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Dummett and Frege on sense and *Selbständigkeit*

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ABSTRACT

As part of his attack on Frege's 'myth' that senses reside in the third realm, Dummett alleges that Frege's view that all objects are *selbständig* ('self-subsistent', 'independent') is an underlying mistake, since some objects depend upon others. Whatever the merits of Dummett's other arguments against Frege's conception of sense, this objection fails. First, Frege's view that senses are third-realm entities is not traceable to his view that all objects are *selbständig*. Second, while Frege recognizes that there are objects that are dependent upon other objects, he does not take this to compromise the *Selbständigkeit* of any objects. Thus, Frege's doctrine that objects are *selbständig* does not make the claim of absolute independence that Dummett appears to have taken it to make. Nevertheless, in order to make a good case against Frege based on the dependency of senses, Dummett need only establish his claim that senses depend upon expressions: appeal to an absolute conception of independence is unnecessary. However, Dummett's arguments for the dependency of senses upon expressions are unsuccessful and they show that Dummett's conception of what it is to be an expression also differs significantly from Frege's.


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1. Introduction

According to Dummett ('Frege's Myth of the Third Realm', 249), Frege's view that senses occupy the 'third realm' is a classic case of 'philosophical mythology'. Dummett aims to retain a broadly Fregean conception of sense *without* commitment to the 'myth' that senses reside in the third realm.

Dummett ('Frege's Myth of the Third Realm', 256) sees Frege as having conceived of senses in two conflicting ways. First, we have the conception of senses as *modes*: senses are modes of presentation of referents. Second, we have the conception of senses as *selbständig*: they are 'self-subsistent objects which we apprehend'. Dummett's strategy, in order to attain his

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aim of retaining a broadly Fregean conception of sense without ‘mythology’, is to retain the conception of senses as modes but to abandon the conception of senses as *selbständig*.

Dummett (‘Frege’s Myth of the Third Realm’, 249) adopts this strategy because he thinks that it is the conception of senses as *selbständig* (and consequently, thinks Dummett, dependent neither upon us nor upon language) that underlies Frege’s view that senses reside in the third realm. Dummett (‘Frege’s Myth of the Third Realm’, 249) has it that it is Frege’s more general ‘assumption’ that ‘all objects are self-subsistent (*selbständig*)’ that underlies the conception of senses as *selbständig*. Dummett takes this more general assumption to be mistaken. According to Dummett (‘Frege’s Myth of the Third Realm’, 249), and contrary, he thinks, to Frege’s more general assumption, ‘some [objects] are intrinsically “of”, or dependent on, other objects’. Evidently, on the conception of *Selbständigkeit* (discussed further in Section 2) that underlies Dummett’s remark, an object can be *selbständig* only if it is absolutely independent: that is, if it is dependent upon no other objects.

This article aims to show that, *contra* Dummett, Frege’s commitment to senses as third-realm entities is not properly traceable to his doctrine, which Dummett appears to have misconstrued, that all objects are *selbständig*. Dummett uses a notion of *Selbständigkeit* that is very different to Frege’s own. On Frege’s conception of *Selbständigkeit*, there is no conflict between the conception of senses as modes and the conception of senses as *selbständig*.

The doctrine that senses are third-realm entities does not require that senses should be considered *selbständig* in Dummett’s sense. Thus, rejection of the doctrine cannot soundly be generated from rejection of the claim that senses are *selbständig* in Dummett’s sense. This does not rule out the possibility of a good argument against the doctrine that senses are third-realm entities deriving directly from the conception of senses as modes. Indeed, if senses *are* modes, then senses are dependent, in a relevant sense, upon referents. Accordingly, those senses that are modes of presentation of referents that are not third-realm entities cannot themselves be third-realm entities, for the senses in question could not be timeless. Of course, this would not establish that no referents reside in the third realm and so it would not establish that the third realm is mythical. (Given this, and that Dummett [‘Frege’s Myth of the Third Realm’] focuses solely on the case of senses, ‘Frege’s Myth of the Third Realm’ is perhaps not a very apt title for the piece.)

It is well-known that Frege (e.g. [‘Über Sinn und Bedeutung’, 205; ‘Was kann ich als Ergebnis meiner Arbeit ansehen?’, 203 German (187 English); ‘Logik in der Mathematik’, 243 (225); ‘Aufzeichnungen für Ludwig Darmstaedter’, 274 (254)]) distinguishes between *complete* (saturated) and *incomplete* (unsaturated) senses, taking a sense to be complete if and only if it is the sense of a proper name (such as, on Frege’s account, an ordinary proper name, a definite description or a sentence). On Frege’s account, the complete senses are

the senses of those expressions that, if they refer at all, refer to objects. It is plausible, e.g. with Zouhar ('The Structure of Frege's Thoughts'), to regard Frege as having held that all and only objects are saturated. On Frege's account, a sense is complete if and only if it either contains a mode of presentation of an object or belongs to a proper name that lacks a referent. Concept words and other function names have senses, but their senses are incomplete. Moreover, Frege has it that thoughts cannot be wholly composed from objects ('Über Sinn und Bedeutung', 205; 'Aufzeichnungen für Ludwig Darmstaedter', 274 (254)). On Frege's account, a thought can only be a unified whole because it has at least one constituent sense that is incomplete and which, thereby, is not an object. Whatever Dummett's own views in the early 1990s about these matters, then, charity would appear to require that his remarks about the supposed tension between the conception of senses as modes and the conception of senses as *selbständig* should be restricted to the senses of Fregean proper names, for Frege does not appear to regard the senses of functional expressions as objects.

