**Sliding Mode Control for High-speed Pantograph-catenary Contact Force with a Disturbance Compensator**

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**Abstract**

In modern high-speed electrified railway, current collection quality between pantograph and catenary is one of the most crucial factors that determine the highest driving speed of an electric locomotive. Apart from parameter optimization of a pantograph-catenary system, active control of a pantograph is another effective measure to reduce the strong fluctuation of the contact force between the pantograph and the contact wire. As is well known that besidesenvironmentalperturbations, the significant variation of the contact force between the pantograph and catenary is mainly caused by two factors: the unsmooth stiffness/mass distribution of the contact wire, and some other nonlinear effect from catenary to pantograph (such as wave propagation along the contact wire). Previous control strategies were mainly focused on the former factor based on a very simple lumped-mass catenary model. Furthermore, most of them were not implemented with a nonlinear validated catenary model. These disadvantages are addressed in this paper. A catenary model is established based on the nonlinear finite element procedure and the pantograph is considered a 2-degree of freedom (DOF) model. In the design stage of the sliding mode controller, the catenary is simplified as a lumped-mass model with a time-varying mass and stiffness, in which, uncertain disturbance is included to compensate the perturbations of nonlinear effect from catenary to pantograph. A sliding mode surface is properly defined to reduce the variation of contact force. In order to decrease the effect of other uncertain disturbance, a disturbance compensator is included to estimate the disturbance in each discrete time step. Several numerical examples indicate that the proposed controller can effectively reduce the contact force fluctuation with a simplified pantograph model and a nonlinear FEM catenary model, as well as eliminate the contact loss.

Keywords

High-speed railway, Pantograph, Catenary, Contact force, Sliding mode control (SMC), Disturbance compensator.

1. Introduction

The recent decades have witnessed a rapid expansion of high-speed electrified railway in many countries around the world. The increase of the driving speed of high-speed trains leads to many new technical issues. One of them is the strong vibration of the pantograph-catenary system resulting in the deterioration of the current collection quality, which has attracted wide attention of many scholars in recent years. Figure 1 describes the schematic of a pantograph-catenary system. The electric power is transmitted from the catenary to the locomotive via a pantograph installed on the car’s roof. Obviously, the sliding contact between the pantograph collector and the contact wire is the most vulnerable point in this system, especially in high-speed driving conditions. In general, the pantograph-catenary contact force (PCCF) is the direct reflection of the contact quality. An excessive contact force can aggravate the wear and fatigue of contact wire and pantograph collector. In contrast, an inadequate contact force may increase the possibility of the separation of the pantograph collector from the contact wire, which may lead to the occurrence of arcing and the interruption of electric power transmission. So, one of the important objectives of studying the pantograph-catenary interaction is to reduce the fluctuation of the contact force and maintain a stable contact between the pantograph collector and the contact wire.

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**Figure 1**. Schematic of the pantograph-catenary system.

Park et al. (2003) and Kim et al. (2007) conducted a state sensitivity analysis for a high-speed pantograph-catenary system and tried to make sense of the influence of the pantograph-catenary parameters on the dynamic response. Then based on the sensitivity analysis results, a design optimization procedure for the pantograph was conducted by Lee et al (2012). Based on a finite element catenary model and a lumped mass-stiffness-damper pantograph model, a parameter optimization for the high-speed pantograph-catenary system was performed by Zhou and Zhang (2011) to improve the pantograph-catenary current collection quality. The suspension characteristics of a pantograph head were studied by Ambrosio and Pombo (2012; 2013), and an optimization procedure was proposed to decrease the standard deviation of the PCCF. Considering the geometrical nonlinearity of catenary wires, Song et al. (2015) proposed a catenary model based on a nonlinear finite element procedure and studied the influence of the contact wire pre-sag and the pre-tension on the standard deviation of PCCF.

Apart from the above parameter optimizations of the high-speed pantograph-catenary system, pantograph control is another solution for this problem. Instead of some passive control devices (Poetsch et al., 1997; Pappalardo et al., 2015), active control strategy has been a popular trend having good potential in reducing the fluctuation of contact force and ensuring a stable current collection quality. An actuator can be installed on the pantograph to provide additional control force. Some scholars (Facchinetti et al., 2009) proposed that the actuator placed in parallel with the collector suspensions could obtain better performance, but this measure seems inconvenient to be realized in an existing railway line. Normally, as shown in Figure 2, the actuator on the lower frame of the pantograph can be used to adjust the uplift of the collector, which is more convenient to implement and does not lead to a large change of the original pantograph structure.



