I heard you the first time: debate in cacophonous surroundings

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Abstract. One often finds in debate involving agents strongly committed to their positions, that argument is promoted not through a rational measured exchange of views but rather through stridency and clamour as proponents try to shout down or otherwise suppress their opponents' opinions. While the presence of moderators may go some way to alleviating the effects of such approaches one has the problems of moderators being ignored and the environment being of a nature that makes the appointment of such infeasible. In this article our concern is, in the first instance, to examine the extent to which an environment where argument is pursued through these means can be modelled. Within this model, we briefly review what techniques may be adopted by participants looking to present their own stance with minimal effort and maximal impact.

Keywords. abstract argumentation frameworks; directed graph spectrum; Perron-Frobenius Theory;

Introduction

Consider a debating arena in which numerous different and conflicting opinions are being championed by several protagonists. There are a number of tactics sometimes adopted by participants that are not intended to progress their stance through rational discourse, but rather since those using such means, mistakenly and naïvely believe them to make their point of view more compelling. Thus one finds, for example in playground or nursery debate, techniques such as wearisome repetition of the same point over and over, this sometimes reduced to single word utterances. Repetition as an indicator of logically "weak" argument, has, of course, long been recognized and studied as one class of fallacious reasoning: e.g. the consequences of *eo ipse* moves in the dialogue protocol of Vreeswijk and Prakken [16], the review of "stone-walling" and other non-cooperative tactics from Gabbay and Woods [12,11]. More generally, strategies whose aim is not to advance but rather to stifle or impede discussion underlie several studies, e.g. Dunne [7,8], Sakama [15], Budzynska and Reed [4].

Participants contributing within (supposedly) more "mature" contexts – such as political debates – will usually recognise the futility of constant repetition as an argumentative tool. To compensate, however, (and often not consciously aware that such measures are being used) they may have recourse to another regressive (or at least non-progressive) technique: that of increasing the force with which their points are delivered. Thus in non-structured debates this will often take the form of increasing vocal volume in an attempt to drown out the arguments of opponents, so rendering them inaudible to neutral observers. This, in turn, may lead to those same opponents adopting identical tactics reiterating their stance at louder and louder volumes. To counteract the deleterious effect on reasoned debate that results from discussions sinking to the level of shambolic shouting contests, in many legislative assemblies a neutral member is recognized as having – among other responsibilities – some authority to intervene and impose a semblance of order. For example, in the U.K. House of Commons, the rôle of Speaker fulfils this function.[1] Nevertheless, despite the presence of a mediator to oversee the conduct of discussions, it can happen (particularly on sensitive issues) that their authority is ignored.[2] Given that, even within structured settings with a recognized moderator, there is the potential for debate to descend to acrimonious discord, the likelihood of *un-mediated* exchanges degenerating to similar levels is so much the greater.

Our aim in this paper is to consider such settings and a number of questions arising therein. In particular the issue of what forms of model amenable to *analytic* investigation can be used in order to treat,

a. Synthesis and discovery of strategies that are intended to impose a point through volubility rather than reason.
b. Differences between moderated and un-moderated discussion, and the susceptibility of the latter to over-strident contributions skewing debate.

We find a basis for our approach by adapting the seminal abstract argumentation frameworks (AFs) of Dung [6]: in their pure form these encapsulate argument interaction as a directed graph structure $\langle \mathcal{X}, \mathcal{A} \rangle$ wherein $\mathcal{X}$ is a (assumed for our purposes to be *finite*) set of *atomic* arguments and $\mathcal{A} \subseteq \mathcal{X} \times \mathcal{X}$ describes an *attack* relationship over these, so that $\langle p, q \rangle \in \mathcal{A}$ captures the concept of the arguments p and q being incompatible by reason of the argument p "attacking" the argument q. For reasons we develop subsequently we augment $\mathcal{A}$ by assigning to each $\langle p, q \rangle \in \mathcal{A}$ a *positive real value*[3] which we will refer to as the *volubility* of $\langle p, q \rangle$ and denoted $\nu(\langle p, q \rangle)$. A triple $\langle p, q, r \rangle$ will be referred to as a *discord*, so that one has an implied relationship $\Delta \subseteq \mathcal{X} \times \mathcal{X} \times \mathbb{R}^+$ in which $\langle p, q, r \rangle \in \Delta$ should $\langle p, q \rangle \in \mathcal{A}$ and $\nu(\langle p, q \rangle) = r$. Before developing our approach in depth it is worth observing that the *atomic* (indivisible) view of "argument" taken in Dung's formalism, although sometimes the source of objections on account of its highly abstract perspective, captures an important aspect relative to the topic of interest in the present article. Specifically it is the forcefulness with which a claim, p is

[1]In the UK, the Speaker although having represented one of the major parties as a member of parliament, on assuming this office, is non-partisan. This status being recognized by the fact that in general elections it is a tradition that the (current) Speaker is unopposed when seeking re-election as M.P. for their local constituency. There have been occasions, however, (the 2015 U.K. Parliamentary election being one) when this tradition has been ignored.

