A Note on the Cogley-Nason-Sims Approach

Syed M. Hussain * Lin Liu †

Abstract

In evaluating an economic model with Structural Vector Auto-Regression (SVAR), the Cogley-Nason-Sims (CNS) approach compares impulse responses estimated from empirical data with those obtained from the identical SVAR run on model generated data. Using Monte-Carlo simulations, this paper examines small sample performance of the CNS approach.

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Key Words: Cogley-Nason-Sims Approach, Small Sample Properties, Structural Vector Auto-Regression, Identification, Monte-Carlo Simulation.

1 Introduction

In macroeconomics, Impulse Responses Functions (IRFs) derived from Structural Vector Auto-Regression (SVAR), are often used to evaluate economic models. Invalid identifications, however, can result in quantitatively large discrepancies between identified and theoretical IRFs (see Carlstrom et al., 2009). The Cogley-Nason-Sims (CNS) approach[1] is meant to be immune to this problem. The reason is that it compares impulse responses estimated from empirical data with those obtained from the identical SVAR run on model generated data. As empirical and model generated data are treated symmetrically, the application of the CNS approach does not require identifications to be valid (see Kehoe, 2006).[2] It may

*Department of Economics, Lahore University of Management Sciences, Lahore, Pakistan, Email: muhammad.hussain@lums.edu.pk

†Management School, University of Liverpool, United Kingdom, Email: Lin.Liu@liverpool.ac.uk

[1] It is advocated by Sims (1989) and applied by Cogley and Nason (1995). It is essentially an application of indirect inference.

[2] The other approach used in macroeconomics is the common approach. It compares impulse responses estimated from empirical data with those directly derived from models. The application of this approach requires identifications to be valid. With Monte-Carlo simulations, Christiano et al. (2006) examines small sample properties of the common approach.

therefore be tempting to use this approach for model evaluations.[3]

In this paper, we investigate and compare finite sample properties of the CNS approach in two scenarios – when identifications are either valid or invalid. We find that, for samples of the size commonly found in macroeconomic applications, when identifications are invalid, the resulting estimates contain considerable bias and are very sensitive to the amount of measurement error included. In particular, when the CNS approach is implemented for parameter estimation, the moments or the estimated IRFs are not informative about structural parameters to be estimated. The poor small sample properties of the CNS approach is due to the added uncertainty from other economic shocks, which in turn is a result of invalid identifications. This paper is a caution against the indiscriminate use of the CNS approach, as the results show that it can still go wrong, especially with invalid identifications.

The paper is organized as follows: Section 2 describes the Monte-Carlo simulations; Section 3 presents results; Section 4 provides discussion and Section 5 concludes.

2 Monte-Carlo Simulations

The data generating processes (DGP) used in the Monte-Carlo simulations are two variants of the New Keynesian (NK) models,[4] which only differ in the assumptions on monetary shocks. One is the standard textbook NK model, where monetary shocks have a contemporaneous effect on the economy (called 'the standard model'). The other adopts the assumption used in Christiano et al. (2005), where monetary shocks do not affect the economy contemporaneously (called 'the CEE model'). Then we derive impulse responses by estimating the three variable SVAR (output, price and interest rate) with the short-run recursive identification, that is, monetary shocks do not affect the current economy. Therefore, with the CEE model the identification is valid, while with the standard model the identification is invalid. We compare finite sample performance of the CNS approach in both of these scenarios.

To avoid from any confusion, some terminologies are clarified here. There are two types of monetary shocks: one is the monetary shocks that appears in the Taylor rule, which we call the *exogenous monetary shocks*, and the other is the monetary shocks recovered using the short-run identification, which we call the *identified shocks*. These two shocks generally are different, and so are their impulse responses. Moreover, there are three types of impulse response functions (IRFs):

[3]The CSN approach has been used in a number of studies: Dupaigne et al. (2007), Mertens and Ravn (2011), Barsky and Sims (2012), Le et al. (2011), Castelnuovo and Surico (2010) and etc.