**Figure 2**. Pantograph with an active actuator.

Generally, the study of pantograph-catenary dynamic interaction belongs to moving-load dynamics problems (Ouyang, 2011). Among them, vibration suppression of beams excited by moving loads has been of great interest to many scholars, as it has wide potentials in various engineering applications, such as bridge-vehicle interaction, and vehicle-track interaction. Abdel-Rohman (2005) proposed a simple feedback controller for a long-span suspended bridge. Moghaddas et al. (2012) attached an optimized tuned mass damper (TMD) to a Timoshenko beam to reduce its vibration excited by moving vehicles. Pi and Ouyang (2015) used Lyapunov-based boundary control method for suppressing the vibration of a multi-span beam subjected to moving masses. Stancioiu and Ouyang (2014) proposed an effective time-varying optimal control strategy for a beam excited by a moving mass. However, compared with the traditional control of the beam subjected to a moving load, pantograph control has some distinctive difficulties:

1) How to realize the control objective? The control objective of active pantograph control is mainly to decrease the fluctuation of the contact force and avoid the occurrence of contact loss, instead of just suppressing the vibration response.

2) How to properly consider the nonlinear effect of the catenary on the pantograph in the design stage of controller? The strong variation of the contact force is mainly caused by the unsmooth stiffness/mass distribution of the catenary, some other uncertain disturbance from catenary to pantograph and some environmental perturbations. Furthermore, the catenary structure exhibits strong flexibility and geometric nonlinearity. And it seems impossible to measure the states of catenary along a very long railway.

3) How to improve the real-time performance of actuators and the sensitivity of sensors?



**Figure 3**. Whole complex pantograph-catenary model

Apart from the work done by Facchinetti and Mauri (2009), who established a hardware-in-the-loop overhead line emulator with an active pantograph, most scholars focused on overcoming the first two difficulties and proposed various kinds of controllers with different pantograph-catenary models. Generally, due to the strong nonlinear effect of the catenary on the pantograph, it is very difficult to design the controller based on the whole complex catenary-pantograph model (shown in Figure 3) directly. The pantograph is widely considered a lumped simple pantograph models with two or three masses. In the stage of the controller design, the catenary is always simplified as a one degree-of-freedom (DOF) model with a time varying stiffness (shown in Figure 4(a)) to facilitate development of the control strategy. Sometimes, a time-varying mass is also included (shown in Figure 4(b)). In Figure 4,  and  are the time-varying stiffness and mass of catenary. ,  and  are the mass, stiffness and damping of the pantograph head, respectively; ,  and  are the mass, stiffness and damping of the pantograph frame, respectively.  is the PCCF.  is the static uplift force.  is the control force.



(a) (b)

Figure 4. Pantograph and simplified catenary model with a time-varying stiffness (a); Pantograph and simplified catenary model with a time-varying stiffness and mass (b)

Based on the simplified model shown in Figure 4(a), Lin and his colleagues (2007) developed an active control law by means of a LQR (linear quadratic regulator). Recently, they considered the influence of actuator time delays and exploited an optimal control approach for solving the general robust control problem of their active pantograph-catenary (Lin et al., 2007). Rachid (2011) developed an LMI (Linear Matrix Inequality) control law to keep the PCCF close to a desired value in various operating conditions. Connor et al. (1997) used the steady state matrix Riccati equation to design a controller for an active pantograph, which obtained an over 50 percent reduction in the contact force variation. Pisano et al. (2004) proposed an output feedback control scheme based on higher-order sliding modes and high-gain observers, which had a good robustness and stability. Walters (2010) indicated that the adaptive fuzzy control was effective for active pantograph control. Based on the simplified model shown in Figure 4(b), Pisano and Usai (2008) implemented a variable structure control (VSC) technique with sliding mode on a wire-actuated symmetric pantograph. Their results indicated that the contact force was very close to the desired value using their proposed controller. Sanchez-Rebollo et al. (2013) designed a PID controller for an active pantograph, which was the first attempt to implement a controller with a validated FEM catenary model.

