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Wilsonian Dark Matter in String Derived Z' Model

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Abstract

The dark matter issue is among the most perplexing in contemporary physics. The problem is more enigmatic due to the wide range of possible solutions, ranging from the ultra–light to the super–massive. String theory gives rise to plausible dark matter candidates due to the breaking of the non–Abelian Grand Unified Theory (GUT) symmetries by Wilson lines. The physical spectrum then contains states that do not satisfy the quantisation conditions of the unbroken GUT symmetry. Given that the Standard Model states are identified with broken GUT representations, and provided that any ensuing symmetry breakings are induced by components of GUT states, leaves a remnant discrete symmetry that forbid the decay of the Wilsonian states. A class of such states are obtained in a heterotic–string derived Z' model. The model exploits the spinor–vector duality symmetry, observed in the fermionic $Z_2 \times Z_2$ heterotic–string orbifolds, to generate a $Z' \in E_6$ symmetry that may remain unbroken down to low energies. The E_6 symmetry is broken at the string level with discrete Wilson lines. The Wilsonian dark matter candidates in the string derived model are $SO(10)$, and hence Standard Model, singlets and possess non–E_6 $U(1)_{Z'}$ charges. Depending on the $U(1)_{Z'}$ breaking scale and the reheating temperature they give rise to different scenarios for the relic abundance, and in accordance with the cosmological constraints.

1 Introduction

The Standard Model provides viable parameterisation of all experimental data at the subatomic scale. Alas, the Standard Model, and point quantum field theories in general, is not compatible with the gravitational interaction that accounts for observations at the celestial, galactic and cosmological scales. Furthermore, the Standard Model contains only a fraction of the stable matter required to explain the data at the galactic and cosmological scales.

String theory provides a viable framework for perturbative quantum gravity, and gives rise to the gauge and matter states that form the core of the Standard Model. Furthermore, these ingredients arise in string theory due to its internal consistency. String theory, therefore, provides a consistent framework to develop a phenomenological approach to quantum gravity. The phenomenological heterotic–string models constructed in the free fermionic formulation are among the most realistic string models constructed to date. These models are obtained in the vicinity of the self–dual point under T–duality, providing plausible symmetry arguments to explain their viability, and correspond to $Z_2 \times Z_2$ toroidal orbifolds, which are among the most symmetric and simplest string compactifications.

The dark matter conundrum is one of the most perplexing puzzles in contemporary observational data. The problem stems from the plethora of possible solutions and the lack of a clear preference for one or the other. Indeed the range of masses for potential candidates extend from 10^{59}GeV in the form of MACHOS [1] to 10^{-31}GeV in the form of ultra–light bosons [2]. It is prudent therefore to seek guidance from string theory. In particular, it is sensible to search for potential candidates in phenomenological string constructions.

String models contain in them the favoured dark matter candidates in the form of stable supersymmetric particles and of axion fields. However, stable supersymmetric dark matter requires reliance on a global symmetry, which ordinarily would be violated in string models [3], whereas recent observational data seem to disfavour a wide range of axion–like candidates [4]. Alternative dark matter candidates in string vacua exist in the form of hidden sector glueball dark matter [5] and Wilsonian dark matter candidates [6]. The latter category arises in string models due to the breaking of the non–Abelian GUT (Grand Unified Theory) gauge symmetries by Wilson lines. The physical spectrum in these string models contains states that do not satisfy the charge quantisation of the unbroken GUT gauge group [7]. Specifically, such states carry fractional charge with respect to some of the unbroken $U(1)$ generators of the original GUT symmetry. Some of these states may carry fractional electric charge, whereas others may carry standard charges under the Standard Model gauge group, but carry fractional charge with respect to an unbroken $U(1)_{Z'}$ gauge symmetry. States that carry fractional electric charge are stable by virtue of electric charge conservation. States that carry fractional charge under an unbroken $U(1)_{Z'}$ symmetry may be stable, depending on the charges of the states that are used to break the

$U(1)_{Z'}$ symmetry. Breaking $U(1)_{Z'}$ with Higgs states that carry the standard GUT charges under $U(1)_{Z'}$ results in a local discrete symmetry that forbids their decay to Standard Model states [6,8]. Such states may therefore be stable and be viable dark matter candidates. We dub such states as Wilsonian matter states due to the fact that they arise from the breaking of non–Abelian GUT symmetries by Wilson lines.

The possibility of Wilsonian matter states forming the dark matter was studied in ref. [6] for a variety of possible states, including fractionally charged states, strongly interacting states and Standard Model singlet states. The least constrained possibility takes into account the states that carry standard Standard Model charges, but carry fractional charge under a $U(1)_{Z'}$ gauge symmetry. The Wilsonian states investigated in ref. [6] arise from the symmetry breaking pattern $SO(10) \rightarrow SU(3) \times SU(2) \times U(1)^2$. However, the string derived models that utilise this symmetry breaking pattern do not contain the required Higgs states with standard GUT charges to break the $U(1)_{Z'}$ gauge symmetry [9]. Consequently, these models necessarily utilise Higgs states with fractional $U(1)_{Z'}$ charges to break the $U(1)_{Z'}$ along supersymmetric flat direction. More specifically, the state which is missing is the $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlet in the $\overline{16}$ representation of $SO(10)$. A scan of a large space of similar standard–like vacua may reveal the existence of models that do include the required states [10]. However, baring these new constructions, the string derived model that we discuss in this paper provides the first concrete example that realises the Wilsonian Standard Model singlet dark matter scenario.

The models under consideration are heterotic–string derived models that admit the symmetry breaking pattern $E_6 \rightarrow SO(10) \times U(1)_\zeta$, with anomaly free $U(1)_\zeta$, in which case $U(1)_\zeta$ can form part of a low scale $U(1)_{Z'}$ combination. This is not the case in most of the phenomenological heterotic–string derived models, in which $U(1)_\zeta$ is anomalous as a generic consequence of the symmetry breaking pattern $E_6 \rightarrow SO(10) \times U(1)_\zeta$. The construction of the heterotic–string derived model in ref. [11] utilises the spinor–vector duality that was observed in fermionic $Z_2 \times Z_2$ orbifolds [12, 13]. The duality operates under the exchange of the total number of $(16 \oplus \overline{16})$ spinorial $SO(10)$ representations with the total number of vectorial 10 representations. The models that admit an anomaly free $U(1)_\zeta$ gauge symmetry are self–dual under the spinor–vector duality. A particular class of models that are self–dual under the spinor–vector duality map are models in which the $SO(10) \times U(1)_\zeta$ symmetry is enhanced to E_6. In these models $U(1)_\zeta$ is anomaly free by virtue of its embedding in E_6. The total number $(16 \oplus \overline{16})$ is equal to the total number of 10 representations due to the fact that the 27 and $\overline{27}$ representations of E_6 contain $16 + 10$ and $\overline{16} + 10$, respectively. Hence, E_6 are self–dual under the exchange of the total number of $(16 \oplus \overline{16})$ and the total number of 10 representations. However, there exist also self–dual models in which the $SO(10) \times U(1)_\zeta$ gauge symmetry is not enhanced to E_6. This is possible if the spinorial and vectorial states are obtained from different fixed points of the $Z_2 \times Z_2$ orbifold.

