USING STATISTICAL ENERGY ANALYSIS TO PREDICT SOUND INSULATION IN BUILDINGS

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1 INTRODUCTION

Sound insulation in the field is determined by both direct and flanking transmission; hence a prediction model such as Statistical Energy Analysis (SEA) is useful at the design stage to determine the overall transmission. This paper gives an overview of classical SEA models which are commonly used to predict airborne and impact sound insulation. Such models sometimes need to incorporate data from measurements or deterministic models (e.g. finite element methods) to give accurate predictions. For example, this occurs when building components, or the coupling/connections between them, are too complex to model with simple idealizations of beams, plates and springs. This is sometimes considered in the low-frequency range because heavyweight walls and floors often have low modal density and low modal overlap which increases the uncertainty in coupling loss factors predicted using wave theory. In this paper, experimental and numerical examples are used to illustrate aspects relating to the inclusion of laboratory measurements in SEA models. Whilst classical SEA gives predictions of steady-state sound and vibration (i.e. L_{eq} values) it is also useful to be able to predict Fast time-weighted sound pressure levels inside buildings. Hence examples are given to show that when the principles of classical SEA are applied in short–time periods using Transient SEA (TSEA) it is possible to predict $L_{p,Fmax}$ from transients such as footsteps on heavyweight floors.

2 STATISTICAL ENERGY ANALYSIS (SEA)

2.1 Overview of SEA

A brief summary of SEA is given here because detailed descriptions of its application to buildings are given in monographs on building acoustics[1,2]. The first step in making the model is to define SEA subsystems that form parts of a building; these are either space subsystems such as rooms, or structural subsystems such as plates or beams. These subsystems need to be defined by their ability to store modal energy; therefore, the boundaries of a subsystem must cause reflections so that the sound or vibration field is reverberant for the specific wave type considered in the subsystem. Reflections occur when there is an impedance change at a boundary. Hence for space subsystems where there is only one wave type, the surfaces that define a room or cavity usually define the subsystem. For structural subsystems it can be slightly more complex. Although bending waves are of primary importance for sound radiation, in-plane waves can be important for structure-borne sound transmission. As these waves have different modal energies, they need to be represented as separate subsystems. For example, a plate can be represented by three subsystems using a separate subsystem for bending, transverse shear and quasi-longitudinal waves. Conversion between these wave types at a junction can therefore be included in an SEA model using coupling loss factors from one subsystem to another. The subsystem boundaries may vary depending upon the wave type under consideration. A plate or beam can be represented by one subsystem for each wave type, although in many situations it is only necessary to consider bending waves and a single subsystem will be sufficient. The analysis is carried out in frequency bands such that the statistical modal density can be used. In building acoustics, it is common to use one-third octave bands or octave bands to assess sound insulation, both of which are reasonable above 100Hz because there are a sufficiently large number of modes in each frequency band[2].

SEA requires knowledge of the dissipative subsystem losses, the coupling losses between subsystems and the power input into the subsystem(s). The losses are described using loss factors; these give the fraction of energy transferred per radian cycle. Three loss factors are defined: internal (dissipative) subsystem losses (η_{ii} for subsystem *i*), coupling losses between subsystems (η_{ij} for subsystem *i* to *j*), and total subsystem losses (η_i for subsystem *i*). These are referred to as the Internal Loss Factor (ILF), the Coupling Loss Factor (CLF) and the Total Loss Factor (TLF).

The output from SEA is the temporal- and spatial-average response of a subsystem in terms of its energy which can be converted to more practical variables such as sound pressure or velocity. By assuming a statistical description for each subsystem, the subsystem response represents the ensemble average of 'similar' subsystems with physical parameters drawn from statistical distributions. Hence it is not necessary to know the exact geometry of a room when representing it as a space subsystem, just the volume is sufficient. This makes SEA an attractive form of analysis from an engineering viewpoint and it simplifies interpretation of the results because (in comparison with finite element models) the predicted subsystem energies show a smooth variation with frequency.

