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# Model-based active control of a continuous structure subjected to moving loads

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**Abstract.** Modelling of a structure is an important preliminary step of structural control. The main objectives of the modelling, which are almost always antagonistic are accuracy and simplicity of the model. The first part of this study focuses on the experimental and theoretical modelling of a structure subjected to the action of one or two decelerating moving carriages modelled as masses. The aim of this part is to obtain a simple but accurate model which will include not only the structure-moving load interaction but also the actuators dynamics. A small scale rig is designed to represent a four-span continuous metallic bridge structure with miniature guiding rails. A series of tests are run subjecting the structure to the action of one or two mini-carriages with different loads that were launched along the structure at different initial speeds. The second part is dedicated to model based control design where a feedback controller is designed and tested against the validated model. The study shows that a positive position feedback is able to improve system dynamics but also shows some of the limitations of state-space methods for this type of system.

## 1. Introduction

The problem of structural control has been investigated for a long time. There are several studies reporting the achievement of significant improvements of the dynamics of a structure. In spite of all these contributions to the general problem of structural control, the problem of controlling a structure subjected to moving loads presents a particular set of challenges [1, 2]. This is mainly due to the fact that the time-invariant structural parameters of the system are significantly affected by additive terms due to the time-varying nature of the excitation. One of the main implications of this particular aspect in what concerns the dynamics of structural interaction in moving load problems becomes the modelling of the controlled structure. This aspect becomes challenging as in many cases the time-varying dynamics needs to be taken into account in designing of the controller action [3, 4].

The research reported in this paper covers the small scale modelling of a continuous structure subjected to one or two loads moving at variable speed. Although the theoretical aspects of the moving load problem are covered extensively in the literature [5, 6, 7] the small-scale experimental research of vehicle-bridge interaction was touched by only a few researchers [8, 9, 10, 11].

Bilello et al. [8] investigated the response of a small-scale bridge model under a moving mass. In the study the bridge was modelled as a single-span beam and the numerical results showed a good agreement with the experimental data. Zhu and Law [10] were concerned with moving forces identification for a continuous beam structure.

A continuous multi-span beam structure was also investigated by Stancioiu et al. [11]. This study presented a set of experimental results for a continuous structure modelled as an Euler-Bernoulli four-span beam under the action of one and two moving masses. The analytical model was validated by experimental tests both in time and frequency domains. Based on probabilistic analysis the paper showed



that the theoretical response-experimental response errors for the moving load-structure interaction could be reduced to an acceptable level of accuracy.

It is generally accepted that the complexity of the theoretical model for interaction dynamics depends upon the problem specifications and a set of valuable studies were presented to deal with different degrees of refinement of the model ranging from simple moving mass [7,8,12] to two-axle systems [6] or even full vehicle [13] or train models [14]. A particular attention was also given to study the response of infinite periodic structures to moving loads with application to railway tracks dynamics [16].

In what concerns the problem of structural control applied directly to the moving mass – beam interaction dynamics this was covered in a couple of studies at theoretical level.

Tsao et al. [4] studied some theoretical aspects of the mass-spring-damper system-bridge interaction and presented a linear parameter variation control method. The time-varying representation of the interaction system was exploited by Stancioiu and Ouyang [3]. They studied a series of objective functions in an optimal control formulation for a simply supported beam – moving mass interaction and determined to what extent the time-varying nature of the parameters influences the control effort. It was concluded that for certain setups where the actuation is placed symmetrically under the bridge the control design was not affected by the time variation of parameters.

Nikkhoo et al. [1] used an iterative method in a linear quadratic formulation to take into account the time variation of the beam-moving load interaction system.

In the present study an analytical model of the beam-moving mass is derived based on an experimental rig which consists of a continuous simply-supported four-span beam subjected to the action of one or two moving carriages modelled as moving masses. The analytical model is cross-validated by experimentally measured data. The objective is not only to obtain a simple model representation for the beam-moving structure interaction but also to model the actuation dynamics which is a novelty of this paper. This approach will enable a control design based on an existing physical model as opposed to a theoretical model without actuation dynamics, which was already analysed in numerous theoretical studies.

## 2. Theoretical Model

### 2.1. Four-span simply supported beam with modal shakers

The main assumption made in this study is that the supporting structure is modelled as a continuous Euler-Bernoulli beam of length  $L$  [11]. A simplified sketch of one span of the supporting beam is presented in figure 1. The beam is traversed by  $n$  point masses  $m_j$  moving from left to right at a speed  $v_j(t)$  which enter the beam at given time instants  $t_j$ .

