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Classification of $SU(4) \times SU(2) \times U(1)$ Heterotic-String Models

Alon E. Faraggi* and Hasan Sonmez†

Dept. of Mathematical Sciences, University of Liverpool, Liverpool L69 7ZL, UK

Abstract

The free fermionic construction of the heterotic string in four dimensions produced a large space of three generation models with the underlying $SO(10)$ embedding of the Standard Model states. The $SO(10)$ symmetry is broken to a subgroup directly at the string scale. Over the past few years free fermionic models with the Pati–Salam and flipped $SU(5)$ subgroups have been classified. In this paper we extend this classification program to models in which the $SO(10)$ symmetry is broken at the string level to the $SU(4) \times SU(2)_L \times U(1)_R$ (SU421) subgroup. The subspace of free fermionic models that we consider corresponds to symmetric $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds. We provide a general argument that shows that this class of SU421 free fermionic models cannot produce viable three generation models.

*alon.faraggi@liv.ac.uk
†Hasan.Sonmez@liv.ac.uk

1 Introduction

The Standard Model of particle physics accounts successfully for all subatomic observational data. The gauge charges of the Standard Model matter states suggest its embedding in $SO(10)$ Grand Unified Theory, which is broken to the Standard Model at the GUT or string scale. The $SO(10)$ unification picture is further supported by: the logarithmic evolution of the Standard Model gauge parameters; the proton longevity; and the suppression of left–handed neutrino masses. The heterotic–string [1] produces chiral $SO(10)$ representations in its perturbative spectrum, and is therefore the one suited to explore the $SO(10)$ GUTs structure underlying the Standard Model. Phenomenological studies of the heterotic–string have been pursued since the mid–eighties [2], using a variety of world–sheet [3, 4, 5] and target space techniques [6, 7].

The free fermionic construction of the heterotic–string in four dimensions produced a rich space of phenomenological three generation models. These models admit the underlying $SO(10)$ GUT embedding of the Standard Model spectrum. However, the $SO(10)$ symmetry is broken directly at the string level. The early studies of these models consisted of isolated examples that shared an underlying NAHE–base structure [8]. Examples in which the $SO(10)$ symmetry is broken to the: flipped $SU(5)$ (FSU5) [9]; $SO(6) \times SO(4)$ Heterotic String Pati–Salam Models (HSPSM) [10]; $SU(3)\times SU(2)\times U(1)^2$ Standard–like Models (SLM) [11]; $SU(3)\times SU(2)^2\times U(1)$ left–right symmetric (LRS) [12]; and $SU(4) \times SU(2) \times U(1)$ (SU421) [13]; subgroups were studied. Among those the FSU5; SLM; HSPSM; LRS cases produced quasi–realistic three generation models, whereas the SU421 case did not produce any viable three generation model. The advantage of the SU421 models compared to the FSU5 and HSPSM is that they admit both the doublet–triplet, as well as the doublet–doublet spitting mechanism [13]. We also note the recent interest in SU421 models from purely phenomenological considerations [14].

The phenomenological free fermionic heterotic–string models are $\mathbb{Z}_2\times\mathbb{Z}_2$ orbifolds that are constructed at enhanced symmetry points in the moduli space [15, 16]. Many of the phenomenological properties of the models are rooted in their underlying $\mathbb{Z}_2 \times \mathbb{Z}_2$ structure [17]. In recent years systematic methods for the classification of symmetric $\mathbb{Z}_2 \times \mathbb{Z}_2$ free fermionic orbifolds were developed in [18] for type II superstrings and in refs. [19, 20] for symmetric $\mathbb{Z}_2 \times \mathbb{Z}_2$ heterotic–string orbifolds with $SO(10)$ GUT symmetry. The classification was extended in refs. [21, 22, 23] and [24] to string vacua in which the $SO(10)$ symmetry is broken to the $SO(6)\times SO(4)$ Pati–Salam and to the flipped $SU(5)$ subgroups, respectively. The Pati–Salam class of free fermionic vacua produced examples of three generation exophobic models in which exotic fractionally charged states only appear in the massive string spectrum [21, 22], whereas the flipped $SU(5)$ class of models did not produce exophobic models with an odd number of generations [24].

In this paper we discuss the classification for the class of SU421 heterotic–string models. We provide a general argument that breaking the $SO(10)$ symmetry to this subgroup cannot produce three chiral generations in the prevalent free fermionic construction which is based on symmetric $\mathbb{Z}_2 \times \mathbb{Z}_2$ toroidal compactification with a $\mathbb{Z}_2 \times \mathbb{Z}_4$ fermionic boundary conditions that break the $SO(10)$ symmetry to $SU(4) \times SU(2) \times U(1)$.