The principles of energy flow between two subsystems are shown here for the simplest situation of relevance to sound insulation. This could represent power input, $W_{\mathrm{in}(1)}$, from the ISO tapping machine into a plate (subsystem 1) representing a concrete floor slab that radiates sound into the receiving room (subsystem 2). From the conservation of energy, the power balance equations for subsystems 1 and 2 are given by

$$W_{\mathrm{in}(1)} + \omega\eta_{21}E_2 = \omega\eta_{11}E_1 + \omega\eta_{12}E_1 \tag{1}$$

$$\omega\eta_{12}E_1 = \omega\eta_{22}E_2 + \omega\eta_{21}E_2 \tag{2}$$

Equations (1) and (2) can be solved to give the subsystem energies with knowledge of the internal loss factors, coupling loss factors and the power input. The energy, E_2, in one-third octave bands can then be converted into sound pressure levels in the receiving room. In practice, buildings are made of many connected walls and floors and because of the importance of flanking transmission it is necessary to consider more than two subsystems. It is then convenient to solve the power balance equations in matrix form to give the energies with

$$\begin{bmatrix} \sum_{n=1}^{N}\eta_{1n} & -\eta_{21} & -\eta_{31} & \cdots & -\eta_{N1} \\ -\eta_{12} & \sum_{n=1}^{N}\eta_{2n} & -\eta_{32} & & \\ -\eta_{13} & -\eta_{23} & \sum_{n=1}^{N}\eta_{3n} & & \\ \vdots & & & \ddots & \\ -\eta_{1N} & & & & \sum_{n=1}^{N}\eta_{Nn} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ \vdots \\ E_N \end{bmatrix} = \begin{bmatrix} \frac{W_{\mathrm{in}(1)}}{\omega} \\ \frac{W_{\mathrm{in}(2)}}{\omega} \\ \frac{W_{\mathrm{in}(3)}}{\omega} \\ \vdots \\ \frac{W_{\mathrm{in}(N)}}{\omega} \end{bmatrix} \tag{3}$$

where η_{ij} indicates the CLF from subsystem *i* to *j*, and η_{ii} indicates the ILF for subsystem *i*. Note that in building acoustics we usually only consider one power input (e.g. one loudspeaker, one tapping machine) such that all the other power inputs would need to be set to zero in equation (3).

The matrix solution gives the response in all of the subsystems by taking account of the energy flow between them. However, one of the real strengths of SEA is the ability to compare the strengths of different sound transmission paths. For any system with power injected into subsystem 1, the energy ratio between subsystem 1 and subsystem N for the transmission path involving the chain of subsystems, $1\to2\to3\to\ldots\to N$, is

$$\frac{E_1}{E_N}=\frac{\eta_2\eta_3\cdots\eta_N}{\eta_{12}\eta_{23}\cdots\eta_{(N-1)N}} \qquad (4)$$

The path with the lowest energy ratio is the strongest path, and the path with the highest energy ratio is the weakest path. In a complete building there are many transmission paths that determine the overall sound insulation so the matrix solution is used to determine the distribution of energy between the subsystems. However, when we want to find ways to increase the sound insulation it is useful to know if there is a dominant transmission path. A dominant path is defined here as giving nominally the same energy level difference as the matrix solution; hence the combination of all the other paths is relatively unimportant. We can then test out various changes to the dominant path that might improve the sound insulation (e.g. adding resilient isolators to a wall lining). However, changes to one path can change the relative importance of the other paths so the overall sound insulation then needs to be checked by re-calculating the sound insulation using the matrix solution.

2.2 Transient SEA (TSEA)

For sleep disturbance and the assessment of impact sounds, building regulations often make use of the maximum Fast time-weighted sound pressure level, $L_{\mathrm{p,Fmax}}$. Recent work[3,4] shows that $L_{\mathrm{p,Fmax}}$ can be predicted in masonry/concrete buildings using Transient SEA (TSEA).

TSEA predicts a time-varying, spatial-average mean-square energy in a given frequency band for a set of SEA subsystems using a defined power input and loss factors. The power balance equations in the time domain are given by Powell and Quartararo[5] and Lyon and DeJong[6] as

$$E_i\left(t_{n+1}\right)=E_i\left(t_n\right)+\Delta t\left[W_{\mathrm{in},i}\left(t_n\right)+\omega\left[\sum_{j(j\neq i)}\eta_{ji}E_j\left(t_n\right)-\eta_i E_i\left(t_n\right)\right]\right] \qquad (5)$$

where $E_i(t_n+1)$ is the energy at the next time step in subsystem i, $E_i(t_n)$ is the energy at the current time step in subsystem i, $W_{\mathrm{in},i}(t_n)$ is the time-varying power input into subsystem i, Δt is the time interval, η_{ij} is the coupling loss factor from subsystem i to subsystem j and η_i is the total loss factor of subsystem i.

