

Exclusive J/ψ and Υ photoproduction and the low x gluon

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We discuss the potential to constrain the small- x PDFs using the exclusive production of heavy vector mesons. The calculation of J/ψ and Υ photoproduction at NLO in collinear factorisation is described. The different behaviour of the NLO corrections for J/ψ and Υ is highlighted and we outline what might be expected from the inclusion of these processes in a PDF fit.

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1. Introduction

Currently there is a large uncertainty in the low $x \lesssim 10^{-3}$ behaviour of the gluon distribution in the global parton distribution function (PDF) analyses, particularly at low scales [1, 2, 3]. A wide variety of data on exclusive heavy vector meson (HVM) production $\gamma^* p \rightarrow V + p$ and ultraperipheral HVM production $pp \rightarrow p + V + p$, with $V = J/\psi, \psi(2S), \Upsilon$, have been measured over the past few decades, for example see Refs. [4, 5, 6, 7, 8, 9], with more data currently under analysis. The data measured at HERA sample the gluon distribution down to $x \sim 10^{-3}$ and data measured by the LHCb collaboration are sensitive to the gluon distribution down to $x \sim 10^{-5}$. However, the exclusive HVM data are not typically included in the global PDF fits. The main reason for this is that the theoretical frameworks used to describe exclusive HVM production differ to varying degrees from the standard framework used to describe more inclusive processes.

In Section 2 we give a brief overview of the description of exclusive HVM production in both the k_T and collinear factorisation frameworks. Our attempts to extract a gluon distribution from the measurements of this process at HERA and the LHC using k_T factorisation are described in Section 3. The process is discussed using the collinear factorisation approach at NLO in Section 4.

2. Overview of Exclusive Heavy Vector Meson Production

The various frameworks describing exclusive HVM production differ in two key ways from that typically used by global analyses.

Firstly, the momentum of the incoming hadron need not be equal to the momentum of the outgoing hadron, therefore, the parton distributions probed are not the conventional PDFs. In the collinear factorisation framework exclusive processes such as GDVCS [10, 11, 12] and exclusive vector meson electroproduction¹ (including HVM electroproduction) [13, 14] are described at amplitude level in terms of a convolution of coefficient functions and Generalized Parton Distributions (GPDs). These distributions and their relation to the PDFs are briefly described in Section 2.1. Note that in this framework exclusive HVM photoproduction has not been proven to factorise, although, at least at NLO this appears to be the case [15]. In the k_T factorisation framework the process is typically described in terms of so-called ‘skewed unintegrated’ PDFs [16, 17].

Secondly, the framework must describe the formation of the HVM. In this work, we use the leading term of the non-relativistic QCD (NRQCD) approach, it depends on an NRQCD matrix element $\langle O_1 \rangle$ which is proportional to the vector meson branching fraction to leptons $\Gamma[V \rightarrow e^+e^-]$ [18]. To this order the description is equivalent to the ‘static’ approximation [19]. The formation of the HVM is subject to modification by relativistic corrections. These corrections have been studied in a framework similar to NRQCD and found to be small [20], they are neglected here. Note, however, that other work, based on integrating over the relative momentum of the outgoing heavy quarks, indicates that the relativistic corrections may be large [21].

For ultraperipheral production there is an additional complication. In this process two protons (or heavy ions) collide. At LO, considering only hard perturbative contributions and neglecting background processes in which the protons break up or enter an excited state, this process can

¹This process is also known as deeply virtual meson production (DVMP).

be modelled in terms of HVM photoproduction. The Equivalent Photon Approximation (EPA), utilised here, is widely used. It allows the ultraperipheral cross-section to be written as

$$\frac{d\sigma(pp)}{dy} = S_+^2(y)N_+\sigma_+(\gamma p) + S_-^2(y)N_-\sigma_-(\gamma p) + \dots \quad (2.1)$$

