Component Importance Measures for Complex Repairable System

G. Feng & E. Patelli
Institute for Risk and Uncertainty, School of Engineering, University of Liverpool, Liverpool, United Kingdom

M. Beer
Institute for Risk and Reliability, Leibniz University Hannover, Hannover, Germany
Institute for Risk and Uncertainty, School of Engineering, University of Liverpool, Liverpool, United Kingdom
School of Civil Engineering & Shanghai Institute of Disaster Prevention and Relief, Tongji University, China

F. P.A. Coolen
Department of Mathematical Sciences, Durham University, Durham, United Kingdom

ABSTRACT: Survival signature is a summary of structure function, which has been presented to perform reliability analysis on system with multiple component types. However, it is pretty intractable and difficult to use analytical methods to perform reliability analysis on repairable system, therefore, numerical simulation method for the analyse of such systems is required. Importance measures in system reliability engineering are applied to find the weakest component in a system. In many practical situation it is a necessary to know the importance of a set of components. The existing component importance measures are calculated through analytical ways which mainly focus on a specific component, however, there are few applications of these measures to repairable systems. In this paper, survival signature-based simulation method has been proposed to analyse reliability on repairable systems. This approach is efficient since the survival signature of the system only need to be calculated once while Monte Carlo simulation is used to generate component transition times. What is more, two component importance measures which based on survival signature are also introduced to estimate the importance degree of a specific component or a set of components. In order to quantify the importance degree of the component, the relative criticality index is presented. Numerical examples are presented to show the applicability of the approaches.

1 INTRODUCTION

In recent years, the system signature has been recognized as an important tool to quantify the reliability of systems consist of independent and identically distributed (iid) or exchangeable components with respect the random failure times (Samaniego 2007). System signature separates the system structure from the component probabilistic failure distribution. However, when it is adopted to solve a complex system with more than one component type, it requires the computation of the probabilities of all possible different ordering statistics of each component failure lifetime distributions (Coolen & Coolen-Maturi 2015), which is often an intractable procedure.

In order to overcome the limitations of the system signature, Coolen & Coolen-Maturi (2012) proposed the use of survival signature. Survival signature method not only reserves the merits of the system signature, but it has been shown to be an effective tool for analysing complex system consisting of more than one single component type. In essence, it does not have the assumption that components of different types are iid, which overcome the long-standing limitation of system signature. Therefore, survival signature is a promising method for application to complex systems and networks.

Recently, Aslett (2012) developed a package to calculate the survival signature. Coolen et al. (2014) proposed a non-parametric predictive inference for system reliability using the survival signature. Aslett et al. (2015) did system reliability within the Bayesian framework of statistics. Based on the above concepts, Feng et al. (2016) developed an analytical method to calculate survival function of systems with uncertain components parameters which belong to exponential distribution. These methods are efficient, however, they do not take the repairable system into consideration.

Repairable components are those that not replaced following the occurrence of a failure; rather, they are repaired and put into operation again. The use of simulation methods for repairable system reliability has attractive features, especially for large or complex systems (Patelli et al. 2012). Moreover, if there exist imprecise probabilities, it is even more complicated to deal with the imprecision (Beer et al. 2013). Most of the current simulation methods for system reliability are based on Monte Carlo simulation and structure function. By generating the state evolution of each component, the structure function is computed to determine the state of the system. However, the calculation of the structure function usually requires the calculation of all the path sets or cut sets. Therefore, it is a difficult task for complex repairable systems.

The survival signature is a summary of the system structure functions, which is not only good for determining the system reliability function, but efficient for complex system with repairable components. This is because it just needs to store survival signature instead of the whole structure functions of the system.

Risk are unavoidable and as such the key challenge in engineering risk analysis is to identify the components of the system that contribute most to risk (Modarres 1992). Component importance measure is a very useful method for the designers and manufactures experts to find how the failure of one component affects the functioning of the system and identify the weakest components in the system. The definition of component importance measure is first introduced by Birnbaum (1968) in 1968, which is obtained by partial differentiation of the system reliability with respect to the given component reliability. An improvement or decline in reliability of the component with the highest importance will cause the greatest increase or decrease in system reliability. Based on this achievement, many other component importance measures have been introduced. e.g., structure importance measure (Borgonovo 2007), Fussell-Vesley importance measure (Vesely 1970, Fussell 1975), failure criticality index (Wang et al. 2004), risk reduction worth and risk achievement worth (Borgonovo and Apostolakis 2001). Dutuit & Rauzy (2014) gave a review for importance factors of coherent systems, which contributes to clarify mathematical and algorithmic foundations of importance factors.