This is used to calculate a set of time-varying subsystem energies in a given frequency band through an iterative calculation of energy in each successive time step. After calculating the energy at each time step over a chosen duration for all subsystems and a desired frequency range, energy can be converted into mean-square pressure for spaces, or mean-square velocity for structures[2]. The result is a time-varying level in each frequency band, from which the maximum level can be determined[3].

3 INCORPORATING MEASURED DATA IN SEA MODELS

Two situations are used to indicate the advantages and disadvantages of including measured data in an SEA model. The first situation is a multi-layer floor structure, and the second concerns vibration transmission between masonry/concrete walls/floors at low-frequencies.

3.1 Multi-layer floor structures

Recent work[7] on predicting airborne sound insulation for a timber-concrete composite floor (see Figure 1) is used here to illustrate how measured data can be incorporated into an SEA model. The complexity in modelling this floor is due to it having (1) a multilayer upper plate formed from concrete and Oriented Strand Board (OSB), (2) multiple types of rigid connector between the upper plate and the timber joists and (3) a resiliently suspended ceiling.

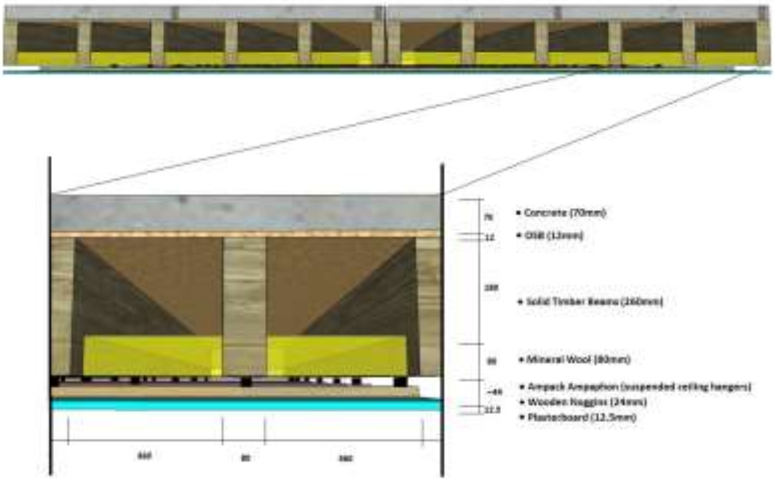

Figure 1. Timber-concrete composite floor.

A five-subsystem model shown in Figure 2 is considered here to treat the combination of concrete, OSB and timber joists as a single plate subsystem. A number of different models have been tried to assess whether this combination gives an orthotropic plate, but it has been shown that it can be treated as an isotropic plate with an effective bending stiffness[7]. However, there were two parts of the SEA model where it was difficult to provide accurate predictions: one was the total loss factor for the cavity hence a measured cavity reverberation time was used, and the other was for the dynamic stiffness of the resilient isolators which was measured in order to calculate a coupling loss factor from subsystem 2 to 4.

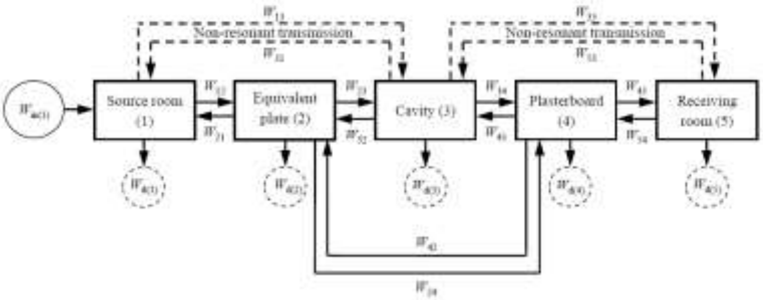

Figure 2. Five subsystem model for the timber-concrete composite floor.

Figure 3(a) shows the sound transmission paths calculated using equation (4). From 50Hz to 100Hz the dominant transmission path is 1→3→5. From 125Hz to 400Hz the strongest paths are 1→2→3(1D)→5 and 1→2→4→5. From 500Hz to 1.6kHz the main paths are 1→2→3(2D/3D)→5 and 1→2→4→5. At and above 2kHz the strongest paths are 1→2→3(2D/3D)→4→5 and 1→2 →4→5. Identification of the dominant paths indicates how the airborne sound insulation could be increased. Above 100Hz, an extra layer of plasterboard on the ceiling and completely filling the cavity with porous material would reduce the strength of paths 1→2→3→5, 1→2→4→5 and 1→2→3→4→5.

