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DRED, UNIVERSALITY AND THE SUPERPARTICLE SPECTRUM *

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ABSTRACT

Recent work on the use of dimensional reduction for the regularisation of non–supersymmetric theories is reviewed. It is then shown that there exists a class of theories for which a universal form of the soft supersymmetry breaking terms is invariant under renormalisation. It is argued that this universal form might be approached as an infra–red fixed point for the unified theory above the unification scale. The superparticle spectrum is calculated for these theories.

1. Introduction

It is commonly assumed that the soft supersymmetry terms in the supersymmetric standard model (SSM) unify at high energies, and are determined ultimately by four parameters: m_0, M, A and B which we will define presently. The calculation of the sparticle spectrum in terms of these parameters is a major industry. At its most basic level, this consists of integrating the set of coupled differential equations for the various running masses and couplings from the scale of gauge unification (M_G) down to M_Z, using the one–loop β–functions. If we wish to refine these calculations by including threshold corrections or using the two–loop β–functions then interesting issues arise, associated with the regularisation of both supersymmetric and non–supersymmetric theories. These issues are explained in Sec. 2.

Even with the universal form for the soft breakings alluded to above, there is still a lot of parameter–space. In Sec. 3 it is explained that with the further assumption that in the underlying theory the universal form of the soft terms is invariant under renormalisation, the sparticle spectrum becomes entirely determined by a single parameter. This *strong* universality might be a property of the fundamental theory, or it might arise to a good approximation in the infra–red limit at M_G, from a more general class of theories at higher scales. The results for the SSM are explored in Sec. 4.

2. DRED (Scylla) and DREG (Charybdis)

Dimensional regularisation (DREG) is inconvenient for supersymmetric theories. The fact that, for example, the quark–quark–gluon and the quark–squark–gluino cou-

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plings are equal (because of supersymmetry) is not preserved under renormalisation, if DREG is employed. If we demand that the two renormalised couplings are the same, then the associated subtractions are different: or, to put it another way, if the couplings are equal at one renormalisation scale, μ, then they are different at another. This point is academic if we are calculating at a single value of μ, but becomes important if we want to relate a given theory at one value of μ to the same theory at another such value: as when we perform the standard running analysis. What this means is that DREG is very inconvenient for the SSM. If we assume "convenient" values for the couplings at unification (such as equality for the couplings mentioned above) then these couplings will be different at M_Z and this difference will have to be accounted for both in the actual evolution analysis and in the calculation of the physical masses.

In 1979 Siegel[1] proposed a modification of DREG designed to render it more compatible with supersymmetry. The essential difference between Siegel's method (DRED[a]) and DREG is that the continuation from 4 to d dimensions is made by *compactification* or *dimensional reduction*. Thus while the momentum (or space-time) integrals are d-dimensional in the usual way, the number of field components remains unchanged and consequently supersymmetry is undisturbed.

Modulo certain ambiguities that do not manifest themselves at ordinary loop levels, DRED is a practical supersymmetric regulator. So practical, in fact that it has sometimes been used as being simpler than DREG even for non–supersymmetric theories such as QCD. That DRED is a viable alternative to DREG has long been believed[2]; but there are subtleties involved that have only been resolved recently[3,4]. These arise due to the effect of Siegel's compactification on the gauge fields. After dimensional reduction to $d = 4 - \epsilon$, it is only the first d components of the gauge field $A_\mu(x)$ that form the actual gauge connection. The remaining ϵ components transform under gauge transformations as a multiplet of scalar fields, called ϵ-scalars.

Now in a straightforward implementation of DRED in, for example, QCD, the quark–quark–gluon and the quark–quark-ϵ-scalar coupling are both equal to the gauge coupling. This equality is not preserved under renormalisation, however, because the latter interaction is independently gauge invariant. We call interactions involving the ϵ-scalars *evanescent* interactions. Only in a supersymmetric theory do they remain equal to their "natural" values under renormalisation. If we denote the genuine masses and couplings of a theory collectively as λ and the evanescent ones as λ_E, then it is possible to show that the S-matrix ($\mathcal{S}$) is independent of λ_E in the sense that there exists a coupling constant redefinition

$$\lambda' = \lambda'(\lambda, \lambda_E) \quad \text{and} \quad \lambda'_E = \lambda'_E(\lambda, \lambda_E) \tag{2.1}$$

such that we have

$$\mathcal{S}(\lambda) = \mathcal{S}_{\text{DRED}}(\lambda', \lambda'_E) \tag{2.2}$$

[a]DRED is a sympathetic antipathy and an antipathetic sympathy (Kierkegaard)

This had to be the case, of course, for DRED to be a consistent regulator. Evidently varying λ_E defines a trajectory in (λ', λ'_E)-space without changing the S-matrix. It follows that we are free to choose a point on this trajectory such that the λ'_E are indeed equal to their natural values. If this is done, however, it should be clear that it would *not* be possible (using DRED) to relate predictions made at different values of the renormalisation scale μ by evolving only the β-functions corresponding to the real interactions.

