# Price-sensitive demand and market entry* 

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#### Abstract

This paper revisits the optimal entry decision in a differentiated product market where customer demand is price-sensitive and depends on a per-unit transport cost. We show that too few firms may enter for high entry cost and high transport cost compared to the socially optimal outcome.

JEL classification: L11; L13. Keywords: Circular city; Horizontal product differentiation; Market entry; Price-sensitive demand.


## 1 Introduction

Spatial models of horizontal product differentiation, such as Hotelling (1929) or Salop (1979), have become workhorse models in regional science, spatial economics as well as in industrial organization. Given this popularity,

[^0]spatial models have been ap plied to a wide variety of applications, and numerous variants of such models have been studied. ${ }^{1}$

In this context, an important question is whether free entry leads to the optimal level of product variety. In spatial models of product differentiation, such as Vickrey (1964) and Salop (1979), a well-known result is the so-called excess entry result suggesting that the equilibrium level of entry exceeds the socially optimal level. ${ }^{2}$

Recently, a number of contributions have challenged this view. In their article, Matsumura and Okamura (2006b) analyze a spatial model of product differentiation under various assumptions on transport costs and production cost functions. They show that, although there is excessive entry under many circumstances, an insufficient level of product variety can also arise. Matsumura (2000) shows that insufficient entry can arise if the integer problem is taken into account. Moreover, Gu and $\operatorname{Wenzel}(2009,2012)$ show that the excess entry result may not hold in a framework where customer demand is price-sensitive and transport costs come as a fixed cost to customers, independent of the quantity that is purchased. ${ }^{3}$ Finally, deliveredprice competition is considered in Matsumura and Okamura (2006a) in a setting with a linear demand function. They find that entry can be insufficient when the entry cost is large.

As van der Weijde et al. (2014) point out, one limitation in these contributions is that, although the distance between customers and the product they buy brings about disutility, the quantity a customer demands is independent of the transport costs. In many applications, however, customer demand depends naturally on both the selling price and the degree of matching between the customer and the product. For example, consider two products $A$ and $B$. For the same price, if product $A$ is a better substitute

[^1]to a customer's ideal product than product $B$, not only does this customer choose $A$ over $B$; she may also buy more of product $A$ compared to having to purchase the more distant product $B$.

To address this limitation, we revisit the question of free and optimal entry in a setting where customer demand is price-sensitive and where, in contrast to previous contributions, transport costs to customers are on a per-unit basis. This means that customers have to incur a transport cost (disutility costs) for each unit they purchase. As such, our setup can thus be interpreted in two ways. First, it may represent product differentiation along a taste dimension where a transport cost per unit representing the mismatch of product characteristic and a customer's preference is a suitable assumption. Second, it may reflect a situation where a customer incurs actual transportation costs per unit shipped (geographical interpretation), i.e., shipping a greater number of items results in an increase in the shipping costs. ${ }^{4}$

We use a model of product differentiation with price-sensitive demand and a per-unit transport cost. For tractability we focus on a linear demand function. In this setting, we characterize the price equilibrium and the level of entry in a free-entry equilibrium. We provide a comparison of equilibrium entry and the socially optimal level of entry. Our main finding is that for a high market entry cost, entry need not be excessive but can be insufficient. More precisely, we show that entry is insufficient if the transport cost per unit is high as in precisely those cases customers would strongly benefit from more entry. At the same time, however, firms' incentives to enter are low due to low profits.

It is instructive to compare the results of this paper with models where the transport costs come as a fixed cost ( Gu and Wenzel, 2012). ${ }^{5}$ In both approaches, insufficient entry occurs for sufficiently high transport costs, but for different reasons. With fixed transport costs, the reason for insufficient product variety comes mainly from the customer side as with high transport

[^2]costs, prices are high and demand is low so that customers would benefit a lot from more entry and the associated lower prices. The incentives to enter, though, are still relatively strong as firm profits increase with the level of transport costs. The effects are different with per-unit transport costs. Here, changes in the transport-cost parameter or changes in the number of firms directly affect customer demand and hence welfare (whereas these parameters affect customer demand only indirectly through the price in Gu and Wenzel, 2012). As a consequence, high transport costs lead to low firm profits (due to low local demand) and low incentives to enter the market. At the same time, welfare strongly increases with entry as local demand rises. Taken together, we show that these two effects can lead to insufficient entry if the transport cost is high enough.

