Stability Analysis of Linear Systems with Two Additive Time-Varying Delays via Delay-Product-Type Lyapunov Functional

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Abstract

This paper is concerned with the stability analysis of continuous linear systems with two additive time-varying delays in the Lyapunov-Krasovskii functional (LKF) framework. Two novel delay-product-type terms are introduced into LKF candidate inspired by our previous research. The Wirtinger-based inequality, together with the reciprocally convex combination technique, is applied to estimate the integral terms arising in the derivative of the LKF. As a result, a new delay- and its-change-rate-dependent stability criterion is established. Its advantage of less conservatism than some existing criteria is demonstrated through a numerical example. Finally, the stability criterion is applied to analyze the stability of the load frequency control scheme of power systems.

Keywords: Linear system, time-varying delay, stability, Writinger-based inequality, delay-product-type functional

1. Introduction

As an increasing number of closed-loop control systems are being implemented using communication networks, time delays inevitably arise in the communication channels, which may degrade the dynamic performance and even destroy the system stability. Hence, the analysis of time-delay systems to understand the effect of those delays on the system stability has become an important topic in the last few decades, and significant research has been devoted to this issue [1, 2, 3]. Most researchers have concentrated on systems with one time-varying delay, which is the combination of all the delays appearing in the total communication network of the control system. For some systems, such as remote control systems and networked control systems, the measured signals transmitted from the sensors to the control center and the control signals sent from the control center may experience different segments of networks, and the time delays arising may have different properties due to variable network transmission conditions [4]. Therefore, it is also an important issue to assess the effects on system stability from different parts of delays.

In [4], the system with two additive time-varying delay components has been proposed firstly to model different properties of delays for different channels, and the free-weighting-matrix (FWM) approach [6] was used to develop a stability criterion. After that, many results for the analysis and design of this model were reported. Robust stability analysis was discussed via taking into account the system uncertainties in [7]. Stability criteria with less conservatism were developed respectively via the improved FWM approach [8], Jensen inequality [11, 13, 15], and other integral

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inequalities [14]. The comparison of several stability criteria was investigated in [9]. In [10], the idea of additive delays modelling was extended to the singular system and several stability criteria were reported. The case that two additive delays of linear systems are constant and have overlapping ranges was studied in [12]. In recent years, the reciprocally convex combination lemma was widely used to develop new stability criteria for systems with additive delays [13, 15]. Stability analysis and stabilization design for the systems with additive delays were discussed via the delay-partitioning-based Lyapunov-Krasovskii functional (LKF) [16], Jensen inequality and cone complementarity linearization algorithm [17], and a relaxed LKF [18]. Robust control design of the systems with both additive delays and parameter uncertainties was studied in [19].

The time delays appearing in the control loops are usually time variant. For the case of time-varying delays, the popular investigation framework is combining the LKF and linear matrix inequality (LMI), and the aforementioned results are all obtained in this framework. It is well known that the effort is to reduce the conservatism of the obtained criteria from the viewpoints of construction of the LKFs and estimation of their derivatives. Up until now, many LKFs with more general forms and integral inequalities with smaller estimation errors were proposed for this task, such as the augmented-based LKFs [13], the delay-partitioning-based LKFs [16], and simple Wirtinger-based inequality [19]. However, there still exists room for further investigation and reduction of the conservatism. On the one hand, in our previous work on discrete-time delayed linear systems [21] and continuous-time delayed neural networks [22], the delay-product-type LKFs were developed and found to be helpful for improving the results. On the other hand, a tighter Wirtinger-based inequality was proposed in [23], while it has not been used for the systems with additive delays. Therefore, it is expected that the stability criterion of system with additive delays will be further improved by combining those two new techniques. This motivates the current research.

This paper provides further study on the stability analysis of linear systems with two additive time-varying delays and develops a new criterion to understand the effect of delays on the system stability. The main contribution of the paper is summarized as follows:

- A new stability criterion with less conservatism in comparison with the existing ones is established. On one hand, inspired by our previous work [21, 40], a new LKF with delay-product-type terms is constructed, and its derivative is estimated via the Wirtinger-based inequality and the reciprocally convex combination technique. Such more general form of LKF, together with the tighter estimation method, leads to a less conservative stability criterion.

- Compared with most literature, the example study part not only applies a numerical example to demonstrate the less conservatism of the proposed criterion, but also applies the proposed criterion to analyze the stability of the load frequency control scheme of power systems, which further shows the significant of our research.

The reminder of paper is organized as follows. Section 2 explains the problem formulation. In Section 3, a new stability criterion is developed through a delay-product-type LKF and the Wirtinger-based inequality. In Section 4, the advantages of the proposed criterion is verified via a numerical example, and the application of the proposed method on the LFC of power system is studied. Conclusions are presented in Section 5.

Notations: Throughout this paper, the superscripts T and -1 mean the transpose and the inverse of a matrix, respectively; $\mathcal{R}^n$ denotes the n-dimensional Euclidean space; $\|\cdot\|$ refers to the Euclidean vector norm; $P > 0$ (≥ 0)

means P is a real symmetric and positive-definite (semi-positive-definite) matrix; I and 0 stand for the identity matrix and the zero-matrix, respectively; the symmetric term in the symmetric matrix is denoted by $*$; and $\mathrm{He}\{X\} = X + X^T$.

2. Problem statement and preliminaries

Consider the following continuous linear system with two additive time-varying delays:

$$\dot{x}(t) = Ax(t) + A_d x(t - d_1(t) - d_2(t)) \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state, A and A_d are known real constant matrices, $d_1(t)$ and $d_2(t)$ are time delays and satisfy the following conditions

$$0 \le d_1(t) \le h_1, \quad 0 \le d_2(t) \le h_2 \tag{2}$$

$$|\dot{d}_1(t)| \le \mu_1, \quad |\dot{d}_2(t)| \le \mu_2 \tag{3}$$

where h_i and μ_i, $i = 1, 2$ are constant. Let $d(t) = d_1(t) + d_2(t)$ and $h = h_1 + h_2$.