To sum up: DREG is inconvenient for a running analysis in a supersymmetric theory because coupling constant relations prescribed by supersymmetry are not preserved, while DRED is inconvenient for non–supersymmetric theories because evanescent couplings do not remain equal to their natural values, and enter into the β–functions for the genuine couplings. This seems to leave us with an obvious choice for any given theory; but, as we shall see in the next section, the case of the SSM presents special problems.

3. The supersymmetric standard model

Let us consider the standard running analysis from M_G to M_Z in the SSM, starting with the dimensionless couplings. If we use the whole SSM as our effective field theory throughout, then there is no need to introduce evanescent dimensionless couplings, because as far as the dimensionless coupling sector is concerned the theory is effectively supersymmetric. We can with confidence proceed to include two–loop contributions to the β–functions. One must ensure that the input values of the couplings at M_Z are those appropriate to the SSM rather than the standard model, which means they will depend through radiative corrections on the sparticle spectrum[5].

There is an alternative approach whereby for scales below any given particle mass, M_S say, the contribution for the corresponding particle is excised from the β–functions; in other words, below each particle mass a new effective theory is defined with the said particle integrated out. Evidently this approach sums to all orders contributions of the form $\alpha \ln(M_S/M_Z)$ but neglects non–logarithmic terms that are equally important unless $M_S >> M_Z$. Within the context of the effective field theory approach it is difficult to recover these non–logarithmic terms; one need only reflect that the true effective theory below M_S contains nonrenormalisable interactions which are suppressed only by powers of M_Z/M_S. Another criticism of this approach is that once we start decoupling particles we lose supersymmetry and thus to go beyond one loop we would need to address the evanescent coupling problem explained in the previous section. It therefore appears preferable to work throughout with the SSM as the effective field theory.

In fact, of course, the SSM is not fully supersymmetric because of the soft breaking terms, and so when we come to run the masses we cannot avoid worrying about the ϵ-scalars[6]. The reason is that since they are indeed scalars, there is no symmetry

which forbids them from having a mass. If we set this mass zero at (say) M_G then it will be non–zero at M_Z, and it will also influence (at two–loops) the evolution of the genuine scalar masses. This is not a problem in principle, but it is more convenient to make a slight change in the regularisation scheme[7] which decouples the ϵ-scalar masses from the β–functions for the genuine scalar masses. The same redefinition renders the one–loop pole masses for the scalars independent of the ϵ-scalar mass.

One might wonder whether it might not be simpler to employ DREG since then the ϵ-scalars do not appear at all. The problem then, however, is that the evolution of the dimensionless couplings would become more complicated, as explained at the beginning of the last section. In subsequent sections we implicitly assume use of the hybrid scheme[7] as indicated above.

4. Universality

In this section we describe how a particular "universal" form for the soft-breaking couplings in a softly broken $N = 1$ theory is renormalisation-group invariant through two loops, provided we impose one simple condition on the dimensionless couplings[8]. The universal form for the trilinear couplings and $\phi^*\phi$ mass terms is identical to that found in a derivation of the soft-breaking terms from string theory[9].

We begin with a general $N = 1$ supersymmetric gauge theory. The Lagrangian $L_{\rm SUSY}(W)$ is defined by the superpotential

$$W = \frac{1}{6}Y^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}\mu^{ij}\Phi_i\Phi_j. \tag{4.1}$$

$L_{\rm SUSY}$ is the Lagrangian for the $N = 1$ supersymmetric gauge theory, containing the gauge multiplet $\{A_\mu, \lambda\}$ (λ being the gaugino) and a chiral superfield Φ_i with component fields $\{\phi_i, \psi_i\}$ transforming as a (in general reducible) representation R of the gauge group $\mathcal{G}$. We assume that there are no gauge-singlet fields and that $\mathcal{G}$ is simple. (The generalisation to a semi-simple group is trivial.) The soft breaking is incorporated in $L_{\rm SB}$, given by

$$L_{\rm SB} = (m^2)_i^j\phi^i\phi_j + \left(\frac{1}{6}h^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + \frac{1}{2}M\lambda\lambda + \text{h.c.}\right) \tag{4.2}$$

(Here and elsewhere, quantities with superscripts are complex conjugates of those with subscripts; thus $\phi^i \equiv (\phi_i)^*$.)

