**Localization of Breathing Cracks in Stepped Rotors Using Super-harmonic Characteristic Deflection Shapes Based on** **Singular Value Decomposition in Frequency Domain**

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**Abstract**

An output-only multiple-crack localization method is proposed in this paper to detect and localize breathing cracks in a stepped rotor, which utilizes the crack induced local shape distortions in super-harmonic characteristic deflection shapes (SCDS’s). To minimize the noise effects on SCDS’s and improve the accuracy of SCDS-based crack localization, singular value decomposition is adopted to estimate the SCDS as the dominant singular vector of output power spectral density matrix at a super-harmonic frequency. Then, in order to better reveal shape distortions in the SCDS’s, an after-treatment technique called gapped smoothing method is applied to derive a damage index. Numerical experiments are carried out to investigate the performance of the proposed method based on a two-disc stepped rotor-bearing system with breathing cracks established by the finite element method. Results show that the method is effective for single and multiple crack localization in stepped rotors and interference of steps can be excluded. Furthermore, the method is robust to noise. Influences of crack depths and rotating speeds are also investigated and how to choose the rotating speed for better crack localization is discussed.

**Keywords:**crack localization; stepped rotor; breathing crack; singular value decomposition; gapped smoothing method; super-harmonic characteristic deflection shapes

## Introduction

Rotors are one of the most important components of rotating machines, such as wheel sets of high-speed trains, motors, generators and aero-engines. Presence of cracks in rotors may lead to catastrophic failures. So crack monitoring for rotors is very important, which includes four levels: crack detection, crack localization, crack severity assessment, remaining service life prediction. Many studies have been carried out based on vibration signature in the past few decades, which has been well documented in the review papers [1-4]. However, most research focusing on crack detection, that is, to determine if any cracks exist in a rotor. As for crack localization, it is still a challenge for operating rotors. A brief review of vibration-based methods for crack localization in rotors will be given first.

Crack localization methods for rotors can be classified as model-based methods, pattern recognition-based methods, modal parameter-based methods, characteristic deflection shape-based methods.

Model-based methods are based on analytical or numerical models to simulate the behaviour of cracked rotors and attempt to correlate the observed vibration signature with the presence of cracks at discrete locations on the rotors [4]. Among the model-based methods, the approach based on equivalent crack forces is well known for crack identification in rotors which considers the effects of cracks as equivalent forces applied in the intact systems. It has been utilized to identify the location and depth of cracks in rotors by many researchers, such as Sekhar [5], Pennacchi et al.[6] and Lees et al.[7]. There are also some other model-based methods. Söffker et al. [8] compared a modern model-based technique based on a proportional-integral observer with a signal-based technique based on support vector machine using features extracted from wavelets to identify a crack in an operating rotor. Singh et al. [9] proposed a model-based method to identify a switching crack in a four-degrees-of-freedom Jeffcott rotor utilising the vibration signal in conjunction with the controller current of the active magnetic bearing. Cavalini Jr et al. [10] put forward a crack identification approach using external diagnostic forces at certain frequencies to obtain the nonlinear combinational resonances which were used as the objective function of a differential evolution optimization code to determine the crack location and depth by minimizing the difference between the measured and modelled rotor systems. Model-based methods were adopted by Saridakis et al. [11], Xiang et al. [12] and He et al. [13] to minimize the difference between real outputs and model outputs to determine the location and depth of a crack in a rotor-bearing system with genetic algorithm. Model-based methods perform well in theory, especially for crack localization and quantification, but they are largely dependent on the accuracy of the established model.

Pattern recognition-based methods are widely used in crack identification whose main idea is to extract features sensitive to crack parameters by advanced signal processing methods or relevant testing, and then the relationship between the extracted features and crack parameters is established by artificial intelligence techniques or some other mapping methods. Artificial neural network was adopted by Zapico-Valle et al. [14], using the first four natural frequencies as features. Support vector machine was used to realize crack identification with features extracted by wavelet transform in [8]. The key for pattern recognition-based methods is how to find the features sensitive to crack parameters.