In Section 2, some salient features of the conception of *Selbständigkeit* adopted in Dummett ('Frege's Myth of the Third Realm'), to which Dummett does not always draw attention himself, are explicitly set out. Section 3 shows that Frege's own conception of *Selbständigkeit* shares none of these features. Though Dummett never says so himself, Dummett and Frege are clearly working with different conceptions of *Selbständigkeit*. Attending to the differences between Dummett's conception and Frege's enables us to see that when Dummett depicts Frege's doctrine that senses occupy the third realm as stemming from the 'mistake' that all objects are *selbständig*, Dummett is confused and does not manage to articulate wherein the mistakenness of this 'mistake' might reside.

In Section 4, the material in earlier sections is brought to bear upon Dummett's critique of Frege's doctrine that senses reside in the third realm. Dummett ('Frege's Myth of the Third Realm') argues, against Frege, that senses are logically dependent upon expressions. Whether or not we ought to agree with Frege's doctrine that senses reside in the third realm, Dummett's arguments for his conclusion that senses are logically dependent upon expressions are unsuccessful. Section 4 shows that Dummett works with a conception of what it is to be an expression that is, as in the case of his conception of *Selbständigkeit*, significantly different from Frege's.

2. Dummettian *Selbständigkeit*

*F*s are ontologically dependent on *G*s if and only if *F*s exist because of the existence of *G*s and not vice versa. (This is a generalized version of what Tahko and Lowe ['Ontological Dependence', §3] call 'explanatory existential dependence'. The 'because' here is explanatory rather than causal.) Dummett construes

Selbständigkeit as involving a notion of 'logical', rather than ontological, independence. In arguing that senses are not *selbständig*, Dummett makes an analogy with chess moves. Dummett thinks that chess moves depend on chess pieces but that this dependency is *not* ontological (existential) dependency. According to Dummett ('Frege's Myth of the Third Realm', 249), while it is 'harmless' to quantify over possible but non-actual chess moves, 'it would be insane to deny that moves are *of* (actual or possible) chess pieces'. When Dummett ('Frege's Myth of the Third Realm', 249) says that moves are *of* pieces, he thinks that the '*of*' is not an '*of*' of ontological dependency, because 'we can speak of moves that have never been assigned to any piece' (i.e. possible but non-actual moves). Rather, the '*of*' is an "'*of*" of logical dependence'.

Dummett explains logical dependence in two ways: in *conceptual* terms and in terms of *reference*. Let us address these explanations in order.

According to Dummett ('Frege's Myth of the Third Realm', 249), to hold that chess moves are logically dependent on chess pieces 'means [...] that to conceive of any move is to conceive of a piece as having that move'. Let *a* be an agent and *F* be a property or a kind. We may distinguish between conceiving *de re* and conceiving *de dicto* as follows.

Conceiving de re: there is an object, *x*, such that *a* conceives of *x* as (an) *F*.

Conceiving de dicto: *a* conceives that there is an object, *x*, such that *Fx*.

It would probably be charitable to interpret Dummett's remark that 'to conceive of any move is to conceive of a piece as having that move' as involving the logically weaker notion of conceiving *de dicto*, rather than that of conceiving *de re*. Dummett's remark, so construed, amounts to the claim that whenever an agent conceives that there is a chess move, the agent thereby conceives that there is a chess piece to which the move belongs. (This reading of the remark also renders superfluous Dummett's – surely hasty – subscription to the view that it is 'harmless to say that "there are" chess moves 'that might have been assigned to pieces in versions of chess that have never been played or thought of' ['Frege's Myth of the Third Realm', 249].) When interpreted as involving conceiving *de dicto*, Dummett's conceptual explanation of logical dependence can, alongside the conceptual explanation of logical independence that is implicit, be specified as follows.

Logical dependence as conceptual dependence

Fs are logically dependent on Gs if and only if:

- (1) it is logically impossible to conceive that there is an object, *x*, such that *Fx*, without conceiving, thereby, that there is something, *y*, distinct (both individually and in kind) from *x*, such that *Gy*; and

- (2) it is logically possible to conceive that there is an object, x , such that Gx , without conceiving, thereby, that there is something, y , distinct (both individually and in kind) from x , such that Fy .

Logical independence as conceptual independence

Fs are *logically independent* if and only if it is logically possible to conceive that there is an object, x , such that Fx , without conceiving, thereby, that there is something, y , distinct (both individually and in kind) from x , such that Gy .¹

Having initially characterized logical dependence in conceptual terms, Dummett later shifts, without remarking upon the shift, to characterizing it in terms of reference:

A self-subsistent, or logically independent, object is [...] one of such a kind that it would be in principle possible to make reference to objects of that kind without referring to objects of any other kind; it is in this respect that senses are logically dependent upon expressions of which they are the senses.

(Dummett, 'Frege's Myth of the Third Realm', 261–2)

Dummett's characterization of logical independence in terms of reference can be compared and contrasted with the remark of Angelelli (*Studies on Gottlob Frege and Traditional Philosophy*, 173) that Frege's doctrine of the unsaturatedness of concepts 'coincides with the tenacious traditional view' that (presumably, reference to) a concept involves 'essential reference to [its] "inferiors" (individuals, etc.)'. The most significant point of comparison is that both appeal to essential reference; that of contrast is that Angelelli is concerned with the dependency of concepts upon objects and Dummett with the dependency of some objects upon others. When Dummett writes of 'reference to objects of [a] kind' he is alluding to the use of singular terms that refer to objects of a given kind (e.g. the 'Socrates' of 'Socrates is mortal', which refers to an individual that is a human being), rather than to the use of sortal terms in subject position (e.g. the 'Men' of 'Men are mortal'). (For Dummett and Frege alike, the latter are, despite grammatical appearance, predicates.) Thus, Dummett's characterization of logical independence as referential independence suggests the following biconditionals.

¹That is, Fs are *logically independent* if and only if it is logically possible to conceive that there is an object, x , such that Fx , without conceiving, thereby, that for some y such that y is not identical to x , and some G such that G is not identical to F , Gy . (It is *de dicto* conceiving that is relevant throughout this.)