Given a certain constraint on the dimensionless couplings, the following relations among the soft breakings are renormalisation group invariant through two–loops[6]:

$$h^{ijk} = -MY^{ijk}, \tag{4.3}$$

$$(m^2)^i{}_j = \frac{1}{3}(1 - \frac{1}{16\pi^2}\frac{2}{3}g^2Q)MM^*\delta^i{}_j \tag{4.4}$$

$$b^{ij} = -\frac{2}{3}M\mu^{ij}. \tag{4.5}$$

The aforementioned constraint is

$$P^i{}_j = \frac{1}{3} g^2 Q \delta^i{}_j, \tag{4.6}$$

where

$$Q = T(R) - 3C(G), \quad \text{and} \quad P^i{}_j = \frac{1}{2} Y^{ikl} Y_{jkl} - 2g^2 C(R)^i{}_j. \tag{4.7}$$

Here

$$T(R)\delta_{AB} = \mathrm{Tr}(R_A R_B), \quad C(G)\delta_{AB} = f_{ACD} f_{BCD} \quad \text{and} \quad C(R)^i{}_j = (R_A R_A)^i{}_j, \tag{4.8}$$

where the f_{ABC} are the structure constants of $\mathcal{G}$.

In the usual SSM notation, Eqs. (4.3)-(4.5) correspond to a universal scalar mass m_0 and universal A and B parameters related (to lowest order in g^2) to the gaugino mass M as follows:

$$m_0 = \frac{1}{\sqrt{3}} M, \tag{4.9}$$

$$A = -M, \tag{4.10}$$

$$B = -\frac{2}{3} M. \tag{4.11}$$

Remarkably, relations of this form can arise in effective supergravity theories motivated by superstring theory, where supersymmetry breaking is assumed to occur purely via vacuum expectation values for dilaton and moduli fields[9]. Eqs. (4.9) and (4.10) are of fairly general validity in this context; the relationship between B and M is more model dependent. Given certain assumptions including dilaton dominance the result is $B = 2M/\sqrt{3}$; this case has been subject to some phenomenological investigation[10]. The similarity between the conditions on the soft-breaking terms which arise from our universality hypothesis and those that emerge from string theory is certainly intriguing. Eqs. (4.9) and (4.10) also arise[11] in the context of *finite* supersymmetric theories (which correspond to a special case of Eq. (4.6), $P = Q = 0$). (Recently Ibáñez has discussed whether emergence of a finite low energy effective field theory from a string theory might be natural[12].)

There is, however, an alternative interpretation of the above results. Consider a unified theory where it would be *possible* to impose Eq. (4.6). The fact that Eqs. (4.6) and (4.9)–(4.11) are renormalisation group invariant is of course equivalent to saying that they are fixed points of the evolution equations; fixed points, moreover, that are approached, given certain conditions, in the infra–red. For example, given a theory based on a simple group with a single Yukawa coupling and a chiral multiplet transforming as an irreducible representation R, then Eqs. (4.6), (4.9) and (4.10) are infra–red attractive as long as $-6C(R) < Q < 6C(R)$, while (4.11) is too if $Q < 0$. At first sight it might appear that the difference between M_P and M_G is insufficient

to allow significant evolution, but it has recently been argued[13] that in the case of the Yukawa couplings the evolution towards the fixed point may occur more rapidly in the unified theory than in the low energy theory. If we believe that this conclusion holds also for the soft terms, then it is possible to argue that for a wide range of input parameters the boundary conditions (4.9)–(4.11) might hold at M_G. (Since, however, $Q > 0$ is favoured for rapid evolution[13] we may have problems with Eq. (4.11).)

Let us turn now to phenomenology[14]. We assume that the SSM is valid below gauge unification, and that the unified theory satisfies Eq. (4.6), either exactly or in the infra–red limit at M_G. We then proceed to impose Eqs. (4.9)-(4.11) as boundary conditions at the gauge unification scale.

