Automatic balancing mechanisms for Notional Defined Contribution Accounts in the presence of uncertainty

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Abstract

The notional defined contribution model combines pay-as-you-go financing and a defined contribution pension formula. This paper aims to demonstrate the extent to which liquidity and solvency indicators are affected by fluctuations in economic and demographic conditions and to explore the introduction of an automatic balancing mechanism into the pension system. We demonstrate that the introduction of an automatic balancing mechanism reduces the volatility of the buffer fund and that, in most cases, the automatic mechanism that re-establishes solvency produces the lowest variance of the notional factor with the lowest expected value.

JEL: E62, H55, J26

Keywords: actuarial analysis, public pensions, retirement, solvency, stochastic processes, risk.

1   Introduction

The introduction of Notional (non-financial) Defined Contribution (NDC) pension accounts as components of modern, multi-pillar pension systems in some countries has been a major innovation of the last two decades of pension reform. NDCs have been established in Italy (1995), Latvia (1996), Poland (1999) and Sweden (1999).[1] Kyrgyzstan (1997), Mongolia (2000) and Russia (2002) have adopted some NDC features, such as life expectancy factors, while Egypt combines both a notional and financial defined contribution schemes for new entrants after 2013. According to Holzmann et al. (2012), countries such as China and Greece are also seriously considering introducing notional defined contribution pension schemes. Holzmann (2006) argues that notional accounts should provide the foundation for a unified pension system in the European Union.

A notional model is a pay-as-you-go (PAYG) system in which the amount of the pension depends on both contributions and returns, that is, the accumulated capital over the course of the participant's employment. The returns on contributions are calculated using a notional rate that reflects the financial health of the system, which is linked to an external index set by law, such as the growth rate of GDP, average salarys, or contribution payments. The account balance is called notional because it is used only for record keeping, that is, the system does not invest funds in financial markets as the system is based on PAYG financing. When an individual reaches retirement age, the fictitious balance is converted into an annuity based on the indexation of benefits and technical interest rate as well as the life expectancy of the cohort.

When designing a pension system, a legislator can obtain two types of equilibria (Bommier and Lee 2003). The first equilibrium is cross-sectional, while the other is longitudinal. A cross-sectional equilibrium implies that all that is received is given. For example, in a pure PAYG pension system, pensions for retirees are paid for by the contributions of the working-age population. In our framework this is referred to as liquidity. Under a longitudinal equilibrium (also known as actuarial fairness) a cohort cannot receive more than their own contribution, that is, at any moment, the present value

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[1]See Chłón-Domińczak et al. (2012) and Holzmann et al. (2012) for updated particularities of NDC countries.

of lifetime contributions equals the present value of lifetime benefits. In this sense, NDCs can be considered actuarially fair at some extent (see Palmer (2006) and Queisser and Whitehouse (2006)). Nevertheless, the underlying design does not produce an automatic cross-sectional equilibrium.

Valdés-Prieto (2000) shows that NDC cannot generally provide financial equilibrium in the short run unless in the unrealistic steady state and with a notional rate equal to the covered salary bill. Therefore, it is also necessary to have other adjustment mechanisms just as in the traditional defined benefit PAYG schemes. Vidal-Meliá et al. (2009) and Vidal-Meliá et al. (2010) define Automatic Balance Mechanisms (ABMs) as a set of pre-determined measures established by law to be applied immediately as required according to an indicator of the financial health of the system, that is defined as the ratio between the contribution asset and the liabilities (solvency ratio) in the Swedish case (Settergren and Mikula 2005). Sweden has also implemented an adjustment mechanism that will be triggered if the solvency indicator that emerges from an actuarial balance sheet reveals insolvency in the system. Some countries with classical Defined Benefit (DB)[2] pension systems, such as Germany, Austria, France, Finland, and Portugal, have also incorporated adjustment mechanisms to assist in calculating or indexing the initial pension payment to mitigate demographic changes, though these adjustment are not usually linked to any solvency indicator.[3]

In this respect, Auerbach and Lee (2006), using U.S demography and economic data under a stochastic macro model, examine the stability of eight NDCs pension systems involving different assumptions of the notional rate and particularities of the Swedish adjustment mechanism. However, their design does not guarantee liquidity or financial equilibrium.