Here y is the rapidity, $\sigma_+(\gamma p)$, $\sigma_-(\gamma p)$ are photoproduction cross-sections computed with different γp centre-of-mass energies depending on the rapidity, $S_+^2(y)$, $S_-^2(y)$ are survival factors and N_+ , N_- are photon fluxes. Note that, without forward proton tagging, it is not possible to distinguish which proton emitted the photon entering the photoproduction amplitude. Therefore, the ultraperipheral cross-section for a given rapidity depends on the photoproduction cross-section evaluated at two γp centre-of-mass energies. The higher energy photoproduction cross-section accounts for photon emission from the forward proton whilst the lower energy photoproduction cross-section accounts for photon emission from the backward proton. The survival factors describe the probability that the rapidity gaps survive soft re-scattering effects between the interacting hadrons. They must be modelled and fitted to data, for this we use the Khoze-Martin-Ryskin model [22]. The photon flux used in this work is described in Ref. [23]. The dependence on the photon flux N_+ , N_- partly cancels against the photon flux appearing in the denominator of the survival factors and so the same photon flux should be used to compute both quantities. The ellipses denote interference terms which are small and are neglected here.

Finally, especially for J/ψ photoproduction, we have a low central scale (squared) of 2.4 GeV^2 and could reasonably question the validity of a perturbative approach. One might expect poor convergence of the perturbative series (due to large α_S) and large effects not captured by perturbation theory. Indeed, NLO results in the collinear factorisation framework seem to indicate a very poor convergence of the perturbative series for reasonable choices of the renormalization and factorisation scales, see Section 4.

2.1 Generalized Parton Distributions

Analogously to the PDFs, the GPDs can be defined through the matrix elements of quark and gluon operators at a light-like separation. For GPDs, unlike for PDFs, the momentum p of the incoming hadron need not be equal to the momentum p' of the outgoing hadron. The GPDs $\mathcal{H}_q(x, \xi, t)$, $\mathcal{H}_g(x, \xi, t)$ depend on the momentum fractions x , ξ of $P^+ = (p^+ + p'^+)/2$ carried by the interacting partons. Conventionally, the incoming parton carries momentum $(x + \xi)P^+$ and the outgoing parton carries momentum $(x - \xi)P^+$, see the right panel of Fig. 1. The GPDs depend also on the Mandelstam variable $t = (p - p')^2$ and, after mass factorisation, on the factorisation scale μ_F^2 which is suppressed here. In the forward limit, defined by $p = p'$ or equivalently $\xi = 0$ and $t = 0$, the GPDs are equal to the PDFs,

$$\mathcal{H}_q(x, 0, 0) = q(x), \quad x > 0, \quad (2.2)$$

$$\mathcal{H}_q(x, 0, 0) = -\bar{q}(-x), \quad x < 0, \quad (2.3)$$

$$\mathcal{H}_g(x, 0, 0) = xg(x). \quad (2.4)$$

Efforts by several groups to directly extract GPDs from GDVCS data are on-going [24, 25, 26]. There also exist GPDs $\mathcal{E}_q(x, \xi, t)$ and $\mathcal{E}_g(x, \xi, t)$ which appear multiplied by the momentum transfer

$(p' - p)$, here we neglect them. In Section 4 we will consider the quantities $F_i(x, \xi, t)$ at $t = 0$ which, neglecting the proton mass, are related to the GPDs as $F_i(x, \xi, 0) = \sqrt{1 - \xi^2} \mathcal{H}_i(x, \xi, 0)$ for $i = q, g$.

It has been conjectured that in the small x and ξ limit, beyond the strict forward limit, the GPDs are related to the PDFs via the Shuvaev transform [27]. This is based on the observation that the anomalous dimensions of the Gegenbauer moments H_N of the GPDs are equal to the anomalous dimensions of the Mellin moments M_N of the PDFs. The polynomiality property of the GPDs $H_N = \sum_{k=0}^{\lfloor (N+1)/2 \rfloor} c_k^N \xi^{2k}$ allows the Gegenbauer moments to be determined from the PDFs up to corrections $\mathcal{O}(\xi)$ at NLO. The Shuvaev transform reconstructs the GPDs from these Gegenbauer moments.

Note that the Shuvaev transform strictly relates the GPDs to some effective forward parton distribution functions and the conjecture consists of equating these distribution functions to the PDFs. The equivalence requires that there are no singularities in the right half N -plane of the PDF input distributions [28]. Further, the transform does not relate the GPDs to the PDFs in the $|x| < \xi$ (time-like) region. In this work we assume that the Shuvaev transform can be used to relate the GPDs to the PDFs in the $\xi < |x|$ (space-like) region.