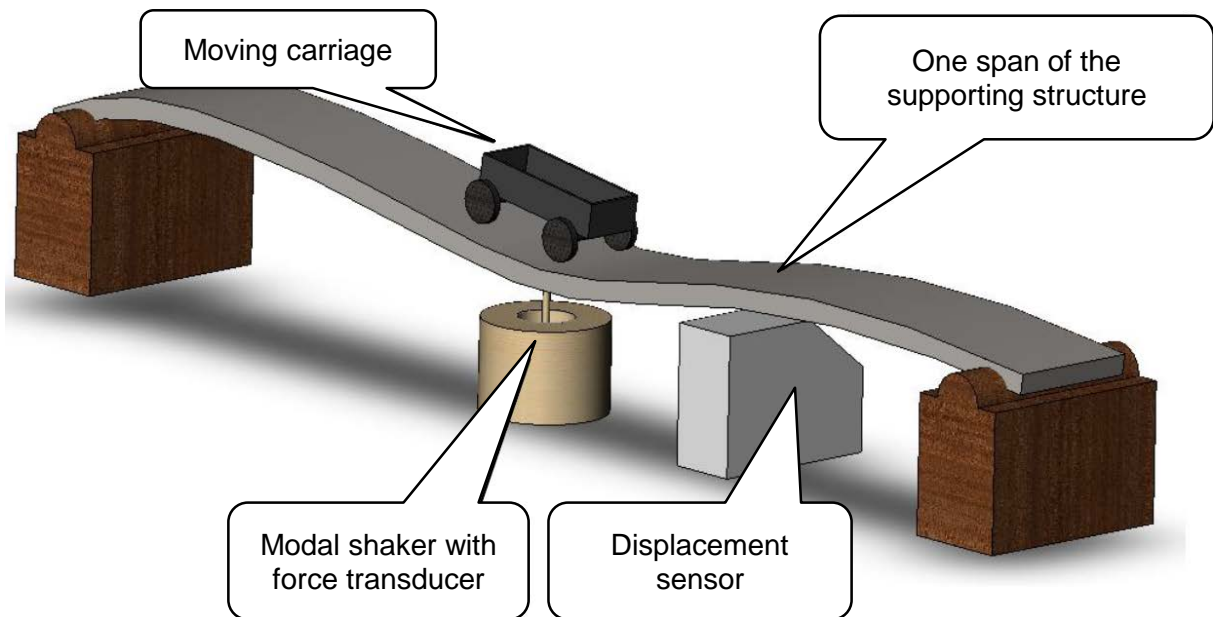
When the modal shakers are positioned under the beam they will modify the structural dynamic response even if they are not active. The simplest way to model inactive shakers effects on the beam's dynamics consists of a spring and a damper.

In the case that  $N_a$  actuators modelled as spring and dampers with stiffness  $k_i$  and damping coefficient  $c_i$  placed at locations  $x_i$  are attached to the beam, the partial differential equation governing the dynamics of the beam is:

$$\begin{aligned}
 EI \frac{\partial^4 w}{\partial x^4}(x, t) + \rho A \frac{\partial^2 w}{\partial t^2}(x, t) + \rho A c \frac{\partial w}{\partial t}(x, t) \\
 = - \sum_{j=1}^n m_j (\ddot{w}(x_j(t), t) + g) G(v_j, L, t_j) \delta(x - x_j(t)) \\
 + \sum_{i=1}^{N_a} (k_i w(x, t) + c_i \dot{w}(x, t)) \delta(x - x_i)
 \end{aligned} \tag{1}$$

The corresponding modal coordinate matrix equation is:

$$(\mathbf{M} + \Delta\mathbf{M}(t))\ddot{\mathbf{q}} + (\mathbf{D} + \Delta\mathbf{D}(t) + \mathbf{D}_a)\dot{\mathbf{q}} + (\mathbf{K} + \Delta\mathbf{K}(t) + \mathbf{K}_a)\mathbf{q} = - \sum_{j=1}^n m_j g \Psi(x_j(t)) \tag{2}$$



