
| K=1.1 | OTM | 0.0995 | 0.1006 | 0.1014 | 0.1022 | 0.1036 |
| K=1.2 | OTM | 0.1398 | 0.1419 | 0.1433 | 0.1448 | 0.1471 |
| K=1.3 | OTM | 0.1862 | 0.1888 | 0.1906 | 0.1925 | 0.1956 |

**Figure 1: Prices of put options for different $\alpha$'s and strike prices K.**




As one can see, when the parameter $\alpha$ increases the value of the put option also increases. On the other hand, one can observe that in the money, the larger the $\alpha$, the smaller the put prices; at the money, all put prices are equal; out of the money, the larger the $\alpha$, the larger the put prices. In other words, for constant $\sigma$ inverse leverage effect implies lower put prices when it is more in the money than out of the money. At the money though, leverage effect does not have any effect on put prices. Finally, it is clear that because of the put/call parity we have the reverse observations for call option prices.

## Demand and the Price Process

In this section, we discuss that the CEV model for $\alpha > 1$ can be supported from a micro-economic perspective by showing that the evolution of the price can be derived from the demand side of the market.

Let $x_t$ denote the demand of a commodity at time $t$. Prices are given by the following power demand function

(13) $$S_t = {x_t}^{\beta},$$

where $\beta < 0$ is a parameter to measure the demand elasticity.

Now we need to give a model for $x_t$. We assume that the demand process needs to be mean reverting because the people's consumption is a dispersion of a normal average consumption.

The most popular mean reverting processes in the literature are Ornstein–Uhlenbeck (OU) and Cox–Ingersoll–Ross (CIR) processes. However, demand needs to be always non-negative, which rules out the OU processes.

Let us consider the following CIR process for $x_t$

$$dx_t = \left(\frac{C^2(1-\beta)}{B}\right)\left(\frac{B}{2} - x_t\right)dt + C\sqrt{x_t}dB_t, \tag{14}$$

where $C > 0$ and $B$ are the model parameters, and $\{B_t\}_{0\le t\le T}$ is a standard Brownian motion. In order to make sure that the process is mean reverting, we have to check the Feller conditions (see Feller (1951))

$$\frac{B}{2} > 0, \frac{C^2(1-\beta)}{B} > 0, 2 \times \frac{C^2(1-\beta)}{B} \times \frac{B}{2} \ge C^2, C > 0. \tag{15}$$

Given that $\beta < 0$ and $C > 0$, one can easily see that these conditions hold if and only if $B > 0$.

Now by Ito calculus one can find the price process as follows

$$\begin{aligned} dS_t &= \beta x_t^{\beta-1} dx_t + \frac{1}{2}\beta(\beta-1)x_t^{\beta-2}(dx_t)^2 \\ &= \beta x_t^{\beta-1}\left(\frac{C^2(1-\beta)}{B}\left(\frac{B}{2} - x_t\right)dt + C\sqrt{x_t}dB_t\right) \\ &+ \frac{1}{2}C^2\beta(\beta-1)x_t^{\beta-1}dt \\ &= \left(\frac{\beta(\beta-1)C^2}{B}\right)x_t^{\beta}dt + \beta C x_t^{\beta-\frac{1}{2}}dB_t \\ &= \left(\frac{\beta(\beta-1)C^2}{B}\right)S_t dt + \beta C S_t^{\frac{\beta-\frac{1}{2}}{\beta}} dB_t. \end{aligned} \tag{16}$$

Therefore, one can rewrite the dynamics of the price as

$$dS_t = \mu S_t dt + \sigma S_t^{\alpha} dW_t, \tag{17}$$

where $\mu = \left(\frac{\beta(\beta-1)C^2}{B}\right)$, $\sigma = -\beta C$, $\alpha = 1 - \frac{1}{2\beta}$ and $W_t = -B_t$. This is apparently a CEV process.