Logical dependence as referential dependence

*F*s are *logically dependent* on *G*s if and only if:

- (1) it is logically necessary that whenever reference is made to an object, x , such that Fx , there is something, y , distinct (both individually and in kind) from x , to which reference is thereby made, such that Gy ; and
- (2) it is logically possible to make reference to an object, x , such that Gx , without there being something, y , distinct (both individually and in kind) from x , to which reference is thereby made, such that Fy .

Logical independence as referential independence

*F*s are *logically independent* if and only if it is logically possible to refer to an object, x , such that Fx , without it being the case that there is something, y , distinct (both individually and in kind) from x , to which reference is thereby made, such that Gy .

Dummettian *Selbständigkeit* is *absolute*, in the sense that it is not independence from any particular object or kind of object, but from all objects whatsoever. It is for this reason that 'of *G*s' is not included on the left-hand side of *logical independence as referential independence*. It is worth noting, however, that it is only the letter of Dummett's explanation of logical independence as referential independence that commits him to this absoluteness. Dummett says nothing that explicitly commits him to the absoluteness of logical independence when explained in terms of conceptual independence: it is just that in treating logical independence so explained as absolute we obtain parity with Dummett's explanation in terms of reference.

Dummett's view that senses are referentially dependent is similar to Angelelli's (albeit stronger) view that, in direct speech, senses can only be designated 'using the phrase "the sense of the name so and so"' (*Studies on Gottlob Frege and Traditional Philosophy*, 49). To name a sense, thinks Angelelli, one must name a name. Names and senses are of course different in kind in that names have, but are not themselves, senses. Angelelli (*Studies on Gottlob Frege and Traditional Philosophy*, 50) thinks that Frege seems to have offered such definite descriptions as 'the pupil of Plato and teacher of Alexander the Great' as *names* of senses (here, of the sense of 'Aristotle'; Frege ['Über Sinn und Bedeutung', 27 note B]). Angelelli (*Studies on Gottlob Frege and Traditional Philosophy*, 50) regards this as a 'curious mistake' on Frege's part, since the definite descriptions concerned name referents (e.g. Aristotle) that are *not* senses. Even if there are two ways of naming senses, and to name a sense is not necessarily thereby to name a name, the kinds of definite descriptions that occur in Frege's footnote include names of objects that are not senses. It would therefore remain the case,

in direct speech, that to name a proper name's sense would, thereby, be to name an object that is not a sense.

While Dummett appears to have intended his conception of *Selbständigkeit* to have been a piece of Frege interpretation, the conception faces difficulties, both intrinsic and *qua* Frege interpretation. Let us deal first with some intrinsic difficulties. Dummett's conception of *Selbständigkeit* as referential independence is ambiguous, with each disambiguation leading to problems.

Strong reading of logical dependence as referential dependence:

*F*s are *logically dependent* on *G*s if and only if:

- (1) it is logically necessary that for every object, *x*, such that *Fx*, every name of *x* should include the name of something, *y*, distinct (both individually and in kind) from *x*, to which reference is thereby made, such that *Gy*; and
- (2) it is not logically necessary that for every object, *x*, such that *Gx*, every name of *x* should include the name of something, *y*, distinct (both individually and in kind) from *x*, to which reference is thereby made, such that *Fy*.

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*F*s are *logically independent* if and only if it is not logically necessary that for every object, *x*, such that *Fx*, every *name* of *x* should include the name of something, *y*, distinct (both individually and in kind) from *x*, to which reference is thereby made, such that *Gy*.

Weak reading of logical dependence as referential dependence:

*F*s are *logically dependent* on *G*s if and only if:

- (1) it is logically necessary that for every object, *x*, such that *Fx*, each *act of reference* to *x* should include an act of reference to something, *y*, distinct (both individually and in kind) from *x*, to which reference is thereby made, such that *Gy*; and
- (2) it is not logically necessary that for every object, *x*, such that *Gx*, each *act of reference* to *x* should include an act of reference to something, *y*, distinct (both individually and in kind) from *x*, to which reference is thereby made, such that *Fy*.

Weak reading of logical independence as referential independence:

*F*s are *logically independent* if and only if it is not logically necessary that for every object, *x*, such that *Fx*, each *act of reference* to *x* should include an act

of reference to something, y , distinct (both individually and in kind) from x , to which reference is thereby made, such that Gy .

The strong reading of referential independence is implausible. Dummett would presumably regard John's smile, for example, as logically dependent upon John. Normally, when we name John's smile, our name for it includes John's name. This, however, is a contingent feature of our language. We could just as well invent, or have had, a name for John's smile, like 'Smohn', that does not include John's name. Charity to Dummett would therefore appear to require adoption of the weak reading.

In order for Dummett's construal of *Selbständigkeit* as referential independence to work, it must be the case that if I refer to John's smile using an expression that does not include a name of John, then it does not follow that my referential act, in referring to John's smile, is not thereby a referential act that also refers to John. A way of defending Dummett here might involve the hope that the deictic element of reference is sufficiently analogous, in the following respect, to pointing. It is possible to point to John without pointing to his smile, since he may not be smiling at the time of pointing. It is impossible, however, to point to John's smile without pointing to John. Dummett's account of logical independence as referential independence works only if it is impossible to refer to John's smile without thereby referring to John. This, in turn, appears to require an externalist account of acts of reference: it is not the pointer's intention to point to John that makes any act of pointing to John's smile an act of pointing to John. Likewise, it is not any intention, on the part of the speaker, to refer to John that makes any act of reference to John's smile an act of reference to John. Rather, it is that there is no way of *individuating* the object of reference (i.e. the intended object of reference, in this case, John's smile) except by reference to *another* object (i.e. in this case, John). In its external aspects, the act of reference can thereby be considered to be an act of reference that picks out (whether intentionally or not) *this other* object, in addition to the object of reference.