[2]Among (many) such examples in the UK, is the incident of the senior Conservative MP, Michael Heseltine, seizing and waving the symbolic mace at Labour members singing the *Red Flag* in the aftermath of a heated 1976 debate on state ownership (nationalization): the Speaker was forced to suspend the sitting.

[3]We distinguish *positive* to indicate > 0 as opposed to *non-negative*, i.e. ≥ 0.

championed over another claim q – the attack $\langle p, q\rangle \in \mathcal{A}$ – as assessed through its volubility that is of interest, rather than any intrinsic merits (or otherwise) of the arguments involved. We note that our model assigns weights to *attacks* rather than to their *source*, i.e. the *argument* from which these originate. There will, of course, be some (implied) relationship between the former (volubility) and the latter (which will be referred to a stridency subsequently). The question of how *exactly* to model the interaction between these two measures is of some interest, however, while we consider some approaches, space does not permit a full consideration of this issue. Our rationale for attack rather than argument weighting is that this explictly recognises that a *single* argument might be exerted with varying levels of force against different arguments, e.g. an "authority" figure might feel confident enough in pushing a "weak" argument without shouting against an argument of a subordinate while feeling the need to be more forceful when the same argument is used to attack arguments of peers.

Our principal intention is to propose and establish some basic properties of one approach. The formal setting raises a number of questions of interest, however, our discussion of these is largely to emphasize the potential for further development rather than propose specific solutions. In the remainder of the paper, we first reprise background from Dung's model in Section 1 which gives a foundation for the structures capturing "debate forms" in Section 2. Section 3 offers the main technical development wherein the concept of a debate being "stable" with respect to some underlying criteria is defined. Together with these criteria we present a broad range of contexts through which a moderator may not only determine whether a current state is "acceptable" but also choose or impose rules enforcing stability. Conclusions are presented within Section 4.

1. Preliminaries

We begin by recalling the concept of abstract argumentation framework and terminology from Dung [6]

Definition 1 *We use $\mathcal{X}$ to denote a* finite *set of arguments with $\mathcal{A} \subseteq \mathcal{X} \times \mathcal{X}$ the so-called* attack *relationship over these. An* argumentation framework *(*AF*) is a pair $\mathcal{H} = \langle \mathcal{X}, \mathcal{A}\rangle$. A pair $\langle x, y\rangle \in \mathcal{A}$ is referred to as 'y* is attacked by *x' or 'x* attacks *y'. Using S to denote an arbitrary subset of arguments for $S \subseteq \mathcal{X}$,*

$$\begin{array}{lcl} S^- & =_{\text{def}} & \{\ p\ :\ \exists\ q \in S \ \ \textit{such that}\ \langle p, q\rangle \in \mathcal{A}\} \\ S^+ & =_{\text{def}} & \{\ p\ :\ \exists\ q \in S \ \ \textit{such that}\ \langle q, p\rangle \in \mathcal{A}\} \end{array}$$

In our subsequent treatment it is assumed for every argument $x \in \mathcal{X}$ that $\langle x, x\rangle \notin \mathcal{A}$: our intention being to consider the effect of x on others, i.e. attacks $\langle x, y\rangle$ stemming from x. Participants are considered not to fight against themselves.

Starting from two concepts – those of *conflict-free* sets S and the arguments *acceptable* to such, Dung offers a number of proposals in order precisely to capture the informal notion of "collection of justifiable arguments". Thus, $x \in \mathcal{X}$ is *acceptable with respect to S* if for every $y \in \mathcal{X}$ that attacks x there is some $z \in S$

that attacks y. Given $S \subseteq \mathcal{X}$, $\mathcal{F}(S) \subseteq \mathcal{X}$ is the set of all arguments that are acceptable with respect to S, i.e.

$$\mathcal{F}(S) \;=\; \{x \in \mathcal{X} \;:\; \forall\, y \text{ such that } \langle y, x\rangle \in \mathcal{A}\;,\; \exists\, z \in S \text{ s.t. } \langle z, y\rangle \in \mathcal{A}\}$$

A subset, S, is *conflict-free* if no argument in S is attacked by any other argument in S, with $\subseteq$-maximal conflict-free sets referred to as *naive* extensions. A conflict-free set S is *admissible* if every $y \in S$ is acceptable w.r.t S. S is a *complete* extension if S is conflict-free and should $x \in \mathcal{F}(S)$ then $x \in S$, i.e. every argument that is acceptable to S is a member of S, so that $\mathcal{F}(S) = S$. The set of $\subseteq$-maximal complete extensions coincide with the set of $\subseteq$-maximal admissible sets, these being termed *preferred extensions*. The set S is a *stable* extension if S is conflict free and $S^{+} = \mathcal{X} \setminus S$.

