String Phenomenology in the Era of the LHC

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by

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Dedicated To My Parents

Abstract

The heterotic string-derived models in the free fermionic formulation give rise to some of the most realistic string models to date, which possess $N = 1$ space-time supersymmetry (SUSY). The characteristics of the required spectrum are well motivated in heterotic string constructions that allow for a light Z'. Anomaly cancellation of the $U(1)_{Z'}$ symmetry requires the existence of the Standard Model (SM) singlet and vector-like states in the vicinity of the $U(1)_{Z'}$ breaking scale. We show that the agreement with the gauge coupling data at one-loop is identical to the case of the Minimal Supersymmetric Standard Model (MSSM), owing to cancellations between the additional states. It is also shown that effects arising from heavy thresholds may push the supersymmetric spectrum beyond the reach of the LHC, while maintaining the agreement with the gauge coupling data.

On the other hand, lack of evidence for SUSY at the LHC has led to the recent interest in non-supersymmetric heterotic string vacua. We explore what may be learned in this context from the quasi–realistic free fermionic models. We show that constructions with a low number of families give rise to proliferation of a priori tachyon producing sectors, compared to the non–realistic examples which typically may contain only one such sector, followed by a concrete example of a non–supersymmetric, non–tachyonic, heterotic string vacuum where we compare the structure of its massless spectrum to the corresponding supersymmetric

vacuum. While in some sectors SUSY is broken explicitly, i.e. the bosonic and fermionic sectors produce massless and massive states, other sectors, and in particular those leading to the chiral families, continue to exhibit Fermi-Bose degeneracy. In these sectors the massless spectrum, as compared to the supersymmetric cases, will only differ in some local or global $U(1)$ charges. Our example model contains an anomalous $U(1)$ symmetry, the cancellation mechanism for which generates a tadpole diagram at one loop–order in string perturbation theory. We entertain the possibility of the cancellation of this tadpole diagram against the corresponding diagram generated at one–loop by the non–vanishing vacuum energy and that in this respect the supersymmetric and non–supersymmetric vacua should be regarded on equal footing.

Declaration

I hereby declare that all work described in this thesis is the result of my own research unless reference to others is given. None of this material has previously been submitted to this or any other university. All work was carried out in the Theoretical Physics Division of the Department of Mathematical Sciences, University of Liverpool, UK, during the period of September 2013 until February 2017.

Publication list

This thesis contains material that has appeared in the following publications by the author:

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2. J. M. Ashfaque, P. Athanasopoulos, A. E. Faraggi and H. Sonmez, "Non-Tachyonic Semi-Realistic Non-Supersymmetric Heterotic String Vacua," *Eur. Phys. Jour.* **C76** (2016) no.4, 208.

Contents

1

Introduction

From time immemorial, man has desired to comprehend the complexity of nature in terms of as few elementary concepts as possible.

Abdus Salam

Quantum field theory (QFT) results from the marriage of quantum mechanics with Einstein's special relativity. A central concept that lies at the heart of these ideas is that of symmetry. And indeed quantum field theories are thought of and classified according to their symmetries.

The most important symmetry is of course the Poincaré group of special relativity. To say that the Poincaré algebra is fundamental in particle physics amounts to assuming that everything falls into some representation of this algebra that is

$$\text{Poincaré} = \text{Translation} \rtimes \text{Lorentz Transformation}.$$

The principle of relativity then asserts that the laws of physics are covariant with respect to this algebra. Other quantum field theories and especially the Standard Model (SM) also have other important symmetries. These symmetries imply that there is an additional algebra, called the Lie algebra, with a commutation relation of the form

$$[T_r, T_s] = i f_{rs}{}^t T_t$$

where the T_r are Hermitian generators and $f_{rs}{}^t$ are the structure constants.[1] This means that every field in the SM Lagrangian also carries a representation of this algebra. If this is a non-trivial representation then there is another 'internal' index on the field. For example the quarks are in the fundamental (i.e. three-dimensional) representation of $SU(3)$ and hence, as they are space-time spinors, the field carries the indices $\psi^a_\alpha(x)$.

Finally, we recall the well-known Noether's theorem which asserts that for every continuous symmetry of the Lagrangian there is a conserved quantity namely the conserved current and therefore one can construct a conserved charge. Consequently, one can think of symmetries and conservation laws as being more or less the same thing. So the SM has several symmetries built into it for example $SU(3)\times SU(2)\times U(1)$ and this means that the various fields carry representations of various algebras. These algebras split up into those associated to space-time (Poincaré) and those which one might call internal (such as the gauge symmetry algebra). In fact the split is a direct product in that

$$[P_\mu, T_a] = [M_{\mu\nu}, T_a] = 0$$

where T_a refers to any internal generator. Physically this means the conserved charges of these symmetries are Lorentz scalars.

[1] see Appendix A for a brief review of Lie algebras.

1.1 *The Standard Model of Particle Physics*

The SM is a theory striving to describe all the known forces of nature apart from gravity, that is the electromagnetic, the weak and the strong interactions, using the tools of quantum field theory especially gauge theories. The model, experimentally, has stood the test of time extremely well, up to energies of a few hundreds of GeV. All carriers of the weak interactions, namely the massive gauge bosons[2], $W_{\pm}$, Z, have been discovered experimentally, and their masses, of order $O(100)$ GeV, have been measured with great accuracy. Moreover, evidence for gluons is very strong, and many precision measurements have been performed that confirm the model as a physically correct up to the energy scales of the electroweak symmetry breaking. The following table captures the particle content of the SM:

spin 0	Higgs	
spin $\frac{1}{2}$	Leptons: $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$	
	Quarks: $\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$	
spin 1	Gluons	Strong Interactions
	Photon	Electromagnetic Interactions
	$W^{\pm}$, Z	Weak Interactions

Table 1.1: The particle content of SM.

In spite of the remarkable success enjoyed by the SM it remains only an

[2]see Appendix B for an instance of electroweak symmetry breaking.

effective field theory, addressing more questions than answers like why the local gauge interactions $SU(3)_C \times SU(2)_L \times U(1)_Y$ and why only 3 families of quarks and leptons? It is worth noting that the SM gauge interactions of the quarks and leptons are completely fixed by their gauge charges. Finally, we would also like to know the origin of CP violation, the solution to the strong CP problem, the origin of the cosmological matter-antimatter asymmetry.

1.2 *Some Unsolved Problems: A Brief Look*

In this section, unsolved problems that render the SM as only an effective field theory are considered which closely match the interests of the author.

1.2.1 *Gravity*

The missing piece of the jigsaw puzzle in the SM are the gravitational interactions. Gravity is by far the weakest of the four fundamental forces of nature, but unlike the other three fundamental forces that are related to gauge symmetries, gravity a universal force. Gravity has its own force-carrier, a spin-2 particle known as the 'graviton', which has not been observed till date. QFT is not consistent with the framework needed to describe the gravitational interactions. The biggest hurdle in the attempt to incorporate gravity in the SM is the fact that it is non-linear with a non-terminating Einstein-Hilbert action due to the presence of $\sqrt{-g}$ and therefore notoriously non-renormalizable. Any attempt for renormalizing gravity leads to an infinite scattering cross-section for interactions. This requires a very fine tuning at each order in the interaction so as to yield a finite result [11]. At the microscopic scale, the gravitational effects are almost negligible, in contrast to the macroscopic scale where they dominate.

1.2.2 *Dark Matter*

Dark matter appears to make up about five times more mass than ordinary matter. Astrophysical and observational constraints indicate that dark matter is non-baryonic, cold and collisionless in nature. A picture that has come to be known as Cold Dark Matter (CDM). One of the major challenges is to understand the very nature of dark matter where ample evidence is provided by astrophysical observations. Dark matter could be made of a completely new, as yet undiscovered particle.

1.2.3 *The Cosmic Coincidence Problem*

In Einstein's original formulation, the gravitational field is a symmetric tensor field $g_{\mu\nu}$. The dynamics is encoded in the Einstein's equations which follow from the Einstein-Hilbert action

$$\mathcal{S} = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}(R - 2\Lambda),$$

where g is the determinant of the metric $g_{\mu\nu}$, R is the Ricci scalar and Λ is the cosmological constant. Whilst theorists were busy in finding a concrete argument that $\Lambda = 0$, refined observations led to the problem of dark energy which is distributed evenly throughout space-time and is associated with the vacuum in space. The even distribution of dark energy causes global gravitational effects resulting in a repulsive force which is believed to drive the accelerating expansion of the universe.

The cleanest argument is that the dark energy represents the cosmological constant. The cosmological constant, however, can not be held responsible for inflation in the early universe because otherwise the accelerated expansion would not end since the cosmological constant, Λ, happens to be of the same order as that of the present matter density of the universe, ρ_M. Nevertheless, it is possible that the cosmological constant is responsible for dark energy because the current cosmic acceleration might indeed continue without end.

1.2.4 *The Strong CP Problem*

Quantum chromodynamics (QCD) is a wonderful theory of the strong interactions. Having said that, however, it suffers from one serious problem: the strong CP problem. There are three known viable solutions to tackle the strong CP problem of which axions are the most plausible solution as they help to keep the strong CP problem in check.

The strong CP problem emerges as a consequence of adding the CP violating term to the QCD Lagrangian

$$\mathcal{L}_{CP} = \frac{\overline{\theta}\alpha_s}{32\pi^2}\tilde{G}_{\mu\nu}G^{\mu\nu}$$

which is a renormalizable and gauge invariant term that violates CP and is allowed in any gauge theory in four dimensions. In the SM it contributes to the CP-odd observables such as the neutron electric dipole moment (nEDM). The very smallness of $\overline{\theta}$ despite large amounts of CP violation in the weak sector of the SM is called the strong CP problem.

1.2.5 *Baryon Asymmetry*

The observed lack of antimatter is in direct contrast with the abundance of ordinary matter in the universe. It is unknown exactly why this is the case, especially since the Big Bang should have created an equal amount of each, but the answer for this asymmetry is believed to lie with CP violation [8].

1.3 *GUTs*

Grand unified theories (GUTs), as the name suggests, focuses on the problem of unification in the SM [13]. Ever since the development of the theories of special and general relativity, symmetries have played an essential role in the construction of physical theories. The main symmetry of the SM and the foundation of its success is the gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$. The central idea of GUTs is to assume that $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ are distinct subgroups of a larger gauge symmetry group in which formerly disconnected fermions of a family, or bosons of different gauge groups, transform in larger fermionic or bosonic multiplets. This larger symmetry is unbroken above a yet-to-be-determined mass scale M_G, that must be broken at presently accessible energies, as it is not observed.

The main requirements which must be satisfied by the GUT, say G, in order to be a viable candidate for a grand unified model are:

- G must embed $SU(3) \times SU(2) \times U(1)$.
- G must be simple or else the product of identical simple factors whose coupling constants can be set equal by a discrete symmetry.
- G must contain complex representations.

The initial work of Georgi and Glashow in 1973 was the much needed drive behind the idea of obtaining a Grand Unified Theory (GUT) influencing much of the research at the time [1].

1.3.1 *The Georgi-Glashow* $SU(5)$

This is the only compact simple Lie group of rank four admitting complex representations. The three low-energy gauge couplings of the SM are still independent arbitrary parameters. This issue is tackled by embedding the SM gauge group into the simple unified gauge group, Georgi-Glashow $SU(5)$, with one universal gauge coupling α_G defined at the grand unification scale M_G where quarks and leptons sit in two irreducible representations with

$$\mathbf{10} = \begin{pmatrix} Q \\ u^c \\ e^c \end{pmatrix}, \qquad \overline{\mathbf{5}} = \begin{pmatrix} d^c \\ L \end{pmatrix}.$$

Nevertheless, the three low-energy gauge couplings are now determined in terms of two independent parameters: the universal gauge coupling α_G and the grand unification scale M_G.

The gauge bosons transform globally under the **24** adjoint representation of the $SU(5)$ which decomposes under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ as

$$\mathbf{24} = (\mathbf{8},\mathbf{1})_{\mathbf{0}} \oplus (\mathbf{1},\mathbf{3})_{\mathbf{0}} \oplus (\mathbf{1},\mathbf{1})_{\mathbf{0}} \oplus (\mathbf{3},\mathbf{2})_{-\frac{\mathbf{5}}{\mathbf{3}}} \oplus (\overline{\mathbf{3}},\mathbf{2})_{+\frac{\mathbf{5}}{\mathbf{3}}}$$

since the $SU(5)$ gauge group is 24 dimensional. Thus, there are 24 gauge bosons: 12 of which are the familiar SM gauge bosons, denoted by the first three multiplets, whilst the last two multiplets are new to GUTs and are referred to as leptoquarks. They have non-trivial quantum numbers under both the $SU(3)_C$ and $SU(2)_L$ gauge groups. Therefore, they can mediate transitions between quarks and leptons as well as between quarks and anti-quarks.

In order to accomplish the spontaneous symmetry breaking Higgs doublets are required which can either be in **24** or **5**: Φ_{24} and Φ_5 respectively. The Φ_{24} can break the $SU(5)$ gauge group to $SU(3) \times SU(2) \times U(1)$ symmetry at a scale $M_\chi \sim \langle \Phi_{24} \rangle$. This also splits Φ_5 into a $SU(3)$ triplet $H_{3,1}$, and $SU(2)$ doublet $h_{(1,2)}$. The later is the Higgs doublet of the SM which spontaneously breaks the electroweak theory. In this way, the SM is obtained at low energies.

However, it should be noticed that the success of $SU(5)$ GUT was short lived as it was ruled out initially due to the increased accuracy in the measurement of $\sin^2\theta_W$ and then by early bounds on the longevity of the proton decay is what prompted the theoretical physicists to attempt the construction of other four-dimensional GUTs in pursuit of a desirable theory.

1.3.2 *The Flipped* $SU(5)$

A central prediction of all GUTs is the decay of the proton via the lepto-quark gauge bosons. Experimentally the lifetime of the proton τ_p [14] is known to be

$$\tau_p \geq 10^{34} \text{ years.}$$

The minimal $SU(5)$ models predict $\tau_p \sim O(10^{31})$ years and is therefore ruled out experimentally. For this very reason, there is an interest in the Flipped $SU(5)$ models. The Flipped $SU(5)$ models differ from the Georgi-Glashow $SU(5)$ GUT as the right-handed neutrino is embedded in the **10** representation of the Flipped $SU(5)$ instead of the adjoint representation.

1.3.3 *The* $SO(10)$ *GUT*

There are two compact simple Lie groups of rank five admitting complex representations of which $SU(6)$ does not yield any new features distinct from those of $SU(5)$. Complete unification is possible with the symmetry group $SO(10)$ with one universal gauge coupling α_G and one family of quarks and leptons sitting in the 16-dimensional spinor representation which can be seen to be composed of the fields q, u^c, d^c, l, e^c and $\mathcal{N}$ which under the $SU(3) \times SU(2) \times U(1)$ has the following decomposition:

	$SU(3)$	$SU(2)$	$U(1)$
q	$\square$	$\square$	$+\frac{1}{6}$
u^c	$\overline{\square}$	1	$-\frac{2}{3}$
d^c	$\overline{\square}$	1	$+\frac{1}{3}$
l	1	$\square$	$-\frac{1}{2}$
e^c	1	1	$+1$
$\mathcal{N}$	1	1	0

The $SO(10)$ GUT has two inequivalent maximal breaking patterns

$$SO(10) \to SU(5) \times U(1)_\chi$$

and

$$SO(10) \to SU(4) \times SU(2)_L \times SU(2)_R.$$

In the first case we obtain Georgi-Glashow $SU(5)$ if Q_{em} is given in terms of $SU(5)$ generators completely or so-called Flipped $SU(5)$ if Q_{em} is in part contained in $U(1)_\chi$. In the latter, we have the Pati-Salam symmetry. If $SO(10)$ breaks directly to the SM at M_G, then we retain the prediction for gauge coupling unification.

1.3.4 *Larger Symmetry Groups*

Lastly, larger symmetry groups can be and have been considered in the literature. For example, E_6 which admits a fundamental representation **27** which transforms as $[\mathbf{16}+\mathbf{10}+\mathbf{1}]$ under $SO(10)$. The problem with exploring such large symmetry groups is that there are many states that as of yet have not been observed in nature and as a result must not appear in the low-energy effective field theory. We briefly review E_6 as it will underlie the analysis of the string-derived Z' model presented in Chapter 3.

1.3.5 E_6 *As GUT*

There are two compact simple Lie groups of rank six admitting complex representations of which $SU(7)$ does not yield any new features whereas E_6 admits

$$SO(10)\times U(1), \qquad SU(6)\times SU(2), \qquad SU(3)\times SU3)\times SU(3)$$

introducing some novelties. All the various E_6 breaking patterns

$$(1a)\ E_6 \ \rightarrow\ SO(10)\times U(1)\rightarrow SU(5)\times U(1)^2 \tag{1.1}$$

$$(1b)\ E_6 \ \rightarrow\ SO(10)\times U(1)\rightarrow SU(4)\times SU(2)\times SU(2)\times U(1) \tag{1.2}$$

$$(2a)\ E_6 \ \rightarrow\ SU(6)\times SU(2)\rightarrow SU(5)\times SU(2)\times U(1) \tag{1.3}$$

$$(2b)\ E_6 \ \rightarrow\ SU(6)\times SU(2)\rightarrow SU(4)\times SU(2)\times SU(2)\times U(1) \tag{1.4}$$

$$(2c)\ E_6 \ \rightarrow\ SU(6)\times SU(2)\rightarrow SU(3)\times SU(3)\times SU(2)\times U(1) \tag{1.5}$$

$$(3)\ E_6 \ \rightarrow\ SU(3)\times SU(3)\times SU(3) \tag{1.6}$$

reduce to one of the two extended MSSM models of rank 6

$$
\begin{aligned}
E_6 &\rightarrow SU(3) \times SU(2) \times [U(1)^3] \\
E_6 &\rightarrow SU(3) \times SU(2) \times [SU(2) \times U(1)^2]
\end{aligned}
$$

which are equivalent up to linear transformations. As will be shown later in Chapter 3, the SM representations are accommodated in the $\mathbf{27}$ of E_6 as

$$
\mathbf{27} = \begin{cases} \mathbf{16}_{+\frac{1}{2}} & F_L + F_R = (q, u^c, d^c, l, e^c, \mathcal{N}) \\ \mathbf{10}_{-1} & D + h \\ \mathbf{1}_{+2} & S \end{cases} .
$$

1.4 *Going Supersymmetric*

Despite the many promising finds, there is a plague of questions which can not be cured in the setting of the four-dimensional theories [2]. Among these issues are the gauge hierarchy problem - the immense ambiguity in stabilising the quantum effects of the GUT scale at the level of the electroweak scale, which take the form of ultraviolet divergences and the fact that the unification of Higgs or other gauge bosons with leptons and quarks seems to be an insuperable obstacle. Throughout the years, various developments in the field altered the face of GUTs. The most fruitful development was that of supersymmetry (SUSY).[3] This is the only existing symmetry which successfully places bosons and fermions on an equal footing. Initially, SUSY was proposed independently and much earlier than string theory,

[3]refer to Appendix C for details.

so indeed the symmetry may be there without strings. Since the breaking scale of SUSY cannot be constrained theoretically, apart from obvious phenomenological lower bounds for the masses of the super-partners, that have to be more massive than the energy scale accessible experimentally to date. But the idea of SUSY gained prominence in the early construction of string theories. There were two approaches of which the celebrated model of Ramond and Neveu-Schwarz [3], proposed SUSY to be a substantial symmetry of the two-dimensional world-sheet.

Even though no signs of space-time SUSY in nature have been observed to date, there is no denying that the four-dimensional world we reside in is non-supersymmetric. Nevertheless, the primary goal of space-time SUSY is to amend one of the greatest flaws in the SM, which is none other than the gauge hierarchy problem [4]. What makes SUSY a superb candidate for this achievement is the fact that it guards the effective field theory (EFT), such as the SM, against unwanted UV completions of any kind. As a result, supersymmetric extensions of the SM have been constructed. A well-known example of such a theory is the simplest extension of the SM, the Minimal Supersymmetric Standard Model (MSSM). This model demands that for every SM particle there exists a supersymmetric counterpart, differing by a half-integer spin. For the MSSM to serve its theoretical purposes, the masses of the superpartners need to be at the `TeV` scale. The following table highlights the particle content of the MSSM:

Non-SUSY Sector	SUSY Partners
2 Higgs (required)	Higgsinos
Leptons Quarks	sLeptons sQuarks
Gluons Photon $W^{\pm}$ Z	Gluinos Photino Winos: $\tilde{W}^{\pm}$ Zino: $\tilde{Z}$

Table 1.2: The particle content of the MSSM.

In fact, SUSY is a global symmetry of space-time, if gravity is not taken into account. Embedding it in a consistent dynamical field theory of gravity, SUSY is elevated into a local space-time symmetry, called supergravity. The latter has far fewer parameters than the supersymmetric models in flat space-times. For instance the non-supersymmetric SM has 28 parameters if one includes neutrino masses, which is a slight extension of the SM, while in its minimal supersymmetric extension there are more than a 100 parameters, 105 to be exact.

1.5 *The Gauge Coupling Unification*

One of the occurrence with respect to the foundations of the SM is that each of the gauge interactions is accompanied by a different coupling constant or coupling strength, with all three of them being independent and seemingly unrelated. In this scenario the crucial idea which arises due to the effect of higher-order quantum corrections in the gauge boson propagators is that of running coupling constants - a terminology used to describe the variation of the coupling strength as a function of a typical energy scale $\tilde{\mu}$. A mathematical account of the running of the gauge couplings is provided by the specification of the renormalization scheme. Variation of the coupling strengths is then solely determined by the particle content and their couplings inside the higher-order loops of the gauge bosons. The value of this variation is expressed in terms of a set of renormalization group equations (RGEs).

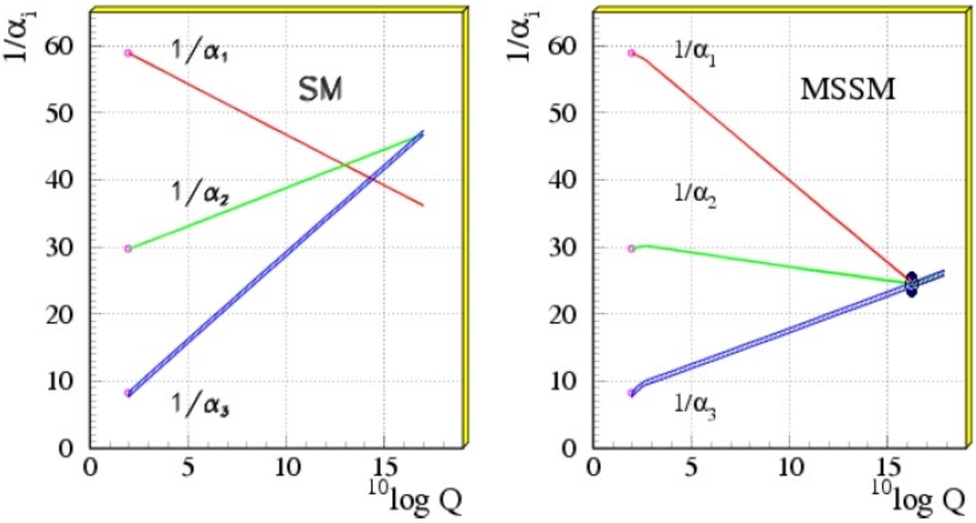


Figure 1.1: Gauge coupling unification in non-SUSY GUTs on the left against SUSY GUTs on the right where the evolution is calculated according to the formulae in [6], p.199 assuming that the masses of the super-partners are in the range of the `TeV` scale. The red, green and blue lines correspond to the running of the electromagnetic, weak and strong couplings respectively.

Based on experimental data, a graphical representation of this result was originally presented in the well-known paper [5]. A modified form of the original graph is adopted by [7] and is shown in Figure 1.1 demonstrating that within the SM the unification of gauge couplings is an unachievable task.

1.5.1 *In The SM*

The set of RGEs at one-loop for the SM gauge couplings, as computed in the $\overline{\mathrm{MS}}$ is

$$\frac{d\alpha_i}{dt} = \frac{1}{4\pi} b_i \alpha_i^2, \qquad t = \log\left(\frac{\tilde{\mu}}{\tilde{\mu}_0}\right), \qquad i = 1,2,3$$

where $\alpha_i \equiv \frac{1}{4\pi} g_i^2$ and $\tilde{\mu}_0$ is a very high energy scale which is chosen arbitrarily. For the SM the coefficients, as can be found in [5], are

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + N_{FAM} \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} + N_{HIGGS} \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{41}{10} \\ -\frac{19}{6} \\ -7 \end{pmatrix}$$

with $N_{FAM} = 3$ being the number of chiral matter families and $N_{HIGGS} = 1$ being the number of Higgs doublets.

1.5.2 *In The MSSM*

In a simplest supersymmetric generalisation of the SM, the MSSM, the calculation of the gauge coupling unification is very similar to that in the non-supersymmetric counterpart. First, it is assumed that the threshold for the supersymmetric particles is somewhere around 1 `TeV` so that up to this scale, the renormalization group equations run just as they do in the SM. Above the 1 `TeV`, there are new contributions to be found from the super-partners of the SM particles and the RG evolution of the three gauge couplings is modified and is now based on a new set of coefficients

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_{FAM} \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_{HIGGS} \begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{33}{5} \\ 1 \\ -3 \end{pmatrix}$$

with $N_{FAM} = 3$ being the number of chiral matter families and $N_{HIGGS} = 2$ being the number of Higgs doublets. It is notable that the running of the strong coupling is much weaker in the case of the MSSM. This occurrence is mainly due to the dominating one-loop b_i contributions. Similarly, the running of the weak coupling has opposite direction while the running of the electromagnetic coupling is faster than in the SM case. Finally, there is a contribution from all the Higgs fields and their supersymmetric counterparts.

1.6 *Thesis Outline*

The chapters of this thesis are organised as follows:

Chapter 2: The set up of the free fermionic formulation of the heterotic string is shown. The one-loop partition function is defined at an arbitrary point in the moduli space. This enables the derivation of specific constraints deduced from modular invariance. These constraints lead to the ABK rules, where phenomenologically realistic model building from string theory can be achieved.

Chapter 3: The heterotic string models in the free fermionic formulation give rise to some of the most realistic string models to date, which possess $N = 1$ space-time supersymmetry. The characteristics of the required spectrum are well motivated in heterotic string constructions that allow for a light Z'. Anomaly cancellation of the $U(1)_{Z'}$ symmetry requires the existence of the SM singlet and vector-like states in the vicinity of the $U(1)_{Z'}$ breaking scale. This chapter contains material that has appeared in publication [80] presented by the author.

Chapter 4: A concrete example of a non–supersymmetric, non–tachyonic, heterotic string vacuum will be presented motivated by the lack of evidence in support of supersymmetry at the LHC. We explore what may be learned in this context from the quasi–realistic free fermionic models. This chapter contains material that has appeared in publication [81] presented by the author.

Appendix: Various elements are covered in this section primarily to help avoid any unnecessary digression as the goal of the thesis is to essentially fulfil the arduous task of bringing together two different worlds that is presenting two different string-derived models in the free fermionic formulation one with and the other without supersymmetry.

2

The Free Fermionic Formalism

Those who dream by day are cognizant of many things which escape those who dream only by night.

Edgar Allan Poe

In this chapter, the framework of the free fermionic construction [38] of the four-dimensional heterotic string is shown. The partition function at an arbitrary point in the moduli space is derived with the description of the self-dual point under T-duality which is then followed by the rewriting of the partition function in the most general way and thence enabling the derivation of the constraints on the form of the partition function. This is further followed by the derivation of all the necessary constraints for the construction of the free fermionic models. Having derived these tools for the construction, a summary of the ABK rules is given. The chapter closes with the discussion of the NAHE model as a prototypical example and the NAHE-based phenomenologically viable models.

2.1 *The Road to Heterotic Strings*

String theory, though initially unpopular, gained prominence due to the works of Green and Schwarz that it was a consistent theory. The theory then followed two revolutionary stages due to its problematic nature. The first one is the "first revolution of string theory" in which string theory was transformed into a realistic theory called superstring theory. The second is called the "second revolution of string theory" where Witten's work identified the five different superstring theories as the different limits of a single theory call M-theory.

