

1 **Population density, mobility, and cultural transmission**

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30 **Abstract**

31 Prompted by the results of a series of recently published simulation models, there is an increasing
32 tendency for archaeologists to invoke demographic variables as explanations for changes in the
33 sophistication or complexity of material culture. Whilst these models are undoubtedly valuable, this
34 paper draws attention to persistent failings in the interpretation and application of these models by
35 archaeologists. Despite having quite different effects, variables such as population size and
36 population density are often used interchangeably; and whilst increasing mobility has an effect
37 broadly equivalent to that of increasing population density, it is rarely given sufficient weight in
38 archaeological explanations of cultural change. The analyses reported here develop a series of new
39 simulations based on the ideal gas model, allowing for an explicit prediction of the encounter rate –
40 the variable for which population density and mobility are proxies, and which ultimately governs the
41 rate of cultural transmission. This model supports the predictions of earlier studies on the effects of
42 population density and mobility, but suggests that population size will have no effect on rates of
43 cultural transmission. These simulations are coupled with analyses that demonstrate a reciprocal
44 correlation between population density and mobility in a large hunter-gatherer dataset. Given this
45 correlation, it is argued that archaeological inferences about cultural transmission based on just one
46 of these variables are unlikely to be valid. These findings are discussed in the context of previous
47 research, and it is suggested that future studies would gain greater explanatory power by focusing
48 explicitly on the social network structures likely to have characterised a particular archaeological
49 population.

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51 **Keywords:** Population density; population size; mobility; encounter rate; cultural transmission;
52 hunter-gatherer; simulation.

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63 Population density, mobility, and cultural transmission

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65 1. Introduction

66 Over the past 15 years, an increasing number of archaeologists have invoked variables such as
67 population density or population size as explanations for changes in material culture. Increases in
68 these variables, it is argued, will lead to greater sophistication or complexity in toolkits, in individual
69 tools, or in methods of tool manufacture (e.g. Shennan 2001; Henrich 2004; James and Petraglia
70 2005; Zilhão 2007; Powell et al. 2009; Langley et al. 2011). The current trend can be traced to the fall
71 of the ‘human revolution’ model, and in particular to the failure of the associated theory that the
72 fluorescence of artistic and symbolic expression during the Upper Palaeolithic of Europe could be
73 attributed to a sudden and dramatic increase in cognitive abilities (e.g. Klein, 2000). Given that
74 *Homo sapiens* appears shortly after 200 ka, the sporadic appearance (and disappearance) of
75 apparently ‘modern’ technologies prior to c.60 ka requires a set of candidate explanations that are
76 extrinsic to the biology of our species; demographic variables are rapidly colonising this niche, to the
77 extent that Palaeolithic archaeology appears to be approaching a new orthodoxy (see Collard et al.
78 2013; French, 2015).

79 Citations demonstrate that three papers have been instrumental in influencing archaeological
80 thought on the relationship between cultural change and demography in recent years: those of
81 Shennan (2001), Henrich (2004), and Powell and colleagues (2009). Shennan (2001) adapted a
82 genetic model of the evolution of sex (Peck et al. 1997), allowing oblique transmission (i.e.
83 transmission of cultural information from an elder who is not necessarily a genetic parent) to
84 influence the ‘fitness’ of a series of cultural traits. Varying the effective population size – taken in
85 this model to be the subset of the overall population that are likely to act as ‘cultural parents’ – and
86 running the models until they reached stationary distributions, Shennan (2001) found that the
87 geometric mean fitness taken across all traits was higher in larger populations. This relationship is
88 approximately logarithmic, with the greatest increases in fitness occurring when very small
89 populations increase in size. Despite the potential archaeological salience of Shennan’s (2001)
90 model, subsequent research has focused on overall skill levels rather than multiple cultural traits.
91 Shennan’s (2001) explicit assumption that cultural change is an evolutionary process, however,
92 remains implicit in more recent work.

93 Henrich (2004) develops a model in which each individual attempts to copy the most skilful member
94 of the population in each iteration. Although the model assumes that the most skilful individual is
95 always accurately identified, copying is subject to error, and more complex skills are harder to copy.
96 By analogy with the genetic process most copying errors are detrimental, but occasionally “through
97 a combination of imperfect imitation, experiments, errors, bad memories and ill fortune” (Henrich
98 2004:200) an individual will produce a copy that is better than the original. Given an original skill
99 value z , the possible outcomes of copying are given by a Gumbel (extreme value) distribution with
100 location parameter $z - \alpha$ and scale parameter β . The complexity of a skill – how hard it is to imitate
101 – is thus measured by α , whilst the extent to which the skill is subject to copying error is governed by
102 β . (For those unfamiliar with the Gumbel distribution, the most relevant description for the
103 purposes of Henrich’s (2004) model is that it is the distribution created by repeatedly choosing the
104 highest value (the ‘extreme value’) from a series of normally distributed random numbers; if one

105 were to generate 100 random numbers from a normal distribution, then discard all but the single
106 highest number among them, and repeat this process multiple times, the numbers retained would
107 follow a Gumbel distribution.) Using a simplified but accurate approximation of the Gumbel
108 distribution, Henrich (2004:202) demonstrates that $\Delta\bar{z} = -\alpha + \beta(\varepsilon + \ln(N))$, where $\Delta\bar{z}$ is the
109 average change in skill level per iteration, N is population size, and ε is the Euler-Mascheroni
110 constant. Skill level thus increases logarithmically with increasing population size. This result is
111 remarkably close to that of Shennan (2001) despite the considerable differences between the two
112 models.

113 Powell and colleagues (2009) introduced a stochastic meta-population simulation analogous to
114 Henrich's (2004) analytical model. This simulation employs a series of equally sized subpopulations
115 and examines the effects of variation in the number and density of subpopulations and the extent of
116 migratory activity between them. Three particularly important results arise from this simulation.
117 Firstly, increasing the number of subpopulations only causes increases in skill level whilst the
118 number of subpopulations is less than approximately 50, indicating that "the accumulation, or
119 maintenance, of culturally inherited skill is not dependent on the absolute meta-population size"
120 (Powell et al. 2009:1300). Whilst 'cultural fitness' increases approximately logarithmically with
121 population size in Shennan's (2001) model, and skill level increases exactly logarithmically with
122 population size in Henrich's (2004) model, Powell and colleagues (2009) demonstrate that in a
123 structured meta-population skill level approaches an asymptote relatively quickly, and is unaffected
124 by further increases in population size.

125 Secondly, Powell and colleagues (2009) demonstrate that sub-population density has a far more
126 consistent effect on the accumulation of skill than does meta-population size; skill levels in high
127 density areas were consistently higher in these simulations, regardless of the values of the
128 parameters α and β . Thirdly, and perhaps most importantly for archaeological interpretation, these
129 authors find that greater migratory activity also leads to higher skill levels, and that this effect is
130 particularly pronounced when the skill being copied is of greater complexity (i.e. when α is higher).
131 These simulations thus provide a series of insights that are of vital importance to archaeological
132 interpretation, and they have, accordingly, been widely utilised.