This paper contributes to fill a gap in the literature investigating how different ABMs, that re-establish liquidity or solvency, react to economic and demographic changes in NDC pension schemes in terms of expected value and volatility of the notional rate and the buffer fund The ABMs considered affect to both the notional rate (in our case linked to the growth of the economy) and indexation of pension benefits in actuarially fair schemes. This paper also aims to shed some light on how the system adapts to one-time exogenous economic and demographic shocks.

Following this introduction, this paper is structured as follows. The subsequent section 2 presents NDCs in a framework with four OLG that allows a dynamic evolution of the main variables that affect the liquidity and solvency indicators of the system. The next section 3, provides two different methods of assessing the financial health of a pension system. The section 4 focuses on the design of an automatic balancing mechanism to restore the liquidity or solvency of the system while the numerical illustration is presented in section 5. The final section of this paper provides the main conclusions and the four appendices provide additional details for the adjustment factor, the accrual and forecasted liabilities, the theoretical framework used and the numerical results for the volatility of the notional compounding rate.

2 A dynamic four OLG model

NDCs have been studied in 2 and 3-period OLG[4] in Valdés-Prieto (2000) and for multiple coexisting generations in Vidal-Meliá and Boado-Penas (2013), Boado-Penas and Vidal-Meliá (2014) and Vidal-Meliá et al. (2015). However, these papers consider that changes in the demographic and economic variables are constant which make their results only valid when the system is in steady state. There

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[2]For instance, Germany's ABM aims to adapt the benefits to ensure a balanced budget at every period for a given increase of the contribution rate (Börsch-Supan et al. 2004). While varying the contribution rate may be a way of seeking sustainability in DB schemes, it is not the case for NDCs as their contribution rate is fixed per construction.

[3]See D'Addio and Whitehouse (2012) for a detailed explanation of current PAYG pension schemes. The paper by Knell (2010) also analyses automatic adjustment factors that can be used to keep the PAYG scheme balanced in the case of fluctuations in cohort sizes.

[4]Defined benefit pension systems have been extensively studied in the existing literature mostly in 2-OLG models in order to derive stylized results (Diamond 1977). However, these results tend to depend strongly on the 2-generations assumption. 3-OLG models seem to incorporate enough complexity to mimic reality more accurately (Samuelson 1958). Also, the paper by Auerbach et al. (2013) provides a general equilibrium setting with multiple generations and evaluates how deterministic shocks are efficiently spread across cohorts of three PAYG pension systems based on the actual US and German systems, though it did not include the Swedish NDC model.

are also some papers studying different features of NDC systems in a multiple OLG setting when the variables are in stochastic steady state. In particular, Auerbach and Lee (2006) study the fiscal sustainability of NDCs with calibrated US data while Auerbach and Lee (2011) analyse the performance of different NDC designs with regard to risk-spreading among generations.

This section examines NDCs using a four OLG model, that includes dynamics of population and salarys, where two generations of contributors, aged 1 and 2, and two generations of pensioners aged 3 and 4 coexist (see Figure 1 for details). At age 5 there are no survivors remaining.

The choice of four generations is not arbitrary; it introduces heterogeneity among contributors because these two generations are characterized by different demographic and salary histories as well as contributions to the system. It also introduces mortality and indexation during the retirement period. This is not possible in a 2 or 3-period model. A minimum of two retirement periods is needed to study explicitly the effect of mortality assumptions on the pension expenditures and to derive results on the 'indexation' that renders the system actuarially fair. Furthermore, it allows us to study different demographic scenarios without relying heavily on computational methods (Bhattacharya and Russell 2003).