As well known, cracks in structures will reduce the local stiffness, thus affect the modal parameters, such as natural frequencies and mode shapes. And the modal parameter-based methods are based on the change of modal parameters or their derivatives to realize crack identification. Rubio et al. [15] used changes in resonant and anti-resonant frequencies to detect crack locations in a two-cracked torsional shaft. Rahman et al. [16] utilized the changes in phase angle of frequency response function to identify the location and depth of an open crack in a rotor. Seo et al. [17] proposed a method for open crack localization of a shaft by comparing the map of the modal constants of the reverse directional frequency response functions with the reference map of the un-cracked model. As for modal parameters, they are almost global parameters without local spatial information except mode shapes. So the method based on global modal parameters will perform well when determining if there is any crack in rotors. However, it may fail to locate the cracks, especially where there are multiple cracks. For the method based on mode shapes, it could locate cracks better, because it contains spatial information. However, mode shapes are often measured on stationery rotors, so they are not suitable for operating rotors.

Characteristic deflection shape (CDS) is a kind of deformation data extracted from responses measured from operating rotors, it can be a mode shape when the rotor is harmonically excited at a resonant frequency, and it also can be an operational deflection shape (ODS) which is the deflection shape of an operating rotor at any time or frequency, but the CDS is not limited to a mode shape or ODS, it is a more general concept. Saravanan et al. [18] proposed a method based on the kurtosis of ODS measured by laser vibrometer for crack localization in operating rotors. In order to amplify the discontinuity introduced by cracks in the ODS, a weighted approximate waveform capacity dimension method was applied for crack localization in rotating rotors by Zhang et al. [19]. Babu et al. [20] proposed a new concept called amplitude deviation curve derived from ODS to implement crack localization in a rotating rotor with two cracks. Singh et al. [21] carried out an experimental investigation for crack localization in rotors based on detecting the discontinuity due to cracks using shaft deflections at regular axial locations of the shaft and at several excitation frequencies, and the method was also applied for multi-crack localization in a stepped shaft with a reference model of the intact shaft [22]. Asnaashari et al. [23] developed a residual ODS based method considering higher harmonic components of exciting frequency to localize cracks in a rotor which eliminated the effect of the fundamental frequency excitation. In view of the low sensitivity of ODS for incipient cracks, some after-treatment techniques were developed to aid crack localization, such as wavelets [24], fractal dimension [25] and gapped smoothing method (GSM) [26].

The proposed method in this paper belongs to the CDS-based methodology which is based on detecting discontinuities introduced by cracks in CDS. It is well known that cracks will reduce the local stiffness of structures, so the presence of cracks in rotors will cause discontinuities of curvature in the elastic line of the shaft. Meanwhile, steps in a shaft which are commonly found in real rotating machines will also introduce discontinuities at the locations of the steps. Therefore, identification of cracks must cope with the interference of the steps, and if there is no reference model of the intact system, the crack localization results will be misleading. However, as well known, cracks in rotors are often due to fatigue and they will breathe with rotation of rotors, which is a main distinction between cracks and steps. So if the crack breathing induced nonlinearity and stiffness reduction can be exploited simultaneously, then cracks can be localized exclusively, regardless of steps in a rotor. This is the main idea of this work.

Inspired from the frequency domain decomposition method [27] for operational modal analysis with outputs only, the approach based on singular value decomposition in frequency domain is proposed to extract the CDS of operating rotors. In order to avoid the interference of steps in rotors, the CDS at super-harmonic frequencies called super-harmonic CDS (SCDS) is used to localize the breathing cracks exclusively. Due to the difficulty in creating breathing cracks experimentally, the research is based on a finite element model of a two-disc stepped rotor-bearing system with breathing cracks which are modelled based on strain energy release rate approach with the crack closure line breathing model proposed by Darpe [28].