We might also attempt to explain the weak reading by generalizing it to the case of functions. A Fregean first-level concept can only be referred to, on Frege's account, within a sentence that includes an expression that refers either to an object or to a second-level concept. This, however, would not provide a means of explaining the logical dependency of concepts because Frege also holds that objects (other than the True and the False) can only be referred to in sentences. (The True and the False are typically not referred to *in* sentences, but *by* sentences.) Moreover, while Frege regards first-level concepts as dependent upon objects, he does not regard first-level concepts as dependent upon second-level concepts. First-level concepts are dependent upon objects because they require objects to saturate them. (It is 'saturate', rather than 'fall under' that is the appropriate terminology here: a concept

requires not that something should *actually* fall under it, but that something should *judgeably* fall under it. This condition is satisfied by empty concepts. Compare Gibson [*From Naming to Saying*, 11–12].) Second-level concepts are dependent upon first-level concepts because second-level concepts do not saturate, but require as their saturators, first-level concepts. (See further McLeod, ‘Frege’s Saturated/Unsaturated Distinction’.)

The weak reading, however, has a problem. For the weak reading, when combined with some other Fregean doctrines, has the anti-Fregean and anti-Dummettian result that *no objects* other than truth values are *selbständig*. The context principle (Frege, *Die Grundlagen der Arithmetik*, x, §§60, 62, 106), when interpreted as a principle with implications for reference, entails that truth value is prior to reference, in the sense that reference to an object other than a truth value can only be secured within a sentence, with the sentence itself referring to its truth value. Truth values are objects that can be referred to without reference thereby being made to other objects, for there are sentences, like ‘There are German men’, that contain no names of objects. Given the context principle, it is the utterance of an entire sentence, rather than that of any of the proper names within it, which constitutes an act of reference to the objects the names of which occur in the sentence. The act of reference for which ‘There are German men’ is used is an act of reference to a truth value. Truth values can be named in acts of reference that are not thereby acts of reference to other objects. Other objects, on the other hand, can only be named using sentences the uses of which involve us in acts of reference to truth values.

The difficulties with the two readings of Dummett’s account of referential independence can now be summarized. The strong reading over-generates, for it counts as logically independent some objects, such as John’s smile, that ought not to be so considered, and which Dummett does not appear to have wanted so to consider. On the other hand, given the context principle, the weak reading under-generates, for it has the surely unintended result that the only Fregean objects that are *selbständig* are, if they are indeed objects, the truth values. The weak reading would, in order to exclude the truth values from those objects upon which other objects might count as depending, need to be amended as follows.

Modified weak reading of logical dependence as referential dependence:

Fs are logically dependent on Gs if and only if:

- (1) it is logically necessary that for every object, x , such that Fx , each *act of reference* to x should include an act of reference to something, y , distinct (both individually and in kind) from x and from the truth values, to which reference is thereby made, such that Gy ; and
- (2) it is not logically necessary that for every object, x , such that Gx , each *act of reference* to x should include an act of reference to something,

y , distinct (both individually and in kind) from x and from the truth values, to which reference is thereby made, such that Fy .

Modified weak reading of logical independence as referential independence:

Fs are *logically independent* if and only if it is not logically necessary that for every object, x , such that Fx , each *act of reference* to x should include an act of reference to something, y , distinct (both individually and in kind) from x and from the truth values (other than x , if x is itself a truth value), to which reference is thereby made, such that Gy .

With this modification in place, both truth values and objects that are not truth values can count as logically independent.

The account of logical independence as conceptual independence would seem to face a more serious under-generation problem. To conceive that there is a composite concrete object is to conceive of something with proper parts, yet only mereological essentialists about composite concreta, among whose number neither Frege (see Section 3) nor, as far as I know, Dummett can properly be counted, hold that every such object is dependent upon its proper parts.

Having considered some intrinsic difficulties, of greater or lesser gravity, with Dummett's conception of *Selbständigkeit*, the rest of this section and the next aim to show that, as Frege exegesis, Dummett's conception goes astray. We cannot get Dummett off the hook here by speculating that he might have been trying to improve upon Frege's notion of *Selbständigkeit*, because Dummett's deployment of the term 'selbständig' is clearly one that he took to be after Frege's fashion: otherwise, it is difficult to see how Dummett could have thought that Frege's view that senses reside in the third realm was traceable to Frege's 'mistake' of having thought that all objects are *selbständig*.

Since Dummettian *Selbständigkeit*, as outlined in Dummett ('Frege's Myth of the Third Realm'), involves a distinction among objects, rather than between objects and functions, it cannot explain (at least the early) Frege's view (discussed in Section 3) that concepts depend upon judgeable contents. Dummett's conception appropriates some terminology from Frege but has the disadvantage that, to the letter of its formulation, it casts no light on Frege's view that objects are *selbständig* but functions are not. Nevertheless, considering some of Dummett's earlier work can enable us to interpret Dummett ('Frege's Myth of the Third Realm') more charitably and to argue, on Dummett's behalf, against the criticism just mentioned. The technical use of 'of' in Dummett ('Frege's Myth of the Third Realm') has a precedent in Dummett (*Frege: Philosophy of Language*):

It is [...] of the essence of a concept or property to be of an object, of a [two-place] relation to be *between* two objects. This [...] does not use Frege's

terminology: but it expresses precisely what he expresses by saying that a concept and a relation are both incomplete [...]

(Dummett, *Frege: Philosophy of Language*, 176)

The primitive conception of a function is given by the vocabulary in which we use the word 'function' only in the context 'function of', where this is used as when we say that 16 is that function of 4 which 9 is of 3 and 4 of 2 [...] there is no way to allude to the function itself, save by talking about objects which it would be senseless to identify with the function, but which are that function of other objects.