Furthermore, one can easily see that $\beta = \frac{1}{2(1-\alpha)}$, $C = \frac{-\sigma}{\beta}$ and $B = \left(\frac{\beta(\beta-1)C^2}{\mu}\right)$. Note that since $\beta < 0$, the Feller conditions hold if and only if $\mu > 0$ and $\alpha > 1$.

Now observe that the demand elasticity parameter $|\beta|$ is in reverse relation with the CEV elasticity parameter $\alpha$. That means, the higher the demand elasticity parameter $|\beta|$, the lower the value of $\alpha$ and therefore, the lower the inverse leverage effect. Based on our observation from the simulations that are reported in Table 1, higher demand elasticity implies higher put option prices in the money, and lower put prices out of the money.

## Data and Estimations

In this paper a Maximum Likelihood Estimator (MLE) is used to estimate the parameters of the CEV model. In order to find the likelihood function first one needs to discretize the price dynamics. Here is the discretization

(18) $$dS_t = \mu S_t + \sigma {S_t}^{\alpha} dW_t \Rightarrow S_{t+\delta} - S_t = \mu S_t \delta + \sigma {S_t}^{\alpha}(W_{t+\delta} - W_t)$$
$$\Rightarrow S_{t+1} - S_t = \mu S_t + \sigma {S_t}^{\alpha} e_{t+1}\sqrt{\delta}$$

where $e_{t+1} = W_{t+1} - W_t \sim N(0,1)$ is a standard normal random variable, and $\delta$ is the time step. Therefore, we have to minimize the minus of the log-likelihood function:

(19) $$\min_{\mu,\sigma,\alpha} \sum_{t=0}^{T} \frac{1}{2\delta} \left(\frac{S_{t+1} - S_t - \mu S_t \delta}{\sigma {S_t}^{\alpha}}\right)^2.$$

In this paper, we use commodity daily futures prices for a set of 13 agricultural commodities across two groups of livestock and non-livestock commodities, as described in Table 2. The first column in Table 2 lists the varieties of commodities and briefly presents data sources. CME: Chicago Mercantile Exchange, CBOT: Chicago Board of Trade, NYBOT: New York Board of Trade, WCE: Winnipeg Commodity Exchange. The second column lists the period of commodity prices data.

**Table 2: Data description and Source**

| Commodities | Date |
|---|---|
| Real W (CBOT Wheat Future) | 1971/12/9 - 2015/6/1 |
| Real BO (CBOT Soybn Oil Future) | 1972/12/23 - 2015/6/1 |
| Real S (CBOT Soybn Future) | 1970/1/2 - 2015/6/1 |
| Real O (CBOT Oats Future) | 1970/1/2 - 2015/6/1 |
| Real C (CBOT Corn Futrue) | 1970/6/26 - 2015/6/1 |
| Real WC (WCE Canola Futrue) | 1982/1/4 - 2015/6/1 |
| Real SB (NYBOT Sugar Future) | 1970/6/26 - 2015/6/1 |
| Real JO (NYBOT Or juice Futrue) | 1970/6/26 - 2015/6/1 |
| Real CC (NYBOT Cocoa Future) | 1970/6/26 - 2015/6/1 |
| Real KC (NYBOT Coffee Future) | 1972/8/26 - 2015/6/1 |
| Real LH (CME Lean Hogs Future) | 1986/4/1 - 2015/6/1 |
| Real LC (CME Live Cattle) | 1982/06/24-2015/6/1 |
| Real FC (CME Feeder Cattle) | 1989/11/8 - 2015/6/1 |

Since our data set consists of daily future prices, we choose $\delta = 1/250$, for considering 250 trading days within a year (the results do not change a lot if we change 250 to 365 trading days).