This paper is concerned with the stability problem of system (1) to understand the effect of time delays therein on the stability. In order to accurately assess the system stability, this paper aims to develop a new stability criterion with as small conservatism as possible.

The following lemmas were used for developing the main results.

Lemma 1. *(Wirtinger-based integral inequality [23]) For a symmetric matrix $R > 0$, scalars a and b with $a < b$, and vector ω such that the integration concerned is well defined, the following inequality holds*

$$(b - a) \int_a^b \dot{\omega}^T(s) R \dot{\omega}(s) ds \ge \chi_1^T R \chi_1 + 3\chi_2^T R \chi_2 \tag{4}$$

where

$$\chi_1 = \omega(b) - \omega(a)$$
$$\chi_2 = \omega(b) + \omega(a) - \frac{2}{b-a} \int_a^b \omega(s) ds$$

Lemma 2. *(Reciprocally convex combination lemma [24]) For given positive integers n and m, a scalar α in the interval $(0, 1)$, a $n \times n$-matrix $R > 0$, two matrices $W_1, W_2 \in \mathcal{R}^{n \times m}$, and any vector $\xi \in \mathcal{R}^m$, define the function $\Theta(\alpha, R)$ with the following form*

$$\Theta(\alpha, R) = \frac{1}{\alpha} \xi^T W_1^T R W_1 \xi + \frac{1}{1-\alpha} \xi^T W_2^T R W_2 \xi$$

If there exists a matrix $X \in \mathcal{R}^{n \times n}$ satisfying $\begin{bmatrix} R & X \\ * & R \end{bmatrix} > 0$, then the following inequality holds

$$\min_{\alpha \in (0,1)} \Theta(\alpha, R) \ge \begin{bmatrix} W_1 \xi \\ W_2 \xi \end{bmatrix}^T \begin{bmatrix} R & X \\ * & R \end{bmatrix} \begin{bmatrix} W_1 \xi \\ W_2 \xi \end{bmatrix} \tag{5}$$

3. Main results

In this section, a new stability criterion and its proof are given as first. Then, some discussions are carried out to show its advantages.

3.1. *A less conservative stability criterion*

A new LKF with delay-product-type terms, together with Lemmas 1 and 2, leads to a novel stability criterion, shown as follows.

Theorem 1. *For given scalars K, h_1, h_2, μ_1, and μ_2, system (1) with the time-varying delay satisfying (2) and (3) is asymptotically stable if there exist $5n \times 5n$-matrix $P = P^T \geqslant 0$, $n \times n$-matrices $Q_i = Q_i^T > 0$, $i = 1, 2, \cdots, 5$, $Z_1 = Z_1^T > 0$, and $Z_2 = Z_2^T > 0$, and $2n \times 2n$-matrices $P_1 = P_1^T > 0$, $P_2 = P_2^T > 0$, $P_3 = P_3^T > 0$, and $P_4 = P_4^T > 0$, and any $2n \times 2n$-matrices X and Y such that the following conditions hold*

$$\Omega_0 = \Phi_1 + \Phi_2 - \frac{1}{h_1} G_a^T \Omega_1 G_a - \frac{1}{h_1 + h_2} G_b^T \Omega_2 G_b < 0 \tag{6}$$

$$\Omega_1 = \begin{bmatrix} \tilde{Z}_1 & X \\ * & \tilde{Z}_1 \end{bmatrix} > 0 \tag{7}$$

$$\Omega_2 = \begin{bmatrix} \tilde{Z}_2 & Y \\ * & \tilde{Z}_2 \end{bmatrix} > 0 \tag{8}$$

where

$$\begin{aligned}
\Phi_1 = {} & \mathrm{He}\{F_1^T P F_a\} - e_3^T (Q_2 - Q_5) e_3 - e_2^T (Q_1 - Q_2 - Q_3) e_2 \\
& + e_1^T Q_1 e_1 + e_4^T (Q_4 - Q_3) e_4 - e_5^T (Q_4 + Q5) e_5 + (Ae_1 + BKe_3)^T (h_1 Z_1 + h Z_2)(Ae_1 + BKe_3) \\
& + \dot{d}_1(t)(e_3^T (Q_2 - Q_5) e_3 + e_2^T (Q_1 - Q_2 - Q_3) e_2) + \dot{d}_2(t) e_3^T (Q_2 - Q_5) e_3
\end{aligned} \tag{9}$$

$$\begin{aligned}
\Phi_2 = {} & \mathrm{He}\{F_2^T P_1 F_b + F_3^T P_2 F_c + F_4^T P_3 F_d + F_5^T P_4 F_e\} \\
& + \dot{d}_1(t)(F_2^T P_1 F_2 - F_3^T P_2 F_3 + F_4^T P_3 F_4 - F_5^T P_4 F_5) + \dot{d}_2(t)(F_4^T P_3 F_4 - F_5^T P_4 F_5)
\end{aligned} \tag{10}$$

$$F_1 = \begin{bmatrix} e_1 \\ d_1(t) e_6 \\ [d_1(t) + d_2(t)] e_8 \\ [h_1 - d_1(t)] e_7 + d_1(t) e_6 \\ d(t) e_8 + [h - d(t)] e_9 - d_1(t) e_6 - [h_1 - d_1(t)] e_7 \end{bmatrix}, \quad F_a = \begin{bmatrix} Ae_1 + BKe_3 \\ e_1 - (1 - \dot{d}_1(t)) e_2 \\ e_1 - (1 - \dot{d}(t)) e_3 \\ e_1 - e_4 \\ e_4 - e_5 \end{bmatrix} \tag{11}$$