(Dummett, *Frege: Philosophy of Language*, 249–50)

In developing these remarks, Dummett (*Frege: Philosophy of Language*, 250–1, 254) proceeds to claim that expressions that can refer to functions (but which would not, of course, be considered by Frege to do so) in subject place (e.g. 'addition' – my example) can only be introduced using the terminology of 'function of'. On this basis, he claims that our understanding of 'what is meant by talking about functions' requires that we should first understand 'the mode of speaking according to which "function" occurs only in "function of"' (Dummett, *Frege: Philosophy of Language*, 251). (Compare Dummett [*Frege: Philosophy of Language*, 251] on directions of lines as incapable 'of being given otherwise than as directions of lines'.)

Interpreting the account of 'of' that is given by Dummett ('Frege's Myth of the Third Realm') charitably, then, it appears to be a localized version of a more general thesis that applies not only within the Fregean category of object, but across the Fregean categories. Dummett ('Frege's Myth of the Third Realm') presumably works with the localized version because he is simply assuming that it is the senses of proper names that are of primary interest when we address the question of the status, as logically dependent or not, of senses. Dummett (*Frege: Philosophy of Language*, 176–7) also serves as evidence for interpreting the notion of referential dependency with which Dummett ('Frege's Myth of the Third Realm') works in terms of acts of reference, rather than in terms of names. This is because Dummett (*Frege: Philosophy of Language*, 176–7) explains the 'intuition' that concepts and properties are of objects in terms of the impossibility of reference to a function on its own, given that function names can only occur in sentences. Thus, while it is not

Table 1. Dummettian and Fregean *Selbständigkeit*.

Dummettian <i>Selbständigkeit</i>	Fregean <i>Selbständigkeit</i>
Logical independence	Ontological independence
Absolute	Relative
Purportedly explicable in terms of reference and/or conceivability	Neither purported by Frege to be explicable in terms of reference and/or conceivability nor so explicable
Typical case: <i>selbständig</i> objects are independent of all other objects that are 'different in kind'	Typical case: all objects are <i>selbständig</i> , being independent of judgeable contents and functions

the case that, for every name of a function, the name must contain the name of an object, it is the case that every act of reference to a function must, thereby, be an act of reference to something else. This something else can of course be another function or it can, in the case of first-level functions, be an object. How can a first-level function be *of* an object given that it is possible to refer to that first-level function in a sentence which does not contain, at sub-sentential level, a name for an object? A natural Fregean answer is that first-level functions are dependent on objects because it is objects that they take as arguments. Second-level functions are dependent on first-level functions and, since dependency is transitive, also dependent on objects.

This piece of advocacy on Dummett's behalf (by way of his earlier work) notwithstanding, in the ways summarized in Table 1 and set out in detail over this and the next section, Frege's own notion of *Selbständigkeit* is significantly different from that of Dummett ('Frege's Myth of the Third Realm').

3. Fregean *Selbständigkeit*

3.1. Fregean *Selbständigkeit* is ontological independence

In describing objects as *selbständig*, and denying that concepts and functions are *selbständig*, Frege employs a notion of ontological independence. Frege is not interested in quantifying over mere possibilities and does not employ the broader notion that Dummett calls 'logical independence'. For Frege, *gesättigt* (when adjectival) and *selbständig* are interchangeable terms. In his surviving works, when Frege first introduces the notion of *unsaturatedness*, he explicitly mentions the *existential* dependency of concepts upon judgeable contents:

A concept is unsaturated [*ungesättigt*] in that it requires something to fall under it; hence it cannot exist on its own. [...] I do not believe that concept formation can precede judgement, because this would presuppose the independent existence [*selbständiges Bestehen*] of concepts, but I think of a concept as having arisen by decomposition from a judgeable content.

(Frege, 'Frege an Marty, 29.8.1882', 164 (101); cf. 'Über die Grundlagen der Geometrie', 372)

When the notion of logical dependency as referential dependency is expanded to cover the case of functions as well as objects, Frege would indeed hold that concepts, though not objects, are *logically* dependent because he took it that concepts can only be named in sentences and that concept words can only occur in sentences. Since concept words do not of themselves name objects, concepts can only be referred to when, at the level of the sentence, a truth value is referred to and, at sub-sentential level, either an object or a higher-level function is referred to. When Frege denies the independent existence of concepts (and, elsewhere, when he asserts the independent existence of objects), however, he is working with an ontological

notion of independence. (For a detailed defence of these claims, and further references, see McLeod, 'Frege's Saturated/Unsaturated Distinction'.)

3.2. *Fregean Selbständigkeit is relative*

By 'exist on its own', the early Frege is not alluding to absolute ontological independence, but to independence from judgeable contents. The later Frege is alluding to independence from functions and logical complexes (i.e. entities with a function–argument structure, such as sentences, thoughts and definite descriptions). Moreover, Frege does not deny that some objects depend ontologically upon others. Frege ('Die Verneinung', 155; cf. *Grundgesetze der Arithmetik, Band II*, §150) contrasts the parts of a thought with parts, such as those of an aggregate, that are 'mutually independent'. Frege ('Gedankengefüge', 36) asks 'how a thought comes to be constructed, and how the parts are so combined together that the whole amounts to something more than the parts taken separately'. For Frege, the parts of an aggregate exist independently of the aggregate; the latter, while an object, and *selbständig*, does not exist independently of its parts. (See further McLeod, 'Frege's Saturated/Unsaturated Distinction'.)

In Frege's philosophy, mereological essentialism plausibly applies to logical complexes as well as to aggregates. A logical complex, however, differs from an aggregate in that the proper parts of an aggregate are 'mutually independent', whereas those of a logical complex are not. Mutual dependence is only the proper contrary of mutual independence, not its contradictory. In a personal, social or economic relationship, you and I are mutually dependent if I depend upon you and you depend upon me. So long as one of us depends on the other, however, it remains the case that we are not mutually independent.