The string derived Z' model of ref. [11] is constructed by first selecting a spinor–

vector self–dual model at the $SO(10)$ level and subsequently breaking the $SO(10)$ symmetry to the Pati–Salam subgroup. The chiral spectrum of the resulting Pati–Salam string model respects the self–duality under the spinor–vector duality. Effectively, the result is that the chiral spectrum forms complete E_6 multiplets and consequently $U(1)_\zeta$ is anomaly free.

An unexpected result that was obtained in ref. [11] is with respect to the type of exotic states that appear in the model. Using the trawling algorithm developed for the classification of free fermionic $Z_2 \times Z_2$ orbifolds [14–18], we fish out a model in which all the exotic fractionally charged states are projected out from the massless spectrum, and appear only as massive states [16]. Such models are dubbed exophobic string models. Therefore, the model does not contain any massless states with fractional charges with respect to the $SO(10)$ subgroup. However, the model contains exotic states with respect to the E_6 subgroup, *i.e.* the model contains states that are $SO(10)$ singlets and carry fractional non–E_6 charge under $U(1)_\zeta$. It is noted that as the gauge symmetries are realised in this model, as level one Kac–Moody algebras the aforementioned exotic charges cannot arise from higher order E_6 representations. Furthermore, the model does contain the required standard E_6 states to break $U(1)_\zeta$ along flat directions. The model of ref [11] therefore, and for the first time, precisely realises the Wilsonian Standard Model singlet dark matter scenario alluded to in ref. [6].

Our paper is organised as follows: in section 2 we discuss and classify the type of exotic states that arise in the phenomenological fermionic $Z_2 \times Z_2$ models. We discuss the structure of the models and their construction. In section 3 we elaborate on the exotic E_6 states that are obtained in the Z' model of ref. [11]. In section 4 we investigate the exotic E_6 states as dark matter candidates, taking into account low and high scale $U(1)_{Z'}$ breaking as well as scenarios with and without inflation. Section 5 concludes our paper.

2 Wilsonian states

The class of string models under consideration are constructed in the free fermionic formulation [19]. The four dimensional heterotic–string in the light–cone gauge requires 20 left–moving, and 44 right–moving, real fermions propagating on the string worldsheet. The sixty–four worldsheet fermions are typically denoted by $\{\psi^{1,2}, (\chi, y, \omega)^{1,\cdots,6} | (\bar{y}, \bar{\omega})^{1,\cdots,6}, \bar{\psi}^{1,\cdots,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,\cdots,8}\}$, where 32 of the right–moving real fermions are grouped into 16 complex fermions that produce the Cartan generators of a rank 16 gauge group. Here $\bar{\psi}^{1,\cdots,5}$ are the Cartan generators of the $SO(10)$ GUT group and $\bar{\phi}^{1,\cdots,8}$ are the Cartan generators of the rank eight hidden sector gauge group. The three complex worldsheet fermions $\bar{\eta}^{1,2,3}$ generate three Abelian currents, $U(1)_{1,2,3}$, in the Cartan subalgebra of the four dimensional gauge group with $U(1)_\zeta$

being their linear combination

$$U(1)_\zeta \;=\; U(1)_1 + U(1)_2 + U(1)_3 \;. \tag{2.1}$$

The worldsheet fermions pick up a phase under parallel transport around one of the non–contractible loops of the vacuum to vacuum torus amplitude. These phases are encoded in forty–eight dimensional vectors $(20_{l.r.} + 12_{r.r.} + 32_{r.c.})$,

$$v = \{v(f_1), \cdots, v(f_{20}) | v(\bar{f}_1), \cdots, v(\bar{f}_{48})\}.$$

Invariance under modular transformations of the one–loop vacuum to vacuum amplitude leads to a set of constrains on the phase assignments. Summation over all the allowed phases, with appropriate phases to render the sum modular invariant, generates the partition function. The string vacua in the free fermionic formulation are obtained by specifying a set of boundary condition basis vectors, $B = \{v_1, v_2, v_3, \cdots\}$, and the one–loop summation phases in the partition function $c\left[\begin{smallmatrix} v_i \\ v_j \end{smallmatrix}\right]$. The basis set spans a space Ξ, which consists of all possible linear combinations of the basis vectors $\Xi = \sum_k n_k v_k$, where $n_k = 0, \cdots, N_{v_k} - 1$, and N_{v_k} denote the order of each of the basis vectors. The physical states in the Hilbert space of a given sector $\xi \in \Xi$ are obtained by acting on the vacuum with fermionic and bosonic oscillators and by imposing the Generalised GSO (GGSO) projections. The $U(1)$ charges with respect to the Cartan generators of the four dimensional gauge group are given by

$$Q(f) = \frac{1}{2}\xi(f) + F_\xi(f),$$

where $\xi(f)$ is the boundary condition of the complex worldsheet fermion f in the sector ξ, and $F_\xi(f)$ is a fermion number operator [19]. The phenomenological properties of the models are extracted by calculating tree-level and higher order terms in the superpotential and by analysing its flat directions. It is important to note that the free fermionic models correspond to toroidal $Z_2 \times Z_2$ orbifolds at special points in the moduli space [20]. Moduli deformations of the six dimensional internal torus are incorporated in the fermionic construction in terms of worldsheet Thirring interactions that are consistent with the transformation properties of the worldsheet fermions.

Early examples of quasi–realistic free fermionic models corresponded to the so–called NAHE–based models. The first set of five basis vectors, dubbed the NAHE–set [21], is common in all these phenomenological models, and the models vary by the addition of three or four basis vectors beyond the NAHE–set. Three generation models with $SU(5)\times U(1)$ (FSU5) [22]; $SO(6)\times SO(4)$ (SO64) [23]; $SU(3)\times SU(2)\times U(1)^2$ (SLM) [9]; and $SU(3) \times U(1) \times SU(2)^2$ (LRS) [24] $SO(10)$ subgroup were obtained, whereas the case with $SU(4) \times SU(2) \times U(1)$ was shown not to produce viable models [25]. In more recent years systematic methods were developed for the classification of large spaces of fermionic $Z_2 \times Z_2$ heterotic–string vacua. The classification methodology uses an appropriate fixed set of boundary condition basis vectors and the space of vacua is spanned by varying the GGSO projection coefficients. In this manner models with unbroken $SO(10)$ gauge group were classified [15], which led to the observation of the spinor–vector duality [12], as well as models with SO64 [16] and FSU5 [17] $SO(10)$ subgroups. Classification of SLM and LRS models is currently underway and will be reported in future publications.