133 A number of researchers have identified problems with the models of Shennan (2001), Henrich
134 (2004), and Powell and colleagues (2009); broadly speaking, these problems can be divided into the
135 theoretical and the empirical. From a theoretical perspective, Vaesen (2012) has noted a series of
136 mathematical issues with the performance of the Henrich (2004) equation. Several authors have also
137 noted that none of these models make reference to the underlying ecology (Vegvari and Foley 2014,
138 Collard et al. 2013). Empirically, Collard and colleagues (2013; Vaesen et al. in press) note that, of the
139 studies examining both demographic variables and variables indexing material culture complexity in
140 extant hunter-gatherer populations, few have found positive correlations between the two. Whilst
141 these problems merit further attention, the purpose of the current paper is to draw attention to a
142 further issue which stems from the interpretation and application of the models by archaeologists
143 rather than from the models themselves. This issue arises from the neglect of migration (or, on a
144 smaller scale, mobility) in the vast majority of archaeological analyses that have made use of the
145 results of these models.

146 In the most comprehensive model published to date, Powell and colleagues (2009:1300) make it
147 abundantly clear that “migratory activity among a set of subpopulations can have the same effect on
148 skill accumulation as increasing the size of a single population”. These authors are also explicit about
149 the limited effect of overall population size. Yet archaeological analyses seeking explanations for
150 cultural change focus primarily on population size, secondarily on population density, and rarely, if
151 at all, discuss mobility (e.g. Zilhão 2007; Langley et al. 2011). While there are some more nuanced
152 applications in the archaeological literature – Riede (2008), for example, stresses the sudden
153 decrease in connectedness following the Lacher See eruption of 12,920 BP, and Hopkinson (2011;
154 Hopkinson et al. 2013) focuses on the spatial interaction of locally and regionally separated
155 populations – the majority of studies opt for population size *or* density (often inter-changeably) as
156 the sole explanation. As the analyses reported below demonstrate, population *density* and mobility
157 must be considered as joint, interacting factors in any valid explanation of cultural change.

158 2. Deriving predictions

159 The hypothesis that high population densities will increase rates of cultural transmission has a clear
160 intuitive appeal: when population densities are high, individuals will encounter one another more
161 often, with each encounter affording an opportunity to transmit cultural information. The same logic
162 underlies the related hypothesis that high mobility rates will increase rates of cultural transmission.
163 Both population density and mobility, therefore, are proxies for the individual encounter rate: it is
164 this latter variable that actually controls the rate of cultural transmission. Following similar logic,
165 although population size is not a direct proxy for an individual’s encounter rate, it might affect the
166 number of encounters that occur at the *population level* per unit time. Encounter rates can be
167 modelled directly via the ideal gas model (IGM) developed originally by particle physicists but
168 employed routinely by primatologists (e.g. Waser 1976; Dunbar 1995; Harcourt and Greenberg 2001;
169 Gursky 2005) and increasingly by anthropologists (e.g. Grove 2010; Grove et al. 2012; Pearce 2014).
170 The IGM is used here to derive predictions about the relationships between population density,
171 mobility, and population size on the one hand, and encounter rates and rates of cultural
172 transmission on the other.

173 The gas model states that an individual’s encounter rate is $E_{ind} = \frac{8\rho Dv}{\pi}$, where ρ is density, v
174 velocity and D is the radius within which the individual can detect other individuals. For current
175 purposes, the constants 8 and π are ignored, and we replace detection distance and velocity with a
176 single mobility parameter $M = Dv$. In this simplified form, $E_{ind} \propto \rho M$, the IGM provides a very basic
177 but useful prediction:

- 178 1. Individual encounter rate has positive, linear relationships with both density and mobility
179 ($E_{ind} \propto \rho, E_{ind} \propto M$);

180 Modifying the Henrich (2004) equation such that the average change in skill level per iteration is
181 related to encounter rate rather than population size gives $\Delta\bar{z} = -\alpha + \beta(\varepsilon + \ln(\rho M))$. This form is
182 henceforth referred to as the Modified Henrich Equation (MHE), and yields a pair of predictions for
183 the scaling of average change in skill level with population density and mobility:

- 184 2. Average change in skill level per iteration has positive, logarithmic relationships with both
185 density and mobility ($E_{ind} \propto \beta \ln(\rho), E_{ind} \propto \beta \ln(M)$).

186 (More fully, $E_{ind} = \underbrace{\beta \ln(\rho)}_{Slope} + \underbrace{\beta(\varepsilon + \ln(M)) - \alpha}_{Constant}$ and $E_{ind} = \underbrace{\beta \ln(M)}_{Slope} + \underbrace{\beta(\varepsilon + \ln(\rho)) - \alpha}_{Constant}$, but it is
187 really the generic scaling relationships that are of interest here).

188 The basic IGM does not include a term for population size. However, the number of individuals
189 inhabiting an area A at density ρ will be ρA . The total number of encounters that occur between
190 individuals in this area per unit time will therefore be the individual encounter rate multiplied by ρA .
191 Maintaining the above simplifications, this gives the total number of encounters per unit time at the
192 population level as $E_{pop} \propto A\rho \cdot \rho M$. The average number of encounters experienced by an
193 individual, however, is E_{pop} divided by population size, $E_{ind} \propto \frac{A\rho \cdot \rho M}{A\rho} = \rho M$; the population size
194 term thus falls out of the equation governing individual encounter rate, as above. The IGM therefore
195 suggests that, whilst the overall rate of encounters within the population increases with increasing
196 population size, the individual encounter rate does not. Since it is the individual encounter rate that
197 determines changes in skill level, this leads to a third prediction:

198 3. Individual encounter rate, and thus average change in skill level per iteration, will have no
199 relationship with population size.

200 Finally, in order to explore this framework using the abundant data on hunter-gatherer population
201 density and mobility, a direct relationship between these two variables is derived. There is
202 considerable variation in both population density and mobility among hunter-gatherer groups, but if
203 encounter rates are important for the maintenance of skill levels, it is to be expected that these will
204 be relatively invariant between groups. Following Henrich (2004), it is considered that a lowering of
205 skill levels could be highly detrimental; it is therefore hypothesized that population density and
206 mobility will covary so as to maintain relatively consistent encounter rates among hunter-gatherers.
207 The simplified IGM shows that encounter rate is the product of population density and mobility. To
208 maintain a constant encounter rate when one of these variables changes, the other variable must
209 scale reciprocally. This yields a fourth prediction to be tested via hunter-gatherer data:

210 4. To maintain a constant encounter rate, the relationship between density and mobility must
211 be reciprocal ($\rho \propto M^{-1}$, or, equivalently, $M \propto \rho^{-1}$).

212 Figure 1 plots this relationship, demonstrating that high density, low mobility populations can
213 experience encounter rates identical to those of low density, high mobility populations. Predictions
214 1-4 are tested below via a combination of simulations and empirical analyses.

215 3. Methods

216 3.1. Simulations

217 n individuals were placed randomly in a square grid with the density of individuals in the upper half
218 of the grid being ten times that in the lower half (following Powell et al. 2009). Each individual
219 interacts only with the neighbours falling within its mobility radius, r . In terms of the hunter-
220 gatherer system, most of these individuals are likely to be kin by blood or marriage, since the degree
221 of kinship will be higher between geographically proximate individuals. Although it does not affect
222 the results of the model, a pattern of isolation by distance as proposed theoretically by Wright
223 (1943; see also Malécot 1969) and confirmed empirically among numerous hunter-gatherer groups

224 by Morton and colleagues (e.g. Morton 1969; Lalouel and Morton 1973; see also Relethford 2004) is
225 assumed. To avoid edge effects, the grid is represented as a torus. Each iteration, each individual
226 copies the skill value, z^* , of the most skilled individual within its radius, obtaining a new value drawn
227 from a Gumbel distribution with location parameter $z^* - \alpha$ and scale parameter β . This is an exact
228 implementation of the model of skill transmission and accumulation described in detail by Henrich
229 (2004).