Superstring theory generalizes the classical bosonic string theory by incorporating SUSY allowing for successful inclusion of the fermions and eliminating the tachyonic state. There are 5 different superstrings. It also includes ghost states which when removed leads to 10 space-time dimensions for the superstrings. Superstrings are also characterized by open and closed strings, oriented and unoriented strings as well as the number of supercharges in the theory. All superstrings eliminate the tachyonic state from the physical spectrum and include a graviton so superstrings describe gravity in a natural way. In short, a relativistic string theory is necessarily

- a theory of general relativity;
- a theory of gauge interactions;
- finite. That is to say that the world-sheet (the area swept out by the string at it moves in space-time) is smooth. This is exactly the reason why in perturbation theory the usual UV divergences of the quantum field theories of relativistic particles do not crop up.

There are two heterotic string theories [9–12] both of which describe closed oriented strings and in both the $N = 1$ SUSY results from the decoupling of the left-moving and right-moving modes. One of these heterotic string theories comes with the $SO(32)$[4] as the gauge group and the other with $E_8 \times E_8$. Such string theories can be regarded as the fusion between the classical bosonic string theory and the superstring theory. The 16 extra dimensions of the bosonic theory are regarded as abstract, mathematical entities, not as space-time dimensions but rather as internal-shift symmetries.

The idea of heterotic strings is a theory of closed superstrings with decoupled left-moving and right-moving modes that preserve the best of both worlds: strings and superstrings. The resulting theory is large enough to incorporate the desired features of the SM. By allowing the left-moving modes to be supersymmetric, fermions can be included in the theory and the tachyons can be projected out. On the other hand, non-Abelian gauge theory is allowed for the right-moving modes by way of adding Majorana-Weyl fermions λ^A in the absence of SUSY.

As will be seen, in the free fermionic construction of the heterotic string in four dimensions, all the extra degrees of freedom needed to cancel the conformal anomaly are represented as free fermions propagating on the string world-sheet. In the light cone-gauge the supersymmetric left-moving sector includes the two transverse space-time fermionic coordinates ψ^μ and 18 internal world-sheet real fermions χ^I, y^I and ω^I whereas the right right-moving bosonic sector contains the 44 real world-sheet fermions λ^a.

[4]see Appendix D for analysis of the $SO(32)$ gauge group. The analysis for $E_8 \times E_8$ is similar.

2.2 *The Free Fermionic Construction*

Our aim is to construct a model with $N = 1$ world-sheet SUSY on the left-movers and no SUSY on the right-movers, which will at the end generate a $N = 1$ space-time SUSY. This could seem quite weird at first glance because cancelling the conformal anomaly requires that the space-time dimension for right-movers is 26 and 10 for the left-movers, although one would expect to have the same space-time dimension.

The critical dimension of the space-time is $D = 26$ for the bosonic string due to no-ghost theorem which simply states that there are no ghosts (negative norm states) if and only if the dimension of space-time is no greater than 26. However, the bosonic string suffers from two main problems. First of all, a tachyon exists in the ground state of the spectrum of the bosonic string which renders the spectrum unphysical. The other problem is that there are no fermions. Introducing SUSY, which is the symmetry that interchanges bosons and fermions where all particles have the same mass and also have the same gauge charge in the supermultiplet. Since SUSY is a gauge symmetry, this allows the writing of the conformal anomaly as

$$C_{anomaly} = C_{bosonic\ ghosts} + C_{fermionic\ ghosts} + C_{bosonic} + C_{fermionic}$$

where $C_{bosonic\ ghosts} = -26$ for the bosonic string, $C_{fermionic\ ghosts} = 11$ which is achieved by introducing fermions in string action, $C_{bosonic} = D$ in D-dimensions which results from the fact that the central charge of the free boson is $c = 1$ and $C_{fermionic} = \frac{D}{2}$ in D-dimensions which results from the fact that the central charge of the free fermion is $c = \frac{1}{2}$. It can be clearly seen that for $C_{anomaly} = 0$ if $D = 10$ which is the critical dimension of the superstring.

As the left-movers and right-movers decouple for the heterotic string, the model can have more fermions added which would be either right-moving or left-moving. The thought is that they would contribute to cancel the central charges

$$C_L = -26 + 11 + D_L + \frac{D_L}{2} + \frac{N_{f_L}}{2} = 0 \qquad (Superstring)$$
$$C_R = -26 + D_R + \frac{N_{f_R}}{2} = 0 \qquad (Bosonic\ String)$$

where D_L and D_R are the left and right space-time dimensions respectively and thus reduce the space-time critical dimension. So if 44 right-moving fermions and 18 left-moving fermions are added, the conformal anomaly would become

$$C_L = -26 + 11 + D_L + \frac{D_L}{2} + \frac{18}{2} = 0 \tag{2.1}$$
$$C_R = -26 + D_R + \frac{44}{2} = 0 \tag{2.2}$$

which for $D_L = D_R = 4$ means that the theory is conformally invariant. These left-moving and right-moving fermions propagate the world-sheet. Moreover, we have the following set of fields

$$X_+^\mu,\ \psi_+^\mu,\ \lambda_+^j$$

in the left-moving sector whilst

$$X_-^\mu,\ \lambda_-^j$$

in the right-moving sector where $\mu = 0, ..., 3,\ i = 1, ..., 44$ and $j = 1, ..., 18$. Adopting complex coordinates defined by

$$z = \tau + i\sigma \text{ and } \overline{z} = \tau - i\sigma$$

the world-sheet fields can now be defined as functions of z and $\overline{z}$ given by the following:

$$
\begin{aligned}
X^\mu(z,\overline{z}) \quad &, \quad \mu = 1,2 \\
\psi^\mu(z) \quad &, \quad \mu = 1,2 \\
\lambda^i(z) \quad &, \quad i = 1, ..., 18 \\
\overline{\lambda}^j(\overline{z}) \quad &, \quad j = 1, ..., 44.
\end{aligned}
$$

In the light-cone gauge the space-time bosons and fermions have only two degrees of freedom, namely the transverse coordinates, where the heterotic action can now take the form

$$S = \frac{1}{\pi}\int d^2z\bigg(\partial_z X_\mu \partial_{\overline{z}} X^\mu - 2i\psi^\mu\partial_z\psi_\mu - 2i\sum_{i=1}^{18}\lambda^i\partial_z\lambda^i - 2i\sum_{i=1}^{44}\overline{\lambda}^j\partial_{\overline{z}}\overline{\lambda}^j\bigg).$$

where $\psi^\mu = \psi^\mu(z)$ and $\overline{\psi}^\mu = \overline{\psi}^\mu(\overline{z})$ corresponds to the left-moving and right-moving fermionic fields respectively.

The world-sheet field content, which forms the basis of the free fermionic construction of the heterotic string in four dimensions, is given in Table 2.1 below.

Sector	Label	Details
Supersymmetric	ψ^μ, χ^i	Real superpartners of the bosonic coordinate X^μ and the six compactified directions in the bosonic formulation.
Left-Moving		
Holomorphic	y^i, ω^i	Real fermions that correspond to the bosons describing the six compactified directions in the bosonic formulation.
Bosonic	$\bar{y}^i$, $\bar{\omega}^i$	Real fermions that correspond to the bosons describing the six compactified dimensions in the orbifold formulation.
Right-Moving	$\bar{\psi}^{1,\dots,5}$,$\bar{\eta}^{1,2,3}$	Complex fermions that describe the visible gauge sector which correspond to the eight internal shifts in T^{16}.
Anti-Holomorphic	$\bar{\phi}^{1,\dots,8}$	Complex fermions that describe the hidden gauge sector which correspond to the other eight internal shifts in T^{16}.

Table 2.1: States that describe the world-sheet, where the internal freely propagating fermions have been separated from those of the space-time coordinates. As can be seen, there are 18 in the left-moving, supersymmetric sector and 44 in the right-moving, bosonic sector in the light-cone gauge.

In the Polyakov picture, string theory is formulated as a perturbative sum over a path integral on the world-sheet, a genus g Riemann surface. The free fermions propagate around the non-contractible loops of this surface. As a result, the boundary conditions need to be specified for each world-sheet fermion. Moreover, the world-sheet SUSY needs to be preserved, which means that the supercurrent T_F must be uniquely defined up to a sign, under the transformation of the world-sheet fermions. The world-sheet supercharge is defined as

$$T_F = \psi^{\mu}\partial X_{\mu} + i\sum_{I=1}^{6}\chi^I y^I \omega^I$$

where $\chi^I(z)$, $y^I(z)$ and $\omega^I(z)$ for $i = 1, ..., 6$ transform as the adjoint representation of $SU(2)^6$. The transport properties of the left-moving and right-moving fermions around a non-contractible loop of the torus show that any configuration of the boundary conditions in some basis consisting of 64 fermions is realized which can either be real or complex where two real fermions in each basis vector can pair to form a single complex fermion

$$\lambda_{ij} = \frac{1}{\sqrt{2}}(\lambda_i + i\lambda_j),$$

$$\lambda^*_{ij} = \frac{1}{\sqrt{2}}(\lambda_i - i\lambda_j).$$

given that they share the same boundary condition.

The free fermionic construction is based on the use of the one-loop partition function defined by a set of boundary condition vectors and a set of projection coefficients associated to each pair of these vectors. Our assumption is that for each set of basis vectors and a set of associated coefficients, referred to as the one-loop phases, there is a consistent model of free fermionic $4D$-heterotic superstring.

2.2.1 *The Torus and Modular Invariance*

At tree level, all reparametrizations are local and quantum corrections are not taken into account, but at higher loops further constraints will arise. Therefore, it is instructive to to look at the one-loop vacuum to vacuum amplitude with no external states. This is precisely the one-loop partition function.

The one-loop partition function includes all the physical states and is sufficient to derive some constraints on the model. It is an integration over all the possible world-sheets, in the case of the one-loop partition function, the world-sheets are all the inequivalent tori. On the world-sheets, two boundary conditions need to be specified for the two non-contractible loops of the torus (poloidal and toroidal), as depicted in Figure 2.1, for each of the free fermionic fields.

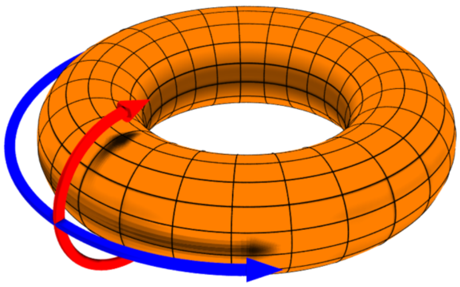


Figure 2.1: A diagram displaying the poloidal direction, represented by the red arrow, and the toroidal direction represented by the blue arrow, the two non-contractible loops of the torus.

The torus can be mapped to the complex plane by cutting it along its two non-contractible loops. It can then be characterized by specifying two finite and non-zero periods in the complex plane λ_1 and λ_2 with a non-real ratio

$$z \sim z + \lambda_1, \qquad z \sim z + \lambda_2.$$

The torus can then be identified with the complex plane modulo a two-dimensional lattice $\Lambda_{\lambda_1,\lambda_2}$ where $\Lambda_{\lambda_1,\lambda_2} = \{m\lambda_1 + n\lambda_2,\ m, n \in \mathbb{Z}\}$. Using the reparametrization $z \to z/\lambda_2$ the torus s equivalent to one whose periods are 1 and $\tau = \frac{\lambda_1}{\lambda_2}$. In other words, the torus is left invariant by the following transformations:

$$
\begin{aligned}
T &: \tau \to \tau + 1, \quad \text{redefines the same torus,} \\
S &: \tau \to -\frac{1}{\tau}, \quad \text{swaps the two coordinates and reorients the torus.}
\end{aligned}
$$

These transformations span a group of transformations known as the modular group

$$
\tau \to \frac{a\tau + b}{c\tau + d}, \quad a, b, c, d \in \mathbb{Z}, \ ad - bc = 1
$$

where any function invariant under these transformations is called modular invariant. The modular group is $PSL(2,\mathbb{Z}) = SL(2,\mathbb{Z})/\mathbb{Z}_2$ where the division by $\mathbb{Z}_2$ takes the equivalence of an $SL(2,\mathbb{Z})$ matrix and its negative into account. The moduli space $\mathcal{M}$ of the torus is

$$
\mathcal{M} \simeq \mathcal{H}/PSL(2,\mathbb{Z}).
$$

The fundamental domain, as illustrated in Figure 2.2, can be taken as

$$
\mathcal{F} = \{\tau \big| |\tau| \geq 1, \ |\mathrm{Re}\,\tau| \leq 1/2, \ \mathrm{Im}\,\tau > 0\}.
$$

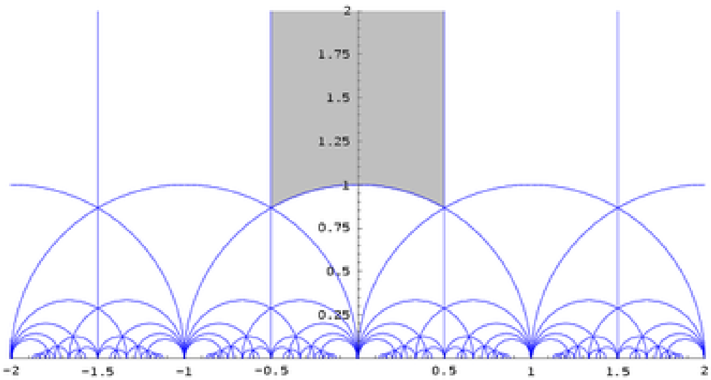


Figure 2.2: The shaded region displays the fundamental domain of the modular group of the torus.

Thus the partition function is a sum over this domain in order to integrate over all conformally inequivalent tori. Consequently, the modular transformations spanned by T and S are invariant. One additional requirement is that the partition function does not depend on the parametrization of the tori.

2.2.2 *The Boundary Conditions*

The boundary conditions express the shifts in the integer phase of the fermionic fields under parallel transport around these non-contractible loops

$$f \to -e^{\,i\pi\alpha(f)} f \tag{2.3}$$

where f is the fermionic field $\alpha(f) = 0$ or 1 for Neveu-Schwarz (NS) and Ramond (R) real fermions respectively and $\alpha(f) \in (-1, +1]$ for the complex fermions. In other words, the fermions which propagate around the string have a boundary condition around the string in the direction of the σ coordinate and they also can pick up a phase by propagating along the τ dimension. As there are two non-contractible loops of the torus, the complete phase assignment for a fermion can be expressed as a set of two phases

$$\begin{bmatrix} \alpha(f) \\ \beta(f) \end{bmatrix}.$$

A set of specified phases for all the fermions for one non-contractible loop is called a spin-structure written as a 64-dimensional vector

$$\alpha = \{\alpha(\psi^1), ..., \alpha(\overline{\phi}^8)\}.$$

Then to complete the spin-structure assignment for all the fermions on the torus, two vectors can then be defined by

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix}.$$

The partition function is given by

$$Z = \sum_{\alpha,\beta\in\Xi} C\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \mathrm{Tr}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \tag{2.4}$$

which is a sum over all spin structures where $C\binom{\alpha}{\beta}$ are Generalized GSO (GGSO) projection coefficients and $\mathrm{Tr}\binom{\alpha}{\beta} \equiv \mathrm{Tr}(\mathrm{e}^{\mathrm{i}\pi\beta\mathrm{F}_\alpha}\mathrm{e}^{\mathrm{i}\pi\tau\mathrm{H}_\alpha})$ with H_α being the hamiltonian, is the trace over the mode excitations of the world-sheet fields in the sector α, subject to the GSO projections induced by the sector β. Requiring invariance under modular transformations results in a set of constraints on the allowed spin structures and the GGSO projection coefficients. Furthermore, the partition function has 68 fields in total made of 64 internal fermions and two each for X^μ_L, X^μ_R and ψ^μ. The bosonic fields have no choice of boundary conditions, they are only periodic. However the fermionic field ψ^μ can be periodic or anti-periodic, so the partition function must include all possible combinations of 64 boundary conditions of the fermions, and this is integrated over all the inequivalent tori. Thus, the boundary conditions take the values $\alpha, \beta = 1, ..., 64$.

In the models considered in this thesis, the following notations for the real and complex fermions, see Table 2.1 for details, will be used

- Real Left Fermions

$$\{\psi^1,\psi^2,\chi^1,y^1,\omega^1,\chi^2,y^2,\omega^2,\chi^3,y^3,\omega^3,\chi^4,y^4,\omega^4,\chi^5,y^5,\omega^5,\chi^6,y^6,\omega^6\}$$

- Real Right Fermions

$$\{\bar{y}^1, \bar{\omega}^1, \bar{y}^2, \bar{\omega}^2, \bar{y}^3, \bar{\omega}^3, \bar{y}^4, \bar{\omega}^4, \bar{y}^5, \bar{\omega}^5, \bar{y}^6, \bar{\omega}^6\}$$

- Complex Left Fermions

$$\{\psi^\mu, \chi^{12}, \chi^{34}, \chi^{56}\}$$

- Complex Right Fermions

$$\{\bar{\psi}^1, \bar{\psi}^2, \bar{\psi}^3, \bar{\psi}^4, \bar{\psi}^5, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3, \bar{\phi}^1, \bar{\phi}^2, \bar{\phi}^3, \bar{\phi}^4, \bar{\phi}^5, \bar{\phi}^6, \bar{\phi}^7, \bar{\phi}^8\}$$

Here, the first 4 complex left and the last 16 complex right fermions are given in complex form and the remaining fermions y and ω are not paired. This is due to the fact that their boundary conditions do not always allow a pairing.

2.2.3 *The One-Loop Partition Function*

Looking at the partition function and thinking of the path integral on a torus of parameter $\tau = \tau_1 + i\tau_2$ as formed by a field on a circle that been evolved for Euclidean time $2\pi\tau_2$, translated by $2\pi\tau_1$ and identified with the initial circle. The generator of the translations in time is the Hamiltonian $H = L_0 + \overline{L}_0 + \frac{1}{24}$ whereas the generator of translation in space is the momentum operator $P = L_0 - \overline{L}_0$. The identification of the ends of the cylinder thus formed is realized by taking the trace over the Hilbert space of states

$$\begin{aligned} Z(\tau_1, \tau_2) &= \sum_{s \in \mathcal{H}} \langle s | e^{2\pi i \tau_1 P} e^{-2\pi i \tau_2 H} | s \rangle \\ &= \mathrm{Tr}\,_{\mathcal{H}} e^{2\pi i \tau_1 P} e^{-2\pi i \tau_2 H} \end{aligned}$$

which can be rewritten using $q \equiv e^{2\pi i\tau}$ as

$$Z(\tau) = q^{-1/48}\overline{q}^{-1/48}\mathrm{Tr}_{\mathcal{H}} q^{L_0}\overline{q}^{\overline{L}_0}. \tag{2.5}$$

This can be calculated for each fermion as we know how L_0 acts on the states space. If the time boundary condition is anti-periodic (NS), then the partition function is just given by the trace with L_0 acting on the appropriate R or NS Fock space:

$$\begin{aligned} Z^{NS}_{NS}(\tau) &= \mathrm{Tr}_{NS}\ q^{L_0-1/48}, \\ Z^{NS}_{R}(\tau) &= \mathrm{Tr}_{R}\ q^{L_0-1/48}. \end{aligned} \tag{2.6}$$

When the time boundary condition is periodic (R) the definition of the trace is modified:

$$\begin{aligned} Z^{R}_{NS}(\tau) &= \mathrm{Tr}_{NS}\ (-1)^F q^{L_0-1/48}, \\ Z^{R}_{R}(\tau) &= \mathrm{Tr}_{R}\ (-1)^F q^{L_0-1/48} \end{aligned} \tag{2.7}$$

where F is the fermion number operator, defined by the relations

$$\begin{aligned} F(f) &= +1, \text{ if f is a fermionic oscillator,} \\ F(f) &= -1, \text{ if f is the complex conjuate of a fermionic oscillator,} \\ F|+\rangle_R &= 0, \\ F|-\rangle_R &= -1, \end{aligned}$$

where $|+\rangle_R = |0\rangle$ is the state of a degenerated vacuum without an oscillator and $|-\rangle_R = f_0^\dagger|0\rangle$ is the state of a degenerated vacuum with zero mode oscillator. The partition function must include all the possible combinations of the boundary conditions and therefore it is a sum over all spin-structures. All the previous work nicely leads to the complete partition function

$$Z = \int_{\mathcal{F}} \frac{d\tau d\overline{\tau}}{(\mathrm{Im}\tau)^2} Z_B^2 \sum_{\substack{spin\\structure}} C\binom{a}{b} \prod_{f=1}^{64} Z_F \begin{bmatrix} \alpha(f) \\ \beta(f) \end{bmatrix},$$

where

- $\frac{d\tau d\overline{\tau}}{(\mathrm{Im}\tau)^2}$ is the invariant measure under the modular transformations of the torus.

- Z_B is the bosonic contribution

$$Z_B = \frac{1}{\sqrt{|\mathrm{Im}\tau|}\eta(\tau)}$$

  where

$$\eta(\tau) = q^{\frac{1}{12}} \prod_n (1 - q^{2n}), \quad \text{with } q = e^{2\pi i\tau}$$

  is the Dedekind eta function.

- $C\binom{\alpha}{\beta}$ are coefficients on the spin-structures that are yet to be determined.

- $Z_F \begin{bmatrix} \alpha(f) \\ \beta(f) \end{bmatrix}$ is the contribution of the fermion f which depends solely on its boundary conditions $\alpha(f)$ and $\beta(f)$. It can be calculated by the use of

Equation (2.5) to obtain the following results:

$$
\begin{aligned}
Z_F \begin{bmatrix} 0 \\ 0 \end{bmatrix} &= \sqrt{\frac{\vartheta_3}{\eta}} \\
Z_F \begin{bmatrix} 0 \\ 1 \end{bmatrix} &= \sqrt{\frac{\vartheta_4}{\eta}} \\
Z_F \begin{bmatrix} 1 \\ 0 \end{bmatrix} &= \sqrt{\frac{\vartheta_2}{\eta}} \\
Z_F \begin{bmatrix} 1 \\ 1 \end{bmatrix} &= \sqrt{\frac{\vartheta_1}{\eta}}
\end{aligned}
$$

where ϑ_i are defined as

$$
\vartheta_1 = \vartheta \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \vartheta_2 = \vartheta \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \vartheta_3 = \vartheta \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \vartheta_4 = \vartheta \begin{bmatrix} 0 \\ 1 \end{bmatrix}
$$

and

$$
\vartheta \begin{bmatrix} a \\ b \end{bmatrix} = \sum_{n \in \mathbb{Z}} q^{\frac{(n-a/2)^2}{2}} e^{2\pi i (n-b/2)(n-a/2)}.
$$

is the Jacobi theta function.

These formulae should be complex conjugated for the right-moving fermions.

2.2.4 *The Modular Invariance Constraints*

The invariance of the partition function under the modular transformations give further constraints for model building. Since the measure element and the bosonic contribution are modular invariant, imposing modular invariance on the remaining terms in the partition function results in additional constraints. Under $\tau \to \tau + 1$, the following transformations are given:

$$\begin{aligned} \eta &\longrightarrow e^{i\pi/12}\eta, \\ \vartheta_1 &\longrightarrow e^{i\pi/4}\vartheta_1 \\ \vartheta_2 &\longrightarrow e^{i\pi/4}\vartheta_2 \\ \vartheta_3 &\longleftrightarrow \vartheta_4 \end{aligned}$$

and under $\tau \to -\frac{1}{\tau}$:

$$\begin{aligned} \eta &\longrightarrow (-i\tau)^{1/2}\eta \\ \frac{\vartheta_1}{\eta} &\longrightarrow e^{-i\pi/2}\frac{\vartheta_1}{\eta} \\ \frac{\vartheta_2}{\eta} &\longleftrightarrow \frac{\vartheta_4}{\eta} \\ \frac{\vartheta_3}{\eta} &\longrightarrow \frac{\vartheta_3}{\eta}. \end{aligned}$$

Since the partition function is a product of the spin-structures of 64 fermions, the modular transformations will take the spin-structure from one to another. Modular invariance requires that both spin-structures related by these transformations need to be present in the partition function with equal weight. This gives the following constraints:

$$C\binom{\alpha}{\beta} = e^{i\frac{\pi}{4}(\alpha\cdot\alpha+\mathbf{1}\cdot\mathbf{1})}C\binom{\alpha}{\beta-\alpha+1}, \tag{2.8}$$

$$C\binom{\alpha}{\beta} = e^{i\frac{\pi}{2}\alpha\cdot\beta}C\binom{\beta}{\alpha}^*, \tag{2.9}$$

where $\mathbf{1}$ is the vector corresponding to periodic boundary conditions for all fermions and the product $\alpha \cdot \beta$ is defined as

$$\alpha \cdot \beta = \left\{ \frac{1}{2} \sum_{real\ left} + \sum_{complex\ left} - \frac{1}{2} \sum_{real\ right} - \sum_{complex\ right} \right\} \alpha(f)\beta(f).$$

Another constraint arises when considering higher order loops

$$C\binom{\alpha}{\beta} C\binom{\alpha'}{\beta'} = \delta_\alpha \delta_{\alpha'} e^{-i\frac{\pi}{2}\alpha\cdot\alpha'} C\binom{\alpha}{\beta+\alpha'} C\binom{\alpha'}{\beta'+\alpha} \tag{2.10}$$

where δ_α is the space-time spin statistics index defined as

$$\delta_\alpha = e^{i\pi\alpha(\psi^\mu_{1,2})} = \begin{cases} +1, & \text{if } \alpha(\psi^\mu_{1,2}) = 0, \\ -1, & \text{if } \alpha(\psi^\mu_{1,2}) = 1. \end{cases}$$

These constraints can be used to derive the rules for constructing the model. Making use of Equation (2.9) and Equation (2.10) with $\alpha' = \alpha$ and $\beta = 0$, implies that

$$C\binom{\alpha}{0}^2 = \delta_\alpha C\binom{\alpha}{0} C\binom{0}{0},$$

which means that either $C\binom{\alpha}{0} = 0$ or $C\binom{\alpha}{0} = \delta_\alpha$where $C\binom{0}{0} = 1$ is normalized. A set of vectors Ξ is then defined to be

$$\Xi = \left\{ \alpha \middle| C\binom{\alpha}{0} = \delta_\alpha \right\}.$$

Using Equation (2.9) and Equation (2.10), Ξ is taken to be an Abelian additive group and the spin-structures contributing to the partition function are pairs of elements in Ξ. Furthermore, if Ξ is taken to be finite and therefore the boundary conditions to be rational, we find the isomorphism

$$\Xi \cong \bigoplus_{i=1}^{k} \mathbb{Z}_{N_i}$$

which means that Ξ is generated by a set of basis vectors $\{b_1, ..., b_k\}$, such that

$$\sum_{i=1}^{k} m_i b_i = 0 \Leftrightarrow m_i = 0 \mod N_i \ \forall i$$

where N_i is the smallest possible integer where $N_i b_i = 0$. Taking the three vectors α, β, $\gamma \in \Xi$, Equation (2.10) can be expressed as

$$C\begin{pmatrix} \alpha \\ \beta + \gamma \end{pmatrix} = \delta_\alpha C\begin{pmatrix} \alpha \\ \beta \end{pmatrix} C\begin{pmatrix} \alpha \\ \gamma \end{pmatrix}. \tag{2.11}$$

Equation (2.8) with $\alpha = \beta$ gives

$$C\begin{pmatrix} \alpha \\ \alpha \end{pmatrix} = e^{-i\frac{\pi}{4}\alpha\cdot\alpha} C\begin{pmatrix} \alpha \\ \mathbf{1} \end{pmatrix}.$$

Manipulating Equation (2.9), Equation (2.10), Equation (2.11) and using the fact that β generates a finite group of order N_β, if N_{ij} is the least common multiple of N_i and N_j, it must satisfy

$$N_{ij} b_i \cdot b_j = 0 \text{ mod } 4.$$

For $i = j$, this constraint holds for odd N_i. However, if N_i is even, then there is an even stronger constraint in place

$$N_i b_i^2 = 0 \text{ mod } 8.$$

When all the constraints that we have derived in this section are satisfied rest assured that the modular invariance condition is satisfied and there are no further obstacles to be faced while assigning coefficients to pairs of elements of Ξ.