For a given semantics σ and AF, $\mathcal{H}(\mathcal{X}, \mathcal{A})$ we use $\mathcal{E}_{\sigma}(\mathcal{H})$ to denote the set of all subsets of $\mathcal{X}$ that satisfy the conditions specified by σ.

2. Debate Arenas & Debate Evolution

It was mentioned earlier that an additional component is added to the basic abstract formalism described by AFs.

Definition 2 *A* debate arena, *$\mathcal{D}$, is formed by a triple $\langle \mathcal{X}, \mathcal{A}, \nu\rangle$ where $\langle \mathcal{X}, \mathcal{A}\rangle$ is an* AF *and $\nu \;:\; \mathcal{A} \to \mathbb{R}^{+}$ is the* debate volubility *function, associating with each $\langle x, y\rangle \in \mathcal{A}$ a* positive *real value.*

The debate volubility function is viewed as describing the force with which its *promoter*, $\pi(\langle x, y\rangle)$, asserts the argument to its *antagonist*, $\alpha(\langle x, y\rangle)$.

Of course the idea of augmenting Dung's *ur*–formalism by allowing quantitative associations with attacks (and, indeed, arguments themselves) has a rich history, being adopted in, amongst others: treatments of so-called "inconsistency tolerance" in Dunne *et. al* [9], Coste-Marquis *et al.* [5]; algorithmic treatments, e.g. Bistarelli and Santini [3]; modelling probabilistic structures, e.g. Li *et al.* [13].

The scenarios of interest to our study involve, however, an aspect which the quantitive formulation of debate arena fails to describe: its treatment of volubility is *static*. In practice, given the context modelled, one would expect the level at which a promoter directs the attack on an antagonist to vary. Such variation need not necessarily be a monotonic increase in $\nu(\langle x, y\rangle)$: hence the often used rhetorical device of reducing the level at which a point is made for emphasis.[4]

Our notion of debate arena can, in essence, be seen as a snapshot within an evolving debate: contributors adjusting their promotion of given arguments over time. In order to reflect *dynamic* elements we formalise this concept via,

[4]For example notice: the contrasting questioning styles in Maximilian Schell's cross-examination of Montgomery Clift and the underspoken manner in which its final observation is delivered (*Judgement at Nuremberg*, Kramer, 1961); the unvarying level of Olivier's repetition of the question "Is it safe?" with finality indicated by only a slight drop in tone. (*Marathon Man*, Schlesinger, 1976).

Definition 3 *An* evolving debate *is a* sequence,

$$\underline{\mathcal{D}} \quad = \quad \langle \mathcal{D}_0,\ \mathcal{D}_1,\ \ldots,\ \mathcal{D}_k,\ \ldots,\ \rangle$$

of debate arenas in which $\mathcal{D}_k = \langle \mathcal{X}, \mathcal{A}_k, \nu_k \rangle$ *with* $\nu_k : \mathcal{A}_0 \rightarrow \mathbb{R}^+ \cup \{0\}$. *This is required to satisfy*

$$\forall\ k \geq 1 \quad \mathcal{A}_k \quad \subseteq \quad \mathcal{A}_{k-1} \quad ; \quad \nu_k \ : \ \mathcal{A}_k \rightarrow \mathbb{R}^+$$

Furthermore, should $\langle x, y \rangle \in \mathcal{A}_{k-1}$ *but* $\langle x, y \rangle \notin \mathcal{A}_k$ *then* $\nu_k(\langle x, y \rangle) = 0$.

Notice that an evolving debate may, in principle, be an infinite sequence of debate arenas. We can, however, prescribe conditions under which $\underline{\mathcal{D}}$ may be treated as finite. One of these follows directly from the subset condition $\mathcal{A}_k \subseteq \mathcal{A}_{k-1}$, namely: an evolving debate is *terminal* if at some point, $t \geq 0$, we have $\mathcal{A}_t = \emptyset$. This condition is, in fact, an extreme form (and thence implied by), the notion of an evolving debate reaching *stasis*.[5] Hence an evolving debate has reached stasis if it contains debate arenas $\mathcal{D}_k$ and $\mathcal{D}_r$ with $r > k$ and $\nu_k = \nu_r$ (note that this implies $\mathcal{A}_k = \mathcal{A}_r$). In general we may be interested in the specific case $r = k + 1$, but in principle given that there is no requirement for monotonicity respecting ν_k and ν_r one could reach the situation where identical arenas appear after some interval. Implicitly, by regarding the occurrence of this as indicative of stasis the implication is that should $\nu_k = \nu_r$ then $\nu_{k+1} = \nu_{r+1}$. Notice this is an *assumption* concerning how $\underline{\mathcal{D}}$ *would* evolve rather than a formal claim of its structure.