The construction of free fermionic models, in either the older trial and error method, or the more recent systematic classification method, can be viewed in two stages. The first part consist of the basis vectors that preserve the $SO(10)$ symmetry. The construction at this stage produces vacua with $(2,0)$ worldsheet supersymmetry, $N=1$ spacetime supersymmetry, and a number of spinorial and vectorial representations of $SO(10)$. The second part consist of the inclusion of the basis vectors that break the $SO(10)$ symmetry to a subgroup.

Correspondingly, the sectors in a free fermionic heterotic–string model can be divided into those that preserve the $SO(10)$ symmetry and those that do not. Physical states that arise from sectors that preserve the $SO(10)$ symmetry correspond to states that may be identified with Standard Model states, or are Standard Model singlets. Sectors that break the $SO(10)$ symmetry produce exotic states, *i.e.* they produce states that carry fractional charge under $U(1)_{\rm e.m.}$ or under $U(1)_{Z'} \in SO(10)$. The exotic states can be further classified according to the $SO(10)$ symmetry breaking pattern in the sector from which they arise.

The Cartan subalgebra of the observable gauge group in the free fermionic models is generated by the complex worldsheet fermions $\{\bar\psi^{1,\cdots,5},\bar\eta^{1,2,3}\}$, with $\bar\psi^{1,\cdots,5}$ producing those of $SO(10)$ and its subgroups and $\bar\eta^{1,2,3}$ producing three $U(1)$ currents. The $SO(10)$ symmetry is broken to one of its subgroups by the following assignments:

$$1.\quad b\{\bar\psi^{1\cdots5}_{\frac{1}{2}}\} = \{\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\} \Rightarrow SU(5)\times U(1), \tag{2.2}$$

$$2.\quad b\{\bar\psi^{1\cdots5}_{\frac{1}{2}}\} = \{1\ 1\ 1\ 0\ 0\} \Rightarrow SO(6)\times SO(4). \tag{2.3}$$

To break the $SO(10)$ symmetry to $SU(3)_C\times SU(2)_L\times U(1)_C\times U(1)_L$ [9] both steps, 1 and 2, are used, in two separate basis vectors[1]. The breaking pattern $SO(10)\rightarrow SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ [24] is obtained with:

$$3.\quad b\{\bar\psi^{1\cdots5}_{\frac{1}{2}}\} = \{\frac{1}{2}\frac{1}{2}\frac{1}{2}00\} \Rightarrow SU(3)_C\times U(1)_C\times SU(2)_L\times SU(2)_R, \tag{2.4}$$

and the breaking pattern $SO(10)\rightarrow SU(4)_C\times SU(2)_L\times U(1)_R$ [25] results from:

$$4.\quad b\{\bar\psi^{1\cdots5}_{\frac{1}{2}}\} = \{000\frac{1}{2}\frac{1}{2}\} \Rightarrow SU(4)_C\times SU(2)_L\times U(1)_R. \tag{2.5}$$

It was shown that the breaking pattern (2.5) does not produce viable models [25].

[1] $U(1)_C = \frac{3}{2}U(1)_{B-L}; U(1)_L = 2U(1)_{T_{3_R}}$.

The states in the free fermionic models that carry exotic charges with respect the Abelian generators of the $SO(10)$ subgroups can be classified according to the $SO(10)$ symmetry breaking pattern in the sectors from which they arise. A basis vector combination that produces exotic states contains in it the $SO(10)$ breaking basis vectors. We focus here on the case of the standard–like models that contain both of the assignments shown in (2.2) and in (2.3) and therefore contain the exotic states that arise in the FSU5 and the SO64 models, as well as those that arise solely in the SLM models. In the following we adapt the notation

$$[(SU(3)_C \times U(1)_C); (SU(2)_L \times U(1)_L)]_{(Q_Y, Q_{Z'}, Q_{\text{e.m.}})}, \tag{2.6}$$

to denote the charges of the states arising in the exotic sectors. Here $U(1)_C$ and $U(1)_L$ are defined in terms of the worldsheet charges by

$$Q_C = Q(\bar{\psi}^1) + Q(\bar{\psi}^2) + Q(\bar{\psi}^3) \text{ and } Q_L = Q(\bar{\psi}^4) + Q(\bar{\psi}^5). \tag{2.7}$$

The FSU5 $U(1)$ combinations are given by

$$U(1)_5 = \frac{1}{3}U(1)_C - \frac{1}{2}U(1)_L \in SU(5) \tag{2.8}$$

$$U(1)_{\tilde{5}} = U(1)_C + U(1)_L \notin SU(5) \tag{2.9}$$

whereas the weak hypercharge and Z' charges in eq. (2.6) are given by

$$U(1)_Y = \frac{1}{3}U(1)_C + \frac{1}{2}U(1)_L = \frac{2}{5}U(1)_5 - \frac{1}{5}U(1)_{\tilde{5}} \tag{2.10}$$

$$U(1)_{Z'} = U(1)_C - U(1)_L = \frac{12}{5}U(1)_5 + \frac{1}{5}U(1)_{\tilde{5}}. \tag{2.11}$$

$U(1)_C$ and $U(1)_L$ are similarly defined in the SO64 models. The electromagnetic charge is given by

$$U(1)_{\text{e.m.}} = T_{3_L} + U(1)_Y. \tag{2.12}$$

Using the notation in eq. (2.6) the Standard Model matter states carry the charges

$$e_L^c \equiv [(1, \frac{3}{2}); (1, 1)]_{(1,1/2,1)}; \tag{2.13}$$

$$u_L^c \equiv [(\bar{3}, -\frac{1}{2}); (1, -1)]_{(-2/3,1/2,-2/3)}; \tag{2.14}$$

$$Q \equiv [(3, \frac{1}{2}); (2, 0)]_{(1/6,1/2,(2/3,-1/3))}; \tag{2.15}$$

$$N_L^c \equiv [(1, \frac{3}{2}); (1, -1)]_{(0,5/2,0)}; \tag{2.16}$$

$$d_L^c \equiv [(\bar{3}, -\frac{1}{2}); (1, 1)]_{(1/3,-3/2,1/3)}; \tag{2.17}$$

$$L \equiv [(1, -\frac{3}{2}); (2, 0)]_{(-1/2,-3/2,(0,1))}, \tag{2.18}$$

and arise from spinorial 16 representations of $SO(10)$, *i.e.* they arise from sectors that preserve the $SO(10)$ gauge symmetry. Similarly, the light Higgs electroweak doublets are obtained from $SO(10)$ vectorial representations, and arise in sectors that preserve the $SO(10)$ symmetry. By contrast the exotic states arise in sectors that break the $SO(10)$ symmetry and can be classified according to the $SO(10)$ symmetry breaking pattern in each sector. Sectors that break the $SO(10)$ symmetry to the FSU5 subgroup contain in them the assignment in eq. (2.2) and produce the states

$$[(3,-\frac{1}{4});(1,\ \frac{1}{2})]_{(1/6,-3/4,1/6)} \quad ; \quad [(\bar{3},\ \frac{1}{4});(1,-\frac{1}{2})]_{(-1/6,\ 3/4,-1/6)}, \quad (2.19)$$

$$[(1,\ \frac{3}{4});(2,-\frac{1}{2})]_{(0,5/4,\pm1/2)} \quad ; \quad [(1,-\frac{3}{4});(2,\ \frac{1}{2})]_{(0,-5/4,\pm1/2)}, \quad (2.20)$$

$$[(1,\ \frac{3}{4});(1,\ \frac{1}{2})]_{(1/2,1/4,\ 1/2)} \quad ; \quad [(1,-\frac{3}{4});(1,-\frac{1}{2})]_{(-1/2,-1/4,-1/2)}. \quad (2.21)$$