2.1 Population and salary dynamics

The demographic-economic structure at any time t is represented as follows:

Age:
$x = 1, 2, 3, 4$
Population at time t:

$$N_t^x = N_{t-x+1}^1 p_t^x = N_0^1 \prod_{i=1}^{t-x+1} (1 + n_i) \, p_t^x \tag{2.1}$$

where

N_t^x denotes individuals aged x for $x =$1, 2, 3 and 4 who are alive at time $t > 0$ and entered the labour market at time $t - x + 1$. New entrants at time t are denoted as N_t^1. At time $t = 0$, the population structure is given at all ages x.

p_t^x is the time-dependent survival probability, that is, the probability that an individual will reach age x by time t. It is assumed that no mortality occurs prior to retirement; therefore, $p_t^x = 1$ for $x = 1, 2$, and 3. For simplicity, the probability that one individual is alive at the age of 4, p_t^4, is denoted by p_t.[5] Furthermore, the survival probability p_t^5 is zero, that is, individuals cannot attain age 5.

n_i is the rate of population variation from period $i - 1$ to period i and is a stochastic process defined in the probability space $(\Omega, \mathcal{F}, P)$.

Table 1 illustrates how the population evolves over three periods by year of birth.
Individual salarys at time t

$$S_t^x = S_0^x \prod_{i=1}^{t} (1 + g_i) \tag{2.2}$$

where

S_t^x denotes the individual salarys for $x =$1 and 2 at time $t > 0$ which are earned by the active population and are assumed to be paid at the beginning of the calendar year. salarys are dependent on time t and age x. Furthermore, S_0^x is the observed salary at time 0 for each active age $x =$1, 2. In our model, the salarys coincide with the contribution bases.

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[5]We ignore the mortality before retirement because it simplifies the notional rate paid to the contributions, based on the rate of increase of the income from contributions. It also highlights the effect of the survival probability on the annuity paid during retirement.

g_i is the rate of salary variation from the period $i-1$ to period i and is a stochastic process defined in the probability space $(\Omega, \mathcal{F}, P)$.

3 Liquidity and solvency indicators

There are various methods of assessing the financial health of an unfunded pension system. This paper focuses on two methods. The first method assesses the liquidity ratio, which indicates the relationship between the income from contributions and the pension expenditures at a specific date.

A second method assesses the solvency of the pension system by compiling an actuarial balance. There are typically two ways for Social Security departments to compile an actuarial balance, namely the 'Aggregate accounting projection' and the 'Swedish' method. The aggregate accounting projection method compares the net present value of the expenditure on pensions and the income from contribution in a long time horizon. This balance uses a forecasted demographic scenario while the macroeconomic scenario is exogenous. In practice, this kind of balance is compiled, on a regular bases, in countries such as U.S (Boards of Trustees of the Federal Old-Age and Survivors Insurance and Disability Insurance Trust Funds 2015), Japan (Sakamoto 2005) or Canada (Office of the Superintendent of Financial Institutions Canada (OSFIC) (2007, 2008)), amongst others.

In Sweden, compiling an official actuarial balance sheet has been normal practice since 2001. This actuarial balance sheet is a traditional accounting balance sheet that lists assets[6] and liabilities over a definite horizon (see Boado-Penas et al. (2008) and Swedish Pension Agency (2015)). The actuarial balance sheet is a financial statement listing the pension system obligations to contributors and pensioners at a particular time and the various assets (financial and contributions) that back these obligations. The solvency ratio, defined as the ratio of assets to liabilities, calculated from the actuarial balance sheet serves the following two purposes in Sweden: to measure whether the system can fulfil its obligations to participants and to decide whether an ABM should be applied. This paper follows the Swedish method, based on cross-sectional verifiable facts, that does not need any projections.

The following subsections provide the calculations of the liquidity and solvency ratios.

3.1 Liquidity ratio

As previously noted, NDCs are financed on a PAYG basis, that is, retiree pensions are paid for by the contributions of the working age population. The liquidity ratio indicates whether the current contributions and financial assets are sufficient to pay current retiree pensions. Formally, the ratio at time t, LR_t is represented as follows:

$$LR_t = \frac{C_t + F_t^-}{P_t} \tag{3.1}$$

where
C_t represents the income from contributions at time t.
P_t represents the total pension expenditures at time t.
F_t^- represents the value of the (buffer) fund at time t, also called reserve fund, before new contributions and benefits payments are considered. F_t^- can be expressed as follows:

$$F_t^- = F_{t-1}^+(1+i_t) = F_0^- \prod_{j=1}^{t}(1+i_j) + \sum_{j=0}^{t-1}(C_j - P_j)\prod_{k=j+1}^{t}(1+i_k) \tag{3.2}$$

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[6]First pillar pension systems, generally based on PAYG, are not backed by financial assets. Although, some countries, such as Chile, apply a funding framework to their first pillar pension systems (Fajnzylber and Robalino 2012). Also, in some former British colonies, such as Singapore or Malaysia, the key pillar of the pension system is based on the Central Provident Fund (CPF) is a compulsory defined-contribution savings plan designed to fund mainly retirement and healthcare.

where

i_t represents the financial rate of return of the fund from period $t-1$ to t and is a stochastic process defined in the probability space $(\Omega, \mathcal{F}, P)$.