**Figure 1.** Simplified sketch of the structural system for one span

In this equation the time invariant matrices  $\mathbf{M}$ ,  $\mathbf{D}$  and  $\mathbf{K}$  can be expressed as functions of the beam's modal shape vectors  $\boldsymbol{\psi}(x)$ , and the material and geometric characteristics of the beam  $\rho A$  and  $EI$ :

$$\begin{aligned}\mathbf{M} &= \rho A \int_0^L \boldsymbol{\psi}(x) \cdot \boldsymbol{\psi}(x) dx \\ \mathbf{D} &= \rho A c \int_0^L \boldsymbol{\psi}(x) \cdot \boldsymbol{\psi}(x) dx \\ \mathbf{K} &= EI \int_0^L \boldsymbol{\psi}(x) \cdot \boldsymbol{\psi}^{IV}(x) dx\end{aligned}\quad (3)$$

Assuming that mass  $m_j$  has a linearly decaying speed  $v_j - a_j t$  instead of a constant speed the moving coordinate  $x_j(t)$  of equation (1) becomes a quadratic function of time and the time dependent matrices in equation (3) are explicitly defined by:

$$\begin{aligned}\Delta \mathbf{M}(t) &= \sum_{j=1}^n m_j \boldsymbol{\psi}(x_j(t)) \boldsymbol{\psi}^T(x_j(t)) \\ \Delta \mathbf{D}(t) &= 2 \sum_{j=1}^n m_j (v_j - a_j t) \boldsymbol{\psi}(x_j(t)) \boldsymbol{\psi}'^T(x_j(t)) \\ \Delta \mathbf{K}(t) &= \sum_{j=1}^n m_j (v_j - a_j t)^2 \boldsymbol{\psi}(x_j(t)) \boldsymbol{\psi}''^T(x_j(t)) - m_j a \boldsymbol{\psi}(x_j(t)) \boldsymbol{\psi}^T(x_j(t))\end{aligned}\quad (4)$$

The action of the actuators is modelled as added stiffness and damping:

$$\begin{aligned}\mathbf{K}_a &= \sum_{i=1}^{N_a} k_i \boldsymbol{\psi}(x_i) \boldsymbol{\psi}^T(x_i) \\ \mathbf{D}_a &= \sum_{i=1}^{N_a} c_i \boldsymbol{\psi}(x_i) \boldsymbol{\psi}^T(x_i)\end{aligned}\quad (5)$$

2.2. Four-span simply supported beam with force input

When the simply supported beam is acted upon by only one shaker at coordinate  $x_m$  with the other three being passive, equation (1) changes to

$$EI \frac{\partial^4 w}{\partial x^4}(x, t) + \rho A \frac{\partial^2 w}{\partial t^2}(x, t) + \rho A c \frac{\partial w}{\partial t}(x, t) = +f_m(t)\delta(x - x_m) + \sum_{\substack{i=1 \\ i \neq m}}^{N_a} (k_i w(x, t) + c_i \dot{w}(x, t))\delta(x - x_i) \tag{6}$$

where force  $f_m(t)$  is an arbitrary time variable force.

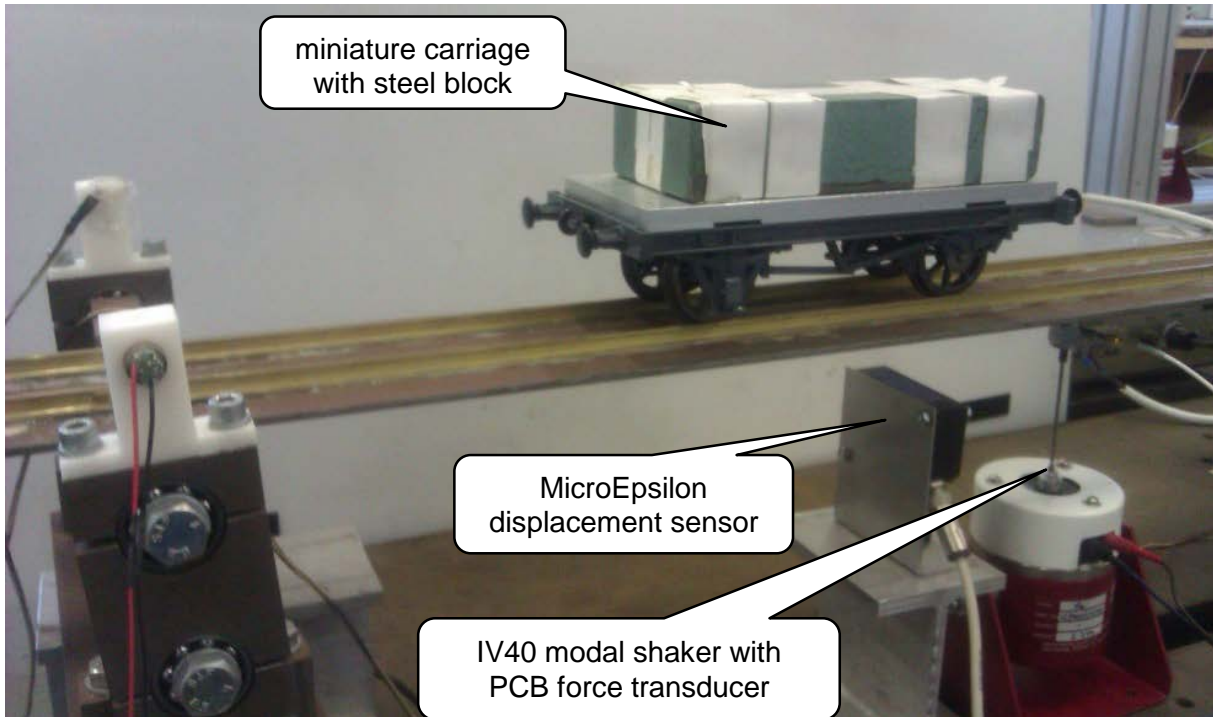
The interest for a control problem is to model directly the action of the shaker and obtain the response as a function of the control action which in this case is the voltage input to the shaker  $u(t)$ .