230 In **Simulation 1**, r is equal for all individuals, irrespective of population density. In **Simulation 2**, the
231 mobility radius in the low density area is set such that the mobility *area* is ten times greater than
232 that in the high density area. This ensures that, on average, individuals in the low and high density
233 areas will have the same number of neighbours from which to copy. **Simulation 3** replicates
234 **Simulation 1**, but with a population four times as large. To maintain population densities identical to
235 those of **Simulation 1**, the increase in population size is achieved by simply increasing the area of the
236 simulation grid (and therefore the number of individuals) by a factor of four. All individuals begin
237 with a skill level of zero, and each simulation is run for 100 iterations. As noted by Powell and
238 colleagues (2009), a duration of 100 iterations is sufficient to yield stable and representative output,
239 and is not prohibitive in terms of computation time.

240 Changes in skill level were recorded for each individual in each iteration, together with the skill level
241 of each individual after 100 iterations. To ensure that comparisons between high and low density
242 areas were not affected by individuals in low density areas who had neighbours in high density areas
243 (or vice versa), individuals that were located at distances less than the mobility radius from the edge
244 of their respective density area were excluded. Statistical analyses were then performed on
245 simulation output so as to test the following predictions of the above listed hypotheses.

246 A significantly higher skill level in the high density area than in the low density area in **Simulation 1**
247 would be consistent with the hypothesis that increasing population density increases rates of
248 cultural transmission. If this first result is confirmed, lack of a significant difference between skill
249 levels of the high and low density areas in **Simulation 2** would be consistent with the hypothesis that
250 increasing mobility increases rates of cultural transmission, negating the differences caused by
251 population density. A lack of significant differences in skill levels between the high population
252 density areas in **Simulations 1** and **3** and between the low population density areas in **Simulations 1**
253 and **3** would be consistent with the hypothesis that increasing population size has no effect on rates
254 of cultural transmission, regardless of variation in population density.

255 As expected from the use of Henrich's (2004) model, the skill levels of individuals in both high and
256 low density areas throughout all simulations were Gumbel-distributed; as such, non-parametric
257 statistics were used to test the above predictions. Running multiple simulations in parallel and
258 performing Wilcoxon Rank-Sum tests on high and low density populations after each iteration
259 demonstrated that relative skill levels were prone to considerable stochastic variance. When running
260 **Simulation 2**, for example, it was not unusual for the individuals in the high density area to have
261 significantly higher skill levels in one iteration, but to have significantly *lower* skill levels a few
262 iterations later (or vice versa), leading to both positive and negative Z-values in the output of the
263 Rank-Sum tests over the course of a simulation. To ensure the statistical analyses tested long-term
264 trends rather than short-term variance, the following procedure was adopted:

- 265 1. A Wilcoxon Rank-Sum test for each of the comparisons listed above was performed after
266 each of the 100 iterations of each simulation;
- 267 2. The Z-values resulting from the Rank-Sum tests were then aggregated for each comparison
268 and subjected to one-sample Sign tests;
- 269 3. If the stochastic variance of a given model during a given iteration was equally likely to lead
270 to a positive or a negative Z-value, the Sign test should fail to reject the null hypothesis that
271 the median of the aggregated Z-values is zero;
- 272 4. If there is a genuine long-term difference between skill levels in any given comparison, by
273 contrast, the Sign test should significantly reject this null hypothesis.

274 Finally, to complement the above pair-wise comparisons of particular population density, mobility,
275 and population size values detailed above, an extensive set of simulations were carried out in which
276 each parameter was varied independently, over a wider set of parameter values. With the other
277 parameters at constant values, population density was varied between 0.1 and 4 individuals per unit
278 square in increments of 0.1, the mobility area between 0.1 and 4 units squared in increments of 0.1,
279 and population size between 100 and 2000 individuals in increments of 50. These analyses were
280 used to test the wider validity of the findings of the pair-wise analyses, and to provide more robust
281 tests of Predictions 1-3. All simulations were programmed in Matlab; code is freely available from
282 the author. Statistical analyses were carried out in SPSS 22.1.

283 3.2. *Ethnographic analyses*

284 Ethnographic data on population density and mobility were taken from Binford's *Frames of*
285 *Reference* database (2001:118-129). This database collates a vast amount of information on hunter-
286 gatherer ecology, subsistence, and sociality, encompassing elements of many earlier databases and
287 publications. Binford's DENSITY variable records the number of individuals per 100km²; this was
288 converted to individuals per km² prior to analysis. Binford's (2001) DISMOV variable records the
289 distance moved per year in miles; these measurements were converted to kilometres prior to
290 analysis. Only cases in which groups were fully mobile (Binford's GRPPAT=1) and had DISMOV values
291 >0 were used, yielding a sample of 175 groups. To test the hypothesis that $M \propto \rho^{-1}$ (or vice versa),
292 the two variables were logarithmically transformed and $\ln(M)$ was regressed on $\ln(\rho)$. The slope of
293 the resulting linear regression equation (the unstandardized B value) thus equates to the exponent
294 of a power law fitted to the raw data, and can be evaluated against the theoretical prediction. The
295 data were considered to be consistent with the theoretical predictions if the predicted exponents
296 fall within the 95% confidence intervals of the empirical, unstandardized B values. As before,
297 statistical analyses were carried out in SPSS 22.1.

298 4. Results

299 4.1. *Simulations*

300 For consistency, all plots and statistics are given for simulations in which $\alpha = 4$ and $\beta = 2$. Repeated
301 simulations with α and β values in the range 2-5 (i.e. with $\frac{\alpha}{\beta}$ values ranging from 0.4 to 2.5) did not
302 affect the directionality or significance of the results presented below. In Figures 2-4, panel A shows
303 the locations and skill levels of all individuals after 100 iterations, whilst panel B shows the

304 distributions of changes in skill level per individual per iteration separately for the high and low
305 density areas.

306 Figure 2 shows a representative result of **Simulation 1**. Figure 5A shows the distribution of Z-values
307 resulting from the Wilcoxon Rank-Sum tests performed after each of the 100 iterations of the
308 simulation. A Sign test performed on this distribution indicates that the median is significantly
309 different from zero ($Z = -9.38, p < 0.001$), confirming the prediction that skill levels are lower in the
310 low density area. Figure 3 shows a representative result of **Simulation 2**. Figure 5B shows the
311 distribution of Z-values resulting from the Wilcoxon Rank-Sum tests. The Sign test demonstrates that
312 the median of this distribution is not significantly different from zero ($Z = 1.16, p = 0.25$), confirming
313 the prediction that higher levels of mobility can negate the effects of low population density and
314 lead to equal skill levels in the high and low density areas.

315 Figure 4 shows a representative result of **Simulation 3**. Figure 5C shows the distribution of Z-values
316 resulting from the Wilcoxon Rank-Sum tests comparing the low density area in **Simulation 1** to the
317 low density area in **Simulation 3**; Figure 5D shows a similar plot comparing the two high density
318 areas. Sign tests performed on these distributions indicate that in neither case is the median
319 significantly different from zero (low densities $Z = -0.95, p = 0.34$; high densities $Z = -0.32, p = 0.75$).
320 This result is consistent with the prediction that skill levels are not affected by population size. Figure
321 6A explicitly compares the distributions of change in skill level per individual per iteration for the low
322 density areas of Simulation 1 and Simulation 3; Figure 6B fulfils a similar function for the high density
323 areas. Figure 6 lends clear visual support to the statistical finding that population size does not affect
324 skill levels in these simulations.

