# - Superstring Phenomenology -
# A Personal Perspective[1]

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**Abstract.** In the first part of this paper I review the construction of the realistic free fermionic models, as well as current attempts to study aspects of these models in the nonperturbative framework of M– and F–theories. I discuss the recent demonstration of a Minimal Superstring Standard Model, which contains in the observable sector, below the string scale, solely the MSSM charged spectrum, and provides further support to the assertion that the true string vacuum is connected to the $Z_2 \times Z_2$ orbifold in the vicinity of the free fermionic point in the Narain moduli space. In the second part I review the recent formulation of quantum mechanics from an equivalence postulate, which offers a new perspective on the synthesis of gravity and quantum mechanics, and contemplate possible relations with string theory and beyond.

## 1. Introduction

Superstring phenomenology aims at achieving two goals. The first task is to reproduce the phenomenological data provided by the Standard Particle Model. The subsequent goal is to extract possible experimental signatures

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which may provide further evidence for the validity of specific string models, in particular, and for string theory, in general. One should however note that, in the lack of substantial experimental evidence for any extension of the Standard Model, the conservative approach would be to derive solely the Standard Model, which we may assume to include non–vanishing neutrino masses. Experimental signatures beyond the Standard Model become firm theoretical predictions once the first goal is achieved and it appears that something extra *unavoidably* remains.

Despite its experimental success, the Standard Model leaves much to be desired. In the first place the Standard Model is made of several disparate sectors. These include the matter, the interaction, and the Higgs, sectors. The Higgs sector is still unobserved experimentally and the least understood. The matter and interaction sectors are made of similar but distinct elements, like the different gauge groups of each interaction and the multiplicity of generations, which are parametrized by various parameters. This enumeration is clearly unappealing and it is reasonable to seek a more economical description. The most important guide in this quest is the multiplet structure of the Standard Model, which is exhibited below,

$$SU(3)_C \times SU(2)_L \times U(1)_Y \; \longrightarrow \; SU(5) \,, \qquad SO(10) \,, \qquad E_6$$

$$
\begin{aligned}
Q \;\;&:\; ( \; 3 \;\;, \;\; 2 \;\;, \;\; \tfrac{1}{6} \; ) \\
U^c_L \;\;&:\; ( \; \bar{3} \;\;, \;\; 1 \;\;, \;\; -\tfrac{2}{3} \; ) \qquad \rightarrow \qquad 10 \\
E^c_L \;\;&:\; ( \; 1 \;\;, \;\; 1 \;\;, \;\; +1 \; ) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \rightarrow \quad 16 \quad \rightarrow \quad 27 \\
L \;\;&:\; ( \; 1 \;\;, \;\; 2 \;\;, \;\; -\tfrac{1}{2} \; ) \\
D^c_L \;\;&:\; ( \; \bar{3} \;\;, \;\; 1 \;\;, \;\; \tfrac{1}{3} \; ) \qquad \rightarrow \qquad \bar{5} \\[1em]
N^c_L \;\;&:\; ( \; 1 \;\;, \;\; 1 \;\;, \;\; 0 \; ) \qquad\qquad\qquad\quad +\;\; 1
\end{aligned}
$$

The matter and gauge multiplets of the Standard Model amazingly fit into representations of larger unifying gauge groups [1]. Most appealing is the framework of $SO(10)$, in which all the Standard Model states (including the right–handed neutrinos which are desirable for neutrino masses and oscillations), in each generation, are embedded in a single representation. A priori there was no reason for this to have been the case. But strikingly all three generations fit, each, into a fundamental representation of $SO(10)$. If we regard (as we should) the quantum numbers of the Standard Model states as experimental observables, then this scheme correlates 18 observable parameters. It seems to me therefore that to deny the evidence for the underlying $SO(10)$ structure of the Standard Model is synonymous to dismissing the Standard Model itself.

An important experimental fact is the unobservation of proton decay. In the Standard Model the proton decay is forbidden by renormalizability and accidental global symmetries. In general, extensions of the Standard Model

produce proton decay mediating operators. In non–SUSY GUTs proton decay is mediated by dimension six operators. In supersymmetric theories dimension four and five operators are also generically allowed. Proton decay becomes an especially acute problem when gravity is unified with the gauge interactions because in that case renormalizability and global symmetries are not expected to be respected. In this context therefore it is expected that proton stability can only be maintained if there exist a gauge symmetry, which after its breaking still leaves a residual discrete symmetry, which forbids proton decay [2]. This is very hard and nontrivial to achieve. The evidence for unification, provided by the Standard Model multiplet structure, together with proton longevity, indicate that the Standard Model cannot be strongly perturbed up to a very large scale. It is difficult to envision how a strong perturbation of the Standard Model at a low scale will not run into conflict with the proton lifetime. These experimental facts therefore indicate the big desert scenario.

Unification and the big desert scenario is then supported by another observation. Namely, if one extrapolates the Standard Model gauge couplings, they are seen to converge at a high scale, which is one or two orders of magnitude below the heterotic string scale [3]. This picture is especially appealing in the case of supersymmetric theories, where the couplings are seen to meet at a scale which is of the order of $10^{16}$GeV [4]. This extrapolation should be taken as qualitative support for the consistency of the big desert scenario. The appealing feature of supersymmetric theories is the fact that when the symmetry is local it necessitates the appearance of a spin two field. We then see that the gauge and gravitational interactions start to converge into a unifying setting. Furthermore, this setting also provides the means to understand how a very small scale such as the electroweak scale can be generated by extrapolation from the Planck scale.

Despite their enormous success point quantum field theories still leave many questions unresolved. Why is a particular gauge group observed at low energies, together with the multiplicity of generations ? The proliferation of Standard Model parameters, in particular in the flavor sector, and the hierarchy between them does not have its origin in GUTs or SUSY GUTs. To understand these issues we must incorporate gravity into the picture. Most importantly, point quantum field theories do not provide for a consistent formulation of quantum gravity. A consistent formulation of quantum gravity requires new conceptual framework and tools. Such a framework will then also shed light on the structure of the Standard Model.

String theory provides a consistent perturbative formulation of quantum gravity. String theory is unique in the sense that it is the only approach to date which gives a consistent common framework for both gravity and the gauge interactions. As such string theory exactly suits our purpose, *i.e.* it provides the tools to study how the Standard Model structure and

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parameters may arise from a theory of quantum gravity.

String theory is defined in perturbation theory. As such it is clear that string theory cannot be the final story. Indeed, over the last few years an important new understanding has emerged in which it is seen that all the different string theories in ten dimensions are in fact perturbative limits of a single theory. This is a very encouraging picture because it tells us that by utilizing string perturbation theory we are truly probing the underlying nonperturbative theory. Now suppose that the situation was reversed and we first had in our hands the full nonperturbative formulation. It is likely that in that case what we would have done in order to study its connection with the real world is to develop perturbation theory in the vicinity of its most relevant limits.