However, the traditional importance measures mainly focus on non-repairable systems, and mainly concern reliability importance of an individual component. In many practical situation it is of interest to evaluate the importance of a set of components instead of just individual component.

A survival signature-based method is proposed in this paper, which is efficient to analysis repairable system reliability. This is essential when dealing with complex repairable systems since they can only be analysed through simulation method. In order to find out the most "critical" component in the system, new component importance measures which based on survival signature are introduced to analysis individual component and component sets respectively. What is more, a new relative criticality index is used to quantify the importance degree of the component. The applicability of the proposed approach is demonstrated by solving the numerical examples.

This paper is organized as follows. Section 2 presents the definition and advantages of survival signature for system with multiple component types. The reliability analytical method for repairable systems which based on survival signature is introduced in Section 3. In Section 4, the component importance measures both for a specific component and component sets are presented. The applicability of the proposed methods is shown by analysing one numerical example in Section 5 and Section 6 closes the paper with conclusions.

2 SURVIVAL SIGNATURE FOR SYSTEM WITH MULTIPLE COMPONENT TYPES

For a system with m components, let state vector $\underline{x} = (x_1, x_2, ..., x_m) \in \{0,1\}^m$, with $x_i = 1$ if the ith component works and $x_i = 0$ if not. The structure function $\phi : \{0,1\}^m \to \{0,1\}$, defined for all possible $\underline{x}$, takes the value 1 if the system functions and 0 if not for state vector $\underline{x}$. In this paper, attention is restricted to coherent systems, for which $\phi(\underline{x})$ is not decreasing in any of the components of $\underline{x}$, so system functioning cannot be improved by worse performance of one or more of its components. It is further assumed that $\phi(\underline{0}) = 0$ and $\phi(\underline{1}) = 1$, so the system fails if all its components fail and functions if all its components function.

Now consider a system with $K \geq 2$ types of components, with m_k components of type $k \in \{1, 2, ..., K\}$ and $\sum_{k=1}^{K} m_k = m$. Assume that the random failure times of components of the system type are exchangeable, while full independence is is assumed for the random failure times of components of different types. The components of the same type can be grouped together due to the arbitrary ordering of the components in the sate vector, which leads to a state vector can be presented as $\underline{x} = (\underline{x}^1, \underline{x}^2, ..., \underline{x}^K)$, with $\underline{x}^k = (x_1^k, x_2^k, ..., x_{m_k}^k)$ illustrating the states of the components of type k. Coolen and Coolen-Maturi (2012) introduced the survival signature for such a system, denoted by $\Phi(l_1, l_2, ..., l_K)$, with $l_k = 0, 1, ..., m_k$ for $k = 1, 2, ..., K$, which is defined to be the probability that the system functions given that l_k of its m_k components of type k work, for each $k \in \{1, 2, ..., K\}$.

There are $\binom{m_k}{l_k}$ state vectors $\underline{x}^k$ with $\sum_{i=1}^{m_k} x_i^k = l_k$ $(k = 1, 2, ..., K)$, and let $S_{l_1, l_2, ..., l_K}$ denote the set of all state vectors for the whole system. Due to independent and identical distributed (iid) assumption, all the state vectors $\underline{x}^k \in S_{l_k}^k$ are equally likely to occur, so the survival signature can be written as

$$\Phi(l_1, ..., l_K) = [\prod_{k=1}^{K} \binom{m_k}{l_k}^{-1}] \times \sum_{\underline{x} \in S_{l_1,...,l_K}} \phi(\underline{x}) \quad (1)$$