In this work, a new breathing crack localization method is proposed for operating rotors with steps using SCDS’s based on singular value decomposition in frequency domain. And the proposed method is validated by numerical simulation. The rest of the paper is organized as follows. In section 2, system modelling is carried out for a two-disc stepped rotor-bearing system considering the static unbalance of discs with breathing cracks based on the finite element method. The method for localization of breathing cracks in stepped rotors using SCDS’s based on singular value decomposition in frequency domain is proposed in section 3. In section 4, numerical experiments are carried out for the cracked rotors with different crack and step configurations. Finally, conclusions are drawn.

## System modelling

A two-disc stepped rotor-bearing system with breathing cracks considering bending-torsion coupling introduced by static unbalance is established based on the finite element method for numerical investigation in this work. A breathing crack model which can represent any crack angles and any types of excitations applied to the rotor is adopted and the stiffness matrix of the crack element is obtained. After that, through assembling the cracked and un-cracked elements, the finite element model of the rotor is established.

### 2.1 Model of a cracked shaft element

Fig.1 shows a cracked shaft element of length *l* and radius *R*. *P*1−*P*12 are the loads acting on the 12 degrees-of-freedom of the two nodes in the element coordinate system. The local coordinate system is defined on the flat crack face to describe the crack cross-section. is the crack angle between the crack face and the shaft centreline (formed by the negative axis turning to the negative *x* axis in the counter-clockwise direction) , is the location of the crack centre in the element coordinate system. CCL (crack closure line) is an imaginary line that separates the open and closed parts of the crack which will be used to simulate the breathing of crack. The hatched area corresponds to the open area of the crack.



Fig.1 Schematic diagram of cracked shaft element

The flexibility matrix of the un-cracked element and the additional flexibility matrix of the cracked element can be derived based on SERR (strain energy release rate) theory [4], and the detailed expressions can be obtained from [28].

According to the assumption of linear elasticity, the total flexibility matrix of the crack element is the sum of  and:

(1)

With the flexibility matrix, the stiffness matrix can be derived considering static equilibrium of crack element (as shown in Fig.1) and definition of the stiffness matrix as:

(2)

(3)

where and are the stiffness matrices of cracked and un-cracked elements respectively, and **T** is the transformation matrix written as:

(4)

### 2.2 Breathing crack model

To consider the breathing phenomenon, what matters the most is to describe the variation of crack section. In this paper, CCL method in [28] is adopted to model the breathing crack. This method assumes that the CCL is perpendicular to the crack tip line and separates the open and closed parts of the crack as can be seen in Fig. 1. And the position of CCL is determined by calculating the opening mode SIF (stress intensity factor)by Eq. (5), which depends on the crack element nodal forces, so the crack is response-dependent nonlinear. A positive corresponds to the open crack state and a negative one to the closed state. And the CCL is located at the position where the sign of changes. Once the CCL is ascertained, the stiffness matrix of the crack element can be obtained.

(5)

where is the opening mode SIF contributed by *Pi* .

### 2.3 Equations of motion of cracked rotor-bearing system with steps

The rotor-bearing system considered in this work is shown in Fig. 2. The rotor is discretized by two-node Timoshenko beam elements. The discs are considered rigid bodies which have three translational and three rotational inertias. And they are added to the mass matrix elements at the corresponding degrees-of-freedom. Gyroscopic effect of the two discs is also included. The ball bearings are simplified as isotropic linear springs and damper, one of which constrains the axial degree-of-freedom. The torsional degree-of-freedom in power input end of the rotor is also constrained. The rotating frequency of the rotor is **. By assembling the system matrix of cracked elements and un-cracked elements, the finite element model can be established.



Fig. 2 (a) Schematic diagram of cracked two discs rotor-bearing system; (b) Definition of rotating and stationary coordinates

Denote as displacement vector of node *i* having 6 degrees-of-freedom:

(6)

The equations of motion in the stationary coordinate system can be written as follows:

(7)

(8)

where is the system mass matrix, is the system damping matrix considering the Rayleigh damping,  is the system gyroscopic matrix, is the system stiffness matrix which will be updated as the crack breathes,  is excitation due to static unbalance of discs,  is excitation due to the gravitational force, and is external excitation during operation.