325 Figure 7 expands the pairwise comparisons discussed above to examine wider ranging differences in
326 population density, mobility, and population size, thus testing Predictions 1-3 over a wider range of
327 parameter values. Figures 7a and 7b show linear increases in encounter rate with increasing
328 population density and mobility respectively, confirming Prediction 1. Figures 7d and 7e show
329 logarithmic increases in skill level with increasing population density and mobility respectively,
330 confirming Prediction 2. Note however that in both Figures 7d and 7e the simulation results depart
331 from the theoretical expectation at very low values; possible reasons for this discrepancy are
332 discussed further below. Figures 7c and 7f show that population size does not affect either
333 encounter rate or skill level, confirming Prediction 3.

334 *4.2. Ethnographic analyses*

335 The gas model suggests that distance moved per year should scale with the reciprocal of population
336 density. Figure 8 plots the relationship between these two variables for the 175 hunter-gatherer
337 groups employed in the analysis. The regression of distance moved per year on population density
338 (performed as a linear regression on logarithmically transformed data) is significant ($\beta = -.794$,
339 $t(173) = -17.184, p < .001$), explaining 63.1% of the variance ($R^2 = .631, F(1,173) = 295.284, p < .001$). As
340 predicted, the relationship is a power law with a negative exponent; however, the exponent is
341 approximately half the predicted value of -1 (unstandardized $B = -.461$, 95% confidence interval [-
342 .514, -.408]). Possible reasons for this discrepancy are discussed below.

343

344 **5. Discussion**

345 The above results are consistent with Predictions 1-3; there is qualitative but not quantitative
346 support for Prediction 4 (see Table 1 for a summary of predictions and results). Accordingly, the
347 section below discusses potential reasons for this quantitative discrepancy. Following sections
348 discuss the implications of the analyses presented here for archaeological interpretation and
349 highlight micro-scale deviations from expectations of the MHE that should inform future research.

350 *5.1. Sociality, subsistence, and mobility in hunter-gatherers*

351 The result shown in Figure 8 suggests that mobility scales with population density as $M \propto \rho^{-0.5}$, a
352 significant departure from the prediction of $M \propto \rho^{-1}$. A possible explanation for this departure
353 stems from a discrepancy in the way mobility is measured between the IGM and the empirical data.
354 Binford's (2001) DISMOV variable is a measure of the *distance* covered, whereas mobility in the IGM
355 is ideally defined as an *area* covered. If DISMOV is converted to an area variable by simply squaring
356 it, the regression of LN(DISMOV²) on LN(Population Density) yields the same statistics as above ($\beta=-$
357 $.794$, $t(173)=-17.184$, $p<.001$; $R^2=.631$, $F(1,173)=295.284$, $p<.001$), but with a unstandardized B value
358 that includes -1 within its 95% confidence interval (unstandardized $B=-0.921$, 95% CI [-0.815, -
359 1.027]). This adjustment therefore supports Prediction 4. Simply squaring DISMOV, however, is
360 unlikely to provide a particularly reasonable estimate of the area covered by a foraging group (and
361 although Binford (2001) gives an AREA estimate, this is the total area inhabited by the whole
362 population).

363 Rather than attempting to convert the DISMOV variable into an area, a more useful approach
364 involves converting the population density estimate into a distance between groups. Thompson
365 (1956) demonstrated that the mean distance D to the n th nearest neighbour in a randomly
366 distributed population at density ρ is $D(n) = \frac{1}{\sqrt{\rho}} \cdot \frac{n \cdot (2n)!}{(2^n \cdot n!)^2}$. Hubbell and colleagues (2008) showed
367 that qualitatively similar patterns hold even when distributions are non-random. The important
368 feature of this equation is that the value of n simply provides a constant to be multiplied by $\frac{1}{\sqrt{\rho}}$;
369 verbally, the distance to any of n neighbouring groups is related linearly to the reciprocal of the
370 square root of population density (henceforth RSPD). Regressing LN(DISMOV) on LN(RSPD) for the
371 hunter-gatherer dataset yields identical statistics again, with the exception that the β , t , and B values
372 are now positive ($\beta=.794$, $t(173)=17.184$, $p<.001$; $R^2=.631$, $F(1,173)=295.284$, $p<.001$). As such, the
373 unstandardized B value now includes 1 within its 95% confidence interval (unstandardized $B=0.921$,
374 95% CI [0.815, 1.027]).

375 That the B value of a linear regression on logarithmically transformed variables is close to 1 suggests
376 that the relationship may be better characterised by a linear regression on the raw (untransformed)
377 data. This regression should pass through the origin, since if $\ln(\text{DISMOV}) = 1 \times \ln(\text{RSPD}) + \ln(\delta)$,
378 then $\text{DISMOV} = \delta \times \text{RSPD}^1 = \delta \times \text{RSPD}$; there is no constant in this equation. Such a relationship
379 would suggest that DISMOV is a simple multiple of RSPD and that, accordingly, hunter-gatherers
380 adjust the distances they move in direct proportion to the distances to neighbouring groups.
381 Regressing DISMOV on RSPD and forcing the line through the origin produces a significant
382 relationship ($\beta=.922$, $t(174)=31.456$, $p<.001$), explaining 85% of the variance in DISMOV ($R^2=.850$,
383 $F(1,174)=989.463$, $p<.001$). This relationship is plotted in Figure 9.

384 Prediction 4 was predicated upon the hypothesis that hunter-gatherers might adjust their
385 population densities and mobility strategies so as to maintain a given number of encounters per unit
386 time. Converting DISMOV into an area metric or, more realistically, converting Population Density
387 into a distance metric, provides a result that is consistent with this hypothesis. Although regression
388 analyses do not permit assessment of causality, the most realistic explanation for this result is that
389 population densities (and hence neighbour distances) are determined by the distribution of
390 resources, with mobility strategies adjusted so as to ensure a sufficient number of encounters with
391 neighbouring groups. However, a further hypothesis – not tested here – must also be considered. It
392 remains possible that both population densities *and* mobility strategies are determined by resource
393 densities, with the relationship between them arising purely from this common dependence. This
394 latter hypothesis has a certain intuitive appeal: when resources are at low density, foragers
395 dependent on those resources should also exist at low density; furthermore, with resources at low
396 density a forager must cover a greater area in order to satisfy a given energetic requirement (e.g.
397 Grove 2009, 2010). Equivalently, when resources are at high density we might expect high
398 population densities and low mobility. Further analyses will be required to determine whether
399 hunter-gatherer mobility is constrained by sociality, subsistence, or some combination thereof.

400 Although the annual distance moved (DISMOV) is the metric most appropriate for use with the ideal
401 gas model, it may also in future be possible to build the number of residential moves per annum
402 (Binford's (2001) NOMOV variable) into models of this kind. Whilst a higher frequency of residential
403 moves would not necessarily impact directly upon the number of encounters an individual
404 experiences (in the current model, two individuals can encounter each other on numerous
405 occasions), it might be expected to increase the number of *different* individuals that a given
406 individual encounters per unit time. This could have interesting additional effects on the speed with
407 which cultural variants spread through a population. Given that there is a positive correlation
408 between DISMOV and NOMOV, at least within the sample employed above, inclusion of this variable
409 in future models would likely accentuate the positive effect of high mobility on rates of cultural
410 transmission described above.