2.2.5 *The Hilbert Space*

Equation (2.7) and Equation (2.8) can be recast in the general case as

$$Z_F \begin{bmatrix} \alpha(f) \\ \beta(f) \end{bmatrix} = \mathrm{Tr}_{\,\alpha} \left[q^{H_\alpha} e^{i\pi\beta\cdot F_\alpha} \right]$$

where H_α is the Hamiltonian and F_α is the fermion number operator in the Hilbert space sector $\mathscr{H}_\alpha$ defined by the vector α. The partition function can then be written as a sum over sectors using the fact that the basis vectors b_i are generators of the discrete group $\mathbb{Z}_{N_i}$ and applying Equation (2.11)

$$\begin{aligned} Z \;\; = \;\; & \int_{\mathcal{F}} \frac{d\tau d\overline{\tau}}{(\mathrm{Im}\tau)^2} Z_B^2 \sum_{\alpha\in\Xi} \delta_\alpha \mathrm{Tr} \left\{ \prod_{b_i} \left(\delta_\alpha C \binom{\alpha}{b_i} e^{i\pi b_i\cdot F_\alpha} + ... \right.\right. \\ & \left.\left. ... + \left\{ \delta_\alpha C \binom{\alpha}{b_i} e^{i\pi b_i \cdot F_\alpha} \right\}^{N_i - 1} + 1 \right) e^{i\pi\tau H_\alpha} \right\}. \end{aligned}$$

The only states that appear in the partition function are those that realize the GGSO projection

$$e^{i\pi b_i\cdot F_\alpha} |S\rangle_\alpha = \delta_\alpha C \binom{\alpha}{b_i}^{*} |S\rangle_\alpha .$$

The full Hilbert space is therefore given as

$$\mathscr{H} = \bigoplus_{\alpha\in\Xi} \prod_{i=1}^{k} \left\{ e^{i\pi b_i\cdot F_\alpha} = \delta_\alpha C \binom{\alpha}{b_i}^{*} \right\} \mathscr{H}_\alpha .$$

2.3 *The Methodology*

It was shown earlier, that for each consistent heterotic string model, there exists a partition function defined by a set of vectors with boundary conditions and a set of coefficients associated to each pair of these vectors. Now that we have all the constraints at hand for model building, using the free fermionic formalism, it will now be shown that for each set of boundary conditions basis vectors and the set of associated coefficients, a set of general rules can be summarized for any model realized in the free fermionic formalism. These rules, originally derived by Antoniadis, Bachas, Kounnas in [38], are known as the ABK rules[5] First, these rules are presented followed by an example model. This will also be the working tool set for all the models carried out in this thesis and any understanding of the derivations presented earlier are not necessary here onwards for what follows. For further convenience, the vectors containing the boundary conditions used to define a model are called the basis vectors and the associated coefficients are called the one-loop phases that appear in the partition function.

2.3.1 *The ABK Rules*

One of the key elements is the set of basis vectors that defines Ξ, the space of all the sectors. For each sector $\beta \in \Xi$ there is a corresponding Hilbert space of states. Each basis vector b_i consists of a set of boundary conditions for each fermion denoted by

$$b_i = \{\alpha(\psi^{\mu}_{1,2}), ..., \alpha(\omega^6)|\alpha(\overline{y}^1), ..., \alpha(\overline{\phi}^8)\}$$

[5]These rules were also developed with a different formalism by Kawai, Lewellen and Tye in [39].

where $\alpha(f)$ is defined by

$$f \to -e^{\,i\pi\alpha(f)} f.$$

The b_i have to form an additive Abelian group and satisfy the constraints. If N_i is the smallest positive integer for which $N_i b_i = 0$ and N_{ij} is the least common multiple of N_i and N_j then the rules for the basis vectors, known popularly as the ABK rules, are given as

$$(1) \quad \sum m_i \cdot b_i = 0 \iff m_i = 0 \mod N_i \ \forall i \tag{2.12}$$

$$(2) \quad N_{ij} \cdot b_i \cdot b_j = 0 \text{ mod } 4 \tag{2.13}$$

$$(3) \quad N_i \cdot b_i \cdot b_i = 0 \text{ mod } 8 \tag{2.14}$$

$$(5) \quad b_1 = \mathbf{1} \iff \mathbf{1} \in \Xi \tag{2.15}$$

$$(4) \quad \text{Even number of real fermions} \tag{2.16}$$

where

$$b_i \cdot b_j = \left(\frac{1}{2} \sum_{\text{left real}} + \sum_{\text{left complex}} - \frac{1}{2} \sum_{\text{right real}} - \sum_{\text{right complex}} \right) b_i(f) \times b_j(f).$$

2.3.2 *Rules for the One-Loop Phases*

The rules for the one-loop phases are

$$C\binom{b_i}{b_j} = \delta_{b_j} e^{\frac{2i\pi}{N_j} n} = \delta_{b_i} e^{\frac{2i\pi}{N_i} m} e^{i\pi \frac{b_i \cdot b_j}{N_j} n} \tag{2.17}$$

$$C\binom{b_i}{b_i} = -e^{\frac{i\pi}{4} b_i \cdot b_j} C\binom{b_i}{1} \tag{2.18}$$

$$C\binom{b_i}{b_j} = e^{\frac{i\pi}{2} b_i \cdot b_j} C\binom{b_i}{1}^* \tag{2.19}$$

$$C\binom{b_i}{b_j + b_k} = \delta_{b_i} C\binom{b_i}{b_j} C\binom{b_i}{b_k} \tag{2.20}$$

where the spin-statistics index is defined as

$$\delta_\alpha = e^{i\alpha(\psi^\mu)\pi} = \begin{cases} 1, & \alpha(\psi_{1,2}) = 0 \\ -1, & \alpha(\psi_{1,2}) = 1 \end{cases} .$$

2.3.3 *The GGSO Projections*

To complete this construction, we have to impose another set of constraints on the physical states called the GGSO projections. The GGSO projection selects the states $|S\rangle_\alpha$ belonging to the α sector satisfying

$$e^{i\pi b_i \cdot F_\alpha} |S\rangle_\alpha = \delta_\alpha C \begin{pmatrix} \alpha \\ b_i \end{pmatrix}^* |S\rangle_\alpha \qquad \forall\ b_i \tag{2.21}$$

where

$$b_i \cdot F_\alpha = \left(\frac{1}{2} \sum_{\text{left real}} + \sum_{\text{left complex}} - \frac{1}{2} \sum_{\text{right real}} - \sum_{\text{right complex}} \right) b_i(f) \times F_\alpha(f) \tag{2.22}$$

where $F_\alpha(f)$ is the fermion number operator given by

$$F_\alpha(f) = \begin{cases} +1, & \text{if } f \text{ is a fermionic oscillator,} \\ -1, & \text{if } f \text{ is the complex conjugate.} \end{cases}$$

2.3.4 *The Massless String Spectrum*

As we are interested in low-energy physics, we are only interested in the massless states. The physical states in the string spectrum satisfy the level matching condition

$$M_L^2 = -\frac{1}{2} + \frac{\alpha_L \cdot \alpha_L}{8} + N_L = -1 + \frac{\alpha_R \cdot \alpha_R}{8} + N_R = M_R^2 \tag{2.23}$$

where $\alpha = (\alpha_L; \alpha_R) \in \Xi$ is a sector in the additive group, and

$$N_L = \sum_f (\nu_L); \qquad\qquad N_R = \sum_f (\nu_R); \tag{2.24}$$

The frequencies of the fermionic oscillators depending on their boundary conditions is taken to be

$$f \to -e^{\,i\pi\alpha(f)} f, \qquad f^* \to -e^{-i\pi\alpha(f)} f^*.$$

The frequency for the fermions is given by

$$\nu_{f,f^*} = \frac{1 \pm \alpha(f)}{2}.$$

Each complex fermion f generates a $U(1)$ current with a charge with respect to the unbroken Cartan generators of the four dimensional gauge group given by

$$\begin{aligned} Q_\nu(f) &= \nu - \frac{1}{2} \\ &= \frac{\alpha(f)}{2} + F \end{aligned}$$

for each complex right–moving fermion f.

2.4 The NAHE Set: A Toy Model

The free fermionic construction is based on the heterotic strings and from here on, we will fix the left-moving sector to be supersymmetric and right-moving sector to be bosonic. In what follows, it will be seen that $\bar{\psi}^{1,\ldots,5}$ are complex fermions which produce the observable $SO(10)$ symmetry whilst $\bar{\phi}^{1,\ldots,8}$ are complex fermions which produce the hidden E_8 gauge group.

The NAHE set [41] is a set of five boundary condition basis vectors $\{\mathbf{1}, \mathbf{S}, \mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3}\}$. With '1' indicating Ramond boundary conditions and '0' indicating Neveu–Schwarz boundary conditions. The NAHE–set basis vectors are given by:

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,\ldots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\ldots,8}$
$\mathbf{1}$	1	1	1	1	1,...,1	1	1	1	1,...,1
$\mathbf{S}$	1	1	1	1	0,...,0	0	0	0	0,...,0
$\mathbf{b_1}$	1	1	0	0	1,...,1	1	0	0	0,...,0
$\mathbf{b_2}$	1	0	1	0	1,...,1	0	1	0	0,...,0
$\mathbf{b_3}$	1	0	0	1	1,...,1	0	0	1	0,...,0

	$y^{3,\ldots,6}$	$\bar{y}^{3,\ldots,6}$	$y^{1,2}, \omega^{5,6}$	$\bar{y}^{1,2}, \bar{\omega}^{5,6}$	$\omega^{1,\ldots,4}$	$\bar{\omega}^{1,\ldots,4}$
$\mathbf{1}$	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1
$\mathbf{S}$	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0
$\mathbf{b_1}$	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0
$\mathbf{b_2}$	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0
$\mathbf{b_3}$	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1

(2.25)

with the set of GGSO phases given by

$$
\begin{array}{c|ccccc}
 & \mathbf{1} & S & b_1 & b_2 & b_3 \\
\hline
\mathbf{1} & 1 & 1 & -1 & -1 & -1 \\
S & 1 & 1 & 1 & 1 & 1 \\
b_1 & -1 & -1 & -1 & -1 & -1 \\
b_2 & -1 & -1 & -1 & -1 & -1 \\
b_3 & -1 & -1 & -1 & -1 & -1
\end{array}
.
$$

2.4.1 *The Non-Supersymmetric Scenario*

To understand how the free fermionic construction works, consider the basis vector $\mathbf{1}$ where all the boundary conditions are periodic as it is required to be in Ξ. Then there are two sectors: $\Xi = \{\mathbf{1}, 2 \cdot \mathbf{1} = \mathbf{0}\}$. Here, the notation NS for the sector $\mathbf{0}$ is used which is the Neveu-Schwarz sector. Given $2 \cdot \mathbf{1} \mod 2 = \mathbf{0}$ then $N_{\mathbf{1}} = 2$ and $\mathbf{1} \cdot \mathbf{1} = -12$ therefore the rules on the basis vectors are satisfied.

To fully specify a model, we need to define the generalized GSO coefficients which in this case are

$$
\begin{aligned}
C\binom{NS}{NS} &= -C\binom{NS}{\mathbf{1}} \\
&= 1 \\
C\binom{NS}{\mathbf{1}} &= -1 \\
C\binom{\mathbf{1}}{NS} &= -1 \\
C\binom{\mathbf{1}}{\mathbf{1}} &= -1
\end{aligned}
$$

where we needed to set $C\binom{\mathbf{1}}{\mathbf{1}}$ by hand while the rest were fixed by modular invariance or hold by definition.

As the states with a mass at M_{string} would have a mass of the order of the

Planck mass, these states are not phenomenologically acceptable and only the massless spectrum is considered where the condition $M = 0$ must be satisfied. Furthermore, all the particle content of the SM should exist in the massless spectrum.

For sector **1**

$$M_L^2 = -\frac{1}{2} + \frac{10}{8} + N_L \quad > \quad 0.$$

Thus this sector contains no massless states and should be excluded. For the NS-sector

$$M_L^2 = -\frac{1}{2} + \frac{0}{8} + N_L$$
$$M_R^2 = -1 + \frac{0}{8} + N_R$$

where for the fermions the frequency is given by

$$\nu_{f,f^*} = \frac{1 \pm 0}{2} = \frac{1}{2}.$$

Recall that the Virasoro level-matching condition $M_L^2 = M_R^2$ must be satisfied. In this case, either a tachyonic negative mass $-\frac{1}{2}$ is obtained by acting on the NS vacuum with 1 fermionic right-moving oscillator such that

$$\overline{\lambda}^j |0\rangle_{NS} \qquad \text{with} \qquad M_L^2 = M_R^2 = -\frac{1}{2}$$

or the following massless states are obtained by acting on the NS vacuum with 1 left-moving fermionic oscillator and either 2 right-moving fermionic oscillators or 1 right-moving bosonic oscillator:

- $\psi^I \partial \overline{X} |0\rangle_{NS}$: These states correspond to the graviton, the dilaton and the antisymmetric tensor.

- $\psi^I\overline{\lambda}^i\overline{\lambda}^j|0\rangle_{NS},\ i,j \in \{1,...,44\}$: Gauge bosons in the adjoint representation of $SO(44)$.
- $\lambda^k\partial\overline{X}|0\rangle_{NS},\ k \in \{1,...,18\}$: Gauge bosons in the adjoint representation of $SU(2)^6$.
- $\lambda^k\overline{\lambda}^i\overline{\lambda}^j|0\rangle_{NS},\ i \in \{1,...,6\}$: Scalars in the adjoint representation of $SU(2)^6\times SO(44)$.

Now the GGSO projection for each state is performed:

State	$e^{i\pi\mathbf{1}\cdot F_{NS}}$	$\delta_{NS}C\binom{NS}{\mathbf{1}}$	Projected
$\psi^I\partial\overline{X}|0\rangle_{NS}$	$e^{i\pi(-1)}$	-1	In
$\psi^I\overline{\lambda}^i\overline{\lambda}^j|0\rangle_{NS}$	$e^{i\pi(-1-1-1)}$	-1	In
$\lambda^k\partial\overline{X}|0\rangle_{NS}$	$e^{i\pi(-1)}$	-1	In
$\lambda^k\overline{\lambda}^i\overline{\lambda}^j|0\rangle_{NS}$	$e^{i\pi(-1-1-1)}$	-1	In
$\overline{\lambda}^j|0\rangle_{NS}$	$e^{i\pi(-1)}$	-1	In

As can be seen, all the states survive the GGSO projection including the tachyonic states. To eliminate the tachyons in order to obtain a physical massless spectrum an additional basis vector with appropriate phases needs to be added. We also desire to include the particle content of the SM and reduce gauge group which leads to further addition of basis vectors. This will be discussed in what follows.

2.4.2 Understanding The SUSY Background

Now we turn our attention to the basis

$$\{\mathbf{1},\mathbf{S}\}$$

giving $2^2 = 4$ sectors, the additive group given by

$$\Xi = \{NS, 1 + S, 1, S\}$$

with space-time spin statistics index

$$\delta_{\mathbf{S}} = -1$$

due to the presence of $\psi_\mu^{1,2}$. Here, we note that in a supersymmetric model where SUSY is generated by the vector $\mathbf{S}$ all the superpartners of a particular sector α are generated by $\alpha + \mathbf{S}$. The tachyonic states arising in this model are obtained by acting on the non–degenerate vacuum with a right–moving fermionic oscillator, which satisfy the level matching condition with $M_L^2 = M_R^2 = -1/2$. These tachyonic states are, however, projected out by the S projection, which is given by

$$\mathrm{e}^{i\pi\mathbf{S}\cdot F_{NS}}|\mathbf{S}\rangle_{NS} = \delta_{\mathbf{S}}|\mathbf{S}\rangle_{NS}. \tag{2.26}$$

As there are no oscillators acting on the left–moving vacuum in the tachyonic untwisted state, and the basis vector $\mathbf{S}$ is blind to the right–moving oscillators, the left–hand side of Equation (2.26) is positive. On the other hand $\delta_{\mathbf{S}} = -1$ because the space-time fermions ψ^μ are periodic in $\mathbf{S}$. The mismatch between the two sides of Equation (2.26) entails that the untwisted NS tachyons are projected out. This argument extends to any free fermionic model that contains the basis vector $\mathbf{S}$ which can be seen as follows:

State	$e^{i\pi \mathbf{S}\cdot F_{NS}}$	$\delta_{NS} C\binom{NS}{\mathbf{S}}$	Projected
$\psi^I \partial \overline{X} \vert 0\rangle_{NS}$	$e^{i\pi(-1)}$	-1	In
$\psi^I \overline{\lambda}^i \overline{\lambda}^j \vert 0\rangle_{NS}$	$e^{i\pi(-1)}$	-1	In
$\{\chi^{1,\ldots,6}, y^{1,\ldots,6}, \omega^{1,\ldots,6}\}\partial \overline{X} \vert 0\rangle_{NS}$	$e^{i\pi(-1)}$	-1	In
$\{\chi^{1,\ldots,6}, y^{1,\ldots,6}, \omega^{1,\ldots,6}\}\overline{\lambda}^i \overline{\lambda}^j \vert 0\rangle_{NS}$	$e^{i\pi(-1)}$	-1	In
$\overline{\lambda}^j \vert 0\rangle_{NS}$	$e^{i\pi(0)}$	-1	Out

where

$$\lambda^k = \{\chi^{1,\ldots,6}, y^{1,\ldots,6}, \omega^{1,\ldots,6}\}.$$

If we were to stop here, we have 1) $SO(44)$ gauge group which is too large, 2) $N = 4$ SUSY and 3) no matter content but what we are interested in is obtaining

- $SU(3) \times SU(2) \times U(1)$ embedding
- $N = 1$ SUSY
- matter

for which we consider adding the basis vectors $\mathbf{b_i}$ for $i = 1, 2, 3$ corresponding to $\mathbb{Z}_2 \times \mathbb{Z}_2$ twists.

2.4.3 *From* $SO(44)$ *To* $SO(10)$*: Step By Step*

The NAHE set consists of five basis vectors. The basis vectors **1** and **S**, generate a model with the $SO(44)$ gauge symmetry and $N = 4$ space–time SUSY. The vectors $\mathbf{b_i}$ for $\mathbf{i} = 1, 2, 3$ correspond to the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold twists. The vector $\mathbf{b_1}$ breaks the $SO(44)$ gauge group to $SO(28) \times SO(16)$ and the $N = 4$ space–time SUSY to $N = 2$. The vector $\mathbf{b_2}$ then reduces the group to $SO(10) \times SO(22) \times SO(6)^2$ gauge group and the $N = 2$ SUSY is further reduced to $N = 1$. Furthermore, the basis vector $\mathbf{b_3}$ gives the decomposition $SO(10) \times SO(16)_1 \times SO(6)^3$

where we fix the GGSO projection coefficient in order to preserve the $N = 1$ space–time SUSY. Moreover, the sector, ξ, given by the linear combination

$$\xi = \mathbf{1} + \mathbf{b_1} + \mathbf{b_2} + \mathbf{b_3} \equiv \{\overline{\phi}^{1,\ldots,8}\}$$

together with the NS–sector form the adjoint representation of E_8 thereby enhancing the $SO(16)_1$. As a result, we obtain

$$SO(10) \times E_8 \times SO(6)^3$$

as the gauge group with $N = 1$ space–time SUSY at the NAHE level.

2.5 *The Phenomenological Models*

The phenomenological free fermionic heterotic string models were constructed following two main routes, the first are the so called NAHE–based models. This set of models utilise a set of eight or nine boundary condition basis vectors. The first five consist of the so–called NAHE set [41] and are common in all these models. The basis vectors underlying the NAHE–based models therefore differ by the additional three or four basis vectors that extend the NAHE set.

The second route follows from the classification methodology that was developed in [42] for the classification of type II free fermionic superstrings and adopted in [25–27, 40] for the classification of free fermionic heterotic string vacua with $SO(10)$ GUT symmetry and its Pati–Salam [26] and Flipped $SU(5)$ [27] subgroups. The main difference between the two classes of models is that while the NAHE–based models allow for asymmetric boundary conditions with respect to the set of internal fermions $\{y, \omega|\bar{y}, \bar{\omega}\}$, the classification method only utilises symmetric boundary conditions. This distinction affects the moduli spaces of the models [45], which can be entirely fixed in the former case [46] but not in the later. On the other hand the classification method enables the systematic scan of spaces of the order of 10^{12} vacua, and led to the discovery of spinor–vector duality [40, 47] and exophobic heterotic string vacua [26].

2.5.1 *The Various* $SO(10)$ *Subgroups*

The $SO(10)$ GUT models generated can be broken to one of its subgroups by the boundary condition assignment on the complex fermion $\overline{\psi}^{1,...,5}$. For the Pati-Salam and the Flipped $SU(5)$ case, one additional basis vector is required to break the $SO(10)$ GUT symmetry. However, in order to construct the $SU(4) \times SU(2) \times U(1)$, the Standard-Like models and the Left-Right Symmetric models, the Pati-Salam breaking is required along with an additional $SO(10)$ breaking basis vector. The following boundary condition basis vectors can be used to construct the necessary gauge groups:

The Pati-Salam Subgroup

$$v_{13} = \alpha = \{\overline{\psi}^{4,5}, \overline{\phi}^{1,2}\}$$

The Flipped $SU(5)$ ***Subgroup***

$$v_{13} = \alpha = \{\overline{\eta}^{1,2,3} = \frac{1}{2}, \overline{\psi}^{1,...,5} = \frac{1}{2}, \overline{\phi}^{1,...,4} = \frac{1}{2}, \overline{\phi}^{5}\}$$

The $SU(4) \times SU(2) \times U(1)$ ***Subgroup***

$$\begin{aligned} v_{13} &= \alpha &&= \{\overline{\psi}^{4,5}, \overline{\phi}^{1,2}\} \\ v_{14} &= \beta &&= \{\overline{\psi}^{4,5} = \frac{1}{2}, \overline{\phi}^{1,...,6} = \frac{1}{2}\} \end{aligned}$$

The Left-Right Symmetric Subgroup

$$
\begin{aligned}
v_{13}=\alpha &= \{\overline{\psi}^{4,5},\overline{\phi}^{1,2}\},\\
v_{14}=\beta &= \{\overline{\eta}^{1,2,3}=\tfrac{1}{2},\overline{\psi}^{1,\ldots,3}=\tfrac{1}{2},\overline{\phi}^{1,2}=\tfrac{1}{2},\overline{\phi}^{3,4}\}
\end{aligned}
$$

The Standard-Like Model Subgroup

$$
\begin{aligned}
v_{13} &= \alpha = \{\overline{\psi}^{4,5},\overline{\phi}^{1,2}\}\\
v_{14} &= \beta = \{\overline{\eta}^{1,2,3}=\frac{1}{2},\overline{\psi}^{1,\ldots,5}=\frac{1}{2},\overline{\phi}^{1,\ldots,4}=\frac{1}{2},\overline{\phi}^{5}\}
\end{aligned}
$$

2.5.2 *The NAHE-Based Models*

The construction of the semi–realistic free fermionic models proceeds by adding three or four additional basis vectors to the NAHE–set. The function of the additional basis vectors is to reduce the forty eight spinorial **16** multiplets to three chiral generations, and at the same time to reduce the $SO(10)$ GUT symmetry to one of its subgroups:

1. $SU(5)\times U(1)$ (FSU5) [17];
2. $SU(3)\times SU(2)\times U(1)^2$ (SLM) [15, 18, 23, 24];
3. $SO(6)\times SO(4)$ (PS) [19];
4. $SU(3)\times U(1)\times SU(2)^2$ (LRS) [20];
5. $SU(4)\times SU(2)\times U(1)$ (SU421) [21].

The first four cases produced viable three generation models, whereas in the last case it was shown that phenomenologically viable models cannot be constructed [21, 28]. The additional basis vectors may each preserve or break the $SO(10)$ symmetry. Basis vectors that preserve the $SO(10)$ symmetry are typically denoted by b_i with $(i = 4, 5, \dots)$, whereas those that break the $SO(10)$ symmetry are denoted by $\{\alpha, \beta, \gamma\}$. The overlap of the additional basis vectors with the SUSY generator basis vector S determine the type of possible SUSY breaking. In Chapter 4, we will focus on the NAHE-based construction of an explicit tachyon-free model.

2.5.3 *The Classification Methodology*

The free fermionic formalism provides an elegant approach to studying the phenomenologically viable properties of the string vacua. The matter content arises from the fundamental representation, **27** of E_6 decomposing under the **16** spinorial and **10** vectorial representations of the $SO(10)$ symmetry achieved by breaking the E_6 at the string scale. The **16** consists of all the left and right-handed fermions, both the known and the predicted, whereas the **10** houses the Higgs states. The $SO(10)$ gauge group is further broken at the string scale to one of its many subgroups.

In Chapter 3, we will turn our focus on free fermionic models where the classification methodology has been used whereby the set of basis vectors is fixed and a large number of string models, of the order of 10^{12} vacua, is explored by enumerating the independent GGSO projection coefficients. In this manner large spaces of string models with $SO(10)$ [25], $SO(6) \times SO(4)$ [26], $SU(5) \times U(1)$ [27], and $SU(4) \times SU(2) \times U(1)$ [28], have been explored. A subset of basis vectors that respect the $SO(10)$ symmetry is given by the set of 12 boundary condition basis vectors $V = \{v_1, v_2, \dots, v_{12}\}$, where

$$
\begin{aligned}
v_1 = \quad & \mathbf{1} & = \{\psi^{1,2}_{\mu}, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,\dots,8}\}, \\
v_2 = \quad & \mathbf{S} & = \{\psi^{\mu}, \chi^{12}, \chi^{34}, \chi^{56}\}, \\
v_{2+i} = \quad & \mathbf{e_i} & = \{y^i, \omega^i | \overline{y}^i, \overline{\omega}^i\}, \ i = 1, \dots, 6, \\
v_9 = \quad & \mathbf{b_1} & = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \overline{y}^{34}, \overline{y}^{56}, \overline{\eta}^1, \overline{\psi}^{1,\dots,5}\}, \\
v_{10} = \quad & \mathbf{b_2} & = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \overline{y}^{12}, \overline{y}^{56}, \overline{\eta}^2, \overline{\psi}^{1,\dots,5}\}, \\
v_{11} = \quad & \mathbf{z_1} & = \{\overline{\phi}^{1,\dots,4}\}, \\
v_{12} = \quad & \mathbf{z_2} & = \{\overline{\phi}^{5,\dots,8}\}.
\end{aligned}
$$

As before, the basis vectors $\mathbf{1}$ and $\mathbf{S}$, generate a model with the $SO(44)$ gauge symmetry and $N = 4$ space–time SUSY with the tachyons being projected out of the massless spectrum. The next six basis vectors: $e_1, ..., e_6$ all correspond to the possible symmetric shifts of the six internal coordinates thus breaking the $SO(44)$ gauge group to $SO(32) \times U(1)^6$ but keeping the $N = 4$ SUSY intact. The vectors $\mathbf{b_i}$ for $\mathbf{i} = 1, 2$ correspond to the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold twists. The vectors $\mathbf{b_1}$ and $\mathbf{b_2}$ play the role of breaking the $N = 4$ down to $N = 1$ whilst reducing the gauge group to $SO(10) \times U(1)^2 \times SO(18)$. The states coming from the hidden sector are produced by $\mathbf{z_1}$ and $\mathbf{z_2}$ left untouched by the action of previous basis vectors. These vectors together with the others generate the following adjoint representation of the gauge symmetry: $SO(10) \times U(1)^3 \times SO(8) \times SO(8)$ where $SO(10) \times U(1)^3$ is the observable gauge group which gives rise to matter states from the twisted sectors charged under the $U(1)$s while $SO(8) \times SO(8)$ is the hidden gauge group gives rise to matter states which are neutral under the $U(1)$s.

2.6 *The* $U(1)$ *Relations*

Here, the definitions and identifications of the observable $U(1)$s are listed which play an important role in constructing viable string inspired models and will be relied upon for the remainder of the thesis.

2.6.1 *Identifications*

$$
\begin{aligned}
U(1)_C &= \frac{3}{2}U(1)_{B-L} \\
U(1)_L &= 2U(1)_{T_{3_R}} \\
U(1)_{e.m.} &= T_{3_L} + U(1)_Y
\end{aligned}
$$

2.6.2 *With* E_6 *Embedding*

$$
\begin{pmatrix} U(1)_Y \\ U(1)_{Z'} \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{1}{2} & 0 \\ \frac{1}{5} & -\frac{1}{5} & -1 \end{pmatrix} \begin{pmatrix} U(1)_C \\ U(1)_L \\ U(1)_\zeta \end{pmatrix}
$$

3

The String-Derived Z' Model

They can because they think they can.

Virgil

In this chapter, we are going to consider a heterotic string-derived Z' model. The characteristics of the required spectrum are well motivated in heterotic string constructions that allow for a light Z' where the anomaly cancellation of the $U(1)_{Z'}$ symmetry requires the existence of the SM singlet and vector-like states in the vicinity of the $U(1)_{Z'}$ breaking scale.