As for the disc located at node *i*, the gravitational excitation vector is:

(9)

The excitation due to static unbalance is:

(10)

where the elements in  can be expressed as [29]:

(11)

As shown in Fig. 2(b), is the relative angular displacement between the rotating coordinate and the stationary coordinate，which will be *t* when the rotor is rotating at a constant speed **.  is the torsional angle. is the unbalance orientation angle of the disc.

From Eq. (11), one can see that the excitation introduced by unbalance is bending-torsion coupled. So, the motion equations of the cracked rotor are response-dependent nonlinear and the excitation term is bending-torsion coupled. The Newmark method [30] is used to solve the equations numerically. The stiffness and damping matrices and the coupled excitation term are updated at each integration step, and the next time step will not start until the response in the current time step reaches convergence.

## Method for localization of breathing cracks in stepped rotors

The crack localization method proposed in this paper belongs to a CDS-based method, and the assumption is that the CDS’s of a healthy un-stepped rotor are smooth.

CSD’s containing spatial information of structures are sensitive to damage and are effective for multi-damage identification in beam- or rotor- type of structures. Nevertheless, one drawback of this kind of methods is that the extracted spatial shape features from experimental data tend to be compromised by noise, which decreases their damage identification accuracy, especially for incipient damage. To overcome this shortcoming, singular value decomposition which is a statistical tool for spatial pattern extraction is applied to the power spectral density (PSD) matrix for robust CDS’s estimation. Moreover, the PSD matrix contains the auto- and cross- spectral correlation of output vibration responses and provides average energy distribution in frequency domain, which is robust to the measurement noise. However, steps in rotors will affect some CDS’s and make the crack localization ambiguous. In order to reduce the interference, SCDS’s are defined for crack localization in stepped rotors based on the fact that steps cannot generate super-harmonics, but cracks can. However, for incipient cracks, the distortions in SCDS’s induced by cracks are still not easy to be detected. In order to reveal the distortions which correspond to crack locations more efficiently, GSM is adopted to derive a damage index. The method will be detailed in the following of this section.

### Singular value decomposition of dynamic responses in frequency domain

Let be the system response matrix which is measured simultaneously by *n* sensors distributed along the shaft:

(12)

where *m* is the sample length and is the response measured by sensor *i .*

In order to use singular value decomposition in frequency domain, the power spectral density matrix is obtained first which can be expressed as:

(13)

here is the Fourier operator and is the correlation matrix which can be written as:

(14)

(15)

whereis the correlation function between and and is the averaging operator.

Then, the power spectral density matrix at frequency can be decomposed by singular value decomposition as:

(16)

Because the power spectral density matrix is a square and positive definite matrix, the above expression equals to:

(17)

where is an orthogonal matrix containing the left singular vectors (); is a pseudo-diagonal matrix with singular values at the diagonal entries; is an orthogonal matrix containing the right singular vectors.

According to singular value decomposition, the first singular vector corresponding to the maximum singular value makes the largest contribution to the structure’s vibration, so is the dominant feature vector of . When there is a harmonic component in the measured response, a peak will be present in the first element of singular value matrix at the corresponding frequency [27]. Therefore, the corresponding can be considered a characteristic property of all the measured responses at this frequency and its shape is defined as a characteristic deflection shape (CDS). And if the harmonic component is a super-harmonic of the driving frequency, then its CDS is called a super-harmonic CDS (SCDS). The super-harmonic components appear due to the nonlinearities induced by cracks (It should be noted that the paper focuses on the more actual breathing cracks usually caused by fatigue, and the open crack case is not discussed.), so they should be more effective for crack localization, while the linear steps which cannot introduce nonlinearities to generate super-harmonics in rotors can be excluded.

