Under consideration for publication in Review of Symbolic Logic

On Truth-Functionality

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Abstract

Benjamin Schnieder has argued that several traditional definitions of truth-functionality fail to capture a central intuition informal characterizations of the notion often capture. The intuition is that the truth-value of a sentence that employs a truth-functional operator depends upon the truth-values of the sentences upon which the operator operates. Schnieder proposes an alternative definition of truth-functionality that is designed to accommodate this intuition. We argue that one traditional definition of 'truth-functionality' is immune from the counter-examples that Schnieder proposes and is preferable to Schnieder's alternative.

1 Schnieder on 'standard' accounts of truth-functionality.

Quine (1982: 8), quoted by Schnieder (2008: 64), characterizes truth-functionality as follows: 'a way of forming compound statements from component statements is truth-functional if the compounds thus formed always have matching truth-value as long as their components have matching truth-value'.

Schnieder (2008: 65) writes that on 'Quine's characterisation ... an operator ζ [is] truth-functional iff ... the truth-value of a complex sentence formed by combining ζ with the appropriate number of sentences is the value of a function of the truth-values of those sentences'.

According to Schnieder (2008: 65), the trouble with this '*simple proposal*', is that it makes 'the truth-functionality of some operators dependent upon arbitrary contingent facts'. Thus, assuming that 'Jeanne d'Arc never uttered any English sentence' (2008: 66), the following counts as a truth-functional operator:

Δ_1 If Jeanne d'Arc ever said in English that ..., then it is true that ...

Schnieder (2008: 66, 67, 70) provides the following further examples of sentential operators that end up counting, by the lights of Quine's simple proposal and some formal definitions developed from it, as truth-functional, but that do not count as truth-functional intuitively.

Δ_2 If France won the World Cup in 1978, then . . .[1]

Δ_3 It is a proposition that . . .

Δ_4 It is expressible in English that . . .

Δ_5 If Jeanne d'Arc ever *actually* said in English that . . . , then it is true that . . .

2 Schnieder's alternative account.

Schnieder (2008: 67) points to some alleged contrasts between the cases of Δ_3 and $\top$, which he specifies as follows:

$\top$ It is either true or it is false that . . .[2]

He contends that these operators have different meanings. He also takes it that both operators correspond 'to the constant truth-function that maps both [input] values to *T*' but that it is 'only in the case of $\top$' that the truth-value of the sentence formed using the operator 'is really *determined* by' the input truth-value.

The alternative account of truth-functionality provided by Schnieder (2008: 69) is to the effect that an *n*-place sentential operator ζ is truth-functional if and only if, for all contexts of evaluation, the truth-value of the sentence *S* that results from concatenating ζ with *n* sentences is a truth-value possessed by *S* '*because of* the truth-values' that the input sentences have in that context. The use of the word 'because' is understood to involve an explanatory, non-evidential but objective notion (2008: 68).

Schnieder (2008: 69–70) applies this definition of truth-functionality to the cases of $\top$ and Δ_3, using the following sentences as examples.

(1) It is a proposition that snow is white.

(2) It is either true or it is false that snow is white.

Schnieder (2008: 69) claims that (2) is true *because* 'snow is white' is true. In a counterfactual situation in which snow is not white, (2) is then true because 'snow is white' is false in that situation. So, in both the actual and the counterfactual scenario, (2) has its truth-value because of the truth-value of 'snow is white'. Now Schnieder (2008: 70) claims a contrast with (1). In the actual scenario, (1) is true. Had the counterfactual scenario

[1] Schnieder (2008: 6–7) discusses how, on one construal of the remark of Edgington (1995: 241) that it is a requirement on truth-functionality that 'in any possible circumstance' the truth-value of the output sentence is 'fixed by the truth value(s)' of the input sentence(s), Δ_2 is debarrred from being a truth-functional operator. Even on that construal, however, the examples we list beneath Δ_2 are not debarred.

[2] We comment later on this specification of $\top$.

obtained, however, '(1) would still have been true but *not because* "snow is white" would have been false'. So, claims Schnieder, his definition by appeal to *because* 'classifies $\top$ as truth-functional but not Δ_3'.