## 2. Superstring constructions

There are two complementary approaches to superstring phenomenology. In one, the general strategy is to first try to understand what is the nonperturbative formulation of string theory. The hope is that the unique string vacuum will be fixed and the low energy predictions unambiguously determined. The second, asserts that we must use low energy data to single out phenomenologically interesting superstring vacua. Such string models will then be instrumental to understand the dynamics which select the string vacuum. These two approaches are in a sense complementary and progress is likely to be made by pursuing both approaches in parallel.

The general goal is therefore to construct superstring models that are as realistic as possible. A realistic model of unification must satisfy a large number of constraints,

1. Gauge group $\longrightarrow SU(3){\times}SU(2){\times}U(1)_Y$ $\qquad$ $U(1) \in SO(10)$
2. Contains three generations
3. Proton stable $\qquad (\tau_\mathrm{P} > 10^{30+}$ years$)$
4. N=1 supersymmetry $\qquad$ (or N=0)
5. Contains Higgs doublets $\oplus$ potentially realistic Yukawa couplings
6. Agreement with $\underline{\sin^2 \theta_W}$ and $\underline{\alpha_s}$ at $M_Z$ (+ other observables).
7. Light left–handed neutrinos
8. $SU(2) \times U(1)$ breaking
9. SUSY breaking
10. No flavor changing neutral currents
11. No strong CP violation
12. Exist family mixing and weak CP violation

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13. +  ...
14. +  **No free exotics**

It is important to emphasize that the $SO(10)$ structure, advocated above, need not be realized in an effective field theory but can be broken directly at the string level. In which case the Standard Model spectrum still arises from $SO(10)$ representations, but the $SO(10)$ non–Abelian spectrum, beyond the Standard Model, is projected out by the GSO projections. However, if we take the Standard Model $SO(10)$ embedding as a necessary requirement this means that the weak hypercharge must have the standard $SO(10)$ embedding with $k_Y = 5/3$. This requirement then already excludes many of the semi-realistic models, which have been constructed to date. Similarly, the requirement that no free exotic particles with fractional electric charge remain in the massless spectrum imposes a highly non–trivial constraint on otherwise valid models. The phenomenological constraints impose very restrictive constraints on the superstring constructions. This is augmented by the fact that, unlike in field theory model building, in string model building both the entire spectrum and symmetries are fixed in a given vacuum. One does not have the freedom to add an additional $U(1)$ or discrete symmetry, or additional matter, to suit one needs. Therefore, string model building is more restrictive than field theory model building. A string model that can satisfy all of the above requirements is likely to be more than an accident.

There are several possible ways to try to construct realistic superstring models. One possibility is to construct superstring models with an intermediate GUT, or semi–GUT gauge group, like $SU(5)$, $SO(10)$, $E_6$, etc [5], or $SU(3)^3$ [6], $SU(5) \times U(1)$ [7] or $SO(6) \times SO(4)$ [8], which are broken to the Standard Model gauge group at an intermediate energy scale. The other possibility is to construct superstring models in which the non–Abelian factors of the Standard–Model gauge group are obtained directly at the string level [9, 10, 11, 12, 13]. The advantage in the second case, as well as in the Pati–Salam type models, is that in these cases the color Higgs triplets, which mediate proton decay through dimension five operators, can be projected out by the GSO projections. Such models then provide a superstring solution to the GUT hierarchy problem [14].

With the advent of superstring duality arguments we can use the different perturbative string limits to try to construct realistic string models. These are all supposedly connected by duality relations and a model in one limit should have a dual model in another limit. Interesting alternatives to the heterotic string are the type I constructions, which allow the unification scale to be lowered as the gauge and gravity multiplets in this case do not arise from the same sector. However, the heterotic string framework still remains the most appealing as it is the only one which naturally gives rise to $SO(10)$ multiplets in the 16 representation.

The construction of realistic superstring vacua proceeds by studying compactification of the heterotic string from ten to four dimensions. Various methods can be used for this purpose which include geometric and algebraic tools, and each has its own advantages and disadvantages. One class of models utilizes compactifications on Calabi–Yau 3–folds that give rise to an $E_6$ observable gauge group, which is broken further by Wilson lines to $SU(3)^3$ [6]. This type of geometrical compactifications correspond at special points to conformal theories which have $(2,2)$ world–sheet supersymmetry. Similar compactifications which have only $(2,0)$ world–sheet supersymmetry have also been studied and can lead to compactifications with $SO(10)$ and $SU(5)$ observable gauge groups [15]. The analysis of this type of compactification is complicated due to the fact that they do not correspond to free world–sheet theories. Therefore, it is difficult to calculate the parameters of the Standard Model in these constructions. On the other hand they provide a sophisticated mathematical window to the underlying geometry.

The next class of superstring vacua are the orbifold models [16]. Here one starts with a compactification of the heterotic string on a flat torus, using the Narain prescription [17], and utilizes free world–sheet bosons. The Narain lattice is moded out by some discrete symmetries which are the orbifold twisting. An important class of models of this type are the $Z_3$ orbifold models [9]. These give rise to three generation models with $SU(3) \times SU(2) \times U(1)^n$ gauge group. A deficiency of this class of models is that they do not give rise to the standard $SO(10)$ embedding of the Standard Model spectrum. Consequently, the normalization of $U(1)_Y$, relative to the non–Abelian currents, is larger than $5/3$, the standard $SO(10)$ normalization. This results generically in disagreement with the observed low energy values for $\sin^2 \theta_W(M_Z)$ and $\alpha_s(M_Z)$.

A special type of string compactifications that has been studied in detail are the free fermionic models. The simplest examples correspond to $Z_2 \times Z_2$ orbifolds at special points in the compactification space. These models give rise to the most realistic superstring models constructed to date. They produce three generation models with the standard $SO(10)$ embedding of the Standard Model spectrum. Hence in these models $U(1)_Y$ has the standard $SO(10)$ embedding, with $k_Y = 5/3$. Consequently these models can be in agreement with the observed low energy values for $\sin^2 \theta_W(M)$ and $\alpha_s(M_Z)$. There are several key features of these models which suggest that the true string vacuum is in the vicinity of these models. First is the fact that the free fermionic models are formulated at a highly symmetric point in the string compactification space. The second is that the emergence of three generations is correlated with the underlying structure of the $Z_2 \times Z_2$ orbifold compactification [18]. Each of the Standard Model generations is obtained from one of the twisted sectors and carries horizontal charges un-

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der one of the orthogonal planes of the $Z_2 \times Z_2$ orbifold. These models then give a reason for the existence of three generation in nature, as originating from the structure of the underlying geometry.

## 3. Free fermionic models

A model in the free fermionic formulation [19] is defined by a set of boundary condition basis vectors, and one–loop GSO phases, which are constrained by the string consistency requirements, and completely determine the vacuum structure of the models. The physical spectrum is obtained by applying the generalized GSO projections. The Yukawa couplings and higher order nonrenormalizable terms in the superpotential are obtained by calculating correlators between vertex operators [20]. The realistic free fermionic models produce an "anomalous" $U(1)$ symmetry, which generates a Fayet–Iliopoulos D–term [22], and breaks supersymmetry at the Planck scale. Supersymmetry is restored by assigning non vanishing VEVs to a set of Standard Model singlets in the massless string spectrum along flat F and D directions. In this process nonrenormalizable terms,

$$\langle V_1^f V_2^f V_3^b \cdots V_N^b \rangle,$$

become renormalizable operators,

$$V_1^f V_2^f V_3^b \langle V_4^b \cdots V_N^b \rangle / M^{N-3}$$

in the effective low energy field theory.