In this case the equation of motion of a beam under the action of the shakers in modal coordinates is:

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{D} + \mathbf{D}_a)\dot{\mathbf{q}} + (\mathbf{K} + \mathbf{K}_a)\mathbf{q} = +\boldsymbol{\Psi}(x_m)\mathbf{C}_{ms}\mathbf{z} \tag{7}$$

$$\dot{\mathbf{z}} = \mathbf{A}_{ms}\mathbf{z} + \mathbf{B}_{ms}u$$

where matrices  $\mathbf{A}_{ms}$ ,  $\mathbf{B}_{ms}$  and  $\mathbf{C}_{ms}$  are the state matrices of the modal shaker modelled as a state space systems with voltage input and force output.



**Figure 2.** Experimental setup. Miniature carriage with mass (total of 4.6kg), DataPhysics IV40 modal shaker and MicroEpsilon displacement laser transducer.

In the case when the moving carriages act on the structure along with externally generated forces from the four shakers acting on the beam at coordinates  $x_{ai}, i = 1$  to 4 and the shakers are all modelled as identical state-space first order systems from input voltage  $u_i$  to output force  $f_i$

$$\dot{z}_i = -\alpha z_i + \beta u_i$$

$$f_i = \gamma z_i \tag{8}$$

the equation of motion in modal coordinates becomes:

$$\ddot{\mathbf{q}} = -(\mathbf{M} + \Delta\mathbf{M}(t))^{-1}(\mathbf{K} + \Delta\mathbf{K}(t) + \mathbf{K}_a)\mathbf{q} - (\mathbf{M} + \Delta\mathbf{M}(t))^{-1}(\mathbf{D} + \Delta\mathbf{D}(t) + \mathbf{D}_a)\dot{\mathbf{q}} + c(\mathbf{M} + \Delta\mathbf{M}(t))^{-1}[\Psi(x_{a1}) \dots \Psi(x_{an})]\mathbf{z} - \sum_{j=1}^n m_j g \Psi(v_j t) \tag{9}$$