$C_k(t) \in \{0, 1, ..., m_k\}$ denotes the number of k components working at time t. Assume that the components of the same type have a known CDF, $F_k(t)$ for type k. Moreover, the failure times of different component types are assumed independent, then:

$$P(\bigcap_{k=1}^{K} \{C_k(t) = l_k\}) = \prod_{k=1}^{K} P(C_k(t) = l_k) = \quad (2)$$

$$\prod_{k=1}^{K} \binom{m_k}{l_k} [F_k(t)]^{m_k - l_k} [1 - F_k(t)]^{l_k}$$

Hence, the survival function of the system with K types of components becomes:

$$P(T_s > t) = \sum_{l_1=0}^{m_1} ... \sum_{l_K=0}^{m_K} \phi(l_1, ..., l_K) * \quad (3)$$

$$P(\bigcap_{k=1}^{K} \{C_k(t) = l_k\})$$

It is obvious from Equation 3 that the survival signature can separate the structure of the system from the failure time distribution of its components, which is the main advantage of the system signature. What is more, the survival signature only need to be calculated once for any system, which is similar to the system signature for systems with only single type of components. It is easily seen that survival signature is closely related with system signature. For a special case of a system with only one type ($K = 1$) of components, the survival signature and the Samaniego's signature (Samaniego 2007) are directly linked to each other through a simple equation, however, the latter cannot be easily generalized for systems with multiple types ($K \geq 2$) of components (Coolen & Coolen-Maturi 2012).

This implies that all attractive properties of the system signature also hold for the method using the survival signature, also the survival signature is easy to apply for systems with multiple types of components, and one could argue it is much easier to interpret than the system signature.

3 REPAIRABLE SYSTEM RELIABILITY ANALYSIS BASED ON SURVIVAL SIGNATURE

If the system with m components is repairable, a schematic diagram of components status and the corresponding system performance are presented in figure 1.

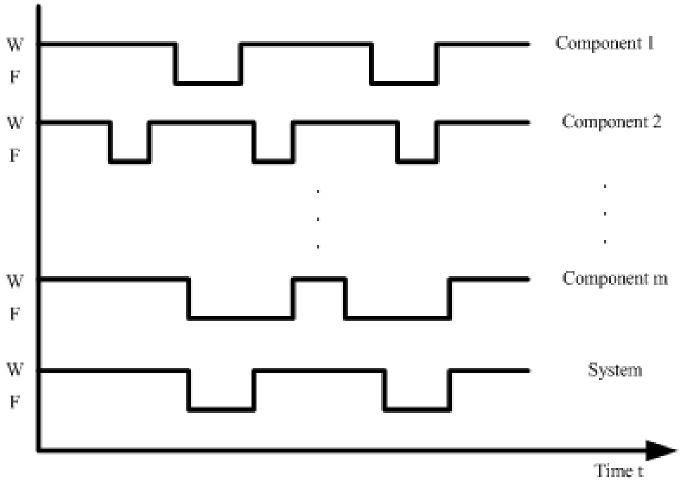

Figure 1: Schematic diagram of components status and the corresponding system performance.

For structure function method, it is a necessary to identify whether the system works or not at each critical time point. The critical time point is the beginning time for each component fails and the finish time for each component repairs.

In this paper, a survival signature-based method is proposed to analyse the repairable system reliability. This method is based on the system production idea proposed by Zio et al. (2006) and Patelli et al. (2016). Assume that there are j_k possible transition for the components of type k. The probability of going from state $s = i$ to state $s' = j$ in given by $p_{kji} = \Pr(X_k = i \mid X_k = j)$. For simplicity, let assume there is only one possible transition to exit from the state $s = i$. For instance, a component in working status $s = 1$ can fail and entering in the state $s' = 2$; the component in the state $s = 2$ can only be repaired and returning in the status $s' = 1$. Let $F_{kj} = \Pr(\cdot \mid X_k = j)$ represents the CDF of the component of type k in the state j to undergo a transition. The Monte Carlo simulation is performed as follows and the evaluation of the survival function of a system are performed using the procedure shown in figure 2.