The novelty of the paper is to analyze equilibrium entry in a setting with price-sensitive demand and a per-unit transport cost. As such, it may contribute to the growing literature on price-sensitive demand in models of horizontal product differentiation ${ }^{6}$ and help to better understand how robust the results are when the impact of the transport costs on customer utility is changed.

## 2 Model

There are $n \geq 2$ price-setting firms which are equidistant from each other along the circular city of unit length, i.e., the distance between any two neighboring firms is equal to $1 / n$ (Salop, 1979). Their marginal costs of production are normalized to zero.

Customers of mass one are uniformly distributed along the circle. All customers buy from one of the firms in the market (see Assumption 1 below). Let $q$ denote the quantity a customer demands at a given price and location. A customer who is located at $x$ and who buys from firm $i$ at location $L_{i}$

[^3](with $i \in\{1, \ldots, n\}$ ) derives the following utility
\[

$$
\begin{equation*}
U\left(x, p_{i}\right)=q-\frac{q^{2}}{2}-q\left(p_{i}+\tau\left|L_{i}-x\right|\right) \tag{1}
\end{equation*}
$$

\]

where $\tau$ is the transport-cost parameter and $p_{i}$ denotes the price charged by firm $i$. The utility specification implies that a customer incurs the transport costs for every unit purchased, i.e., transport costs increase linearly in the quantity purchased. One may interpret $\tau$ as the cost a customer incurs when shipping one unit of the product to his location. Alternatively, it may be viewed as the disutility the customer suffers when he buys a product that does not fully match his ideal product. Then, $q \tau\left|L_{i}-x\right|$ represents the total transport costs borne by a customer located at $x$ when buying a product located at $L_{i}$.

As customers are assumed to be utility maximizers, the above utility specification implies that a customer who is located at $x$ and buys from firm $i$ has the following local demand

$$
\begin{gather*}
\max _{q} U\left(x, p_{i}\right) \Rightarrow \frac{\partial U}{\partial q}=1-q-p_{i}-\tau\left|L_{i}-x\right|=0 \\
\Rightarrow q\left(x, p_{i}\right)=1-p_{i}-\tau\left|L_{i}-x\right| \tag{2}
\end{gather*}
$$

Note that with a per-unit transport cost, individual demand decreases as the difference between the customer's location (preferences) and the actual product location (characteristics) grows. In contrast, the quantity a customer demands when transport costs are on a lump-sum basis depends only on the price.

We make the following assumption:

Assumption 1. Transport costs are such that the market is covered, i.e., $0<$ $\tau \leq 8 / 3=: \bar{\tau}$.

The assumption that the market is covered means that in equilibrium for any number of firms, all customers along the circle buy from one firm, i.e., not buying at all is not optimal. However, as can be seen from expression
(2), this is only true if transport costs are not too high. ${ }^{7}$

In what follows, we will analyze the following two-stage free-entry game. In the first stage, firms decide whether to enter the market. In the second stage, firms compete in prices.

## 3 Results

In this section, we first derive the findings from the free-entry game to compare them to the first-best and the second-best outcomes.

## Free entry

We start by analyzing customers' purchasing decisions. As we are interested in a symmetric equilibrium, suppose that all firms but firm $i$ located at $L_{i}=0$ charge the same price $p$. Then, plugging the local demand specified in expression (2) into the utility expression (1) implies that the indifferent customer between firm $i$ and its neighboring firm at $1 / n$ is located at a distance $\tilde{x}$ :

$$
U\left(\tilde{x}, p_{i}\right)=U(\tilde{x}, p) \Leftrightarrow \hat{x}=\frac{1}{2 n}-\frac{p_{i}-p}{2 \tau} .
$$

Total demand at firm $i$ by customers from the left and the right in the second stage is thus given by

$$
Q_{i}=2 \int_{0}^{\tilde{x}}\left(1-p_{i}-\tau x\right) d x
$$

Firm $i^{\prime}$ s profits amount to $\pi_{i}=p_{i} Q_{i}$. Let $A:=\sqrt{4 n^{2}-4 n \tau+13 \tau^{2}}$. Proceeding in the standard way to derive the optimal symmetric equilibrium prices (i.e., $p_{i}=p$ ), we have the following price in the second stage of the game ${ }^{8}$

$$
\begin{equation*}
p=\frac{2 n+3 \tau-A}{4 n} \tag{3}
\end{equation*}
$$

[^4]It is useful to analyze the comparative statics properties of the equilibrium price.