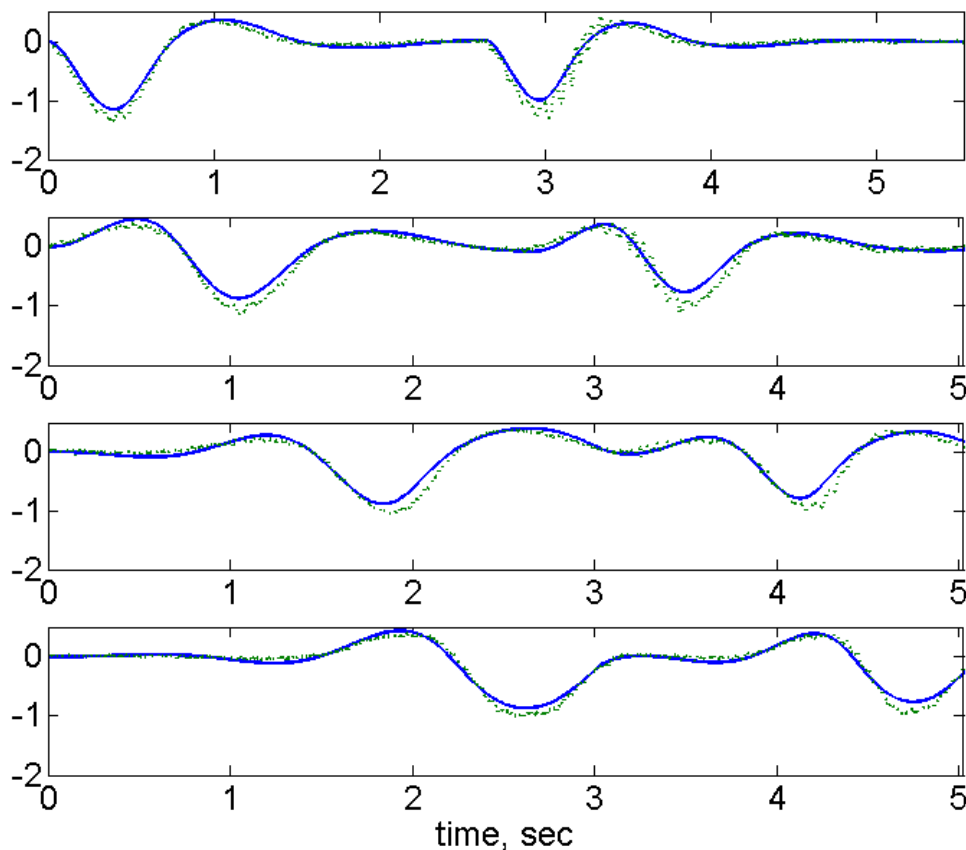
$$\dot{\mathbf{z}} = \mathbf{A}_{ms}\mathbf{z} + \mathbf{B}_{ms}\mathbf{u}$$

where  $\mathbf{A}_{ms} = -\alpha\mathbf{I}_4$  and  $\mathbf{B}_{ms} = \beta\mathbf{I}_4$  with  $\mathbf{I}_4$  the identity matrix of order 4.

### 3. Experimental validation

#### 3.1. Four-span simply supported beam under the action of one or two moving carriages

The supporting structure is a 3.64 m long four-span simply supported thin plate with a constant section of 101×3 mm<sup>2</sup> similar to the structure described and modelled in [11]. In this case the flexural rigidity of the beam has been increased by addition of a set of brass guiding rails on top of the plate. The moving structures are miniature carriages with rigid suspensions loaded with steel blocks (figure 2).