### Damage indexes from super-harmonic characteristic deflection shapes by gapped smoothing method

Singularities will be introduced in the SCDS at the positions of cracks rather than of steps. This phenomenon is used to realize crack localization in stepped rotors. However, it is still difficult to localize cracks by the SCDS directly, because the singularities introduced by cracks are quiet weak, especially for incipient cracks. In order to amplify singularities in the SCDS without a reference intact model, a kind of polynomial curve fitting method called GSM is adopted.

A polynomial function with *n* orders ()at thegapped point can be written as [26]:

(18)

where are determined at .

After trial and error, the gapped linear interpolation is found to be more efficient in the application of crack localization in rotors. In this case, the first-order gapped polynomial function can be expressed as:

(19)

where and are determined at .

Then, a damage index (DI) is put forward as the squared difference between the gapped polynomial function and the corresponding value of the actual SCDS:

(20)

## Numerical experiments

In order to investigate the proposed localization methods, numerical experiments are carried out for the stepped rotor-bearing system shown in Fig. 2 and its detailed parameter values are given in Tab. 1, where andare calculated by assuming modal damping ratios of the first two modes being 0.005 and 0.01. And the first critical speed is calculated in the no-crack condition.

The rotor is discretized into 28 equivalent two-node twelve-degree-of-freedom Timoshenko beam elements, and cracks with different configurations are embedded using the cracked shaft elements.

The excitations applied on the rotor are the gravity and unbalance excitations of discs. All the cracks considered are transverse ones (they can also be slant cracks) and the cracks are assumed not to propagate during the short period of excitation while measurement is made. Newmark method is adopted to obtain the responses in time domain. The Newmark constants are 0.25 and 0.5 respectively, the sampling frequency is 5000Hz to capture breathing of cracks and the accuracy of convergence for each step is set to 10-11.

Tab. 1 Parameters of the rotor-bearing system

|  |  |  |  |
| --- | --- | --- | --- |
| Parameter | Value(units) | Parameter | Value(units) |
| Shaft length (*l*0) | 0.56m | Density of steel | 7.8x103kg/m3 |
| Shaft diameter (*D*) | 0.01m | Young’s modulus | 2.11x1011Pa |
| Stepped shaft diameter (*d*) | 0.008m | Poisson’s ratio | 0.3 |
| Position of disc 1 (*l*1) | 0.12m | Gravitational acceleration | 9.8m/s2 |
| Position of disc 2 (*l*2) | 0.44m | Rayleigh damping coefficient (*a*) | 0.49 |
| Position of stepped shaft (*l*3) | 0.24m | Rayleigh damping coefficient (*b*) | 4.28x10-5 |
| Length of stepped shaft (*l*4) | 0.04m | Bearing stiffness | 2.5x105N/m |
| Disc diameter | 0.074m | Bearing damping | 100Ns/m |
| Disc thickness | 0.025m | First critical speed | 1567r/min |
| Disc eccentricity (*e*) | 2x10-5m | Fundamental natural frequency | 26.11Hz |
| Unbalance orientation angle | 0o |  |  |

### Super-harmonic characteristics of the cracked rotor system

Responses are obtained numerically by Newmark method based on the established finite element model of the cracked rotor system. Vertical responses ‘measured’ by 29 sensors distributed along the rotor corresponding to 29 nodes are used for crack localization.

In order to localize cracks by SCDS’s, the super-harmonic characteristics of the cracked rotor system are investigated firstly. Fig. 3 is the waterfall plot of vertical response obtained by the 14th sensor of the stepped rotor of varying rotating speed with a crack located in the 10th element with crack depth of 0.2*D*, and the stepped shaft is arranged from the 13th to the 14th element.

Fig. 3 Typical waterfall plot of vertical response

From Fig. 3, 1X, 2X and 3X components can be clearly seen (where X represents the frequency corresponding to the rotating speed). And the amplitude of 1X component increases with the rotating speed and will reach the maximum when the rotating speed reaches the first critical speed. As for the 2X and 3X components which are generated by breathing of cracks, their amplitudes all first increase and reach the maximum at the speed of 1/2 and 1/3 of the first critical speed respectively, and then decrease. The super-harmonics and their behaviour are often utilized to determine if there is a crack or not. Because the crack localization by SCDS’s requires clearly recognisable super-harmonic components, the rotating speed of 540r/min which is near the 1/3 critical speed is chosen to investigate properties of the proposed method, and the effect of rotating speed will be studied later.