Observation 1. Define $\hat{\tau}:=2(1+3 \sqrt{3}) n / 13 \approx 0.95325 n$ and $\hat{n}:=(3 \sqrt{3}-$ 1) $\tau / 4 \approx 1.04904 \tau$.
(i) Comparative statics with respect to transport costs $\tau$ give

$$
\frac{\partial p}{\partial \tau} \begin{cases}>0 & \text { if } 0<\tau<\hat{\tau} \\ =0 & \text { if } \tau=\hat{\tau} \\ <0 & \text { if } \hat{\tau}<\tau \leq \bar{\tau}\end{cases}
$$

(ii) Comparative statics with respect to the number of firms $n$ give

$$
\frac{\partial p}{\partial n} \begin{cases}>0 & \text { if } 2 \leq n<\hat{n} \\ =0 & \text { if } n=\hat{n} \\ <0 & \text { if } n>\hat{n} .\end{cases}
$$

Proof: Ad (i): it holds that $\partial p / \partial \tau=(2 n-13 n \tau+3 A) / 4 n A$. Solving $\partial p / \partial \tau=0$ for $\tau$ gives $\hat{\tau}$ as the only solution that lies within the bounds specified by Assumption 1. Furthermore, it holds that $\partial p / \partial \tau \rightarrow 1 / n>0$ for $\tau \rightarrow 0$ and $\partial^{2} p / \partial \tau^{2}=-\sqrt{3} / 6 n^{2}<0$ for $\tau=\hat{\tau}$.
Ad (ii): it holds that $\partial p / \partial n=-(2 n-13 \tau+3 A) / 4 n^{2} A$. Solving $\partial p / \partial n=$ 0 for $n$ gives $\hat{n}$ as the only solution that satisfies the condition that $n \leq$ 2. Moreover, for $n \rightarrow \infty$, it is true that $\partial p / \partial n \rightarrow 0$. Also, it holds that $\partial^{2} p / \partial n^{2}=-128 \sqrt{3} /(3 \sqrt{3}-1)^{4} \tau^{2}<0$ for $n=\hat{n}$.

The properties of the equilibrium price differ from the standard model with inelastic demand (see, e.g., Tirole, 1988) as well as from the setup where elastic demand depends only on the price but not on transport costs (see, at an equal distance of $1 / 2 n$ to two neighboring firms demands a quantity of

$$
q\left(\frac{1}{2 n}, p\right)=1-\frac{2 n+5 \tau-A}{4 n} .
$$

Solving $q(1 / 2 n, p) \geq 0$, which is required to ensure a covered market, for $\tau$ gives two solutions: 0 and $4 n / 3$. For $0<\tau \leq 4 n / 3$, it holds that $q(1 / 2 n, p) \geq 0$. Setting $n=2$ yields the upper bound given in Assumption 1.
e.g., Gu and Wenzel, 2009). Whereas in those cases, the price unambiguously increases in the transport-cost parameter $\tau$, here the price may increase or decrease in $\tau$. This is because transport costs have two effects. On the one hand, larger values of $\tau$ relax competition leading firms to increase prices. On the other hand, larger values of $\tau$ also reduce individual demand which forces firms to reduce their prices. For low values of $\tau$, the first effect dominates whereas for high values of $\tau$, the second effect dominates. ${ }^{9}$

Similarly, the effect of additional competitors on prices is ambiguous. As in the standard model, more firms means tougher competition because firms are located closer to each other. On the other hand, additional competitors also allow firms to target only those customers with high local demand. For markets with few firms, the second effect dominates. However, as more firms enter the market, the first effect becomes stronger and this effect eventually dominates.