$$F_2 = \begin{bmatrix} e_1 \\ e_6 \end{bmatrix}, \quad F_b = \begin{bmatrix} d_1(t)(Ae_1 + BKe_3) \\ e_1 - (1 - \dot{d}_1(t)) e_2 - \dot{d}_1(t) e_6 \end{bmatrix}, \quad F_4 = \begin{bmatrix} e_1 \\ e_8 \end{bmatrix}, \quad F_d = \begin{bmatrix} d(t)(Ae_1 + BKe_3) \\ e_1 - (1 - \dot{d}(t)) e_3 - \dot{d}(t) e_8 \end{bmatrix} \tag{12}$$

$$F_3 = \begin{bmatrix} e_1 \\ e_7 \end{bmatrix}, \quad F_c = \begin{bmatrix} (h_1 - d_1(t))(Ae_1 + BKe_3) \\ (1 - \dot{d}_1(t)) e_2 - e_4 + \dot{d}_1(t) e_7 \end{bmatrix}, \quad F_5 = \begin{bmatrix} e_1 \\ e_9 \end{bmatrix}, \quad F_e = \begin{bmatrix} (h - d(t))(Ae_1 + BKe_3) \\ (1 - \dot{d}(t)) e_3 - e_5 + \dot{d}(t) e_9 \end{bmatrix} \tag{13}$$

$$G_a = \begin{bmatrix} G_3^T & G_4^T & G_1^T & G_2^T \end{bmatrix}^T, \quad G_b = \begin{bmatrix} G_7^T & G_8^T & G_5^T & G_6^T \end{bmatrix}^T, \quad \tilde{Z}_1 = \begin{bmatrix} Z_1 & 0 \\ * & 3Z_1 \end{bmatrix}, \quad \tilde{Z}_2 = \begin{bmatrix} Z_2 & 0 \\ * & 3Z_2 \end{bmatrix} \tag{14}$$

$$\begin{aligned}
& G_1 = e_2 - e_4, \quad G_2 = e_2 + e_4 - 2e_7, \quad G_3 = e_1 - e_2, \quad G_4 = e_1 + e_2 - 2e_6, \qquad h = h_1 + h_2 \\
& G_5 = e_3 - e_5, \quad G_6 = e_3 + e_5 - 2e_9, \quad G_7 = e_1 - e_3, \quad G_8 = e_1 + e_3 - 2e_8, \qquad d(t) = d_1(t) + d_2(t) \\
& e_i = \begin{bmatrix} 0_{n \times (i-1)n}, & I_n, & 0_{n \times (9-i)n} \end{bmatrix}, \ i = 1, 2, \cdots, 9
\end{aligned}$$

Proof: Inspired by our previous research [21], the following LKF candidate with four delay-product-type terms is constructed

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) + V_4(x_t) + V_5(x_t) + V_6(x_t) \tag{15}$$

where

$$
\begin{aligned}
V_1(x_t) &= \xi_0^T(t)P\xi_0(t)\\
V_2(x_t) &= d_1(t)\xi_1^T(t)P_1\xi_1(t) + d(t)\xi_2^T(t)P_3\xi_2(t)\\
V_3(x_t) &= [h_1 - d_1(t)]\xi_3^T(t)P_2\xi_3(t) + [h - d(t)]\xi_4^T(t)P_4\xi_4(t)\\
V_4(x_t) &= \int_{t-d_1(t)}^{t} x^T(s)Q_1x(s)ds + \int_{t-d(t)}^{t-d_1(t)} x^T(s)Q_2x(s)ds\\
V_5(x_t) &= \int_{t-h_1}^{t-d_1(t)} x^T(s)Q_3x(s)ds + \int_{t-h}^{t-h_1} x^T(s)Q_4x(s)ds + \int_{t-h}^{t-d(t)} x^T(s)Q_5x(s)ds\\
V_6(x_t) &= \int_{-h_1}^{0}\int_{t+s}^{t} \dot{x}^T(\alpha)Z_1\dot{x}(\alpha)d\alpha ds + \int_{-h}^{0}\int_{t+s}^{t} \dot{x}^T(\alpha)Z_2\dot{x}(\alpha)d\alpha ds
\end{aligned}
$$

with

$$
\begin{aligned}
\xi_0(t) &= \left[x^T(t),\ \ \int_{t-d_1(t)}^{t} x^T(s)ds,\ \ \int_{t-d(t)}^{t} x^T(s)ds,\ \ \int_{t-h_1}^{t} x^T(s)ds,\ \ \int_{t-h}^{t-h_1} x^T(s)ds\right]^T\\
\xi_1(t) &= \left[x^T(t),\ \ \int_{t-d_1(t)}^{t} \tfrac{x^T(s)}{d_1(t)}ds\right]^T,\ \ \xi_2(t) = \left[x^T(t),\ \ \int_{t-d(t)}^{t} \tfrac{x^T(s)}{d(t)}ds\right]^T\\
\xi_3(t) &= \left[x^T(t),\ \ \int_{t-h_1}^{t-d_1(t)} \tfrac{x^T(s)}{h_1-d_1(t)}ds\right]^T,\ \ \xi_4(t) = \left[x^T(t),\ \ \int_{t-h}^{t-d(t)} \tfrac{x^T(s)}{h-d(t)}ds\right]^T
\end{aligned}
$$

On one hand, if the matrices in $V(x_t)$ satisfying $P > 0$, $P_1 > 0$, $P_2 > 0$, $P_3 > 0$, $P_4 > 0$, $Q_i = Q_i^T$ $(i = 1, 2, \cdots, 5)$, $Z_1 > 0$, and $Z_2 > 0$, then $V(x_t) \geq \varepsilon_1\|x(t)\|$ for a sufficiently small scalar $\varepsilon_1 > 0$.