2 $SU(4) \times SU(2) \times U(1)$ Phenomenology

The field theory content of the $N = 1$ supersymmetric $SU(4)_C \times SU(2)_L \times U(1)_R$ model* was discussed in ref. [13]. The SU421 class of heterotic–string models differs from the HSPSM models in the breaking of $SU(2)_R \to U(1)_R$ directly at the string level. Similar to the HSPSM, the SU421 heterotic–string models admit the $SO(10)$ embedding and the chiral states are obtained from the spinorial **16** representations of $SO(10)$ which decomposes in the following way:

$$\begin{aligned}
F_L^i &= (4, 2, \; 0) &&= (3, 2, \; \frac{1}{3}, \quad 0) + (1, 2, -1, \quad 0) &&= \begin{pmatrix} u \\ d \end{pmatrix}^i + \begin{pmatrix} \nu \\ e \end{pmatrix}^i, &(2.1)\\
U_R^i &= (\overline{4}, 1, -\frac{1}{2}) &&= (\overline{3}, 1, -\frac{1}{3}, -\frac{1}{2}) + (1, 1, +1, -\frac{1}{2}) &&= u^{ci} + N^{ci}, &(2.2)\\
D_R^i &= (\overline{4}, 1, +\frac{1}{2}) &&= (\overline{3}, 1, -\frac{1}{3}, +\frac{1}{2}) + (1, 2, +1, +\frac{1}{2}) &&= d^{ci} + e^{ci}. &(2.3)
\end{aligned}$$

The first and second equalities show the decomposition under $SU(4)_C \times SU(2)_L \times U(1)_R$ and $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_R$, respectively. The electroweak $U(1)_Y$ current is given by

$$U(1)_Y = \frac{1}{2} U(1)_{B-L} + U(1)_R. \qquad (2.4)$$

From eq. (2.1) we note that F_L produces the quarks and leptons weak doublets, and that U_R and D_R produces the right–handed weak singlets. The two Higgs multiplets of the Minimal Supersymmetric Standard Model, h^u and h^d, are given by,

$$\begin{aligned}
h^d &= (1, 2, -\frac{1}{2}), &(2.5)\\
h^u &= (1, 2, +\frac{1}{2}). &(2.6)
\end{aligned}$$

The heavy Higgs states that are responsible for breaking $SU(4)_C \times U(1)_R$ gauge symmetry to the Standard Model groups $SU(3) \times U(1)_Y$ are given by the fields

$$\begin{aligned}
H &= (\overline{4}, 1, -\frac{1}{2}) &(2.7)\\
\overline{H} &= (4, 1, +\frac{1}{2}) &(2.8)
\end{aligned}$$

*we note that $U(1)_R$ as defined here is equal to $1/2\,U(1)_L$ as defined in ref. [11].

The SU421 heterotic–string models may also contain states that transform as

$$(6,1,0) = (3,1,\frac{1}{3},0) + (\overline{3},1,-\frac{1}{3},0).$$

These multiplets arise from the vectorial **10** representation of $SO(10)$. These coloured states generate proton decay from dimension five operators, and therefore must be sufficiently heavy to be in agreement with the proton lifetime limits. An important benefit of the SU421 symmetry breaking pattern is that these colour triplets may be projected out by the Generalised GSO (GGSO) projections [25], and need not be present in the low energy spectrum. The string doublet–triplet mechanism works in all models that include the symmetry breaking pattern $SO(10) \to SO(6) \times SO(4)$. The HSPSM heavy Higgs states, which break $SU(4) \times SU(2)_R \to SU(3)_C \times U(1)_Y$, contain colour triplets with the charges of the states in (2.3) that may give rise to dimension five proton decay mediating operators. In the HSPSM the superpotential terms $\lambda_2 HHD + \lambda_3 \bar{H}\bar{H}\bar{D}$ couples the colour triplets from the vectorial representation $(6,1,1)$ to the colour triplets arising from the heavy Higgs field. The GUT scale VEVs of the heavy Higgs fields H and $\bar{H}$ are used to give heavy mass to the Higgs colour triplets. However, the heavy Higgs representations in the SU421 heterotic–string models, eq. (2.8), do not contain the states with the charges of eq. (2.3). Consequently, the stringy doublet–triplet splitting mechanism works only in models in which the $SO(10)$ symmetry is broken to $SU(3)_C \times SU(2)_L \times U(1)^2$, $SU(4)_C \times SU(2)_L \times U(1)_R$, or $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

Another important advantage of the SU421 class of models versus the PS and LRS models is with respect to the light Higgs representations. In the LRS and PS models, the light Higgs states exist in bi–doublet representations and couple simultaneously to the up– and down–type quarks, which may give rise to Flavor Changing Neutral Currents (FCNC) at an unacceptable rate [26]. This introduces a bi–doublet splitting problem. The solutions that have been proposed in the literature [27] use a $SU(2)_L$ triplet representation that are not present in string models in which the gauge symmetry is realised as a level one Kac–Moody algebra. On the other hand, in SU421 models $SU(2)_R$ is broken at the string level and consequently the Higgs bi–doublet is split at the string level.

The solutions to the doublet–doublet as well as the doublet–triplet splitting problems are the two appealing properties offered by the SU421 free fermionic heterotic–string models. However, as we argue in the next section the free fermionic $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold models, with additional $\mathbb{Z}_2 \times \mathbb{Z}_4$ basis vectors that are used to break the $SO(10)$ symmetry to $SU(4)_C \times SU(2)_L \times U(1)_R$, cannot in fact produce three complete chiral generations and therefore, like the NAHE–based free fermionic models [13], these models do not produce viable SU421 string models.