The fact that the equilibrium price can be increasing in the number of firms is an unexpected result in circular models. When disutility from mismatching is independent of the quantity consumed as in Gu and Wenzel (2012), the equilibrium price unequivocally decreases in the number of firms. The reason for this new result is that, in previous models, individual demand is independent of the distance traveled although they may be price dependent. In contrast, in the current model, both per-unit transport cost and the distance traveled have an impact on individual demand given by expression (2). As a consequence, the overall demand can be less elastic when customers travel less as the number of firms increases. ${ }^{10}$

Firm profits in the second stage amount to

$$
\pi=\frac{(2 n+3 \tau-A)(2 n-4 \tau+A)}{16 n^{3}} .
$$

[^5]Observation 2. Define $B:=\sqrt[3]{7552+273 \sqrt{993}}$ and $\check{\tau}:=\left(-257+B^{2}+\right.$ $4 B) n / 39 B \approx 0.49008 n$.
(i) Comparative statics with respect to transport $\operatorname{costs} \tau$ give

$$
\frac{\partial \pi}{\partial \tau} \begin{cases}>0 & \text { if } 0<\tau<\check{\tau} \\ =0 & \text { if } \tau=\check{\tau} \\ <0 & \text { if } \check{\tau}<\tau \leq \bar{\tau}\end{cases}
$$

(ii) Comparative statics with respect to the number of firms $n$ give

$$
\frac{\partial \pi}{\partial n}<0
$$

Proof: Ad (i): it holds that $\partial \pi / \partial \tau=\left(14 n^{2}+n A-21 n \tau-25 \tau A+91 \tau^{2}\right) / 8 n^{3} A$. Solving $\partial \pi / \partial \tau=0$ for $\tau$ gives $\check{\tau}$ as the only solution that lies within the bounds specified by Assumption 1. Next, it is true that $\partial \pi / \partial \tau \rightarrow 1 / n^{2}>0$ for $\tau \rightarrow 0$ and $\partial^{2} \pi / \partial \tau^{2} \approx-0.5641 / n^{3}<0$ for $\tau=\check{\tau}$.
Ad (ii): it holds that $\partial \pi / \partial n=\tau\left(-56 n^{2}-4 n A+70 n \tau+75 \tau A-273 \tau^{2}\right) / 16 n^{4} A$. Solving $\partial \pi / \partial n=0$ for $n$ gives two (real) solutions: $\tau / 4$ and $3 \tau / 4$. Note that not even the greater of the two solutions satisfies the condition that $n \geq 2$ as $3 \tau / 4 \geq 2 \Leftrightarrow \tau \geq 8 / 3$ which contradicts Assumption 1. Note last that for $n=2$ and $\tau=1, \partial \pi / \partial n=(67-17 \sqrt{21}) / 256 \approx-0.0426$.

The parameter $\tau$ also has an ambiguous impact on firm profits. Note that profits are particularly low when $\tau$ is relatively large. This observation will later be helpful to explain why entry may be insufficient. Interestingly, an increase in the number of competitors may lead to higher prices for customers but firms' profits do not increase at the same time. This means that the negative effect on profits from a lower market share outweighs the positive price effect.

Turning to the firms' entry decision in the first stage, assume that market entry comes at a fixed cost $f$ (with $f>0$ ). We focus on those cases where at least two firms enter which can be guaranteed by assuming sufficiently low values of the entry cost $f$. As firm profits are strictly decreasing in the number of firms, the maximum relevant entry cost is given for $n=2$.

Denote this cost by $\bar{f}$. Hence, at least two firms enter the market as long as

$$
f \leq \frac{\tau\left(4-25 \tau+7 \sqrt{16-8 \tau+13 \tau^{2}}\right)}{128}=: \bar{f}
$$