Sectors that break the $SO(10)$ symmetry to the SO64 subgroup contain in them the assignment in eq. (2.3) and produce the states

$$[(3,\ \frac{1}{2});(1,\ 0)]_{(1/6,\ 1/2,\ 1/6)} \quad ; \quad [(\bar{3},-\frac{1}{2});(1,\ 0)]_{(-1/6,-1/2,-1/6)}, \quad (2.22)$$

$$[(1,\ \frac{3}{2});(1,\ 0)]_{(1/2,\ 3/2,\ 1/2)} \quad ; \quad [(1,-\frac{3}{2});(1,\ 0)]_{(-1/2,-3/2,-1/2)}, \quad (2.23)$$

$$[(1,\ 0);(1,\ 1)]_{(1/2,\ -1,\ 1/2)} \quad ; \quad [(1,\ 0);(1,\ -1)]_{(-1/2,\ 1,-1/2)}, \quad (2.24)$$

$$[(1,\ 0);(2,\ 0)]_{(0,\ 0,\ \pm1/2)} \quad . \quad (2.25)$$

Sectors that break the $SO(10)$ symmetry to the SLM subgroup contain a linear combination of both assignments in eq.(2.2) and eq. (2.3). These sectors produce states that carry standard GUT charges with respect to the Standard Model gauge group but carry fractional non–GUT charges with respect to the $U(1)_{Z'}$ combination in eq. (2.11):

$$[(3,-\frac{1}{4});(1,-\frac{1}{2})]_{(-1/3,\ 1/4,-1/3)} \quad ; \quad [(\bar{3},\ \frac{1}{4});(1,\ \frac{1}{2})]_{(1/3,-1/4,\ 1/3)}, \quad (2.26)$$

$$[(1,\ \frac{3}{4});(2,-\frac{1}{2})]_{(1/2,\ 3/4,\ (1,0))} \quad ; \quad [(1,-\frac{3}{4});(2,\ \frac{1}{2})]_{(-1/2,-3/4,(0,-1))}, \quad (2.27)$$

$$[(1,\ \frac{3}{4});(1,-\frac{1}{2})]_{(0,\ 5/4,\ 0)} \quad ; \quad [(1,-\frac{3}{4});(1,-\frac{1}{2})]_{(0,-5/4,\ 0)}. \quad (2.28)$$

The exotic states appearing in eqs. (2.26,2.27, 2.28) may therefore produce viable dark matter candidates. This would be the case if the heavy Higgs states that break $U(1)_{Z'}$ carry the standard GUT charges with respect to $U(1)_{Z'}$. In that case a remnant discrete symmetry forbids the formation of unsuppressed terms that can lead to decay of the exotic states to the Standard Model states [6]. In the FSU5 and SO64 heavy Higgs states necessarily arise from GUT representation in order to break the remaining non–Abelian symmetry. However, in the SLM models this need not be the case. The remnant unbroken symmetry $U(1)_{Z'}$ of eq. (2.11) can be broken by heavy Higgs states with the standard GUT charges of eq. (2.16) and its conjugate $\bar{N}^c_L$, or by using the exotic states and charges in eq. (2.28). In the SLM models of [9], a state with the quantum numbers of $\bar{N}^c_L$ does not appear in the massless spectrum. Consequently, in these models, breaking $U(1)_{Z'}$ and preserving supersymmetry at a high scale forces the exotic states in eq. (2.28) to get a non–trivial VEV at the high scale. Alternatively, we may contemplate that either the $U(1)_{Z'}$ symmetry is broken at the low scale, or that supersymmetry is broken at the high scale. Suppression of left–handed neutrino masses by the seesaw mechanism disfavours the first possibility. Breaking supersymmetry at the high scale also introduces a plethora of new questions that we do not consider in this paper. The upshot is that the Wilsonian singlet dark matter scenario of [6], with the type singlets in eq. (2.28), is not realised in the existing SLM heterotic–string free fermionic models.

3 E_6 Wilsonian states in Z' string model

We next turn to discuss the Wilsonian matter states in the string derived model of ref. [11]. The $SO(10)$ symmetry is broken in this model to the Pati–Salam subgroup. The chiral spectrum of the model forms complete E_6 multiplets. Consequently, the $U(1)_\zeta$ combination, which possesses the embedding $SO(10) \times U(1)_\zeta \in E_6$ is anomaly free. The complete massless spectrum of the model, and its charges under the four dimensional gauge group, is given in ref. [11]. Here a glossary of the states in the model and the charges under the $SU(4) \times SO(4) \times U(1)_\zeta$ are shown in tables 1 and 2. The heterotic–string model in ref. [11] is an exophobic Pati–Salam model. The type of massless exotic states that can appear in this model are those in eqs. (2.22, 2.25, 2.23, 2.24). However, none of these states appear in the massless spectrum. In fact, none of the exotic states discussed in section 2 appear in this model.

The model contains, however, a new type of exotic states. These exotic states carry standard $SO(10)$ charges and are in fact $SO(10)$ singlets. They are exotic with respect to $U(1)_\zeta$ as they carry 1/2 of the charge of the standard $SO(10)$ singlets in the 27 and $\overline{27}$ representations of E_6. Inspection of table 1 shows that the exotic states of this type are $\{\phi_{1,2}, \bar{\phi}_{1,2}\}$, which are also singlets of the rank 8 hidden sector gauge group. The $SO(10)$ singlet states H^+ and H^- in table 2 carry similar $U(1)_\zeta$ charges and transform under the hidden sector gauge group. These exotic states arise in the string models due to the breaking of the E_6 symmetry by Wilson lines. However, as the Wilson line is realised in the free fermionic construction in terms of a GGSO phase, its precise identification is not a simple exercise. Its imprint is revealed due to the exotic charges, which will not have been generated otherwise. Furthermore, we note from table 1 that the string model does contain the $SO(10)$ singlet states S and $\overline{S}$, with standard E_6 charges to break $U(1)_\zeta$ along flat directions. Therefore, this