In order to test the goodness of fit of the CEV model (compared to the BSM model) we run a likelihood-ratio test. For that, we need to set $\alpha = 1$ for the restricted model, find the restricted log likelihood function, and form the following test statistics:

(20) $D = 2 \times (\log(unrestricted\ likelihood\ ) - \log(likelihood\ with\ \alpha = 1)).$

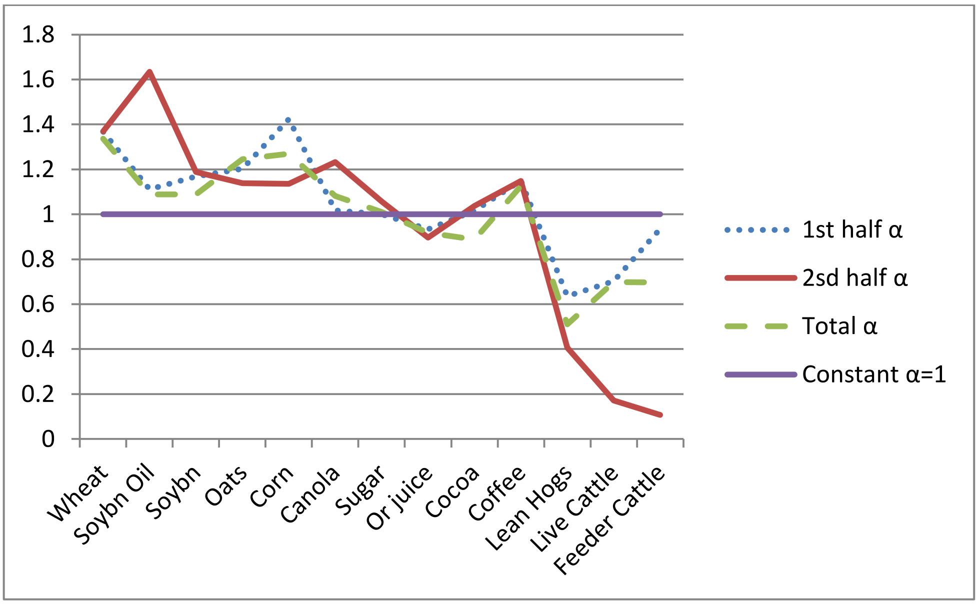
Given that the unrestricted model has 3 degrees of freedom and the restricted model has 2 degrees of freedom, the distribution of $D$ is $\chi^2$ with degrees of freedom 1.

In order to verify the stability of the estimations, we divide the time interval of any commodity prices into two equal time intervals and then we estimate the parameters of a given commodity for the 1st half, 2nd half and the total data. We report the results of the estimations in Table 3 and visualise the results for the $\alpha$ estimations in Figure 2. In order to ensure robustness of our estimations and that we do not report the local minimums, we have used several initial values for $\alpha$: 0.001, 0.01, 0.75, 1, 2, 3, 4 and 10. We observed that beyond values of 0.75, estimations become more stable and the optimal likelihood increases until 4. Thus, we chose an initial value of 3 for estimation of $\alpha$.

**Table 3: Estimations of the CEV model parameters. Estimations in bold indicate rejection of the null hypothesis BS model against the CEV model at 5 % significant level.**