3. The k_T factorisation approach

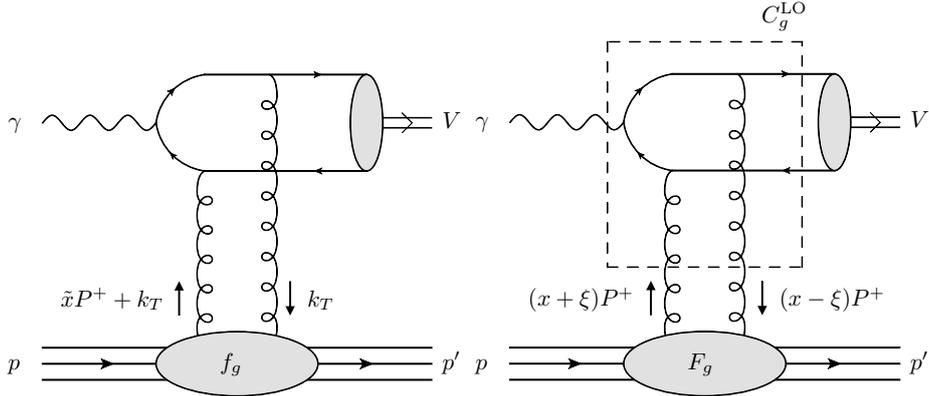


Figure 1: Framework used in (left panel) our k_T factorisation approach and (right panel) the collinear factorisation approach. The quarks connected to the photon are the massive (c or b) valance quarks which form the vector meson $V = J/\psi, \psi(2S), \Upsilon$. The dashed box in the right panel encloses a (5-leg) diagram that contributes to the coefficient function C_g^{LO} .

Previously we have studied exclusive HVM production in the context of k_T factorisation [23, 29]. In our work we calculated at tree level (LO) the dominant imaginary part of the amplitude for photoproduction and electroproduction. At tree level only gluon exchange between the proton and photon contributes. We considered only the high energy ‘maximal skew’ approximation in which the incoming gluon carries plus momentum fraction $\tilde{x} \approx 2x = 2\xi$ and the outgoing gluon carries no momentum in the plus direction. In k_T factorisation the incoming and outgoing gluons can be off-shell and have momentum k_T transverse to the plus direction, see the left panel of Fig. 1.

In this framework the ‘skewed unintegrated’ gluon distribution was parametrised in terms of a gluon PDF and a ‘skewing factor’, which can be derived from the Shuvaev transform using

the ‘maximal skew’ approximation and assuming the gluon PDF obeys a simple power law form $xg(x, \mu_F^2) \sim x^{\lambda(\mu_F^2)}$. Note that in Ref. [30] it was shown that the parametrisation used in our work can introduce an $\mathcal{O}(20 - 30\%)$ effect on the total cross-section as compared to using the full Shuvaev transform. Under the assumption that the Shuvaev transform is valid, that non-relativistic corrections to the HVM formation are small and neglecting contributions higher order in α_S , this is the dominant uncertainty in our analysis.

Two models were considered. In the first model the approximation $k_T^2 \ll \bar{Q}^2$ was used along with a simple power law form for the gluon PDF. In the second model the full k_T dependence was maintained and a double-leading logarithm approximation was used for the gluon PDF which mimics DGLAP evolution over the relevant small range of scales. Maintaining the full k_T dependence in the second model includes some contributions that would only appear beyond LO in collinear factorisation. In both models the gluon PDF was parametrised with 3 variables that were fitted to HERA J/ψ data [4, 5] (both photoproduction and electroproduction) and ultraperipheral data from CDF [6] and the LHCb 2013 data [7], but no other data were included in these fits. The fits obtained are in good agreement with the data, for the first model we obtain $\chi^2/\text{d.o.f} = 41/79$ and for the second model we obtain $\chi^2/\text{d.o.f} = 50/79$. The extracted gluons are broadly similar to the global PDFs but the uncertainty bands at low $x \sim 10^{-5}$ for scales 2–6 GeV² are much narrower.

Further, the predictions from these models compare well to subsequent data on J/ψ , $\psi(2S)$ and Υ measured at LHCb after our publications [8, 9]. Note that our Υ prediction does not rely on any Υ data but is just the result obtained from (approximate) DGLAP evolution of the gluon fitted from J/ψ photoproduction and electroproduction.

Unfortunately, going beyond tree level in this framework appears not to be straightforward, further, the connection between the ‘skewed unintegrated’ distribution is less direct than in the collinear factorisation framework.