The first five basis vectors of the realistic free fermionic models consist of the NAHE set [21, 11, 18]. The gauge group after the NAHE set is $SO(10) \times E_8 \times SO(6)^3$ with $N = 1$ space–time supersymmetry, and 48 spinorial 16 of $SO(10)$, sixteen from each sector $b_1$, $b_2$ and $b_3$. The three sectors $b_1$, $b_2$ and $b_3$ are the three twisted sectors of the corresponding $Z_2 \times Z_2$ orbifold compactification. The $Z_2 \times Z_2$ orbifold is special precisely because of the existence of three twisted sectors, with a permutation symmetry with respect to the horizontal $SO(6)^3$ symmetries. The NAHE set is depicted in the table below which highlights its cyclic permutation symmetry.

**THE NAHE SET**

|   | $\psi^\mu$ | $\chi^{12}$ | $\chi^{34}$ | $\chi^{56}$ | $\bar{\psi}^{1,\dots,5}$ | $\bar{\eta}^1$ | $\bar{\eta}^2$ | $\bar{\eta}^3$ | $\bar{\phi}^{1,\dots,8}$ |
|---|---|---|---|---|---|---|---|---|---|
| **1** | 1 | 1 | 1 | 1 | 1,...,1 | 1 | 1 | 1 | 1,...,1 |
| $S$ | 1 | 1 | 1 | 1 | 0,...,0 | 0 | 0 | 0 | 0,...,0 |
| $b_1$ | 1 | 1 | 0 | 0 | 1,...,1 | 1 | 0 | 0 | 0,...,0 |
| $b_2$ | 1 | 0 | 1 | 0 | 1,...,1 | 0 | 1 | 0 | 0,...,0 |
| $b_3$ | 1 | 0 | 0 | 1 | 1,...,1 | 0 | 0 | 1 | 0,...,0 |

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| | $y^{3,...,6}$ | $\bar{y}^{3,...,6}$ | $y^{1,2},\omega^{5,6}$ | $\bar{y}^{1,2},\bar{\omega}^{5,6}$ | $\omega^{1,...,4}$ | $\bar{\omega}^{1,...,4}$ |
|---|---|---|---|---|---|---|
| $\mathbf{1}$ | 1,...,1 | 1,...,1 | 1,...,1 | 1,...,1 | 1,...,1 | 1,...,1 |
| $S$ | 0,...,0 | 0,...,0 | 0,...,0 | 0,...,0 | 0,...,0 | 0,...,0 |
| $b_1$ | 1,...,1 | 1,...,1 | 0,...,0 | 0,...,0 | 0,...,0 | 0,...,0 |
| $b_2$ | 0,...,0 | 0,...,0 | 1,...,1 | 1,...,1 | 0,...,0 | 0,...,0 |
| $b_3$ | 0,...,0 | 0,...,0 | 0,...,0 | 0,...,0 | 1,...,1 | 1,...,1 |

The NAHE set is common to a large class of three generation free fermionic models. The construction proceeds by adding to the NAHE set three additional boundary condition basis vectors which break $SO(10)$ to one of its subgroups, $SU(5) \times U(1)$, $SO(6) \times SO(4)$ or $SU(3) \times SU(2) \times U(1)^2$, and at the same time reduces the number of generations to three, one from each of the sectors $b_1$, $b_2$ and $b_3$. The various three generation models differ in their detailed phenomenological properties. However, many of their characteristics can be traced back to the underlying NAHE set structure. One such important property to note is the fact that as the the generations are obtained from the three twisted sectors $b_1$, $b_2$ and $b_3$, they automatically possess the Standard $SO(10)$ embedding. Consequently the weak hypercharge, which arises as the usual combination $U(1)_Y = 1/2 U(1)_{B-L} + U(1)_{T_{3_R}}$, has the standard $SO(10)$ embedding. To date, of the orbifold models that have been constructed, only the free fermionic models have yielded such a structure.

The massless spectrum of the realistic free fermionic models then generically contains three generations from the three twisted sectors $b_1$, $b_2$ and $b_3$, which are charged under the horizontal symmetries. The Higgs spectrum consists of three pairs of electroweak doublets from the Neveu–Schwarz sector plus possibly additional one or two pairs from a combination of the two basis vectors which extend the NAHE set. Additionally the models contain a number of $SO(10)$ singlets which are charged under the horizontal symmetries and a number of exotic states.

Exotic states arise from the basis vectors which extend the NAHE set and break the $SO(10)$ symmetry. Consequently, they carry either fractional $U(1)_Y$ or $U(1)_{Z'}$ charge. Such states are generic in superstring models and impose severe constraints on their validity. In some cases the exotic fractionally charged states cannot decouple from the massless spectrum, and their presence invalidates otherwise viable models. In the NAHE based models the fractionally charged states always appear in vector–like representations. Therefore, in general mass terms are generated from renormalizable or nonrenormalizable operators. However, the mass terms which arise from non–renormalizable terms will in general be suppressed, in which case the fractionally charged states may have intermediate scale masses. Here I

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describe the analysis of a model in which all the fractionally charged states decouple from the massless spectrum at the cubic level of the superpotential and receive mass of the order of the string scale.

## 4. Minimal Superstring Standard Model

The superstring model under consideration [10] is a typical three generation free fermionic model. It is generated by the NAHE–set plus three additional basis vectors $\{b_4, \alpha, \beta\}$, where $b_4$ preserves the $SO(10)$ symmetry, $\alpha$ breaks $SO(10) \rightarrow SO(6) \times SO(4)$ and $\beta$ breaks $SO(6) \times SO(4) \rightarrow SU(3) \times SU(2) \times U(1)^2$. The massless spectrum consist of three generations from the sectors $b_1$, $b_2$ and $b_3$, The Neveu–Schwarz (NS) sector produces, the gravity and gauge multiplets, three pairs of electroweak doublets $\{h_1, h_2, h_3, \bar{h}_1, \bar{h}_2, \bar{h}_3\}$, seven pairs of $SO(10)$ singlets with observable $U(1)$ charges, $\{\phi_{12}, \bar{\phi}_{12}, \phi_{23}, \bar{\phi}_{23}, \phi_{13}, \bar{\phi}_{13}, \phi_{56}, \bar{\phi}_{56}, \phi'_{56}, \bar{\phi}'_{56}, \phi_4, \bar{\phi}_4, \phi'_4, \bar{\phi}'_4\}$, and three scalars that are singlets of the entire four dimensional gauge group, $\phi_1, \phi_2, \phi_3$. The states from the NS sector and the sectors $b_1$, $b_2$ and $b_3$ are the only ones that transform solely under the observable, $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{T_{3_R}} \times U(1)_{1,\cdots,6}$ gauge group. The sectors $(I) + b_{1,2,3,4} + 2\beta$ produce $SO(10)$ singlet matter states in the 16 vector representation of the hidden $SO(16)$ gauge group, decomposed under the final hidden group. The sectors with some combination of $\{\mathbf{1}, b_1, b_2, b_3, b_4, \alpha\}$ plus $\beta$ or $2\beta$ produce states that are $SU(3)_C \times SU(2)_L$ singlets, but carry fractional charge under $U(1)_Y$, $U(1)_{Z'}$, or $U(1)_{em}$. These exotic states, which do not fit into $SO(10)$ representations, arise due to the $SO(10)$ symmetry breaking, by the basis vectors $\alpha$ and $\beta$, and carry fractional electric charge $\pm 1/2$ or fractional $U(1)_{Z'}$ charge. The full massless spectrum and charges are given in ref. [10, 23].