[4]The NK model setup closely follows Carlstrom et al. (2009). Please refer to Appendix for more details.

- The *theoretical IRFs* give the effects of the exogenous monetary shocks. They are derived directly from models (see Christiano et al., 2005);
- The *population IRFs* describe the effects of the identified shocks in the population, which are immune from random sampling uncertainties. In the standard model, the population IRFs are obtained from the analytical VAR representation of model dynamics with the short-run identification (see Carlstrom et al., 2009). In the CEE model, the population IRFs are obtained by applying SVAR with the short-run identification on model generated data with sufficiently large sample size and number of lags.[5]
- The *estimated IRFs* describe the effects of the identified shocks in finite samples. They are estimated by applying the SVAR with the short-run identification to model generated data with sample size commonly found in macroeconomic applications. Since the length of simulated data sets are limited, they suffer from finite sample problem (see Christiano et al., 2006).

Carlstrom et al. (2009) examines the difference between the theoretical and population IRFs, due to the mis-identification of monetary shocks. In this paper, we investigate the difference between the estimated and population IRFs, due to small sample size.

3 Results

In this section, we evaluate finite sample performance of the CNS approach with two estimators – estimated IRFs and estimated model parameter. Throughout the paper, all the responses are normalized so that the initial rise in interest rate is 25 basis points, and here we only report results for output responses.[6]

3.1 Estimated IRFs

In each scenario, we generate $N = 500$ simulated data sets, with length equal to 180 periods each. To derive the estimated IRFs, we apply SVAR with short-run recursive assumption to each data set. Then we obtain N sets of estimated IRFs of the identified shocks.

[5]Since the CEE model does not have a pure finite VAR representation, we could not derive the population IRFs analytically. So we derive the population IRFs with sufficiently large sample size and number of lags. As shown in Figure B.1 in the Appendix, for the CEE model, the population IRFs match closely with the theoretical IRFs, which does not suffer from finite sample problems. Moreover, it implies that the SVAR with short-run identification can correctly identify the exogenous monetary shocks in the CEE model.

[6]Please refer to the Appendix for the full sets of results.

The first row in Figure 1 presents the mean estimated IRFs for both scenarios – the average of all the estimated responses, along with the population IRFs for easy comparison. Since the population IRFs are not subject to sampling uncertainties, they provide us criteria for evaluating the estimated IRFs. We find that, for the CEE model, the mean estimated IRFs are very close to the population IRFs, while for the standard model, the mean responses are markedly different from the population IRFs. Furthermore, in order to show the magnitudes of sampling uncertainties associated with the estimated IRFs, in the second row of Figure 1 we look at both sample probability intervals and confidence intervals.[7] We can see that for both models the confidence bands and probability intervals are very similar. This confirms the findings in Christiano et al. (2006) that confidence intervals correctly reveal the amount of sampling uncertainties contained in probability intervals. However, we find that for the CEE model, the bands are very narrow at the initial few periods, suggesting that the drop in output is statistically significant. In contrast, for the standard model, the bands are too wide to provide any useful inference. In other words, they support a broad range of empirical results, and are not very informative.

3.2 Estimated Parameter

The CNS approach is often used to estimate model parameters by matching impulse responses derived from empirical observations and model generated data. We choose the auto-correlation of monetary shocks as the targeted parameter to be estimated.[8] The true parameter value is 0.5. To proceed, for each scenario, with the true persistence we simulate one data set from which we estimate the impulse responses. These are treated as the empirical IRFs, and the parameter is then estimated by the simulated method of moments. We repeat this procedure for 500 times, and obtain a series of estimates.

The third row in Figure 1 plots the probability density functions for the parameter estimates. Clearly, the estimates of the CEE model center around the true parameter value. The mean of all the estimates is 0.491 and the median is 0.502, suggesting the estimation is neither biased nor skewed. In contrast, for the standard model, the mean of all the estimates

[7] Probability intervals are those estimated IRFs that are two standard deviations away from the mean. They describe the extent of uncertainties associated with random realization of economic shocks. Moreover, for each data set we derive 95 percentage confidence intervals of its estimated IRFs. The average of all these confidence intervals are the confidence bands presented in Figure 1.