Symbol	Fields in [11]	$SU(4)\times SU(2)_L\times SU(2)_R$	$U(1)_\zeta$
F_L	F_{1L}, F_{2L}, F_{3L}	$(\mathbf{4},\mathbf{2},\mathbf{1})$	$+\frac{1}{2}$
F_R	F_{1R}	$(\mathbf{4},\mathbf{1},\mathbf{2})$	$-\frac{1}{2}$
$\bar{F}_R$	$\bar{F}_{1R}, \bar{F}_{2R}, \bar{F}_{3R}, \bar{F}_{4R}$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$	$+\frac{1}{2}$
h	h_1, h_2, h_3	$(\mathbf{1},\mathbf{2},\mathbf{2})$	-1
Δ	$D_1,\dots,D_7$	$(\mathbf{6},\mathbf{1},\mathbf{1})$	-1
$\bar{\Delta}$	$\bar{D}_1, \bar{D}_2, \bar{D}_3, \bar{D}_6$	$(\mathbf{6},\mathbf{1},\mathbf{1})$	$+1$
S	$\Phi_{12}, \Phi_{13}, \Phi_{23}, \chi_1^+, \chi_2^+, \chi_3^+, \chi_5^+$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$+2$
$\bar{S}$	$\bar{\Phi}_{12}, \bar{\Phi}_{13}, \bar{\Phi}_{23}, \bar{\chi}_4^+$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	-2
ϕ	ϕ_1, ϕ_2	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$+1$
$\bar{\phi}$	$\bar{\phi}_1, \bar{\phi}_2$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	-1
ζ	$\Phi_{12}^-, \Phi_{13}^-, \Phi_{23}^-, \bar{\Phi}_{12}^-, \bar{\Phi}_{13}^-, \bar{\Phi}_{23}^-$	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0
	$\chi_1^-, \chi_2^-, \chi_3^-, \bar{\chi}_4^-, \chi_5^-$		
	$\zeta_i, \bar{\zeta}_i, i=1,\dots,9$		
	$\Phi_i, i=1,\dots,6$		

Table 1: Observable sector field notation and associated states in [11].

Symbol	Fields in [11]	$SU(2)^4\times SO(8)$	$U(1)_\zeta$
H^+	H_{12}^3	$(\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	$+1$
	H_{34}^2	$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1})$	$+1$
H^-	H_{12}^2	$(\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	-1
	H_{34}^3	$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1})$	-1
H	H_{12}^1	$(\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	0
	$H_{13}^i, i=1,2,3$	$(\mathbf{2},\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1})$	0
	$H_{14}^i, i=1,2,3$	$(\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1})$	0
	H_{23}^1	$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1})$	0
	H_{24}^1	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{2},\mathbf{1})$	0
	$H_{34}^i, i=1,4,5$	$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1})$	0
Z	$Z_i, i=1,\dots,$	$(\mathbf{1},\mathbf{1},\mathbf{8})$	0

Table 2: Hidden sector field notation and associated states in [11].

model can realise the Wilsonian singlet dark matter scenario articulated in ref. [6]. In the next section we turn to examine this question.

4 $SO(10)$ singlet Wilsonian dark matter

The low energy spectrum of the string model consists of the states in table 3, where we also allow for the possibility of completely neutral states that may correspond to light hidden sector states.

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{Z'}$
Q_L^i	3	2	$+\frac{1}{6}$	$-\frac{2}{5}$
u_L^i	$\bar{3}$	1	$-\frac{2}{3}$	$-\frac{2}{5}$
d_L^i	$\bar{3}$	1	$+\frac{1}{3}$	$-\frac{4}{5}$
e_L^i	1	1	$+1$	$-\frac{2}{5}$
L_L^i	1	2	$-\frac{1}{2}$	$-\frac{4}{5}$
D^i	3	1	$-\frac{1}{3}$	$+\frac{4}{5}$
$\bar{D}^i$	$\bar{3}$	1	$+\frac{1}{3}$	$+\frac{6}{5}$
H^i	1	2	$-\frac{1}{2}$	$+\frac{6}{5}$
$\bar{H}^i$	1	2	$+\frac{1}{2}$	$+\frac{4}{5}$
S^i	1	1	0	-2
$\mathcal{D}$	3	1	$-\frac{1}{3}$	$+\frac{4}{5}$
$\bar{\mathcal{D}}$	$\bar{3}$	1	$+\frac{1}{3}$	$-\frac{4}{5}$
ϕ	1	1	0	-1
$\bar{\phi}$	1	1	0	$+1$
ζ^i	1	1	0	0

Table 3: *Spectrum and $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Z'}$ quantum numbers, with $i = 1,2,3$ for the three light generations. The charges are displayed in the normalisation used in free fermionic heterotic–string models.*

The trilinear superpotential embedding the supersymmetric Z' model and the relevant interactions of the ϕ and $\bar{\phi}$ is

$$\begin{aligned} \mathcal{W} &= {Y^u}_{i,j,k} \bar{u}_R^i \, Q_L^j \, \bar{H}^k - {Y^d}_{i,j,k} \bar{d}_R^i \, Q_L^j \, H^k - {Y^e}_{i,j,k} \bar{e}_R^i \, L_L^j \, H^k \\ &+ \lambda_{i,j,k} \, S^i \, H^j \, \bar{H}^k + \kappa_{i,j,k} \, S^i \, \bar{D}^j \, D^k + {\lambda^s}_i S^i \, \bar{\phi} \, \bar{\phi} \ . \end{aligned} \quad (4.1)$$

As seen from eq. (4.1) there are no terms that allow for the $\phi, \bar{\phi}$ states to decay to lighter states at leading order. Breaking the $U(1)_{Z'}$ symmetry with the VEV of S^i leaves a remnant discrete symmetry [6, 8] which forbids their decay at any order in the superpotential. A potential mass term for $\bar{\phi}$ arises in the trilinear superpotential, eq. (4.1). Additional mass terms, that are invariant under all gauge and discrete symmetries, may be generated from higher order terms. Therefore, the Wilsonian matter states, namely the fermionic component arising from the $\phi, \bar{\phi}$ states, in the string derived Z' model are heavy and stable. Their mass density may overclose the universe if they are over abundant. We refer to these states as Wilsonian singlet dark matter, or W_s for short.

The Wilsonian singlet interacts with the non–exotic states in table 3 only via the Z' gauge charges. Its number density can change only by annihilations via the diagrams in figures 1 and 2 into fermions and their superpartners, and, depending on the $U(1)_{Z'}$ symmetry breaking scale, into the gauge bosons and their superpartners, as in figures 3 and 4.