Figure 3(b) compares the sound reduction index in terms of the measurement and SEA matrix prediction. The close agreement demonstrates that SEA can be used to model direct transmission across this relatively complex floor construction.

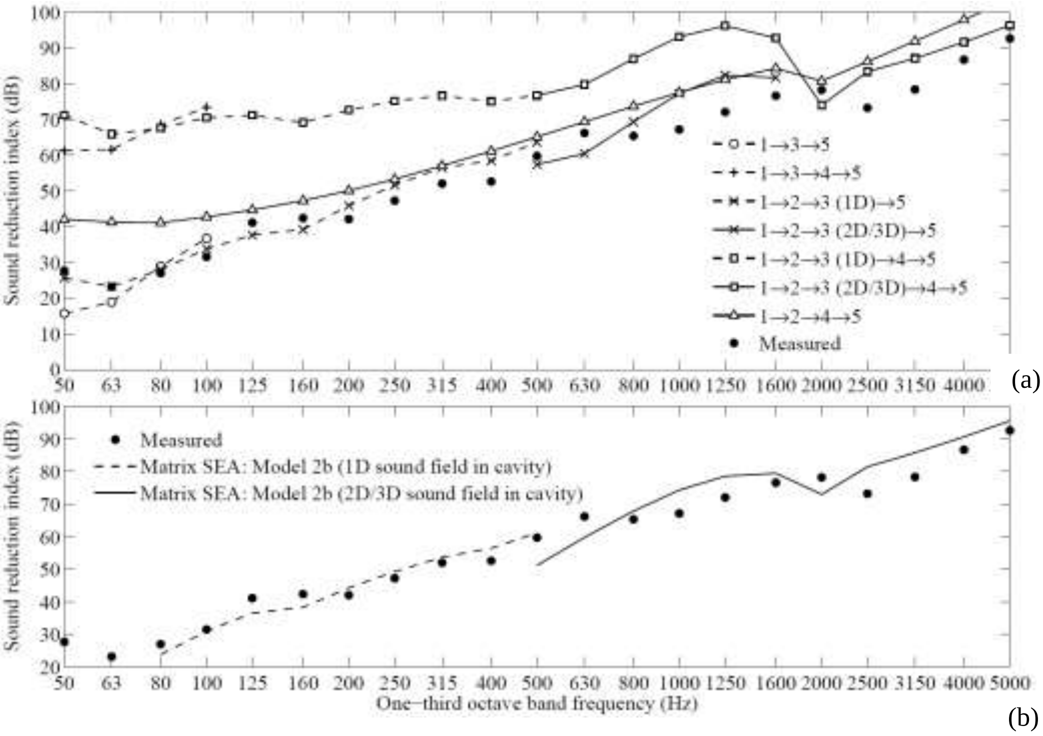

Figure 3. Timber-concrete composite floor: (a) comparison of SEA transmission paths with measurements, (b) comparison of matrix SEA with measurements.

3.2 Coupling between walls and/or floors at low-frequencies

Wave theory is commonly used to predict coupling loss factors for junctions of connected masonry walls and concrete floors[1,2]. In the low-frequency range (50Hz to 200Hz) these wall and floor subsystems tend to have low mode counts in one-third octave bands and wave theory tends to overestimate the coupling loss factor in some frequency bands, e.g. see [2,8]. A potential solution is to use Finite Element Methods (FEM) or analytical models to determine coupling loss factors for the low-frequency range which follow the modal fluctuations that occur due to low mode counts[9,10].

For EN 12354[11], improved estimates of the vibration reduction index (which is simply related to the coupling loss factor[2]) have been determined by combining wave theory and FEM[12,13]. However, it has been shown that problems occur with EN 12354 as it is effectively the same as SEA path analysis but only considering paths involving one structural junction for the flanking paths[2,14,15]. In practice there are so many paths involving transmission across two or more junctions that are of collective significance, that matrix SEA provides more accurate predictions than EN12354. For this reason it is worth querying whether the inclusion of low-frequency coupling loss factors (measured or predicted with other models) improves SEA predictions when there are many similar plate junctions in a building.