If the proper parts of a logical complex are not mutually independent, this entails only that some of them are dependent on others. On a set of proper parts, mutual independence is evidently symmetrical. When Frege denies that the parts of a thought are mutually independent, he means to suggest that functional parts depend existentially on objectual parts. This ontological notion of dependency is the one to which Frege is alluding when he says that the parts of an aggregate are 'mutually independent': that is to say, none depend ontologically on others.

In the case of an aggregate, it is not the case that the whole and its parts are mutually independent: rather, the whole depends upon the proper parts. Nevertheless, the ontological dependency of an aggregate upon its proper parts is *not* an example of the sort of ontological dependency Frege has in mind when he denies that the parts of a logical complex are mutually independent. For Frege considers *both* an aggregate and each of its parts to be *objects* and he holds that *all* objects are *selbständig* (i.e. independent) complete wholes.

Frege (*Die Grundlagen der Arithmetik*, §60) remarks that

[t]he self-subsistence [*Selbständigkeit*] which I am claiming for number is not to be taken to mean that a number word signifies something when removed from the context of a proposition, but only to preclude the use of such words as predicates or attributes.

Numbers are *selbständig* not because each number is ontologically independent of everything that is not a number (even if that is the case) but merely because numbers are not functions. The reason why number words cannot be used predicatively, in Frege's view, is that they refer to entities that are complete wholes: that is, entities that do not have argument places. Since Frege holds that all objects are *selbständig* but *also* that some depend ontologically on others, the Fregean *Selbständigkeit* of an object does not require that the object should exist independently of all other objects (whether different in kind or not).

3.3. Fregean *Selbständigkeit* is neither purported by Frege to be explicable in conceptual/referential terms nor in fact so explicable

Frege himself never explains the dependency of functions, or the independence of objects, in conceptual or referential terms. For Frege, the *Selbständigkeit* of objects and the dependency of functions are written into the nature of things. It is not our inability to conceive of functions without conceiving, thereby, of objects that *ultimately explains* the dependent nature of functions. Rather it is that functions, but not objects, have argument places.

If, as Frege believed, numbers are objects and they are infinite in number, then there are numbers with no names and of which it is impossible for us to conceive. Dummett's attempts to formulate a notion of *Selbständigkeit* that is explicable in terms of conceptual or referential independence cannot therefore encompass the *Selbständigkeit* of all independently existent Fregean objects.

3.4. Typical cases of Fregean *Selbständigkeit*

Prior to Frege's explicit introduction, in 1891, of the distinction between sense and reference ('Frege an Husserl, 24.5.1891', 96; *Funktion und Begriff*, 14), the typical case of Fregean *Selbständigkeit* is the existential independence of objects from judgeable contents. At this time, the typical case of dependency is the existential dependence of functions upon logical complexes (e.g. of concepts upon judgeable contents). After 1891, the typical case of *Selbständigkeit* is the independence of objects from functions. The typical case of dependency is the dependency of functions upon objects. The dependency of one object upon another is *not* a case in which there is a failure of Fregean

Selbständigkeit, for Frege would freely admit, without compromise to his doctrine that all objects are *selbständig* as he understands it, that aggregates are objects that depend upon their proper parts.

4. Senses, expressions and the third realm

This section addresses two questions. First, what are the implications of the different conceptions of *Selbständigkeit* for the question of whether senses reside in the third realm? Second, when Dummett argues that, since senses are senses *of* (i.e. logically dependent upon) expressions, senses do *not* reside in the third realm, is his argument against Frege successful?

We have seen that, unlike Frege, Dummett construes *Selbständigkeit* as an absolute notion of logical independence. In doing so, Dummett goes beyond what is required for his main critical purpose of trying to show that senses are logically dependent upon expressions. According to Dummett, since expressions are not logically independent of our language and of our thinking, neither can be senses. It is implicit in Dummett ('Frege's Myth of the Third Realm') that our language and our thinking *do not* reside in the third realm (perhaps because they are practices). Instead of appeal to an absolutist conception of *Selbständigkeit*, Dummett's argument requires the modest thesis on an entity that does not so reside. Though Dummett identifies Frege's general doctrine that all objects are *selbständig* as a 'mistake' that underlies Frege's 'myth' that senses reside in the third realm, this general doctrine is actually irrelevant to the issue of whether senses reside in the third realm: as Frege propounds the doctrine, it is not an allusion to absolute independence.

Moving to our second question, are Dummett's arguments that senses are senses *of* expressions enough to secure, in turn, his conclusion that Frege's doctrine that senses reside in the third realm is false? In considering this question, it will come to light that it is not only Dummett's conception of *Selbständigkeit* that is non-Fregean: so is his conception of what it is to be an expression.

Dummett ('Frege's Myth of the Third Realm') contains two broad lines of argument for the conclusion that senses are logically dependent upon expressions. First, there is an indirect argument, albeit sketchy and rhetorical, to the effect that various matters become 'mysterious' so long as senses are conceived of, instead, as belonging to the third realm:

As long as this perspective [according to which senses reside in the third realm] is dominant, all is mysterious. There is no way of explaining how thoughts relate to things in other realms of reality, that is, what makes them about anything. There is no way of explaining how we grasp them [...]. Above all, there is no way of explaining how we attach senses to words or expressions, that is, what makes them senses *of* those words and expressions.

(Dummett, 'Frege's Myth of the Third Realm', 251–2)

While it appears to be an exaggeration to say that ‘all is mysterious’ on Frege’s view, our critical focus will be on the last sentence in this quotation, to which Dummett attaches the most significance. This last sentence can figure in a non-circular argument for Dummett’s conclusion only if its ‘of’ is *not* an ‘of’ of logical dependence. The sentence is also ambiguous. It might be alluding to the question of how it is that expressions, in general, can come to have senses. Alternatively, it might be concerned with how it is that particular expressions can come to have the senses that they actually have. Whichever way we construe it, and supposing its coherence, the question of how expressions come to have senses cannot, despite the impression given by Dummett’s wording, be the same question as that of how senses are logically dependent on expressions, for the second question presupposes the dependency that is at issue, while the first does not. So it appears that Dummett’s indirect argument either begs the question or brings an equivocation over ‘of’ into play.