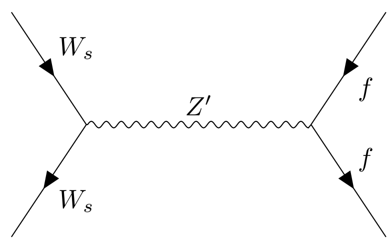

Figure 1: $W_s\bar{W}_s \longrightarrow f\bar{f}$ *decay.*

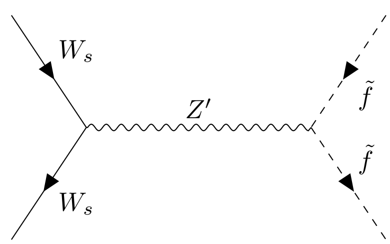

Figure 2: $W_s\bar{W}_s \longrightarrow \tilde{f}\tilde{f}^*$ *decay.*

The annihilation into fermion and sfermion in figures 1 and 2 leads to the two contributions

$$\sigma_{W_sW_s\to ff} = \frac{4\,\pi}{3}\frac{N_{Z'}\sqrt{S}\,\left(2\,M_{W_s}^2+S\right)}{\sqrt{S-4\,M_{W_s}^2}\,\left(M_{Z'}^2-S\right)^2} \tag{4.2}$$

and

$$\sigma_{W_sW_s\to \tilde{f}\tilde{f}} = \frac{\pi}{3}\frac{N_{Z'}\sqrt{S}\,\left(2\,M_{W_s}^2+S\right)}{\sqrt{S-4\,M_{W_s}^2}\,\left(M_{Z'}^2-S\right)^2} \tag{4.3}$$

where we defined the parameter

$$N_{Z'} = \frac{g'^4}{16\,\pi^2}\,Q_f^2\,Q_{W_s}^2 \tag{4.4}$$

with Q_i and g' being the $U(1)_{Z'}$ charge and the corresponding coupling constant, respectively. The $N_{Z'}$ parameter accounts for the strength of a Z' exchange. The total cross-section for the annihilation into fermions and sfermions $W_s\bar{W}_s \to f\bar{f}, \tilde{f}_L\bar{\tilde{f}}_L, \tilde{f}_R\bar{\tilde{f}}_R$ is now easily reached

$$\sigma_{W_sW_s\to ff,\tilde{f}\tilde{f}} = \sigma_{W_sW_s\to ff} + 2\,\sigma_{W_sW_s\to\tilde{f}\tilde{f}} = \frac{2\pi N_{Z'}\sqrt{S}\,\left(2\,M_{W_s}^2 + S\right)}{\sqrt{S - 4\,M_{W_s}^2}\,\left(M_{Z'}^2 - S\right)^2} \tag{4.5}$$

where the contribution of (4.3) has been doubled to account for the separate events $W_s\bar{W}_s \to \tilde{f}_L\bar{\tilde{f}}_L$ and $W_s\bar{W}_s \to \tilde{f}_R\bar{\tilde{f}}_R$ which have identical cross-sections.

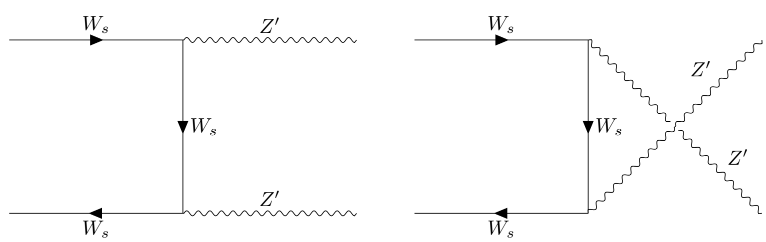

Figure 3: $W_s\bar{W}_s \longrightarrow Z'Z'$ *decay.*

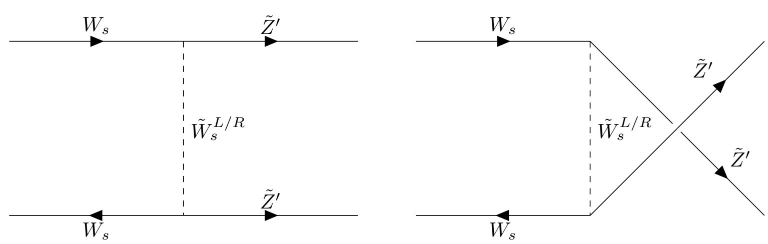

Figure 4: $W_s\bar{W}_s \longrightarrow \tilde{Z}'\tilde{Z}'$ *decay.*

The computation of the annihilations into vector bosons in figure 3 and into their superpartners in figure 4 leads to the cross-sections

$$\sigma_{W_sW_s\to Z'Z'} = 4\,\pi\,\tilde{N}_{Z'}\frac{\left(S^2 + 4M_{W_s}^2\,S - 8\,M_{W_s}^4\right)\ln\left(\frac{S+A}{S-A}\right) - A\left(S + 4M_{W_s}^2\right)}{A^2 S} \tag{4.6}$$

and

$$\sigma_{W_sW_s\to\tilde{Z}'\tilde{Z}'} = 2\,\pi\,\tilde{N}_{Z'}\frac{A - 2M_{W_s}^2\ln\left(\frac{S+A}{S-A}\right)}{A^2} \tag{4.7}$$

where we have defined $\tilde{N}_{Z'}$ as

$$\tilde{N}_{Z'} = \frac{g'^4}{16\,\pi^2}\,Q^4_{W_s} \tag{4.8}$$

and the kinematical parameter $A = \sqrt{S(S-4M^2_{W_s})}$. Since W_s is heavy and stable, its mass density is constrained by the requirement that it does not overclose the universe. Alternatively, we may extract the regions of parameter space where the non–baryonic dark matter abundance can be explained in terms of the Wilsonian singlet dark matter. After $U(1)_{Z'}$ symmetry breaking the W_s interactions are suppressed by $1/M^2_{Z'}$ and it can be classified as weakly interacting massive particle. It decouples from the thermal bath when its annihilation rate falls behind the expansion rate of the universe. The annihilation rate of a particle is

$$\Gamma = \langle\sigma_{\rm ann}|v|\rangle n_{_{EQ}}, \tag{4.9}$$

where the number density at the equilibrium, $n_{_{EQ}}$, is given by

$$n_{_{EQ}} = \begin{cases} g_{\rm eff}\left(\dfrac{\zeta(3)}{\pi^2}\right)T^3 & \text{relativistic} \\ g_{\rm eff}\left(\dfrac{mT}{2\pi}\right)^{3/2}\exp(-M/T) & \text{non-relativistic} \end{cases} \tag{4.10}$$

and $\zeta(3) = 1.20206$ is the Riemann zeta function of 3, and $g_{\rm eff}$ is the effective number of degrees of freedom of the particle. In the expanding universe the evolution equation of the number density is described by the Boltzmann equation [27]. In terms of the number density in a comoving volume $Y = n/s_{\rm e}$, where $s_{\rm e}$ is the entropy density, and the dimensionless parameter $x = M/T$, the Boltzmann equation takes the form