Since profits are strictly decreasing in the number of firms, the unique equilibrium number of firms $n^{*}$ is implicitly characterized as the solution to ${ }^{11}$

$$
\pi\left(n^{*}\right)-f=0 .
$$

## First-best entry

We define social welfare as the sum of customer surplus and firm profits. Unlike models with inelastic demand, prices matter due to their impact on the quantities purchased. In the first-best welfare benchmark, social welfare consists of total entry costs and customer welfare where the product price is set equal to the marginal cost of production, i.e., $p^{F}=0$ (where superscript $F$ stands for first-best). Social welfare can then be expressed as

$$
\begin{equation*}
W^{F}=2 n \int_{0}^{\frac{1}{2 n}} \frac{(1-\tau x)^{2}}{2} d x-n f=\frac{12 n^{2}-6 n \tau+\tau^{2}}{24 n^{2}}-n f . \tag{4}
\end{equation*}
$$

Maximizing expression (4) with respect to the number of firms and defining $C:=\sqrt[3]{\tau f(-3 \tau f+\sqrt{-3 \tau f(1-3 \tau f)})}$, the first-best level of entry $n^{F}$ is given by

$$
\begin{equation*}
n^{F}=\frac{\sqrt[3]{3}\left(\sqrt[3]{3} \tau f+C^{2}\right)}{6 f C} \tag{5}
\end{equation*}
$$

As the free-entry level is only given implicitly, we cannot directly compare the entry decision in the free-entry equilibrium and in the first-best scenario. Instead, we consider $\pi\left(n^{F}\right)-f$. As firm profits strictly decrease in the number of firms, it holds that whenever $\pi\left(n^{F}\right)-f<0$, there is insufficient entry in the free-entry equilibrium and vice versa. Since the analytical expressions are cumbersome, we present the comparison graphically in Figure 1 below. Note that, in the figure, $f^{F}$ is defined as the fixed entry cost where

[^6]the scope of entry in the first best is equal to the free-entry equilibrium. For any $f<f^{F}$, there is excessive entry; for any $f^{F}<f \leq \bar{f}$, an insufficient number of firms enter the market.

The figure shows that for relatively low levels of the fixed cost $f$, there is always excessive entry. For higher levels of the fixed cost, whether entry is excessive or insufficient depends on the transport costs. If the transport costs are sufficiently large (small), we observe insufficient (excessive) entry. The occurrence of insufficient entry can be explained by the observation that firm profits are decreasing in the transport-cost parameter for high values of $\tau$ (see Observation 2). With large values of $\tau$ customers would benefit to a large extent from additional market entry but the incentives to enter are low. As a consequence, the equilibrium level of entry falls short compared to the optimal outcome.

We summarize this finding:

Result 1. Market entry is excessive compared to the first-best outcome for low values of the entry cost $f$ and/or low values of the transport costs $\tau$. For high values of these parameters, market entry is insufficient.

## Second-best entry

We now consider the optimal entry level under a second-best benchmark where prices are set according to expression (3). Second-best social welfare (superscript $S$ ) is given as

$$
\begin{align*}
W^{S} & =2 n \int_{0}^{\frac{1}{2 n}} \frac{(1-p-\tau x)^{2}}{2} d x+2 n p \int_{0}^{\frac{1}{2 n}}(1-p-\tau x) d x-n f  \tag{6}\\
& =\frac{12 n^{2}+6 n A-24 n \tau+9 \tau A-31 \tau^{2}}{48 n^{2}}-n f . \tag{7}
\end{align*}
$$

Since we cannot derive an explicit expression for $n^{S}$ from the maximization of expression (6), we instead consider $\partial W^{S} / \partial n$ at $n=n^{F}$ to compare the socially optimal scope of entry in the first-best and second-best regimes. It holds that whenever $\partial W^{S} / \partial n$ at $n=n^{F}$ is larger than zero, more entry is
desirable in the second best and vice versa. ${ }^{12}$
We state the following result:

Result 2. The second-best level of entry is higher than the first-best level.


Figure 1: Entry comparison in the three scenarios: entry is insufficient (excessive) compared to the first-best benchmark if $f^{F}<f \leq \bar{f}\left(f<f^{F}\right)$ and insufficient (excessive) compared to the second-best benchmark if $f^{S}<f \leq$ $\bar{f}\left(f<f^{S}\right)$.