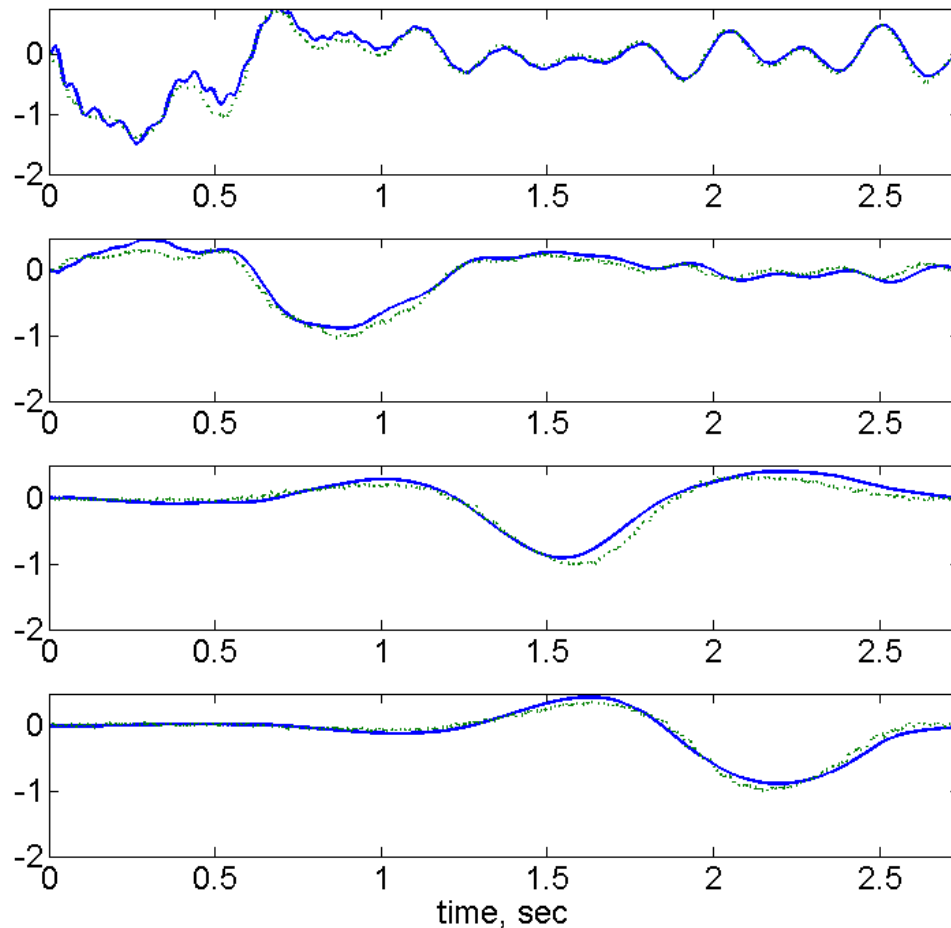


**Figure 3.** Comparison of the experimentally obtained deflection (mm) under the action of two moving carriages (dashed line) and theoretical results (continuous line) when the modal shakers are attached under the beam and modelled as spring-damper supports.

The time history of the beam’s deflections at positions of 0.54, 1.23, 2.39 and 2.98 m along the beam for the case of two miniature carriages, the second one launched after 2.6 s are presented in Figure 3. The estimated starting speeds are 1.43 and 1.7 m/s and the estimated deceleration are 0.15 and 0.2 m/s<sup>2</sup>. The masses of the two moving carriages are 4.6 and 4.2 kg.

These results show that the rig and the miniature carriages can be modelled as continuous beam-moving masses with a good accuracy. It is also observed that the constant speed model which was valid for the case presented in [11] when the moving mass was a ball is no longer valid as the carriages in this

investigation have a couple of moving components affected by friction. In this case the model needs to account for speed variation which is reflected by changing the added time variable components of the stiffness, damping and mass matrices in equation (4).



**Figure 4.** Time history of beam's deflection response (in mm) under the action of a moving mass and a modal shaker. The modal shaker located at coordinate  $x_1 = 54$  cm is modelled as a first order filter (continuous line – analytical results, dotted line – experimental results).

### 3.2. Four-span simply supported beam under one moving carriage and one external force

A complete model of the beam structure under the simultaneous action of one moving carriage and one active external shaker located at the first span is presented in figure 4. The 4.6-kg miniature carriage starts moving at an estimated initial velocity of 1.7 m/s and decelerates at a rate of 0.2 m/s<sup>2</sup>. The input voltage used for modelling is the experimental signal measured directly at a constant sampling rate from the input to the shaker and linearly interpolated in time to complete the variable time steps required for analysis.

The analytical model shows a good level of accuracy and again a low transmissibility of the shaker's effect to the adjacent beam spans as the dynamic action of the shaker reduces considerably for spans 3 and 4 and the response looks similar to that of the beam with no externally applied forces (figure 3).

## 4. Feedback controller design based on the analytical model

A reduced mode model is used to test the feasibility of designing a controller for the analytical model obtained in section 3. A representation for a single span of the structural system is shown in figure 5. The displacement output of the system is fed back to the structure via a first order system which models

