Reply to Comment on "Duality of x and ψ in Quantum Mechanics"

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The purpose of this note is to prevent possible confusion that may arise from the misunderstanding in [1] whose content is the derivation of Eq.(13) in [2] by direct differential calculus: which is precisely the same method we used to derive it (it is in fact difficult to imagine any other possible derivation). Because of the triviality of the derivation the details were not given in [2] and only the sentence "In particular, by inverting the Schrödinger equation we obtain" was given before Eq.(13). On the other hand, it is well known that the inversion of a differential equation is made by using $\psi'(x) = 1/x'(\psi)$ etc. (it would be bizarre to look for other methods). The fact that Eq.(12) in [2] precedes the explanation of its derivation presumably caused the misunderstanding in [1] where it is erroneously stated that Eq.(13) was derived in [2] from "techniques inspired by field theory duality". Actually, according to [2], substituting

$$\frac{\sqrt{2m}}{\hbar}x(\psi_E) = \frac{1}{2}\psi_E\frac{\partial \mathcal{F}_E}{\partial \psi_E} - \mathcal{F}_E,$$

in the (inverted) Schrödinger equation $\frac{\hbar^2}{2m}\partial^2_{\psi_E}x = \psi_E(E - V(x))(\partial_{\psi_E}x)^3$ (=Eq.(13) in [2]), one obtains $4\mathcal{F}'''_E + (V(x) - E))\left(\mathcal{F}'_E - \psi_E\mathcal{F}''_E\right)^3 = 0$. We therefore clarify that no use of the prepotential $\mathcal{F}_E$ was made in the derivation of (13), which is only one step in [2] and from which (after substituting (6) into (13)) the differential equation for $\mathcal{F}_E$ follows, and not vice versa (which is, of course, well understood, e.g. [3]). Finally, we note that in [4] no reference to the pilot–wave interpretation of quantum mechanics has been made. Referring in such a way to [4] is the interpretation of the authors of [1].

References

[1] J.L. Lucio and M. Ruiz–Altaba, hep-th/9712219.

[2] A.E. Faraggi and M. Matone, *Phys. Rev. Lett.* **78**, 163 (1997).

[3] M. Matone, *Phys. Lett.* **B357**, 342 (1995); *Phys. Rev.* **D53**, 7354 (1996).
G. Bonelli and M. Matone, *Phys. Rev. Lett.* **76**, 4107 (1996).
K. Ito and S.–K. Yang, hep-th/9603073.
R. Carroll, *Nucl. Phys.* **B502**, 561 (1997).

[4] A.E. Faraggi and M. Matone, hep-th/9705108.