5. The running analysis

We start with the superpotential:

$$W = \mu_s H_1 H_2 + \lambda_t H_2 Q \overline{t} + \lambda_b H_1 Q \overline{b} + \lambda_\tau H_1 L \overline{\tau} \tag{5.1}$$

where we neglect Yukawa couplings except for those of the third generation. The Lagrangian for the SSM is defined by the superpotential of Eq. (5.1) augmented with soft breaking terms as follows:

$$L_{\rm SSM} = L_{\rm SUSY}(W) + L_{\rm SOFT} \tag{5.2}$$

where

$$\begin{aligned} L_{\rm SOFT} = & - m_1^2 H_1^\dagger H_1 - m_2^2 H_2^\dagger H_2 + [m_3^2 H_1 H_2 + \text{h.c.}] \\ & - \sum_i \left(m_Q^2 |Q|^2 + m_L^2 |L|^2 + m_{\overline{t}}^2 |\overline{t}|^2 + m_{\overline{b}}^2 |\overline{b}|^2 + m_{\overline{\tau}}^2 |\overline{\tau}|^2 \right) \\ & + [A_t \lambda_t H_2 Q \overline{t} + A_b \lambda_b H_1 Q \overline{b} + A_\tau \lambda_\tau H_1 L \overline{\tau} + \text{h.c.}] \\ & - \tfrac{1}{2} [M_1 \lambda_1 \lambda_1 + M_2 \lambda_2 \lambda_2 + M_3 \lambda_3 \lambda_3 + \text{h.c.}] \end{aligned} \tag{5.3}$$

and the sum over i for the m^2 terms is a sum over the three generations. The running analysis of the SSM has been performed many times. The novel feature here is the restricted set of boundary conditions at gauge unification, where we impose (in the usual notation)

$$m_1 = m_2 = m_Q = m_L = m_{\overline{t}} = m_{\overline{b}} = m_{\overline{\tau}} = \frac{1}{\sqrt{3}} M, \tag{5.4}$$

$$A_\tau = A_b = A_t = -M, \quad M_1 = M_2 = M_3 = M, \tag{5.5}$$

$$m_3^2 = -\frac{2}{3} \mu_s M \tag{5.6}$$

where Eq. (5.4) includes the squarks and sleptons of all three generations.

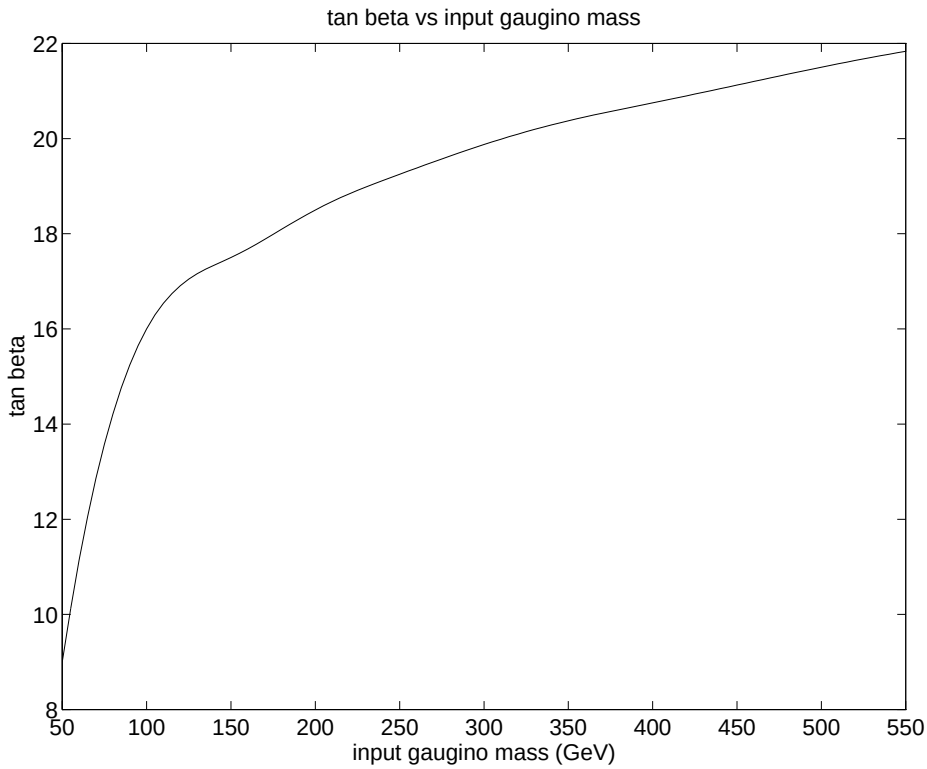

Fig. 1. $\tan\beta$ vs. M for $m_t = 175$ GeV.