The value of the fund at time t after contributions and payments is given by $F_t^+ = F_t^- + C_t - P_t$.

3.1.1 Income from contributions

The income from contributions received by the pension system at time t, C_t, is represented as follows:

$$
\begin{aligned}
C_t &= \pi \cdot N_t^1 \cdot S_t^1 + \pi \cdot N_t^2 \cdot S_t^2 \\
&= \pi \cdot N_0^1 \prod_{i=1}^{t-1}(1+n_i) \prod_{i=1}^{t}(1+g_i) \underbrace{\left(S_0^1(1+n_t) + S_0^2\right)}_{K_t^C}
\end{aligned} \tag{3.3}
$$

where

π is the fixed contribution rate of the NDC pension scheme.

K_t^C is a factor that takes into account the heterogeneity of salarys and births changes.

The notional compounding factor at time t, $1+r_t$, is chosen such that it reflects the financial health of the system through changes in the total contribution base of the system[7] and is represented as follows:

$$
1 + r_t = \frac{C_t}{C_{t-1}} = (1+n_{t-1})(1+g_t)\frac{K_t^C}{K_{t-1}^C} \tag{3.4}
$$

where

r_t is the notional rate for the period $t-1$ to t. Note that the notional rate is affected by both salary and population processes, such as new active individuals or salary trends.

The rate, r_t, is usually known as the 'natural rate' (Valdés-Prieto (2000) and Börsch-Supan (2006)) or 'canonical rate' of the NDC scheme (Gronchi and Nisticò 2006). It is also known as the 'biological rate' of the economy (Samuelson 1958). In practice, only Latvia and Poland apply this notional rate during the accumulation phase while Sweden applies the growth of the contribution base per capita[8] and Italy the three-year GDP growth average rate.

Note that if the population is in a steady state, the notional factor reduces to $1+r_t=(1+n)(1+g)$ which means that the notional rate, r_t equals $(1+n)(1+g)-1$. In our framework, steady state holds if $n_t=n$ and $g_t=g \; \forall \; t$ and if the survival probabilities are constant over time.

3.1.2 Pension expenditures

At retirement, the accumulated notional capital for each individual is converted into an annuity for the next two periods. In this model, all individuals in the same birth cohort have the same contribution history and mortality. Therefore, the accumulated notional capital or notional pension wealth, NPS_t^x for $x = 1, 2, 3, 4$, will be calculated for each generation.[9]

To calculate the accumulated notional capital for each cohort we consider their past contributions and assign them a return that corresponds to the notional rate (3.4) until retirement. The total notional accumulated capital for the cohort aged 3 retiring at time t, NPS_t^3, is represented as follows:

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[7]Given a contribution rate, π, the growth of the total contribution base of the system coincides with the growth of the income from contributions

[8]The notional rate in Sweden is, equivalent to the rate of increase of the salaries $1 + g_t$, when salarys are age-independent.

[9]Note that this calculation method allows surviving contributors to receive eventual inheritance gains from the deceased (Boado-Penas and Vidal-Meliá 2014). In our setting, however, there are no inheritance gains because there is no mortality prior to retirement.

$$NPS_t^3 = \pi S_{t-2}^1 N_{t-2}^1 (1 + r_{t-1})(1 + r_t) + \pi S_{t-1}^2 N_{t-1}^2 (1 + r_t) \tag{3.5}$$

The notional capital for the retiring cohort can be also rewritten as follows:

$$NPS_t^3 = C_t K_t^N \tag{3.6}$$

where
$K_t^N = \frac{S_0^1(1+n_{t-2})}{S_0^1(1+n_{t-2})+S_0^2} + \frac{S_0^2}{S_0^1(1+n_{t-1})+S_0^2}$ is a function that depends on the notional rate during contributors' career.