[8] We could have chosen to estimate more parameters or some other parameters. The reason we choose to estimate auto-correlation of monetary shocks is that it is one of the key determinant factors for the impulse responses of the identified shocks in both models. By concentrating on estimating this parameter, on the one hand we want to give the CNS approach its best shot in uncovering the true parameter value, and on the other hand the results are easily comparable between the two models.

is 0.75, which is above the true parameter value. The median is 0.835 so that the estimation has a heavier right tail.

3.3 Sensitivity Analyses

In applying the CNS approach, measurement error is often added to model simulated data to avoid the singularity problem in the estimation. Here, we examine how the estimation results change, when we vary the amount of measurement error added. Following Chari et al. (2005), a measurement error of 0.01% or 0.04% is added. Since these numbers are extremely small, they do not affect the overall fluctuations and the relative importance of monetary shocks.

Figure 2 shows that for the CEE model, the mean estimated IRFs, probability intervals and the distribution of the estimated parameters do not exhibit any noticeable change. However, the results are remarkably different for the standard model, and very sensitive to the amount of measurement error added. As size of measurement error increases, the mean estimated IRFs become more pronounced and the probability intervals become much narrower. The estimated parameter values are closer to the true value and less skewed.

4 Discussion

As shown in Fernandez-Villaverde et al. (2007), the models here admit the following VAR representation:

$$z_t = H_1 z_{t-1} + \cdots + H_i z_{t-i} + \cdots + F\,\nu_t, \tag{1}$$

where z_t is a vector of endogenous variables – output, price and interest rate, ν_t is a vector of innovations – productivity shocks, price markup shocks and monetary shocks, and $H_1, \cdots H_i, \cdots\ F$ are matrices determined by model parameters. In practice, however, we estimate:

$$z_t = B_1 z_{t-1} + B_2 z_{t-2} + \cdots + B_4 z_{t-4} + B_0\,\tilde{\nu}_t,$$

where $B_1, \cdots B_4$ are matrices to be estimated, B_0 is assumed to be lower-triangular and $\tilde{\nu}_t$ is a vector of identified shocks. It is different from equation (1) in that, in order to uncover monetary shocks, B_0 is assumed to be lower-triangular, while F may or may not be lower-triangular depending on the model specifications.[9] Thus, the identified monetary shocks are not the same as the exogenous monetary shocks. In fact, the identified monetary shocks are a weighted sum of all the three exogenous shocks, and so are the estimated IRFs.[10]

Figure 3 provides the contributions of each exogenous shocks to the estimated IRFs of the identified monetary shocks. For the CEE model, the effects from both productivity shocks and price markup shocks are negligible. The estimated IRFs are entirely dominated by the exogenous monetary shocks, and thus subject mainly to sampling uncertainties from random realization of the exogenous monetary shocks. In contrast, for the standard model the responses to both productivity and monetary shocks are very important for shaping the overall IRFs of the identified shocks. This brings the added uncertainty, as they are subject to sampling uncertainties from other economic shocks, in addition to exogenous monetary shocks. With a small sample size, these additional influences impose tremendous bias and uncertainty for the estimators.

When the CNS approach is implemented to estimate the auto-correlation of the monetary shocks, it amounts to essentially matching a very particular moment of the data. Whether that moment is informative about the structural parameter to be estimated is shown to depend on the identification scheme. With the CEE model, the estimated IRFs correctly identify the theoretical IRFs from the structural model and hence it is clear that they are very informative. With the standard model, the estimated IRFs that we match are not informative about the auto-correlation of the monetary shocks, because to a large extent those responses are driven by other shocks. Hence, there is also no reason to expect that this approach should be informative about the monetary policy shock auto-correlation coefficient.