In the model of ref. [10] it is noticed that all the fractionally charged states couple at the cubic level of the superpotential to the set of $SO(10)$ singlets $\{\phi_4, \phi'_4, \bar{\phi}_4, \bar{\phi}'_4\}$ [24],

$$\frac{1}{\sqrt{2}}\{H_1 H_2 \phi_4 + (H_3 H_4 + H_5 H_6)\bar{\phi}_4 + (H_7 H_8 + H_9 H_{10})\phi'_4 + H_{11} H_{13} \bar{\phi}'_4$$
$$+ (V_{41} V_{42} + V_{43} V_{44})\bar{\phi}_4 + V_{45} V_{46} \phi_4 + (V_{47} V_{48} + V_{49} V_{50})\bar{\phi}'_4 + V_{51} V_{52} \phi'_4\}.$$

where F–flatness imposes $(\phi_4 \bar{\phi}'_4 + \bar{\phi}_4 \phi'_4) = 0$. The problem then is to find flat F and D solutions, which incorporates the set of fields $\{\phi_4, \phi'_4, \bar{\phi}_4, \bar{\phi}'_4\}$. In the last couple of years the search for F and D flat solutions in superstring models was systematized [25], following similar developments in supersymmetric field theory models. Applying these methods to the model of ref. [10] we indeed found solutions with the desired properties. One example is given by the set of fields,

$$\{\phi_{12}, \phi_{23}, \bar{\phi}_{56}, \phi_4, \phi'_4, \bar{\phi}_4, \bar{\phi}'_4, H_{15}, H_{30}, H_{31}, H_{38}\}.$$

Furthermore, it was shown that with this solution also the extra Higgs multiplets, beyond the MSSM, as well as an additional pair of Higgs triplets receive mass by the same set of VEVs from cubic and quintic order terms. Therefore, in this solution, we have in the observable sector, solely the MSSM charged spectrum. Moreover, the F and D flat solutions have been completely classified and it was shown that solutions with such properties are in fact abundant, which encourages the prospect for obtaining realistic values for the Standard Model parameters. Another important property of the F and D flat solutions is that the set of VEVs necessarily includes fields that break the $U(1)_{Z'}$, which is embedded in $SO(10)$. Thus, in this case $SO(10)$ symmetry is necessarily broken directly to $SU(3)_C \times SU(2)_L \times U(1)_Y$. Finally, the model of ref. [10] supplemented with the flat F and D solutions provides the first example in the literature with solely the MSSM charged spectrum below the string scale. Thus, for the first time we have an example of a long–sought Minimal Superstring Standard Model !

## 5. Phenomenology

The model of ref. [10] and its success in the terms of producing solely the MSSM charged spectrum at low energies, should be viewed as a prototype example of a realistic free fermionic model. The lesson that should be extracted is that the underlying structure of these models, provided by the NAHE set, produces the right features for obtaining realistic phenomenology. It provides further evidence for the assertion that the true string vacuum is connected to the $Z_2 \times Z_2$ orbifold in the vicinity of the free fermionic point in the Narain moduli space. With this in mind we note that many of the important issues relating to the phenomenology of the Standard Model and supersymmetric unification have been addressed in the past in similar prototype free fermionic heterotic string models. These studies have been reviewed in the past and I refer to the original literature and review references [26]. These include among others: top quark mass prediction [12], several years prior to the actual observation by the CDF/D0 collaborations; generations mass hierarchy [27]; CKM mixing [28]; superstring see–saw mechanism [29]; Gauge coupling unification [30]; Proton stability [14]; and supersymmetry breaking and squark degeneracy [31].

## 6. Exotic signatures

After establishing the phenomenological viability of free fermionic heterotic–string models, it makes sense to seek possible experimental signals that may provide evidence for specific models in particular, and for string theory in general. This is in essence a secondary task as the first duty

is to reproduce the parameters of the Standard Model. Obtaining the full structure of the Standard Model from a string model will be an everlasting achievement. With this in mind there are several possible exotic signatures that have been discussed in the past. These include the possibility of extra $U(1)$'s [32]; specific supersymmetric spectrum scenarios [33]; R–parity violation [34]; and exotic matter [35]. R–parity violation is an intriguing but somewhat remote possibility. The problem is that in string models if R–parity is violated at the same time one expects to get fast proton decay. The model of ref. [36] provides an example how R–parity violation can arise in superstring theory. This string model gives rise to custodial symmetries which allow lepton number violation while forbidding baryon number violation.

The second possibility is that of exotic matter, which arises in superstring models because of the breaking of the non–Abelian symmetries by Wilson–lines. It is therefore a unique signature of superstring unification, which does not arise in field theory GUTs. While the existence of such states imposes severe constraints on otherwise valid string models [13], provided that the exotic states are either confined or sufficiently heavy, they can give rise to exotic signatures. For example, they can produce heavy dark matter candidates, possibly with observable consequences [37].

## 7. Toward M(F)–theory embedding

Over the past few years a remarkable new understanding of string theory has emerged. In this picture all the ten dimensional perturbative string theories as well as 11 dimensional supergravity are all perturbative limits of a single theory, referred to as M (or F) theory [38]. In ref. [40] we have undertaken the task of trying to understand how the phenomenological free fermionic models may fit in the nonperturbative framework of M(F)–theory. As discussed above the NAHE set, which corresponds to $Z_2 \times Z_2$ orbifold compactification, plays a pivotal role in the realistic free fermionic models. Its correspondence with a $Z_2 \times Z_2$ orbifold is made more explicit by adding to the NAHE set an additional boundary condition basis vector $X$ [18], which extends the $SO(10) \times SO(6)^3$ symmetry to $E_6 \times U(1)^2 \times SO(4)^3$. This model contains (27,3) multiplets in the $(27, \overline{27})$ representations of $E_6$. The same model is generated in the orbifold language by moding out an $SO(12)$ Narain lattice by a $Z_2 \times Z_2$ discrete symmetry with standard embedding [18]. The fermionic $Z_2 \times Z_2$ orbifold model differs from the one which has usually been examined in the literature, with $(h_{11}, h_{21}) = (51, 3)$, which corresponds to twisting of an $SO(4)^3$ lattice. Many discussions on F–theory have focused on compactifications on the (51,3) $Z_2 \times Z_2$ Calabi–Yau 3–fold to six dimensions. The first task then is to connect the (51,3) model to the (27,3) model, and then to implement this connection in the

F–theory compactification on the (51,3) model. It should be emphasized that the aim here is not to extract direct phenomenological data from these investigations. The goal is rather to try to bridge between basic structures which appear in the phenomenological free fermionic models and structures which appear in M(F)–theory, hoping that it will eventually yield further phenomenological insight.