It is obvious that the simplified models shown in Figure 4 can only reflect the unsmooth stiffness and mass distribution of the catenary without considering some other nonlinear disturbance from the catenary to the pantograph in the stage of controller design. On the other hand, except the work done by Sanchez-Rebollo et al. (2013), none of the other works implemented their controllers with a more realistic nonlinear validated catenary model. However, the validation of the controller with a realistic nonlinear catenary model is very important in the theoretical study of the active pantograph control before it can be evaluated through a field experiment in a real railway line.

This paper addresses these difficulties and weakness of the previous studies and proposes a sliding mode control (SMC) strategy with a disturbance compensator for a high-speed pantograph. Compared with other control strategies, SMC seems more effective and convenient to realize the control objective of keeping the contact force close to a desired value through defining a proper sliding mode surface. SMC was shown to be very suitable for variable structures (Gao et al., 1995). Recently, Pi and Ouyang (2015) also found that SMC had a good performance in suppressing the vibration of a beam excited by a moving mass. In (Pisano and Usai, 2008), SMC was firstly introduced in the active pantograph control and a very simple control law was proposed. But the over-simplified model used in that work cannot guarantee its validation with a more realistic pantograph-catenary model.

In this paper, a simplified pantograph-catenary model shown in Figure 4(b) with uncertain disturbance is adopted to design the controller. A discrete disturbance compensator is introduced to reduce the influence of the nonlinearity and uncertain disturbance exerted on the pantograph by the catenary. Then in order to evaluate the performance of the proposed controller, a nonlinear FEM catenary model is established, which is traversed by an active pantograph with the proposed controller. The performance of the controller in reducing the PCCF variation and the possibility of contact loss is evaluated through several examples. Some of the results are compared with those from other publications.

The outline of this paper is as follows. A catenary model based on a nonlinear finite element procedure is presented in Section II. The design procedure of the sliding mode controller with a disturbance compensator is established in section III. Section IV presents some numerical examples to evaluate the performance of the proposed controller. Section V draws conclusions of this paper and suggests some further improvements.

2. Establishment of nonlinear pantograph-catenary model

In this section, a complex catenary model is established based on a nonlinear finite element procedure. As other scholars used to do, the pantograph is considered a multi-rigid-body system with 2 DOFs. The sliding contact between the pantograph and the catenary is implemented by the prevalent “penalty function method”.

## Nonlinear FEM catenary model

It is well known that the catenary is comprised of three main components. As shown in Figure 3, the contact wire is responsible for providing the electrical energy through a pantograph to the high-speed train. The messenger wire and droppers support the contact wire to keep a certain amount of pre-sag in the contact wire. The nonlinear modelling method in the authors’ previous work (Song et al., 2015) is adopted here to establish the catenary model, in which, the nonlinear effect of the slacking of droppers, the geometrical nonlinearity of the messenger/contact wires and the exact initial configuration of catenary can be properly reflected.



**Figure 5**. Nonlinear cable element

The contact/messenger wire is discritized as a number of nonlinear cable elements based on the FEM. Figure 5 presents a segment of the nonlinear cable with the unstrained length of . The relationship between the relative positions , , of the two nodes and the nodal forces  is governed by Eq. (1) (Jung et al., 2013).

 (1)

in which,  is the self-weight per unit length. *E* and *A* denote the Young’s modulus and the cross-sectional area of the messenger/contact wire. The relatinship between the nodal forces  at node I and  at node Jcan be expressed as

   (2)

By differentiating both sides of Eq. (1), the flexibility matrix can be obtained.

 (3)

where *,* *,*  are the increments of the relative positions in,  and directions, respectively. , ,  represent the increments of the corresponding internal force. The stiffness matrix can be obtained by inverting the flexibility matrix , which is updated in each iterative step according to the nodal forces  to ensure accuracy when large deformation of the messenger/contact wire occurs. The vector  related to  is used to solve the initial state. So Eq. (3) can be rewritten as

 (4)

where

 (5)

It is worthwhile to note that after the initial configuration is determined, the initial length  of the cable does not change. So, in the dynamic simulation, Eq. (4) can be written as the following equation with a constant initial length .

 (6)



**Figure 6**. Nonlinear string element

Droppers can only work in tension and have no resistance to compression force, which behave like one-sided springs. In this paper, the following string element is utilized to model the nonlinear behaviour of droppers, as shown in Figure 6. The equilibrium equation can be expressed as (Kim et al., 2012):

 (7)

in which,  represent the internal forces of each node in different directions.  and  denote the elastic modulus and the cross-sectional area of the dropper. and are the strained and unstrained lengths of the string element. When , the dropper works in tension. Otherwise, the dropper provides no stiffness and hence .Similar to the deriviation of the cable element, the element stiffness matrix and the internal force incremental vector  have the following relationship:

 (8)

where  is the stiffness matrix related to the displacement incremental vector .  is the generalized stiffness matrix related to the initial length incremental vector .