We have shown that, with a small sample size, the CNS approach can go wrong, especially with invalid identifications. Nevertheless, when sample size becomes sufficiently large, the performance of the CNS approach improves substantially. In Figure 4 we can see that when the sample size increases to 5000, the mean estimated IRFs closely tracks the population IRFs and the parameter estimates center around the true value. Therefore, the poor small sample properties of the CNS approach is due to the added uncertainties from other economic

[9]The other difference between the two equations is that the analytical VAR representation may be of infinite order, while the estimation equation is of finite order. It is not a problem for the standard model, as it admits a VAR(2) analytical VAR representation. It turns out that the truncation bias is not an issue for the CEE model either. Throughout the paper, we use four lags in the estimation. As shown in the Appendix, change in the number of lags included in the estimation does not change the qualitative results in the paper.

[10]We regress the identified monetary shocks on all three exogenous shocks. With the estimated weights and the theoretical IRFs of each exogenous shocks, we calculate the contribution of each exogenous shock to the estimated IRFs of the identified monetary shocks.

shocks, which in turn is a result of invalid identification used in the SVAR.

5 Conclusion

To evaluate economic models using SVAR, the CNS approach is recommended as it treats empirical observations and model generated data symmetrically, and is immune to mis-identification issue. However, we show in finite samples, with invalid identifications the resulting estimates contain considerable bias. So ironically, the very reason for adopting the CNS approach is also the cause for its poor small sample performance. This paper is a caution against the indiscriminate use of the CNS approach.

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References

Barsky, Robert B. and Eric R. Sims, "Information, Animal Spirits, and the Meaning of Innovations in Consumer Confidence," *American Economic Review*, June 2012, *102* (4), 1343–77.

Carlstrom, Charles T., Timothy S. Fuerst, and Matthias Paustian, "Monetary Policy Shocks, Choleski Identification, and DNK Models," *Journal of Monetary Economics*, October 2009, *56* (7), 1014–1021.

Castelnuovo, Efrem and Paolo Surico, "Monetary Policy, Inflation Expectations and The Price Puzzle," *The Economic Journal*, 2010, *120* (549), 1262–1283.

Chari, Varadarajan, Patrick Kehoe, and Ellen R. McGrattan, "A Critique of Structural VARs Using Real Business Cycle Theory," Working Papers 631, Federal Reserve Bank of Minneapolis 2005.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, February 2005, *113* (1), 1–45.

_ , _ , **and Robert Vigfusson**, "Assessing Structural VARs," in "NBER Macroeconomics Annual 2006," Vol. 21 2006, pp. 1–106.

Cogley, Timothy and James M. Nason, "Output Dynamics in Real-Business-Cycle Models," *American Economic Review*, 1995, *85* (3), 492–511.

Dupaigne, Martial, Patrick Feve, and Julien Matheron, "Avoiding pitfalls in using structural VARs to estimate economic models," *Review of Economic Dynamics*, 2007, *10* (2), 238 – 255.

Fernandez-Villaverde, Jesus, Juan F. Rubio-Ramirez, Thomas J. Sargent, and Mark W. Watson, "A, B, C's (and D)'s for Understanding VARs," *American Economic Review*, June 2007, *97* (3), 1021–1026.

Kehoe, Patrick J., "Assessing Structural VARs: Comment," in "NBER Macroeconomics Annual 2006," Vol. 21 2006, pp. 1–106.

Le, Vo Phuong Mai, David Meenagh, Patrick Minford, and Michael Wickens, "How much nominal rigidity is there in the US economy? Testing a new Keynesian DSGE model using indirect inference," *Journal of Economic Dynamics and Control*, 2011, *35* (12), 2078 – 2104.

Mertens, Karel and Morten O. Ravn, "Understanding the aggregate effects of anticipated and unanticipated tax policy shocks," *Review of Economic Dynamics*, 2011, *14* (1), 27 – 54.