3. Stable debates: detection and enforcement

We recall that one issue of interest concerns responding to situations where the force with which arguments are promoted, reaches a level sufficient to obstruct other participants. In fact this scenario has similarities to the well-studied problems of dealing with *power control* in mobile communications, see e.g. Bertoni [2].[6]

A significant distinction from our setting and such as these is the fact that the latter occurs within a rather more "cooperative" context: levels of signal strength being assigned externally having been determined at optimal levels through analysis and, once fixed, no deviation occurs.

To these ends the following factors are relevant: the force with which argument x_j is being pressed upon the promoter of argument x_i; the stridency with which the champion of argument x_i is proclaiming this to others.

The first of these, which we will denote by F_{ij} is,

$$F_{ij} \quad = \quad \begin{cases} 0 & \text{if } \langle j, i \rangle \notin \mathcal{A} \\ \nu(\langle j, i \rangle) & \text{otherwise} \end{cases}$$

[5] We, intentionally, avoid the, potentially misleading, term "agreement" (which might reflect a specific form of "stasis") and rather overloaded words such as "equilibrium".

[6] For example, when there are several competing "mobile phone networks" each using transmitter stations whose signal strength must be high enough to enable good reception by the network users but not at such a level as to cause excessive interference with other networks.

The latter, denoted S_i is described by a positive real value.[7]

For each x_i the quantity S_i represents the overall volume that is being used to press its merits upon others. Conversely, F_{ij} captures the interference with this case being inflicted by the promoter of x_j. One might reasonably claim, therefore, that x_i is being "promoted too forcefully" should its stridency S_i "significantly" exceed the total interference that it must tolerate as inflicted by the other actors in the system. This interpretation raises the following questions: how to assess whether the agent championing x_i is "too strident", informally, how is it determined if this agent is shouting too loudly? Secondly, *what* level of promotion is considered "excessive"?

For the moment let us assume that each argument has assigned to it a *non-negative* real value, μ_i, that defines (in some sense) the "acceptable" level of force with which x_i can be promoted without this being considered detrimental to the interests of others. Then should the *ratio* between S_i and the amount of interference dealt with, violate the levels set by μ_i then one can conclude that the actor promoting x_i is "shouting too much". We have, however, one point of detail to consider, namely how to describe what is measured as "the amount of interference dealt with". In principle one could simply fix this as the total of the forces (F_{ij}) directed against it. The problem with this, however, is its failure to take into account how an agent promoting x_j (with $\langle x_j, x_i\rangle \in \mathcal{A}$) might manipulate the system. Suppose, in a moderated system, the sanction for "shouting too loud" is (perhaps temporary) expulsion. Then measuring "acceptable" noise level via $(S_i/\sum_{j\neq i} F_{ij}) \leq \mu_i$ allows the agent pushing x_j to (for the time being) fix $\nu(\langle j, i\rangle)$ at a "token minimum" whilst compensating, for instance, by increasing the level with which x_j is forced upon other agents. Such manouevres lead to an increase in $(S_i/\sum_{j\neq i} F_{ij})$ (even more so if conducted in conjunction with other allied agents) with the possible result that the agent promoting x_i is suspended even though there has been no increase in stridency *from this agent*. Despite this, the agent pressing x_j benefits (x_i is taken out of the system) even though it may be pushing some arguments "harder" (in order to maintain its – presumably considered acceptable – level of stridency). To moderate such manipulative effects (although as we discuss later, it is uncertain whether these can be entirely eliminated), in gauging whether an agent is "shouting too loudly" we view the interference from x_j it must contend with relative to the overall volume with which x_j is being announced. That is to say, the relevant ratio we examine in deciding if x_i is being pushed "too hard" is *not* $S_i/(\sum_{j\neq i} F_{ij})$ but rather

$$\frac{S_i}{\sum_{j\neq i} F_{ij} S_j}$$

This now leads to

Definition 4 *Let* $\underline{\mu} = \langle \mu_1, \mu_2, \ldots, \mu_n\rangle$ *and* $\mathcal{D} = \langle \mathcal{X}, \mathcal{A}, \nu\rangle$ *be a debate arena. We say that* $\mathcal{D}$ *is* stable with respect to stridency $\underline{\mu}$ *(or simply* $\underline{\mu}$–stable*) if*

[7]We defer, for the moment, issues arising in relating S_i to the volubility in promoting x_i.

$$\forall\ 1 \leq i \leq n \quad \frac{S_i}{\sum_{j \neq i} F_{ij} S_j} \quad \leq \quad \mu_i$$

In other words the debate represented by $\mathcal{D} = \langle \mathcal{X}, \mathcal{A}, \nu \rangle$ is being "harmoniously" conducted should the maximum level of noise (μ_i) set for each participant *not* be exceeded by any.