4. The Collinear Factorisation Approach at NLO

More recently we considered exclusive HVM photoproduction at NLO in collinear factorisation. In this framework the NLO result has already been known for more than a decade [15] but contained errors corrected only recently [31, 15]. Our re-calculation, which confirms the corrected result, uses a different technique than the original authors, it is described in Section 4.1.

At NLO exclusive HVM production receives additional contributions from a quark exchange between the proton and photon. In the collinear factorisation framework the amplitude can be written as

$$A \propto \int_{-1}^1 dx \left[C_g(x, \xi) F_g(x, \xi) + \sum_{q=u,d,s} C_q(x, \xi) F_q(x, \xi) \right], \quad (4.1)$$

where C_g, C_q are perturbatively calculable coefficient functions and F_g, F_q depend on the GPDs. Here the dependence of the coefficient functions on the renormalization scale μ_R^2 and the factorisation scale μ_F^2 is suppressed. The dependence of F_g, F_q on the Mandelstam t and μ_F^2 has also been suppressed.

4.1 Calculation

To obtain the LO coefficient function we must compute the 5-leg diagrams contained in the dashed box of Fig. 1. The Lorentz or spin indices of the light partons connected to the GPD are contracted with GPD projectors whilst the heavy quarks which enter the NRQCD matrix element are contracted with a projector onto the HVM state, see Refs. [15, 32]. There are 6 gluon diagrams at tree level and 97 gluon + 12 quark diagrams at 1-loop. We compute using dimensional regularisation, the diagrams are generated using QGRAF [33] then processed using FORM [34]. The factorisation imposes that the momenta of the incoming and outgoing light partons are proportional whilst the leading term of the NRQCD expansion imposes that the momenta of the outgoing heavy quarks are equal. The naïve application of standard loop techniques will therefore lead to a singular decomposition and/or an incomplete reduction to the master integrals.

To circumvent these problems we perform a Sudakov decomposition of all external momenta $p_i^\mu = (p_i \cdot n)p^\mu + (p_i \cdot p)n^\mu + p_{iT}^\mu$. Due to the kinematic constraints we see that all transverse components p_{iT} can be chosen to vanish. All external momenta can therefore be constructed from just two linearly independent vectors which can be chosen as the Sudakov basis vectors p, n . Thus, we can decompose our integrals in terms of just p, n (and the metric).

Further, the linear dependence of the external momenta implies that there are relations between the propagators. These relations allow us to reduce N point integrals to $N - 1$ point integrals by eliminating linearly dependent propagators from each integral. Since the kinematics of the process impose that at most two external momenta can be linearly independent there can be at most three different propagators appearing in any integral.

After eliminating the linear dependencies and using integration by parts identities, as implemented in REDUZE [35], we obtain scalar triangle, bubble and tadpole master integrals only. These integrals depend on only one scaleless ratio. The analytic continuation of our result for the coefficient functions is checked against the numerical implementation of the integrals in Ref. [36].

4.2 Results

To obtain the amplitude the coefficient functions calculated as described in Section 4.1 must be convoluted with the GPDs according to Eq. (4.1). Rather than fitting a gluon to the J/ψ data as in Section 3 we instead extract GPDs from the global PDFs using the full Shuvaev transform implemented as described in Ref. [28]. Since the Shuvaev transform limits us to the $\xi < |x|$ space-like region we can consider only the imaginary part of the amplitude which is zero outside this region (this can be seen explicitly from the coefficient functions). Further, it should be noted that, even if valid, the Shuvaev transform may be a poor approximation for large ξ (small γp centre-of-mass energy W). The result for the born and sum of quark and gluon 1-loop contributions to the amplitude for J/ψ and Υ are shown in Fig. 2. For the central value of the scale for the J/ψ the 1-loop contributions dominate the born contribution and have the opposite sign which leads to a (nonsensical) negative cross-section. For Υ the situation is not as severe, for the central value of the scale the born contribution dominates the 1-loop correction leading to a positive cross-section which is, however, very sensitive to the scale choice (for W greater than a few hundred GeV the scale variation band encompasses negative cross-sections). This poor perturbative stability is a major obstacle for any attempt to extract either GPDs or PDFs from exclusive HVM photoproduction.