A shape-finding method is adopted to solve the initial configuration of catenary, which has been verified by several numerical examples in (Song et al., 2015). The global equilibrium equation of catenary can be obtained by assembling the element stiffness matrices in Eqs. (4) and (8) according to the FEM. In each interactive step, the following global equilibrium equation can be obtained as:

 (10)

where the superscript *k* denotes the *k*th iterative step for solving the equilibrium state.  and  denote the global stiffness matrices related to the incremental displacement vector  and the incremental initial length vector , respectively.  denotes the unbalanced force vector, which can be calculated by

 (11)

where  denotes the external applied nodal force vector,  denotes the internal force vector, which can be calculated by the element nodal forces  and  in the *k*th interactive step. Eq. (11) can be solved by applying some additional constraint conditions that meet the design requirement of catenary.

After the initial configuration is determined, a nonlinear finite element dynamic procedure is performed to solve the dynamic response of the catenary. In Eq. (10), the incremental initial length vector  equals zero. The global stiffness matrix  of the catenary can be formulated through the FEM at any time instant *t*. In combination with the global lumped mass matrix  and the global damping matrix  (which are considered constant matrices in the dynamic simulation procedure), the structural dynamic equation can be written as

 (12)

in which, ,  and  are the incremental vectors of the global acceleration, velocity and displacement for the catenary.  on the right side is the incremental vector of the excitation, which includes the contact force exerted on the contact point and the internal force of the structure.

## Pantograph model and its contact with catenary

A two-DOFs pantograph model as shown in Fig. 7 is adopted.  is the vertical displacement of the pantograph head.  is the vertical displacement of the pantograph frame.  is the vertical displacement of the contact wire at the contact point. The contact between the pantograph head and the contact wire is implemented by the penalty function method, which assumes that the pantograph head connects with the contact wire through a contact stiffness *k*s. The contact force can be calculated as



**Figure 7**. Lumped pantograph model

 (13)

## Verification of pantograph-catenary model

Two series of pantograph-catenary parameters are adopted here to verify the validation of the dynamic performance of the present model. The first one is the benchmark model in EN 50318 (European Committee Electrotechnical Standardization, 2002), which is adopted to implement the pantograph-catenary dynamic simulation. The statistics of the numerical results are shown in Table. 1. It is found that the results of the present model show good agreement with EN 50318.

**Table 1**. Validation of the present mode according to EN 50318

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Statistical index | Standard ranges | | Computation results | |
| Speed [km/h] | 250 | 300 | 250 | 300 |
| Mean contact force (N) | 110~120 | 110~120 | 115.30 | 115.45 |
| Standard deviation (N) | 26~31 | 32~40 | 26.72 | 34.56 |
| Max. statistic value (N) | 190~210 | 210~230 | 195.46 | 219.13 |
| Min. statistic value (N) | 20~40 | -5~20 | 35.14 | 11.77 |
| Max. real value (N) | 175~210 | 190~225 | 191.13 | 196.16 |
| Min. real value (N) | 50~75 | 30~55 | 65.79 | 37.37 |
| Max. uplift at support(mm) | 48~55 | 55~65 | 52.06 | 60.73 |
| Contact loss | 0% | 0% | 0% | 0% |

The second one is the newest benchmark proposed by Bruni et al. (2015). Using the parameters of the benchmark model, the result of the initial configuration of one-span catenary is shown in Figure 8(a) and the corresponding pre-sag of the contact wire is shown in Figure 8(b). It can be observed that the maximum value of the pre-sag is 56.1mm, which has only little difference from the benchmark value (55mm). For the verification of the dynamic performance, Table 2 shows the dynamic simulation results compared with the benchmark values. It is observed that only a small difference exists between the two sets of results.