Step 1. Sample the transition times t_i for $i = 1, 2, \ldots, m$ for each component from the corresponding CDF F_{kj} and stored in a vector $Vt = \{t_1, t_2, \ldots, t_m\}$, set $t_{old} = 0$;

Step 2. Identify the first transition time $min(Vt)$ and the corresponding component z. Hence, t_1 represents the first transition of a system, t_2 the second transition and so on;

Step 3. At each transition time, the number of components in working status is computed: $C = (C_1, C_2, \ldots, C_K)$ and the corresponding "production level" by evaluating the survival signature Φ_C;

Step 4. Collect the value of the survival signature Φ_C in the vector Vr representing the survival function as follows: $Vr(j) = Vr(j) +$

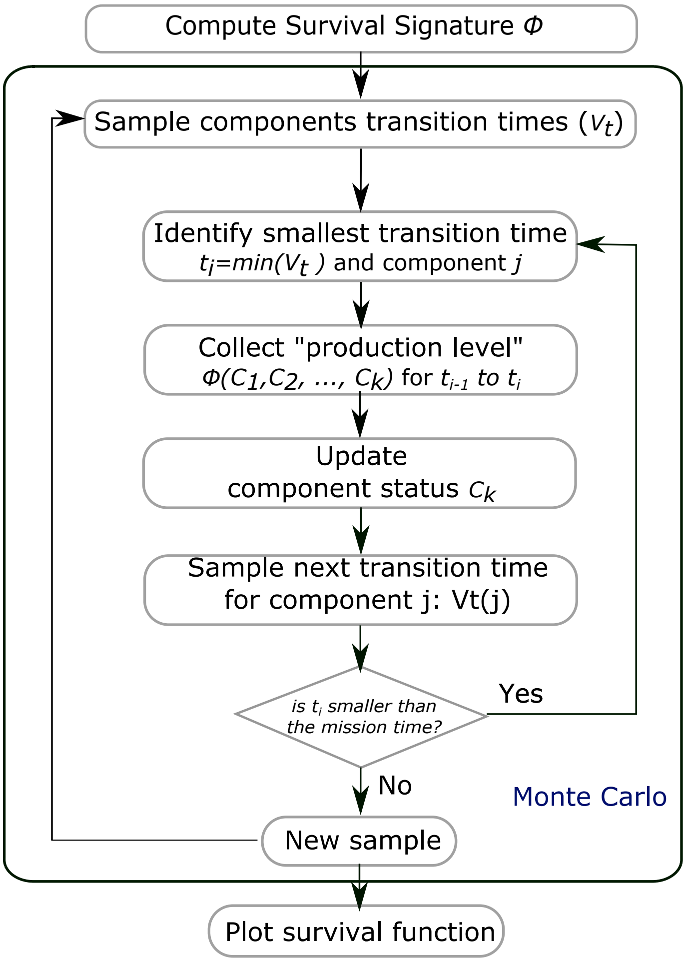

Figure 2: Flow chart of survival signature based survival function evaluation method for repairable system.

$\Phi_{t_i} \quad \forall j : t_{old} \leq j \cdot dt < min(Vt)$, Set $t_{old} = min(Vt)$.

Step 5. Update the status j of component z;

Step 6. Update the vector of transition time Vt by sampling the next transition time t'_z of the component z from F_{kj} where k is the component type of the component z and j its status. Hence: $Vt(z) = t_z + t'_z$;

Step 7. If $min(Vt) < T_{Mission}$, return to point 2.

4 COMPONENT IMPORTANCE MEASURES

4.1 *Importance measure of a specific component*

Based on the results of Feng et al. (2016), the relative important index ($RI_i(t)$) of the ith component at time t can be used to analyse repairable component importance. It is the survival function probability differences between the repairable system works if the ith component functions and the repairable ith component failures. The mathematical expression formula of $RI_i(t)$ is

$$RI_i(t) = P(T_S > t \mid T_i > t) - P(T_S > t \mid T_i \leq t) \quad (4)$$

Where, $P(T_S > t \mid T_i > t)$ represents the probability that the repairable system works knowing that the ith component functions; $P(T_S > t \mid T_i \leq t)$ denotes the probability that the repairable system functions if the ith component has failed.

4.2 *Importance measure of a set of components*

It is sometimes important to evaluate the importance of a set of components instead of a specific one in real engineering world. Therefore, the relative importance index for a specific component can be extended for components of type k, which can be denoted by $RI_k(t)$. It is the probability difference values between the repairable system functions if the components of type k are repairable and the components of type k cannot be repaired. $RI_k(t)$ can be expressed as follows

$$RI_k(t) = P(T_S > t \mid T_k > t) - P(T_S > t \mid T_k \leq t) \quad (5)$$

Where, $P(T_S > t \mid T_k > t)$ represents the probability that the repairable system works if the components of type k are repairable; $P(T_S > t \mid T_k \leq t)$ denotes the probability that the repairable system functions knowing that the components in type k cannot be repaired.