411 *5.2. Implications of results for archaeological analyses*

412 Regardless of its ultimate causes, the result that there exists a reciprocal correlation between
413 population density and mobility among extant hunter-gatherers has serious consequences for
414 archaeological interpretation. This result leads to a conclusion that can be stated in strong or weak
415 forms. The weak form states that archaeological analyses that consider only population density *or*
416 mobility as proxies for rates of cultural transmission are invalid; only joint consideration of these
417 variables can lead to informed analyses. The strong form states that since population density and
418 mobility covary so as to equalise encounter rates, neither is a useful proxy for rates of cultural
419 transmission. The weak form of the conclusion is fully supported by the analyses reported above; the
420 strong form is also supported by these analyses, but it should be noted that the results highlight only
421 a general pattern. The scatters around the regression lines in Figures 8 and 9 demonstrate that there
422 does exist some variation in encounter rates, and that as such population density and mobility may
423 retain some explanatory power in certain specific contexts.

424 The simulation results reported above demonstrate the independent effects of population density,
425 mobility, and population size on rates of cultural transmission. These results are in broad agreement

426 with those of Powell and colleagues (2009), despite the fact that these authors consider subgroups
427 interacting within a meta-population structure, whereas the model presented above considers
428 individuals interacting within a single continuous population. The broad agreement between the two
429 models is therefore reassuring, and suggests that when fine-grained differences in encounter rates
430 between archaeological societies can be discerned their consequences can be reliably inferred.

431 The results reported here do, however, differ markedly from those of Shennan (2001) and Henrich
432 (2004) in terms of the effects of population size. The disparity between the above results and those
433 of Henrich (2004) is in fact more apparent than real, and can be easily resolved. It is established
434 above that population density and mobility are simply proxies for the encounter rate, with the latter
435 being the actual determinant of rates of cultural transmission. Population size is not a proxy for
436 encounter rate simply because an individual does not encounter all other members of her
437 population on a timescale relevant to cultural transmission, if ever. Instead, the cultural influence of
438 distant members of a population occurs via diffusion, and does not depend on the adopter of an
439 idea coming into direct contact with its originator. Henrich's (2004) equation does, however, have
440 the potential to provide a very useful method for examining the combined effects of encounter rate
441 and imperfect copying. By replacing 'population size' with 'encounter rate', in what is referred to
442 above as the MHE, the benefits of this model can be maintained and strengthened.

443 The disparity between the above results and those of Shennan (2001) is more complex. Whilst the
444 central problem is again the assumption that an individual will interact with all members of her
445 population, the structure of Shennan's (2001) model is quite different in that it considers the
446 'fitness' of multiple cultural traits rather than a broad measure of skill. As outlined above, the IGM
447 suggests that the size of the population will affect the number of encounters that occur at the
448 population level per unit time for given population density and mobility combinations, even though
449 it will not affect the individual encounter rate. This implies that when a given innovation appears it
450 may spread to a *greater number* of individuals per unit time in a larger population, but that the
451 *proportion* of the population that the innovation spreads to per unit time is unaffected by
452 population size. If the rapid spread of an innovation to a large number of people is important to that
453 innovation's longer-term survival, therefore, it remains possible that innovations occurring in larger
454 populations will be more likely to survive. If it is also assumed that there is a per capita innovation
455 rate, and therefore that innovations are more likely per unit time in larger populations, it is possible
456 that population size could influence rates of cultural innovation and transmission in subtle ways not
457 accounted for in the model presented here.

458 *5.3. Networks and higher-order effects*

459 The results of the simulations presented here are largely consistent with the predictions of the MHE
460 as regards the relationship between encounter rate and skill level. There are some discrepancies,
461 however, and these are most notable towards the ends of simulations that compare high and low
462 density or high and low mobility groups. Figure 10 plots the number of neighbours an individual has
463 against skill level after 1, 10, and 100 iterations of a single run of **Simulation 1**. Unlike Figure 2, in
464 which individuals were discounted if they were positioned within a mobility radius of the boundary
465 of their respective high or low density area, Figure 10 plots all 1100 individuals regardless of
466 location. Figure 10a shows that the MHE is a good model for skill level after a single iteration; Figures
467 10b and 10c, however, show that the MHE is less effective at predicting skill level as the simulations

468 progress. Two observations concerning Figure 10c are particularly important: firstly, some
469 individuals with just one neighbour have a skill level as high as those with as many as 19 neighbours;
470 secondly, the median skill level is relatively constant for individuals with ≥ 4 neighbours.

471 Both these observations are explained by the higher-order effects of skill diffusion along networks of
472 individuals in the simulation. Although an individual can only copy her neighbours, her neighbours
473 can copy their neighbours, and so on throughout the population. This leads to a diffusion of higher
474 skill levels throughout the population which is beyond the scope of either Henrich's (2004) original
475 equation or the MHE presented here. To give an example, with $\alpha = 4$ and $\beta = 2$, an individual (A)
476 with just one neighbour (B) should have a change in skill after one iteration of $z_A = -4 +$
477 $2(\varepsilon + \ln(2)) \approx -1.46$ (an individual with one neighbour has two possible models, herself and her
478 neighbour). This negative number amounts to a maladaptive loss of skill in Henrich's (2004)
479 terminology. If her one neighbour (B) has four neighbours, B's change in skill after one iteration
480 should be $z_B = -4 + 2(\varepsilon + \ln(5)) \approx 0.37$; that is, B experiences an adaptive gain in skill. Assuming
481 that none of B's neighbours has more than four neighbours, B's skill level will continue to grow at a
482 rate of ≈ 0.37 per iteration. Since B will always have a skill level higher than A, A will simply be trying
483 to copy B from iteration 2 onwards; her skill level in subsequent iterations will be $z_A(t) =$
484 $z_B(t - 1) - \alpha + \beta\varepsilon$. This demonstrates that A's skill level will become positive when $z_B(t - 1) +$
485 $\beta\varepsilon > \alpha$; in the example above, this occurs in the fifth iteration. Thus an individual with just one
486 neighbour, and who is predicted by the MHE to have a steadily decreasing skill level, will in fact have
487 a positive skill level after just five iterations due to the presence of a well connected neighbour.

488 Higher-order effects such as this will be common in continuous populations, with high skill levels
489 diffusing further through populations over larger numbers of iterations. Generalising further, if you
490 are connected to a neighbour n vertices away who has a skill level in the current iteration that is
491 greater than $n(-\alpha + \beta\varepsilon)$, your skill level in the next iteration will be positive, regardless of the
492 number of immediate (first-order) neighbours you have. This effect explains the majority of the
493 discrepancies between the MHE and the simulation results shown in Figures 10b and 10c. It also
494 explains the fact that simulated skill levels are higher than those predicted by the MHE at low
495 population densities in Figure 7d and at low mobility values in Figure 7e.

496 There is, however, another effect to be considered at the opposite end of the 'connectedness'
497 spectrum. Whilst those individuals with the fewest neighbours will often be copying their best
498 connected neighbour, those with the most neighbours will often be copying themselves. This effect
499 occurs because in a given iteration the individual with the most neighbours will have the greatest
500 number of models from whom to select; in the subsequent iteration, that individual will inevitably
501 be among the best possible models herself. A clear analogy comes from the work of Pulliam (1988)
502 on 'sources' and 'sinks' in population regulation. In Pulliam's (1988) terms, source areas
503 demonstrate higher birth rates than death rates and thus add to the regional population, whereas in
504 sink areas the reverse is true. In the current model, well-connected individuals act as sources of skill,
505 with their advances radiating out towards less-connected individuals; these less-connected
506 individuals act as sinks, because they absorb these advances but rarely produce any of their own. A
507 necessary corollary is that such well connected 'source' individuals will not be buoyed by their
508 neighbours in the way that 'sink' individuals are; hence the MHE remains a relatively good model for
509 those individuals who have the greatest number of neighbours, even after many iterations (see
510 Figures 10b and 10c). The particular source-sink effects that occur will depend on the topology of the

511 network, and future research into demographic effects on cultural transmission in archaeology
512 would benefit from studies that explicitly compare different network structures (see Cowan and
513 Jonard 2004; Allen et al. 2013).