We found, quite remarkably, that in the Z' models the compatibility of gauge coupling unification with the data at the electroweak scale is identical to that of the MSSM and present our findings. We further show that effects arising from heavy thresholds may push the supersymmetric spectrum beyond the reach of the LHC, while maintaining the agreement with the gauge coupling data.

As further data did not substantiate the observation of the diphoton excess [77, 78] indicating that the initial observation was a statistical fluctuation the discussion has been relegated to Appendix E. However, it is still worth noting that all the key ingredients that form the basis of the discussion are readily available in the string-derived Z' model.

3.1 *The Extra Z' Stringy Model*

We begin by noting that the construction of heterotic string models that allow for a light Z' is highly non–trivial [55,72,73] . The reason being that the extra family universal $U(1)$ symmetries that are typically discussed in the string–inspired literature tend to be anomalous and are therefore broken near the string scale [74]. The relevant symmetries tend to be anomalous due to the symmetry breaking pattern $E_6 \to SO(10) \times U(1)_\zeta$, induced at the string level by the Gliozzi–Scherk–Olive (GSO) projection [75]. In [55], th spinor–vector duality property of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds [69, 70] was utilized to construct a string-derived model with anomaly free $U(1)_\zeta$, thus enabling it to remain unbroken down to low scales.

The difficulty in constructing heterotic string models with light Z' symmetries arises due to the breaking of the observable E_6 symmetry in the string constructions by discrete Wilson lines to $SO(10) \times U(1)_\zeta$. Application of the symmetry breaking at the string level results in the projection of some states from the physical spectrum. The consequence is that $U(1)_\zeta$ is in general anomalous in the string vacua, and cannot remain unbroken to low scales. The extra $U(1)$ symmetry which is embedded in $SO(10)$, and is orthogonal to the Standard Model weak hypercharge, is typically broken at the high scale to generate sufficiently light neutrino masses.

The string-derived model of [55] was constructed in the free fermionic formulation [38] of the four-dimensional heterotic string. The details of the construction, the massless spectrum of the model and its superpotential are given in [55] and will not be repeated here. We review here the properties of the model that are relevant for the anomaly free extra Z' symmetry.

The model utilises the spinor–vector duality symmetry that was observed in the space of fermionic $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactifications [69, 70]. The spinor vector duality operates under exchange of the total number of spinorial $(\mathbf{16}\oplus\overline{\mathbf{16}})$ representations of $SO(10)$ with the total number of vectorial $\mathbf{10}$ representations. For every string vacuum with a $\#_1$ of $(\mathbf{16}\oplus\overline{\mathbf{16}})$ representations and $\#_2$ of $\mathbf{10}$ representations there is a dual vacuum in which $\#_1 \leftrightarrow \#_2$. The understanding of this duality is facilitated by considering the vacua in which the $SO(10) \times U(1)_\zeta$ symmetry is enhanced to E_6. The chiral representations of E_6 are the $\mathbf{27}$ and $\overline{\mathbf{27}}$ and their decomposition under $SU(10) \times U(1)_\zeta$ is

$$\begin{aligned}\mathbf{27} &= \mathbf{16}_{+1/2} + \mathbf{10}_{-1} + \mathbf{1}_{+2},\\ \overline{\mathbf{27}} &= \overline{\mathbf{16}}_{-1/2} + \mathbf{10}_{+1} + \mathbf{1}_{-2},\end{aligned}$$

where the subscript denotes the $U(1)_\zeta$ charge. Thus, the string vacua with E_6 symmetry are self–dual with respect to the spinor–vector duality, *i.e.* in these vacua $\#_1(\mathbf{16}\oplus\overline{\mathbf{16}}) = \#_2(\mathbf{10})$. In this case $U(1)_\zeta$ is anomaly free by virtue of its embedding in E_6. There exist a discrete Wilson line that reduce E_6 symmetry to $SO(10) \times U(1)_\zeta$ with $\#_1(\mathbf{16}\oplus\overline{\mathbf{16}})$ & $\#_2(\mathbf{10})$, and a corresponding discrete Wilson line with $\#_2(\mathbf{16}\oplus\overline{\mathbf{16}})$ & $\#_1(\mathbf{10})$ [70].

The string vacua with enhanced E_6 symmetry correspond to heterotic string vacua with $(2,2)$ world-sheet SUSY. We can realise the E_6 symmetry by breaking the ten dimensional untwisted gauge symmetry to $SO(8)^4$ [69]. One of the $SO(8)$ factors is reduced further to $SO(2)^4$ and the E_6 symmetry is generated from additional sectors in the string vacua. In parallel to the spectral flow operator on the supersymmetric side of the heterotic string that maps between different space-time spin representations, there exists a spectral flow operator on the bosonic side. In the vacua with enhanced E_6 symmetry the spectral flow operator exchanges between the spinorial and vectorial components in the E_6 representations. The spectral flow operator is the $U(1)$ generator of the $N = 2$ world-sheet SUSY on the bosonic side of the heterotic string. In the vacua with broken E_6 symmetry, the $N = 2$ world-sheet SUSY on the bosonic side is broken and the spectral flow operator induces the map between the spinor–vector dual vacua.

The construction of [55] utilises the classification methods developed in [42] for type IIB string and in [25] for heterotic string vacua with unbroken $SO(10)$ gauge group. The heterotic string classification was extended to vacua with the Pati–Salam and Flipped $SU(5)$ subgroups of $SO(10)$ in [43] and [44], respectively. In this method a space of the order of 10^{12} is spanned and models with specific phenomenological characteristics can be extracted. The string vacuum with anomaly free $U(1)_{Z'}$ is obtained by first trawling a self–dual $SO(10)$ model with six chiral families and subsequently breaking the $SO(10)$ symmetry to the Pati–Salam subgroup [55]. The chiral spectrum of the models forms complete E_6 representations, whereas the additional vector–like multiplets may reside in incomplete multiplets. This is in fact an additional important property of the string, which affects compatibility with the gauge coupling data. The complete massless spectrum of the model was presented in [55]. The Z' model under consideration here was obtained in the class of Pati-Salam heterotic string models which are generated by a set of thirteen boundary condition basis vectors $B = \{v_1, ..., v_{13}\}$. To recapitulate, a subset of basis vectors that respect the $SO(10)$ symmetry is given by the set of 12 boundary condition basis vectors $V = \{v_1, v_2, \ldots, v_{12}\}$

$$
\begin{aligned}
v_1 = &\quad 1 &&= \{\psi_\mu^{1,2}, \chi^{1,\ldots,6}, y^{1,\ldots,6}, \omega^{1,\ldots,6} | \bar{y}^{1,\ldots,6}, \bar{\omega}^{1,\ldots,6}, \bar{\psi}^{1,\ldots,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,\ldots,8}\},\\
v_2 = &\quad S &&= \{\psi^\mu, \chi^{12}, \chi^{34}, \chi^{56}\},\\
v_{2+i} = &\quad e_i &&= \{y^i, \omega^i | \overline{y}^i, \overline{\omega}^i\},\ i = 1, \ldots, 6,\\
v_9 = &\quad b_1 &&= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \overline{y}^{34}, \overline{y}^{56}, \overline{\eta}^1, \overline{\psi}^{1,\ldots,5}\},\\
v_{10} = &\quad b_2 &&= \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \overline{y}^{12}, \overline{y}^{56}, \overline{\eta}^2, \overline{\psi}^{1,\ldots,5}\},\\
v_{11} = &\quad z_1 &&= \{\overline{\phi}^{1,\ldots,4}\},\\
v_{12} = &\quad z_2 &&= \{\overline{\phi}^{5,\ldots,8}\}
\end{aligned}
$$

where the basis vectors 1 and S, generate a model with the $SO(44)$ gauge symmetry and $N = 4$ space–time SUSY with the tachyons being projected out of the massless spectrum. The next six basis vectors: $e_1, ..., e_6$ all correspond to the possible symmetric shifts of the six internal coordinates thus breaking the $SO(44)$ gauge group to $SO(32) \times U(1)^6$ but keeping the $N = 4$ SUSY intact. The vectors b_i for $i = 1, 2$ correspond to the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold twists. The vectors b_1 and b_2 play the role of breaking the $N = 4$ down to $N = 1$ whilst reducing the gauge group to $SO(10) \times U(1)^2 \times SO(18)$. The states coming from the hidden sector are produced by z_1 and z_2 left untouched by the action of previous basis vectors. These vectors together with the others generate the following adjoint representation of the gauge symmetry: $SO(10) \times U(1)^3 \times SO(8) \times SO(8)$ where $SO(10) \times U(1)^3$ is the observable gauge group which gives rise to matter states from the twisted sectors charged under the $U(1)$s while $SO(8) \times SO(8)$ is the hidden gauge group gives rise to matter states which are neutral under the $U(1)$s. With the addition of an additional basis vector

$$v_{13} = \alpha = \{\overline{\psi}^{4,5}, \overline{\phi}^{1,2}\}$$

yields the Pati-Salam subgroup of the $SO(10)$ at the string scale with the set of GGSO phases given by

$$\begin{array}{c} \\ 1 \\ S \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ b_1 \\ b_2 \\ z_1 \\ z_2 \\ \alpha \end{array}
\begin{array}{c} \begin{array}{ccccccccccccc} 1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & b_1 & b_2 & z_1 & z_2 & \alpha \end{array} \\
\left(\begin{array}{ccccccccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1
\end{array}\right) \end{array} .$$

The space-time vector bosons are obtained solely from the untwisted sector and generate the observable and hidden gauge symmetries, given by:

$$\begin{aligned} \text{observable} &: \quad \overbrace{SO(6) \times SO(4)}^{Pati-Salam} \times U(1)_1 \times U(1)_2 \times U(1)_3 \\ \text{hidden} &: \quad SO(4)^2 \times SO(8) \ . \end{aligned}$$

The additional space-time vector bosons may arise from

$$\left\{\begin{array}{ccccc} z_1, & z_2, & \alpha, & x, & z_1+z_2 \\ \alpha+z_1, & \alpha+z_2, & \alpha+x, & \alpha+z_1+z_2, & \alpha+x+z_1 \end{array}\right\}$$

and enhance the four dimensional gauge group where

$$x = 1 + S + \sum_{i=1}^{6} e_i + \sum_{i=1}^{2} z_i$$

may enhance the $SO(16)$ to E_8. The E_6 combination,

$$U(1)_\zeta = \sum_{i=1}^{3} U(1)_i \tag{3.1}$$

is anomaly free whereas the orthogonal combinations of $U(1)_{1,2,3}$ are anomalous. The matter states in the Pati-Salam string-derived models are embedded in the $SU(4) \times SU(2)_L \times SU(2)_R$ representations as follows:

$$\begin{aligned}
F_L(\mathbf{4},\mathbf{2},\mathbf{1}) &\rightarrow q\left(\mathbf{3},\mathbf{2},-\frac{1}{6}\right) + l\left(\mathbf{1},\mathbf{2},\frac{1}{2}\right) \\
\overline{F}_R(\overline{\mathbf{4}},\mathbf{1},\mathbf{2}) &\rightarrow u^c\left(\overline{\mathbf{3}},\mathbf{1},\frac{2}{3}\right) + d^c\left(\overline{\mathbf{3}},\mathbf{1},-\frac{1}{3}\right) + e^c\left(\mathbf{1},\mathbf{1},-1\right) + \mathcal{N}\left(\mathbf{1},\mathbf{1},0\right) \\
h(\mathbf{1},\mathbf{2},\mathbf{2}) &\rightarrow h^u\left(\mathbf{1},\mathbf{2},-\frac{1}{2}\right) + h^d\left(\mathbf{1},\mathbf{2},\frac{1}{2}\right) \\
D(\mathbf{6},\mathbf{1},\mathbf{1}) &\rightarrow d_3\left(\mathbf{3},\mathbf{1},\frac{1}{3}\right) + \overline{d}_3\left(\overline{\mathbf{3}},\mathbf{1},-\frac{1}{3}\right)
\end{aligned} \tag{3.2}$$

where F_L and $\overline{F}_R$ contain the SM generation; h^u and h^d are the electroweak Higgs doublets; and D contains the vector-like triplets with the following electric charge definition

$$Q_{e.m.} = \frac{1}{\sqrt{6}} T_{15} + \frac{1}{2} T_{3_L} + \frac{1}{2} T_{3_R}$$

where $T_{15} = \text{diag}\,(1, 1, 1, -3)$ is the diagonal generator of $SU(4)$ and T_{3_L}, T_{3_R} are the diagonal generators of $SU(2)_L$ and $SU(2)_R$ respectively. The complete massless spectrum of the string model and the charges under the gauge symmetries are given in [55]. Table 3.1 and Table 3.2 show a glossary of the states in the model and their charges under the $SU(4) \times SO(4) \times U(1)_\zeta$ group factors, where we adopt the notation of [71]. The sextet states are in vector–like representations with respect to the Standard Model, but are chiral under $U(1)_\zeta$. Thus, if $U(1)_\zeta$ is part of an unbroken $U(1)_{Z'}$ combination down to low scales, it protects the sextets, and corresponding bi–doublets, from acquiring a mass above the $U(1)_{Z'}$ breaking scale. The model also contains vector–like states that transform under the hidden $SU(2)^4 \times SO(8)$ group factors, with charges $Q_\zeta = \pm 1$ or $Q_\zeta = 0$.

Symbol	Fields in [55]	$SU(4) \times SU(2)_L \times SU(2)_R$	$U(1)_\zeta$
F_L	F_{1L}, F_{2L}, F_{3L}	$(\mathbf{4}, \mathbf{2}, \mathbf{1})$	$+\frac{1}{2}$
F_R	F_{1R}	$(\mathbf{4}, \mathbf{1}, \mathbf{2})$	$-\frac{1}{2}$
$\bar{F}_R$	$\bar{F}_{1R}, \bar{F}_{2R}, \bar{F}_{3R}, \bar{F}_{4R}$	$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$	$+\frac{1}{2}$
h	h_1, h_2, h_3	$(\mathbf{1}, \mathbf{2}, \mathbf{2})$	-1
Δ	$D_1, \ldots, D_7$	$(\mathbf{6}, \mathbf{1}, \mathbf{1})$	-1
$\bar{\Delta}$	$\bar{D}_1, \bar{D}_2, \bar{D}_3, \bar{D}_6$	$(\mathbf{6}, \mathbf{1}, \mathbf{1})$	$+1$
S	$\Phi_{12}, \Phi_{13}, \Phi_{23}, \chi_1^+, \chi_2^+, \chi_3^+, \chi_5^+$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$+2$
$\bar{S}$	$\bar{\Phi}_{12}, \bar{\Phi}_{13}, \bar{\Phi}_{23}, \bar{\chi}_4^+$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	-2
ϕ	ϕ_1, ϕ_2	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$+1$
$\bar{\phi}$	$\bar{\phi}_1, \bar{\phi}_2$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	-1
ζ	$\Phi_{12}^-, \Phi_{13}^-, \Phi_{23}^-, \bar{\Phi}_{12}^-, \bar{\Phi}_{13}^-, \bar{\Phi}_{23}^-$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	0
	$\chi_1^-, \chi_2^-, \chi_3^-, \bar{\chi}_4^-, \chi_5^-$		
	$\zeta_i, \bar{\zeta}_i, i = 1, \ldots, 9$		
	$\Phi_i, i = 1, \ldots, 6$		

Table 3.1: Observable sector field notation and associated states in [55].

Symbol	Fields in [55]	$SU(2)^4 \times SO(8)$	$U(1)_\zeta$
H^+	H^3_{12}	$(\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	$+1$
	H^2_{34}	$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1})$	$+1$
H^-	H^2_{12}	$(\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	-1
	H^3_{34}	$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1})$	-1
H	H^1_{12}	$(\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})$	0
	$H^i_{13}, i=1,2,3$	$(\mathbf{2},\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1})$	0
	$H^i_{14}, i=1,2,3$	$(\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1})$	0
	H^1_{23}	$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1})$	0
	H^1_{24}	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{2},\mathbf{1})$	0
	$H^i_{34}, i=1,4,5$	$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1})$	0
Z	$Z_i, i=1,\ldots,$	$(\mathbf{1},\mathbf{1},\mathbf{8})$	0

Table 3.2: Hidden sector field notation and associated states in [55].

As noted from Table 3.1 the string model contains the Higgs representations required to break the non–Abelian Pati–Salam gauge symmetry [56]. These are $\mathcal{H} = F_R$ and $\bar{\mathcal{H}}$, being a linear combination of the four $\bar{F}_R$ fields. The decomposition of these fields under the SM group is given by:

$$\bar{\mathcal{H}}(\bar{\mathbf{4}},\mathbf{1},\mathbf{2}) \rightarrow u^c_H\left(\bar{\mathbf{3}},\mathbf{1},\frac{2}{3}\right) + d^c_H\left(\bar{\mathbf{3}},\mathbf{1},-\frac{1}{3}\right) + \bar{\mathcal{N}}\left(\mathbf{1},\mathbf{1},0\right) + e^c_H\left(\mathbf{1},\mathbf{1},-1\right)$$
$$\mathcal{H}\left(\mathbf{4},\mathbf{1},\mathbf{2}\right) \rightarrow u_H\left(\mathbf{3},\mathbf{1},-\frac{2}{3}\right) + d_H\left(\mathbf{3},\mathbf{1},\frac{1}{3}\right) + \mathcal{N}\left(\mathbf{1},\mathbf{1},0\right) + e_H\left(\mathbf{1},\mathbf{1},1\right)$$

The suppression of the left–handed neutrino masses favours the breaking of the Pati–Salam (PS) gauge symmetry at the high scale [57, 58]. The possibility of breaking the PS symmetry at a low scale was considered in refs. [59, 64]. Here we will take the PS breaking scale to be in the vicinity of the string scale or slightly below. The VEVs of the heavy Higgs fields that break the PS gauge group leave an unbroken $U(1)_{Z'}$ symmetry given by

$$U(1)_{Z'} \;=\; \frac{1}{2}U(1)_{B-L} - \frac{2}{3}U(1)_{T_{3_R}} - \frac{5}{3}U(1)_{\zeta} \;\notin\; SO(10), \tag{3.3}$$

that may remain unbroken down to low scales provided that $U(1)_\zeta$ is anomaly free. Cancellation of the anomalies requires that the additional vector–like quarks and leptons, that arise from the **10** representation of $SO(10)$, as well as the $SO(10)$ singlet in the **27** of E_6, remain in the light spectrum. The three right–handed neutrino states are neutral under the low scale gauge symmetry and receive mass of the order of Pati–Salam breaking scale. The spectrum below the PS breaking scale is displayed schematically in Table 3.3. The spectrum is taken to be supersymmetric down to the `TeV` scale. As in the MSSM, compatibility of gauge coupling unification with the experimental data requires the existence of one vector–like pair of Higgs doublets, beyond the number of vector–like triplets.

Field	$SU(3)_C$	$\times SU(2)_L$	$U(1)_Y$	$U(1)_{Z'}$
Q_L^i	3	2	$+\frac{1}{6}$	$-\frac{2}{3}$
u_L^i	$\bar{3}$	1	$-\frac{2}{3}$	$-\frac{2}{3}$
d_L^i	$\bar{3}$	1	$+\frac{1}{3}$	$-\frac{4}{3}$
e_L^i	1	1	$+1$	$-\frac{2}{3}$
L_L^i	1	2	$-\frac{1}{2}$	$-\frac{4}{3}$
D^i	3	1	$-\frac{1}{3}$	$+\frac{4}{3}$
$\bar{D}^i$	$\bar{3}$	1	$+\frac{1}{3}$	2
H^i	1	2	$-\frac{1}{2}$	2
$\bar{H}^i$	1	2	$+\frac{1}{2}$	$+\frac{4}{3}$
S^i	1	1	0	$-\frac{10}{3}$
h	1	2	$-\frac{1}{2}$	$-\frac{4}{3}$
$\bar{h}$	1	2	$+\frac{1}{2}$	$+\frac{4}{3}$
ϕ	1	1	0	$-\frac{5}{3}$
$\bar{\phi}$	1	1	0	$+\frac{5}{3}$
ζ^i	1	1	0	0

Table 3.3: Spectrum and $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Z'}$ quantum numbers, with $i = 1, 2, 3$ for the three light generations. The charges are displayed in the normalisation used in free fermionic heterotic string models.

3.2 *The Gauge Coupling Unification*

In this section, analysis of the compatibility of gauge coupling unification in the string inspired model with the low energy gauge coupling data, where we may assume that the unification scale is either at the GUT or string scales [65]. We examine the case in which the PS symmetry is broken at the string scale as well as the case in which is broken at an intermediate scale. We take the following values for the input parameters at the Z–mass scale [66]:

$$M_Z = 91.1876 \pm 0.0021 \ \texttt{GeV} \qquad \sin^2\theta_W\left(M_Z\right)\big|_{\overline{\mathrm{MS}}} = 0.23116 \pm 0.00012$$
$$\alpha^{-1} \equiv \alpha^{-1}_{\text{e.m.}}\left(M_Z\right) = 127.944 \pm 0.014 \qquad \alpha_3\left(M_Z\right) = 0.1184 \pm 0.0007. \tag{3.4}$$

We also include the top quark mass of $M_t \sim 173.5$ GeV [66] and the Higgs boson mass of $M_H \sim 125$ GeV [67] in our analysis. String unification implies that the Standard Model gauge couplings are unified at the heterotic string scale where the tree level relation between the Planck and string scales is given by

$$M_S^2 = \frac{k_i}{16}\alpha_i M_{Pl}^2$$

where α_i are the corresponding gauge interactions and $k_i = \{1, 1, 5/3\}$ for $i = 3, 2, 1$ are the corresponding Kac-Moody level. The one–loop renormalisation group equations (RGEs) for the gauge couplings are given by

$$\frac{1}{\alpha\left(M_X\right)} = \frac{1}{k_i\alpha_i\left(\mu\right)} - \frac{b_i}{2\pi}\log\frac{M_X}{\mu^2} + \Delta_i^{(\text{total})}, \tag{3.5}$$

where b_i are the one–loop beta–function coefficients, $\Delta_i^{(\text{total})}$ represents corrections two–loop and mixing effects, and $k_i = \{1, 1, 5/3\}$ for $i = 3, 2, 1$. The analysis is most revealing at the one–loop level. Therefore, for the most part we limit our exposition to the one–loop investigation and give an estimate of the higher order corrections, which do not affect the overall picture. We obtain algebraic expressions for $\sin^2\theta_W\left(M_Z\right)$ and $\alpha_3\left(M_Z\right)$ by solving the one–loop RGEs. In our analysis, we initially assume the full spectrum of the Z' model between the unification scale, M_X, and the Z–boson scale, M_Z, and treat all perturbations as effective threshold terms. At the unification scale we have

$$\alpha_S \equiv \alpha_3(M_X) = \alpha_2(M_X) = k_1 \alpha_Y(M_X), \tag{3.6}$$

where $k_1 = 5/3$ is the canonical $SO(10)$ normalisation. We initially study the case in which the PS symmetry is broken at the string scale. In this case the expression for $\sin^2\theta_W\,(M_Z)\big|_{\overline{MS}}$ takes the general form

$$\sin^2\theta_W\,(M_Z)\big|_{\overline{MS}} = \Delta_{Z'}^{\sin^2\theta_W} + \Delta_{\text{L.T.}}^{\sin^2\theta_W} + \Delta_{\text{T.C.}}^{\sin^2\theta_W} \tag{3.7}$$

with $\alpha_3\,(M_Z)|_{\overline{MS}}$ having a similar form with corresponding Δ^{α_3} corrections. Here $\Delta_{Z'}$ is the one–loop contribution from the states of the Z' model between the unification scale and the Z–boson mass scale. $\Delta_{\text{L.T.}}$ are corrections from the light thresholds, which consist of the light supersymmetric thresholds; the Higgs and the top mass thresholds; and the mass thresholds of the heavy vector–like matter states in the Z' model. The last term,

$$\Delta_{\text{T.C.}}^{\sin^2\theta_W} = \Delta_{\text{Yuk.}}^{\sin^2\theta_W} + \Delta_{\text{2-loop}}^{\sin^2\theta_W} + \Delta_{\text{Conv.}}^{\sin^2\theta_W}, \tag{3.8}$$

includes the two–loop; kinetic mixing; Yukawa couplings and scheme conversion corrections. These corrections are found to be small and do not affect the overall picture. These effects can be absorbed into modifications of the light thresholds, which in any case are not fixed and can be varied. For $\sin^2\theta_W\,(M_Z)$ we obtain

$$
\begin{aligned}
\Delta_{Z'}^{\sin^2\theta_W} &= \frac{3}{8} + \frac{5\alpha}{16\pi}\left(b_2^{Z'} - b_1^{Z'}\right)\log\frac{M_X}{M_Z}; \\
\Delta_{\text{L.T.}}^{\sin^2\theta_W} &= \frac{5\alpha}{16\pi}\sum_i\left(b_{1_i}^{\text{L.T.}} - b_{2_i}^{\text{L.T.}}\right)\log\frac{M_i}{M_Z},
\end{aligned}
\tag{3.9}
$$

where M_i are the light mass thresholds and $\alpha = \alpha_{\text{e.m.}}\,(M_Z)$. Similarly for $\alpha_3\,(M_Z)$, we have:

$$
\begin{aligned}
\Delta_{Z'}^{\alpha_3} &= \frac{3}{8\alpha} + \frac{1}{2\pi}\left(b_3^{Z'} - \frac{3}{8}b_2^{Z'} - \frac{5}{8}b_1^{Z'}\right)\log\frac{M_S}{M_Z}; \\
\Delta_{\text{L.T.}}^{\alpha_3} &= \frac{1}{2\pi}\sum_i\left(\frac{5}{8}b_{1_i}^{\text{L.T.}} + \frac{3}{8}b_{2_i}^{\text{L.T.}} - b_{3_i}^{\text{L.T.}}\right)\log\frac{M_i}{M_Z}.
\end{aligned}
\tag{3.10}
$$

The predictions for gauge coupling observables at the Z–scale can therefore be seen to correspond to 0^{th} order predictions consisting of the first lines of Equation (3.9) and Equation (3.10) plus the threshold corrections due to the decoupling of the different particles at their mass thresholds. The values of the beta function coefficients of these light thresholds are shown in Table 3.4. The zeroth order coefficients are given by

$$
\begin{aligned}
b_3^{Z'} &= 0 = b_3^{\text{MSSM}} + 3, \\
b_2^{Z'} &= 4 = b_2^{\text{MSSM}} + 3, \\
b_1^{Z'} &= \tfrac{48}{5} = b_1^{\text{MSSM}} + 3.
\end{aligned}
$$

Hence, the $b_i^{Z'}$ are identical to the b_i^{MSSM}, see Section 1.5.2, up to a common shift by 3 arising from the vector–like colour triplets and electroweak doublets. As the 0^{th} order predictions for $\sin\theta(M_Z)$ and $\alpha_3(M_Z)$ only depend on the differences of the beta function coefficients, the zeroes order predictions are identical to those that are obtained in the MSSM.

R	$b_1(R)$	$b_2(R)$	$b_3(R)$	$b_1 - b_2$	$\frac{5}{8}b_1 + \frac{3}{8}b_2 - b_3$	factor
$\tilde{g}$	0	0	2	0	-2	$\frac{2}{3}$
$\tilde{w}$	0	$\frac{4}{3}$	0	$-\frac{4}{3}$	$\frac{1}{2}$	$\frac{2}{3}$
$\tilde{\ell}_\ell$	$\frac{1}{10}$	$\frac{1}{6}$	0	$-\frac{1}{15}$	$\frac{1}{8}$	$\frac{1}{3}$
$\tilde{\ell}_r$	$\frac{1}{5}$	0	0	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{3}$
$\tilde{Q}$	$\frac{1}{30}$	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{7}{15}$	$-\frac{1}{8}$	$\frac{1}{3}$
$\tilde{d}_r$	$\frac{1}{15}$	0	$\frac{1}{6}$	$\frac{1}{15}$	$-\frac{1}{8}$	$\frac{1}{3}$
$\tilde{u}_r$	$\frac{4}{15}$	0	$\frac{1}{6}$	$\frac{4}{15}$	0	$\frac{1}{3}$
$\tilde{h}$	$\frac{1}{5}$	$\frac{1}{3}$	0	$-\frac{2}{15}$	$\frac{1}{4}$	$\frac{2}{3}$
h	$\frac{1}{10}$	$\frac{1}{6}$	0	$-\frac{1}{15}$	$\frac{1}{8}$	$\frac{1}{3}$
t	$\frac{17}{30}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{15}$	$-\frac{1}{8}$	$\frac{2}{3}$
$D + \tilde{D}$	$\frac{1}{5}$	0	$\frac{1}{2}$	$\frac{1}{5}$	$-\frac{3}{8}$	1
$\bar{D} + \tilde{\bar{D}}$	$\frac{1}{5}$	0	$\frac{1}{2}$	$\frac{1}{5}$	$-\frac{3}{8}$	1
$H + \tilde{H}$	$\frac{3}{10}$	$\frac{1}{2}$	0	$-\frac{1}{5}$	$\frac{3}{8}$	1
$\bar{H} + \tilde{\bar{H}}$	$\frac{3}{10}$	$\frac{1}{2}$	0	$-\frac{1}{5}$	$\frac{3}{8}$	1

Table 3.4: Beta function coefficients of the light thresholds in the string inspired Z' model. The factor in the last column indicates the spin degeneracy factor.