$$\frac{dY}{dx} = -\lambda x^{-2}\left(Y^2 - Y^2_{EQ}\right)\,, \tag{4.11}$$

with λ related to the interaction rate of the particle through

$$\lambda = 0.26 < \sigma_{ann}|v| > M_{W_s} m_{Pl}\frac{g_{*s}}{\sqrt{g_*}} \tag{4.12}$$

with Y_{EQ} the density at thermal equilibrium and m_{Pl} the Planck Mass. In (4.12) the variables g_* and g_{*s} count, for different purposes, the effective relativistic degrees of freedom at a given T

$$\begin{aligned} g_* &= \sum_{i=bosons} g_i\left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_{i=fermions} g_i\left(\frac{T_i}{T}\right)^4\,, \\ g_{*s} &= \sum_{i=bosons} g_i\left(\frac{T_i}{T}\right)^3 + \frac{7}{8}\sum_{i=fermions} g_i\left(\frac{T_i}{T}\right)^3\,, \end{aligned} \tag{4.13}$$

where g_i are the complete internal degrees of freedom and T_i the temperature of the given particle i. The annihilation cross-section σ_{ann} determines the evolution of the relic density via its thermal average, $<\sigma_{ann}|v|>$, and its computation must be performed to calculate the relic abundance left by W_s. We can distinguish different scenarios for the Wilsonian singlet dark matter, depending on the Z' symmetry breaking scale, $M_{Z'}$, the mass scale of W_s, $M_{W_s} \equiv M$, and the reheating temperature, T_R, following inflation. The Z' breaking scale can vary from the experimental LHC mass limits, of the order of a few TeV, up to the Planck scale. The constraints on the W_s mass can vary from being ultra light [2] to being super–massive [6,28], depending on the Z' breaking scale, and the reheating temperature. There are several possible dark matter scenarios for the Wilsonian singlet W_s:

- $M_{W_s} >> M_{Z'}$ **without inflation.** In this case the Wilsonian singlet is strongly interacting in the early universe and remains in thermal equilibrium until it becomes non–relativistic. The solution of the Boltzmann equation leads to a density value of

$$Y_0 = \frac{3.79}{m_{Pl}\, T_{dec}\, <\sigma_{ann}|v|>} \left(\frac{\sqrt{g_*}}{g_{*s}}\right) . \tag{4.14}$$

In this scenario all the annhilation channels depicted in figures 1, 2, 3, 4 are open. The s-wave expansion of the Boltzmann equation [27] yields the thermal average of (4.5), (4.6) and (4.7) as:

$$<\sigma_{W_s W_s \to f f, \tilde{f}\tilde{f}}|v|> = \frac{8\pi N_{Z'}}{3\, M^2_{W_s}} + O(v^2) \tag{4.15}$$

and

$$<\sigma_{W_s W_s \to Z' Z'}|v|> + <\sigma_{W_s W_s \to \tilde{Z}'\tilde{Z}'}|v|> = \frac{4\pi \tilde{N}_{Z'}}{M^2_{W_s}} + O(v^2) \tag{4.16}$$

which sums with (4.15) for the total annihilation cross-section of the singlet in the regime $M_{W_s} >> M_{Z'}$

$$<\sigma_{ann}|v|> = \sigma_0 + O(v^2) = \frac{4\pi}{M^2_{W_s}} \left(\frac{2}{3}\, N_{Z'} + \tilde{N}_{Z'}\right) + O(v^2) \ . \tag{4.17}$$

To complete the computation of the number density (4.14) the decoupling temperature is inferred by exploiting, from the Boltzmann equation, the condition $dY/dx \simeq 0$ which results in [27]

$$\frac{T_{dec}}{M_{W_s}} \simeq \left(\ln\left(0.038\,(g/\sqrt{g_*})\, m_{Pl} M_{W_s} \sigma_0\right) - \frac{1}{2}\, \ln\left(\ln\left(0.038\,(g/\sqrt{g_*})\, m_{Pl} M_{W_s} \sigma_0\right)\right)\right)^{-1} .$$

Its easy to show that the decoupling temperature has a very mild dependence over the model dependent factors $N_{Z'}$ and $\tilde{N}_{Z'}$, as well as over realistic values of the variable g_*. A solid and simple fit is found using the formula [6]

$$T_{dec} = \frac{M_{W_s}}{\ln\left(m_{Pl}/M_{W_s}\right)}\,, \tag{4.18}$$

which allows to find the relation for the density Y_0 in eq. (4.14)

$$Y_0 \simeq \frac{0.3 M_{W_s} \ln(m_{Pl}/M_{W_s})}{\sqrt{g_*}\, m_{Pl}\, (\frac{2}{3}\, N_{Z'} + \tilde{N}_{Z'})} \tag{4.19}$$

where the approximation $g_* \sim g_{*s}$ can be used as the decoupling temperature is sufficiently high in the relevant region of parameter space. Finally, to draw an estimate for the value of $\Omega\, h^2$, we first find the current energy density ρ_0 by multiplying for the entropy s_{e_0}, $\rho_0 = s_{e0}\, Y_0\, M$, and then divide by the critical energy ρ_c to arrive at the final expression

$$\Omega\, h^2 = \frac{\rho_0\, h^2}{\rho_c} = \frac{2970\, M_{W_s}\, Y_0\, cm^{-3}}{1.05\, 10^4\, eV\, cm^{-3}}\,. \tag{4.20}$$

Taking $\Omega h^2 \sim 1$ we obtain an upper bound

$$M_{W_s} < 10^5 \sqrt{\frac{\sqrt{g_*}\,\mathcal{N}}{\ln(m_{Pl}/M_{W_s})}} GeV \tag{4.21}$$

with $\mathcal{N} = \left(\frac{2}{3}\, N_{Z'} + \tilde{N}_{Z'}\right)$ determined by the $U(1)_{Z'}$ couplings in table 3.

- $M_{W_s} >> M_{Z'}$ **with inflation.** The current relic density could also be generated by an out-of-equilibrium production after reheating if such process takes place after singlet decoupling. In this case inflation will dilute the singlet density and, from the Boltzmann equations (4.11), we can study the subsequent evolution with the approximate knowledge of the density derivative,

$$\frac{d\,Y}{d\,x} = \lambda x^{-2}\, Y_{EQ}^2\,, \tag{4.22}$$

with $Y_{EQ} = 0.145 g_{eff}/g_{*s}\, x^{3/2}\, e^{-x}$ (non relativistic case). When λ is independent of x, eq. (4.22) can be promptly integrated to the present temperature, providing the density produced by the singlet after reheating [6]

$$Y_0 = \frac{\lambda g_{eff}^2}{2} \left(\frac{0.145}{g_*}\right)^2 \left(\frac{M_{W_s}}{T_R} + \frac{1}{2}\right) e^{-2\,\frac{M_{W_s}}{T_R}}\,, \tag{4.23}$$

with T_R the reheating temperature. Such value, inserted in (4.20) and requiring that it does not exceed the measured cosmological relic density, results in a bound, involving the Z' and W_s masses as well as T_R, whose specific form depends on the process through which the final density is regenerated. As long as the reheating temperature is greater than the Z' mass, the generation of the singlets will take place with all the channels investigated above, so that the relevant (thermal) cross-section is (4.17). Using (4.23), the cosmological upper bound over the relic is obtained with the condition

$$M_{W_s} > T_R \left(26 + \frac{1}{2} \ln \left(\frac{M_{W_s}}{T_R} + \frac{1}{2} \right) + \frac{1}{2} \ln (\mathcal{N}) \right) \tag{4.24}$$

and $\mathcal{N} = \left(\frac{2}{3} N_{Z'} + \tilde{N}_{Z'} \right)$.