It immediately follows that free entry can also be insufficient compared to a second-best benchmark and that the parameter range where entry is insufficient is larger than for the first-best comparison. Figure 1 provides a graphical illustration. In the figure, $f^{S}$ is defined as the fixed entry cost where the level of entry in the second best is equal to the free-entry equilibrium. For any $f<f^{S}$, too many firms enter the market from a social-welfare

[^7]

Figure 2: Entry comparison under free entry and in the second-best scenario for $f=0.055$.
perspective; for any $f^{S}<f \leq \bar{f}$, there is insufficient entry. From the figure we can thus summarize:

Result 3. Market entry is excessive for low values of the entry cost $f$ and/or low values of the transport costs $\tau$ and insufficient for high values of these parameters compared to the second-best outcome.

Note that this result extends previous findings. In settings with fixed transport costs, entry levels can also be insufficient if the entry cost or transport costs are high ( Gu and Wenzel, 2012). ${ }^{13}$ Thus, a more general picture emerges in models with price-sensitive demand. Independent of who bears the transport costs and independent of whether the transport costs are on a fixed or a per-unit basis, insufficient entry may arise for high levels of transport costs. The reason for insufficient entry differ, however, between models with fixed transport costs and with per-unit transport costs. With fixed transport costs, insufficient entry arises as with high transport costs, prices are high and customer demand is low whereas firm profits are high. Contrary to that, with per-unit transport costs, insufficient entry arises as the incentives to enter the market (that is, firm profits) are very low with

[^8]high transport costs because of the resulting downward shift of the local demand function (see Observation 2). Note, however, that in both types of models, the same main message (though for different reasons) emerges, i.e., insufficient entry arises under high transport costs.

Figure 2 provides an alternative illustration of the comparison of market vs. optimal entry. For a given fixed cost of entry of $f=0.055$, the left panel shows equilibrium and second-best entry; the right panel plots the difference between these two entry levels. The figure shows that the degree of insufficient entry can become quite large for high values of the transport cost parameter $\tau .{ }^{14}$

## Extension: More general utility function

In this short extension, we provide some evidence that assuming a more general utility function does not result in qualitatively different results. A customer who is located at $x$ and who buys from firm $i$ at location $L_{i}$ derives the following utility

$$
U\left(x, p_{i}\right)=\frac{\alpha q}{\beta}-\frac{q^{2}}{2 \beta}-q\left(p_{i}+\tau\left|L_{i}-x\right|\right),
$$

where $\alpha, \beta>0$.
Given that customers are utility maximizers, this customer has the following local demand when buying from firm $i$

$$
q\left(x, p_{i}\right)=\alpha-\beta p_{i}-\beta \tau\left|L_{i}-x\right| .
$$

Proceeding in the exact same way as in the previous part, we can compare the scope of entry in the different scenarios. ${ }^{15}$ Figure 3 illustrates the scope of entry in the private optimum and the second best for two concrete examples ( $\alpha=1, \beta=0.5 ; \alpha=1.5, \beta=1$ ). In the figure, the upper bound

[^9]on the entry cost $\bar{f}$ as well as the critical $f^{S}$ where the scope of entry is the same under free entry and in the second-best benchmark are given for the two cases. As can be seen from the figure, there is again insufficient market entry whenever both entry and transport costs are not too low. Moreover, the relative scope of insufficient entry remains the same in these two alternative scenarios. ${ }^{16}$ We thus conclude that introducing parameters $\alpha$ and $\beta$ to allow for a more general utility specification does not qualitatively affect our results concerning insufficient entry.


Figure 3: Comparison of entry decisions in the free-entry equilibrium and the second-best benchmark for two cases: (i) $\alpha=1, \beta=0.5$ (solid curves); (ii) $\alpha=1.5, \beta=1$ (dashed curves).

## 4 Conclusion

This paper revisits market and optimal entry in a circular-city model where customer demand is price-sensitive and also depends on the transport costs (per unit) a customer incurs. The main finding is that entry is not always

[^10]excessive. So far, this has only been shown for the case where transport costs are on a lump-sum basis independent of the quantity. The current paper extends this result to the case where transport costs are incurred for each unit. Given that circular models of product differentiation are widely used, the results of this paper may be of practical use for researchers in the fields of industrial organization, regional science, and transport economics.