The corrections due to the light thresholds are given by

$$\begin{aligned}\delta\sin^2\left(\theta_W\right)_{light} &= \frac{5\alpha}{16\pi}\Bigg(-\frac{4}{3}\log\frac{M_{\tilde{w}}}{M_Z}-\frac{1}{5}\log\frac{M_{\tilde{\ell}_\ell}}{M_Z}+\frac{3}{5}\log\frac{M_{\tilde{\ell}_r}}{M_Z}+\frac{1}{5}\log\frac{M_{\tilde{d}_r}}{M_Z}\\ &\quad -\frac{7}{5}\log\frac{M_{\tilde{Q}_r}}{M_Z}+\frac{4}{5}\log\frac{M_{\tilde{u}_r}}{M_Z}-\frac{4}{15}\log\frac{M_{\tilde{h}}}{M_Z}-\frac{2}{15}\log\frac{M_h}{M_Z}\\ &\quad +\frac{1}{15}\log\frac{M_t}{M_Z}+\frac{6}{5}\log\frac{M_D}{M_Z}-\frac{6}{5}\log\frac{M_H}{M_Z}\Bigg),\end{aligned} \tag{3.11}$$

$$\begin{aligned}\delta\left(\alpha_3^{-1}\right)_{light} &= \frac{1}{2\pi}\Bigg(-2\log\frac{M_{\tilde{g}}}{M_Z}+\frac{1}{2}\log\frac{M_{\tilde{w}}}{M_Z}-\frac{3}{8}\log\frac{M_{\tilde{\ell}_\ell}}{M_Z}+\frac{3}{8}\log\frac{M_{\tilde{\ell}_r}}{M_Z}\\ &\quad -\frac{3}{8}\log\frac{M_{\tilde{d}_r}}{M_Z}-\frac{3}{8}\log\frac{M_{\tilde{Q}_r}}{M_Z}+\frac{1}{2}\log\frac{M_{\tilde{h}}}{M_Z}+\frac{1}{4}\log\frac{M_h}{M_Z}\\ &\quad -\frac{1}{8}\log\frac{M_t}{M_Z}-\frac{9}{4}\log\frac{M_D}{M_Z}+\frac{9}{4}\log\frac{M_H}{M_Z}\Bigg).\end{aligned} \tag{3.12}$$

It is noted from Equation (3.11) and Equation (3.12) that if the vector–like colour triplets are degenerate in mass with the vector–like electroweak doublets, then their threshold corrections exactly cancel. In that case the predictions for $\sin^2\theta_W(M_Z)$ and $\alpha_3(M_Z)$ coincide exactly with those of the MSSM. The exact masses of these states depend of course on the details of their couplings to the Z' breaking VEV. Allowing for mass splitting of the order of a few `TeV` may be compensated by contributions from the supersymmetric states. Imposing the experimental limits on the supersymmetric particles and allowing for such mass differences Figure 3.1 shows a scatter plot of $\sin^2\theta_W(M_Z)$ and $\alpha_3(M_Z)$, where the masses of the supersymmetric particles are varied independently.

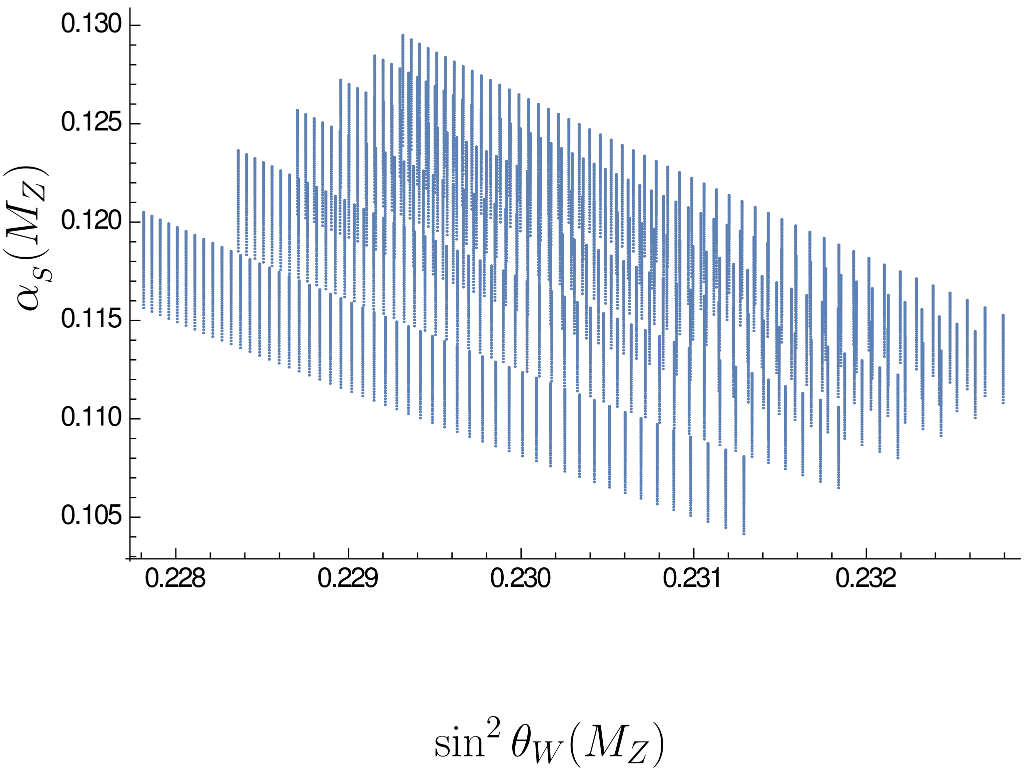


Figure 3.1: Gauge coupling data at the electroweak scale in the presence of a light Z' and assuming unification at the heterotic string scale.

Next we study the predictions for the gauge coupling parameters with Pati–Salam breaking at an intermediate energy scale M_{PS}. The gauge symmetry is $SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_\zeta$, and $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Z'}$, above and below the intermediate Pati–Salam breaking scale, respectively. The weak hypercharge is given by[6]

$$U(1)_Y \;=\; \frac{1}{3}U(1)_C + T_{3_R} \tag{3.13}$$

with $k_C = 6$. When solving the RGEs for the low scale predictions we have to

[6] $U(1)_C = 3U(1)_{B-L}/2$; $U(1)_{\hat{C}} = U(1)_C/\sqrt{3}$.

distinguish the running above and below the intermediate breaking scale. The RGEs and beta function coefficients below the symmetry breaking scale coincide with those of the Z' model discussed above. Above the symmetry breaking scale the spectrum differs from the standard Pati–Salam model due to the anomaly cancellation requirement of $U(1)_\zeta$. To ensure that $U(1)_\zeta$ is anomaly free, all the additional states above the intermediate breaking scale have to be vector–like with respect to $U(1)_\zeta$. The Pati–Salam model contains an additional sextet field required for the missing–partner–like mechanism that gives heavy mass to the heavy Higgs states [68]. Hence, anomaly cancellation with respect to $U(1)_\zeta$ demands another sextet in the spectrum with opposite $U(1)_\zeta$ charge. Similarly, the spectrum above the intermediate symmetry breaking scale contains two bi–doublet states with opposite $U(1)_\zeta$ charges, whereas only one pair of Higgs doublets remain below the intermediate scale. The beta function coefficients above the intermediate breaking scale are therefore

$$b_4^{\rm PS} \;=\; 1 \quad , \quad b_2^{\rm PS} \;=\; 5 \quad , \quad b_{\rm R}^{\rm PS} \;=\; 9 \; , \tag{3.14}$$

which also takes into account the contribution of the heavy Higgs states, and $b_2^{\rm PS}$, $b_{\rm R}^{\rm PS}$ are the beta function coefficients of $SU(2)_L$, $SU(2)_R$, respectively. The effect of the intermediate symmetry breaking scale is to add correction terms to Equation (3.9) and Equation (3.10), given by

$$\Delta_{\rm I.S.}^{\sin^2\theta_W} \;=\; \frac{5\alpha}{16\pi}\left(b_1^{Z'} - \frac{3}{5}b_{\rm R}^{\rm PS} - \frac{2}{5}b_4^{\rm PS} - b_2^{Z'} + b_2^{\rm PS}\right)\log\frac{M_X}{M_{PS}}, \tag{3.15}$$

$$\Delta_{\rm I.S.}^{\alpha_3} \;=\; \frac{1}{2\pi}\left(\frac{3}{4}b_4^{\rm PS} - b_3^{Z'} - \frac{3}{8}b_{\rm R}^{\rm PS} + \frac{5}{8}b_1^{Z'} + \frac{3}{8}b_2^{Z'} - \frac{3}{8}b_2^{\rm PS}\right)\log\frac{M_X}{M_{PS}}. \tag{3.16}$$

We may also consider the case of the LRS model in which the $SO(10)$ symmetry is broken to $SU(3) \times U(1)_C \times SU(2)_L \times SU(2)_R$. We assume that $U(1)_\zeta$ charges admit the E_6 embedding. In this case the heavy Higgs states consists of the pair $\mathcal{N}\left(\mathbf{1}, \mathbf{\frac{3}{2}}, \mathbf{1}, \mathbf{2}, \mathbf{\frac{1}{2}}\right)$, $\bar{\mathcal{N}}\left(\mathbf{1}, -\mathbf{\frac{3}{2}}, \mathbf{1}, \mathbf{2}, -\mathbf{\frac{1}{2}}\right)$. The VEV along the electrically neutral component leaves unbroken the SM gauge group and the $U(1)_{Z'}$ combination in Equation (3.3). We remark, however, that in the free fermionic LRS models [20] the $U(1)_\zeta$ charges do not admit the E_6 embedding. Here, we consider such models as purely field theory models and study the effect on the low scale gauge coupling parameters. Above the symmetry breaking scale the spectrum coincides with that of Table 3.3 with the right–handed fields arranged into doublet representations of $SU(2)_R$. Additionally, the spectrum contains the heavy Higgs states and a pair of Higgs bi–doublets with opposite $U(1)_\zeta$ charges. Crucially, here, the intermediate symmetry breaking does not require the existence of coloured states in the interval between M_R and M_X, which may be incorporated in non–minimal extensions. Consequently, the beta function coefficients above the intermediate symmetry breaking scale M_R are

$$b_3^{\mathrm{R}} = 0 \quad , \quad b_2^{\mathrm{R}} = 5 \quad , \quad b_{\mathrm{R}}^{\mathrm{R}} = 6 \quad , \quad b_{\hat{C}}^{\mathrm{R}} = \frac{21}{2} \; , \tag{3.17}$$

whereas the $b_i^{Z'}$ below the intermediate breaking scale coincide with those given above. Here, b_2^{R} is the beta function coefficient of $SU(2)_L$; $b_{\mathrm{R}}^{\mathrm{R}}$ is that of $SU(2)_R$; and $b_{\hat{C}}^{\mathrm{R}}$ is that of the normalised $U(1)_C$ generator. The effect of the intermediate scale symmetry breaking is to add correction terms for $\sin^2\theta_W(M_Z)$ and $\alpha_3(M_Z)$ given by

$$\Delta_{\text{I.S.}}^{\sin^2\theta_W} = \frac{5\alpha}{16\pi}\left(b_1^{Z'} - \frac{3}{5}b_{\mathrm{R}}^{\mathrm{R}} - \frac{2}{5}b_{\hat{C}}^{\mathrm{R}} - b_2^{Z'} + b_2^{\mathrm{R}}\right)\log\frac{M_X}{M_R}, \tag{3.18}$$

$$\Delta_{\text{I.S.}}^{\alpha_3} = \frac{1}{2\pi}\left(\frac{3}{8}\left(b_2^{Z'} - b_2^{\mathrm{R}} - b_{\mathrm{R}}^{\mathrm{R}} - \frac{2}{3}b_{\hat{C}}^{\mathrm{R}}\right) + \frac{5}{8}b_1^{Z'}\right)\log\frac{M_X}{M_R}. \tag{3.19}$$

3.3 *The Impact Of D-Terms*

The presence of an extra Abelian factor together with the dynamical generation of a μ-term supply our model with the minimal set of tools to relieve the tree-level MSSM hierarchy between the Z and Higgs masses. In the low-energy regime the superpotential [55] provides different interaction terms of the singlet fields S_i and ζ_i which can be extracted from Table 3.3, among them we have

$$\lambda_D^{ijk} S_i D_j \bar{D}_k + \lambda_H^{ijk} S_i H_j \bar{H}_k + \lambda_h^{ij} S_i H_j \bar{h} + \eta_{\mathcal{D}}^i \zeta_i \mathcal{D}\bar{\mathcal{D}} + \eta_h^i \zeta_i h \bar{h}\,. \tag{3.20}$$

To explore the low-energy scalar spectrum that can be naturally covered by the parameter space, we focus on the simple scenario involving only the fields interacting through the coupling λ_H^{ijk} in Equation (3.20). The neutral scalar components will then include 9 supermultiplets; 6 from $H, \bar{H}$ plus other 3 from the SM singlet S. Among different possible settings a viable one is achievable from

$$\langle H_{1,2}\rangle \;=\; \langle \bar{H}_{1,2}\rangle \;=\; \langle S_{1,2}\rangle \;=\; 0, \tag{3.21}$$

with non–zero VEVs concerning only the third generation

$$\langle H_3\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_d \\ 0 \end{pmatrix}, \qquad \langle \bar{H}_3\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_u \end{pmatrix}, \qquad \langle S_3\rangle = \frac{v_S}{\sqrt{2}}, \tag{3.22}$$

where $v_u = v\sin\beta$ and $v_d = v\cos\beta$. The setting in Equations (3.21-3.22) is not the only one capable to minimise the scalar potential and break the symmetry down to $SU(3)\times U(1)_{e.m.}$. It is nevertheless the one with the simplest and more MSSM-like structure. Given the illustrative purpose of this section, we take λ_H^{ijk}

and the soft-SUSY masses to be flavour-diagonal and real parameters. The part of the potential relevant to the spontaneous breaking analysis contains only the (scalar component of the) fields $H_3, \bar{H}_3$, and S_3

$$
\begin{aligned}
V_{Higgs} &= V_{charged} + V_{neutral} && (3.23)\\
&= V_{soft} + V_F + V_D \\
&= \overbrace{\tilde{m}_H^2|H|^2 + \tilde{m}_{\bar{H}}^2|\bar{H}|^2 + \tilde{m}_S^2|S|^2 - \left(\lambda_H\, A_\lambda H\, \bar{H}\, S + \text{h.c.}\right)}^{V_{soft}} \\
&+ \overbrace{\lambda_H^2\left(|H\bar{H}|^2 + |H|^2|S|^2 + |\bar{H}|^2|S|^2\right)}^{V_F} \\
&+ \overbrace{\frac{1}{2}\,g_2^2\left(H^\dagger\frac{\sigma^\mu}{2}H + \bar{H}^\dagger\frac{\sigma^\mu}{2}\bar{H}\right)^2 + \frac{1}{2}g_1^2\left(\frac{1}{2}|\bar{H}|^2 - \frac{1}{2}|H|^2\right)^2}^{V_D} \\
&+ \overbrace{\frac{1}{2}{g'_1}^2\left(Q'_{\bar{H}}|\bar{H}|^2 + Q'_H|H|^2 + Q'_S|S|^2\right)^2}^{V_D} && (3.24)
\end{aligned}
$$

where at the charge-breaking minimum $\langle V_{charged}\rangle = 0$ and

$$
\begin{aligned}
\langle V_{Higgs}\rangle = \langle V_{charged}\rangle + \langle V_{neutral}\rangle &= \lambda_H^2(v_S^2(v_1^2+v_2^2) + v_1^2v_2^2) \\
&\quad + m_{H_1}^2v_1^2 + m_{H_2}^2v_2^2 + m_S^2v_S^2 - 2\lambda_H A_\lambda v_1v_2v_S \\
&\quad + \frac{g^2+g'^2}{8}\left(v_1^2 - v_2^2\right)^2 + 2g''^2\left[v_1^2 + \frac{2}{3}v_2^2 - \frac{5}{3}v_S^2\right]^2
\end{aligned}
$$

As is customary, the trilinear coefficient has been written in the form $\lambda_H\, A_\lambda$. The three soft-masses $\tilde{m}^2_{H\,3,3}, \tilde{m}^2_{\bar{H}\,3,3}, \tilde{m}^2_{S\,3,3}$ non–trivially solve the tadpole–conditions to accommodate for the VEVs structure of Equations (3.21-3.22). Putting such values in the neutral-boson mass matrices and considering the large v_S limit we obtain

$$m_Z^2 = \frac{v^2}{4}\left(g_1^2 + g_2^2\right) \ , \ m_{Z'}^2 = \left(Q'_S\, g'_1\, v_S\right)^2 = \left(Y'_S\, g'_1\, v_S\right)^2 \ . \tag{3.25}$$

By requiring

$$\tilde{m}^2_{H\,1,1} = \tilde{m}^2_{H\,2,2} \ , \tilde{m}^2_{\bar{H}\,1,1} = \tilde{m}^2_{\bar{H}\,2,2} \ , \tilde{m}^2_{S\,1,1} = \tilde{m}^2_{S\,2,2} \ , \tag{3.26}$$

the 9×9 CP-odd mass matrix can be analytically diagonalised. In the Landau gauge the two massless Goldstone bosons are promptly found and the remaining 7 masses are a degenerate ensemble of the independent set:

$$\left(m_1^2, m_2^2, m_3^2, m_{A_\lambda}^2\right) \ . \tag{3.27}$$

The eigenvalues m_{1-3}^2 are uniquely linked to the three independent soft masses of Equation (3.26) and consequently are all double degenerate. The eigenvalue dubbed as $m_{A_\lambda}^2$ is connected to the trilinear soft term. In the limit of large v_S we find

$$m_{A_\lambda}^2 = \sqrt{2}\, v_S\, \lambda_H \frac{A_\lambda}{\sin(2\beta)} \ , \tag{3.28}$$

where $\tan\beta = v_u/v_d$. The correspondence with the MSSM is clear once we identify the effective μ-term $\mu_{eff} = v_S\, \lambda_H\, /\sqrt{2}$. All the soft-masses in Equation (3.23) can thus be traded for the CP-odd eigenvalues and, via tadpole conditions, for the non-zero VEVs. The mass matrix for the charged Higgs scalars[7] can similarly be analytically diagonalised. The eigenvalues are simply linked to the W mass and

[7]We are always considering only the supermultiplets H, $\bar{H}$ and S.

the CP-odd masses. In the Landau gauge we find one massless Goldstone while the remaining independent masses are given by (for $v_S \gg v$)

$$\left(m_1^2 + M_W^2\,\cos(2\beta),\, m_2^2 - M_W^2\,\cos(2\beta), m_{A_\lambda}^2 + M_W^2 - \frac{\lambda^2 v^2}{2}\right), \quad (3.29)$$

with degeneracy inherited from the CP-odd structure. The CP-even mass matrix is mostly diagonal with mixing involving only the third generations of $H, \bar{H}$, and S. The remaining 3×3 block to be diagonalised includes the matrix elements

$$\begin{aligned}
m_{1,1}^2 &= M_Z^2\,\cos^2\beta + 4\,M_Z^2\,\left(\frac{g_1'\,Q_H'}{\bar{g}}\right)^2\,\cos^2\beta + \Delta\;\sin^2\beta, \\
m_{2,2}^2 &= M_Z^2\,\sin^2\beta + 4\,M_Z^2\,\left(\frac{g_1'\,Q_{\bar{H}}'}{\bar{g}}\right)^2\,\sin^2\beta + \Delta\;\cos^2\beta, \\
m_{3,3}^2 &= {M_Z'}^2 + \Delta\left(\frac{M_Z\,\sin(2\beta)}{\bar{g}\,v_S}\right)^2,
\end{aligned}$$

$$\begin{aligned}
m_{1,2}^2 &= \cos\beta\,\sin\beta\,\left(-M_Z^2 - \Delta + \frac{4\,M_Z^2}{\bar{g}^2}\,\left(\lambda^2 + {g_1'}^2\,Q_H'\,Q_{\bar{H}}'\right)\right), \\
m_{1,3}^2 &= \cos\beta\,\left(2\,\frac{M_Z\,v_S}{\bar{g}}\right)\,\left(-\frac{\Delta}{v_S^2}\,\sin^2\beta + \lambda^2 + {g_1'}^2\,Q_H'\,Q_S'\right), \\
m_{2,3}^2 &= \sin\beta\,\left(2\,\frac{M_Z\,v_S}{\bar{g}}\right)\,\left(-\frac{\Delta}{v_S^2}\,\cos^2\beta + \lambda^2 + {g_1'}^2\,Q_S'\,Q_{\bar{H}}'\right) \qquad (3.30)
\end{aligned}$$

where

$$\bar{g}^2 = g_1^2 + g_2^2\,, \qquad \Delta = \frac{\bar{g}^2\,M_{A_\lambda}^2\,v_S^2}{M_Z^2\,\sin^2(2\beta) + \bar{g}^2\,v_S^2}\,. \quad (3.31)$$

The numerical diagonalisation of the previous mass matrices easily reveals large branches of the parameter space with tree-level eigenvalues that elude the MSSM hierarchy between the lightest scalar (LS) and M_Z (Figure 3.2). To obtain an analytical estimation of the impact of the D–terms we minimise the expectation value of the CP-even mass matrix with the vector $(\cos\beta, \sin\beta, 0)$ [62]. The result represents an upper limit for its smallest eigenvalue

$$M_h^2 \leq M_Z^2 \cos^2(2\,\beta) + \frac{v^2}{2}\,\lambda^2 \sin^2(2\,\beta) + {g'_1}^2\, v^2 \,\left(Q'_H \cos^2\beta + Q'_{\bar{H}} \sin^2\beta\right)^2 . \quad (3.32)$$

In the formal limit $g'_1, \tilde{g} \to 0$ we recover the upper bound of the NMSSM [62]-

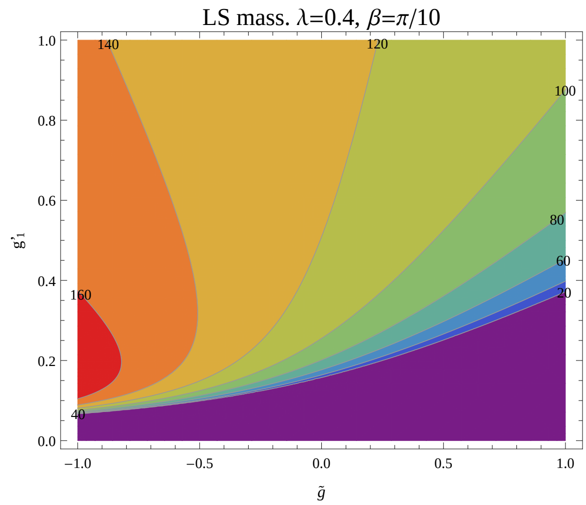
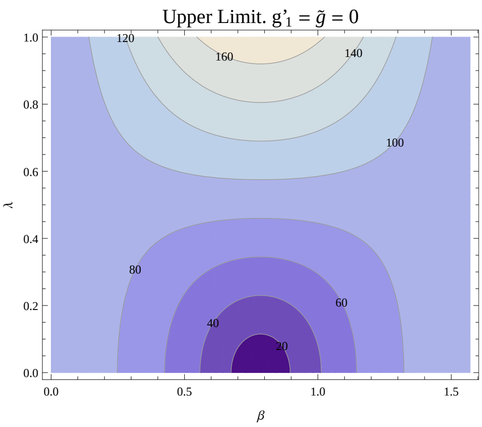
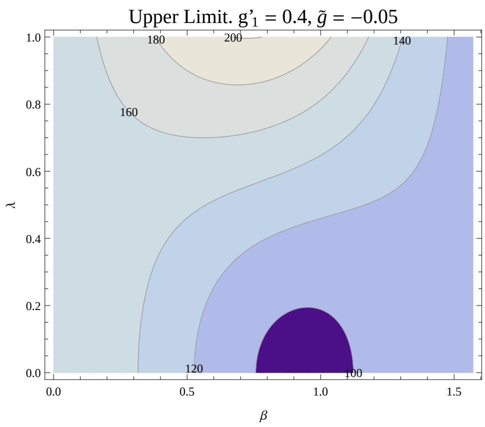


Figure 3.2: Contour plot of lightest scalar eigenvalue of matrix (3.30). $v_S = 2.5$ TeV $M_{A_\lambda} = 500$ GeV.

[63] and a further limit, $\lambda_H \to 0$, we obtain the MSSM one. As known, the singlet extension of the MSSM is a first step to increase the tree-level value of the LS. The positive contribution of the $U(1)_{Z'}$-related D–terms in Equation (3.32) allows even larger upper bounds (Figure 3.3).



(a)



(b)

Figure 3.3: Contour plot of upper bounds for LS mass.

4

No Sign of SUSY...Thus Far

A smooth sea never made a

skilled sailor.

Franklin D. Roosevelt

In this chapter, recent interest in the non-supersymmetric heterotic string vacua is presented, which has led to several formal developments [76], ignited primarily by the lack of evidence in favour of SUSY at the LHC. We show that phenomenologically viable free fermionic models with a low number of families give rise to proliferation of a priori tachyon producing sectors, compared to the nonrealistic examples, which typically may contain only one such sector. An example of a quasirealistic, nonsupersymmetric, nontachyonic, heterotic string vacuum is then presented and the structure of its massless spectrum is then compared to the corresponding supersymmetric vacuum. While in some sectors we observed that SUSY is broken explicitly, i.e. the bosonic and fermionic sectors produce massless and massive states, other sectors, and in particular those leading to the chiral families, continue to exhibit Fermi-Bose degeneracy.

A brief discussion on string models with a split SUSY structure, has been relegated to Appendix F, which did not prove to be fruitful where the basic idea there was to use two basis vectors to generate space-time SUSY. By recalling that in the semi-realistic free fermionic models the SUSY generators arise from the basis vector S. The aim was then to construct two basis vectors that generate SUSY with the focus to construct models in which the gaugini are obtained from one generator, whereas those of the second generator are projected out, as well as the the scalar superpartners of the twisted matter fermionic states.

4.1 *Why The Non-Supersymmetric Vacua?*

String theory provides the most developed contemporary approach to study how the SM parameters may arise from a unified theory of the gauge and gravitational interactions. For this purpose several models that reproduce the spectrum of the MSSM have been produced [15, 16]. Amongst them the free fermionic models [15, 17–22, 25–27] are the most studied examples. The majority of semi–realistic heterotic string models constructed to date possess $N = 1$ space-time SUSY, while non–supersymmetric vacua were investigated sporadically [29–32].

The lack of experimental evidence in favour of SUSY at the LHC has led to the recent interest in non–supersymmetric heterotic string vacua [33–37, 76]. It is therefore prudent to examine what may be learned in this context from the quasi–realistic free fermionic models and the different avenues that may be used to break SUSY directly at the string scale in light of the recent analysis [34].