2.1 The $SU(4) \times SU(2) \times U(1)$ Free Fermionic Construction

The string vacuum in the free fermionic formulation [3] is defined in terms of a set of boundary condition basis vectors and the Generalised GSO projection coefficients, which span the one–loop partition function. The basis vectors generate a finite additive group $\Xi = \sum_k n_k b_k$ where $n_k = 0, \cdots, N_{z_k} - 1$. The physical states in the Hilbert space of a sector $\alpha \in \Xi$ are obtained by acting on the vacuum with fermionic and bosonic oscillators and by applying the GGSO projections. Each fermionic complex oscillator acting on the vacuum is counted by a fermion number operator as $F_\alpha(f) = 1$ and $\alpha(f^*) = -1$. For periodic complex fermions with $\alpha(f) = 1$, the vacuum is in a doubly degenerate spinorial representation $|\pm\rangle$, annihilated by the zero modes f_0 and ${f_0}^*$ and with fermion numbers $F(f) = 0, -1$, respectively. The $U(1)$ charges $Q(f)$ of the unbroken Cartan generators of the right–moving gauge group are given in terms of the boundary conditions and fermion numbers of the complex right–moving world–sheet fermions by

$$Q(f) = \frac{1}{2}\alpha(f) + F(f). \tag{2.9}$$

In the light–cone gauge, the free fermionic heterotic–string models in four dimensions require 20 and 44, left–moving and right–moving real world–sheet fermions respectively, to cancel the conformal anomaly. In the usual notation these are denoted as: $\psi^\mu, \chi^{1,\ldots,6}, y^{1,\ldots,6}, \omega^{1,\ldots,6}$ and $\overline{y}^{1,\ldots,6}, \overline{\omega}^{1,\ldots,6}$, $\overline{\psi}^{1,\ldots,5}$, $\overline{\eta}^{1,2,3}$, $\overline{\phi}^{1,\ldots,8}$.

2.2 The $SU(4) \times SU(2) \times U(1)$ Gauge Group

In the following we set up the necessary ingredients for the classification of the SU421 free fermionic heterotic–string models. The analysis is along similar lines to the one performed in the classification of the $SO(10)$ [19]; heterotic–string Pati–Salam models [21]; and flipped $SU(5)$ models [24]. The novelty compared to these cases is that the SU421 models employ two basis vectors that break the $SO(10)$ symmetry, whereas the HSPSM and FSU5 models use only one. However, we argue below that this class of heterotic–string vacua cannot in fact produce phenomenologically viable models. The basis vectors that generate our $SU(4) \times SU(2) \times U(1)$ heterotic–string models are given by the following 14 basis vectors

$$
\begin{aligned}
v_1 = \mathbf{1} &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \overline{y}^{1,\dots,6}, \overline{\omega}^{1,\dots,6}, \overline{\eta}^{1,2,3}, \overline{\psi}^{1,\dots,5}, \overline{\phi}^{1,\dots,8}\},\\
v_2 = S &= \{\psi^\mu, \chi^{1,\dots,6}\},\\
v_{2+i} = e_i &= \{y^i, \omega^i | \overline{y}^i, \overline{\omega}^i\},\ i = 1,\dots,6,\\
v_9 = b_1 &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \overline{y}^{34}, \overline{y}^{56}, \overline{\eta}^1, \overline{\psi}^{1,\dots,5}\},\\
v_{10} = b_2 &= \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \overline{y}^{12}, \overline{y}^{56}, \overline{\eta}^2, \overline{\psi}^{1,\dots,5}\},\\
v_{11} = z_1 &= \{\overline{\phi}^{1,\dots,4}\},\\
v_{12} = z_2 &= \{\overline{\phi}^{5,\dots,8}\},\\
v_{13} = \alpha &= \{\overline{\psi}^{4,5}, \overline{\phi}^{1,2}\},\\
v_{14} = \beta &= \{\overline{\psi}^{4,5} = \tfrac{1}{2}, \overline{\phi}^{1,\dots,6} = \tfrac{1}{2}\}.
\end{aligned}
\tag{2.10}
$$

The basis vector $\mathbf{1}$ generates models with $SO(44)$ gauge group from the Neveu–Schwarz sector. The vector S produces $N = 4$ space–time supersymmetry. The vectors $e_1,\dots,e_6$ break the $SO(44)$ gauge group to $SO(32) \times U(1)^6$ and preserve the $N = 4$ space–time supersymmetry. The e_i basis vectors correspond to all the possible symmetric shifts of the six internal bosonic coordinates. The basis vectors b_1 and b_2 correspond to $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold twists and break $N = 4$ space–time supersymmetry to $N = 1$. Additionally, they reduce the rank of gauge group by breaking the $U(1)^6$ symmetry. Combined with the projections of the basis vectors z_1 and z_2 the $SO(32)$ gauge group is reduced to $SO(10) \times U(1)^3 \times SO(8)_1 \times SO(8)_2$, where $SO(10) \times U(1)^3$ and $SO(8)_1 \times SO(8)_2$ correspond to the observable and hidden gauge groups, respectively. The combined projection of the basis vectors α and β breaks the $SO(10)$ GUT symmetry to $SU(4) \times SU(2) \times U(1)$, where α is identical to the basis vector used in the classification of the Pati–Salam models, and hence breaks the $SO(10)$ symmetry to $SO(6) \times SO(4)$ and finally using the β basis vector with fractional boundary conditions reduces the $SO(10)$ gauge symmetry to $SU(4) \times SU(2) \times U(1)$.