### 4.2 Localization of breathing cracks by the proposed method

In order to evaluate the performance of the proposed crack localization method, different cases of a stepped or a plain rotor with one or two cracks at different locations as shown in Tab. 2 are studied. And the crack depth ratios in each case are from 0.05 to 0.2 which are the ratios between actual crack depths to the corresponding shaft diameter. All the responses obtained are the vertical steady-state responses at the rotating speed of 540r/min except when the effect of rotating speed is studied in section 4.2.4.

Tab. 2 Cases of the stepped rotor with cracks at different locations

|  |  |  |  |
| --- | --- | --- | --- |
| Case | Crack 1 | Crack 2 | Position of the stepped shaft |
| 1 | 180-200mm (the 10th element) | -- | -- |
| 2 | 180-200mm (the 10th element) | -- | 240-280mm (element 13 to 14) |
| 3 | 260-280mm (the 14th element) | -- | 240-280mm (element 13 to 14) |
| 4 | 180-200mm (the 10th element) | 360-380mm (the 19th element) | 240-280mm (element 13 to 14) |

#### 4.2.1 Localization results

In order to validate the proposed method and to choose the SCDS for crack localization for the stepped rotor, SCDS’s at the super-harmonic frequencies of 2X and 3X from rotors in case 1 and case 2 with a crack depth of 0.1 are first compared. From Fig. 4 and Fig. 5, one can see that at the positions of the crack and steps, there are singularities which will be amplified by GSM for 3X SCDS. On the other hand, only the position of the crack presents an evident singularity and peak for 2X SCDS and the effect of steps can be neglected. It can be concluded that only the crack is sensitive to the 2X SCDS at rotating speed of 540r/min. So by using the 2X SCDS, the cracks can be localized exclusively and the interference introduced by stiffness reduction at a step can be eliminated.

The reason why the step influences 3X SCDS but not the 2X SCDS is given here. As an SCDS is a kind of shape at a particular frequency, it can be considered the superposition of linear modes from the linear part of the structure and the nonlinear characteristic modes which are a kind of major feature vectors induced by nonlinearity from the nonlinear part of the same structure. It should be noted that the term ‘nonlinear characteristic modes’ is used in this paper to express the influence of the nonlinear part of the structure on an SCDS, which is different from ‘nonlinear normal modes’ for a nonlinear system. For the 3X SCDS, its frequency is near the fundamental frequency of the rotor and thus the first linear mode is excited, so the characteristic shape is dominated by the first linear mode whose curvature has a singularity at the location of the step due to the stiffness reduction. However, for the 2X SCDS, because the 2X frequency is away from the fundamental frequency and there is no driving frequency near the 2X frequency, the characteristic shape is mainly determined by the nonlinear characteristic mode from the nonlinear part of the rotor and the effect of the first linear mode can be ignored. Therefore the effect of a step is very limited.



Fig. 4 Comparison of localization results of rotor using 2X SCDS between case 1 and case 2 with a crack of depth 0.1



Fig. 5 Comparison of localization results of rotor using 3X SCDS between case 1 and case 2 with a crack of depth 0.1

In order to investigate the performance of the proposed crack localization method for rotors with a crack near or at a step and with two cracks respectively, the 2X SCDS from rotors in case 3 and case 4 are extracted to localize the cracks. The results are shown in Fig. 6 and Fig. 7.