(a)

 (b)

**Figure 8.** Initial configuration of catenary (a); Initial pre-sag of contact wire (b)

Table 2. Validation of present model according to the benchmark results

|  |  |  |  |
| --- | --- | --- | --- |
| Indicators | Benchmark | Simulation | Error (%) |
| Speed (km/s) | 320 | 320 | 0 |
| Mean contact force (N) | 169 | 171.25 | 1.3 |
| Standard deviation (N) | 53.91 | 55.21 | 2.4 |
| Max. real value (N) | 313.22 | 321.79 | 2.7 |
| Min. real value (N) | 60.40 | 56.84 | -5.9 |
| Banded SD 0-2Hz (N) | 38.27 | 39.11 | 2.2 |
| Banded SD 0-5Hz (N) | 41.04 | 40.52 | -1.3 |
| Banded SD 5-20Hz (N) | 34.80 | 36.07 | 3.6 |
| PPA of the vertical displacement (mm) | 47.58 | 45.31 | 4.8 |
| Max. uplift at support (mm) | 59.12 | 60.17 | 1.8 |

3. Design of sliding mode controller for active pantograph

In this section, the sliding mode controller for the active pantograph with a disturbance compensator is derived and its stability is verified. Due to the high flexibility and nonlinearity of the catenary structure, it is very difficult to design the controller directly using the complex model constructed in section II. So for the controller design, the catenary is temporarily simplified as a one-DOF model with time-varying stiffness and mass as shown in Figure 4(b). Simultaneously, an disturbance is included to compensate for the perturbations of other nonlinear effect from the catenary to the pantograph. For the convenience to compare with other papers, the parameters of catenary and pantograph are chosen according to the benchmark model of European Standard EN 50318 (European Committee Electrotechnical Standardization, 2002) in the design of controller and the numerical simulations in the next section.

## 3.1 Simplified catenary model

The time-varying mass and stiffness of the catenary can be determined from the FEM model constructed in Section II based on the equivalence of elastic energy and kinetic energy (Lopez-Garcia et al., 2007). According to EN 50318, a catenary is constructed with 10 spans. The equivalent mass and stiffness of one span are shown in Figure 9(a-b), which are identical for other spans.

 (a)



(b)

**Figure 9**. Mass distribution along one span (a) and stiffness distribution along one span (b)

After obtaining the equivalent mass and stiffness, the equation of motion of the simplified catenary-pantograph system (as shown in Figure 4(b)), considering an uncertain disturbance *d*(*t*)can be derived as follows:

 (14)

in which, the contact force  is determined by Eq. (13).

## 3.2 Sliding mode control law design

In reality, only the contact force and the states of pantograph head and frame can be measured (Facchinetti et al., 2009). So, according to the penalty function method, the contact wire displacement  can be expressed by the contact force  as

 (15)

By differentiating both sides, one can get

 (16)

Substitute Eq. (15-16) to Eq. (14) , the contact wire displacement , velocity  and acceleration  can be eliminated. Then the equation of motion can be rewritten as

 (17)

where,



Eq. (17) is a generalized equation of motion, in which, the contact wire displacement  and its derivatives are all replaced by the contact force . Eq. (17) can be converted to the state-space form as

 (18)

in which,



in which,  is a 3×3 identity matrix. Then the continuous state-space equation can be transformed to a discrete form by a very small time step  as

 (19)

where, ,  and  are the discrete forms of ,  and  from a very small sampling period  .

According to the control objective, a vector  containing the desired values of all the states is defined as

 (20)

in which,  is the desired value of the contact force. Generally  is equal to the mean value of contact force, which is normally equal to the static uplift force . In addition, it is required that , so that the third element of  is chosen as zero. So let , the error equation can be written as

 (21)

in which, . The sliding surface can be defined as

 (22)

 should be a relatively larger number to guarantee the primary control objective. It can be found from Eq. (22), that only the contact force and its derivative are required in this controller.

Then in order to improve the dynamic performance, a constant control law in discrete form is adopted (Eun et al., 1999):

 (23)

in which,  is the factor of the reaching law in SMC. A bigger  can make the response of the system quicker but may increase the possibility of chatting. From Eq. (22), the following can be deduced:

 (24)

Substituting Eq. (21) into Eq. (24), and combining Eq. (23), the control force  can be obtained as

 (25)

where



Obviously,  can be calculated directly at each time instant. However,  is determined by the disturbance , which can be estimated by a disturbance compensator. Before the disturbance compensator is introduced, the stability of the sliding mode controller is proved as follows:

The conditions for the existence of SMC and for reaching it can be written as

 (26)

For the specific system, the existence condition is:

 (27)

The reaching condition is

 (28)

It can be found that only if , the stability can be guaranteed.