If we examine the condition described in Defn. 4 then (for the limits defined by $\underline{\mu}$) the debate arena is $\underline{\mu}$-stable if

$$\forall\ 1 \leq i \leq n \quad S_i \quad \leq \quad \mu_i \sum_{i \neq j} F_{ij} S_j$$

Now consider the $n \times n$ *force*, $\mathbf{F}$ and *constraint*, $\mathbf{C}$, matrices defined through

$$\mathbf{F}_{ij} \quad = \quad \begin{cases} 0 & \text{if } i = j \\ F_{ij} & \text{otherwise} \end{cases} \qquad\qquad \mathbf{C}_{ij} \quad = \quad \begin{cases} \mu_i & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

together with the $n \times 1$ column vector, $\underline{\mathbf{S}}$ formed by the transpose of $\langle S_1, S_2, \ldots, S_n \rangle$. The n required relations are then expressed as:

$$\mathbf{C} \times \mathbf{F} \times \underline{\mathbf{S}} \quad \geq \quad \underline{\mathbf{S}}$$

or, writing $\mathbf{B}$ for the product of $\mathbf{C} \times \mathbf{F}$:

$$\mathbf{B} \times \underline{\mathbf{S}} \quad \geq \quad \underline{\mathbf{S}} \qquad (1)$$

Now, in the scenario we have presented, at any given instant (for $\mathcal{D}$ within an evolving debate $\underline{\mathcal{D}}$): $\mathbf{F}$ is determined by the current volubility function; $\mathbf{C}$ has been fixed (possibly through a moderator); and we have assumed $\underline{\mathbf{S}}$ is within the control of the agents involved.

Thus within a given $\mathcal{D}$ the component $\mathbf{B} = \mathbf{C} \times \mathbf{F}$ of (1) is unchanging and provided that the contribution from $\underline{\mathbf{S}}$ satisfies $\mathbf{B} \times \underline{\mathbf{S}} \geq \underline{\mathbf{S}}$ the debate is $\underline{\mu}$–stable.

This summary reduces the issue being considered to the following question

> Are there conditions for $\mathbf{B}$ that (if satisfied) allow some "suitable" $\underline{\mathbf{S}}$ to be adopted?

This question is simply a rephrasing of a classical linear algebra question, as captured by

Fact 1 [8] *For an $n \times n$ real-valued matrix, $\mathbf{A}$ and $\lambda \in \mathbb{R}^+$, the $n \times 1$ (non-zero) vector $\underline{\mathbf{z}}$ satisfies $\mathbf{A}\underline{\mathbf{z}} = \lambda\underline{\mathbf{z}}$ if and only if λ is an eigenvalue of $\mathbf{A}$ and $\underline{\mathbf{z}}$ an associated (right) eigenvector.*

Of course, should any $\underline{\mathbf{z}}$ satisfy $\mathbf{A}\underline{\mathbf{z}} = \lambda\underline{\mathbf{z}}$, (with $\lambda \geq 1$) then we find infinitely many such solutions simply by using any scalar multiple of $\underline{\mathbf{z}}$.

Now it is easily seen that $\mathbf{B}$ is non-negative. If it is also *irreducible*[9] we have,

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[8] What is stated as "fact" here, is often used as a formal definition of eigenvalue and eigenvector w.r.t. to a matrix $\mathbf{A}$.

[9] An $n \times n$ non-negative real-valued matrix $\mathbf{A}$ is said to be *irreducible* if for each $\langle i, j \rangle$ there is some $k \in \mathbb{N}$ for which $[\mathbf{A}^k]_{ij} > 0$.

Theorem 1 *(Perron-Frobenius Theorem [14,10])*

If $\mathbf{A}$ *is an irreducible* $n \times n$ *matrix then,*

PF1. There is a positive real eigenvalue, λ^A_{pf}*, of* $\mathbf{A}$ *with positive eigenvectors.*

PF2. If λ *is any other*[10] *eigenvalue of* $\mathbf{A}$ *then* $|\lambda| < \lambda^A_{pf}$*. Notice that, writing* $\lambda = x + \mathrm{i}y$ *with* $y \neq 0$ *in the case of complex values,* $|\lambda|$ *is the (positive) square root of* $(x^2 + y^2)$*.*

In total Thm. 1, prescribes *sufficient* conditions for $\mathcal{D}$ to be $\underline{\mu}$-stable.