An example is shown in Figure 4 for five adjacent rooms formed from masonry/concrete walls and floors which compares predictions from FEM and SEA. As there will be natural variation in the material properties, FEM is used to run a Monte Carlo simulation with uncertainty in the Young's modulus for each plate. This is then compared against two different SEA models. The first SEA model uses coupling loss factors calculated from angular average wave theory which assumes a diffuse field on each plate. The second SEA model uses coupling loss factors that are calculated from analytical models for finite plates that form the isolated L- and T-junctions; these analytical models captures the modal fluctuations at low-frequencies.

The results indicate that when the receiving plate is directly connected to the source plate, both SEA models are close to the FEM ensemble average. When the receiving plate is far from the source plate, SEA wave theory tends to underestimate the transmission. However, there is some improvement by using coupling loss factors determined from a finite plate approach (although this is marginal below 200Hz). The reason for lower energy level differences with FEM compared to SEA is partly due to the effect of spatial filtering across successive junctions which is not accounted for in SEA[16]. This indicates that extending the approach used in EN 12354 to incorporate measured data from isolated junctions may be problematic for transmission across several rooms in large buildings. In practice it is sound transmission between adjacent rooms that is most important and this is not an issue. However, it is relevant to the prediction of machinery noise transmission in large buildings where the noise problem can occur in non-adjacent rooms.

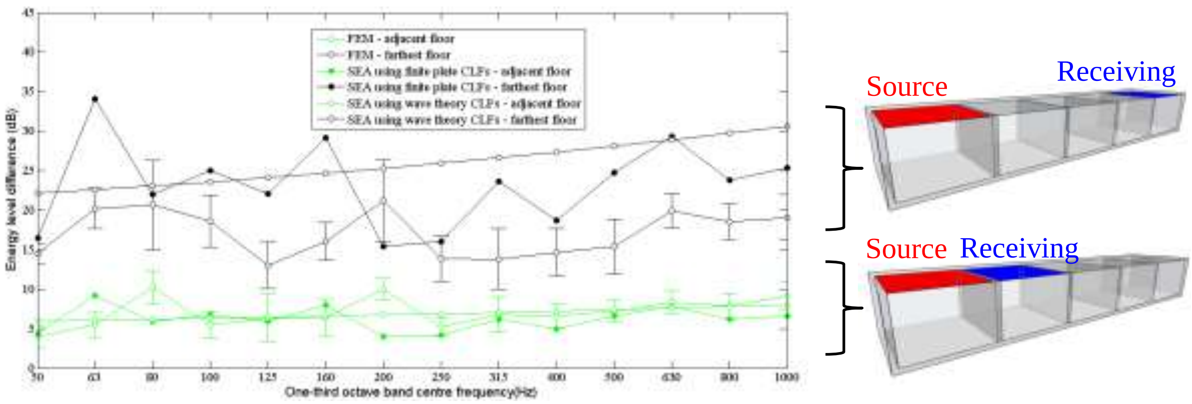

Figure 4. Energy level difference between source and receiving floors predicted using FEM and SEA either using coupling loss factors from wave theory or from a finite plate approach.

4 USING TSEA TO PREDICT MAXIMUM FAST TIME-WEIGHTED SOUND PRESSURE LEVELS

Prediction of maximum fast time-weighted sound pressure levels is relevant to impacts on floors in buildings, such as with transient excitation from the ISO rubber ball (commonly used in South Korea and Japan) that can be used to measure impact sound insulation, and human footsteps which are a common source of disturbance. For both the rubber ball and footsteps it is necessary to measure the ‘transient power’ using a force plate to provide input data for the TSEA model[17].

Examples are shown in Figure 5 for a rubber ball dropped from 1m height and a footstep from a male walker in socks. The latter provides a significantly more demanding test of TSEA because there are three distinct parts of the footstep: the heel strike, mid-stance and toe off. For the TSEA model, transient power was applied over these three consecutive time periods each of which corresponded to the actual duration of each phase.

Validation of this approach is shown in Figure 6 using measurements in a heavyweight laboratory structure with a 140mm concrete floor excited by the ISO rubber ball and footsteps from a male walker in socks. For the footsteps, the walker took five steps to walk diagonally across the floor; hence the transient power measured for one footstep using the force plate was assembled into a sequence of transient power inputs representing this walk across the floor. Close agreement between measurements and TSEA confirms correct implementation of transient power from the measured force time-history in the TSEA model. Comparisons with a male walker in hard- and soft-soled shoes show similarly good agreement[17].