## 3.3 Design of the disturbance compensator

To deal with the uncertain disturbance exerted on the simplified pantograph-catenary model shown in Figure 4(b), a discrete disturbance compensator is designed as follows:

An estimated disturbance  is used to replace the real disturbance  in each time instant to calculate the control force  related to the disturbance. So  in Eq. (25) can be calculated according to the estimated disturbance  as

 (29)

A constant *g* can be introduced to estimate the disturbance in each time instant. The estimated disturbance  can be calculated by

 (30)

At each time instant , the real disturbance  can be determined according to Eq. (21) as:

 (31)

Then Eq. (31) can be used to estimate the disturbance at time instant *t*. According to (Eun et al., 1999), only if the following two conditions hold for all time instants, the stability can be guaranteed.

1)  ;

2) 

## 3.4 Saturated controller design

The above description only gives an optimal theoretical value of the control force , which sometimes has a value exceeding the maximum limit of the pantograph actuator force. So, a saturated controller is employed to avoid excessive control force (Yokoyama et al., 2010):

 (32)

in which,  is the real control force.  is the bound of the saturated controller. According to EN 50318, the static uplift is 120N, so the maximum bound of control force is normally defined as no more than 100N in the simulation conducted in this paper.

4. Evaluation of control performance

In this section, several numerical examples are implemented with both the simplified model and nonlinear FEM model of the catenary, respectively, to evaluate the performance of the proposed control strategy in reducing the PCCF variation. And some of the results with nonlinear FEM model are compared with (Sanchez-Rebollo et al. 2013). Then the contact wire irregularity is included in the FEM model, which is one of the major anomalies of the catenary infrastructure normally caused by the manufacture installation errors and wear of the contact wire. Under such a perturbation, the control performance is evaluated in reducing the possibility of contact loss. It should be noted that the pantograph-catenary parameters used in the following analysis are chosen from the benchmark model in EN 50318. A 10-span catenary model is established and the numerical results of the central two spans are adopted as the subject of analysis and optimization. The FEM results of PCCF are filtered through 0-20Hz, as the requirements of EN 50318.

## 4.1 Implementation with a simplified model

Firstly, to avoid huge calculation cost, the proposed control strategy is implemented with a simplified model shown in Fig. 4(b). Besides the unsmooth distribution of mass and stiffness, other disturbances  to the catenary in the state-space equation are considered a white Gaussian noise (WGN) with intensity of 0.5dB. In this simulation, the bound *f*m in Eq. (31) of the saturated controller are chosen as 50N and 100N, respectively. As the sliding mode surface is completely determined by factor ** in Eq. (22), the simulations are conducted with different values of *.* Consider that the driving speed of vehicle is 300km/h and time step , factor *g* in Eq. (30) is 0.9, and *q* in Eq. (23) is 0.6.

The results of the contact force at different values of ** and different controller bounds  are shown in Figure 10. It can be observed that the proposed controller has a good performance in the reduction of the PCCF variation. However, with each controller bound, the factor of the sliding mode surface ** also has an obvious influence on the control performance. Generally, the increase of ** can lead to a quicker system response but also an increase of the possibility of chattering. So the influence of ** on the control performance should be studied deeply.

  (a)

 (b)

**Figure 10**. Contact force with *f*m=50N (a); Contact force with *f*m=100N (b)

Figure 11 shows the results of the PCCF variation with different values of ** and different controller bounds . It is found that in general, the proposed controller has a significant effect in reducing the variance of the PCCF. The controller with  has a better performance compared with that with a smaller bound (). The best performance of the controller with  appears at **, where the PCCF variance decreases from 1238.3 N2 to 899.8 N2 by 27%. When the controller bound , the lowest variation appears at **, which decreases the PCCF variance by over 46%. The control force under these two conditions are shown in Figure 12(a) and Fig. 11(b), respectively.

**Figure 11.** PCCF variation with different values of *β*



(a)



(b)

**Figure 12**. Control force with *f*m=50N, *β*=8 (a); Control force with *f*m=100N, *β*=7 (b)

## Implementation with a nonlinear FEM model

As explained in the introduction, the implementation of the controller with a nonlinear FEM model of the catenary is a very important step in designing an active pantograph. Similar to the analysis with the simplified model, the results of the PCCF variation with different values of ** and different controller bounds  are shown in Figure 13. A similar conclusion can be obtained that with each controller bound, the factor ** plays an important role in influencing the control performance.