Theorem 2 *The debate arena* $\mathcal{D} = \langle \mathcal{X}, \mathcal{A}, \nu \rangle$ *is* $\underline{\mu}$*-stable if the product of constraint and force matrices* $\mathbf{C} \times \mathbf{F}$ *is irreducible and* $\lambda^{\mathbf{C}\times\mathbf{F}}_{pf} \geq 1$*.*

Proof: Immediate from definitions and consequences of Thm. 1. The stridency vector $\underline{\mathbf{S}}$ can be chosen as *any* (positive) eigenvector for $\lambda^{\mathbf{C}\times\mathbf{F}}_{pf}$. These properties and choices ensure

$$(\mathbf{C} \times \mathbf{F}) \times \underline{\mathbf{S}} \;=\; \lambda^{C\times F}_{pf} \times \underline{\mathbf{S}} \;\geq\; \underline{\mathbf{S}}$$

□

The requirement in Thm. 2 that the supporting matrix $\mathbf{C} \times \mathbf{F}$ be irreducible may seem unduly limiting: in fact this is not the case.

Theorem 3 *If the structure* $\langle \mathcal{X}, \mathcal{A} \rangle$ *describes a* strongly-connected[11] *directed graph then,*

$$\forall\ \nu\ :\ \mathcal{A} \rightarrow \mathbb{R}^+,\ \underline{\mu} \in \langle \mathbb{R}^+ \rangle^{|\mathcal{X}|} \qquad \mathbf{C} \times \mathbf{F} \textit{ is irreducible.}$$

Proof: (Outline) Let $\mathbf{B}$ denote $\mathbf{C} \times \mathbf{F}$. Then,

$$[\mathbf{B}]_{ij} \quad = \quad \sum_{k=1}^{n} C_{ik} F_{kj} \quad = \quad \mu_i F_{ij}$$

Thus it suffices to establish that $\langle \mathcal{X}, \mathcal{A} \rangle$ being strongly-connected implies $\mathbf{F}$ is irreducible. Consider any $\langle x_i, x_j \rangle \in \mathcal{X}^2$. If $\langle x_i, x_j \rangle \in \mathcal{A}$ then $F_{ij} > 0$ so that the choice $k = 1$ witnesses $[\mathbf{F}^k]_{ij} > 0$. If $\langle x_i, x_j \rangle \notin \mathcal{A}$ (so that $F_{ij} = 0$) let t be the number of arguments in any path from x_i to x_j (where $t > 2$)[12] that is

$$x_i \;\equiv\; y_1 \rightarrow y_2 \rightarrow \cdots \rightarrow y_t \;\equiv\; x_j$$

[10] In general, the *spectrum* $\{\lambda_1, \ldots, \lambda_m\}$ of eigenvalues for $\mathbf{A}$ could contain n elements, some of which may be complex.

[11] A directed graph $\langle V, E \rangle$ is said to be *strongly-connected* if for every pair $\langle v_i, v_j \rangle \in V^2$ there is a directed path of edges in E by which v_j can be reached from v_i.

[12] Note these do *not* have to be distinct, so a path from x_1 to x_1 might be witnessed by $x_1 \equiv y_1 \rightarrow y_2 \rightarrow y_3 \equiv x_1$ in the event of $\mathcal{A}$ containing symmetric attacks $\langle x_1, y_2 \rangle$ and $\langle y_2, x_1 \rangle$.

so that $\langle y_k, y_{k+1}\rangle \in \mathcal{A}$ for all $1 \le k < t$. We show by induction on $t \ge 2$ that when such a path exists from x_i to x_j then $[\mathbf{F}^{t-1}]_{ij} > 0$. The base ($t = 2$) is already established via $x_i \equiv y_1 \to y_2 \equiv x_j$, i.e. the case $\langle x_i, x_j\rangle \in \mathcal{A}$. Assuming the property holds for all $t < k$ with $k \ge 3$, i.e. $[\mathbf{F}^{k-1}]_{ij} > 0$, suppose

$$x_i \;\equiv\; y_1 \to y_2 \to \cdots \to y_k \;\equiv\; x_j$$

is a path linking x_i to x_j and that no path with at least one attack and at most $k-1$ between the two exists. By definition,

$$[\mathbf{F}^{k-1}]_{ij} \quad = \quad [\mathbf{F}^{k-2} \times \mathbf{F}]_{ij} \quad = \quad \sum_{r=1}^{n} \; [\mathbf{F}^{k-2}]_{ir} F_{rj}$$

and (with a slight notational abuse)

$$\sum_{r=1}^{n} \; [\mathbf{F}^{k-2}]_{ir} F_{rj} \quad \ge \quad [\mathbf{F}^{k-2}]_{i(t-1)} F_{(t-1)j}$$