2.3 The String Spectrum

The space–time vector bosons that are obtained from the Neveu–Schwarz (NS) sector and that survive the GGSO projections, defined by the basis vectors in (2.10) generate the observable and hidden gauge groups given by:

$$
\begin{aligned}
\text{Observable} &: \quad SU(4) \times SU(2)_L \times U(1)_R \times U(1)^3\\
\text{Hidden} &: \quad SU(2)_A \times U(1)_A \times SU(2)_B \times U(1)_B \times SU(2)_C \times U(1)_C \times SO(4)_2
\end{aligned}
$$

The string states arising in other sectors transform under these gauge group factors. Additional space–time vector bosons that enhance the NS observable and/or hidden gauge groups may arise from additional sectors. In order to preserve the above gauge groups, all these additional space–time vector bosons need to be projected out. These additional space–time vector bosons arise from the following 36 sectors

$$
\mathbf{G_{Enh}} = \left\{ \begin{array}{ccc}
z_1, & z_1+\beta, & z_1+2\beta, \\
z_1+\alpha, & z_1+\alpha+\beta, & z_1+\alpha+2\beta, \\
z_2, & z_2+\beta, & z_2+2\beta, \\
z_2+\alpha, & z_2+\alpha+\beta, & z_2+\alpha+2\beta, \\
z_1+z_2, & z_1+z_2+\beta, & z_1+z_2+2\beta, \\
z_1+z_2+\alpha, & z_1+z_2+\alpha+\beta, & z_1+z_2+\alpha+2\beta, \\
\beta, & 2\beta, & \alpha, \\
\alpha+\beta, & \alpha+2\beta, & x, \\
z_1+x+\beta, & z_1+x+2\beta, & z_1+x+\alpha, \\
z_1+x+\alpha+\beta, & z_2+x+\beta, & z_2+x+\alpha+\beta, \\
z_1+z_2+x+\beta, & z_1+z_2+x+2\beta, & z_1+z_2+x+\alpha+\beta, \\
x+\beta, & x+\alpha, & x+\alpha+\beta,
\end{array} \right\}, \qquad (2.11)
$$

where $x = 1 + S + \sum_{i=1}^{6} e_i + z_1 + z_2$.

2.4 The Matter Content

The observable matter states in heterotic–string vacuum with $(2,2)$ world–sheet supersymmetry is embedded in the **27** representation of E_6. In the free fermionic construction that we adopt here, and using the basis vectors in (2.10), the E_6 is first broken to the $SO(10) \times U(1)$ symmetry. Therefore, the **27** of E_6 decomposes in the following way

$$
\mathbf{27} \;\; = \;\; \mathbf{16} + \mathbf{10} + \mathbf{1}. \qquad (2.12)
$$

Where the **16** transforms under the spinorial representation of $SO(10)$ and **10** transforms in the vectorial representation of the $SO(10)$, and similarly for $\overline{\mathbf{27}}$. The following 48 sectors produce states that give the spinorial **16** or $\overline{\mathbf{16}}$ of $SO(10)$

$$
\begin{aligned}
B^{(1)}_{pqrs} &= S + b_1 + pe_3 + qe_4 + re_5 + se_6 \\
&= \{\psi^\mu, \chi^{12}, (1-p)y^3\overline{y}^3, p\omega^3\overline{\omega}^3, (1-q)y^4\overline{y}^4, q\omega^4\overline{\omega}^4, \\
&\qquad (1-r)y^5\overline{y}^5, r\omega^5\overline{\omega}^5, (1-s)y^6\overline{y}^6, s\omega^6\overline{\omega}^6, \overline{\eta}^1, \overline{\psi}^{1,\dots,5}\}, \qquad (2.13) \\
B^{(2)}_{pqrs} &= S + b_2 + pe_1 + qe_2 + re_5 + se_6, \\
B^{(3)}_{pqrs} &= S + b_3 + pe_1 + qe_2 + re_3 + se_4,
\end{aligned}
$$

where $p, q, r, s = 0, 1$ and $b_3 = b_1 + b_2 + x$. In order to distinguish between the spinorial **16** and $\overline{\mathbf{16}}$ in the states given above, the following chirality operators are used

$$
\begin{aligned}
X^{(1)SO(10)}_{pqrs} &= C\begin{pmatrix} B^{(1)}_{pqrs} \\ b_2+(1-r)e_5+(1-s)e_6 \end{pmatrix}, \\
X^{(2)SO(10)}_{pqrs} &= C\begin{pmatrix} B^{(2)}_{pqrs} \\ b_1+(1-r)e_5+(1-s)e_6 \end{pmatrix}, \\
X^{(3)SO(10)}_{pqrs} &= C\begin{pmatrix} B^{(3)}_{pqrs} \\ b_1+(1-r)e_3+(1-s)e_4 \end{pmatrix}.
\end{aligned} \tag{2.14}
$$