| Commodities | 1st half | | | 2sd half | | | Total | | |
|---|---|---|---|---|---|---|---|---|---|
| | μ | σ | α | μ | σ | α | μ | σ | α |
| Wheat | **-0.15** | **0.032** | **1.368** | **0.099** | **0.032** | **1.369** | **0.047** | **0.04** | **1.336** |
| Soybn Oil | **0.035** | **0.18** | **1.112** | **0.072** | **0.039** | **1.634** | **0.041** | **0.174** | **1.089** |
| Soybn | **0.123** | **0.085** | **1.169** | **0.147** | **0.08** | **1.188** | **-0.034** | **0.139** | **1.088** |
| Oats | **0.103** | **0.118** | **1.202** | **0.144** | **0.16** | **1.138** | **0.164** | **0.097** | **1.245** |
| Corn | **0.089** | **0.024** | **1.424** | **0.016** | **0.123** | **1.135** | **0.097** | **0.058** | **1.269** |
| Canola | **0.02** | **0.184** | **1.017** | **0.036** | **0.049** | **1.233** | **0.098** | **0.127** | **1.082** |
| Sugar | 0.113 | 0.425 | 1 | **0.171** | **0.436** | **1.057** | 0.072 | 0.332 | 1.01 |
| Or juice | **0.067** | **0.427** | **0.933** | **0.059** | **0.465** | **0.897** | **0.07** | **0.49** | **0.918** |
| Cocoa | 0.094 | 0.284 | 1.016 | **0.197** | **0.253** | **1.038** | **0.095** | **0.694** | **0.888** |
| Coffee | **0.11** | **0.183** | **1.15** | **0.119** | **0.172** | **1.149** | **0.099** | **0.218** | **1.125** |
| Lean Hogs | **0.082** | **1.459** | **0.635** | **0.005** | **3.326** | **0.406** | **0.011** | **2.635** | **0.51** |
| Live Cattle | **-0.01** | **0.618** | **0.703** | **0.016** | **5.524** | **0.172** | **0.04** | **0.65** | **0.699** |
| Feeder Cattle | 0.067 | 0.18 | 0.935 | **-0.017** | **5.343** | **0.107** | **0.086** | **0.627** | **0.697** |

**Figure 2: Estimated $\alpha$'s for different data sets.**




As one can see, the results are relatively stable for different time intervals. Estimated parameters in bold indicate the rejection of the BSM model in favour of the CEV model at the 5% significance level. The estimations for the BSM model are also reported in Table 4.

**Table 4: Estimations of the BSM model parameters.**

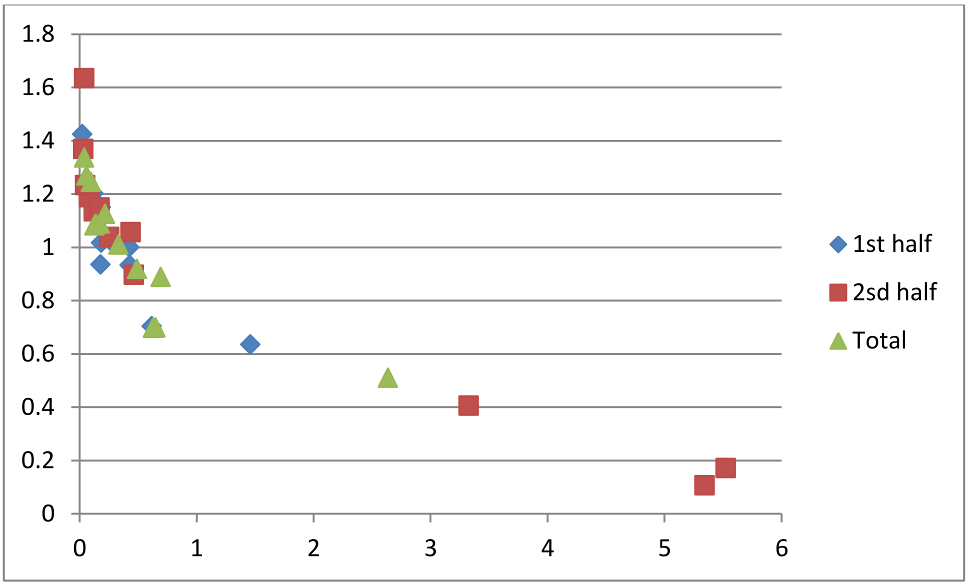
| Commodities | 1st half | | 2sd half | | Total | |
|---|---|---|---|---|---|---|
| | μ | σ | μ | σ | μ | σ |
| Wheat | 0.07 | 0.295 | 0.066 | 0.276 | 0.074 | 0.313 |
| Soybn Oil | 0.028 | 0.258 | 0.02 | 0.281 | 0.036 | 0.233 |
| Soybn | 0.064 | 0.259 | 0.072 | 0.268 | 0.056 | 0.25 |
| Oats | 0.088 | 0.338 | 0.089 | 0.318 | 0.086 | 0.356 |
| Corn | 0.061 | 0.27 | 0.054 | 0.258 | 0.068 | 0.282 |
| Canola | 0.035 | 0.202 | 0.027 | 0.191 | 0.043 | 0.212 |
| Sugar | 0.113 | 0.425 | 0.156 | 0.496 | 0.071 | 0.34 |
| Or juice | 0.076 | 0.313 | 0.072 | 0.291 | 0.079 | 0.334 |
| Cocoa | 0.092 | 0.32 | 0.08 | 0.336 | 0.105 | 0.302 |
| Coffee | 0.089 | 0.375 | 0.095 | 0.351 | 0.081 | 0.398 |
| Lean Hogs | 0.074 | 0.335 | 0.07 | 0.334 | 0.079 | 0.336 |
| Live Cattle | 0.039 | 0.17 | 0.011 | 0.171 | 0.067 | 0.169 |
| Feeder Cattle | 0.046 | 0.132 | 0.005 | 0.115 | 0.088 | 0.148 |