Now $F_{(t-1)j} > 0$ since $\langle y_{t-1}, x_j\rangle \in \mathcal{A}$ and (via the Inductive Hypothesis) $[\mathbf{F}^{k-2}]_{i(t-1)} > 0$ (since $x_i \equiv y_1 \to \cdots \to y_{k-1}$ is a path from x_i to y_{k-1}). Hence we deduce $[\mathbf{F}^{k-1}]_{ij} > 0$ and $\mathbf{F}$ is irreducible. □

In combination, Thms. 2 and 3 indicate that for any strongly-connected AF, a moderator is able to carry out some very basic determination of a corresponding debate arena's stability regardless of $\nu : \mathcal{A} \to \mathbb{R}^+$ and the moderator's desired constraint settings, $\underline{\mu}$.

Thus, given the starting point, $\mathcal{D}_0$, in (what will proceed as) an evolving debate $\underline{\mathcal{D}}$ an initial analysis could proceed by:

a. The moderator decides what they consider to be the maximal acceptable levels of noise, i.e fixes $\underline{\mu} \in \langle \mathbb{R}^+\rangle^n$ in such a way that for all $\underline{\tau} \in \langle \mathbb{R}^+\rangle^n$, should $\tau_i > \mu_i$ (irrespective of other components), then $\underline{\tau}$ is considered to be unreasonable.

b. Using $\mathbf{C}^{\underline{\mu}}$ and $\mathbf{F}^0$ (the constraint and force matrices resulting from $\underline{\mu}$ and $\mathcal{D}_0$) compute $\lambda_{pf}^{0,\underline{\mu}}$ the (unique) maximal positive eigenvalue for $\mathbf{C}^{\underline{\mu}} \times \mathbf{F}^0$.

c. If $\lambda_{pf}^{0,\underline{\mu}} \ge 1$, set $\langle S_1, S_2, \ldots, S_n\rangle$ – the permitted stridency levels – as

$$\left(\frac{\sum_{\langle x_i, x_j\rangle \in \mathcal{A}_0} \; \nu_0(\langle x_i, x_j\rangle)}{\sum_{i=1}^{n} \; w_i} \right) \; \langle w_1, \; w_2, \; \ldots, \; w_n\rangle$$

where $\underline{w}$ is a (transposed) eigenvector of $\lambda_{pf}^{0,\underline{\mu}}$. The multiplicative term preceding this is just a normalizing factor. This gives $w_i > 0$ for all $1 \le i \le n$.

d. Notify agents of the limits on total volubility.

The steps outlined in (a)–(d), raise several further issues. Amongst the most pressing of these we have the following questions.

Q1 Our derivation did not assume any relations between values of S_i and the volubility used when x_i is promoted. This, however, would not typically be the case, i.e. one would expect to see *some* relationship between S_i and

$$\{ \ \nu(\langle x_i, x_j\rangle) \ : \ \langle x_i, x_j\rangle \in \mathcal{A}\}$$

Q2 What steps could be taken if, for the choices colouring the computation in (b), the outcome is $\lambda_{pf}^{0,\underline{\mu}} < 1$. In other words $\mathcal{D}_0$ is "inherently unstable" with respect to $\underline{\mu}$?

Q3 What effects on overall coordination of debate would arise, should some subset $S \subset \mathcal{X}$ act collectively to exploit some common grounds, e.g. $S \in \mathcal{E}_\sigma(\langle \mathcal{X}, \mathcal{A}\rangle)$ for some semantics σ?

We consider the first of these in a little more detail here.

Suppose, instead of being an arbitrary positive real, the stridency S_i is *directly* related to $\{\nu(\langle x_i, x_j\rangle \ : \ \langle x_i, x_j\rangle \in \mathcal{A}\}$, via

$$S_i \ =_{\text{def}} \ \sum_{\langle x_i, x_j\rangle \in \mathcal{A}} \nu(\langle x_i, x_j\rangle)$$

That is, the *total* volume emanating in defending x_i is the sum of the efforts put into the individual attacks with x_i as their source. It is easily seen that,

$$S_i \ = \ \sum_{\langle x_i, x_j\rangle \in \mathcal{A}} \nu(\langle x_i, x_j\rangle) \ = \ \sum_{j \neq i} F_{ji}$$

Recalling that $F_{pq} = 0$ when $\langle x_q, x_p\rangle \notin \mathcal{A}$) the relevant ratio is now,

$$\frac{\sum F_{ji}}{\sum \ (F_{ij} \sum \ F_{kj})}$$

How does this affect the matrix representation of the system of inequalities considered earlier? Letting $\underline{\mathbf{1}}$ denote the $n \times 1$ column vector, each of whose elements is 1, it is easy to see that

$$\underline{\mathbf{S}} \ = \ \mathbf{F}^{\mathrm{T}} \times \underline{\mathbf{1}}$$

($\mathbf{A}^{\mathrm{T}}$ denoting the transpose of $\mathbf{A}$, i.e. the $n \times n$ matrix for which $[\mathbf{A}^{\mathrm{T}}]_{ij} = [\mathbf{A}]_{ji}$).