514 **6. Conclusions**

515 The analyses reported above demonstrate that population density and mobility have independent,
516 equivalent effects on individual encounter rates, and therefore on rates of cultural transmission;
517 conversely, population size has no effect on either of these variables. Population density and
518 mobility are reciprocally correlated in extant hunter-gatherers, suggesting that archaeological
519 inferences about cultural transmission based on just one of these variables are unlikely to be valid.
520 Future research should redress the imbalance caused by a tendency to postulate population density
521 as the sole explanation for variation in the rate of cultural change; mobility should be viewed as an
522 equal partner in this relationship, with the ultimate goal being the explicit characterisation of the
523 network structures characteristic of particular archaeological populations. This endeavour will be of
524 particular importance in future analyses of the emergence and spread of particularly innovative
525 behaviours such as those thought to characterise ‘behaviourally modern’ *Homo sapiens*. Accurately
526 provenanced raw materials act as ideal markers of archaeological mobility (e.g. Pearce and Moutsiou
527 2014), whilst commonalities between assemblages give potential insights into the social connections
528 between groups and how these relate to patterns of geographic separation (e.g. Coward 2013).
529 Consideration of the archaeological proxies for mobility and network structure alongside those for
530 population density will considerably advance our understanding of this critical phase in human
531 evolution.

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640 **Table Caption:**

641 **Table 1.** Summary of the hypotheses and results, with references to the relevant figures.
642 Accompanying statistics are given in the text. *Hypothesis 4 is supported only after conversion of the
643 original distance metric to an area metric; see Section 5.1.

644

645 **Figure Captions:**

646 **Figure 1.** The relationship between population density, mobility, and encounter rate as predicted by
647 the ideal gas model (IGM). The diagonals are encounter rate isoclines. Individual A is a member of a
648 low population density, high mobility population, whilst individual B is a member of a high
649 population density, low mobility population. The two individuals have identical encounter rates.

650 **Figure 2.** Results of Simulation 1. Skill levels after 100 iterations (A) and changes in skill per iteration
651 (B) are significantly higher in the high population density area.

652 **Figure 3.** Results of Simulation 2. With mobility increased so as to equate the average number of
653 neighbours an individual interacts with in the high and low density areas, skill levels after 100
654 iterations (A) and changes in skill per iteration (B) are no longer significantly different.

655 **Figure 4.** Results of Simulation 3. Skill levels after 100 iterations (A) and changes in skill per iteration
656 (B) are significantly higher in the high population density area. Population size is four times that
657 depicted in Figure 2.

658 **Figure 5.** Distributions of Z values from Wilcoxon rank-sum tests performed after each of the 100
659 iterations of Simulations 1-3. (A) compares the low and high density areas of Simulation 1, and (B)
660 the low and high density areas of Simulation 2; (C) compares the low density areas of Simulations 1
661 and 3, and (D) compares the high density areas of these two simulations.

662 **Figure 6.** Comparison of change in skill level per iteration between the small population of
663 Simulation 1 and the large population of Simulation 3. (A) compares the low density areas, (B) the
664 high density areas.

665 **Figure 7.** Results of further simulations demonstrating the wider relationships between population
666 density, mobility, and population size and encounter rates and skill levels. All simulations conform to
667 the expectations of the modified Henrich equation (MHE) with the exception of the relationships
668 between skill level and very low population density (D) and very low mobility (E). Simulation results
669 are shown as median \pm median absolute deviation.

670 **Figure 8.** The relationship between population density (individuals per km²) and mobility (km / year)
671 for the 175 groups in the hunter-gatherer dataset. The inset shows the same relationship on
672 logarithmic axes. Data from Binford (2001: 118-129).

673 **Figure 9.** The relationship between the reciprocal of the square root of population density (km) and
674 mobility (km / year) for the 175 groups in the hunter-gatherer dataset. The regression line show is
675 forced through the origin. Data from Binford (2001: 118-129).

676 **Figure 10.** Skill levels after (A) 1, (B) 10, and (C) 100 iterations of Simulation 1. The red ellipse in (C)
677 highlights individuals with only one neighbour who have skill levels indistinguishable from those with
678 as many as 19 neighbours.

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704 **Tables**

705 **Table 1**

	Hypothesis	Supported?	Figures
	1 Encounter rate has a positive, linear relationship with population density	Yes	7a
	Encounter rate has a positive, linear relationship with mobility	Yes	7b
	2 Average change in skill level has a positive, logarithmic relationship with population density	Yes	2, 4, 7d
	Average change in skill level has a positive, logarithmic relationship with mobility	Yes	3, 7e
	3 Encounter rate has no relationship with population size	Yes	7c
	Average change in skill level has no relationship with population size	Yes	5, 6, 7f
706	4 To maintain constant encounter rate, population density and mobility must scale reciprocally	Yes*	8, 9

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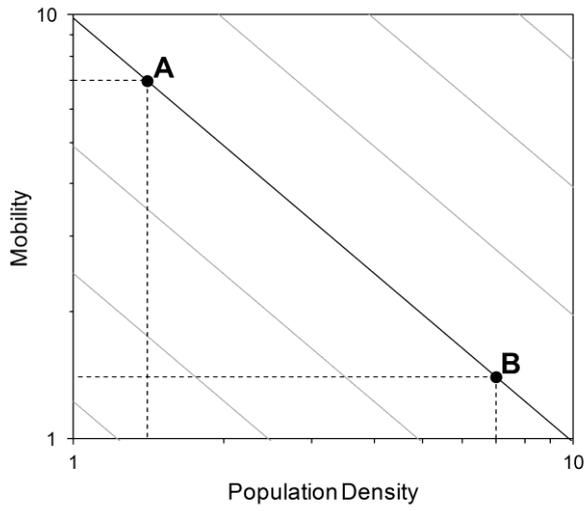
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729 **Figures**