Both $RI_i(t)$ and $RI_k(t)$ is a function of time and they reveals the trend of the importance degree of a specific component or a set of components within the repairable system. The bigger the value of $RI_i(t)$ or $RI_k(t)$, the more "critical" of the ith component or the set of components on the repairable system reliability at a specific time t, and vice versa. This helps to allocate resources, which might including resources for reliability improvement, surveillance and maintenance, design modification, security, operating procedure, training, quality control requirements, and a wide variety of other resource expenditures. By using the importance of a specific component or a set of components, resources expenditure can be properly optimized to reduce the total life-cycle resource expenditures while keeping the risk as low as possible. In other words, for a given resource expenditure such as for maintenance, the importance measure of a specific component or set of components can be used to allocate resources to minimize the total system risk. This approach allows the risk manager to offer the "biggest bang for the buck" (Modarres 2006).

4.3 *Quantify importance degree of the component*

In order to quantify importance degree of the component, a new index which called relative criticality index (RC) is introduced in this paper. The numerically obtained index for a repairable system is through Monte Carlo simulation which based on Section 3. The failure times of the system can be got through each trial, after having simulated many histories of

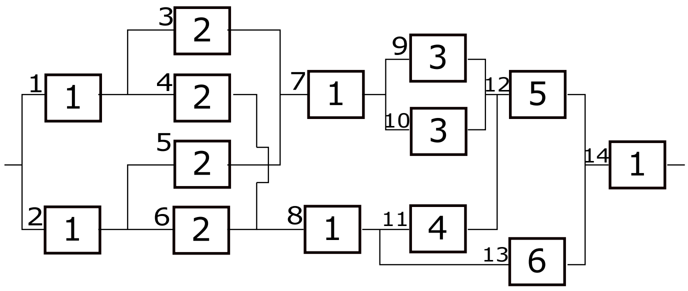

Figure 3: The complex repairable system with six types of components. The numbers inside the component boxes indicate the component type. The numbers above the component boxes indicate the component indices.

Table 1: Distribution for failure and repair process of components

Component Type	Process	Distribution	Parameters
1	$1 \to 2$	Exponential	2.3
1	$2 \to 1$	Constant	0.5
2	$1 \to 2$	Exponential	1.2
2	$2 \to 1$	Constant	1.0
3	$1 \to 2$	Weibull	(1.7,3.6)
3	$2 \to 1$	Constant	0.7
4	$1 \to 2$	Lognormal	(1.5,2.6)
4	$2 \to 1$	Constant	1.1
5	$1 \to 2$	Weibull	(3.2,2.5)
5	$2 \to 1$	Constant	1.3
6	$1 \to 2$	Gamma	(3.1,1.5)
6	$2 \to 1$	Constant	1.2

the system, estimates are made of the desired relative criticality index statistically. For a system with m components, given component i, this index is expressed as

$$RC_i = \frac{N_i^f}{max\{N_{1,\dots,m}^f\}} \tag{6}$$

Where, N_i^f represents average number of system failures if the ith component cannot repair; and $max\{N_{1,\dots,m}^f\}$ denotes the maximum average number of system failures if the each component cannot repair respectively.

This index can quantify the importance degree of each component, and the relative criticality index values of all the components are compared with the biggest RC value. Therefore, the bigger the value is, the bigger influence of the ith component on the repairable system.

5 NUMERICAL EXAMPLE

The purpose of this numerical example is to show the efficiency of the survival signature-based reliability analysis on repairable system. What is more, the component importance measures presented in this paper are also used to analyse the same system. The reliability block diagram representation of a simplified auxiliary feedwater system can be seen in figure 3, it consists of fourteen components which belong to six component types.

Table 1 shows the distribution for failure process ($1 \to 2$) and repair process ($2 \to 1$) of components.

Let first perform importance measure of a specific component. The results can be seen in figure 4.