On the other hand, the conditions guaranteeing the negative definite of the derivative of $V(x_t)$ are discussed. At first, for simplifying the representation of the subsequent part, the following notations are defined:

$$
\begin{aligned}
\zeta(t) = \Big[&x^T(t),\ \ x^T(t-d_1(t)),\ \ x^T(t-d(t)),\ \ x^T(t-h_1),\ \ x^T(t-h)\\
&\frac{1}{d_1(t)}\int_{t-d_1(t)}^{t} x^T(s)ds,\ \ \frac{1}{h_1-d_1(t)}\int_{t-h_1}^{t-d_1(t)} x^T(s)ds,\ \ \frac{1}{d(t)}\int_{t-d(t)}^{t} x^T(s)ds,\ \ \frac{1}{h-d(t)}\int_{t-h}^{t-d(t)} x^T(s)ds\Big]^T\\
e_i = \Big[&0_{n\times(i-1)n},\ \ I_n,\ \ 0_{n\times(9-i)n}\Big],\ \ i = 1, 2, \cdots, 9
\end{aligned}
$$

Calculating the derivative of the $V_1(x_t)$ along the solutions of system (1) leads to

$$
\begin{aligned}
\dot{V}_1(x_t) &= 2\xi_0^T(t)P\dot{\xi}_0(t)\\
&= 2\begin{bmatrix} x(t)\\ \int_{t-d_1(t)}^{t} x(s)ds\\ \int_{t-d(t)}^{t} x(s)ds\\ \int_{t-h_1}^{t-d_1(t)} x(s)ds + \int_{t-d_1(t)}^{t} x(s)ds\\ \int_{t-d(t)}^{t} x(s)ds + \int_{t-h}^{t-d(t)} x(s)ds - \int_{t-d_1(t)}^{t} x(s)ds - \int_{t-h_1}^{t-d_1(t)} x(s)ds \end{bmatrix}^T P \begin{bmatrix} \dot{x}(t)\\ x(t) - [1-\dot{d}_1(t)]x(t-d_1(t))\\ x(t) - [1-\dot{d}(t)]x(t-d(t))\\ x(t) - x(t-h_1)\\ x(t-h_1) - x(t-h) \end{bmatrix}\\
&= \zeta^T(t)\mathrm{He}\{F_1^TPF_a\}\zeta(t) \qquad (16)
\end{aligned}
$$

where F_1 and F_a are defined in (11).

Calculating the derivative of the $V_2(x_t)$ along the solutions of system (1) leads to

$$
\dot{V}_2(x_t) = \dot{d}_1(t)\xi_1^T(t)P_1\xi_1(t) + 2d_1(t)\xi_1^T(t)P_1\dot{\xi}_1(t) + \dot{d}(t)\xi_2^T(t)P_3\xi_2(t) + 2d(t)\xi_2^T(t)P_3\dot{\xi}_2(t) \qquad (17)
$$

where

$$\xi_1(t) = \begin{bmatrix} e_1 \\ e_6 \end{bmatrix} \zeta(t) = F_2\zeta(t), \qquad d_1(t)\dot{\xi}_1(t) = \begin{bmatrix} d_1(t)(Ae_1 + BKe_3) \\ e_1 - (1 - \dot{d}_1(t))e_2 - \dot{d}_1(t)e_6 \end{bmatrix} \zeta(t) = F_b\zeta(t)$$

$$\xi_2(t) = \begin{bmatrix} e_1 \\ e_8 \end{bmatrix} \zeta(t) = F_4\zeta(t), \qquad d(t)\dot{\xi}_2(t) = \begin{bmatrix} d(t)(Ae_1 + BKe_3) \\ e_1 - (1 - \dot{d}(t))e_3 - \dot{d}(t)e_8 \end{bmatrix} \zeta(t) = F_d\zeta(t)$$

Thus, $\dot{V}_2(x_t)$ is rewritten as

$$\begin{aligned} \dot{V}_2(x_t) &= \dot{d}_1(t)\zeta^T(t)F_2^T P_1 F_2\zeta(t) + \zeta^T(t)\text{He}\{F_2^T P_1 F_b\}\zeta(t) + \dot{d}(t)\zeta^T(t)F_4^T P_3 F_4\zeta(t) + \zeta^T(t)\text{He}\{F_4^T P_3 F_d\}\zeta(t) \\ &= \zeta^T(t)\left[\dot{d}_1(t)(F_2^T P_1 F_2 + F_4^T P_3 F_4) + \dot{d}_2(t)F_4^T P_3 F_4 + \text{He}\{F_2^T P_1 F_b + F_4^T P_3 F_d\}\right]\zeta(t) \end{aligned} \tag{18}$$

where F_2, F_b, F_4, and F_d are defined in (12).

Calculating the derivative of the $V_3(x_t)$ along the solutions of system (1) leads to

$$\dot{V}_3(x_t) = -\dot{d}_1(t)\xi_3^T(t)P_2\xi_3(t) + 2(h_1 - d_1(t))\xi_3^T(t)P_2\dot{\xi}_3(t) - \dot{d}(t)\xi_4^T(t)P_4\xi_4(t) + 2(h - d(t))\xi_4^T(t)P_4\dot{\xi}_4(t) \tag{19}$$

where

$$\xi_3(t) = \begin{bmatrix} e_1 \\ e_7 \end{bmatrix} \zeta(t) = F_3\zeta(t), \qquad (h_1 - d_1(t))\dot{\xi}_3(t) = \begin{bmatrix} (h_1 - d_1(t))(Ae_1 + BKe_3) \\ (1 - \dot{d}_1(t))e_2 - e_4 + \dot{d}_1(t)e_7 \end{bmatrix} \zeta(t) = F_c\zeta(t)$$

$$\xi_4(t) = \begin{bmatrix} e_1 \\ e_9 \end{bmatrix} \zeta(t) = F_5\zeta(t), \qquad (h - d(t))\dot{\xi}_4(t) = \begin{bmatrix} (h - d(t))(Ae_1 + BKe_3) \\ (1 - \dot{d}(t))e_3 - e_5 + \dot{d}(t)e_9 \end{bmatrix} \zeta(t) = F_e\zeta(t)$$

Thus, $\dot{V}_3(x_t)$ is rewritten as

$$\begin{aligned} \dot{V}_3(x_t) &= -\dot{d}_1(t)\zeta^T(t)F_3^T P_2 F_3\zeta(t) + \zeta^T(t)\text{He}\{F_3^T P_2 F_c\}\zeta(t) - \dot{d}(t)\zeta^T(t)F_5^T P_4 F_5\zeta(t) + \zeta^T(t)\text{He}\{F_5^T P_4 F_e\}\zeta(t) \\ &= \zeta^T(t)\left[-\dot{d}_1(t)(F_3^T P_2 F_3 + F_5^T P_4 F_5) - \dot{d}_2(t)F_5^T P_4 F_5 + \text{He}\{F_3^T P_2 F_c + F_5^T P_4 F_e\}\right]\zeta(t) \end{aligned} \tag{20}$$

where F_3, F_c, F_5, and F_e are defined in (13).