The $(51,3)$ $Z_2 \times Z_2$ orbifold model is obtained by twisting a $(T_2)^3$ 3–dimensional complex manifold parametrized by $\{z_1, z_2, z_3\}$. The first and second $Z_2$ twists take

$$\{z_1, z_2, z_3\} \rightarrow \{-z_1, -z_2, z_3\}$$

and

$$\{z_1, z_2, z_3\} \rightarrow \{z_1, -z_2, -z_3\}.$$

Calculating the cohomology of this manifold, yields $(h_{11}, h_{21}) = (51, 3)$. Connecting this model to the $(27, 3)$ can be done using the Landau–Ginzburg formalism with a freely acting twist [40] or by adding the freely acting shift

$$\{z_1, z_2, z_3\} \rightarrow \{z_1 + 1/2, z_2 + 1/2, z_3 + 1/2\},$$

which identifies points by the shift on all three complex tori simultaneously. Under this shift the number of fixed points from the twisted sectors is reduced by half, hence producing the spectrum of the (27,3) model.

The next step is to implement this freely acting shift or twist in the F–theory compactification to six dimensions. The key here is that the models should admit and elliptic fibration with a global section, in which a Calabi–Yau 3–fold is identified as a two complex–dimensional base manifold $B$ with a fiber. The compactification is then defined by specifying the toroidal fiber in the Weierstrass form,

$$y^2 = x^3 + f(z_1, z_2)x + g(z_1, z_2),$$

where $f$ and $g$ are polynomials of degrees 8 and 12, respectively and are functions of the base coordinates. The number of neutral hypermultiplets, tensor multiplets and the rank of the vector multiplets are then given by: $H^0 = h^{21}(X) + 1$, $T = h^{11}(B) - 1$ and

$$r(V) = h^{11}(X) - h^{11}(B) - 1,$$

respectively, in terms of $h_{11}$, $h_{21}$ of the CY 3–fold and the base. Cancelation of the gravitational anomaly in six dimensions requires that

$$H^0 - V = 273 - 29T,$$

where $V$ is the number of vector multiplets.

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The Weierstrass representation of F–theory compactification on the (51,3) model is given by

$$y^2 = x^3 + f_8(w, \tilde{w})xz^4 + g_{12}(w, \tilde{w})z^6,$$

where

$$f_8 = \eta - 3h^2, \text{ and } g_{12} = h(\eta - 2h^2),$$

$$h = K \prod_{i,j=1}^{4} (w - w_i)(\tilde{w} - \tilde{w}_j)$$

and

$$\eta = C \prod_{i,j=1}^{4} (w - w_i)^2 (\tilde{w} - \tilde{w}_j)^2.$$

Taking $w \to w_i$ (or $\tilde{w} \to \tilde{w}_i$) we have a $D_4$ singular fiber. Thus, we have an enhanced $SO(8)^8$ gauge symmetry, since $i, j = 1, \ldots, 4$. These $D_4$ singularities intersect in 16 points, $(w_i, \tilde{w}_j)$, $i, j = 1, \ldots 4$, in the base and give rise to 16 additional tensor multiplets. With the equations given above we see that this fits the data of F–theory compactification on the $Z_2 \times Z_2$ CY 3–fold with $(h_{11}, h_{21}) = (51, 3)$.

To find the F–theory compactification on the corresponding $(27, 3)$ model we implement the freely acting twist in the elliptic fibration. For this purpose it is more convenient to represent the fiber in quartic form, which is given by

$$\hat{y}^2 = \hat{x}^4 + \hat{x}^2 \hat{z}^2 \hat{f}_4 + \hat{x}\hat{z}^3 \hat{g}_6 + \hat{z}^4 \hat{h}_8,$$

with $\hat{f}_4 = -3h$, $\hat{g}_6 = 0$, and $\hat{h}_8 = -1/4\eta$. The freely acting twist which acts simultaneously on the base and the fiber is given by

$$(\hat{y}, \hat{x}, \hat{z}, w, \tilde{w}) \to (-\hat{y}, -\hat{x}, \hat{z}, -w, -\tilde{w}).$$

We now see that to implement this identification $h$ and $\eta$ are modified and are given by

$$h = K \prod_{i,j=1}^{2} (w^2 - w_i^2)(\tilde{w}^2 - \tilde{w}_j^2)$$

and

$$\eta = C \prod_{i,j=1}^{2} (w^2 - w_i^2)^2 (\tilde{w}^2 - \tilde{w}_j^2)^2.$$

We see that there are now only 4 $D_4$ singularities, and similarly the number of intersecting singularities is reduced by 2. Hence, in this F–theory compactification the enhanced symmetry is $SO(8)^4$ with $T = 9$, $H^0 = 4$ and $V = 112$. This matches the data of the $(27,3)$ model with $h_{11}(B) = 10$.

However, we see that in this case the gravitational anomaly is apparently not satisfied.

A plausible interpretation of the above result is that the (27,3) model does not provide a consistent background for F–theory compactification to six dimension. However, this will still be a strange situation because the freely acting twist is a consistent operation which should not destroy the fibration. To study the issue further, we examine the effect of the freely acting shift on the $(T_2)^3$ manifold. It is then seen that although the shift is freely acting on the CY 3–fold, it is not freely acting on the base when the CY is regarded as a fibration. Hence, the base in the fibered CY has four singular points that are not singular points of the CY 3–fold. Therefore, there are no $h_{11}$ and $h_{21}$ forms that can be used to resolve these singularities. The elliptic fibration and the global section are destroyed at those singular points.

The existence of these special singular points is quite interesting. Another plausible interpretation for the resolution of the puzzles is that due to these special singularities there exist additional massless states that can only be seen nonperturbatively. Furthermore, the appearance of the special singularities is closely tied to the action of the freely acting shift on the fiber. To see that we implement another shift on the elliptically fibered (51,3) model, induced by the shift

$$(z_1, z_2, z_3) \rightarrow (z_1 + 1/2, z_2 + 1/2, z_3).$$

This shift is not freely acting on the CY 3–fold and there is an additional sector yielding $(h_{11}, h_{21}) = (31, 7)$. This shift is not freely acting on the base but there are now four additional $(h_{11}, h_{21})$ pairs that can be used to resolve the base singularities in the usual manner. For F–theory compactification on the (31,7) CY 3–fold: $T = 13$; $H^0 = 8$ and $V = 112$ and it is checked that it is consistent with the gravitational anomaly.