It is clear that component 14 always has higher relative importance index than the other thirteen components, which means it is the most “critical” component in the repairable system. Then it comes to component 8. Component 13 has litter relative importance index values at the first time, however, its relative importance index values become bigger and bigger as time goes on, which just follows the components 14 and 8. Component 1 and component 2 has similar relative importance values, which are sometimes crossover. The same circumstance occurs on components 4 and 6. The relative importance of the five components (3, 5, 7, 9, 10) always within within 0.1, which means they has less importance influence degree than other components on the repairable system.

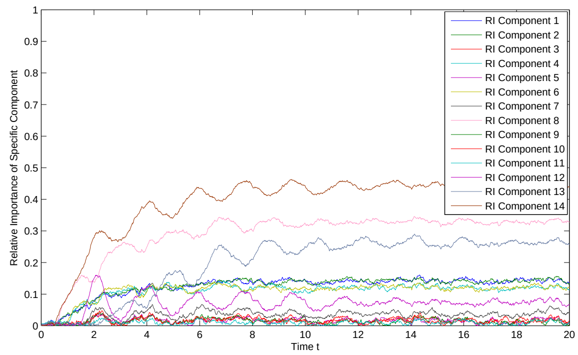

Figure 4: Relative importance index of the specific component in system.

For application in the real world, sometimes people want to know the importance degree of a set of components. i.e., the relative importance index of components of set 1 to set 6 in this repairable system. Figure 5 shows the results of them.

It can be seen that the relative importance index values of component sets 1 and 2 are bigger than other component sets, therefore, components of type 1 and 2 are more important than components of other types in this repairable system. On the contrary, component set 4 is the least important within the system because it has the smallest values of relative importance index. The values of component set 1 are higher than 2 at the beginning time, however, their values are the same as

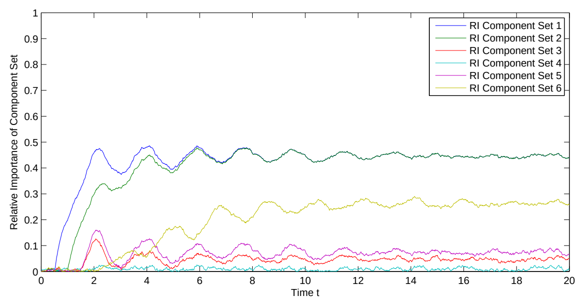

Figure 5: Relative importance index of the component sets in system.

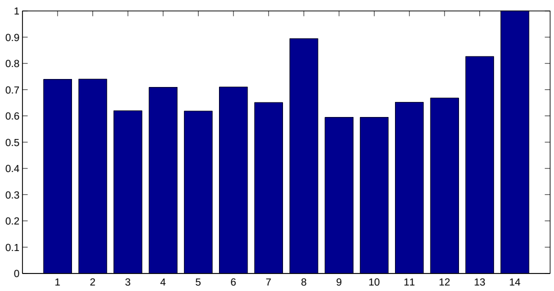

Figure 6: Relative criticality index of the component in system.

time goes on. Component set 6 has lower relative importance values than component sets 3 and 5, but the values go up and rank the third within the six component sets in the last. Component set 3 and 5 has the similar relative importance values trend, although the value of set 5 is bigger than set 3.

If using the relative criticality index to quantify the importance degree, the RC of each component can be seen in figure 6.

It shows that component 14 has the highest importance degree to the repairable system, then comes component 8 and 13. The components 9 and 10 has the similar relative criticality index and they are also the “critical” ones.

6 CONCLUSIONS

Survival signature is an efficient approach for analysing complex system with more than one component type, and it is just need to be calculated once when perform reliability analysis on a specific system, which represents a significant computational advantage. Sensitivity analysis is an important concept in system reliability, it helps to quantify the importance of the component and identify the weakest component, also it gives guidance on how to allocate the resources. Some of the component importance measures can be calculated through analytical methods, while for complex systems or repairable systems, simulation ways are much more efficient and useful.

In this paper, survival signature-based methods for repairable system reliability analysis and component importance measures have been presented. The efficient simulation method which based on survival signature has been used for reliability analysis on repairable system, and survival signature only need to be calculated once for the same system whether it is repairable or not. What is more, the proposed importance measures can not only find out the importance degree of the exact component, but can be used for analysing importance of a set of components. In order to quantify the importance degree of the component within the repairable system, a new relative criticality index is introduced in this paper. The application of these approaches presented in this paper is illustrated by performing an analysis on a complex repairable system.

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