Taking the derivative of $V_4(x_t)$ along the solutions of system (1) yields

$$\begin{aligned} \dot{V}_4(x_t) &= x^T(t)Q_1x(t) - (1 - \dot{d}_1(t))x^T(t - d_1(t))Q_1x(t - d_1(t)) \\ &\quad + (1 - \dot{d}_1(t))x^T(t - d_1(t))Q_2x(t - d_1(t)) - (1 - \dot{d}(t))x^T(t - d(t))Q_2x(t - d(t)) \end{aligned} \tag{21}$$

The derivative of $V_5(x_t)$ along the solutions of system (1) can be obtained as

$$\begin{aligned} \dot{V}_5(x_t) &= (1 - \dot{d}_1(t))x^T(t - d_1(t))Q_3x(t - d_1(t)) - x^T(t - h_1)Q_3x(t - h_1) + x^T(t - h_1)Q_4x(t - h_1) - x^T(t - h)Q_4x(t - h) \\ &\quad + (1 - \dot{d}(t))x^T(t - d(t))Q_5x(t - d(t)) - x^T(t - h)Q_5x(t - h) \end{aligned} \tag{22}$$

Taking the derivative of $V_6(x_t)$ yields

$$\dot{V}_6(x_t) = h_1\dot{x}^T(t)Z_1\dot{x}(t) + h\dot{x}^T(t)Z_2\dot{x}(t) - \int_{t-h_1}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-h}^{t} \dot{x}^T(s)Z_2\dot{x}(s)ds \tag{23}$$

Combining (16), (18), (20)-(23) yields

$$\dot{V}(x_t) = \zeta^T(t)(\Phi_1 + \Phi_2)\zeta(t) - \int_{t-h_1}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-h}^{t} \dot{x}^T(s)Z_2\dot{x}(s)ds \tag{24}$$

where Φ_1 and Φ_2 are defined in (9) and (10), respectively.

By applying Lemmas 1 and 2 to estimate the Z_1-dependent integral term, the following is obtained

$$\begin{aligned}
&-\int_{t-h_1}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds = -\int_{t-d_1(t)}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-h_1}^{t-d_1(t)} \dot{x}^T(s)Z_1\dot{x}(s)ds \\
&\le -\frac{1}{d_1(t)}\begin{bmatrix} x(t)-x(t-d_1(t)) \\ x(t)+x(t-d_1(t))-\frac{2}{d_1(t)}\int_{t-d_1(t)}^{t} x(t)ds \end{bmatrix}^T \begin{bmatrix} Z_1 & 0 \\ 0 & 3Z_1 \end{bmatrix} \begin{bmatrix} x(t)-x(t-d_1(t)) \\ x(t)+x(t-d_1(t))-\frac{2}{d_1(t)}\int_{t-d_1(t)}^{t} x(t)ds \end{bmatrix} \\
&-\frac{1}{h_1-d_1(t)}\begin{bmatrix} x(t-d_1(t))-x(t-h_1) \\ x(t-d_1(t))+x(t-h_1)-\int_{t-h_1}^{t-d_1(t)} \frac{2x(t)}{h_1-d_1(t)}ds \end{bmatrix}^T \begin{bmatrix} Z_1 & 0 \\ 0 & 3Z_1 \end{bmatrix} \begin{bmatrix} x(t-d_1(t))-x(t-h_1) \\ x(t-d_1(t))+x(t-h_1)-\int_{t-h_1}^{t-d_1(t)} \frac{2x(t)}{h_1-d_1(t)}ds \end{bmatrix} \\
&= -\zeta^T(t)\left\{\frac{1}{d_1(t)}\begin{bmatrix} e_1-e_2 \\ e_1+e_2-2e_6 \end{bmatrix}^T \begin{bmatrix} Z_1 & 0 \\ * & 3Z_1 \end{bmatrix} \begin{bmatrix} e_1-e_2 \\ e_1+e_2-2e_6 \end{bmatrix} + \frac{1}{h_1-d_1(t)}\begin{bmatrix} e_2-e_4 \\ e_2+e_4-2e_7 \end{bmatrix}^T \begin{bmatrix} Z_1 & 0 \\ * & 3Z_1 \end{bmatrix} \begin{bmatrix} e_2-e_4 \\ e_2+e_4-2e_7 \end{bmatrix}\right\}\zeta(t) \\
&\le -\frac{1}{h_1}\zeta^T(t)G_a^T\begin{bmatrix} \tilde{Z}_1 & X \\ * & \tilde{Z}_1 \end{bmatrix} G_a\zeta(t)
\end{aligned} \tag{25}$$

where G_a and $\tilde{Z}_1$ are defined in (14) and $\begin{bmatrix} \tilde{Z}_1 & X \\ * & \tilde{Z}_1 \end{bmatrix} > 0$.

Similarly, the Z_2-dependent integral term in (24) being estimated through Lemmas 1 and 2 leads to

$$-\int_{t-h}^{t} \dot{x}^T(s)Z_2\dot{x}(s)ds \le -\frac{1}{h_1+h_2}\zeta^T(t)G_b^T\begin{bmatrix} \tilde{Z}_2 & Y \\ * & \tilde{Z}_2 \end{bmatrix} G_b\zeta(t) \tag{26}$$

where G_b and $\tilde{Z}_2$ are defined in (14) and $\begin{bmatrix} \tilde{Z}_2 & Y \\ * & \tilde{Z}_2 \end{bmatrix} > 0$.