Where $X^{(1,2,3)SO(10)}_{pqrs}=1$ implies the states corresponds to the $\mathbf{16}$ of $SO(10)$ and $X^{(i)SO(10)}_{pqrs}=-1$ to the $\overline{\mathbf{16}}$ of $SO(10)$. Moreover, we note that the states here can be projected in or out depending on the GGSO projections of the basis vectors $e_1, ..., e_6$, z_1 and z_2. Therefore, we define below a projector P, such that $P=1$ implies the state is projected in and $P=0$ implies the state is projected out. The projector P is given by

$$
\begin{aligned}
P^{(1)}_{pqrs} &= \frac{1}{16}\left(1-C\begin{pmatrix} e_1 \\ B^{(1)}_{pqrs}\end{pmatrix}\right)\cdot\left(1-C\begin{pmatrix} e_2 \\ B^{(1)}_{pqrs}\end{pmatrix}\right)\cdot\left(1-C\begin{pmatrix} z_1 \\ B^{(1)}_{pqrs}\end{pmatrix}\right)\cdot\left(1-C\begin{pmatrix} z_2 \\ B^{(1)}_{pqrs}\end{pmatrix}\right), \\
P^{(2)}_{pqrs} &= \frac{1}{16}\left(1-C\begin{pmatrix} e_3 \\ B^{(2)}_{pqrs}\end{pmatrix}\right)\cdot\left(1-C\begin{pmatrix} e_4 \\ B^{(2)}_{pqrs}\end{pmatrix}\right)\cdot\left(1-C\begin{pmatrix} z_1 \\ B^{(2)}_{pqrs}\end{pmatrix}\right)\cdot\left(1-C\begin{pmatrix} z_2 \\ B^{(2)}_{pqrs}\end{pmatrix}\right), \\
P^{(3)}_{pqrs} &= \frac{1}{16}\left(1-C\begin{pmatrix} e_5 \\ B^{(3)}_{pqrs}\end{pmatrix}\right)\cdot\left(1-C\begin{pmatrix} e_6 \\ B^{(3)}_{pqrs}\end{pmatrix}\right)\cdot\left(1-C\begin{pmatrix} z_1 \\ B^{(3)}_{pqrs}\end{pmatrix}\right)\cdot\left(1-C\begin{pmatrix} z_2 \\ B^{(3)}_{pqrs}\end{pmatrix}\right).
\end{aligned} \tag{2.15}
$$

These projectors above can in fact be expressed as matrix equations given by

$$
\begin{aligned}
&\begin{pmatrix} (e_1|e_3) & (e_1|e_4) & (e_1|e_5) & (e_1|e_6) \\ (e_2|e_3) & (e_2|e_4) & (e_2|e_5) & (e_2|e_6) \\ (z_1|e_3) & (z_1|e_4) & (z_1|e_5) & (z_1|e_6) \\ (z_2|e_3) & (z_2|e_4) & (z_2|e_5) & (z_2|e_6) \end{pmatrix}\begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} (e_1|b_1) \\ (e_2|b_1) \\ (z_1|b_1) \\ (z_2|b_1) \end{pmatrix}, \\
&\begin{pmatrix} (e_3|e_1) & (e_3|e_2) & (e_3|e_5) & (e_3|e_6) \\ (e_4|e_1) & (e_4|e_2) & (e_4|e_5) & (e_4|e_6) \\ (z_1|e_1) & (z_1|e_2) & (z_1|e_5) & (z_1|e_6) \\ (z_2|e_1) & (z_2|e_2) & (z_2|e_5) & (z_2|e_6) \end{pmatrix}\begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} (e_3|b_2) \\ (e_4|b_2) \\ (z_1|b_2) \\ (z_2|b_2) \end{pmatrix}, \\
&\begin{pmatrix} (e_5|e_1) & (e_5|e_2) & (e_5|e_3) & (e_5|e_4) \\ (e_6|e_1) & (e_6|e_2) & (e_6|e_3) & (e_6|e_4) \\ (z_1|e_1) & (z_1|e_2) & (z_1|e_3) & (z_1|e_4) \\ (z_2|e_1) & (z_2|e_2) & (z_2|e_3) & (z_2|e_4) \end{pmatrix}\begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} (e_5|b_3) \\ (e_6|b_3) \\ (z_1|b_3) \\ (z_2|b_3) \end{pmatrix}.
\end{aligned} \tag{2.16}
$$

Writing the projectors as matrix equations given above entails solving systems of linear equations. These algebraic equations can be solved using a computerised code, which can be used to scan a vast space of models.

Similar to the spinorial representations singlet and vectorial **10** representations of $SO(10)$ are obtained from the following 48 sectors

$$
\begin{aligned}
B^{(4)}_{pqrs} &= B^{(1)}_{pqrs} + x \\
&= \{\psi^\mu, \chi^{12}, (1-p)y^3\overline{y}^3, p\omega^3\overline{\omega}^3, (1-q)y^4\overline{y}^4, q\omega^4\overline{\omega}^4, \\
&\qquad (1-r)y^5\overline{y}^5, r\omega^5\overline{\omega}^5, (1-s)y^6\overline{y}^6, s\omega^6\overline{\omega}^6, \overline{\eta}^{2,3}\}, \qquad (2.17) \\
B^{(5,6)}_{pqrs} &= B^{(2,3)}_{pqrs} + x.
\end{aligned}
$$

Massless states that arise in these sectors are obtained by acting on the vacuum with a NS oscillator. The type of states therefore depend on the type of oscillator, and may correspond to $SO(10)$ singlets or vectorial **10** representation of $SO(10)$, which is needed for electroweak symmetry breaking. The different type of $SO(10)$ singlets arising from eq. (2.17) are

- $\{\overline{\eta}^i\}|R\rangle^{(4,5,6)}_{pqrs}$ or $\{\overline{\eta}^{*i}\}|R\rangle^{(4,5,6)}_{pqrs}$, $i = 1,2,3$, where $|R\rangle^{(4,5,6)}_{pqrs}$ is the degenerated Ramond vacuum of the $B^{(4,5,6)}_{pqrs}$ sector. These states transform as a vector–like representations under the $U(1)_i$'s.