4.2 *The Classification Set*

Once again, to reiterate, additional basis vectors are added to the set

$$\begin{aligned}
v_1 = &\quad 1 &&= \{\psi_\mu^{1,2}, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,\dots,8}\},\\
v_2 = &\quad S &&= \{\psi^\mu, \chi^{12}, \chi^{34}, \chi^{56}\},\\
v_{2+i} = &\quad e_i &&= \{y^i, \omega^i | \overline{y}^i, \overline{\omega}^i\},\ i = 1,\dots,6,\\
v_9 = &\quad b_1 &&= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \overline{y}^{56}, \overline{\eta}^1, \overline{\psi}^{1,\dots,5}\},\\
v_{10} = &\quad b_2 &&= \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \overline{y}^{56}, \overline{\eta}^2, \overline{\psi}^{1,\dots,5}\},\\
v_{11} = &\quad z_1 &&= \{\overline{\phi}^{1,\dots,4}\},\\
v_{12} = &\quad z_2 &&= \{\overline{\phi}^{5,\dots,8}\}
\end{aligned}$$

to construct vacua with $SO(10)$ subgroups [26, 27]. In this notation, the world-sheet fermions appearing in the curly brackets have periodic boundary conditions, whereas all other world-sheet fermions are anti–periodic. The entries in the matrix of GGSO phases $c\left[\begin{smallmatrix} v_i \\ v_j \end{smallmatrix}\right]$ with $i > j$ then span the space of string vacua. Additional constraints that are imposed on the string vacua, like the existence of space-time SUSY leave 40 independent phases of the original 66. One can then resort to a complete [40] or statistical sampling[8] of the total space [25], and classify the models by their twisted matter spectrum. The classification is facilitated by expressing the GGSO projections in algebraic form [25, 40]. The analysis of the entire spectrum of the string models is computerised and vacua with specific phenomenological characteristics can be fished our from the larger

[8] We note that analysis of large sets of string vacua has also been carried out by other groups [48].

space of models.

In terms of space-time SUSY breaking, as with the NAHE–set based models the space-time SUSY generator is the basis vector S. The subset $\{\mathbf{1}, S\}$ gives rise to $N = 4$ space-time SUSY, which is broken by b_1 and b_2 to $N = 2$ space-time SUSY and their combined action breaks $N = 4 \to N = 1$. As with the NAHE–based models imposing $c\left[{S \atop v_i}\right] = -\delta_{v_i}$ ensures the preservation of $N = 1$ SUSY. Projecting the remaining SUSY in this model is obtained by relaxing this condition. Furthermore, the basis vectors $\{e_i, z_1, z_2\}$ satisfy $S \cdot e_i = 0$ and $S \cdot z_i = 0$. These basis vectors therefore act as projectors on the S–sector. These basis vectors can be used to project all the states from the S–sector and hence induce the breaking $N = 4 \to N = 0$ space-time SUSY.

4.3 *Tachyons In The Free Fermionic Models*

String models, heterotic string models in particular, generally give rise to tachyonic states in their spectra. Any sector that satisfies

$$M_L^2 \;<\; -\frac{1}{2} \quad \text{and} \quad M_R^2 \;<\; -1 \tag{4.1}$$

may produce tachyonic physical states. Tachyonic states can be obtained by acting on the vacuum with fermionic oscillators. They satisfy the level matching condition and survive all the GGSO projections. Their presence in the physical spectrum indicates the instability of the string vacuum. The existence of space-time SUSY guarantees that all tachyonic states are projected out. The situation is altered if SUSY is broken to $N = 0$ space-time SUSY by projecting all the states from the S–sector. One then has to check in each model whether tachyonic states exist.

The existence of non–supersymmetric non–tachyonic string vacua has been known since the mid–eighties [29]. The gauge symmetry of this model is $SO(16)\times SO(16)$, and its non–perturbative extension was considered in [32]. In the free fermionic formalism the model is constructed by the set of boundary condition basis vectors $\{\mathbf{1}, S, X, I\}$ where $X = \{\bar\psi^{1,\cdots,5}, \bar\eta^{1,2,3}\}$ and $I = \{\bar\phi^{1,\cdots,8}\}$ with the set of GGSO phases given by

$$\begin{array}{c c} & \begin{array}{cccc} \mathbf{1} & S & X & I \end{array} \\ \begin{array}{c} \mathbf{1} \\ S \\ X \\ I \end{array} & \begin{pmatrix} +1 & -1 & +1 & +1 \\ -1 & -1 & -1 & -1 \\ +1 & -1 & +1 & \pm 1 \\ +1 & -1 & \pm 1 & +1 \end{pmatrix} \end{array}. \tag{4.2}$$

In ten dimensions the choice of the GGSO phase $c\left[{X \atop I}\right] = \pm 1$ yields either the supersymmetric $E_8\times E_8$, or the non–supersymmetric $SO(16)\times SO(16)$, heterotic string. This is necessarily the case in ten dimensions because the SUSY generator is given by $S = \mathbf{1} + X + I$ and therefore the projections on the three sectors are correlated. In the four dimensional models the same phase can be used to reduce the gauge symmetry from $E_8 \times E_8$ to $SO(16) \times SO(16)$ without breaking SUSY. The same vacua can be constructed in the orbifold representation and can be connected by interpolations [30]. Hence, the supersymmetric and non-supersymmetric vacua exist on the boundary of the same moduli space.

It is instructive to examine the case of the non-supersymmetric $SO(16) \times SO(16)$ model first. In the four dimensional model SUSY may be broken from $N = 4 \to N = 0$ by the I or X projections. The only sector that may produce tachyons in this model is the NS sector. We are lead to conclude that in any non–supersymmetric free fermionic model that includes the S–sector the untwisted tachyons are always projected out, irrespective of the choice of the SUSY projecting phases.

The best case scenario would be a model in which the only tachyon producing sector is the NS–sector. In this case we are guaranteed that tachyons do not exist in the physical spectrum. However, a model with this property has not been found to date. The next best case scenario is a model that gives rise only to one type of tachyon producing sectors. Existence of a model with this characteristic may depend on further detailed phenomenological properties of the string vacua. For example, we were not able to find such a model in the class of NAHE–based free fermionic models with reduced Higgs spectrum [46], whereas the class of left–right symmetric models [20] did produce a model with the desired property. The set of boundary condition basis vectors, beyond the NAHE–set, generating the string vacuum is given by

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,\ldots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\ldots,8}$
α	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
β	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
γ	0	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0 0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0

$$
\begin{array}{c|cccc|cccc|cccc}
 & y^3y^6 & y^4\bar{y}^4 & y^5\bar{y}^5 & \bar{y}^3\bar{y}^6 & y^1\omega^5 & y^2\bar{y}^2 & \omega^6\bar{\omega}^6 & \bar{y}^1\bar{\omega}^5 & \omega^2\omega^4 & \omega^1\bar{\omega}^1 & \omega^3\bar{\omega}^3 & \bar{\omega}^2\bar{\omega}^4 \\
\hline\hline
\alpha & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\
\beta & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
\gamma & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1
\end{array}
\tag{4.3}
$$

This model gives rise only to one type of tachyon producing sectors with

$$
\alpha_L^2 \ = \ 2 \quad \& \quad \alpha_R^2 \ = \ 6 \quad \Rightarrow \quad N_R \ = \ 0 \tag{4.4}
$$

The supersymmetric version of this model was presented in [20] with the set of GGSO phases given by

$$
\begin{array}{c}
 \\ \mathbf{1} \\ S \\ \\ b_1 \\ b_2 \\ b_3 \\ \\ \alpha \\ \beta \\ \gamma
\end{array}
\begin{array}{c}
\begin{array}{cccccccc}
\mathbf{1} & S & b_1 & b_2 & b_3 & \alpha & \beta & \gamma
\end{array} \\
\left(\begin{array}{cccccccc}
1 & 1 & -1 & -1 & -1 & 1 & 1 & i \\
1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\
 & & & & & & & \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & i \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & i \\
-1 & -1 & -1 & -1 & -1 & -1 & 1 & i \\
 & & & & & & & \\
1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & -1 & 1
\end{array}\right)
\end{array} . \tag{4.5}
$$

The full mass spectrum of this model together with the cubic level superpotential was presented in [20]. The modification

$$c\begin{bmatrix} S \\ \alpha \end{bmatrix} = -1 \to +1 \quad \text{and} \quad c\begin{bmatrix} S \\ \beta \end{bmatrix} = -1 \to +1 \tag{4.6}$$

projects the remaining gravitino and induces $N = 1 \to N = 0$. It can be checked that all the tachyonic states are projected out in this model. Furthermore, it can be verified that making the modification

$$c\begin{bmatrix} S \\ \alpha \end{bmatrix} = -1 \to +1 \quad \text{and} \quad c\begin{bmatrix} S \\ \beta \end{bmatrix} = -1 \to -1 \tag{4.7}$$

$i.e.$ modifying only $c\left[\begin{smallmatrix} S \\ \alpha \end{smallmatrix}\right]$ but not $c\left[\begin{smallmatrix} S \\ \beta \end{smallmatrix}\right]$ results in a model that contains tachyonic states. The reason is that in this model all the sectors that may produce tachyons appear with the combination $m(\alpha + \beta)$, where $m = 0, 1$. Hence, with the modification given by Equation (4.6). the S–projection on the tachyonic sectors is the same as in the corresponding SUSY preserving choice given in Equation (4.5), whereas with the modification given by Equation (4.7), the S–projection in some sectors is modified in comparison to the supersymmetric model and some tachyonic states are not projected out. We note that the construction of tachyonic free semi–realistic vacua is highly nontrivial. In the next section we present an explicit example of the tachyon-free model.

4.4 *An Explicit Tachyon-Free Model*

We consider the model defined by the set of basis vectors

$$
\begin{aligned}
1 &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}, \\
S &= \{\psi^\mu, \chi^{1,..,6}\} \\
b_1 &= \{\psi^\mu, \chi^{1,2}, y^{3,\dots,6} | \overline{y}^{3,\dots,6}, \overline{\psi}^{1,\dots,5}, \overline{\eta}^1\} \\
b_2 &= \{\psi^\mu, \chi^{3,4}, y^{1,2}, \omega^{5,6} | \overline{y}^{1,2}, \overline{\omega}^{5,6}, \overline{\psi}^{1,\dots,5}, \overline{\eta}^2\} \\
b_3 &= \{\psi^\mu, \chi^{5,6}, \omega^{1,\dots,4} | \overline{\omega}^{1,\dots,4}, \overline{\psi}^{1,\dots,5}, \overline{\eta}^3\} \\
b_4 &= \alpha \\
&= \{y^{1,\dots,6}, \omega^{1,\dots,6} | \overline{\omega}^1, \overline{y}^2, \overline{\omega}^3, \overline{y}^{4,5}, \overline{\omega}^6, \overline{\psi}^{1,2,3}, \overline{\phi}^{1,\dots,4}\} \\
b_5 &= \beta \\
&= \{y^2, \omega^2, y^4, \omega^4 | \overline{y}^{1,\dots,4}, \overline{\omega}^5, \overline{y}^6, \overline{\psi}^{1,2,3}, \overline{\phi}^{1,\dots,4}\} \\
b_6 &= \gamma \\
&= \{y^1, \omega^1, y^5, \omega^5 | \overline{\omega}^{1,2}, \overline{y}^3, \overline{\omega}^4, \overline{y}^{5,6}, \overline{\psi}^{1,2,3} = \frac{1}{2}, \overline{\eta}^{1,2,3} = \frac{1}{2}, \overline{\phi}^{2,\dots,7} = \frac{1}{2}\}
\end{aligned}
$$

with the set of GGSO phases given by

$$
\begin{array}{c}
\\ 1 \\ S \\ b_1 \\ b_2 \\ b_3 \\ \alpha \\ \beta \\ \gamma
\end{array}
\begin{array}{c}
\begin{array}{cccccccc} 1 & S & b_1 & b_2 & b_3 & \alpha & \beta & \gamma \end{array} \\
\left(\begin{array}{cccccccc}
1 & 1 & -1 & -1 & -1 & 1 & 1 & i \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & i \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & i \\
-1 & -1 & -1 & -1 & -1 & -1 & 1 & i \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & -1 & 1
\end{array}\right)
\end{array} .
$$

This is a 3 generation model, with one generation appearing in each of the twisted sectors b_1, b_2 and b_3. The full spectrum can be found in the table of appendix G, with the exception of the gauge bosons which have been omitted in the interest of space. It is sufficient to state that the gauge group is

$$
\underbrace{SU(3)_C \times U(1)_C \times SU(2)_L \times SU(2)_R \times \prod_{i=1}^{6} U_i}_{\text{observable sector}} \times \underbrace{SU(3)_{H_1} \times SU(3)_{H_2} \times \prod_{j=7}^{10} U_j}_{\text{hidden sector}} .
$$

This model exhibits many interesting features regarding SUSY. Firstly, we observe that the model is manifestly non-supersymmetric. The gravitino and the gaugini are projected out and there is a clear mismatch between the number of states in the NS and S sectors. Furthermore, there are eight sectors with only scalars and the sectors that contain the would-be superpartners are massive. These are

$$\begin{aligned}
&\beta+\gamma, && \beta+3\gamma,\\
&\alpha+\gamma, && \alpha+3\gamma,\\
&1+b_1+b_2+b_3+\beta+\gamma, && 1+b_1+b_2+b_3+\beta+3\gamma,\\
&1+b_1+b_2+b_3+\alpha+\gamma, && 1+b_1+b_2+b_3+\alpha+3\gamma.
\end{aligned} \tag{4.8}$$

Such sectors would not remain in the spectrum in the supersymmetric choice of phases. The reason is that the space-time SUSY generator in the supersymmetric model is the basis vector S, *i.e.* for a given sector $\rho \in \Xi$, the supersymmetric superpartners are obtained from the sector $S+\rho$. All the sectors in Equation (4.8) have $(\rho)_L^2 = 4$, whereas $(S+\rho)_L^2 = 8$, *i.e.* in these sectors the would-be superpartners are massive. In the supersymmetric vacua the states from the sectors in Equation (4.8) are necessarily projected out, as they break SUSY explicitly. However, once SUSY is broken they may appear in the spectrum, as is seen in our model. It is a highly non-trivial task to find a model with 3 generations in which sectors of these type, that only appear when SUSY is broken, contain no tachyons, with this model being such an example.

On the other hand, there are (pairs of) sectors that are completely supersymmetric. This is due to the modification, Equation (4.6), not affecting the GGSO projections in any sectors where none of the vectors S, α or β appear. Therefore such sectors will be identical to the corresponding sectors of the supersymmetric model. Nonetheless, for some of these sectors to remain supersymmetric as claimed above, the superpartners should be unchanged as well, or at least the effect must be (at most) a change in the R–charges of the superpartners. Sectors b_i and $1+b_i+b_j+2\gamma$ are of this type.

Finally, there are sectors that do not fit any of the above categories. In these sectors the number of bosons and fermions is the same, but on the other hand some of the gauge charges of these states are different which in principle prevents us from grouping them together into supermultiplets. Most of the sectors are of this type. We use the term sectors in which SUSY is "nicely broken" when referring to this case.

Thus, while SUSY is broken, some segments of the string vacuum still respect the underlying supersymmetric degeneracy. This is in accordance with the findings in [34], which showed that the partition function of string vacua with spontaneously broken SUSY can be divided into several orbits, some of which preserve the original SUSY.

Furthermore, we would like to comment in our model the fermionic states from the sectors b_1, b_2 and b_3, as well as the bosonic states from the NS–sector, are not affected by the GGSO phases that project the gravitino and gaugini from the S–sector, and therefore break space-time SUSY. Therefore, the untwisted scalar states of our non–supersymmetric model as well as the fermionic states from the sectors b_1, b_2 and b_3 are identical to those in the corresponding supersymmetric model. Consequently, the leading twisted–twisted–untwisted couplings in the non–supersymmetric model, which are obtained by using the methods developed in [49], are identical to those of the supersymmetric model. The model generated by the Equations (4.3,4.5) contains electroweak doublet scalar representations from the twisted sectors that may be used as Higgs doublets. However, in this model the untwisted Higgs bi–doublets, which couple at leading order to the twisted sector states, are projected out and consequently the leading mass term which is identified with the top mass is absent. Other LRS models [20], as well as the FSU5 [17], PS [19] and SLM [15, 18] models, do contain the untwisted Higgs doublets and in those cases a leading top mass term is obtained.

It is also worth noting that even for non-supersymmetric models the cosmological constant can be exponentially suppressed. As discussed in [35], this can be achieved if the massless spectrum has an equal number of bosons and fermions (irrespectively of their charges). Even though our model is not of this type and will therefore have an unsuppressed vacuum energy, our construction hints at how one might go about achieving such a goal. It is clear for example, that we do not have to worry about sectors that either respect SUSY or in which SUSY is nicely broken.

On the other hand, sectors that badly break SUSY will have to be carefully engineered. There are a few ways one might go about such a task. For example, one might entertain the idea that the addition of further basis vectors could project such sectors out of the spectrum. The biggest problem with this approach is that the removal of the gaugini from the S sector, even if some fermions transforming in a different than the adjoint representation are preserved, will create a mismatch of states in the S and NS sectors turning them into sectors that break SUSY badly; and it is impossible to project out the NS sector no matter what basis vectors are added. It is *a priori* possible that further basis vectors will remove exactly the correct number of bosons from the NS sector to match the remaining fermionic states in the S sector, but this method seems unnecessarily restricting.

4.5 *The Anomalous* $U(1)$

Another interesting point to note is the existence of an anomalous $U(1)$ symmetry in this model. The anomalous $U(1)$ is cancelled by the Green–Schwarz–Dine–Seiberg–Witten mechanism [50, 51], in which a potentially large Fayet-Iliopoulos

D-term is generated by the VEV of the dilaton field giving rise to a tadpole diagram at one–loop order in string perturbation theory [52], which reflects the instability of the string vacuum. The mismatch between the fermionic and bosonic states at different mass levels gives rise to a non–vanishing vacuum energy, which similarly gives rise to a tadpole diagram, indicating the instability of the string vacuum. We may therefore entertain the possibility of employing one against the other so that they conspire to cancel. The anomalous $U(1)$ contribution is proportional to the trace over the massless fermionic states and the sign can be altered by the GGSO projections [52, 53]. It is proportional to the gauge coupling and consequently only depends on the dilaton moduli. On the other hand, the vacuum amplitude contribution depends on other moduli [35], and may be tuned to obtain cancellation of the two contributions. In general, other background fields will be affected by the shift of the vacuum, and to demonstrate the existence of a stable vacuum one would need to solve the set of equations affecting those fields in the shifted vacuum. However, in this regard the same constraints would apply in the case of the supersymmetric vacua, where the Fayet–Iliopoulos term [51,54], which is generated from the anomalous $U(1)$ tadpole diagram [51, 52], is cancelled by assigning VEVs to some massless scalar fields, along flat supersymmetric directions. As a result, we suggest that the non–supersymmetric non–tachyonic string vacua should be considered on equal footing to the supersymmetric examples.

Another approach providing more freedom is to aim for an equality in the number of bosons and fermions not in each sector, but among different sectors. To cancel the surplus of bosons from the NS sector this would imply the existence of surviving fermionic states in different sectors, the bosonic counterpart of which has been projected out. It is now easy to see that since the model presented does not have any sectors α with more fermions than bosons in the sector $S + \alpha$, it does not have a suppressed cosmological constant. Finding a semi-realistic model with suppressed cosmological constant appears to be very challenging, but still is of great interest.

5

Conclusion and Outlook

Thou seest not, in the creation of the All-merciful any imperfection, Return thy gaze, seest thou any fissure? Then Return thy gaze, again and again. Thy gaze, Comes back to thee dazzled, aweary.

The Noble Qur'an, 67:3-4

Hitherto SM continues to reign supreme in providing viable parametrization for subatomic observational data. Incorporating gravitational phenomena mandates the extension of the SM with string theory providing for the minimal departure from the point particle hypothesis underlying the SM; mathematically self–consistent framework for perturbative quantum gravity and develop a phenomenological approach to explore the synthesis of the gauge and gravitational interactions. Detailed phenomenological models that incorporate the key features of the SM have been constructed. These detailed phenomenological constructions contain a new symmetry, *i.e.* $N = 1$ space-time SUSY.

The heterotic string models in the free fermionic formulation give rise to some of the most realistic string models to date, which possess $N = 1$ space-time SUSY. The characteristics of the required spectrum are well motivated in heterotic string constructions that allow for a light Z'. Anomaly cancellation of the $U(1)_{Z'}$ symmetry requires the existence of the SM singlet and vector-like states in the vicinity of the $U(1)_{Z'}$ breaking scale. We showed that the agreement with the gauge coupling data at one-loop is identical to the case of the MSSM, owing to cancellations between the additional states. Above the intermediate breaking scale the weak hypercharge is embedded in a non–Abelian group and kinetic mixing cannot arise. Below the intermediate breaking scale kinetic mixing arises due to the extra pair of electroweak doublets, but it is found to be small and does not affect the results.

On the other hand, very recently, formal developments [76] have emerged as SUSY has not been observed experimentally to date at the LHC, with such models having a sound theoretical background as well as fascinating phenomenological prospects and it is therefore crucial to explore the consequences of breaking space-time SUSY directly at the string scale. Pursuing this direction, we see that the generic feature of non–supersymmetric string vacua is the existence of tachyonic states in the physical spectrum. Non–supersymmetric string vacua, such as the $SO(16) \times SO(16)$ heterotic string in ten dimensions, do not contain tachyonic states, but are typically connected in the moduli space to supersymmetric vacua, and tend to have large moduli states and group factors. More realistic constructions on the other hand, typically have reduced moduli spaces and contain more sectors that may a priori give rise to tachyons.

It is therefore important to examine the structure of non–supersymmetric string vacua in a more realistic setting. We have shown that while generically the quasi–realistic non–supersymmetric vacua do contain tachyons, there also exist examples in which all the tachyonic states are projected out by the GGSO projections. Furthermore, given that the moduli spaces of the quasi–realistic constructions may be much reduced [45,46], one may entertain the possibility that the tachyon free non–supersymmetric quasi–realistic vacua may not be connected to supersymmetric solutions. We have shown with a concrete example that the non–supersymmetric quasi–realistic vacuum may retain some of the structure of the corresponding supersymmetric solution. Finally, we suggested the possibility of entertaining the cancellation between the tadpole diagram generated at one-loop order in string perturbation theory due to a potentially large Fayet-Iliopoulos D-term being generated by the VEV of the dilaton field, associated with the cancellation of the anomalous $U(1)$, against the tadpole diagram generated at one-loop associated to the non-vanishing vacuum energy. The model presented did not have a suppressed cosmological constant as finding a semi-realistic model with suppressed cosmological constant is a challenging task but still is of great interest.

To conclude, the current status of the unification of gravity and the gauge interactions are heavily motivated by string-derived models which continue to provide a feasible contemporary framework. Consequently, three generation models need to be obtained for phenomenological reasons, however, a detailed example is still not in sight. Nevertheless, string theory provides a sea of well established models which promising phenomenological prospects and therefore can be considered as acceptable candidates for providing a good description of nature, especially at the low energies where our four-dimensional, observable world handsomely resides.

A

Lie Algebras

Let G denote a simple Lie group and $\mathfrak{g}$ its corresponding Lie algebra with generators T_a for $a = 1, ..., \dim(G)$. Then the nature of the algebra is defined by structure constants f_{ab}^c as

$$\left[T_a, T_b\right] = f_{ab}^c T_c.$$

Now define the rank of the Lie algebra $\mathfrak{g}$ as the number of simultaneously diagonalisable generators which is also the dimension of the associated maximal Cartan subalgebra $\mathfrak{h} \subset \mathfrak{g}$, that is the algebra of all generators H_i with $i = 1, ..., r$ satisfying

$$\left[H_i, H_j\right] = 0.$$

The remaining generators are then defined to be the eigenfunctions of the Cartan generators H_i denoted by $E_{\vec{a}}$ which satisfy the relation

$$\left[H_i, E_{\vec{a}}\right] = \alpha_i E_{\vec{a}}$$

where r-component vectors $\vec{a}$ are known as roots living in an r-dimensional Euclidean space.

For an n-dimensional representation ρ of the rank r Lie algebra $\mathfrak{g}$, the generators H_i of the Cartan subalgebra, can be represented in terms of $n \times n$ diagonal matrices with elements ν_i^a for $a = 1, ..., n$. These matrices act naturally on n-vectors and we fix the basis to be the canonical basis: $\{e_1, ..., e_n\}$. We can now define the Cartan generators as

$$H_i = \nu_i^a e_a.$$

Then the rank r vector $|\lambda^a\rangle$ simply gives

$$H_i|\lambda^a\rangle = \nu_i^a|\lambda^a\rangle.$$

Given a set of roots λ, we define a positive root to be such that its first non-zero element in the specified basis is positive.

We define simple roots as positive roots that cannot be expressed as a sum of other roots with positive coefficients. There is a theorem which states that the number of simple roots is equal to the rank of the Lie algebra. In particular, the simple roots form a basis of the Cartan subalgebra.

Note A.0.1 . The rank of the Lie algebra $\mathfrak{su}(n)$ is $n - 1$.

The Lie algebra $\mathfrak{su}(3)$ has rank 2, so there are two Cartan generators which we can define as two traceless matrices given by

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Then the states are given by $\{(1,1),(1,0),(-2,-1)\}$. It can be seen that $(1,1)$ and $(1,0)$ are simple, positive roots.

Remark A.0.2 . A standard choice of generators in this case is $T_a = \frac{1}{2}\lambda_a$ where λ_a are the traceless 3×3 Gell-Mann matrices.

A.1 *Examples of Lie Algebras*

Example 1. The Lie algebra $\mathfrak{sl}(2)$, the set of 2×2 traceless matrices, has the basis

$$e = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \ f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \ h = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

with relations

$$[h,e] = 2e, \ [h,f] = -2f, \ [e,f] = h.$$

Example 2. The Heisenberg Lie algebra $\mathcal{H}$ of matrices

$$\begin{pmatrix} 0 & * & * \\ 0 & 0 & * \\ 0 & 0 & 0 \end{pmatrix}$$

has the basis

$$i = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \ j = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ k = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

with relations

$$[j,i] = k, \ [i,k] = [j,k] = 0.$$

B

An Instance of Electroweak Symmetry Breaking

The Glashow-Salam-Weinberg model is a minimal model which uses quarks to build the hadrons and the hadrons constructed from each generation participate in both the weak and the strong interactions in effect realizing the universality of the weak Fermi coupling G_F. In order to construct hadron with their known electric charges, the quarks must carry fractional charges. For example, suppressing their color index, the two quarks u and d form the first generation and can be expressed as a doublet

$$q = \begin{pmatrix} u \\ d \end{pmatrix}.$$

The u quark possesses electric charge

$$\frac{2}{3}e$$

while the d quark has

$$-\frac{1}{3}e.$$

The composite state $u\bar{d}$ has charge e and constitute a charged π meson. The baryon number of the composite is derived by associating baryon number

$$\frac{1}{3}$$

to each quark and

$$-\frac{1}{3}$$

to each anti-quark. So $u\bar{d}$ has baryon number 0.

There are also leptons in the model. Leptons participate only in the electromagnetic and weak interactions. The first generation is composed of the electron and its neutrino so that

$$l = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}.$$

This at first might seem questionable as the neutrino is only left-handed and has negligible mass whereas the electron is observed with both chiralities and carries mass. Moreover, the neutrino is electrically neutral but the electron carries a charge e.

The gauge theory approach to uniting the weak interactions with the electromagnetic interactions requires that the left-handed constituents of the fields be treated differently from the right-handed ones. Due to the doublet nature of the generations the simplest non-Abelian gauge group is $SU(2)$ in its fundamental representation.

Note B.0.1 . As the weak interactions only involve the left-handed constituents, the gauge group will act only on the left-handed quarks and leptons and therefore it is denoted

$$SU(2)_L.$$

Using $SU(2)$ introduces 3 gauge bosons. The presence of these intermediate vector bosons in the weak interactions require that at least two of these carry an electric charge. To meet the experimental observation of the neutral current events, introduce a $U(1)$ gauge field that interacts with both the left and right sectors in order to obtain the desired electromagnetic coupling. The strength of the $U(1)$ coupling to the various left and right sectors can then be varied to make contact with the observed electromagnetic charges of each field.

Form the left-handed constituents of the quark and lepton doublets

$$L_L = \frac{1}{2}(1 - \gamma_5)L, \qquad\qquad Q_L = \frac{1}{2}(1 - \gamma_5)Q$$

which will transform under $SU(2)_L$. The left-handed sector is therefore described by the action

$$\mathcal{L}_L = i\bar{Q}_L\gamma^\mu\left(\partial_\mu+\frac{1}{2}igW^j_\mu\sigma_j+\frac{1}{2}ig'Y_QB_\mu\right)Q_L+i\bar{L}_L\gamma^\mu\left(\partial_\mu+\frac{1}{2}igW^j_\mu\sigma_j+\frac{1}{2}ig'Y_LB_\mu\right)L_L.$$

The factor of $\frac{1}{2}$ is used for convenience.