The discussion above illustrates that the puzzles in the F–theory compactification on the (27,3) model precisely arise because of the action of the freely acting shift on the fiber

$$z_3 \rightarrow z_3 + 1/2.$$

While one possible interpretation is that the (27,3) model does not provide a consistent background for F–theory compactification, the situation is non trivial and still unresolved. But we may contemplate that some new physics is associated with the action of the shift on the fiber, which in the case of the type IIB string theory is identified with the dilaton. All in all we see that the free fermionic $Z_2 \times Z_2$ orbifold is special and perhaps more surprises lie in store.

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## 8. Equivalence postulate of quantum mechanics – embarking on a novel approach to quantum gravity

Over the last few years important new insight has been gained on the fundamental structure of string theory. We now know that different theories, that are classically distinct, are in fact related quantum mechanically by various duality transformations. Many of the duality relations have an inherent geometrical description. What is the lesson to be extracted from this new understanding ? It seems to me that what is needed is a new fundamental principle. This is the approach which is being pursued by Matone and myself. We have imposed the basic postulate that all physical systems labelled by a potential function can be connected by a coordinate transformation and showed that consistency of this postulate necessitates the appearance of quantum mechanics and is intimately connected to phase–space duality. The Planck constant appears in this context as a covariantizing parameter. We then have a fundamental geometrical principle behind quantum mechanics and $\hbar \neq 0$. I will follow here the historical path of this development.

Working in Seiberg–Witten theory [41] Matone [42] noted that the Picard–Fuchs equation for the duals, $a(u)$ and $a_D(u)$, can be inverted. Yielding $u(a) = \frac{1}{2}a\partial_a \mathcal{F} - \mathcal{F}$, which is an exact non–perturbative relation and the prepotential function is given by $a_D = \partial_a \mathcal{F}$. Written in the form $u(a) = a^2 \partial_{a^2} \mathcal{F} - \mathcal{F}$, it is noted that it has the form of the Legendre transformation. This relation is not particular to Seiberg–Witten theory and can be applied to other theories which are described by a second order linear differential equation. We first applied this idea to the Schrödinger equation, where we introduced a prepotential function defined by the relation

$$\psi_D = \partial_\psi \mathcal{F},$$

with $\psi$ and $\psi_D$ being the two linearly independent solutions of the Schrödinger equation [43]. One then obtains the inverted form

$$x(\psi) = \psi^2 \partial_{\psi^2} \mathcal{F} - \mathcal{F},$$

which offers the possibility of a coordinate free formulation of quantum gravity. A direction which is being pursued primarily in ref. [44, 45].

The inversion relation is a general relation between dual variables related by a generating function. The natural step is to apply it to the phase–space coordinates related by Hamilton's generating function $p = \partial_q \mathcal{S}_0$. One then obtains the dual Legendre transformations [46],

$$\mathcal{S}_0 = p\partial_p \mathcal{T}_0 - \mathcal{T}_0$$

and

$$\mathcal{T}_0 = q\partial_q \mathcal{S}_0 - \mathcal{S}_0$$

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where $\mathcal{T}_0(p)$ is a new generating function defined by $q = \partial_p \mathcal{T}_0$. Two observations are important to note. The first is that the Legendre transformation is undefined for linear functions, *i.e.* for $\mathcal{S}_0 = A + Bq$. The second is that similar to the case of Seiberg–Witten theory, and the case of the Schrödinger equation, one can associate a second order differential equation with each Legendre transformation, which we call the "canonical equation" [46]. The potential function in the "canonical equation" for $\mathcal{S}_0$ is given by $\{q, \mathcal{S}_0\}$, where $\{,\}$ denotes the Schwarzian derivative. Choosing that the reduced action transforms as a scalar function under coordinate transformations, it is noted that by construction the $2^{nd}$–order differential equation is covariant. The Schwarzian derivative, however, is invariant under Möbius transformations, but not under general coordinate transformations. This fact suggests that different physical systems labelled by different potentials can be connected by coordinate transformations. Given the new insight gained in the context of string dualities, and the discussion above on Legendre duality, it is natural to promote this new insight to the level of a fundamental physical principle. In ref. [46] we posed the following postulate:

*Given two physical systems with $\mathcal{W}^a(q^a) \in \mathcal{H}$ and $\mathcal{W}^b(q^b) \in \mathcal{H}$, where $\mathcal{H}$ denotes the space of all possible $\mathcal{W}$'s, there always exists a coordinate transformation $q^a \to q^b = v(q^a)$ such that $\mathcal{W}^a(q^a) \to \mathcal{W}^{av}(q^b) = \mathcal{W}^b(q^b)$.*

We note that this postulate also implies that there should always exist a coordinate transformation connecting any state to the state $\mathcal{W}^0(q^0) = 0$. Inversely, this means that any state $\mathcal{W} \in \mathcal{H}$ can be reached from the state $\mathcal{W}^0(q^0)$ by a coordinate transformation.

A natural application of this postulate is in the context of the classical Hamilton–Jacobi formalism. There one solves the dynamical problem by performing canonical transformations which map a dynamical system, governed by a Hamiltonian $H$, to a trivial dynamical system with vanishing Hamiltonian. The solution is given by the Classical Hamilton–Jacobi Equation (CHJE), and the functional relation between $p$ and $q$ is only extracted after the Hamilton–Jacobi equation is solved. We aim to pose a similar question, but with the novelty that we consider the transformation $q \to \tilde{q}(q)$, while imposing the functional relation $p = \partial_q \mathcal{S}_0$, reducing to the free system with vanishing energy. Motivated from the Legendre duality discussion we impose that under the transformation $\tilde{\mathcal{S}}_0(\tilde{q}) = \mathcal{S}_0(q)$. It follows that $p$ transforms as $\partial_q$.

The CSHJE,
$$1/2m(\partial_q \mathcal{S}_0)^2 + \mathcal{W}(q) = 0,$$
fixes the transformation

$$\mathcal{W}(q) \to \tilde{\mathcal{W}}(\tilde{q}) = (\partial_{\tilde{q}} q)^2 \mathcal{W}(q).$$

It is observed that the state $\mathcal{W}^0(q^0) = 0$ is a fixed point under the co-ordinate transformation. That is we cannot reach all possible states by coordinate transformation from the state $\mathcal{W}^0(q^0) = 0$. Consistency of the equivalence postulate then implies that the CSHJE should be deformed. The most general form would be,

$$1/2m(\partial_q \mathcal{S}_0)^2 + \mathcal{W}(q) + Q(q) = 0,$$

where the nature of $Q(q)$ is to be determined by the consistency of the equivalence postulate, which imposes that the combination $(\mathcal{W} + Q)$ transforms as a quadratic differential. On the other hand all states should be connected to the state $\mathcal{W}^0(q^0) = 0$ by a coordinate transformation. The basic transformation properties are,

$$\begin{aligned}
\mathcal{W}^v(q^v) &= (\partial_{q^v} q^a)^2 \, \mathcal{W}^a(q^a) + (q^a; q^v), \\
Q^v(q^v) &= (\partial_{q^v} q^a)^2 \, Q^a(q^a) - (q^a; q^v),
\end{aligned}$$