 **Figure 13**. PCCF variation with different values of *β*

**Figure 14**. Computation results of PCCF with nonlinear FEM model.

With the controller bounds  and , the lowest PCCF variances respectively appear at ** and **. The results of PCCF in these two conditions are shown in Figure 14(a) and (b), respectively. It is worthwhile to note that Sanchez-Rebollo et al. (2013) implemented a minimum variance PID control strategy with a nonlinear FEM model, which can decrease the PCCF variance by 10% with a maximum control force of about 100N. In contrast, in this work, when  and **, the PCCF variance can decrease by over 14%. It can also be observed from Figure 14 that due to the effect of the controller saturation, the PCCF obtained in this investigation generally has a smaller fluctuation compared with the uncontrolled result, but the value of PCCF at each time instant has a slight increase, which can decrease the possibility of contact loss. A similar effect can also be found in (Sanchez-Rebollo et al. 2013). The results of corresponding control force from 3.6s to 3.8s are shown in Figure 15(a-b), similar results are found throughout the whole time duration.

 (a)

 (b)

**Figure 15**. Control force with *f*m=50N, *β*=18 (a); Control force with *f*m=100N, *β*=15 (b).

## Influence of wire geometry

The contact wire irregularity is one of the major ‘perturbations’ in the daily operation of a pantograph-catenary system. As the train speed increases, the effect of contact wire irregularity on the pantograph-catenary interaction becomes more noticeable (Collina et al., 2007). Zhang et al. (2010) indicated that the contact wire irregularity can be described as

 (33)

where,  is the vehicle speed,  is the wave depth of the contact wire irregularity,  is the wavelength of the contact wire irregularity, and  is the length of the catenary. Here, the contact wire irregularity is included to evaluate the performance of the controller. Consdier that the wavelength  is 6.5m (which often appears in reality, equalling to the interval of droppers), the drving speed is 300km/h, the wave depth is chosen as 2mm, 4mm and 6mm, respectively. The controller parameters are defined as  and *β*=15.

The results of contact force are shown in Figure 16. And the relevant statistics (contact loss and variation) are summarized in Table 3. It can be found that the increase of the wave depth of the contact wire irregularity leads to a deterioration of the contact quality between the pantograph and catenary. The proposed controller not only has a good performance in reducing the PCCF variation, but also eliminates the contact loss between the pantograph and catenary. It can also be observed that the controller leads to a slight increase of the PCCF in each time instant, which effectively decreases the possibility of contact loss between the pantograph and catenary.

 (a)

 (b)

 (c)

**Figure 16**. Contact force with  (a); Contact force with  (b); Contact force with  (c);

**Table 3**. Statistics of PCCF with contact wire irregularity

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Wave depth (mm) | PCCF variation (N2) | | Contact loss (%) | |
| With control | Without control | With control | Without control |
| 2 | 1671.1 | 1686.6 | 0 | 0 |
| 4 | 2484.7 | 2522.8 | 0 | 0.56 |
| 6 | 3436.1 | 3455.2 | 0 | 1.53 |

5. Conclusions

In this paper, to address the weakness of the previous studies, an effective sliding mode controller for high-speed pantograph is proposed in combination with a disturbance compensator. Through several numerical simulation examples with both the simplified model and the nonlinear FEM model of the catenary, it is found that the proposed controller has a good performance in reducing the pantograph-catenary contact force variation, as well as eliminating the contact loss. In the future, more complex factors will be considered to evaluate the performance of the controller. And a more detailed pantograph model will be established. Experimental validation of on a laboratory bench will be constructed in the research work, which is another key step before the controller can be implemented in a real railway line.

It should be indicated that the control performance with a nonlinear FEM model is far from satisfactory compared with that with a simplified model. The maximum reduction of the contact force variance reaches only 14%, by using the proposed controller, which is a small improvement over a previous investigation of using a minimum variance PID control strategy which could get only 10% reduction of the contact force variance with a nonlinear FEM model. It is suspected that some of the past controllers proposed based on the simplified catenary model may not be validities when they are implemented with a complex nonlinear catenary model. In the future, more adaptive controllers should be developed to deal with the disturbance of strong nonlinear effect from the catenary to the pantograph.

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**Conflict of interest**

None declared.

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