Dummett’s second, direct, argument for his conclusion that senses are logically dependent on expressions is as follows:

If the sense is the route to the referent, or the way the referent is given, the general notion of sense cannot be explained except by appeal to that of reference; we must therefore have the latter notion first. If we have the notion of reference in advance of that of sense, we cannot be conceiving of the possession of a reference as a property of the sense, but only of the expression: it is to the expression that reference is therefore primarily to be ascribed, even though it has the specific reference that it has in virtue of its sense. On this conception, then, it is an expression which primarily has a reference: in particular, utterances of sentences are the primary bearers of truth and falsity. Since a sense is a way of referring to something, and it is the expression which refers, senses are intrinsically the senses *of* expressions.

(Dummett, ‘Frege’s Myth of the Third Realm’, 255)

We may reconstruct this argument as follows, with the hidden premises being rendered explicit.

Dummett’s Direct Argument for the Dependency of Senses on Expressions

1. A sense is a way that a referent is given. (Premise)
2. If a sense is a way a referent is given, the general notion of sense cannot be explained except by appeal to that of reference. (Premise)
3. If the general notion of sense cannot be explained except by appeal to that of reference then ‘we must have the latter notion first’: i.e. the notion of reference is conceptually prior to that of sense. (Premise)

4. The possession of a reference is a property either of senses or of expressions. (Premise)
5. If the notion of reference is prior to that of sense, we cannot conceive of the possession of a reference as a property of the sense. (Premise)
6. If we cannot conceive of the possession of a reference as a property of the sense, then it is not a property of the sense. (Premise)
7. If a sense is a way that a referent is given and it is the expression which refers, then senses are intrinsically *of* (i.e. logically dependent on) expressions. (Premise)
8. The notion of reference is prior to that of sense. (From 1, 2, 3)
9. We cannot conceive of the possession of a reference as a property of the sense. (From 5, 8)
10. The possession of a reference is not a property of the sense. (From 6, 9)
11. The possession of a reference is a property of the expression. (From 4, 10)
12. A sense is a way that a referent is given and it is the expression which refers. (From 1, 11)
13. Senses are intrinsically *of* (i.e. logically dependent on) expressions. (From 7, 12)

In relation to Premise 1, and setting aside the obvious problem of empty names, it is perhaps worth noting that Frege ('Über Sinn und Bedeutung', 26–7) does not say that senses *are* modes of presentation of referents: rather, he says that a sense *contains* a mode of presentation of a referent. While what Frege says here is obscure, the difference is relevant to Dummett's initial setting-up of a supposed tension between the conception of senses *as* modes of presentation and the conception of senses as *selbständig*. For it is only if a sense is one-and-the-same as a mode that this alleged tension can readily appear plausible.

The dubiousness of Premise 3 can be seen by analogy with the cases of the notions of necessity and possibility. If the general notion of necessity cannot be explained except by appeal to that of possibility then it does not follow that we must have the latter notion first: for both notions may be primitive, with each being explicable only by appeal to the other. If one notion cannot be grasped without grasp of another, then this does not entail the conceptual priority of the latter notion upon the former. The premise can be amended, as follows, to fix this shortcoming.

- 3.* If the general notion of sense cannot be explained except by appeal to that of reference *and the general notion of reference can be explained without appeal to that of sense* then 'we must have the latter notion first': i.e. the notion of reference is conceptually prior to that of sense.

If Dummett's claim that reference is prior to sense is to be well-founded, he must show that an agent can grasp the general notion of reference without grasp of the general notion of sense. His attempt to do this, which appears before the direct argument, is as follows.

In [...] part I of *Grundgesetze der Arithmetik*, the notion of reference appears as prior to that of sense. The primitive symbols are explained by stipulating what their references are to be. These serve jointly to determine the conditions under which the referent of any formula of the system is the value *true*. The formula will then express the thought that these conditions are fulfilled: and the sense of each part of the formula consists in its contribution to the thought so expressed, that is, in its role in determining the condition for the reference of the formula to be *true* [...] The original stipulations did not mention sense: they did not lay down the *senses* of the primitive symbols, but their referents. [...] it is because we can understand what is involved in assigning a reference to an expression in advance of our having the general notion of sense that we could understand the stipulations.

(Dummett, 'Frege's Myth of the Third Realm', 254–5;
see also, Frege, *Grundgesetze der Arithmetik, Band I*, 50–1)

Even if Dummett's argument here is successful, a problem remains. It concerns Premise 7 and an underlying issue relating to it (and to Dummett's discussion more generally). Premise 7 was included in the reconstruction of the direct argument as a way of rendering formally valid what would otherwise appear to be a formally invalid inference: 'a sense is a way of referring to something, and it is the expression which refers, [so] senses are intrinsically the senses of expressions' ('Frege's Myth of the Third Realm', 255). The truth of Premise 7's antecedent would appear to be compatible with the falsehood of its consequent. There is no contradiction in holding (again leaving the senses of empty names aside) that senses are ways that referents are given, that it is expressions, rather than senses, that refer *and* that it is not the case that senses are dependent on expressions. Someone who holds that an expression is an expression of a sense (rather than being, for example, a mere symbol string) is not debarred from accepting that senses are ways that referents are given, and that it is expressions, rather than senses, that refer.