This now indicates the conditions on $\mathbf{C}$ and $\mathbf{F}$ must satisfy,

$$\mathbf{C} \times \mathbf{F} \times \mathbf{F}^{\mathrm{T}} \times \underline{\mathbf{1}} \ \geq \ \mathbf{F}^{\mathrm{T}} \times \underline{\mathbf{1}}$$

In other words *sufficient* conditions for the debate arena, $\mathcal{D}$, to be $\underline{\mu}$-stable is that $\mathbf{F}^{\mathrm{T}} \times \underline{\mathbf{1}}$ is an eigenvector for $\lambda_{pf}^{\mathbf{C}\times\mathbf{F}}$ with this eigenvalue being at least 1. In which event,

$$\begin{aligned} \mathbf{C} \times \mathbf{F} \times \mathbf{F}^{\mathrm{T}} \times \underline{\mathbf{1}} \ &= \ \lambda_{pf}^{\mathbf{C}\times\mathbf{F}} \times \mathbf{F}^{\mathrm{T}} \times \underline{\mathbf{1}} \\ &\geq \ \mathbf{F}^{\mathrm{T}} \times \underline{\mathbf{1}} \end{aligned}$$

Regarding our second issue – possible actions in the event that the combination of constraint and force matrices do not allow a suitable stridency assignment to be made – one can posit two approaches: firstly to weaken the desired conditions and adjust $\mathbf{C}$ upwards according to some convention; secondly to consider approaches whereby some subset of existing arguments are "suspended" in the hope that the reduced set-up will allow some degree of harmony. Of course, in both of these approaches a large number of further questions arise. In the first solution approach:

a. what are good bases for adjusting $\mathbf{C}$?
b. If agents (or a subset of these) view such increased tolerance of noise as an indicator of "weakness" on the part of a moderator, what is to prevent such increasing their contribution to $\underline{\mathbf{S}}$ so that even more generous commitments within $\mathbf{C}$ have no effect?

Similarly the second solution raises,

c. The "obvious" candidates to remove are those corresponding to agents for which $S_i > \mu_i$. We observed earlier, in choosing $\sum_{j\neq i} F_{ij}S_j$ to measure the degree of interference that x_i is subjected to, that naive mechanisms might allow agents to manipulate the system, (for example if we defined "interference" by $\sum_{j\neq i} F_{ij}$). From the moderator's perspective such manipulation ought to be ignored. Nevertheless there are many possibilities for choosing the subset of agents to suspend ranging from "the agent for which $(\sum_{j\neq i} F_{ji} - \mu_i)$ is largest", to all agents exceeding μ_i.
d. A rather more subtle problem with "brute-force" suspension can, however, appear. Removal of x_i from $\langle \mathcal{X}, \mathcal{A}, \nu \rangle$ will induce a sub-graph of $\langle \mathcal{X}, \mathcal{A} \rangle$. Our discussion of $\mathbf{C} \times \mathbf{F}$ and its properties, was predicated on this being irreducible: property gauranteed in the event of $\langle \mathcal{X}, \mathcal{A} \rangle$ being strongly-connected. Strong-connectivity of $\langle \mathcal{X}, \mathcal{A} \rangle$ does not, however, ensure strong-connectivity of the framework induced by $\mathcal{X} \setminus \{x_i\}$. In principle this may create complications with dominant (i.e. maximal) eigenvalues and existence of positive associated eigenvectors.

As a final issue we, briefly, consider the assumption of "strong-connectivity". While this is useful in guaranteeing the conditions of Thm. 1 are met, it is not an essential prerequisite of our approach. In particular, by considering the strongly-connected component decomposition of $\langle \mathcal{X}, \mathcal{A} \rangle$ – whose benefits have been studied in Baroni *et al.* [1] – similar analyses of acceptable levels of volubility are possible.

4. Conclusions

The main intention of this paper has been to offer a model (based on Dung's classical AF formalism) by which problems arising from over-heated debates can be studied. Such models may offer a vehicle for considering divers strategies that could be adopted by moderators in controlling debates with minimal intervention being required. Underpinning the problems of interest is the concern that the *force* with which an argument is made can seem (to observers) at least as significant factor in gauging its merits as the argument's intrinsic logic and rationale. Our principal aim in this paper has been to highlight an important "non-logical" facet of real-world debate and argument together with a possible modelling approach. It is, of course, the case that this is rather crude and raises a number of directions for future research: a number of these are the focus of work currently in progress.

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