Thus, based on (24)-(26), the following is true

$$\dot{V}(x_t) < \zeta^T(t)\left(\Phi - \frac{1}{h_1}G_a^T\Omega_1G_a - \frac{1}{h_1+h_2}G_b^T\Omega_2G_b\right)\zeta(t) = \zeta^T(t)\Omega_0\zeta(t) \tag{27}$$

Therefore, $\Omega_0 < 0$ leads to $\dot{V}(x_t) \le -\epsilon_2\|x(t)\|^2$ for a sufficient small scalar $\epsilon_2 > 0$. Hence, when (6)-(8) hold, system (1) with the time-varying delay satisfying (2) and (3) is asymptotically stable. This completes the proof. ■

3.2. Discussions

Theorem 1 gives the relationship between the time delays (including the bounds of delays and their derivatives, h_i and μ_i, $i = 1, 2$) and the stability of system. Thus, it can be used to analyze the effect of delays on the system stability and find how big of time-varying delays the system can tolerate (i.e., delay margins calculation problem).

- Although the construction of the LKF is complex, there is no need to obtain the detailed form of LKF to check the stability of system. The conditions of Theorem 1, (6)-(8), are a set of matrix inequalities with respect to the Lyapunov matrices (P, P_1, P_2, P_3, P_4, Q_1, Q_2, Q_3, Q_4, Q_5, Z_1, and Z_2). If there are feasible solutions of those matrix inequalities, then the LKF ensuring the system stability can be constructed by using the form of (15) and the feasible solutions of (6)-(8). Therefore, the LKF-constructing problem is transformed into the feasibility-checking problem of inequalities (6)-(8).

- Among inequalities (6)-(8), $\Omega_1 < 0$ and $\Omega_2 < 0$ are LMIs. While the condition, $\Omega_0 < 0$, is not an LMI due to its dependence on the time-varying delays, $d_1(t)$ and $d_2(t)$, and their change rate, $\dot{d}_1(t)$ and $\dot{d}_2(t)$. In fact, this condition can be rewritten as the following form:

$$\Omega_0(d_1(t), d_2(t), \dot{d}_1(t), \dot{d}_2(t)) = d_1(t)[\Upsilon_1 + \dot{d}_1(t)\Upsilon_2 + \dot{d}_2(t)\Upsilon_3] + d_2(t)[\Upsilon_4 + \dot{d}_1(t)\Upsilon_5 + \dot{d}_2(t)\Upsilon_6] < 0 \qquad (28)$$

  where $\Upsilon_i, i = 1, 2, \cdots, 6$ are time-independent matrix-combinations. By using the convex combination technique [25] and following the same proof procedure in [26] [proof of Theorem 1 therein], the condition $\Omega_0 < 0$ holds if the following LMI-based conditions hold

$$\Omega_0|_{(d_1(t),d_2(t),\dot{d}_1(t),\dot{d}_2(t))\in([0,h_1]\times[0,h_2]\times[-\mu_1,\mu_1]\times[-\mu_2,\mu_2])} < 0 \qquad (29)$$

  Thus, the feasibility-checking of inequalities (6)-(8) becomes the solving of the LMIs, which can be easily solved by using MATLAB/LMI Toolbox.

- Finally, the delay margins for guaranteeing the system stability can be obtained by presetting various values of h_i and μ_i, $i = 1, 2$ and checking the feasibility of LMI conditions [36].

Several new techniques are applied to develop Theorem 1 and they contribute to the reduction of conservatism of Theorem 1.

- The LKF used in this paper is different from the ones reported in the literature. It not only contains some augmented terms similar to the one used in [13] but also introduces four delay-product-type terms, $V_2(x_t)$ and $V_3(x_t)$, which are inspired by our previous work for discrete-time time delay system [21]. Those terms contribute to the conservatism reduction from two aspects. Firstly, the introduction of those terms makes the candidate LKF be more general, which means that more choices are provided for finding an LKF ensuring the system stability. Secondly, the derivative of those terms leads to additional information into the condition guaranteeing the negative definite of the LKF, i.e., Φ_2 in (6), which relaxes the condition of $\Omega_0 < 0$ in comparison with $\Omega_0 - \Phi_2 < 0$.

- During the proof of Theorem 1, the Wirtinger-based integral inequality and the reciprocally convex combination technique are applied to estimate the integral terms arising in the derivative of the LKF, as shown in (25). It is well known that the Wirtinger-based integral inequality is tighter than the Jensen inequality used in [11, 13, 15], which means that the conservatism of the proposed criterion can be reduced. Moreover, the usage of the reciprocally convex combination avoids some enlargement treatments, such as $d(t)$ is directly enlarged to its upper bound h [4]. As a result, the conservatism of the proposed criterion is further reduced.

In order to easily demonstrate the contribution of delay-product-type terms, the following corollary is directly obtained from Theorem 1 by setting $P_i = 0, i = 1, 2, 3, 4$.

Corollary 1. *For given scalars K, h_1, h_2, μ_1, and μ_2, system (1) with the time-varying delay satisfying (2) and (3) is asymptotically stable if there exist $5n \times 5n$-matrix $P = P^T \geqslant 0$, $n \times n$-matrices $Q_i = Q_i^T > 0 (i = 1, 2, \cdots, 5)$, and $Z_1 = Z_1^T > 0$, and $Z_2 = Z_2^T > 0$, and any $2n \times 2n$-matrices X and Y such that LMIs (7) and (8) and the following condition hold*

$$\Phi_1 - \frac{1}{h_1} G_a^T \Omega_1 G_a - \frac{1}{h_1 + h_2} G_b^T \Omega_2 G_b < 0 \tag{30}$$

where the related notation are given in Theorem 1.