The right-handed sector is formed from the three $SU(2)$ singlets

$$e_R = \frac{1}{2}(1 + \gamma_5)e, \qquad u_R = \frac{1}{2}(1 + \gamma_5)u, \qquad d_R = \frac{1}{2}(1 + \gamma_5)d$$

and the action consistent with their electric charges is given by

$$\mathcal{L}_R = i\bar{e}_R\gamma^\mu\left(\partial_\mu + ig'B_\mu\right)e_R + i\bar{u}_R\gamma^\mu\left(\partial_\mu - \frac{2}{3}g'B_\mu\right)u_R + i\bar{d}_R\gamma^\mu\left(\partial_\mu + \frac{1}{3}g'B_\mu\right)d_R.$$

The gauge fields in addition to possessing the standard free actions are coupled to a doublet of complex scalar fields

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}.$$

Since the Goldstone modes G^0 and $G^\pm$ must be combined with the gauge fields to form the massive bosons, two of the degrees of freedom for ϕ must carry an electric charge while the other two must be neutral. This is achieved by choosing the action for the gauge-scalar coupling to be

$$\mathcal{L}_s = \left(\partial_\mu - \frac{1}{2} i g W_\mu^j \sigma_j - \frac{1}{2} i g' B_\mu \right) \phi^\dagger \left(\partial_\mu + \frac{1}{2} i g W_\mu^j \sigma_j + \frac{1}{2} i g' B_\mu \right) \phi.$$

All that is left to do is to break the symmetry by assuming that the effective potential at the tree level is given by

$$V = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2.$$

The vacuum expectation of ϕ is assumed to be real in order that the vacuum carry no electric charge so that the VEV must take the form

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

where

$$v = \sqrt{\frac{\mu^2}{\lambda}}, \qquad \mu^2 > 0.$$

Translating the scalar field by the VEV generates mass terms for the vector fields. It is easily seen that

$$\left[\left(\frac{1}{2} g W_\mu^j \sigma_j + \frac{1}{2} g' B_\mu \right) \langle \phi \rangle \right]^2 = \frac{1}{4} g^2 v^2 W_\mu^+ W^{-\mu} + \frac{1}{8} (g^2 + g'^2) v^2 Z_\mu Z^\mu$$

where

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\Big(W_\mu^{(1)} \pm iW_\mu^{(2)}\Big)$$

and

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}\Big(g'W_\mu^{(3)} - gB_\mu\Big).$$

It follows that the fourth gauge boson must be a combination of $W_\mu^{(3)}$ and B_μ that is orthonormal to Z_μ in isospin space given by

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\Big(gW_\mu^{(3)} + g'B_\mu\Big).$$

The field A_μ is massless and therefore corresponds to the photon field, the generator that remains unbroken and which is associated with the $U(1)_{e.m.}$ gauge symmetry. Equivalently, this can be simply expressed as

$$\frac{SU(2)\times U(1)}{U(1)_{e.m.}}.$$

The masses of the gauge fields are given by

$$M_W = \frac{1}{2}gv, \qquad M_Z = \frac{1}{2}v\sqrt{g^2+g'^2}$$

where $v = 246.22$ `GeV` is the VEV. Setting

$$g'\cos\theta_W = e, \qquad gsin\theta_W = e, \qquad \frac{g'}{g} = \tan\theta_W$$

the following relation holds

$$\frac{M_W}{M_Z} = \cos\theta_W$$

which is known as the weak-mixing angle. The inequality $M_Z \neq M_W$ is due to th mixing between W_μ^3 and B_μ fields.

The unphysical Goldstone bosons can be removed by a transformation to the unitary gauge. In this gauge, their degrees of freedom become the longitudinal components of the Z^0 and $W^\pm$ physical gauge bosons respectively. At the same time degrees of freedom of the Higgs field are absorbed by the $W^\pm$ and one by the Z^0 gauge bosons which now become massive.

The final degree of freedom of the Higgs field becomes the new fundamental scalar particle, namely the Higgs boson, that appears in the SM since all other scalars are quark composites.

Note B.0.2 . The parameter ρ, known as the Veltmann parameter, specifies the relative strength of the neutral and charged current interactions which is given by

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W}.$$

In the Glashow-Salam-Weinberg minimal model fixes this parameter to be

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1.$$

C

Why SUSY?

The Poincaré algebra is so central to our understanding of space-time it is natural to ask if the direct product

$$[P_\mu, T_a] = [M_{\mu\nu}, T_a] = 0$$

where T_a refers to any internal generator, is necessarily the case or if there is some deeper underlying symmetry that has a non-trivial commutation relation with the Poincaré algebra. This question was answered by Coleman and Mandula:

In any space-time dimension, $D > 2$, interacting quantum field theories have Lie algebra symmetries that are

$$\mathfrak{g} \times \text{Poincaré}$$

where $\mathfrak{g}$ is the Lie algebra generated by T_a. This is to say that there is no Lie algebra that is a symmetry of interacting quantum field theories that is not a Lorentz scalar.

One key assumption of the Coleman-Mandula No-Go theorem is that the additional symmetry is a Lie algebra symmetry. This theorem can be avoided by relaxing this assumption and Lie-algebras are inevitable whenever you have continuous symmetries. The way to proceed is to note that quantum field theories such as the SM contain two types of fields: bosons and fermions. These are distinguished by the representation of the field under the Lorentz group. In particular a fundamental theorem in quantum field theory: the spin-statistics theorem which asserts that bosons must carry representations of the Lorentz group with integer spins and their field operators must commute whereas fermions carry half-integer spins and their field operators are anti-commuting. This means that the fields associated to fermions must be Grassmann variables that satisfy

$$\psi_1(x)\psi_2(x) = -\psi_2(x)\psi_1(x).$$

The idea is now to consider a Grassmann generator which also carries a spinor index and which requires a Grassmann valued spinorial parameter. One then is lead to something called a $\mathbb{Z}_2$-graded Lie-algebra. This means that the generators can be labelled as either even and odd. The even generators behave just as the generators of a Lie-algebra and obey commutation relations. An even and an odd generator will also obey a commutator relation. However two odd generators will obey an anti-commutation relation. The even-ness or odd-ness of this generalized Lie-bracket is additive modulo two: the commutator of two even generators is even, the anti-commutator of two odd generators is also even, whereas the commutator of an even and an odd generator is odd.

Schematically, let $\mathfrak{g}$ be a Lie algebra. Then $\mathfrak{g}$ decomposes as

$$\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$$

where $\mathfrak{g}_0$ represents even part and $\mathfrak{g}_1$ represents the odd part.

For the linear map

$$[\ ,\ ] : \mathfrak{g} \times \mathfrak{g} \to \mathfrak{g}$$

we have

$$\begin{aligned}
\mathfrak{g}_0 \times \mathfrak{g}_0 &\rightarrow \mathfrak{g}_0 \\
\mathfrak{g}_0 \times \mathfrak{g}_1 &\rightarrow \mathfrak{g}_1 \\
\mathfrak{g}_1 \times \mathfrak{g}_0 &\rightarrow \mathfrak{g}_1 \\
\mathfrak{g}_1 \times \mathfrak{g}_1 &\rightarrow \mathfrak{g}_0
\end{aligned}$$

where it can be seen that the linear map on $\mathfrak{g}_0$ acts as a commutator but on $\mathfrak{g}_1$ acts as as an anti-commutator.

D

The $SO(32)$ *Heterotic String Action*

For the left-moving sector, we consider the superstring fields X_+^μ and ψ_+^μ with $\mu = 0, ..., 9$. The critical dimension is $D = 10$. Then for the right-moving sector we have the ten bosonic fields X_-^μ. Since a space-time boson contributes a unit to the central charge and a free fermion contributes half a unit, 32 Majorana-Weyl right-moving free fermions λ_-^i are needed to cancel the conformal anomaly $c = -26$ in the bosonic string. The theory is still ten-dimensional because the space-time indices $\mu = 0, ..., 9$ are carried by the coordinates X^μ in both the right- and left-moving sectors whilst the internal fermions λ_-^i do not carry space-time indices.

To summarize, in such a construction, we have considered X_+^μ and ψ_+^μ fields in the left-moving sector and X_-^μ and λ_-^i fields in the right-moving sector where $\mu = 0, ..., 9$ and $i = 1, ..., 32$.

The action for the heterotic string is therefore

$$S = \frac{1}{\pi} \int d^2\xi \left(2\partial_- X_\mu \partial_+ X^\mu + i\psi^\mu \partial_- \psi_\mu + i \sum_{i=1}^{32} \lambda^i \partial_+ \lambda^i \right).$$

E

An Event No More

The observation of di–photon excess at the LHC reported both by the ATLAS [77] and CMS [78] collaborations could have been understood through the production and the subsequent decay of a SM singlet via heavy vector–like colour triplets and electroweak doublets in one–loop diagrams had sparked significant interest [79].

The excess, and the absence of any other observed signatures are well understood as a resonance of a SM singlet scalar field, which is produced and subsequently decays via triangular loops incorporating heavy vector–like states can be seen in Figure E.1.

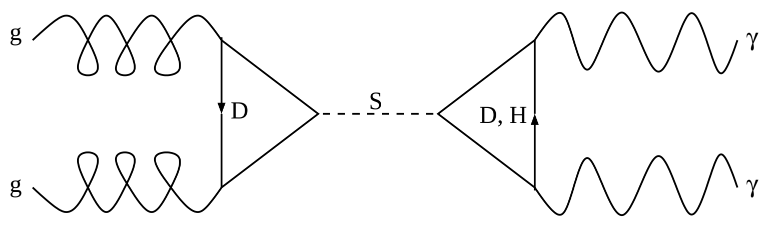


Figure E.1: Production and di–photon decay of the Standard Model singlet scalar state.

All the crucial elements that form the basis of the schematic diagram in Figure E.1 are readily found in the string-derived Z' model [55, 71].

In the low-energy regime the superpotential [55] provides different interaction terms of the singlet fields S_i and ζ_i which can be extracted from Table 3.3, among them we have

$$\lambda_D^{ijk} S_i D_j \bar{D}_k + \lambda_H^{ijk} S_i H_j \bar{H}_k + \lambda_h^{ij} S_i H_j \bar{h} + \eta_{\mathcal{D}}^i \zeta_i \mathcal{D}\bar{\mathcal{D}} + \eta_h^i \zeta_i h \bar{h}\,. \tag{E.1}$$

All these terms may comply with the di–photon excess reported by both the ATLAS and CMS experiments with a resonance around 750 GeV described by either the singlets S_i or ζ_i. Indeed, the presence of vector-like quarks, which is natural in heterotic string models, facilitates the production of these states at the LHC. In the following discussion we will consider the most simple and economic scenario in order to highlight the effects of the vector-like coloured states $D, \bar{D}$ and their role in the explanation of the di–photon excess. For this reason we assume that the resonance is reproduced by exchange of one of the singlet S_i and we ignore the contribution of the ζ_i fields and of the coupling $SH\bar{H}$. The real scalar component of one of the S_i superfields acquires a VEV v_S and breaks the extra $U(1)_{Z'}$ symmetry thus providing the mass of the Z' gauge boson and of the $D, \bar{D}$ field through the coupling λ_D in the superpotential, Equation (E.1). Provided v_S around the TeV scale, the mass of the singlet S_i, of the vector-like states $D, \bar{D}$ and of the Z' lay in the TeV ballpark thus establishing a intimate relationship between the 750 GeV di–photon resonance and the presence of an additional spontaneously broken $U(1)_{Z'}$ gauge symmetry. Interestingly this can also be probed at the LHC in the lepto-production channel [61, 64]. Moreover, as we have already stated, in order to reproduce the di–photon excess it is enough to consider the impact of the vector-like coloured superfields $D, \bar{D}$ only. Therefore we assume $\lambda \equiv \lambda_D^{3ii}$ and we neglect all the other couplings. The fermionic components of D_i and $\bar{D}_i$ can be rearranged into three Dirac spinors ψ_{D_i}, while the scalar components will

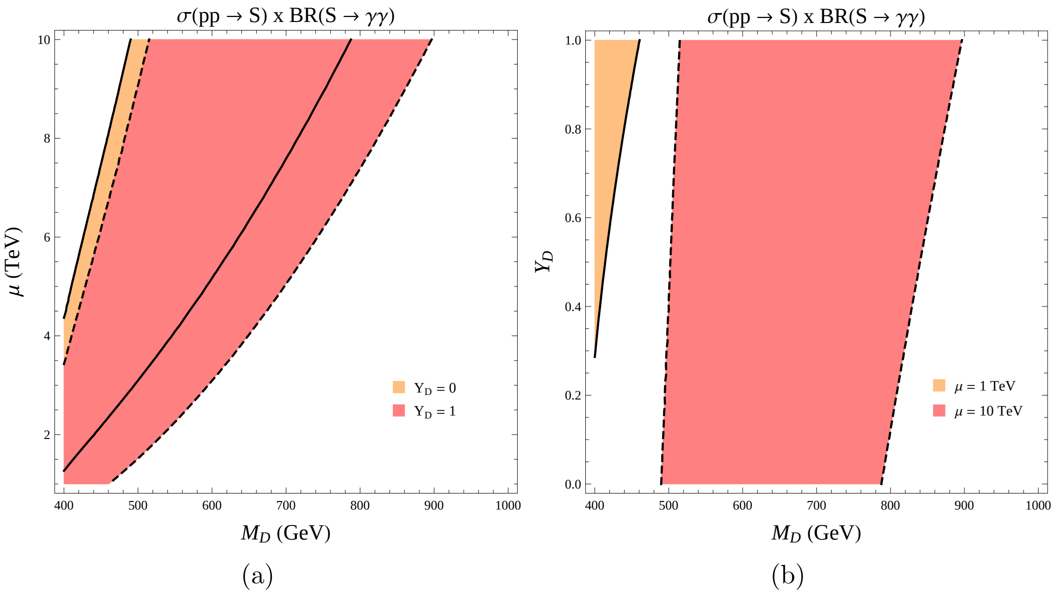


Figure E.2: $\sigma(pp \to S) \times \mathrm{BR(S \to \gamma\gamma)}$ at 13 TeV LHC in (a) the (M_D, μ) plane for two values of the Yukawa coupling Y_D and (b) in the (M_D, Y_D) plane for two values of the scalar coupling μ. The coloured regions corresponds to a 2σ region of the measured cross section 4.5 ± 1.9 fb.

provide six complex scalars $\tilde{D}_j$. The corresponding interaction Lagrangian can be parameterised as

$$\mathcal{L} = -Y_D\, S\bar{\psi}_{D_i}\psi_{D_i} - \mu\, S|\tilde{D}_j|^2\,, \tag{E.2}$$

where S is the real scalar component of one of the S_i singlet whose mass M_S is identified with the 750 GeV resonance, $Y_D = \lambda/\sqrt{2}$ and μ is the corresponding soft-breaking term.

The LHC cross section of the di–photon production through the exchange of a scalar resonance in the s–channel is, in the narrow width approximation,

$$\sigma(pp \to S \to \gamma\gamma) = \frac{1}{M_S\, s} C_{gg} \Gamma(S \to gg)\mathrm{Br(S \to \gamma\gamma)} \tag{E.3}$$

where M_S is the resonance mass, C_{gg} the luminosity factor in the gluon–gluon channel and $\sqrt{s}$ the centre-of-mass energy. We assume that the main production mechanism occurs via gluon fusion with the corresponding luminosity factor at 13 $\texttt{TeV}$ given by

$$C_{gg} = \frac{\pi^8}{8} \int_{M_S^2/s}^{1} \frac{dx}{x} g(x) g\left(\frac{M_S^2}{sx}\right) \simeq 2137\,, \tag{E.4}$$

where $g(x)$ is the gluon distribution function and the value has been computed for $\sqrt{s} = 13$ $\texttt{TeV}$ and for $M_S = 750$ $\texttt{GeV}$ using MSTW2008NLO [60].

The partial decay widths of S into gluons and photons are

$$\Gamma(S \to gg) = \frac{\alpha_S^2}{128\pi^3} M_S^3 \left| \sum_f \frac{y_f}{m_f} A_{1/2}(\tau_f) + \sum_s \frac{\mu_s}{2m_s^2} A_0(\tau_s) \right|^2 , \tag{E.5}$$

$$\Gamma(S \to \gamma\gamma) = \frac{\alpha^2}{256\pi^3} M_S^3 \left| \sum_f N_c^f q_f^2 \frac{y_f}{m_f} A_{1/2}(\tau_f) + \sum_s N_c^s q_s^2 \frac{\mu_s}{2m_s^2} A_0(\tau_s) \right|^2 \tag{E.6}$$

where m_f and m_s are the masses of a generic fermion and scalar running in the loops, y_f and μ_s the corresponding couplings to S and N_c the colour factor. As $D,\ \bar{D}$ are singlets of $SU(2)_L$, their electric charge q coincides with the hypercharge Y. The fermionic and scalar loop functions are given by

$$A_{1/2}(\tau) = 2[\tau + (\tau-1) f(\tau)]/\tau^2, \qquad A_0(\tau) = -[\tau - f(\tau)]/\tau^2 \tag{E.7}$$

with $\tau_i = M_S^2/(4m_i^2)$ and

$$f(\tau) = \begin{cases} \arcsin^2 \sqrt{\tau}, & \text{if } \tau \le 1 \\ -\frac{1}{4}\left[\log \frac{1+\sqrt{1-\tau^{-1}}}{1-\sqrt{1-\tau^{-1}}} - i\,\pi\right]^2, & \text{if } \tau > 1 \end{cases} . \tag{E.8}$$

Assuming $\Gamma_{\text{tot}} = \Gamma(S \to gg) + \Gamma(S \to \gamma\gamma)$, we show in Figure E.2 the portion of the parameters space in which the di–photon excess can be reproduced in a 2σ region around the measured value $\sigma = 4.5 \pm 1.9$ fb reported by the ATLAS and CMS collaborations at 13 `TeV`. For simplicity we assume $M_{\psi_{D_i}} \simeq M_{\tilde{D}_i} \simeq M_D$ and we present our results in the (M_D, μ) and (M_D, Y_D) planes. The cross section is dominated by the complex scalar loops while the fermionic components of the supermultiplets D, $\bar{D}$ only provide a small contribution. Therefore, a huge Yukawa coupling is not strictly necessary as is usually required in the literature, since its effect is compensated by a large soft–breaking term and relatively light squark–like states. We stress again that this analysis is far from being exhaustive, while its only purpose is to show how the di–photon excess can be naturally accommodated in heterotic string scenarios where the $U(1)_{Z'}$ gauge symmetry is broken around the `TeV` scale. We have neglected, for instance, the impact of the $SH\bar{H}$ interaction which would increase, in general, the partial decay width into photons and thus broaden the preferred parameter space and will provide more involved decay patterns through the mixing with the H and $\bar{H}$ fields.

F

The Split SUSY models

In this section, we present a rather brief discussion on string models with a split SUSY structure. The basic idea is to use two basis vectors to generate space-time SUSY. We recall that in the semi-realistic free fermionic models the SUSY generators arise from the basis vector S. Here, the aim is to construct two basis vectors that generate SUSY with the focus to construct models in which the gaugini are obtained from one generator, whereas those of the second generator are projected out, as well as the the scalar superpartners of the twisted matter fermionic states. Our construction proceeds by keeping our previous basis vector $S = \{\psi^{1,2}, \chi^{1,...,6}\} \equiv S_1$. The second generator is given by

$$S_2 = \{\psi^{1,2}, \chi^{1,2}, \omega^{3,4}, \omega^{5,6}\}. \tag{F.1}$$

The basis vectors b_1 and b_2 of the NAHE–set are added as well as the basis vectors $\mathbf{1}$ and X, which is used to project the supersymmetric generators from S_1, as discussed in section 4.3. Shift basis vectors similar to the e_i basis vectors can be added, and variations that include the basis vector I. We consider the set of six basis vectors given by

$$
\begin{aligned}
v_1 = 1 &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \\
&\qquad \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}, \\
v_2 = S_1 &= \{\psi^\mu, \chi^{1,\dots,6}\}, \\
v_3 = S_2 &= \{\psi^\mu, \chi^{1,2}, \omega^{3,\dots,6}\}, \\
v_4 = b_1 &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \\
v_5 = X &= \{\bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}\}, \\
v_6 = I &= \{\bar{\phi}^{1,\dots,8}\}.
\end{aligned}
\tag{F.2}
$$

with the set of GGSO phases given by

$$
\begin{array}{c}
\\ \mathbf{1} \\ S_1 \\ S_2 \\ b_1 \\ X \\ I
\end{array}
\begin{array}{c}
\begin{array}{cccccc} \mathbf{1} & S_1 & S_2 & b_1 & X & I \end{array} \\
\begin{pmatrix}
-1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & -1 & 1 & 1 & 1 \\
-1 & 1 & -1 & 1 & -1 & -1 \\
-1 & -1 & -1 & -1 & 1 & -1 \\
-1 & 1 & -1 & -1 & 1 & -1 \\
-1 & 1 & -1 & -1 & -1 & 1
\end{pmatrix}
\end{array}.
\tag{F.3}
$$

In this model the NS–sector is the only sector that produces space-time vector bosons. Hence the gauge symmetry in four dimensions is $SO(8) \times SO(4) \times SO(4) \times SO(12) \times SO(16)$. The sector b_1 gives rise to space-time fermions in the spinor and anti–spinor representations of $SO(12)$. The SUSY generators of S_1 are projected out, whereas the gaugini from S_2 are retained. The model retains the scalar superpartners from the sector $S_2 + b_1$, and projects those from the sector

$S_1 + b_1$. Our general aim in the construction of models with split SUSY is to construct models that retain the gaugini and space-time fermions from S_2 and b_1, while projecting the gaugini (and hence the gravitini) from S_1, as well as the superpartners from the sectors $S_1 + b_1$ and $S_2 + b_1$. However, variations of the model in Equations (F.2,F.3), including adding the e_i projectors did not yield the desired result. The models in which SUSY is entirely broken that is those models in which the SUSY generators from S_1 as well as S_2 are projected out, typically contain tachyons.

G

The Massless Spectrum

Recall that the gauge group of the model is

$$\underbrace{SU(3)_C \times U(1)_C \times SU(2)_L \times SU(2)_R \times \prod_{i=1}^{6} U_i}_{\text{observable sector}} \times \underbrace{SU(3)_{H_1} \times SU(3)_{H_2} \times \prod_{j=7}^{10} U_j}_{\text{hidden sector}} .$$

The notation for the table is the following: The first column describes if the states correspond to space-time bosons or space-time fermions. The second column is the name of the sector. The third column gives the dimensionality of the states under $SU(3)_C \times SU(2)_L \times SU(2)_R$ and the fourth the charges of the observable $U(1)$s: $Q_{\bar{\eta}^1}$, $Q_{\bar{\eta}^2}$, $Q_{\bar{\eta}^3}$, $Q_{\bar{y}^{3,6}}$, $Q_{\bar{y}^1\bar{w}^5}$, $Q_{\bar{w}^{2,4}}$. Columns 5 and 6 describe the hidden sector. The only charges appearing in the table that do not have a self–evident name are:

$$\begin{aligned} Q_C &= Q_{\overline{\psi}^1} + Q_{\overline{\psi}^2} + Q_{\overline{\psi}^3} \ , \\ Q_8 &= Q_{\overline{\phi}^2} + Q_{\overline{\phi}^3} + Q_{\overline{\phi}^4} \ , \\ Q_9 &= Q_{\overline{\phi}^5} + Q_{\overline{\phi}^6} + Q_{\overline{\phi}^7} \ . \end{aligned} \tag{G.1}$$

To avoid writing fractional numbers all the charges in the table have been multiplied by 4. Finally, for every state the CPT conjugate is also understood to be in the spectrum and has not be written explicitly. The massless spectrum is given below:

F	SEC	$(C;L;R)$	Q_C	$Q_{\bar{\eta}^1}$	$Q_{\bar{\eta}^2}$	$Q_{\bar{\eta}^3}$	$Q_{\bar{y}^{3,6}}$	$Q_{\bar{y}^1\bar{w}^5}$	$Q_{\bar{w}^{2,4}}$	$SU(3)_{H_{1,2}}$	$Q_{\bar{\Phi}^1}$	Q_8	Q_9	$Q_{\bar{\Phi}^8}$
b	NS	$(1,1,1)$	0	0	0	0	0	0	0	$(1,1)$	0	0	0	0
		$(1,1,1)$	0	0	0	0	0	0	0	$(1,1)$	0	0	0	0
		$(1,1,1)$	0	0	0	0	0	0	0	$(1,1)$	0	0	0	0
		$(1,1,1)$	0	-4	4	0	0	0	0	$(1,1)$	0	0	0	0
		$(1,1,1)$	0	4	-4	0	0	0	0	$(1,1)$	0	0	0	0
		$(1,1,1)$	0	0	-4	4	0	0	0	$(1,1)$	0	0	0	0
		$(1,1,1)$	0	0	4	-4	0	0	0	$(1,1)$	0	0	0	0
		$(1,1,1)$	0	-4	0	4	0	0	0	$(1,1)$	0	0	0	0
		$(1,1,1)$	0	4	0	-4	0	0	0	$(1,1)$	0	0	0	0

Table G.1: The untwisted Neveu-Schwarz sector matter states and charges.

F	SEC	$(C;L;R)$	Q_C	$Q_{\bar\eta^1}$	$Q_{\bar\eta^2}$	$Q_{\bar\eta^3}$	$Q_{\bar y^{3,6}}$	$Q_{\bar y^1\bar w^5}$	$Q_{\bar w^{2,4}}$	$SU(3)_{H_{1,2}}$	$Q_{\bar\Phi^1}$	Q_8	Q_9	$Q_{\bar\Phi^8}$
f	S	$(1,1,1)$	0	0	0	0	0	0	0	$(3,\overline{3})$	0	-4	4	0
		$(1,1,1)$	0	0	0	0	0	0	0	$(3,\overline{3})$	0	4	-4	0
		$(1,1,1)$	0	0	0	0	0	0	0	$(1,1)$	4	0	0	4
		$(1,1,1)$	0	0	0	0	0	0	0	$(1,1)$	4	0	0	-4
		$(1,1,1)$	0	0	0	0	0	0	0	$(1,1)$	-4	0	0	4
		$(1,1,1)$	0	0	0	0	0	0	0	$(1,1)$	-4	0	0	-4
		$(3,1,1)$	-4	4	0	0	0	0	0	$(1,1)$	0	0	0	0
		$(\overline{3},1,1)$	4	-4	0	0	0	0	0	$(1,1)$	0	0	0	0
		$(3,1,1)$	-4	0	4	0	0	0	0	$(1,1)$	0	0	0	0
		$(\overline{3},1,1)$	4	0	-4	0	0	0	0	$(1,1)$	0	0	0	0
		$(3,1,1)$	-4	0	0	4	0	0	0	$(1,1)$	0	0	0	0
		$(\overline{3},1,1)$	4	0	0	-4	0	0	0	$(1,1)$	0	0	0	0
		$(1,2,2)$	0	0	0	0	0	0	0	$(1,1)$	0	0	0	0
		$(1,2,2)$	0	0	0	0	0	0	0	$(1,1)$	0	0	0	0
		$(1,1,1)$	0	0	0	0	4	0	0	$(1,1)$	0	0	0	0
		$(1,1,1)$	0	0	0	0	-4	0	0	$(1,1)$	0	0	0	0
		$(1,1,1)$	0	0	0	0	0	4	0	$(1,1)$	0	0	0	0
		$(1,1,1)$	0	0	0	0	0	-4	0	$(1,1)$	0	0	0	0
		$(1,1,1)$	0	0	0	0	4	0	0	$(1,1)$	0	0	0	0
		$(1,1,1)$	0	0	0	0	-4	0	0	$(1,1)$	0	0	0	0
		$(1,1,1)$	0	0	0	0	0	4	0	$(1,1)$	0	0	0	0
		$(1,1,1)$	0	0	0	0	0	-4	0	$(1,1)$	0	0	0	0

Table G.2: The untwisted S matter states and charges.