When Dummett ('Frege's Myth of the Third Realm', 252) raises the question of 'how we attach senses to words or expressions' he gives the impression that, for Frege, senses and expressions are mutually independent. It is because Dummett denies this, taking senses to depend upon expressions, that Dummett thinks he either has an answer to, or avoids, the question,

which is one that he thinks Frege is unable to answer. Dummett's view is that senses are not detachable from expressions. This is not to suggest that, for any expression, it essentially has the sense that it has. Rather, Dummett thinks that just as the moves that are associated with a chess piece can change, and it remains numerically the same piece, so the sense that is attached to an expression can change, and it remains numerically the same expression. Dummett's presupposition here is that expressions are to be individuated syntactically, rather than semantically.

Dummett's account of what it is to be an expression and Frege's are significantly different. For Frege, while all expressions are names, only some expressions can have names and only some senses are nameable: object names are nameable, but function names are not. (For a defence of this controversial exegetical claim, see McLeod, 'Frege's Saturated/Unsaturated Distinction'.) Moreover, Frege has no conception of what an expression is on which an expression is nameable in detachment from its sense. Frege ('Über Sinn und Bedeutung', 26) distinguishes between signs syntactically individuated (in the case of written language, as mere shapes or strings of them) and signs semantically individuated (i.e. individuated as designators). Angelelli (*Studies on Gottlob Frege and Traditional Philosophy*, 44) calls signs individuated as mere shapes 'figures', reserving 'sign' for usage as a synonym of 'expression' (in Frege's sense). Sundholm ('What is an Expression?', 181–2) makes a distinction, similar to Angellelli's figure/sign distinction, between expressions that can only be spoken about and those that can be used to say something. For Frege, an expression is never what Sundholm ('What is an Expression?', 184) describes as 'an element of a freely generated algebra of strings', which is 'an object *without meaning*' (original emphasis). Rather, a Fregean expression is essentially meaningful and can therefore directly feature in an assertion (rather than it just being the case that the expression's *name* can so feature). Frege would agree with Sundholm ('What is an Expression?', 185) that 'whatever an expression may be, it must express, or have, content'. (For references to Frege on this point, see Sundholm ['What is an Expression?', 187]).

Dummett ('Frege's Myth of the Third Realm', 260) appears to take his claims that 'senses are *of* expressions' and expressions are 'logically prior' to senses to be alternative ways of saying the same thing. Since (in the case of proper names and their senses), expressions and senses are objects, senses are not Dummett-*selbständig*. Now since logical dependence is an asymmetrical matter, it can only be the case that 'senses are *of* expressions', in Dummett's sense, if it is not the case that expressions are (in the same sense of '*of*') *of* senses.

Given the asymmetry of logical dependence, Dummett's contention that senses are logically dependent upon expressions is not consistent with

Frege's account, on which to conceive of something as an expression is to conceive of it as having a sense.

- (1) *F*s are logically dependent on *G*s if and only if: (i) it is logically impossible to conceive that there is an object, *x*, such that *Fx*, without conceiving, thereby, that there is something, *y*, distinct (both individually and in kind) from *x*, such that *Gy*; and (ii) it is logically possible to conceive that there is an object, *x*, such that *Gx*, without conceiving, thereby, that there is something, *y*, distinct (both individually and in kind) from *x*, such that *Fy*. *(Logical dependence as conceptual dependence, from Section 2 above.)*
- (2) Senses are logically dependent on expressions. *(Dummett's view)*
- (3) To conceive that there is an expression is to conceive that there is some sense that the expression expresses. *(Frege's view)*

(1) and (2) together entail the falsehood of (3), while (1) and (3) together entail the falsehood of (2).

In Dummett's arguments for his view that senses are *of* (i.e. logically dependent on) expressions, and not the other way round, Dummett employs a conception of what it is to be an *expression* that is non-Fregean and which, when taken together with a relativized version of Dummett-*Selbständigkeit*, would appear to beg the question against Frege's view that expressions are expressions of senses. Frege's view that expressions are expressions of senses does not, of itself, presuppose that senses reside in the third realm; it merely embodies a semantic conception of what it is to be an expression. Whether or not we share Dummett's distaste for the doctrine that senses reside in the third realm, Dummett's arguments for the logical dependency of senses upon expressions provide insufficient reason to do so.

5. Conclusion

As we have seen, Frege by no means denies that some objects depend on others. Frege rightly takes his doctrine that all objects are *selbständig* to be consistent with the contention, which he also accepts, that some objects (e.g. aggregates) are ontologically dependent upon others. Frege's doctrine that objects are *selbständig* does not admit of the construal that Dummett gives it, on which no object depends logically on any other object. Rather, Frege's doctrine simply means that objects are, in his technical sense, and as opposed to functions, complete wholes. Frege's claim that all objects are *selbständig* does not, on Frege's conception of *Selbständigkeit*, entail that for every object it is either ontologically or logically independent of all others.

Whatever the inadequacies of Frege's conception of sense and of his commitment to the third realm, they are by no means traceable to Frege's view that all objects are *selbständig*.

When Dummett accuses Frege of the 'mistake' of having regarded all objects as *selbständig*, Dummett identifies no difficulty with Frege's own thesis that all objects are *selbständig*. Instead, Dummett supplants Frege's conception of *Selbständigkeit* with Dummett's own. There is no evidence that Frege ever held that all objects were logically independent in Dummett's sense.

Concerning the supposed logical dependency upon expressions of those senses that are objects, Dummett's thesis is inconsistent with Frege's conception of what it is to be an expression. Frege would not have been able to retain the notion of sense but relinquish commitment to the third realm by embracing Dummett's view about the logical dependency of sense without also abandoning his conception, in itself neutral about whether senses inhabit the third realm, of what it is to be an expression.

When *Selbständigkeit* is understood as Dummett understands it, Frege would agree with Dummett that some objects depend upon others. On Frege's conception of *Selbständigkeit*, there is no conflict between the conception of senses as (or, to the letter of his formulation, as containing) modes and the conception of senses as *selbständig*.

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