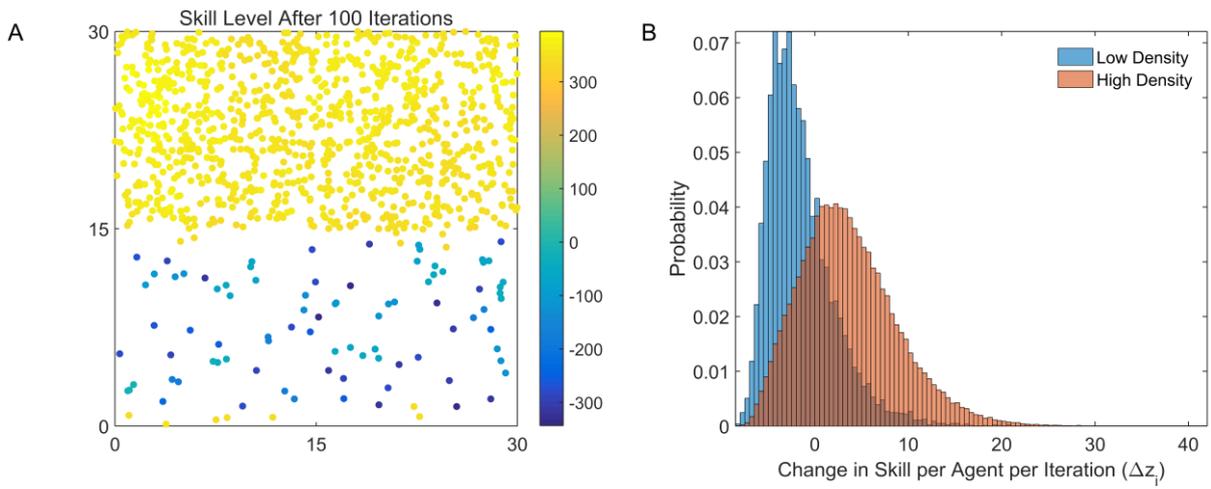
730 **Figure 1**



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733 **Figure 2**



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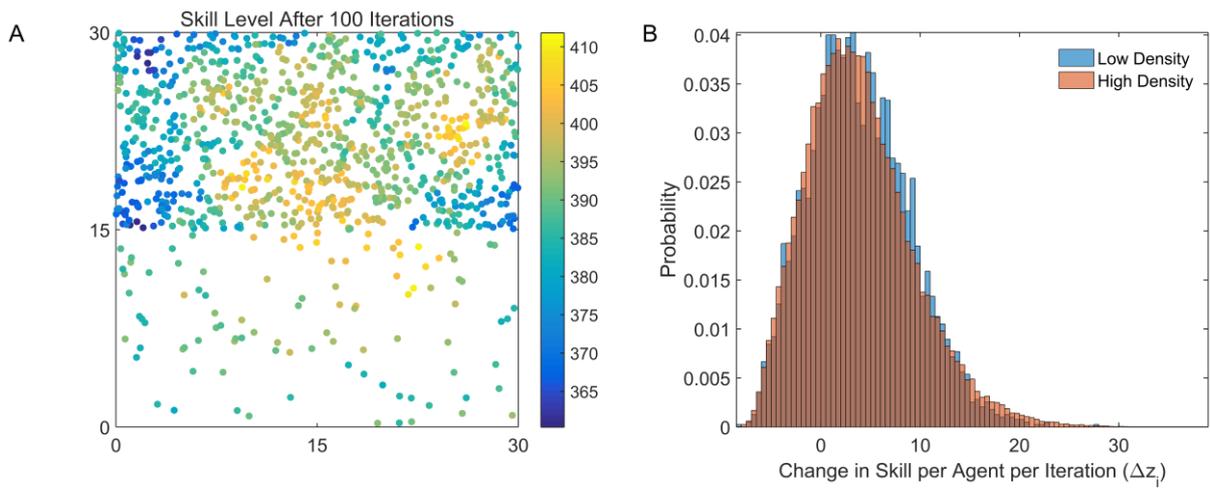
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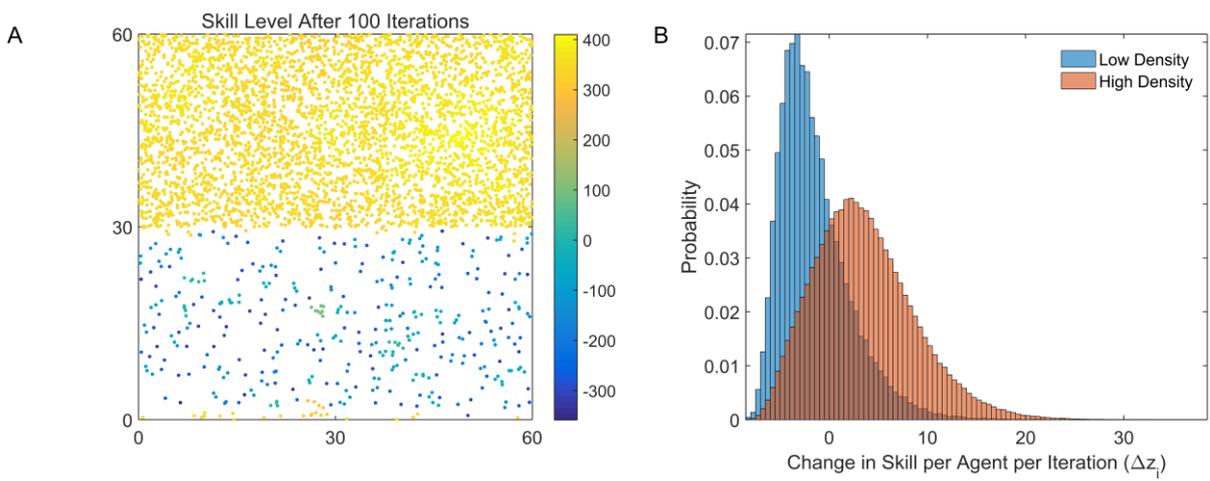
743 **Figure 3**



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746 **Figure 4**



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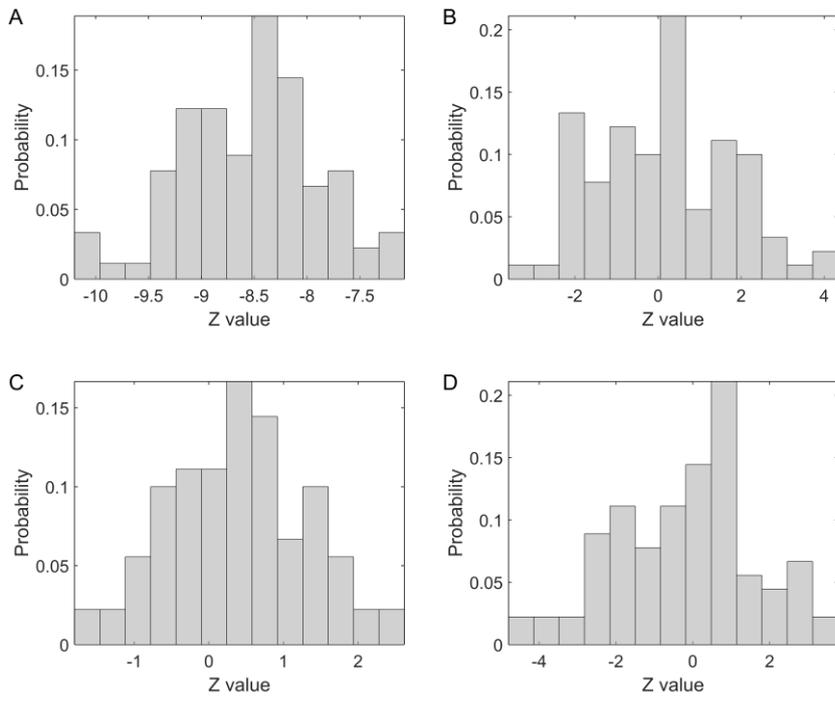
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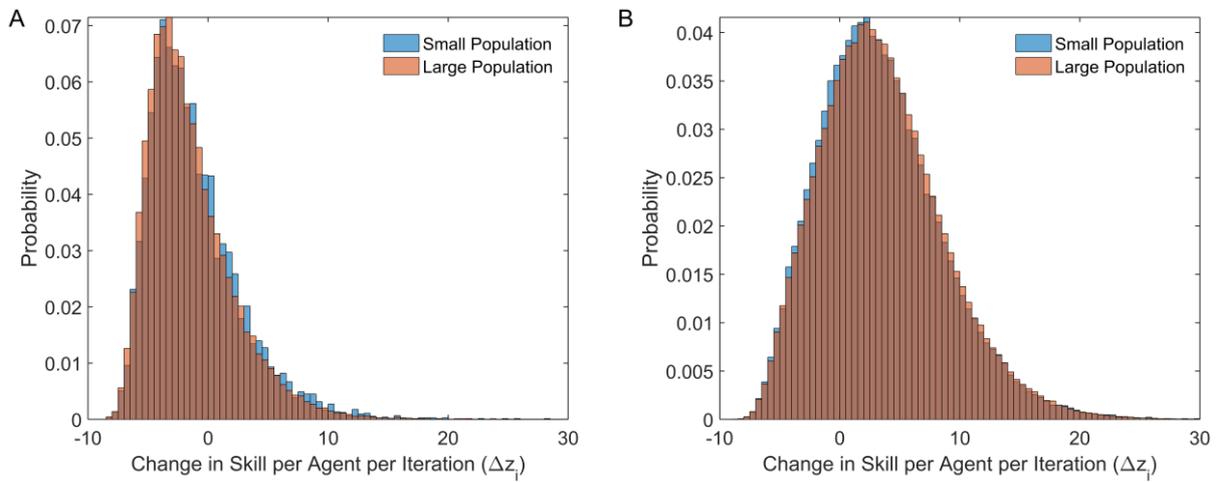
757 **Figure 5**