3 A classical-logical alternative

We agree with Schnieder that Δ_3 and $\top$ have different meanings. In this section, we argue that, by the lights of classical logic, Δ_3 is neither a truth-functional operator nor a threat to a proper *classical-logical* understanding of truth-functionality. Schnieder's alternative account appeals to the intensional notion of explanation that many classical logicians will eschew. It is a strategic advantage in logical theory, therefore, if truth-functionality can be characterized in a manner that appeals to notions that are less disputed, and yet does not count as truth-functional operators that ought not to be treated as truth-functional. In that category, we include Schnieder's Δ_1, Δ_2, Δ_3, Δ_4, and Δ_5. Schnieder's appeal was to informal characterizations as capturing something intuitively important about truth-functionality, namely *dependency* of output truth-value on input truth-value(s). He then proceeded to allege that the 'standard proposal' of Quine (and other proposals developed from it) did not capture these intuitions, delivering the wrong results about what is a truth-functional operator in some cases.

We contend that another, fairly standard, way of defining 'truth-functionality' avoids Schnieder's counter-examples and appeals to notions that are less disputed than those that feature in his own account. On our account, a sentential operator (which we here take, with Schnieder,[3] to be a linguistic item) is truth-functional if and only if its meaning is exhaustively given by its truth-table.[4] Note that we do not say '*exclusively* given by its truth-table'; we are quite open to the operator's having other, equivalent, definitions that do not mention the truth-table as such. By 'truth-table' we simply mean the associated function that takes input truth-values to an output truth-value, or the familiar graphical representation of this function. For our purposes it does not really matter which is meant; the important point is that this fully specifies or exhausts the meaning of the operator. At this point it may be objected that 'meaning' is also an intensional term, like Schnieder's 'because'. This is true, but we maintain that the classical logician is already committed to the notion that operators have meaning; we do not share Quine's scepticism on this front. We do not think, however, that the classical logician is already committed to relations of explanation (as expressed in Schnieder's 'because') between the truth-values of his sentences.

The standard zero-place operators $\top$ and $\perp$ are truth-functional. For any n, where n is a natural number, there is an n-place truth-functional operator that always returns the value T on the input sentence(s) and an n-place truth-functional operator that always returns the value F on the input sentence(s).

[3] Schnieder writes 'An n-ary sentential operator or connective is any expression whose combination with n arbitrary sentences yields a complex sentence' (2008: 64).

[4] Our view compares to that of Forbes (1993: 46): 'Any sentential connective whose meaning can be captured in a truth-table is . . . a *truth-functional* connective'. Compare also Tomassi (1999: 127): 'any connective which can be completely defined by a truth-table' is truth-functional.

If Schnieder takes the meaning of $\top$ to involve semantic predicates, as his natural-language specification of it suggests, then we take it to have a meaning different from the one Schnieder gives it. We take it that its meaning is fully specified by its truth-table: no appeal to semantic predicates is required. $\top$ is the operator whose meaning is exhausted by the fact that it always returns the value 'T' (or '1').[5]

The zero-place operators can be treated as variable-place operators such that for any *n* they return the same value as in their zero-place incarnations. Alternatively, we can regard $\top_0, \top_1, \top_2, \ldots, \top_n$ and $\bot_0, \bot_1, \bot_2, \ldots, \bot_n$ as two families of operators, where the subscripts indicate the number of inputs to the function.

The examples Schnieder uses to try to show standard accounts to be too liberal with respect to our intuitions about the dependency of output value on input values are all examples that return either *true* for all input values or *false* for all input values. All the examples face the following dilemma. Let ζ be any such example. Either the meaning of ζ is exhausted by its truth-table or the meaning of ζ is not exhausted by its truth-table. If the meaning of ζ is not exhausted by its truth-table, then ζ is *not*, by our lights, a truth-functional operator (even if ζ *is* an extensional operator). If the meaning of ζ *is* exhausted by its truth-table then ζ is merely a notational variant of $\top$ or $\bot$, where these are construed as variable-place operators, or of one of the subscripted versions of these operators, where the subscript corresponds to the number of input places. (The dilemma can obviously be generalized for examples that do not return either *true* for all input values or *false* for all input values.)

Consider Δ_2, Δ_3, Δ_4, Δ_5, and Schnieder's version of $\top$. Either a sentence employing such an operator has proper semantic content that is contained neither in the input sentence nor in the truth-table for the operator, or it does not. If it does then the operator is not truth-functional. If it does not then the operator *is* truth-functional, expressing just the same truth-function as $\top_1$.

The first horn of this dilemma can also be drawn by reference to the concept of expressive adequacy for a language of sentential logic. A set of sentential operators, Γ, is expressively adequate if and only if every truth-function can be expressed using the elements of Γ. In standard sentential logic, the following are among the expressively adequate sets: $\{\neg, \wedge\}, \{\neg, \vee\}, \{\neg, \rightarrow\}$.