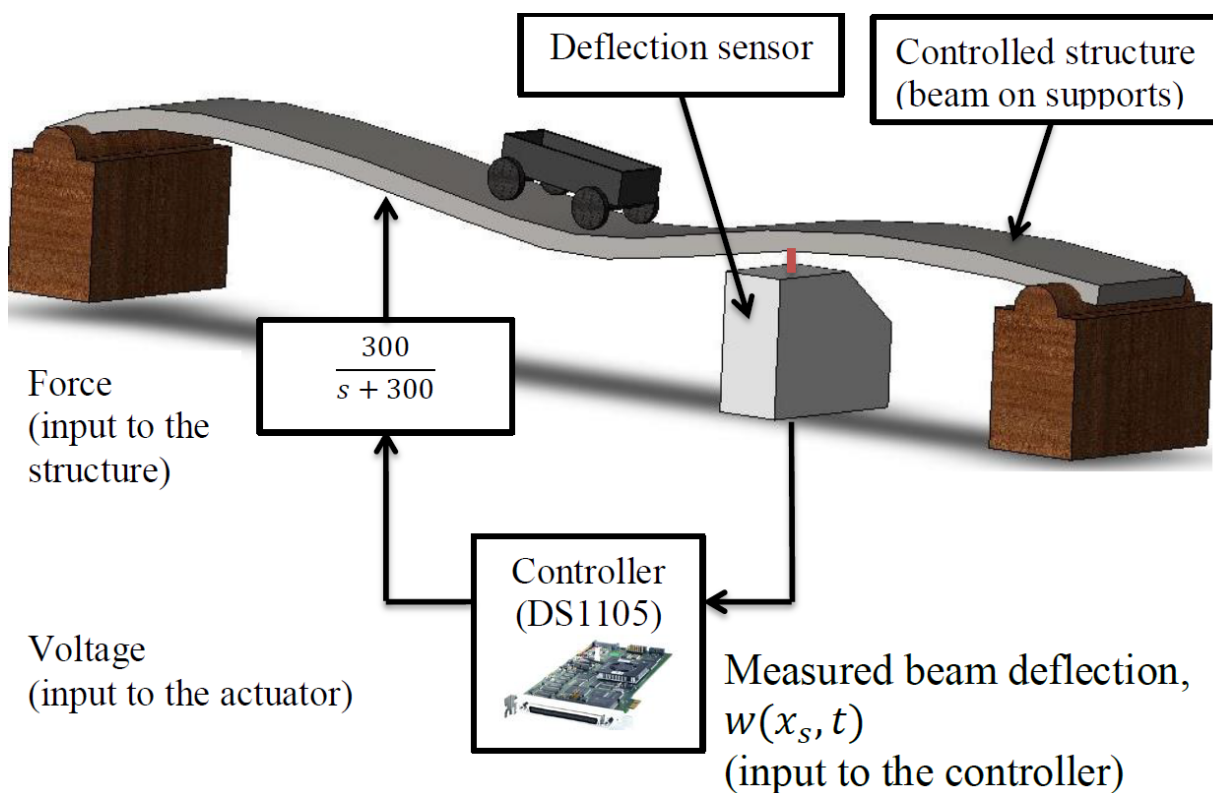
the actuator as a linear system from input voltage to output force. For the hardware in the loop simulation the controller is created in Simulink and uploaded as a real-time system in dSpace. For the purpose of this study the real structure (input voltage, output deflection) will be replaced by the analytical model.

The simplest possible architecture of a controller for this type of system is based on constant positive position feedback. The equation of the states of the system in modal coordinates for the first  $n$  modes becomes:

$$\ddot{\mathbf{q}} = -(\mathbf{M} + \Delta\mathbf{M}(t))^{-1}(\mathbf{K} + \Delta\mathbf{K}(t) + \mathbf{K}_a)\mathbf{q} - (\mathbf{M} + \Delta\mathbf{M}(t))^{-1}(\mathbf{D} + \Delta\mathbf{D}(t) + \mathbf{D}_a)\dot{\mathbf{q}} + \gamma(\mathbf{M} + \Delta\mathbf{M}(t))^{-1}[\Psi(x_{a1}) \dots \Psi(x_{a4})]\mathbf{z} - \sum_{j=1}^n m_j g \Psi(v_j t) \tag{10}$$

$$\dot{\mathbf{z}} = \beta K[\Psi(x_{s1}) \dots \Psi(x_{s4})]^T \mathbf{q} + \mathbf{0}_{4 \times N} \dot{\mathbf{q}} - \alpha \mathbf{I}_4 \mathbf{z}$$

where  $K$  is the constant feedback gain and the displacement sensor locations are  $x_{sj}, j=1$  to 4.