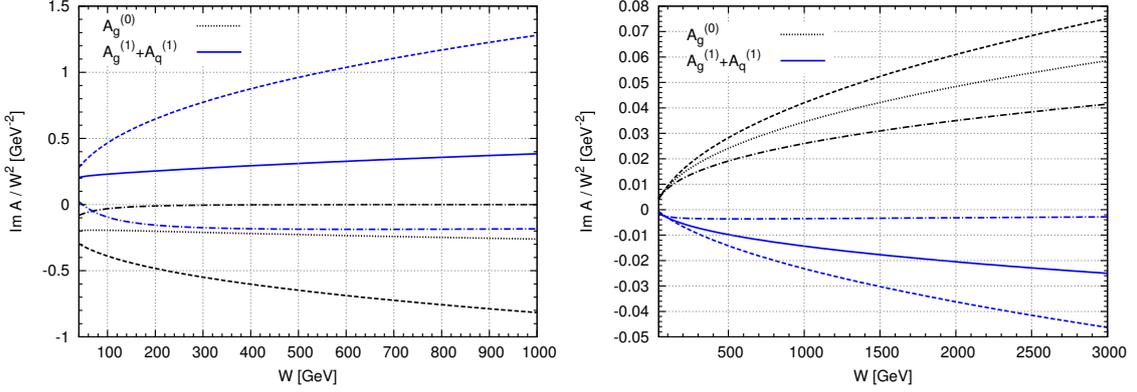


Figure 2: The imaginary part of the born and 1-loop contributions to the amplitude as a function of the γp centre-of-mass energy W for (left panel) $\gamma p \rightarrow J/\psi + p$ and (right panel) $\gamma p \rightarrow \Upsilon + p$ produced using CTEQ6.6 partons [37]. The bands show the $\mu^2 \equiv \mu_R^2 = \mu_F^2$ variation about the central scale. For the left panel $\mu^2 = 1.2, 2.4, 4.8 \text{ GeV}^2$ and for the right panel $\mu^2 = 11.9, 22.4, 44.7 \text{ GeV}^2$. The dot-dashed, solid and dashed lines correspond to the low, central and high values of the scale μ , respectively.

Nevertheless, the sensitivity to the scale for $W \gg M_V^2$, where M_V^2 is the HVM mass, can be understood. It originates from terms containing $\ln(m^2/\mu_F^2) \ln(1/\xi)$, where m is the heavy quark mass, generated by mass factorisation, which can become large for small ξ . It was argued in Ref. [38] that a scale fixing procedure can resum these double log terms and reduce the sensitivity to the factorisation scale. Another approach being pursued is to resum the leading $\ln(1/\xi)$ à la BFKL [39]. Furthermore, for J/ψ one might reasonably suspect that the low central scale is at least partly responsible for the poor perturbative stability.

5. Conclusions

We have described our LO results for exclusive HVM in the k_T factorisation framework. In this framework the gluon distribution extracted from J/ψ exclusive and ultraperipheral production has a reduced uncertainty at low $x \sim 10^{-5}$ and low scales $2 - 6 \text{ GeV}^2$ as compared to the global analyses [23, 29].

Recently we have also re-calculated and confirmed the (recently corrected) NLO result for exclusive photoproduction [15]. For both the J/ψ and the Υ , the 1-loop contribution is large, comparable to the born contribution, and very sensitive to the factorisation scale choice. The loop contribution has opposite sign to the born contribution which can lead even to negative cross-sections for seemingly reasonable scale choices. For the Υ , the correction is large but the cross-section is positive for γp centre-of-mass energies less than a few hundred GeV. The poor behaviour of the correction at large centre-of-mass energies for both J/ψ and Υ is at least partly due to the appearance of large $\ln(1/\xi)$ and several groups are working, with some success, to address this primarily by attempting to resum these logarithms [38, 39].