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760 **Figure 6**



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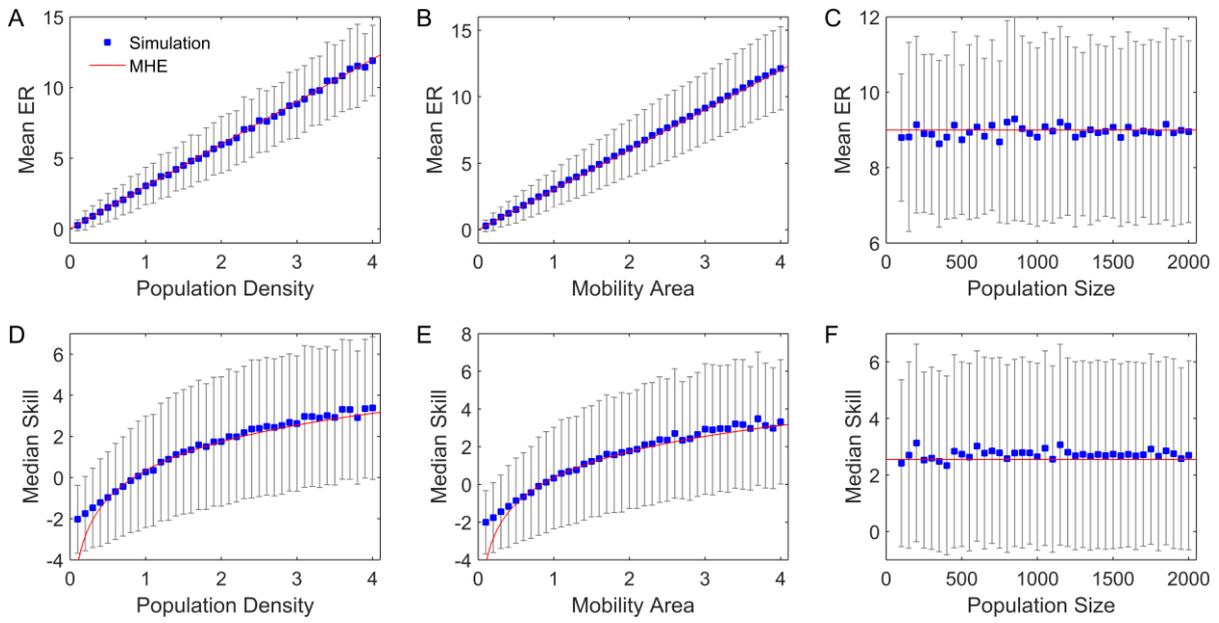
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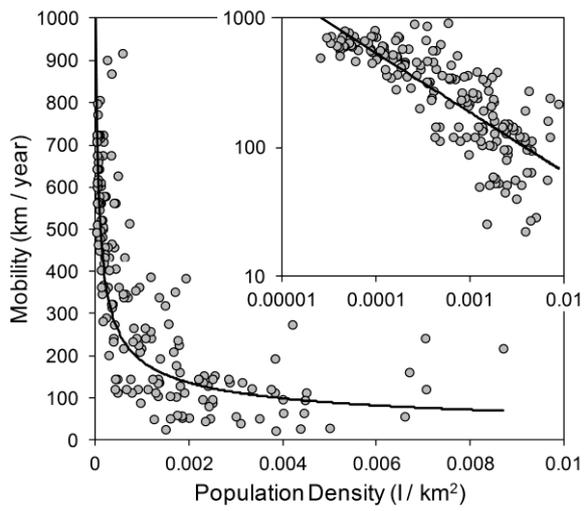
768 **Figure 7**



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771 **Figure 8**



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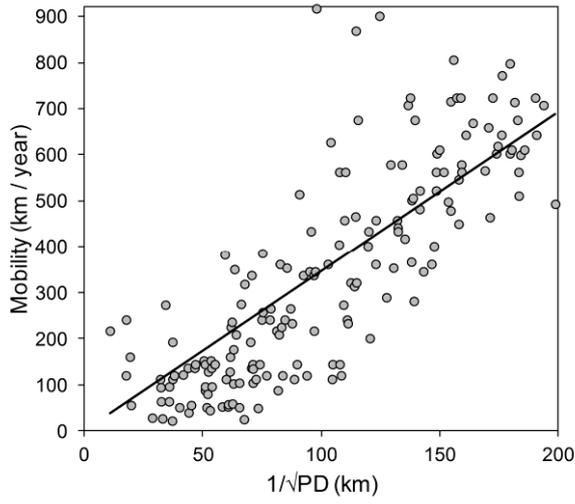
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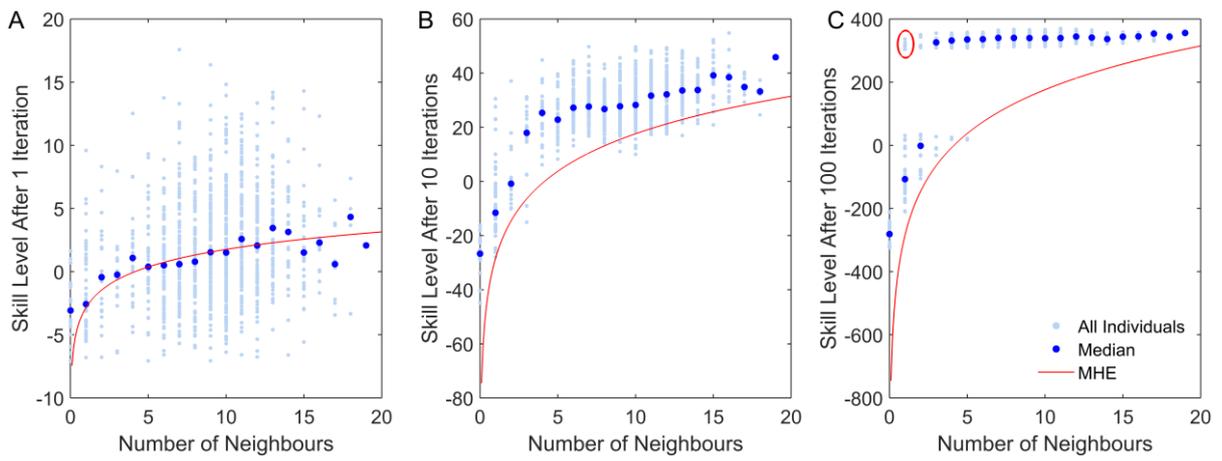
780 **Figure 9**



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783 **Figure 10**



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