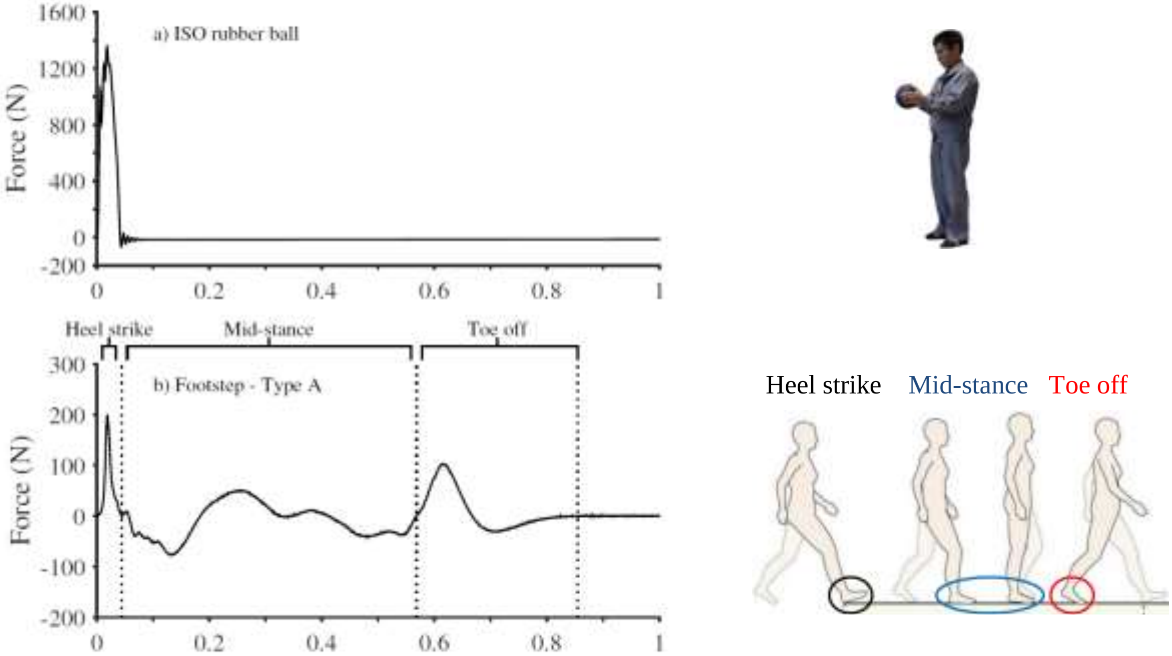

Figure 5. Force versus time in seconds for (a) ISO rubber ball dropped from 1m, and (b) footstep from a male walker in socks.