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References

- [1] R. D. Ball *et al.* [NNPDF Collaboration], *Parton distributions for the LHC Run II*, JHEP **1504** (2015) 040 [arXiv:1410.8849 [hep-ph]].
- [2] L. A. Harland-Lang, A. D. Martin, P. Motylinski and R. S. Thorne, *Parton distributions in the LHC era: MMHT 2014 PDFs*, Eur. Phys. J. C **75** (2015) 204 [arXiv:1412.3989 [hep-ph]].
- [3] S. Dulat *et al.*, *New parton distribution functions from a global analysis of quantum chromodynamics*, Phys. Rev. D **93** (2016) 033006 [arXiv:1506.07443 [hep-ph]].
- [4] C. Alexa *et al.* [H1 Collaboration], *Elastic and Proton-Dissociative Photoproduction of J/ψ Mesons at HERA*, Eur. Phys. J. C **73** (2013) 2466 [arXiv:1304.5162 [hep-ex]].
- [5] S. Chekanov *et al.* [ZEUS Collaboration], *Exclusive electroproduction of J/ψ mesons at HERA*, Nucl. Phys. B **695** (2004) 3 [hep-ex/0404008].
- [6] T. Aaltonen *et al.* [CDF Collaboration], *Observation of exclusive charmonium production and $\gamma + \gamma$ to $\mu^+ \mu^-$ in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV*, Phys. Rev. Lett. **102** (2009) 242001 [arXiv:0902.1271 [hep-ex]].
- [7] R. Aaij *et al.* [LHCb Collaboration], *Exclusive J/ψ and $\psi(2S)$ production in pp collisions at $\sqrt{s} = 7$ TeV*, J. Phys. G **40** (2013) 045001 [arXiv:1301.7084 [hep-ex]].
- [8] R. Aaij *et al.* [LHCb Collaboration], *Updated measurements of exclusive J/ψ and $\psi(2S)$ production cross-sections in pp collisions at $\sqrt{s} = 7$ TeV*, J. Phys. G **41** (2014) 055002 [arXiv:1401.3288 [hep-ex]].
- [9] R. Aaij *et al.* [LHCb Collaboration], *Measurement of the exclusive Υ production cross-section in pp collisions at $\sqrt{s} = 7$ TeV and 8 TeV*, JHEP **1509** (2015) 084 [arXiv:1505.08139 [hep-ex]].
- [10] A. V. Radyushkin, *Scaling limit of deeply virtual Compton scattering*, Phys. Lett. B **380** (1996) 417 [hep-ph/9604317].
- [11] X. D. Ji, *Deeply virtual Compton scattering*, Phys. Rev. D **55** (1997) 7114 [hep-ph/9609381].
- [12] J. C. Collins and A. Freund, *Proof of factorization for deeply virtual Compton scattering in QCD*, Phys. Rev. D **59** (1999) 074009 [hep-ph/9801262].
- [13] A. V. Radyushkin, *Asymmetric gluon distributions and hard diffractive electroproduction*, Phys. Lett. B **385** (1996) 333 [hep-ph/9605431].
- [14] J. C. Collins, L. Frankfurt and M. Strikman, *Factorization for hard exclusive electroproduction of mesons in QCD*, Phys. Rev. D **56** (1997) 2982 [hep-ph/9611433].
- [15] D. Y. Ivanov, A. Schafer, L. Szymanowski and G. Krasnikov, *Exclusive photoproduction of a heavy vector meson in QCD*, Eur. Phys. J. C **34** (2004) 297, Erratum Eur. Phys. J. C **75** (2015) 75, Erratum hep-ph/0401131v3.
- [16] A. D. Martin and M. G. Ryskin, *Unintegrated generalized parton distributions*, Phys. Rev. D **64** (2001) 094017 [hep-ph/0107149].