Our model fits markets where the quantities demanded by individual customers depend on a unit price as well as the degree of matching between product characteristics and individual preferences (such as food and drinks; visits to restaurants, pubs, and cinemas; travel demands). Our analysis suggests that the price in these markets is not necessarily monotonic in transport costs or the number of firms. As a policy implication, our paper suggests that entry in markets, where product differentiation is of high importance to consumers (high transport cost), may be insufficient, and subsidizing entry in such market may be welfare-enhancing.

## References

Anderson, S. P., de Palma, A., and Thisse, J.-F. (1992). Discrete Choice Theory of Product Differentiation. MIT Press.

Biscaia, R. and Mota, I. (2013). Models of spatial competition: A critical review. Papers in Regional Science, 92(4):851-871.

Braid, R. (2014). Search costs decrease prices in a model of spatial competition. Papers in Regional Science, 93(1):125-139.

Chen, Y. and Riordan, M. H. (2007). Price and variety in the spokes model. Economic Journal, 117(522):897-921.

Colombo, S. (2011). Spatial price discrimination in the unidirectional Hotelling model with elastic demand. Journal of Economics, 102(2):157169.

Colombo, S. (2014). Location choices with a non-linear demand function. Forthcoming Papers in Regional Science.

Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic competition and optimum product diversity. American Economic Review, 67(3):297-308.

Esteves, R.-B. and Reggiani, C. (2014). Elasticity of demand and behaviourbased price discrimination. International Journal of Industrial Organization, 32:46-56.
$\mathrm{Gu}, \mathrm{Y}$. and Wenzel, T. (2009). A note on the excess entry theorem in spatial models with elastic demand. International Journal of Industrial Organization, 27(5):567-571.
$\mathrm{Gu}, \mathrm{Y}$. and Wenzel, T. (2012). Price-dependent demand in spatial models. B. E. Journal of Economic Analysis \& Policy, 12(1). Article 6.

Hamilton, J. H., Thisse, J.-F., and Weskamp, A. (1989). Spatial discrimination: Bertrand vs Cournot in a model of location choice. Regional Science and Urban Economics, 19:87-102.

Hotelling, H. (1929). Stability in competition. Economic Journal, 39:41-57.
Janssen, M. C. W., Karamychev, V. A., and van Reeven, P. (2005). Multi-store competition: Market segmentation or interlacing? Regional Science and Urban Economics, 35(6):700-714.

Matsumura, T. (2000). Entry regulation and social welfare with an integer problem. Journal of Economics, 71(1):47-58.

Matsumura, T. and Okamura, M. (2006a). Equilibrium number of firms and economic welfare in a spatial price discrimination model. Economics Letters, 90(1):396-401.

Matsumura, T. and Okamura, M. (2006b). A note on the excess entry theorem in spatial markets. International Journal of Industrial Organization, 24(5):1071-1076.

Rasch, A. and Herre, J. (2013). Customer-side transparency, elastic demand, and tacit collusion under differentiation. Information Economics and Policy, 25(1):51-59.

Rothschild, R. (1997). Product differentiation and cartel stability: Chamberlin versus Hotelling. Annals of Regional Science, 31(3):259-271.

Salop, S. (1979). Monopolistic competition with outside goods. Bell Journal of Economics, 10(1):141-156.

Sattinger, M. (1984). Value of an additional firm in monopolistic competition. Review of Economic Studies, 51(2):321-332.

Shilony, Y. (1981). Hotelling's competition with general customer distributions. Economics Letters, 8:39-45.

Tirole, J. (1988). The Theory of Industrial Organization. MIT Press.
van der Weijde, A. H., Verhoef, E. T., and van den Berg, V. A. C. (2014). A Hotelling model with price-sensitive demand and asymmetric distance costs: The case of strategic transport scheduling. Journal of Transport Economics and Policy, 48(2):261-277.

Vickrey, W. S. (1964). Microstatics. Harcourt, Brace and World, New York.