F	SEC	$(C;L;R)$	Q_C	$Q_{\bar\eta^1}$	$Q_{\bar\eta^2}$	$Q_{\bar\eta^3}$	$Q_{\bar y^{3,6}}$	$Q_{\bar y^1\bar w^5}$	$Q_{\bar w^{2,4}}$	$SU(3)_{H_{1,2}}$	$Q_{\bar\Phi^1}$	Q_8	Q_9	$Q_{\bar\Phi^8}$
Q_{L_1}	b_1	$(3,2,1)$	2	2	0	0	-2	0	0	$(1,1)$	0	0	0	0
Q_{R_1}		$(\overline{3},1,2)$	-2	-2	0	0	-2	0	0	$(1,1)$	0	0	0	0
L_{L_1}		$(1,2,1)$	-6	2	0	0	-2	0	0	$(1,1)$	0	0	0	0
L_{R_1}		$(1,1,2)$	6	-2	0	0	-2	0	0	$(1,1)$	0	0	0	0
b	$S+b_1$	$(3,1,2)$	2	2	0	0	-2	0	0	$(1,1)$	0	0	0	0
		$(\overline{3},2,1)$	-2	-2	0	0	-2	0	0	$(1,1)$	0	0	0	0
		$(1,2,1)$	6	-2	0	0	-2	0	0	$(1,1)$	0	0	0	0
		$(1,1,2)$	-6	2	0	0	-2	0	0	$(1,1)$	0	0	0	0
Q_{L_2}	b_2	$(3,2,1)$	2	0	2	0	0	-2	0	$(1,1)$	0	0	0	0
Q_{R_2}		$(\overline{3},1,2)$	-2	0	-2	0	0	-2	0	$(1,1)$	0	0	0	0
L_{L_2}		$(1,2,1)$	-6	0	2	0	0	-2	0	$(1,1)$	0	0	0	0
L_{R_2}		$(1,1,2)$	6	0	-2	0	0	-2	0	$(1,1)$	0	0	0	0
b	$S+b_2$	$(3,1,2)$	2	0	2	0	0	-2	0	$(1,1)$	0	0	0	0
		$(\overline{3},2,1)$	-2	0	-2	0	0	-2	0	$(1,1)$	0	0	0	0
		$(1,2,1)$	6	0	-2	0	0	-2	0	$(1,1)$	0	0	0	0
		$(1,1,2)$	-6	0	2	0	0	-2	0	$(1,1)$	0	0	0	0
Q_{L_3}	b_3	$(3,2,1)$	2	0	0	2	0	0	-2	$(1,1)$	0	0	0	0
Q_{R_3}		$(\overline{3},1,2)$	-2	0	0	-2	0	0	-2	$(1,1)$	0	0	0	0
L_{L_3}		$(1,2,1)$	-6	0	0	2	0	0	-2	$(1,1)$	0	0	0	0
L_{R_3}		$(1,1,2)$	6	0	0	-2	0	0	-2	$(1,1)$	0	0	0	0
b	$S+b_3$	$(3,1,2)$	2	0	0	2	0	0	-2	$(1,1)$	0	0	0	0
		$(\overline{3},2,1)$	-2	0	0	-2	0	0	-2	$(1,1)$	0	0	0	0
		$(1,2,1)$	6	0	0	-2	0	0	-2	$(1,1)$	0	0	0	0
		$(1,1,2)$	-6	0	0	2	0	0	-2	$(1,1)$	0	0	0	0

Table G.3: The observable matter sectors. All sectors, fermionic and bosonic, have CPT conjugates which are not displayed.

F	SEC	$(C;L;R)$	Q_C	$Q_{\bar{\eta}^1}$	$Q_{\bar{\eta}^2}$	$Q_{\bar{\eta}^3}$	$Q_{\bar{y}^{3,6}}$	$Q_{\bar{y}^1\bar{w}^5}$	$Q_{\bar{w}^{2,4}}$	$SU(3)_{H_{1,2}}$	$Q_{\bar{\Phi}^1}$	Q_8	Q_9	$Q_{\bar{\Phi}^8}$
f	$S+$	$(1,1,1)$	0	2	-2	0	0	0	0	$(1,1)$	0	0	0	4
	b_1+b_2	$(1,1,1)$	0	2	-2	0	0	0	0	$(1,1)$	0	0	0	-4
	$+\alpha+\beta$	$(1,1,1)$	0	-2	2	0	0	0	0	$(1,1)$	0	0	0	4
		$(1,1,1)$	0	-2	2	0	0	0	0	$(1,1)$	0	0	0	-4
		$(1,1,1)$	0	2	2	0	0	0	0	$(3,1)$	0	4	0	0
		$(1,1,1)$	0	-2	-2	0	0	0	0	$(3,1)$	0	4	0	0
		$(1,1,1)$	0	2	2	0	0	0	0	$(\overline{3},1)$	0	-4	0	0
		$(1,1,1)$	0	-2	-2	0	0	0	0	$(\overline{3},1)$	0	-4	0	0
b	b_1+b_2	$(1,1,1)$	0	2	-2	0	0	0	0	$(1,1)$	4	0	0	0
	$+\alpha+\beta$	$(1,1,1)$	0	2	-2	0	0	0	0	$(1,1)$	-4	0	0	0
		$(1,1,1)$	0	-2	2	0	0	0	0	$(1,1)$	4	0	0	0
		$(1,1,1)$	0	-2	2	0	0	0	0	$(1,1)$	-4	0	0	0
		$(1,1,1)$	0	2	2	0	0	0	0	$(1,3)$	0	0	4	0
		$(1,1,1)$	0	-2	-2	0	0	0	0	$(1,3)$	0	0	4	0
		$(1,1,1)$	0	2	2	0	0	0	0	$(1,\overline{3})$	0	0	-4	0
		$(1,1,1)$	0	-2	-2	0	0	0	0	$(1,\overline{3})$	0	0	-4	0

Table G.4: The vector-like $SO(10)$ singlet states. All sectors, fermionic and bosonic, have CPT conjugates which are not displayed.

F	SEC	$(C;L;R)$	Q_C	$Q_{\bar{\eta}^1}$	$Q_{\bar{\eta}^2}$	$Q_{\bar{\eta}^3}$	$Q_{\bar{y}^{3,6}}$	$Q_{\bar{y}^1\bar{w}^5}$	$Q_{\bar{w}^{2,4}}$	$SU(3)_{H_{1,2}}$	$Q_{\bar{\Phi}^1}$	Q_8	Q_9	$Q_{\bar{\Phi}^8}$
f	$S+$	$(1,1,1)$	0	-2	0	0	0	-2	-2	$(1,1)$	0	6	0	2
	$1+b_1$	$(1,1,1)$	0	2	0	0	0	-2	-2	$(1,1)$	0	-6	0	-2
	$+\alpha+2\gamma$	$(1,1,1)$	0	2	0	0	0	-2	-2	$(3,1)$	0	2	0	-2
		$(1,1,1)$	0	-2	0	0	0	-2	-2	$(\overline{3},1)$	0	-2	0	2
b	$1+b_1$	$(1,1,1)$	0	-2	0	0	0	2	-2	$(1,1)$	0	6	0	2
	$+\alpha+2\gamma$	$(1,1,1)$	0	2	0	0	0	2	-2	$(1,1)$	0	-6	0	-2
		$(1,1,1)$	0	2	0	0	0	2	-2	$(3,1)$	0	2	0	-2
		$(1,1,1)$	0	-2	0	0	0	2	-2	$(\overline{3},1)$	0	-2	0	2
f	$S+$	$(1,1,1)$	0	0	-2	0	-2	0	-2	$(1,1)$	0	6	0	2
	$1+b_2$	$(1,1,1)$	0	0	2	0	-2	0	-2	$(1,1)$	0	-6	0	-2
	$+\alpha+2\gamma$	$(1,1,1)$	0	0	2	0	-2	0	-2	$(3,1)$	0	2	0	-2
		$(1,1,1)$	0	0	-2	0	-2	0	-2	$(\overline{3},1)$	0	-2	0	2
b	$1+b_2$	$(1,1,1)$	0	0	-2	0	2	0	-2	$(1,1)$	0	6	0	2
	$+\alpha+2\gamma$	$(1,1,1)$	0	0	2	0	2	0	-2	$(1,1)$	0	-6	0	-2
		$(1,1,1)$	0	0	2	0	2	0	-2	$(3,1)$	0	2	0	-2
		$(1,1,1)$	0	0	-2	0	2	0	-2	$(\overline{3},1)$	0	-2	0	2
f	$S+$	$(1,1,1)$	0	0	-2	0	-2	0	-2	$(1,1)$	-2	0	6	0
	b_1+b_3	$(1,1,1)$	0	0	2	0	-2	0	-2	$(1,1)$	2	0	-6	0
	$+\alpha+2\gamma$	$(1,1,1)$	0	0	2	0	-2	0	-2	$(1,3)$	2	0	2	0
		$(1,1,1)$	0	0	-2	0	-2	0	-2	$(1,\overline{3})$	-2	0	-2	0
b	b_1+b_3	$(1,1,1)$	0	0	-2	0	2	0	-2	$(1,1)$	-2	0	6	0
	$+\alpha+2\gamma$	$(1,1,1)$	0	0	2	0	2	0	-2	$(1,1)$	2	0	-6	0
		$(1,1,1)$	0	0	2	0	2	0	-2	$(1,3)$	2	0	2	0
		$(1,1,1)$	0	0	-2	0	2	0	-2	$(1,\overline{3})$	-2	0	-2	0
f	$S+$	$(1,1,1)$	0	0	0	-2	-2	-2	0	$(1,1)$	-2	0	6	0
	b_1+b_2	$(1,1,1)$	0	0	0	2	-2	-2	0	$(1,1)$	2	0	-6	0
	$+\alpha+2\gamma$	$(1,1,1)$	0	0	0	2	-2	-2	0	$(1,3)$	2	0	2	0
		$(1,1,1)$	0	0	0	-2	-2	-2	0	$(1,\overline{3})$	-2	0	-2	0
b	b_1+b_2	$(1,1,1)$	0	0	0	-2	2	-2	0	$(1,1)$	-2	0	6	0
	$+\alpha+2\gamma$	$(1,1,1)$	0	0	0	2	2	-2	0	$(1,1)$	2	0	-6	0
		$(1,1,1)$	0	0	0	2	2	-2	0	$(1,3)$	2	0	2	0
		$(1,1,1)$	0	0	0	-2	2	-2	0	$(1,\overline{3})$	-2	0	-2	0

Table G.5: The vector-like $SO(10)$ singlet states. All sectors, fermionic and bosonic, have CPT conjugates which are not displayed.

F	SEC	$(C;L;R)$	Q_C	$Q_{\bar\eta^1}$	$Q_{\bar\eta^2}$	$Q_{\bar\eta^3}$	$Q_{\bar y^{3,6}}$	$Q_{\bar y^1\bar w^5}$	$Q_{\bar w^{2,4}}$	$SU(3)_{H_{1,2}}$	$Q_{\bar\Phi^1}$	Q_8	Q_9	$Q_{\bar\Phi^8}$
f	$S+$	$(1,1,1)$	0	-2	0	0	0	-2	-2	$(1,1)$	-2	0	6	0
	b_2+b_3	$(1,1,1)$	0	2	0	0	0	-2	-2	$(1,1)$	2	0	-6	0
	$+\alpha+2\gamma$	$(1,1,1)$	0	2	0	0	0	-2	-2	$(1,3)$	2	0	2	0
		$(1,1,1)$	0	-2	0	0	0	-2	-2	$(1,\overline{3})$	-2	0	-2	0
b	b_2+b_3	$(1,1,1)$	0	-2	0	0	0	2	-2	$(1,1)$	-2	0	6	0
	$+\alpha+2\gamma$	$(1,1,1)$	0	2	0	0	0	2	-2	$(1,1)$	2	0	-6	0
		$(1,1,1)$	0	2	0	0	0	2	-2	$(1,3)$	2	0	2	0
		$(1,1,1)$	0	-2	0	0	0	2	-2	$(1,\overline{3})$	-2	0	-2	0
f	$S+$	$(1,1,1)$	0	0	0	-2	-2	-2	0	$(1,1)$	0	6	0	2
	$1+b_3$	$(1,1,1)$	0	0	0	2	-2	-2	0	$(1,1)$	0	-6	0	-2
	$+\alpha+2\gamma$	$(1,1,1)$	0	0	0	2	-2	-2	0	$(3,1)$	0	2	0	-2
		$(1,1,1)$	0	0	0	-2	-2	-2	0	$(\overline{3},1)$	0	-2	0	2
b	$1+b_3$	$(1,1,1)$	0	0	0	-2	2	-2	0	$(1,1)$	0	6	0	2
	$+\alpha+2\gamma$	$(1,1,1)$	0	0	0	2	2	-2	0	$(1,1)$	0	-6	0	-2
		$(1,1,1)$	0	0	0	2	2	-2	0	$(3,1)$	0	2	0	-2
		$(1,1,1)$	0	0	0	-2	2	-2	0	$(\overline{3},1)$	0	-2	0	2

Table G.6: Table 5 continued.

F	SEC	$(C;L;R)$	Q_C	$Q_{\bar\eta^1}$	$Q_{\bar\eta^2}$	$Q_{\bar\eta^3}$	$Q_{\bar y^{3,6}}$	$Q_{\bar y^1\bar w^5}$	$Q_{\bar w^{2,4}}$	$SU(3)_{H_{1,2}}$	$Q_{\bar\Phi^1}$	Q_8	Q_9	$Q_{\bar\Phi^8}$
b	$\alpha\pm\gamma$	$(1,1,1)$	-3	1	1	1	-2	0	-2	$(1,1)$	2	-3	3	0
		$(1,1,1)$	-3	1	1	1	2	0	2	$(1,1)$	2	-3	3	0
		$(1,1,1)$	-3	1	1	1	2	0	2	$(1,1)$	2	-3	3	0
		$(1,1,1)$	-3	1	1	1	-2	0	-2	$(1,1)$	2	-3	3	0
		$(1,1,1)$	3	-1	-1	-1	-2	0	-2	$(1,1)$	-2	3	-3	0
		$(1,1,1)$	3	-1	-1	-1	2	0	2	$(1,1)$	-2	3	-3	0
		$(1,1,1)$	3	-1	-1	-1	2	0	2	$(1,1)$	-2	3	-3	0
		$(1,1,1)$	3	-1	-1	-1	-2	0	-2	$(1,1)$	-2	3	-3	0
b	$\beta\pm\gamma$	$(1,1,1)$	-3	1	1	1	0	2	2	$(1,1)$	-2	-3	3	0
		$(1,1,1)$	-3	1	1	1	0	-2	-2	$(1,1)$	-2	-3	3	0
		$(1,1,1)$	-3	1	1	1	0	-2	-2	$(1,1)$	-2	-3	3	0
		$(1,1,1)$	-3	1	1	1	0	2	2	$(1,1)$	-2	-3	3	0
		$(1,1,1)$	3	-1	-1	-1	0	2	2	$(1,1)$	2	3	-3	0
		$(1,1,1)$	3	-1	-1	-1	0	-2	-2	$(1,1)$	2	3	-3	0
		$(1,1,1)$	3	-1	-1	-1	0	-2	-2	$(1,1)$	2	3	-3	0
		$(1,1,1)$	3	-1	-1	-1	0	2	2	$(1,1)$	2	3	-3	0
b	$1+b_1$	$(1,1,1)$	-3	1	1	1	0	2	2	$(1,1)$	0	3	-3	2
	$+b_2+b_3$	$(1,1,1)$	-3	1	1	1	0	-2	-2	$(1,1)$	0	3	-3	2
	$+\beta\pm\gamma$	$(1,1,1)$	-3	1	1	1	0	-2	-2	$(1,1)$	0	3	-3	2
		$(1,1,1)$	-3	1	1	1	0	2	2	$(1,1)$	0	3	-3	2
		$(1,1,1)$	3	-1	-1	-1	0	2	2	$(1,1)$	0	-3	3	-2
		$(1,1,1)$	3	-1	-1	-1	0	-2	-2	$(1,1)$	0	-3	3	-2
		$(1,1,1)$	3	-1	-1	-1	0	-2	-2	$(1,1)$	0	-3	3	-2
		$(1,1,1)$	3	-1	-1	-1	0	2	2	$(1,1)$	0	-3	3	-2
b	$1+b_1$	$(1,1,1)$	-3	1	1	1	2	0	-2	$(1,1)$	0	3	-3	-2
	$+b_2+b_3$	$(1,1,1)$	-3	1	1	1	-2	0	2	$(1,1)$	0	3	-3	-2
	$+\alpha\pm\gamma$	$(1,1,1)$	-3	1	1	1	2	0	-2	$(1,1)$	0	3	-3	-2
		$(1,1,1)$	-3	1	1	1	-2	0	2	$(1,1)$	0	3	-3	-2
		$(1,1,1)$	3	-1	-1	-1	2	0	-2	$(1,1)$	0	-3	3	2
		$(1,1,1)$	3	-1	-1	-1	-2	0	2	$(1,1)$	0	-3	3	2
		$(1,1,1)$	3	-1	-1	-1	2	0	-2	$(1,1)$	0	-3	3	2
		$(1,1,1)$	3	-1	-1	-1	-2	0	2	$(1,1)$	0	-3	3	2

Table G.7: The table displays all the massless sectors for which the "would-be superpartners" are massive and do not form part of the massless spectrum. The "would-be superpartners" arise from the sectors that are obtained by adding the basis vector S to a given sector and are the fermionic counterparts.

F	SEC	$(C;L;R)$	Q_C	$Q_{\bar\eta^1}$	$Q_{\bar\eta^2}$	$Q_{\bar\eta^3}$	$Q_{\bar y^{3,6}}$	$Q_{\bar y^1\bar w^5}$	$Q_{\bar w^{2,4}}$	$SU(3)_{H_{1,2}}$	$Q_{\bar\Phi^1}$	Q_8	Q_9	$Q_{\bar\Phi^8}$
f	$S+$	$(1,1,1)$	-3	-3	-1	-1	0	0	0	$(1,1)$	-2	-3	3	0
	b_2+b_3	$(1,1,1)$	-3	1	3	-1	0	0	0	$(1,1)$	-2	-3	3	0
	$+\beta\pm\gamma$	$(1,1,1)$	-3	1	-1	3	0	0	0	$(1,1)$	-2	-3	3	0
		$(1,1,1)$	3	-1	1	1	0	0	0	$(1,\overline{3})$	-2	3	1	0
		$(1,1,1)$	3	3	1	1	0	0	0	$(1,1)$	2	3	-3	0
		$(1,1,1)$	3	-1	-3	1	0	0	0	$(1,1)$	2	3	-3	0
		$(1,1,1)$	3	-1	1	-3	0	0	0	$(1,1)$	2	3	-3	0
		$(1,1,1)$	-3	1	-1	-1	0	0	0	$(1,3)$	2	-3	-1	0
b	b_2+b_3	$(\overline{3},1,1)$	1	1	-1	-1	0	0	0	$(1,1)$	-2	-3	3	0
	$+\beta\pm\gamma$	$(1,1,1)$	-3	1	-1	-1	0	0	0	$(\overline{3},1)$	2	1	3	0
		$(3,1,1)$	-1	-1	1	1	0	0	0	$(1,1)$	2	3	-3	0
		$(1,1,1)$	3	-1	1	1	0	0	0	$(3,1)$	-2	-1	-3	0
f	$S+$	$(1,1,1)$	-3	3	1	-1	0	0	0	$(1,1)$	2	-3	3	0
	b_1+b_3	$(1,1,1)$	-3	-1	-3	-1	0	0	0	$(1,1)$	2	-3	3	0
	$+\alpha\pm\gamma$	$(1,1,1)$	-3	-1	1	3	0	0	0	$(1,1)$	2	-3	3	0
		$(1,1,1)$	3	1	-1	1	0	0	0	$(1,\overline{3})$	2	3	1	0
		$(1,1,1)$	3	-3	-1	1	0	0	0	$(1,1)$	-2	3	-3	0
		$(1,1,1)$	3	1	3	1	0	0	0	$(1,1)$	-2	3	-3	0
		$(1,1,1)$	3	1	-1	-3	0	0	0	$(1,1)$	-2	3	-3	0
		$(1,1,1)$	-3	-1	1	-1	0	0	0	$(1,3)$	-2	-3	-1	0
b	b_1+b_3	$(\overline{3},1,1)$	1	-1	1	-1	0	0	0	$(1,1)$	2	-3	3	0
	$+\alpha\pm\gamma$	$(1,1,1)$	-3	-1	1	-1	0	0	0	$(\overline{3},1)$	-2	1	3	0
		$(3,1,1)$	-1	1	-1	1	0	0	0	$(1,1)$	-2	3	-3	0
		$(1,1,1)$	3	1	-1	1	0	0	0	$(3,1)$	2	-1	-3	0

Table G.8: The vector-like exotic states. All sectors, fermionic and bosonic, have CPT conjugates which are not displayed.

F	SEC	$(C;L;R)$	Q_C	$Q_{\bar\eta^1}$	$Q_{\bar\eta^2}$	$Q_{\bar\eta^3}$	$Q_{\bar y^{3,6}}$	$Q_{\bar y^1\bar w^5}$	$Q_{\bar w^{2,4}}$	$SU(3)_{H_{1,2}}$	$Q_{\bar\Phi^1}$	Q_8	Q_9	$Q_{\bar\Phi^8}$
f	$S+$	$(\overline{3},1,1)$	-3	-1	1	-1	0	0	0	$(1,1)$	0	3	-3	-2
	$1+b_2$	$(1,1,1)$	3	-1	1	-1	0	0	0	$(1,\overline{3})$	0	3	3	2
	$+\alpha\pm\gamma$	$(3,1,1)$	3	1	-1	1	0	0	0	$(1,1)$	0	-3	3	2
		$(1,1,1)$	-3	1	-1	1	0	0	0	$(1,3)$	0	-3	-3	-2
b	$1+b_2$	$(1,1,1)$	-3	3	1	-1	0	0	0	$(1,1)$	0	3	-3	-2
	$+\alpha\pm\gamma$	$(1,1,1)$	-3	-1	-3	-1	0	0	0	$(1,1)$	0	3	-3	-2
		$(1,1,1)$	-3	-1	1	3	0	0	0	$(1,1)$	0	3	-3	-2
		$(1,1,1)$	-3	-1	1	-1	0	0	0	$(\overline{3},1)$	0	-3	-3	2
		$(1,1,1)$	3	-3	-1	1	0	0	0	$(1,1)$	0	-3	3	2
		$(1,1,1)$	3	1	3	1	0	0	0	$(1,1)$	0	-3	3	2
		$(1,1,1)$	3	1	-1	-3	0	0	0	$(1,1)$	0	-3	3	2
		$(1,1,1)$	3	1	-1	1	0	0	0	$(3,1)$	0	3	3	-2
f	$S+$	$(1,1,1)$	-3	-3	-1	-1	0	0	0	$(1,1)$	0	3	-3	2
	$1+b_1$	$(1,1,1)$	-3	1	3	-1	0	0	0	$(1,1)$	0	3	-3	2
	$+\beta\pm\gamma$	$(1,1,1)$	-3	1	-1	3	0	0	0	$(1,1)$	0	3	-3	2
		$(1,1,1)$	-3	1	-1	-1	0	0	0	$(3,1)$	0	-3	-3	-2
		$(1,1,1)$	3	3	1	1	0	0	0	$(1,1)$	0	-3	3	-2
		$(1,1,1)$	3	-1	-3	1	0	0	0	$(1,1)$	0	-3	3	-2
		$(1,1,1)$	3	-1	1	-3	0	0	0	$(1,1)$	0	-3	3	-2
		$(1,1,1)$	3	-1	1	1	0	0	0	$(\overline{3},1)$	0	3	3	2
b	$1+b_1$	$(3,1,1)$	3	1	-1	-1	0	0	0	$(1,1)$	0	3	-3	2
	$+\beta\pm\gamma$	$(1,1,1)$	-3	1	-1	-1	0	0	0	$(1,3)$	0	3	3	-2
		$(\overline{3},1,1)$	-3	-1	1	1	0	0	0	$(1,1)$	0	-3	3	-2
		$(1,1,1)$	3	-1	1	1	0	0	0	$(1,\overline{3})$	0	-3	-3	2

Table G.9: The vector-like exotic states. All sectors, fermionic and bosonic, have CPT conjugates which are not displayed.

F	SEC	$(C;L;R)$	Q_C	$Q_{\bar{\eta}^1}$	$Q_{\bar{\eta}^2}$	$Q_{\bar{\eta}^3}$	$Q_{\bar{y}^{3,6}}$	$Q_{\bar{y}^1\bar{w}^5}$	$Q_{\bar{w}^{2,4}}$	$SU(3)_{H_{1,2}}$	$Q_{\bar{\Phi}^1}$	Q_8	Q_9	$Q_{\bar{\Phi}^8}$
f	$1+b_2$	$(1,2,1)$	0	0	-2	-2	2	0	0	$(1,1)$	-2	0	0	2
	$+b_3+2\gamma$	$(1,2,1)$	0	0	-2	-2	-2	0	0	$(1,1)$	2	0	0	-2
		$(1,1,2)$	0	0	2	2	2	0	0	$(1,1)$	2	0	0	-2
		$(1,1,2)$	0	0	2	2	-2	0	0	$(1,1)$	-2	0	0	2
b	$S+$	$(1,2,1)$	0	0	2	2	-2	0	0	$(1,1)$	-2	0	0	2
	$1+b_2$	$(1,2,1)$	0	0	2	2	2	0	0	$(1,1)$	2	0	0	-2
	$+b_3+2\gamma$	$(1,1,2)$	0	0	-2	-2	-2	0	0	$(1,1)$	2	0	0	-2
		$(1,1,2)$	0	0	-2	-2	2	0	0	$(1,1)$	-2	0	0	2
f	$1+b_1$	$(1,2,1)$	0	-2	0	-2	0	2	0	$(1,1)$	-2	0	0	2
	$+b_3+2\gamma$	$(1,2,1)$	0	-2	0	-2	0	-2	0	$(1,1)$	2	0	0	-2
		$(1,1,2)$	0	2	0	2	0	2	0	$(1,1)$	2	0	0	-2
		$(1,1,2)$	0	2	0	2	0	-2	0	$(1,1)$	-2	0	0	2
b	$S+$	$(1,2,1)$	0	2	0	2	0	-2	0	$(1,1)$	-2	0	0	2
	$1+b_1$	$(1,2,1)$	0	2	0	2	0	2	0	$(1,1)$	2	0	0	-2
	$+b_3+2\gamma$	$(1,1,2)$	0	-2	0	-2	0	-2	0	$(1,1)$	2	0	0	-2
		$(1,1,2)$	0	-2	0	-2	0	2	0	$(1,1)$	-2	0	0	2
f	$1+b_1$	$(1,2,1)$	0	-2	-2	0	0	0	2	$(1,1)$	-2	0	0	2
	$+b_2+2\gamma$	$(1,2,1)$	0	-2	-2	0	0	0	-2	$(1,1)$	2	0	0	-2
		$(1,1,2)$	0	2	2	0	0	0	2	$(1,1)$	2	0	0	-2
		$(1,1,2)$	0	2	2	0	0	0	-2	$(1,1)$	-2	0	0	2
b	$S+$	$(1,2,1)$	0	2	2	0	0	0	-2	$(1,1)$	-2	0	0	2
	$1+b_1$	$(1,2,1)$	0	2	2	0	0	0	2	$(1,1)$	2	0	0	-2
	$+b_2+2\gamma$	$(1,1,2)$	0	-2	-2	0	0	0	-2	$(1,1)$	2	0	0	-2
		$(1,1,2)$	0	-2	-2	0	0	0	2	$(1,1)$	-2	0	0	2
f	$S+$	$(1,1,1)$	-6	0	0	-2	0	0	0	$(1,1)$	2	0	0	2
	$1+b_3$	$(\bar{3},1,1)$	-2	0	0	2	0	0	0	$(1,1)$	-2	0	0	-2
	$+\alpha+\beta$	$(1,1,1)$	6	0	0	2	0	0	0	$(1,1)$	-2	0	0	-2
	$+2\gamma$	$(3,1,1)$	2	0	0	-2	0	0	0	$(1,1)$	2	0	0	2
b	$1+b_3$	$(1,1,1)$	6	0	0	2	0	0	0	$(1,1)$	-2	0	0	-2
	$+\alpha+\beta$	$(1,1,1)$	-6	0	0	-2	0	0	0	$(1,1)$	2	0	0	2
	$+2\gamma$	$(3,1,1)$	2	0	0	-2	0	0	0	$(1,1)$	2	0	0	2
		$(\bar{3},1,1)$	-2	0	0	2	0	0	0	$(1,1)$	-2	0	0	-2

Table G.10: The vector-like exotic states. All sectors, fermionic and bosonic, have CPT conjugates which are not displayed.

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