Fig. 6 Localization results of rotor in case 3 using 2X SCDS with a crack of depth 0.1



Fig. 7 Localization results of rotor in case 4 using 2X SCDS with two cracks of depth 0.1

Fig. 6 shows the localization results of a rotor with one crack just in the stepped area. And from Fig. 6 one can see that an abrupt distortion appears in the 2X SCDS at the position of the crack corresponding to the peak of damage index. The reason why the abrupt distortion in the 2X SCDS is due to crack instead of the step is that a step will not introduce 2X harmonics. And Fig. 7 is the localization result of a stepped rotor with two cracks. Large changes of slope in the 2X SCDS can be seen in the positions of the two cracks which correspond to the two peaks of damage index. All the identified crack locations in Figs. 6 and 7 are in quite agreement with those of cases 3 and 4 in Tab. 2, and the effects of steps are negligible. So, one can say that the proposed method is suitable for both one crack and two-crack localization in stepped rotors.

#### 4.2.2 Effects of crack depth

According to the crack localization results above, one can see that the proposed method based on the SCDS can localize the cracks accurately. In order to investigate the feasibility of the method to quantify the severity of cracks, the damage index curves for varying depths of the crack in case 2 are compared in Fig. 8.



Fig. 8 Effect of crack depth on the localization result of rotor in case 2 using 2X SCDS

Fig. 8 indicates that the deeper the crack is, the higher the damage index peak is, which means that the damage index is sensitive to crack depth, so the severity of a crack can be also assessed. In addition, the effect of steps decreases with crack depths. The reason for this phenomenon is that with the increase of crack depths, the degree of nonlinearity increases and the SCDS is more influenced by the nonlinear characteristic mode from the nonlinear part of the rotor.

#### 4.2.3 Robustness of the method

In practice, the measured responses are always polluted by noises. So in order to evaluate the robustness of the proposed method, rotors in case 2 and case 4 with crack depth ratio of 0.1 under different level of noise are studied.

White Gaussian noise is added to the original response **y**, so the noise-polluted response can be expressed as [31]:

(21)

where *N* is the length of vector **y**. is a number within (0, 1) that represents the noise level. and are the mean value and standard deviation of **y** respectively. is an *N*-length vector of normally distributed random numbers with zero mean and unit variance.

The typical responses at a measurement point (node 14) without noise and contaminated with different levels of noise are shown in Fig. 9. Signal and noise ratios (SNR’s) of the responses with noise level of 3%, 5% and 10% are 73.1dB, 68.6dB and 62.6dB respectively.



Fig. 9 Typical responses without and with noise

Fig. 10 and Fig. 11 are the localization results of rotors in case 2 and case 4 respectively.



Fig. 10 Effect of noise on the localization result of rotor in case 2 with one crack using 2X SCDS



Fig. 11 Effect of noise on the localization result of rotor in case 4 with two cracks using 2X SCDS

From Figs. 10 and 11, one can see that with the increase of noise level, more fluctuations appear in the corresponding DI’s, and the effects of the steps are almost drown by the noise and so they are negligible. And for case 2 with different levels of noise in Fig.10, the identified locations are all between 180mm and 200mm and the identified crack locations of case 4 in Fig. 11 are between 180mm and 200mm and between 360mm and 380mm, which are in agreement with the true locations in Tab. 2. Therefore, one can say that the proposed method is robust to measurement noise to some degree.

Though the proposed method is proved to be effective for cracks localization in stepped rotors by comparing the identified crack locations and the true locations and also by its good performances, in order to validate the proposed method further, a published method based on ODS in [18, 20] is adopted for comparison. The main idea of the published method for crack localization is based on detecting the discontinuities in the ODS, and to reveal the discontinuities better, the amplitude deviation curve (ADC) is derived from the ODS. Rotors in case 2 and case 4 are reanalysed using the published method. Results are shown in Fig. 12 and Fig. 13 respectively.