As one can see, except sugar 1$^{st}$ half and total, cocoa 1$^{st}$ half and total, and Feeder Cattle 1$^{st}$ half, CEV model is a better fit. Another important observation is that all livestock commodities and orange juice have $\alpha$'s less than 1, whereas the other commodities have $\alpha$'s greater than or equal to 1. This is interesting since it indicates that the livestock and orange juice prices behave more like stock prices; see Beckers (1980).

Here we discuss some important implications form the observations: first livestock commodities and orange juice have leverage effect, whereas the rest have inverse leverage effect. However, as it is shown in Figure 3, there is a reverse relation between $\sigma$ and the elasticity parameter $\alpha$. This shows that for agricultural commodities, inverse leverage effect is associated with lower uncertainty. Note that larger $\alpha$'s are associated with smaller $|\beta|$'s; therefore, commodities with larger $\alpha$ are more inelastic. This means that for more inelastic commodities, there is less uncertainty in prices, whereas prices themselves can have larger effect on volatility.

The second implication is that the put option prices can be dependent on the type of the commodities. As it has been discussed earlier, since the estimated $\alpha$ for livestock commodities and orange juice is less than one, theses commodities have leverage effect and as a result their put option prices in the money are expected to be higher than the other commodities with equal $\sigma$.

**Figure 3: Estimation $\alpha$'s versus $\sigma$'s for different data sets.**




Finally, the last implication we want to discuss here is that for all non-livestock commodities, except orange juice and cocoa-total, we can provide a proper demand function. In Table 5 we report the estimated β's. Note that only the negative β's can be considered valid.

**Table 5: Estimated β**

| | β | | |
|---|---|---|---|
| | 1st half | 2sd half | Total |
| Wheat | -0.184 | -0.185 | -0.168 |
| Soybn Oil | -0.056 | -0.317 | -0.045 |
| Soybn | -0.084 | -0.094 | -0.044 |
| Oats | -0.101 | -0.069 | -0.122 |
| Corn | -0.212 | -0.068 | -0.134 |
| Canola | -0.008 | -0.116 | -0.041 |
| Sugar | 0 | -0.028 | -0.005 |
| Or juice | 0.034 | 0.051 | 0.041 |
| Cocoa | -0.008 | -0.019 | 0.056 |
| Coffee | -0.075 | -0.074 | -0.063 |
| Lean Hogs | 0.182 | 0.297 | 0.245 |
| Live Cattle | 0.149 | 0.414 | 0.15 |
| Feeder Cattle | 0.032 | 0.447 | 0.151 |

## Conclusion

In this paper, the CEV model was adopted to model options on commodity prices. It was argued that the CEV model for commodity prices can be supported by the mean reverting CIR model for demand process and a power demand function. We then examined the effect of the CEV elasticity parameter on the derivative prices. In addition, the parameters of the

CEV model for agricultural commodities were estimated and the implications on derivative prices were discussed. Finally, we observed that all non-livestock commodities, except orange juice, have an inverse leverage effect on prices, whereas livestock commodities and orange juice have leverage effect.

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