Suppose that Δ_3 always returns the value *true*, in virtue of its truth-table semantics alone. It is then, on our account, the same operator as $\top_1$ and is expressible using the elements of any expressively adequate set of sentential operators. If it is taken that Δ_3 always returns the value *true* but denied that it is the same operator as $\top_1$, then something over and above the truth-table semantics for these operators must underwrite that judgment. Putting the point another way, some meaning other than that captured by an expressively adequate set of operators is being imported.

Our definition admits as truth-functional the Peirce arrow and the Sheffer stroke, but does not so admit any of Schnieder's purported counter-examples, if taken in the natural way as having a meaning over and above the truth-table. Δ_1 and Δ_5 contain embedded

[5] Any two distinct values can be used for truth-table construction in a bivalent logic and any *n* distinct values can be used in an *n*-valued logic.

sentences dealing with Jeanne D'Arc; they thus are naturally taken as having a meaning over and above the truth-table. Δ_2 also contains an embedded sentence, dealing with France and the World Cup, so it too is naturally taken as having a meaning over and above the truth-table.

Δ_3 is not a truth-functional operator because the expression 'proposition' imports meaning over and above the simple correlation of truth-values encoded in the truth-table.

In Δ_4 the word 'expressible' is modal. Thus, 'It is expressible in English that ...' contains, in its logical syntax, a modal operator. Since the semantics for modal operators is not captured by truth-tables, Δ_4 breaches our definition. (Δ_4 also fails to meet our definition since it employs the notions of expressibility and of the English language, bringing it about that its meaning fails to consist just in the information in the truth-table.) The modality might be expunged by changing the example to 'It will be expressed in a moment in English that ...' but this just shifts the problem. In the new example, we have either a temporal operator or a quantifier over moments featuring in the logical syntax. Since the semantics of neither temporal operators nor quantifiers is specifiable via truth-tables, we again have a breach of our definition. An objection here might be that there are no non-tensed sentences, so our definition, or at least our current application of it, precludes the existence of truth-functional operators. The objection, however, is mistaken. When specifying the classical-logical sentential operators in natural language, the classical logician should not employ semantic predicates. Thus, disjunction is not specified as 'Either it is true that ... or it is true that _ _ _' but as 'Either ... or _ _ _'. Were the semantic predicates employed, the classical logician would be breaching our definition. The case of negation, however, is different. Here, the classical logician typically specifies negation as 'It is not the case that ...', in which we do have a verb, and which the objector will contend is a tensed one. One answer to this objection appeals to the fact that 'not' is not a verb. In natural language, negation is an operation often performed by qualifying a verb with an adverb, such as the word 'not'. The word 'not', or whatever is doing the negating in a sentence, does not do any temporal work: that is done by the verbs. Classical propositional logic does not recognize a distinction between internal and external negation, and classical quantificational logic does not recognize a very strong distinction between internal and external negation. In both, negation is a sentential modifier (rather than a verb-phrase modifier). Since the word 'not' is not a verb, one way in which we may circumvent the objection to our account is by specifying negation as follows:

$\neg$: Not ...

4 Conclusion

Although Schnieder may be right that some accounts of truth-functionality, e.g. Quine's, do not accommodate all our intuitions about truth-functionality, we have contended that the traditional account of a truth-functional operator as one whose meaning is entirely specified by the truth-table is immune to Schnieder's objections and superior to his own suggestion. Its superiority lies in the fact that the classical logician is (or should be) already committed to the notion that the truth-functional operators have a meaning, and is not (or should not be) committed to the notion that explanatory relations hold between the truth-values of the sentences of first-order logic. Those logicians that discount the notion of meaning will probably be happy to continue to use Quine's definition and bite the bullet concerning Schnieder's counter-examples, since all they will be concerned with will be the input and output truth-values; the fact that the output truth-value might depend on contingent facts concerning Jeanne d'Arc will not bother them.

The reader may be worried about the traditional English expressions of truth-functions 'if', 'and', 'or' etc. It is true that sometimes these are used in normal English to mean more than is encapsulated in their associated truth-tables, e.g. 'if' is sometimes used to indicate a causal connection of some kind between antecedent and consequent, and 'and' is sometimes used to indicate that the event described in the first conjunct predated that described in the second conjunct. If the English expressions are used that way, then they are not being used as truth-functional operators. On the other hand, sometimes we do use 'if', 'and', 'or' etc. to do no more than to express the logical relationships encoded in the truth-table. In this case they *are* being used as truth-functional operators.[6]

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[6] We thank for their comments Dorothy Edgington, Benjamin Schnieder, Timothy Williamson, and an anonymous referee for *Review of Symbolic Logic*.