Our procedure is as follows. We input α_1, α_2, α_3, m_t and $\tan\beta$ at M_Z, and calculate the unification scale M_G (defined as the meeting point of α_1 and α_2) by running the dimensionless couplings. Then we input the gaugino mass M at M_G, and run the dimensionful parameters (apart from m_3^2 and μ_s) down to M_Z. We can then determine m_3^2 and μ_s^2 as usual at M_Z by minimising the (one–loop corrected) Higgs potential. Then we run m_3^2 and μ_s back up to M_G (for the two possibilities of sign μ_s) and calculate $B' = B/M = (m_3^2)/(M\mu_s)$. By plotting B' against the input value of $\tan\beta$ we can then determine whether (for a given input M) there exists a value of $\tan\beta$ such that Eq. (4.11) is satisfied. Given a set $M, \tan\beta$ satisfying our boundary conditions we can calculate the sparticle spectrum in the usual way and plot the resulting masses against M. We have included one–loop corrections in the minimisation of the Higgs potential, and in the calculation of the mass (m_h) of the lighter CP–even Higgs boson. Our results for other masses are based on the tree mass matrices but again with all running parameters evaluated at the scale M. Since the two–loop corrections to the β–functions are now available [6,15] we incorporate these. In general their effect is very small, being most noticeable in the Higgs sector; although the mass of the lightest Higgs is essentially unchanged, the other Higgs masses are increased by up to 10% by the two loop corrections. Of course for precise predictions, we should also include threshold corrections.

In Fig. (1) we plot $\tan\beta$ against the input gaugino mass, M, having satisfied Eq. (4.11). We find that the results for the masses of the various particles exhibit linear behaviour for a wide range of input gaugino masses. Rather than give more figures, we therefore summarise our results in Table 1, which gives a good approximation (within a few GeV) for 100 GeV $< M <$ 500 GeV.

The phenomenology of our results is fairly typical. For $M \approx 150$ GeV, for example, we have a stable neutralino at 55GeV, a τ-slepton at 80GeV, and the light Higgs at 115 GeV. Notice that m_h is almost independent of M. The main distinguishing feature of our scenario lies in the relationship between $\tan\beta$ and M, as shown in Fig. (1). At first sight this appears to disfavour $b-\tau$ unification. This is of course in any case sensitive to the nature of the unified theory which according to our scenario is required to satisfy Eq. (4.6).

Table 1. Linear approximations of the form $m = aM + b$ to the mass spectrum for $m_t = 175$ GeV, $m_t = 185$ GeV and $m_t = 190$ GeV.

m_t	175		185		190	
$m = aM + b$	a	b	a	b	a	b
m_h	0.048	109	0.060	108	0.070	106
m_H	1.613	15	1.800	7	1.870	5
m_A	1.585	8	1.782	4	1.855	2
$m_{H^\pm}$	1.555	25	1.755	20	1.829	17
$m_{\tilde{e}_1}$	0.872	12	0.873	12	0.874	11
$m_{\tilde{e}_2}$	0.666	12	0.667	12	0.668	12
$m_{\tilde{\nu}_e}$	0.930	-22	0.930	-21	0.930	-21
$m_{\tilde{\tau}_1}$	0.830	31	0.852	22	0.861	18
$m_{\tilde{\tau}_2}$	0.615	-11	0.644	1	0.657	5
$m_{\tilde{\nu}_\tau}$	0.903	-21	0.917	-21	0.923	-20
$m_{\chi_1^+}$	1.527	48	1.580	46	1.601	45
$m_{\chi_2^+}$	0.793	-21	0.799	-23	0.805	-25
$m_{\chi_1^0}$	1.532	44	1.583	44	1.603	45
$m_{\chi_2^0}$	1.566	22	1.622	20	1.645	18
$m_{\chi_3^0}$	0.789	-19	0.793	-20	0.797	-21
$m_{\chi_4^0}$	0.410	-7	0.413	-8	0.417	-9
$m_{\tilde{u}_1}$	2.264	26	2.266	26	2.269	26
$m_{\tilde{u}_2}$	2.189	29	2.191	30	2.194	30
$m_{\tilde{d}_1}$	2.245	37	2.247	37	2.251	37
$m_{\tilde{d}_2}$	2.175	33	2.177	33	2.180	33
$m_{\tilde{t}_1}$	1.829	143	1.849	143	1.861	142
$m_{\tilde{t}_2}$	1.645	0	1.615	18	1.609	27
$m_{\tilde{b}_1}$	2.040	56	2.113	46	2.142	42
$m_{\tilde{b}_2}$	1.963	20	1.992	28	2.009	30

6. Outlook

We have shown that the restrictions imposed by the conjecture of renormalisation–invariant universality at M_G leaves a viable and well determined supersymmetric phenomenology. What we need now is a compelling unified theory that satisfies Eq. (4.6), (either exactly or in the infra–red).

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