Remark 1. *In recent years, other integral inequalities, tighter than Wirtinger-based integral inequality, have been developed, for example, free-matrix-based inequality [27], auxiliary function-based inequalities [28, 29, 30, 31, 32, 33, 34], and Bessel-Legendre inequality [35], The further improved stability criteria can be obtained by combining those inequalities and the proposed delay-product-type LKF, which will be studied in future research.*

4. Example studies

This section gives a typical numerical example to show the less conservatism of the proposed criterion in comparison with the existing ones. Moreover, the proposed method is applied to analyze the delay-dependent stability of the load frequency control (LFC) for a singe area power system, which further shows the significant of our research.

4.1. A numerical example

The less conservatism of the proposed criterion is demonstrated by a numerical example. The key issue of delay-dependent stability analysis for system (1) is to find the maximal delay bound $h_i, i = 1, 2$ such that for all $d_i(t) \in [0, h_i], i = 1, 2$ system (1) is stable. The stability criteria obtained in the framework of the LKF and the LMI are usually sufficient conditions, which means that the calculated values ($h_{cal,i}, i = 1, 2$) are less than their analytical values ($h_{ana,i}, i = 1, 2$). Thus, the comparison of conservatism among different stability criteria is based on the results calculated based on different criteria, and the criterion providing bigger value has less conservatism.

Consider system (1) with the following parameters

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}.$$

This example is widely used for checking the conservatism of the stability criteria. Only the ones that consider the system with additive delays [4, 5, 7, 8, 17, 11, 41] are given for comparison in this paper. It is assumed that the bounds of the delay change rates are respective 0.1 and 0.8, i.e., $|\dot{d}_1(t)| \leq 0.1$ and $|\dot{d}_2(t)| \leq 0.8$. For given different upper bounds of $d_1(t)$, i.e., $h_1 \in \{1.0, 1.2, 1.5\}$, the upper bounds of $d_2(t)$ guaranteeing the stability of system calculated by Theorem 1 and Corollary 1 are listed in Table 1, where the results reported in other literature are also given for comparison. Note that '—' indicates that the results for corresponding cases are not reported in other literatures. It can be found that Theorem 1 can provide less conservative results than the existing criteria do. Moreover, it is found find the the results from Theorem 1 are bigger than those by Corollary 1, namely, Theorem 1 leads to a less conservatism result (i.e., calculated value is more closed to its analytical value), which means that the proposed delay-product-type terms indeed contribute to reduce the conservative of results.

Table 1: Upper bounds of $d_2(t)$ for given h_1 and upper bounds of $d_1(t)$ for given h_2

Criteria	Delay bound h_2 for given h_1			Delay bound h_1 for given h_2		
	$h_1 = 1.0$	$h_1 = 1.2$	$h_1 = 1.5$	$h_2 = 0.3$	$h_2 = 0.4$	$h_2 = 0.5$
[4]	0.415	0.376	0.248	1.324	1.039	0.806
[5]	0.512	0.406	0.283	1.453	1.214	1.021
[8]	0.519	0.453	0.378	—	—	—
[17]	0.596	0.463	0.313	1.532	1.313	1.140
[7]	0.872	0.672	0.371	1.572	1.472	1.372
[11]	0.873	0.673	0.373	1.573	1.473	1.373
[41]	0.982	0.782	0.482	1.682	1.582	1.482
Corollary 1	1.075	0.824	0.416	1.827	1.727	1.626
Theorem 1	1.163	0.965	0.669	1.875	1.773	1.671

4.2. Application to the stability analysis of LFC

Unlike most literature that only carries out numerical example study, the application of the proposed method to a single area LFC is also investigated. As mentioned in our previous work [36], the LFC scheme of singe area power systems, which is designed for maintaining the frequency at its required value, need transmit the frequency derivation from the remote power plant to the control center and send the calculated power reference signal from the control center to the power generation plant. The time delays arising in open communication channels may affect the performance of the LFC scheme [36]. Delay-dependent stability analysis and control design are investigated by using single delay to model all time delays arising in communication channels [36, 37, 39]. In fact, The time delays arising in the feedback measurement channel and those in the forward control channel may have different properties, and more useful guidelines can be obtained when considering such difference.

The basic diagram of the simplified LFC of single area power system is shown in Fig. 1, where e^{-sd_1} and e^{-sd_2} are time delays, respectively, arising during the measured signal Δf transmitted from sensor to the load frequency control center and the control signal u sent from the control center to the governor [38].

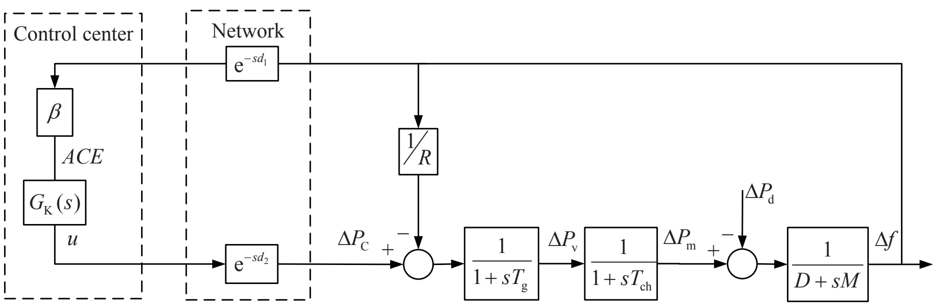

Figure 1: Diagram of the LFC for single area power system

The open-loop system can be expressed as follows

$$\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{B}\Delta P_C(t) \tag{31}$$

where

$$\bar{x}(t)=\begin{bmatrix}\Delta f\\ \Delta P_m\\ \Delta P_v\end{bmatrix},\quad \bar{y}(t)=ACE,\quad \bar{A}=\begin{bmatrix}-\frac{D}{M} & \frac{1}{M} & 0\\ 0 & -\frac{1}{T_{ch}} & \frac{1}{T_{ch}}\\ -\frac{1}{RT_g} & 0 & -\frac{1}{T_g}\end{bmatrix},\quad \bar{B}=\begin{bmatrix}\beta & 0 & 0\end{bmatrix}$$