which fixes the cocycle condition for the inhomogeneous term [46]

$$(q^a; q^c) = (\partial_{q^c} q^b)^2 \, [(q^a; q^b) - (q^c; q^b)].$$

The importance of the cocycle condition is that it uniquely fixes the transformation properties of the inhomogeneous term, and hence fixes its functional form. It is then proven that the inhomogeneous term $(q^a; q^b)$ is invariant under the Möbius transformation, and is uniquely given by the Schwarzian derivative $\{q^a, q^b\}$. The cocycle condition is generalizable to higher dimensions and fixes that the inhomogeneous term is invariant under the D–dimensional Möbius transformations [48]. The cocycle condition univocally implies [46],

$$\mathcal{W}(q) = V(q) - E = -\frac{\hbar^2}{4m}\{\mathrm{e}^{(i2\mathcal{S}_0/\hbar)}, q\}, \tag{1}$$

$$Q(q) = \frac{\hbar^2}{4m}\{\mathcal{S}_0, q\}, \tag{2}$$

where in demonstrating this we used the basic identity,

$$(\partial_q \mathcal{S}_0)^2 = \hbar^2/2 \left(\{\exp(i2\mathcal{S}_0/\hbar, q)\} - \{\mathcal{S}_0, q\}\right). \tag{3}$$

$\mathcal{S}_0$ is solution of the Quantum Stationary Hamilton–Jacobi Equation

$$\frac{1}{2m}\left(\frac{\partial \mathcal{S}_0}{\partial q}\right)^2 + V(q) - E + \frac{\hbar^2}{4m}\{\mathcal{S}_0, q\} = 0, \tag{4}$$

which can be obtained from the Schrödinger equation by taking,

$$\psi(q) = \frac{1}{\sqrt{\mathcal{S}_0'}} \mathrm{e}^{\pm \frac{i\mathcal{S}_0}{\hbar}}$$

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Note that the QSHJE is a non–linear third–order differential equation. The Schrödinger equation in this context can be regarded as linearization of the QHJE [46], in the following sense. From eq. (1) and the Möbius invariance of the Schwarzian derivative it is seen that the solution of the QHJE is given in terms of the ratio of the two real linearly independent solutions of the stationary Schrödinger equation, $w = \psi_D/\psi$, by

$$\mathrm{e}^{\frac{i2}{\hbar}\mathcal{S}_0\{\delta\}} = \mathrm{e}^{i\alpha}\frac{w + i\bar{\ell}}{w - i\ell} \tag{5}$$

where $\delta = \{\alpha, \ell\}$ with $\alpha \in R$ and $\mathrm{Re}\ell \neq 0$, which is equivalent to the condition $\mathcal{S}_0 \neq const$. We note that the trivializing map to the $\mathcal{W}^0(q^0) = 0$ system is given by $q \rightarrow q^0 = w$.

Several points are important to note. First is that also for the state $\mathcal{W}^0 = 0$ we have $\mathcal{S}_0 \neq const$. Therefore, $\mathcal{S}_0 = const$ is not in the space of solutions and we have that the equivalence postulate is consistent with quantum mechanics but is inconsistent with classical mechanics. This fact also allows for the definability of the Legendre transformation for all physical states. Thus we have that the definability of the Legendre duality and the consistency of the equivalence postulate are intimately related.

It is further shown [46] that consistency of the equivalence postulate implies both energy quantization for bound states with a square integrable wave–function, as well as the tunnelling effect, without assuming the probability interpretation of the wave–function. Thus, we have that the main characteristics of quantum mechanics arise from the self–consistency of the equivalence postulate. This is of fundamental importance as we see that the main phenomenological features of quantum mechanics are reproduced starting from the equivalence postulate without further assumptions. I refer the reader to the original papers [46] for details of this fascinating avenue. Here I focus on the characteristics of the formulation which are related to the Planck scale, and hence may be related to string theory.

There are two consequences of the formulation that are clearly related to the Planck scale and hence to gravity and possibly to string theory. The first is the appearance of a fundamental length scale which is identified with the Planck length and the second is the existence of equivalence classes of the wave–function which depend on this length scale [46].

First it is noted that the formulation provides a trajectory representation of quantum mechanics [47], which due to the Möbius symmetry depends on the constant $\ell$. From the solution for the QHJE we get $p = \partial_q \mathcal{S}_0 = p(q)$, which depends on the integration constants of the QSHJE.

$$p_E = \pm\frac{\hbar(\ell_E + \bar{\ell}_E)}{2|k^{-1}\sin kq - i\ell_E \cos kq|^2}, \tag{6}$$

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where $k = \sqrt{2mE}/\hbar$. The existence of a fundamental length scale, identified with the Planck length, can already be inferred from the basic Legendre duality and consistency of the equivalence postulate which require that $\mathcal{S}_0 \neq 0$. It is seen more explicitly by considering the consistency of the classical limit, $\hbar \to 0$. In this limit we have that for $E \to 0$ we should have $p_0 \to 0$. This shows that $\mathrm{Re}\ell_0 \sim \hbar^\gamma$ with $-1 < \gamma < 1$. Thus, consistency of these limits implies the identification $\mathrm{Re}\ell_0 \sim \lambda_P \neq 0$ [46].

The next important property of the formulation is the existence of equivalence classes of the wave–function. As the QHJE is a third–order differential equation whereas the Schrödinger equation is a second order one, more initial conditions are needed to be specified in the case of the QHJE. It follows that the wave function remains invariant under suitable transformations of $\delta = \{\alpha, \ell\}$, corresponding to different trajectories. The implication is that there are hidden variables which depend of the Planck length and that these can suitably change without affecting the wave–function. Recently, t'Hooft [49] has advocated that hidden variables must play a role in the implementation of the holographic principle [50].


## 9. Is there a connection with string theory ?

At the outset I would state that I do not know the answer to this question. The aim is to try to find some overlaps in the physical and mathematical characteristics in the equivalence postulate derivation and in string theory. Nevertheless, it should be stressed that it is very natural to expect that the correct theory of quantum gravity would arise from a principle such as the equivalence postulate. Already the appearance of the Schwarzian derivative in the framework of quantum mechanics should be regarded as tantalizing evidence for a possible connection with string theory and hence with quantum gravity. Below I enumerate other possible relations.

1) Quadratic differential: In string theory elimination of the world–sheet conformal anomaly is necessary in order for the energy–momentum tensor to transform as a quadratic differential and for obtaining diffeomorphism invariance, $i.e.$ Einstein equations, in target space. Thus, the fact that the energy–momentum tensor transforms as a quadratic differential plays an important role. Similarly, the equivalence postulate derivation imposes that the Hamiltonian (Hamilton–Jacobi equation) transforms as a quadratic differential. The similarity is not complete because in the string case we require that cancelation of a quantum anomaly restores a classical symmetry, whereas in the equivalence postulate derivation we required that the quantum modification enables that the HJ equation transforms as a quadratic differential. Nevertheless, it seems that there should be a deep reason why in both cases the quadratic differential transformation plays a crucial role. Another caveat is that we have not yet included fermions in

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the formalism. However, we may envision that the square of the fermionic Hamiltonian transforms as a quadratic differential.