**Figure 5.** Schematic of the structural control system for one span.

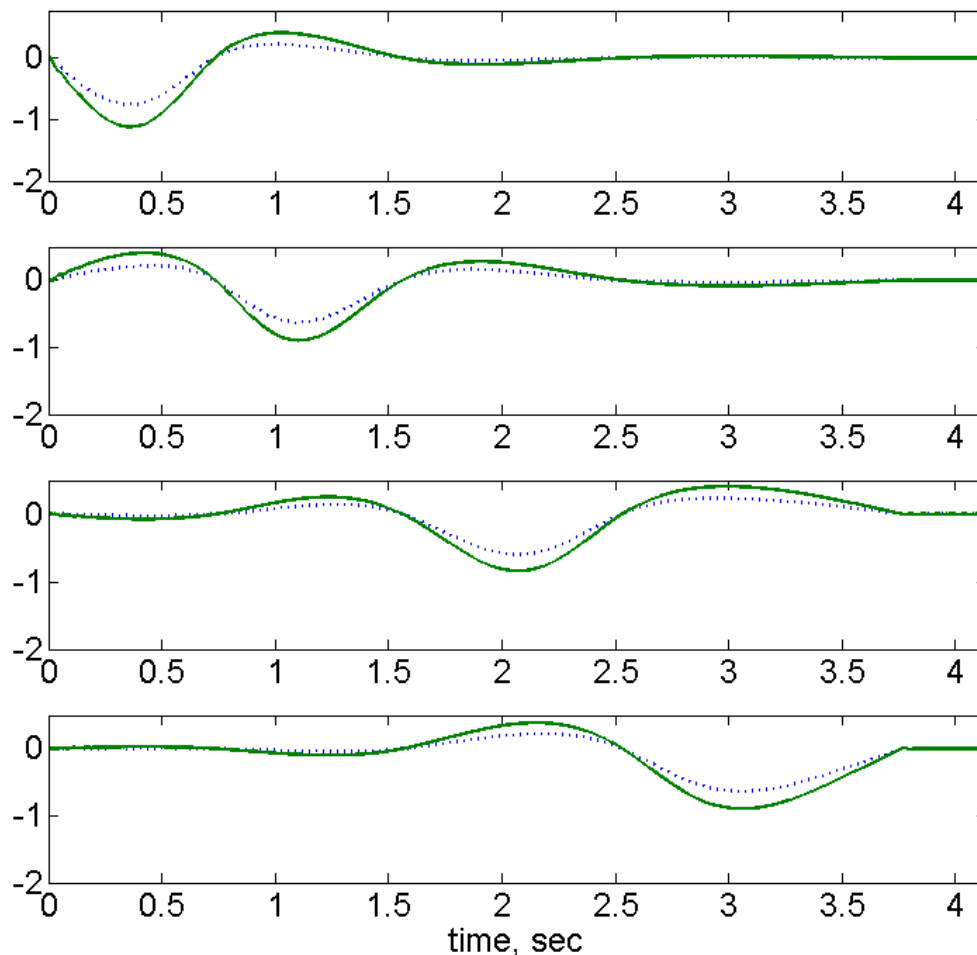
The response of the system improves for a wide range of possible values of  $K$ . In theory the selection of  $K$  is restricted by the loss of stability but in practice the value of  $K$  can be also limited by the saturation of the actuators. A comparison of the response of the original system with the controlled system is shown in figure 6 where the gain is close to the limit of saturation.

It can be seen that the deflection response reduces with an average of 10-15% per span. In this study the actuator location is close to the middle of each span. For a real structure this arrangement might not be feasible. A velocity feedback control for this system will not improve the dynamics and worse still the controlled system will lose stability at very small gains.

The disadvantage of this type of control is that for a physical system, the deflection values which are used to provide the voltage control of the system are usually measured with noise. This noise is also amplified by the constant gain and applied as control to the system. In a real implementation this may come with the supplementary requirement of designing a set of low-pass filters which can produce delays. Even in the case of an appropriate set of filters the accidental damage of one of the sensors can produce high levels of noise which may destabilize the system.



A state-space approach based on linear quadratic regulation (LQR) may not be applicable even with an accurate model of the system. One recognized and often mentioned problem when applying modal based techniques to structural systems is the loss of controllability for higher modes which can lead to instabilities. In fact the LQR technique was studied by many researchers for moving load problems and it provided good results particularly for single span beams but for this type of model showed its limitations.



**Figure 6.** Deflection response at sensor locations for the uncontrolled and feedback controlled system. Continuous line – uncontrolled system, dashed line – controlled system

## 5. Conclusions

This study presents a series of results related to experimental model validation and active structural control applied to the moving load problem. The model of the structure is based on an existing physical model and takes into account not only the structure itself but also the dynamics of the actuators which are used for control. In this respect the present study is one of the first to use physical models for active control of small-scale experimental structures subjected to moving loads.

A couple of simple control methods are tested but some of the state-space methods based on the modal space representation do not provide a good and reliable solution. This is partly due to the complexity of the model of a four-span continuous beam structure. The system loses the controllability due to the high number of modes used in the model. On the other hand the modelling of the actuators means the control force does no longer apply directly upon the structure but through the dynamics of the actuators which in theory adds a new set of state-variables.

Methods based on position feedback provide good results but may depend on the quality of the signal processing and conditioning capability available for a particular implementation. The control effort for a position feedback controller is relatively high for a heavy structure. The dynamic effect of a control solution depends on the supporting structure geometry and configuration, particularly on the actuator positions therefore it is relevant to look for particular actuator locations that will make a better use of the control effort.

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