Sims, Christopher A., "Models and Their Uses," *American Journal of Agricultural Economics*, 1989, *71* (2), 489–494.

Figure 1. The Estimated IRFs and Estimated Model Parameter

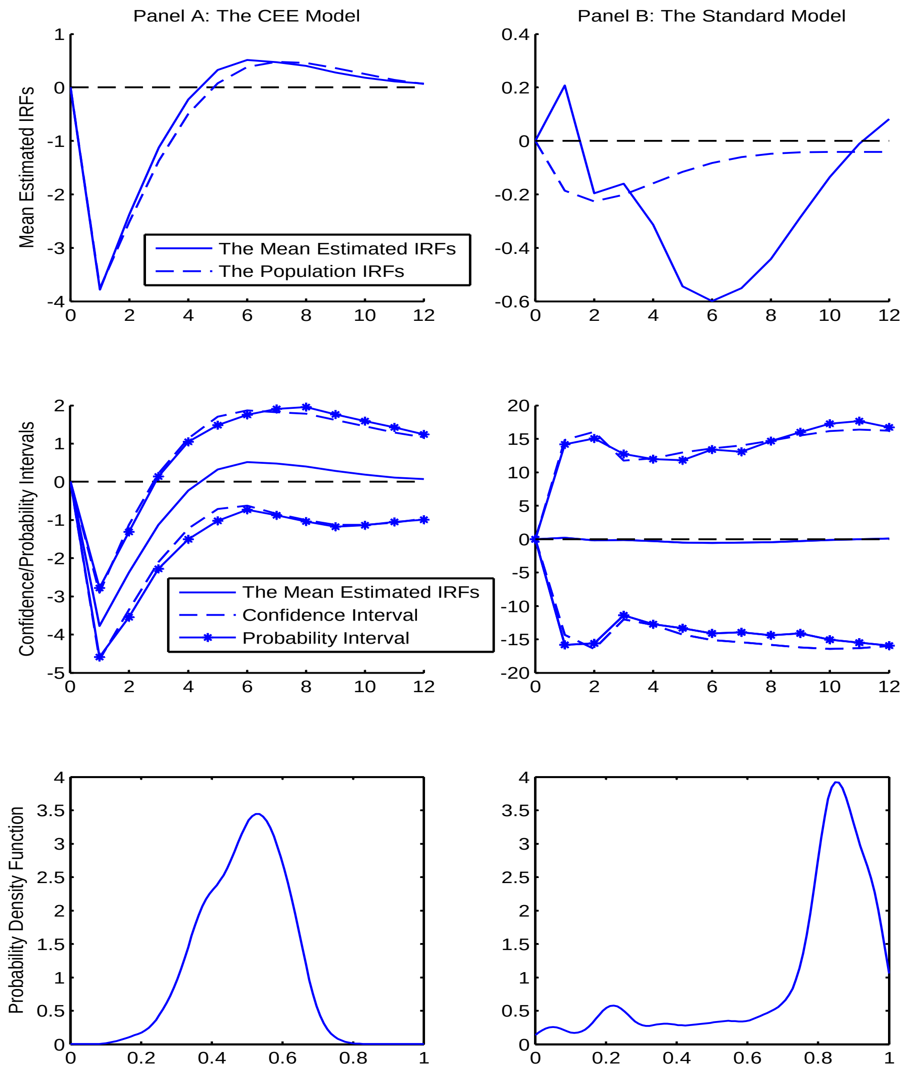

Figure 2. Sensitivity Analysis Varying Measurement Error