- $\{\overline{\phi}^{1,2}\}|R\rangle^{(4,5,6)}_{pqrs}$ or $\{\overline{\phi}^{*1,2}\}|R\rangle^{(4,5,6)}_{pqrs}$. These states transform as a vector–like representations of $SU(2)_A \times U(1)_A$.

- $\{\overline{\phi}^{3,4}\}|R\rangle^{(4,5,6)}_{pqrs}$ or $\{\overline{\phi}^{*3,4}\}|R\rangle^{(4,5,6)}_{pqrs}$. These states transform as a vector–like representations of $SU(2)_B \times U(1)_B$.

- $\{\overline{\phi}^{5,6}\}|R\rangle^{(4,5,6)}_{pqrs}$ or $\{\overline{\phi}^{*5,6}\}|R\rangle^{(4,5,6)}_{pqrs}$. These states transform as a vector–like representations of $SU(2)_C \times U(1)_C$.

- $\{\overline{\phi}^{7,8}\}|R\rangle^{(4,5,6)}_{pqrs}$ or $\{\overline{\phi}^{*7,8}\}|R\rangle^{(4,5,6)}_{pqrs}$. These states transform as a vector–like representations of $SO(4)$.

Similarly, for the matrix equations given above in eq. (2.15), we can write algebraic equations for the sectors in eq. (2.17) given as follows:

$$\begin{pmatrix} (e_1|e_3) & (e_1|e_4) & (e_1|e_5) & (e_1|e_6) \\ (e_2|e_3) & (e_2|e_4) & (e_2|e_5) & (e_2|e_6) \\ (z_1|e_3) & (z_1|e_4) & (z_1|e_5) & (z_1|e_6) \\ (z_2|e_3) & (z_2|e_4) & (z_2|e_5) & (z_2|e_6) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} (e_1|b_1+x) \\ (e_2|b_1+x) \\ (z_1|b_1+x) \\ (z_2|b_1+x) \end{pmatrix},$$

$$\begin{pmatrix} (e_3|e_1) & (e_3|e_2) & (e_3|e_5) & (e_3|e_6) \\ (e_4|e_1) & (e_4|e_2) & (e_4|e_5) & (e_4|e_6) \\ (z_1|e_1) & (z_1|e_2) & (z_1|e_5) & (z_1|e_6) \\ (z_2|e_1) & (z_2|e_2) & (z_2|e_5) & (z_2|e_6) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} (e_3|b_2+x) \\ (e_4|b_2+x) \\ (z_1|b_2+x) \\ (z_2|b_2+x) \end{pmatrix}, \tag{2.18}$$

$$\begin{pmatrix} (e_5|e_1) & (e_5|e_2) & (e_5|e_3) & (e_5|e_4) \\ (e_6|e_1) & (e_6|e_2) & (e_6|e_3) & (e_6|e_4) \\ (z_1|e_1) & (z_1|e_2) & (z_1|e_3) & (z_1|e_4) \\ (z_2|e_1) & (z_2|e_2) & (z_2|e_3) & (z_2|e_4) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} (es_5|b_3+x) \\ (e_6|b_3+x) \\ (z_1|b_3+x) \\ (z_2|b_3+x) \end{pmatrix}.$$

3 The Observable Matter Spectrum

The basis vectors α and β given in eq. (2.10) break the $SO(10)$ symmetry to $SU(4) \times SU(2)_L \times U(1)_R$. Following the α and β GGSO projections, the decomposition of the spinorial $\mathbf{16}$ and $\overline{\mathbf{16}}$ representations of $SO(10)$, under the $SU(4) \times SU(2)_L \times U(1)_L$ gauge group is given as follows:

$$\begin{aligned} \mathbf{16} &= (\mathbf{4}, \mathbf{2}, 0) + \left(\bar{\mathbf{4}}, \mathbf{1}, -1\right) + \left(\bar{\mathbf{4}}, \mathbf{1}, +1\right), \\ \overline{\mathbf{16}} &= \left(\bar{\mathbf{4}}, \mathbf{2}, 0\right) + (\mathbf{4}, \mathbf{1}, -1) + (\mathbf{4}, \mathbf{1}, +1). \end{aligned}$$

Here to break the $SU(4) \times SU(2)_L \times U(1)_L$ gauge group to the standard model group, we require the heavy higgs pair. This pair is given by

$$\left(\bar{\mathbf{4}}, \mathbf{1}, -1\right) + (\mathbf{4}, \mathbf{1}, -1).$$

Similarly, the vectorial representation $\mathbf{10}$ of $SO(10)$ decomposed under the $SU(4) \times SU(2)_L \times U(1)_L$ gauge group is given as follows

$$\mathbf{10} = (\mathbf{6}, \mathbf{1}, 0) + (\mathbf{1}, \mathbf{2}, -1) + (\mathbf{1}, \mathbf{2}, +1),$$