and Δf, ΔP_m, and ΔP_v are the frequency deviation, the mechanical output change, and the valve position change, respectively; M and D are the moment of inertia of the generator and generator damping coefficient, respectively; T_g and T_{ch} are the time constant of the governor and the turbine, respectively; R is the speed drop; ΔP_C is the setpoint; and β is the frequency bias factor. The following PI controller is used as the LFC scheme:

$$u=-K_pACE-K_I\int ACE \tag{32}$$

where K_P and K_I are PI gains; and the ACE is the area control error. Due to the existence of time delays, $d_1(t)$ and $d_2(t)$, in feedback and forward channels, respectively, the following is obtained

$$\Delta P_C(t)=u(t-d_2(t)),\quad ACE(t)=\beta\Delta f(t-d_1(t)) \tag{33}$$

By defining virtual state and measurement output vectors as $x(t)=[\Delta f,\ \Delta P_m,\ \Delta P_v,\ \int ACE]^T$ and $y(t)=[ACE,\ \int ACE]^T$, respectively, and using (31)-(33), the closed-loop LFC system can be expressed as follows:

$$\dot{x}(t)=Ax(t)+A_dx(t-d_1(t)-d_2(t)) \tag{34}$$

where

$$x(t)=\begin{bmatrix}\Delta f\\ \Delta P_m\\ \Delta P_v\\ \int ACE\end{bmatrix},\quad A=\begin{bmatrix}-\frac{D}{M} & \frac{1}{M} & 0 & 0\\ 0 & -\frac{1}{T_{ch}} & \frac{1}{T_{ch}} & 0\\ -\frac{1}{RT_g} & 0 & -\frac{1}{T_g} & 0\\ \beta & 0 & 0 & 0\end{bmatrix},\quad A_d=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ -\frac{K_p\beta}{T_g} & 0 & 0 & -\frac{K_I}{T_g}\\ 0 & 0 & 0 & 0\end{bmatrix}$$

with the parameters given in [36]: $M=10$, $D=1$, $T_{ch}=0.3$, $T_g=0.1$, $R=0.05$, and $\beta=21$.

Based on the discussion of Section 3.2 and the similar procedure of [36], the bounds of time delays the system can tolerate for different cases can be obtained. Due to the page limitation, only the results for the case of $K_I=0.2$, $K_p=0.1$, $|\dot{d}_1(t)|\le 0.1$ and $|\dot{d}_2(t)|\le 0.8$ are given in Table 2. The results show that the proposed criterion can provide less conservative results compared with the ones reported in [36]. A simple simulation verification is carried out for this case: assume an increase step load of 0.1 pu happen at 1s and time-varying delays be $d_1(t)=\frac{1.5}{2}\sin\left(\frac{20}{1.5}x(t)\right)+\frac{1.5}{2}$ and $d_2(t)=\frac{5.683}{2}\sin\left(\frac{2.5}{5.683}x(t)\right)+\frac{5.683}{2}$ (satisfying $d_1(t)\le 1.5, d_2(t)\le 5.683, |\dot{d}_1(t)|\le 0.1, |\dot{d}_2(t)|\le 0.8$). The simulation results are shown in Fig. 2, in which the LFC has achieve its objective and the control system is stable, which verifies the effectiveness of the proposed method.

5. Conclusions

This paper has investigated the stability of linear systems with two additive time-varying delays. A delay-product-type LKF has been developed and its derivative has been estimated through Wirtinger-based inequality. Those techniques have led to a stability criterion with less conservatism in comparison with the existing criteria. Then the effect of the delays on system stability can be assessed accurately by using the proposed stability criterion. A numerical example and an application of the LFC have been used to demonstrate the advantages of proposed method.

Table 2: Upper bounds of $d_2(t)$ for given h_1 and upper bounds of $d_1(t)$ for given h_2

Criteria	h_1			h_2		
	1.0	1.2	1.5	2.0	3.0	4.0
[36]	4.803	4.603	4.303	3.803	2.803	1.803
Theorem 1	5.882	5.682	5.383	4.892	3.886	2.885

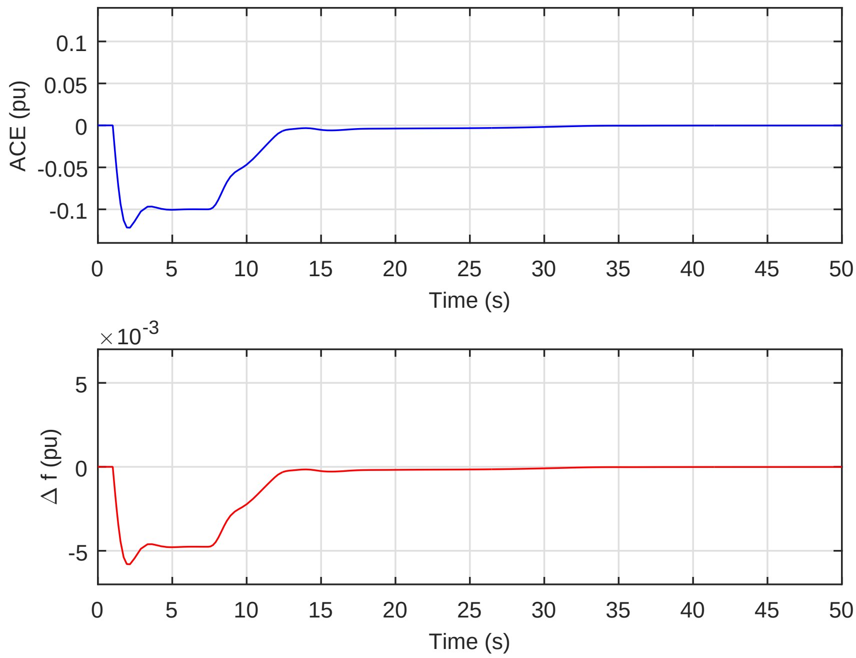

Figure 2: Frequency deviation and ACE of the LFC under a step load change (0.1 pu)

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