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[^1]:    ${ }^{1}$ For instance, studies have focused on the structure of transport costs and customer distribution (e.g, Shilony, 1981; Matsumura and Okamura, 2006b), the introduction of search costs (Braid, 2014), delivered pricing models (Hamilton et al., 1989; Colombo, 2014), or multiproduct competition (Janssen et al., 2005). A recent overview is provided by Biscaia and Mota (2013).
    ${ }^{2}$ This is different from representative customer and discrete choice models. There, it is understood that equilibrium product variety could either be excessive, insufficient, or optimal depending on the model configuration (see, e.g., Dixit and Stiglitz, 1977; Sattinger, 1984). For an overview, see Anderson et al. (1992).
    ${ }^{3}$ Note that in the original setup and in many applications, it is assumed that each customer demands one unit of a differentiated product, independent of the price.

[^2]:    ${ }^{4}$ This is different from another geographical view which is not covered here. For instance, regarding spatial competition among supermarkets or fast-food restaurants with an identical product range on offer, fixed transport costs covering the actual transportation (the cost of driving to a supermarket/restaurant) is a reasonable modeling assumption.
    ${ }^{5}$ Note, however, that the magnitude and parameter values across the two models are not directly comparable since the two approaches are not nested.

[^3]:    ${ }^{6}$ Recent analyses which explore the impact of price-sensitive demand on market outcomes while employing either a per-unit or a fixed transport cost approach are Colombo (2011) (on the effects of spatial price discrimination), Rasch and Herre (2013) (on the stability of collusion), and Esteves and Reggiani (2014) (on the effects of behavior-based pricing). van der Weijde et al. (2014) analyze the scheduling decisions of competing transport providers when customers' demand depends on timing.

[^4]:    ${ }^{7}$ Specifically, Assumption 1 ensures that, for any number $n \geq 2$ of firms, even the customer with the largest travel distance to a firm will buy a non-negative amount at the equilibrium price (3). Note that this assumption is more restrictive than necessary but we use it for simplicity. As we will point out below (see the next footnote), the upper bound is increasing in the number of firms $n$, i.e., $\tau \leq 8 / 3$ represents the lowest value. As the number of firms is determined endogenously, the upper bound could be endogenized too.
    ${ }^{8}$ Note that in the symmetric equilibrium with $n$ firms, the indifferent customer located

[^5]:    ${ }^{9}$ See also Rothschild (1997) or Rasch and Herre (2013) for a related discussion.
    ${ }^{10}$ We note that an increase in the number of firms can also lead to a higher equilibrium price for a range of reservation prices in the spokes model (Chen and Riordan, 2007). In such cases in the spokes model, as the number of firms becomes higher, the monopoly segmentwhere demand is surprisingly more elastic-shrinks and the competitive segment expands and as a result, the overall demand elasticity reduces. In contrast, in the current paper, as the number of firms increases, marginal customers are located closer to the firms and demand a higher quantity at any given price. Hence, individual demand becomes less elastic, in particular, when there are just a few firms active in the market $(n<\hat{n})$.

[^6]:    ${ }^{11}$ Note that we have shown in Observation 2 that $\partial \pi(n, \tau, f) / \partial n<0$. Hence, the implicit function theorem guarantees the existence of a unique $n^{*}(\tau, f)$ such that $\pi\left(n^{*}(\tau, f), \tau, f\right)$ $f=0$.

[^7]:    ${ }^{12}$ Analytical results are not obtainable, so we numerically verified that indeed $\partial W^{S} / \partial n>$ 0 holds at $n=n^{F}$ for all permissible parameter values. The result is summarized in Figure 1. The figure shows that $f^{S}<f^{F}$ and that, in particular, both lines do not intersect. This implies that the second-best level of entry is higher than the first-best level.

[^8]:    ${ }^{13}$ With delivered-price competition, insufficient entry also arises if fixed costs are large (Matsumura and Okamura, 2006a).

[^9]:    ${ }^{14}$ Again, we stress that Assumption 1 is stronger than needed as even at the highest level of transport costs, the marginal customer would demand a strictly positive quantity. Indeed, we could extend the analysis to higher values of the transport costs without violating the assumption of a covered market. This would enlarge the area with insufficient entry.
    ${ }^{15}$ Note that the modified utility function requires a slightly adapted assumption regarding the transport-cost parameter. It must hold that $0<\tau \leq 8 \alpha / 3 \beta$.

[^10]:    ${ }^{16}$ Note that we have not found a meaningful parameter set where (qualitative) results changed.