Furthermore, we take the following normalizations of the hypercharge and electromagnetic charge

$$\begin{aligned} Y &= \frac{1}{3}(Q_1 + Q_2 + Q_3) + \frac{1}{2}(Q_4 + Q_5), \\ Q_{em} &= Y + \frac{1}{2}(Q_4 - Q_5). \end{aligned}$$

where the Q_i charges of a state arise due to ψ^i for $i = 1, ..., 5$. The following table summaries the charges of the colour $SU(3)$ and electroweak $SU(2) \times U(1)$ Cartan generators of the states which form the $SU(4) \times SU(2)_L \times U(1)_L$ matter representations:

Representation	$\overline{\psi}^{1,2,3}$	$\overline{\psi}^{4,5}$	Y	Q_{em}
	$(+,+,-)$	$(+,-)$	1/6	2/3
	$(+,+,-)$	$(-,+)$	1/6	-1/3
$(\mathbf{4}, \mathbf{2}, 0)$	$(-,-,-)$	$(+,-)$	-1/2	0
	$(-,-,-)$	$(-,+)$	-1/2	-1
	$(+,-,-)$	$(-,-)$	-2/3	-2/3
$(\overline{\mathbf{4}}, \mathbf{1}, -1)$	$(+,+,+)$	$(-,-)$	0	0
	$(+,-,-)$	$(+,+)$	1/3	1/3
$(\overline{\mathbf{4}}, \mathbf{1}, +1)$	$(+,+,+)$	$(+,+)$	1	1
	$(+,-,-)$	$(+,-)$	-1/6	-2/3
	$(+,-,-)$	$(-,+)$	-1/6	1/3
$(\overline{\mathbf{4}}, \mathbf{2}, 0)$	$(+,+,+)$	$(+,-)$	1/2	0
	$(+,+,+)$	$(-,+)$	1/2	1
	$(+,+,-)$	$(+,+)$	2/3	2/3
$(\mathbf{4}, \mathbf{1}, -1)$	$(-,-,-)$	$(+,+)$	0	0
	$(+,+,-)$	$(-,-)$	-1/3	-1/3
$(\mathbf{4}, \mathbf{1}, +1)$	$(-,-,-)$	$(-,-)$	-1	-1

Here " + " and " − ", label the contribution of an oscillator with fermion number $F = 0$ or $F = -1$, to the degenerate vacuum. These states correspond to particles of the Standard Model. More precisely we can decompose these representations under $SU(3) \times SU(2) \times U(1)$ as

$$\begin{aligned}
(\mathbf{4}, \mathbf{2}, 0) &= \left(\mathbf{3}, \mathbf{2}, +\frac{1}{6}\right)_Q + \left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right)_L , \\
\left(\overline{\mathbf{4}}, \mathbf{1}, -1\right) &= \left(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}\right)_{u^c} + (\mathbf{1}, \mathbf{1}, 0)_{\nu^c} , \\
\left(\overline{\mathbf{4}}, \mathbf{1}, +1\right) &= \left(\overline{\mathbf{3}}, \mathbf{1}, +\frac{1}{3}\right)_{d^c} + (\mathbf{1}, \mathbf{1}, +1)_{e^c} .
\end{aligned}$$

Where L is the lepton–doublet; Q is the quark–doublet; d^c, u^c, e^c and ν^c are the quark and lepton singlets. Because of the α- and β-projections, which projects on incomplete $\mathbf{16}$ and $\overline{\mathbf{16}}$ representations, complete families and anti–families are formed by combining states from different sectors.

4 Nonviability of the $SU(4) \times SU(2) \times U(1)$ model

We now discuss why in our free fermionic construction, the $SU(4)\times SU(2)\times U(1)$ GUT models are not viable. As mentioned in the previous section, the matter content comes from the **16** of $SO(10)$. However, with the addition of the α and β basis vectors from eq. (2.10), the **16** representation is broken by the GGSO projections that are in general given by

$$e^{i\pi v_i \cdot F_\xi}|S_\xi >= \delta_\xi \; C\begin{pmatrix} \xi \\ v_i \end{pmatrix}^* |S_\xi > . \tag{4.1}$$

Here $\delta_\xi = \pm 1$ is a spacetime spin statistics index and F_ξ is the fermion number operator. In the SU421 models spanned by eq. (2.10) the GGSO projection coefficients $C\binom{\xi}{v_i}$ can take the values $\pm 1; \pm i$. Therefore, firstly considering the α GGSO projection, we decompose the **16** into the Pati-Salam group representation. Moreover, using the following chirality operators

$$\begin{aligned} X^{(1)_{SO(6)}}_{pqrs} &= C\begin{pmatrix} B^{(1)}_{pqrs} \\ \alpha \end{pmatrix}, \\ X^{(2)_{SO(6)}}_{pqrs} &= C\begin{pmatrix} B^{(2)}_{pqrs} \\ \alpha \end{pmatrix}, \\ X^{(3)_{SO(6)}}_{pqrs} &= C\begin{pmatrix} B^{(3)}_{pqrs} \\ \alpha \end{pmatrix}, \end{aligned} \tag{4.2}$$

we deduce that for $X^{(i)_{SO(6)}}_{pqrs} = 1$ we get the $Q_R \equiv (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ states under $SU(4) \times SU(2)_L \times SU(2)_R$, whereas the $Q_L \equiv (\mathbf{4}, \mathbf{2}, \mathbf{1})$ states correspond to $X^{(i)_{SO(6)}}_{pqrs} = -1$. Next, considering the β GGSO projection, the operators