- $M_{W_s} << M_{Z'}$ **without inflation.** In this case W_s is a WIMP and it can only annihilate into the matter supermultiplets in table 3 via the diagrams in figures 1 and 2, which are suppressed by the heavy Z' vector boson mass. The interaction between W_s and other matter states will be kinematically suppressed when T is below $M_{Z'}$. Decoupling therefore occurs when W_s is still relativistic at freeze-out. The resulting density in the regime $T \gg M$ can be analytically extracted from the Boltzmann equation [27]

$$Y_0 = 0.2 \, \frac{g_{Ws}}{g_{*s(T_{dec})}} \, . \tag{4.25}$$

To obtain an estimate for the value of $\Omega \, h^2$ we use the expression in eq. (4.20). We first find the current energy density $\rho_0 = s_{e0} \, Y_0 \, M$, and then divide by the critical energy ρ_c to arrive at the expression in eq. (4.20). The resulting formula can then be used to obtain a constraint on M_{W_s}

$$M_{W_s} < \frac{\Omega \, h^2 \, 3.5 \, eV}{Y_0} \simeq (17.7 eV) \, \frac{g_{*s(T_{dec})}}{g_{Ws}} \, \Omega \, h^2 \, . \tag{4.26}$$

If we consider a scenario with $M_{Z'}$ at the TeV scale, with a similar range for the decoupling temperature, the number of the degrees of freedom still relativistic will be of order 10^2. Taking $\Omega \, h^2 \sim 1$ we obtain the limit

$$M_{W_s} \lesssim 1 KeV \, . \tag{4.27}$$

Such tight constraint is typical of over abundant relativistic WIMP particle, as the condition $M_{W_s} < M_{Z'}$ forces the singlet to be light because of the suppression of the interactions by a factor $1/M_{Z'}$.

- $M_{W_s} << M_{Z'}$ **with inflation.**

The constraint in eq. (4.27) is relaxed in the presence of inflation. In this case W_s is completely diluted by inflation and is regenerated after reheating. In the limit $M < M_{Z'}$ and $T_R < M_{Z'}$, we approximate the Z'–mediated interaction by a four–point Fermi interaction. In this case W_s can only annihilate via the diagrams in figures 1 and 2 into matter states, but not those in figures 3 and 4 into gauge bosons and gauginos. The thermal cross-section in the non-relativistic limit $T_R < M_{W_s}$ is given by

$$\sigma_0 = \frac{32\pi \, M_{W_s}^2}{M_{Z'}^4} \, N_{Z'} \; , \tag{4.28}$$

and by

$$\sigma_0 = \frac{2\pi \, s}{M_{Z'}^4} \, N_{Z'} \; . \tag{4.29}$$

in the relativistic limit $T_R > M_{W_s}$. The non–relativistic case is a replication of the analysis for the non-relativistic, out-of-equilibrium production W_s when $T_R > M_{Z'}$. The resulting bound is

$$M_{W_s} > T_R \left(27 + \frac{1}{2} \ln \left(\frac{M_{W_s}^5}{M_{Z'}^4 \, T_R} + \frac{M_{W_s}^4}{2 \, M_{Z'}^4} \right) + \frac{1}{2} \ln \left(N_{Z'} \right) \right) \; . \tag{4.30}$$

In the relativistic limit, the thermal cross section defines a temperature-dependent λ parameter and the integration of the Boltzmann equations requires more care. To proceed we can express the Center of Mass energy of the process as a thermal average, yielding its dependence on the temperature as

$$< s >= 4 < E^2 > \simeq \left(\frac{5}{4} \right) \, 40 \, T^2 \, . \tag{4.31}$$

The integration of (4.22), with a relativistic equilibrium density, is now at hand and the final constraint on the Wilsonian singlet mass bound takes the form

$$\frac{M_{W_s} \, T_R^3}{M_{Z'}^4} < 1.6 \times 10^{-28} \left(\frac{g_*^{3/2}}{N_{Z'} \, g_{eff}^2} \right) \; . \tag{4.32}$$

As there are three unknown parameters (M_{W_s}, $M_{Z'}$, T_R) in Eq. (4.32) we cannot infer a definite value for the mass of W_s and Z'. We may nevertheless conclude that Z' should be very heavy.

5 Conclusions

The issue of dark matter is one of important enigmas of modern physics. The problem is exacerbated as plausible particle candidates can vary from the ultra–light [2] to the super–massive [6, 28]. It is then sensible to seek guidance from string theory, which is the only contemporary approach that consistently unifies gravity with the gauge interactions.

It is then amply rewarding that string constructions indeed contain in them the intrinsic ingredients to produce heavy and stable dark matter. The stabilisation mechanism arises due to the breaking of non–Abelian gauge symmetries in string theory by Wilson line, which gives rise to exotic states that do not satisfy the quantisation conditions of the unbroken non–Abelian gauge symmetry. Well known examples of such states include those that carry fractional electric charge that are highly constrained by experiments. The favoured class of Wilsonian states that can constitute the dark matter are those that are neutral under the Standard Model, but carry fractional charge under an extra $U(1)_{Z'}$ symmetry. While this possibility has been entertained before in [6], the string derived Z' model of ref. [11] is the first concrete string derived model in which the Wilsonian singlet dark matter can be realised. This model contains all the ingredients needed to break the $U(1)_{Z'}$ symmetry at a high or low scale, while maintaining a discrete symmetry that forbids the decay of the Wilsonian singlet matter state. The Wilsonian dark matter singlets in the model of ref. [11] are $SO(10)$ singlets and arise from the breaking of $E_6 \to SO(10) \times U(1)$ by Wilson lines. Even within the constraints of the string construction, as we discussed in section 4, there are varied possibilities depending on the $U(1)_{Z'}$ breaking scale, the reheating temperature T_R and the mass of W_s itself. We can then all but hope that forthcoming experiments will narrow down the possibilities by, for example, discovering a extra vector boson Z' in the vicinity of the multi–TeV scale.

Acknowledgments

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