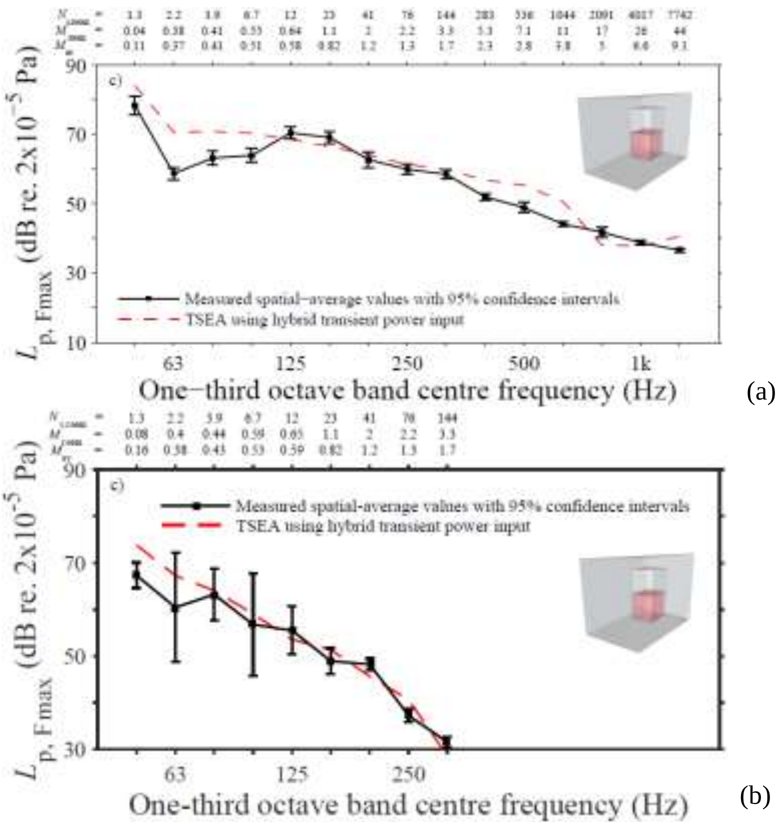

Figure 6. Measured and TSEA predicted maximum Fast time-weighted sound pressure levels in a receiving room from different impacts on a 140mm concrete floor: (a) a single impact from the ISO rubber ball, (b) footsteps from a male walker in socks.

5 CONCLUSIONS

This paper gives an overview of how SEA models can be used to predict airborne and impact sound insulation in buildings. Examples illustrate the usefulness of path analysis to assess the strength of transmission paths and how measured data can be incorporated into SEA models to improve the accuracy. Examples using TSEA show that prediction of L_{pFmax} is possible for transient sources such as the rubber ball used to measure impact sound in South Korea and Japan, and footsteps.

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7 REFERENCES

1. R.Craik. Sound transmission through buildings using statistical energy analysis. Gower (1996).
2. C. Hopkins. Sound insulation. Elsevier, Oxford, UK. ISBN: 978-0-7506-6526-1 (2007).
3. M. Robinson and C. Hopkins. Prediction of maximum time-weighted sound and vibration levels using transient statistical energy analysis. Part 1: Theory and numerical implementation. Acta Acustica united with Acustica, 100(1), 46-56 (2014).
4. M. Robinson and C. Hopkins. Prediction of maximum time-weighted sound and vibration levels using transient statistical energy analysis. Part 2: Experimental validation. Acta Acustica united with Acustica, 100(1), 57-66 (2014).
5. R.E. Powell and L.R. Quartararo. Statistical energy analysis of transient vibration. Proceedings of the American Society of Mechanical Engineers winter meeting, Boston, USA (1987).
6. R.H. Lyon and R.J. DeJong. Theory and application of statistical energy analysis, Second ed., Butterworth-Heinemann, Boston (1995).
7. C. Churchill and C. Hopkins. Prediction of airborne sound transmission across a timber–concrete composite floor using Statistical Energy Analysis. Applied Acoustics, 110, 145-159 (2016).
8. C. Hopkins. Statistical energy analysis of coupled plate systems with low modal density and low modal overlap, Journal of Sound and Vibration, 251 (2), 193–214 (2002).
9. J.A. Steel and R.J.M. Craik. Statistical energy analysis of structure-borne sound transmission by finite element methods, Journal of Sound and Vibration, 178 (4), 553–561 (1994).
10. C. Hopkins. Vibration transmission between coupled plates using finite element methods and statistical energy analysis. Part 1: Comparison of measured and predicted data for masonry walls with and without apertures, Applied Acoustics, 64, 955–973 (2003).
11. EN 12354:2000 Building acoustics – Estimation of acoustic performance of buildings from the performance of elements – Parts 1 and 2.
12. C. Hopkins. Determination of vibration reduction indices using wave theory for junctions in heavyweight buildings. Acta Acustica united with Acustica, 100(6), 1056-1066 (2014).
13. C. Hopkins, C. Crispin, J. Poblet-Puig, C. Guigou-Carter. Regression curves for vibration transmission across junctions of heavyweight walls and floors based on finite element methods and wave theory. Applied Acoustics 113, 7-21 (2016).
14. R.J.M. Craik. The contribution of long flanking paths to sound transmission in buildings, Applied Acoustics 62 (1), 29–46 (2001).
15. L. Galbrun. The prediction of airborne sound transmission between two rooms using first-order flanking paths. Applied Acoustics 69(12), 1332-1342 (2008).
16. D. Wilson and C. Hopkins. Analysis of bending wave transmission using beam tracing with advanced statistical energy analysis for periodic box-like structures affected by spatial filtering. Journal of Sound and Vibration, 341, 138-161 (2015).
17. M. Robinson and C. Hopkins. Prediction of maximum fast time-weighted sound pressure levels due to transient excitation from the rubber ball and human footsteps. Building and Environment, 94, 810-820 (2015).