2) The existence of Möbius symmetry represents an invariance under finite diffeomorphism. The $SL(2, C)$ symmetry plays a central role in string theory and a central role in the formulation of quantum mechanics from the equivalence postulate. The Möbius symmetry is the origin of the existence of equivalence classes of the wave–function and of a fundamental length scale. The presence of a Möbius symmetry should suggest a connection with string theory. Indeed, one may expect that performing infinitesimal diffeomorphism would recover some of the features of 2D-CFT's, including the Virasoro algebra, vector spaces, etc. In higher dimensions the symmetry of the cocycle is the higher dimensional Möbius group [48]. In that case, we may envision that performing infinitesimal diffeomorphism should entail some generalization of the Virasoro algebra, possibly in the form of W–algebras ?

3) The hidden variables in the equivalence postulate formulation can be identified with a fundamental length scale, most naturally with the Planck length [46]. Furthermore, the equivalence classes of the wave–function can be parametrized in terms of this fundamental length scale. The emergence of a fundamental length scale should give rise to the suspicion of a connection with quantum gravity and possibly with string theory. An intriguing thought is that the emerging length scale and its role in the equivalence classes of the wave–function may somehow be related to the internal string dimension. The analogy of the formalism with uniformization theory [46] suggests that the hidden variables may be associated with Riemann surfaces, further indicating possible connections with string theory.

## 10. Does it address the vacuum energy problem ?

Again I would state that I do not know the answer to that question. We may however contemplate how the equivalence postulate may affect the standard picture. Surely, if the equivalence postulate is a fundamental law of Nature, as we may infer from the understandings gained in the context of string dualities, then it will by default also shed light on this issue.

We see that in the quantum HJ equation there is an additional term which is identified with a quantum motion, or quantum potential, which is nonzero also when the potential and the energy vanish

$$\frac{1}{2m} \left( \partial_q \mathcal{S}_0 \right)^2 + \frac{\hbar^2}{4m} \{\mathcal{S}_0, q\} = 0 \tag{7}$$

The implication is that, unlike the classical case, $\mathcal{S}_0$ is never vanishing and the state $\mathcal{S}_0 = constant$ is excluded from the space of allowed solutions.

This is the fundamental characteristic of quantum mechanics in our approach. Thus, we see that the state with $V(q) = 0$ and $E = 0$ is indeed pointed out in the equivalence postulate approach. The solution for the ground state in the quantum case is given by $\mathcal{S}_0 = i\hbar/2 \ln q$, up to Möbius transformations. Consequently, the conjugate momentum for the ground state is also non–trivial. Being the characteristic property of quantum mechanics, we can regard this as the quantum trajectory version of the uncertainty principle. All in all, we see that the vacuum state is singled out relative to its role in the classical case. Furthermore, the quantum ground solution is also the self–dual state of the Legendre phase–space transform and its dual. That is, it is the unique simultaneous solution of the two second order linear differential equations associated with each Legendre transformations [46]. Thus, we have that the vanishing of the vacuum state may be intimately related to the Legendre phase–space duality. This is already one hint that the equivalence postulate and Legendre phase–space duality may shed light on the vacuum energy issue.

The equivalence postulate point of view is that the fundamental equation is the QHJE, which is a third–order non–linear differential equation. It is equivalent to the Schrödinger equation (in the sense discussed above eq. (5)), but requires specifying more initial conditions than for the Schrödinger equation. We have that there is a moduli space of solutions of the QHJE, which corresponds to the same wave function. That is, there are hidden variables which depend on the Planck length and are not detected in the solutions of the Schrödinger equation. This means that the Schrödinger equation with its related apparatus provides an effective description, albeit an extremely successful one from the experimental point of view. Now, the vacuum energy in conventional quantum mechanics is an artifact of the Hilbert space construction, *i.e.* it is an artifact of the effective description. But from the point of view of the equivalence postulate the more complete solution is given by the QHJE, which admits a non–trivial solution also for the state with vanishing energy and vanishing potential. The existence of such a specialized state already indicates that it may have something to do with the vacuum energy, as according to the equivalence postulate all other states are connected to this special state by coordinate transformations. This leads to the existence of a fundamental length scale with all the expected implications of modifications of the uncertainty relations, and space–time uncertainty relations, etc. However, the important fact is the existence of the additional term in the quantum HJ equation, $Q(q) = (\hbar/2m)\{\mathcal{S}_0, q\}$, which is never vanishing. This term can be interpreted as a curvature term [46], which means that the existence of the state itself is associated with a sort of "quantum curvature". So it seems artificial to speak of particles and curvature as distinct entities. It seems, that from the equivalence postulate point of view there is no meaning to talk about vacuum energy. Rather we should speak about the vacuum cur-

vature, associated with the existence of the particles themselves, which is never vanishing, and is associated with the state for which the potential and the energy vanish.

Another observation on the equivalence postulate derivation is the way in which mass appears in the formalism. In quantum field theories, the vacuum energy problem is tightly related to the generation of mass through symmetry breaking. In the equivalence postulate derivation, it is intriguing to note that mass appears only after making the identification in eq. (1). In the identity, eq. (3), which is related to the QHJE eq. (4), the mass is only a multiplicative constant, which can be dropped from the equation. This feature also persists in the higher dimensional generalization [48]. Thus, we see that mass in the equivalence postulate derivation is an artificial effect of representing the potential in the functional form $\mathcal{W}(q) = V(q) - E$. The fact that the Schwarzian derivative is associated with a curvature term suggests that mass in this formalism is related to intrinsic curvature associated with a particle state. This property of the formalism is reminiscent of the gravitational equivalence principle and is further evidence that the equivalence postulate formalism provides a natural framework for quantum gravity.

## 11. Conclusions

Should we believe in the relevance of string theory in nature ? The most urgent issue in particle physics is the nature of the electroweak symmetry breaking mechanism. The basic question is whether fundamental scalar states exist in nature, or whether a more intricate, yet unperceived, mechanism plays a role. This question must and will be resolved by our experimental colleagues, who have already provided us with a glorious confirmation of the Standard Model gauge and matter sectors. The Standard Model multiplet structure strongly indicates the realization of grand unifying structures in nature, and augmented with supersymmetry is the leading candidate for a theory of electroweak symmetry breaking, to be tested by future experiments. String theory provides the most advanced tools to study quantum, gravity and gauge, unification. The fact that one finds string models that closely resemble the real world, and exactly where expected, namely near a maximally symmetric point, leads to the intriguing suspicion that string theory is indeed relevant in nature.

A plausible view of recent years deeper understandings in string theory is that different theories that are classically distinct, are in fact connected quantum mechanically. It is then natural to promote this new insight to a level of a fundamental physical principle. From such a principle the

correct theory of quantum gravity should be derivable, as well as the phenomenological characteristics of quantum field theories, and the fundamental vexing problems, like the vanishing of the cosmological constant and the problem of mass. At the closing of one millennium, it seems that the new one may still offer plenty of surprises.

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