- [17] G. Watt, A. D. Martin and M. G. Ryskin, *Unintegrated parton distributions and electroweak boson production at hadron colliders*, Phys. Rev. D **70** (2004) 014012, Erratum Phys. Rev. D **70** (2004) 079902 [hep-ph/0309096].
- [18] G. T. Bodwin, E. Braaten and G. P. Lepage, *Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium*, Phys. Rev. D **51** (1995) 1125, Erratum Phys. Rev. D **55** (1997) 5853 [hep-ph/9407339].
- [19] E. L. Berger and D. L. Jones, *Inelastic Photoproduction of J/ψ and Upsilon by Gluons*, Phys. Rev. D **23** (1981) 1521.
- [20] P. Hoodbhoy, *Wave function corrections and off forward gluon distributions in diffractive J/ψ electroproduction*, Phys. Rev. D **56** (1997) 388 [hep-ph/9611207].
- [21] L. Frankfurt, W. Koepf and M. Strikman, *Diffractive heavy quarkonium photoproduction and electroproduction in QCD*, Phys. Rev. D **57** (1998) 512 [hep-ph/9702216].
- [22] V. A. Khoze, A. D. Martin and M. G. Ryskin, *Photon exchange processes at hadron colliders as a probe of the dynamics of diffraction*, Eur. Phys. J. C **24** (2002) 459 [hep-ph/0201301].
- [23] S. P. Jones, A. D. Martin, M. G. Ryskin and T. Teubner, *Probes of the small x gluon via exclusive J/ψ and Υ production at HERA and the LHC*, JHEP **1311** (2013) 085 [arXiv:1307.7099].
- [24] K. Kumericki, S. Liuti and H. Moutarde, *GPD phenomenology and DVCS fitting - Entering the high-precision era*, contribution to Eur. Phys. J. A topical issue on *3D Structure of the Nucleon* arXiv:1602.02763 [hep-ph].
- [25] M. Boer and M. Guidal, *Generalized Parton Distributions and Deeply Virtual Compton Scattering*, J. Phys. G **42** (2015) 034023 [arXiv:1412.4651 [hep-ph]].
- [26] J. O. Gonzalez-Hernandez, S. Liuti, G. R. Goldstein and K. Kathuria, *Interpretation of the Flavor Dependence of Nucleon Form Factors in a Generalized Parton Distribution Model*, Phys. Rev. C **88** (2013) 065206 [arXiv:1206.1876 [hep-ph]].
- [27] A. Shuvaev, *Solution of the off forward leading logarithmic evolution equation based on the Gegenbauer moments inversion*, Phys. Rev. D **60** (1999) 116005 [hep-ph/9902318].
- [28] A. D. Martin, C. Nockles, M. G. Ryskin, A. G. Shuvaev and T. Teubner, *Generalised parton distributions at small x* , Eur. Phys. J. C **63** (2009) 57–67 [arXiv:0812.3558 [hep-ph]].
- [29] S. P. Jones, A. D. Martin, M. G. Ryskin and T. Teubner, *Predictions of exclusive $\psi(2S)$ production at the LHC*, J. Phys. G **41** (2014) 055009 [arXiv:1312.6795 [hep-ph]].
- [30] L. A. Harland-Lang, *Simple form for the low- x generalized parton distributions in the skewed regime*, Phys. Rev. D **88** (2013) 034029 [arXiv:1306.6661 [hep-ph]].
- [31] C. J. Nockles, The University of Liverpool, PhD thesis 2009.
- [32] E. Braaten and J. Lee, *Exclusive double charmonium production from $e^+ e^-$ annihilation into a virtual photon*, Phys. Rev. D **67** (2003) 054007, Erratum Phys. Rev. D **72** (2005) 099901 [hep-ph/0211085].
- [33] P. Nogueira, *Automatic Feynman graph generation*, J. Comput. Phys. **105** (1993) 279.
- [34] J. Kuipers, T. Ueda, J. A. M. Vermaseren and J. Vollinga, *FORM version 4.0*, Comput. Phys. Commun. **184** (2013) 1453 [arXiv:1203.6543 [cs.SC]].
- [35] C. Studerus, *Reduze-Feynman Integral Reduction in C++*, Comput. Phys. Commun. **181** (2010) 1293 [arXiv:0912.2546 [physics.comp-ph]].

- [36] T. Hahn and M. Perez-Victoria, *Automatized one loop calculations in four-dimensions and D-dimensions*, Comput. Phys. Commun. **118** (1999) 153 [hep-ph/9807565].
- [37] P. M. Nadolsky, H. L. Lai, Q. H. Cao, J. Huston, J. Pumplin, D. Stump, W. K. Tung and C.-P. Yuan, *Implications of CTEQ global analysis for collider observables*, Phys. Rev. D **78** (2008) 013004 [arXiv:0802.0007 [hep-ph]].
- [38] S. P. Jones, A. D. Martin, M. G. Ryskin and T. Teubner, *Exclusive J/ψ and Υ photoproduction and the low x gluon*, J. Phys. G **43** (2016) 035002 [arXiv:1507.06942 [hep-ph]].
- [39] D. Y. Ivanov, B. Pire, L. Szymanowski and J. Wagner, *Probing GPDs in photoproduction processes at hadron colliders*, to appear in proceedings of *PHOTON2015* [arXiv:1510.06710 [hep-ph]].