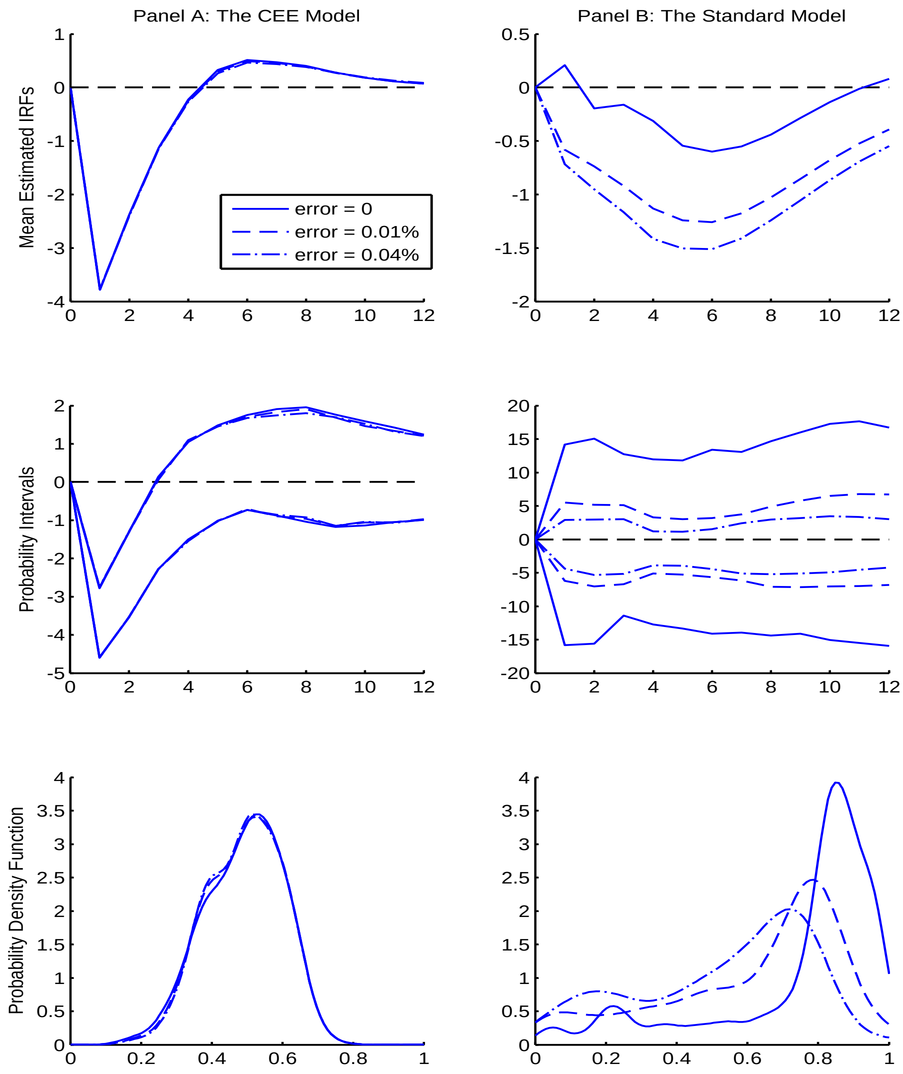

Figure 3. The Contributions of Exogenous Shocks to the Estimated IRFs of the Identified Shocks

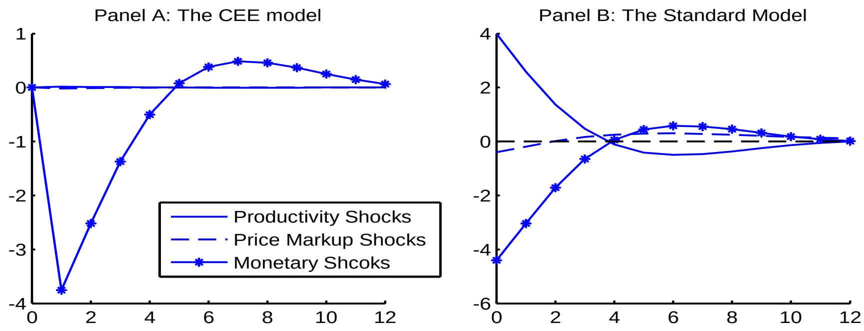

Figure 4. The Mean Estimated IRFs and Model Parameters Varying Sample Size (the Standard Model)