$$\begin{aligned} X^{(1)_{421}}_{pqrs} &= C\begin{pmatrix} B^{(1)}_{pqrs} \\ \beta \end{pmatrix}, \\ X^{(2)_{421}}_{pqrs} &= C\begin{pmatrix} B^{(2)}_{pqrs} \\ \beta \end{pmatrix}, \\ X^{(3)_{421}}_{pqrs} &= C\begin{pmatrix} B^{(3)}_{pqrs} \\ \beta \end{pmatrix}. \end{aligned} \tag{4.3}$$

determine the decomposition of the Q_L and Q_R states under $SU(4) \times SU(2) \times U(1)$. Here, the product $\beta \cdot B_j^{pqrs} = -1$ with $(j = 1, 2, 3)$, and the modular invariance constraints, impose that $X^{(1,2,3)_{421}}_{pqrs} = \pm\, i$. Therefore, this implies the states cannot be completed to form a family. Thus, to complete the **16** the states: $(\mathbf{4}, \mathbf{2}, 0)$, $(\overline{\mathbf{4}}, \mathbf{1}, -1)$ and $(\overline{\mathbf{4}}, \mathbf{1}, +1)$ under the $SU(4)\times SU(2)_L \times U(1)_R$ group all need to survive the GGSO projections, but in order for the $(\overline{\mathbf{4}}, \mathbf{1}, -1)$ and $(\overline{\mathbf{4}}, \mathbf{1}, +1)$ states to survive, we need $X^{(1,2,3)_{421}}_{pqrs} = \pm\, 1$, which is forbidden in this case by modular invariance. To see more clearly why this is the case we consider the decomposition of the **16** representation in the combinatorial notation of ref. [28]

$$
\begin{aligned}
\mathbf{16} \;&\equiv\; \left[\binom{5}{0}+\binom{5}{2}+\binom{5}{4}\right] && (4.4)\\
&\equiv\; \left[\binom{3}{0}+\binom{3}{2}\right]\left[\binom{2}{0}+\binom{2}{2}\right] \;+\; \left[\binom{3}{1}\right]\left[\binom{2}{1}\right] && (4.5)\\
&\equiv\; \left[\binom{3}{0}+\binom{3}{2}\right]\left[\binom{2}{0}\right] \;+\; \left[\binom{3}{0}+\binom{3}{2}\right]\left[\binom{2}{2}\right] \;+\; \left[\binom{3}{1}\right]\left[\binom{2}{1}\right] && (4.6)
\end{aligned}
$$

where the combinatorial factor counts the number of periodic fermions in the $|-\rangle$ state. The second line in eq. (4.5) corresponds to the decomposition of the **16** under the Pati–Salam subgroup, whereas eq. (4.6) shows its decomposition under the SU421 subgroup. The key point here, as seen from eq. (4.6), is the even number of fermions in the $|-\rangle$ vacuum of the Q_R states, resulting in ± 1 projections on the left–hand side of eq. (4.1), whereas the right–hand side is fixed by the product $\beta \cdot B_j^{pqrs} = -1$ to be $\pm i$. Thus, the exclusion arises because the β projection fixes the chirality of the vacuum of the world–sheet fermions $\overline{\psi}^{4,5}$ that generate the $SU(2)_L \times U(1)_R$ symmetry. We note that the situation here is different from the case of the SU421 models of ref. [13]. The reason is that our classification method only allows for symmetric boundary conditions for the set of internal fermions $\{y, \omega|\overline{y}, \overline{\omega}\}^{1,\cdots,6}$, whereas the models of ref. [13] introduce additional freedom by allowing asymmetric boundary conditions. Thus, while the NAHE–based models of ref. [13] did not yield any model with three complete generations they contain both the Q_L and Q_R states in their spectra, whereas vacua with only symmetric boundary conditions with respect to the set $\{y, \omega|\overline{y}, \overline{\omega}\}^{1,\cdots,6}$ do not contain Q_R states and are therefore categorically excluded. It is of further interest to note that in the case of the LRS models the chirality of the $Q_L + L_L$ and $Q_R + L_R$ is similarly affected [12]. However, there it is compensated by the chirality of the $\overline{\eta}^j$ worldsheet fermions leading to opposite charges under the $U(1)_j$ gauge symmetries. The SLM models [11] are obtained by combining the PS and FSU5 breaking vectors. Therefore, the SLM models produce complete **16** multiplets decomposed under the SLM group and with equal $U(1)_j$ charges. The SU421 class of models is the only case that is excluded in vacua with symmetric internal boundary conditions.

5 Conclusion

In this paper we discussed the classification of the SU421 models with symmetric internal boundary conditions. This continues the development of the classification program initiated in ref. [19], which led to the discovery of spinor–vector duality [29] and exophobic string vacua [21, 22, 30]. The novel feature in the classification of the SU421 models compared to the PS and FSU5 vacua is the introduction of two basis vectors that break the $SO(10)$ symmetry. An appealing feature of the SU421 models is the admission of both the triplet–doublet as well as the doublet–doublet splitting mechanism, which is shared only with the standard–like models. However, as we showed in section 4 these models cannot accommodate the weak $SU(2)$ singlet states of the Standard Model and are therefore excluded. The next step in our classification program is the classification of standard–like models that will be reported in a future publication.

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