Fig. 12 Localization results for rotor in case 2 with one crack of depth 0.1 by the published method in [18, 20] (a) ODS of the rotor; (b) ADC derived from the ODS



Fig. 13 Localization results for rotor in case 4 with two cracks of depth 0.1 by the published method in [18, 20] (a) ODS of the rotor; (b) ADC derived from the ODS

From Figs. 12 and 13 above one can see that when there is no noise, cracks in both cases 2 and 4 can be localized accurately by the ADC of ODS, but the ODS and the ADC are sensitive to noise even when the noise level is 3%. In comparison, when using the proposed method in this paper, the localization results are accurate and the method is robust to noise even when the noise level is 10%, which can be seen in Figs. 10 and 11.Therefore, the effectiveness of the proposed method can be validated, and at the same time the advantages are obvious.

#### 4.2.4 Effects of rotating speed

The proposed method based on SCDS’s can or cannot eliminate the interference of steps in rotors depends on whether the SCDS’s are mainly determined by the nonlinear characteristic mode from the nonlinear part of the rotor or not. And the nonlinear behaviour will change with rotating speed which will affect the performance of the proposed method. So the effects of rotating speed and how to choose the right rotating speed for crack localization are studied for case 2 with a crack depth ratio of 0.1 at different rotating speeds.



Fig. 14 Effect of rotating speed on the localization results of rotor in case 2 with one crack using 2X SCDS



Fig. 15 Effect of rotating speed on the localization results of rotor in case 2 with one crack using 3X SCDS

As one can see from Figs. 14 and 15, DI from SCDS’s will change with the rotating speed. Specifically, for the 3X SCDS in Fig. 15, the influence of steps is very obvious, and with the increase of rotating speed, the influence becomes more serious; however when there is a relatively low speed, the interference of steps can be ignored. For the 2X SCDS in Fig. 14, the crack localization results are all quite good at different rotating speeds, and the interference of steps can be neglected. However, with the increase of rotating speed, the peak at the location of steps becomes higher which means that the 2X SCDS may degrade when the rotating speed is high. So much attention should be paid to choose the right rotating speed when measuring responses for crack identification using the proposed method.

All the phenomena mentioned above can be explained as follows. For 2X SCDS, it is dominated by the nonlinear characteristic mode from the nonlinear part of the rotor, so the influence of linear features such as steps is quite limited. But for 3X SCDS, because its corresponding frequency (for example, at **=540rpm) is near the fundamental natural frequency, the first linear mode is excited, so the first linear mode is dominant. However, when the rotating speed is low enough for the frequency of 3X to be away from the fundamental natural frequency, the influence of steps can be ignored, since the corresponding SCDS is controlled by the nonlinear characteristic mode from the nonlinear part of the rotor.

Based on the above finding, the following summary can be made: For , if *n*X SCDS is chosen, then the frequency of *n*X should be made away from the fundamental natural frequency as far as possible; and the *n*X frequency component in the response should be strong enough.

## Conclusions

A new crack localization method for stepped rotors is proposed by detecting the crack induced local shape distortions in super-harmonic characteristic deflection shapes (SCDS’s) based on singular value decomposition in frequency domain. The noise robust SCDS’s are estimated by applying singular value decomposition (SVD) to the output response power spectrum density (PSD) matrix. A damage index is derived by gapped smoothing method to better reveal the distortions in SCDS’s. The performance of the proposed method is investigated numerically based on a finite element model of a two-disc rotor-bearing system with breathing cracks and steps. Results show that the method is suitable for single or multiple crack localization in stepped rotors and the localization results are quite good when a proper rotating speed and super-harmonic component(s) are chosen. Furthermore, regardless of input excitations, only responses are needed by the proposed method and no reference models of intact rotors are required which will be useful for rotors with complex structures and complicated boundaries that are difficult to model. Besides steps, the proposed method is not affected by the presence of some common nonlinear factors in rotors which can generate super-harmonics, such as misalignment and initial bow, because the two factors will not affect the smoothness of the deflection of a rotor. Though there are several merits, the main limitation of the proposed method for real applications lies in the measurement of SCDS, because responses from a set of measurement points are required which could be achieved by embedded sensor technology, wireless sensor network, smart sensing skin technology and non-contact laser scanning sensors. Future work will be carried out to validate the proposed approach by experiments in laboratory.

## Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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