The time-dependent factor K_t^N equals 1 when $n_{t-2} = n_{t-1}$, that is, the notional capital for the retiring cohort - pensioners who have just retired- is equal to the system's income from contributions at t. It is remarkable that, in steady state, the income from contributions, which is a cross-sectional measure, is equivalent to the commitments the system takes on with pensioners who are just retired, which is a longitudinal measure.[10]

The individual pension P_t^3 to be paid at time t to the new retirees aged 3 is represented as follows:

$$P_t^3 = \frac{NPS_t^3}{a_t \ N_t^3} \tag{3.7}$$

with

$$a_t = \mathbb{E}_t \left[1 + p_{t+1}^* \frac{1 + \lambda_{t+1}^*}{1 + r_{t+1}^*} \right] \tag{3.8}$$

where
a_t represents the annuity that transforms the accumulated notional capital into a pension considering indexation, λ_{t+1}^*, probability of survival, p_{t+1}^*, and discount (or notional rate), r_{t+1}^*, processes.

Calculating the annuity at time t requires assumptions about future processes, which are made in accordance with known information at time of calculation. The pension system is exposed to risk in so far as we cannot know accurately the evolution of the survival rates. The systemic or undiversifiable risk has therefore an impact on the financial health of pension systems.

Therefore, the ex-ante or theoretical mortality p_{t+1}^* does not necessarily coincide with the ex-post or observed mortality p_{t+1}, presented in the previous section. The first is the choice of the government, while the latter represents the observed survival probability. In practice, Italy uses period tables with a revision every three years (Belloni and Maccheroni 2013), Poland use annual period tables (Chłón-Domińczak et al. 2012), while Latvia uses projected tables (Vanovska 2006). Sweden uses life span statistics for the five-year period preceding the year when the individual reached age 60, if the pension is withdrawn before age 65, and age 64 if the pension is withdrawn after 65 (Swedish Pension Agency 2015), which do not necessarily coincide with the mortality observed during retirement.

The pension for the second generation of retirees aged 4, P_t^4, corresponds to the indexed first period pension, that is:

$$P_t^4 = P_{t-1}^3 \underbrace{(1 + \lambda_t) A_t}_{\text{adjusted indexation}} \tag{3.9}$$

The first component of the adjusted indexation, λ_t, is the observed value at time t of the government's indexation process, λ_t^*, used in the annuity calculation. This value may differ from the

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[10]Valdés-Prieto (2000) shows that the notional capital for the retiring cohort is proportional to the income from contributions in a continuous setting. Vidal-Meliá et al. (2015) show that the income from contributions coincides with both the accrued and forecasted debt in a discrete setting when the system is in steady state.

expected value $\mathbb{E}_{t-1}\left[1 + \lambda_t^*\right]$ considered when the annuity is calculated at time $t - 1$, disturbing the effective longitudinal cohort equilibrium. The same reasoning holds for the theoretical discounting rate r_t^* and observed notional rate r_t.

The second component, A_t, corresponds to the actuarial adjustment applied to the observed indexation which ensures actuarial fairness or longitudinal equilibrium. We add this adjustment to the observed indexation here to later show that an actuarially fair system is not necessarily liquid or solvent.[11] The adjustment, A_t, is derived in Appendix A and is represented by the following expression:

$$A_t = \frac{\mathbb{E}_{t-1}\left[p_t^* \frac{1+\lambda_t^*}{1+r_t^*}\right]}{p_t \frac{1+\lambda_t}{1+r_t}} \tag{3.10}$$

As can be seen, this adjustment factor A_t, which captures the differences between the theoretical and the observed values, is a combination of an economic and a longevity factor. The economic adjustment arises due to the difference between the observed notional rate and observed indexation rate, i.e., r_t and λ_t respectively, with the theoretical discount and indexation rates, i.e., r_t^* and λ_t^* respectively. This adjustment is used in practice in Sweden. The annuity, as calculated by the Swedish Pension Agency, assumes a discounting rate of 1.6% (Swedish Pension Agency 2015). This front-loading decreases the value of the annuity and provides a higher initial pension at the expense of a potential lower indexation during retirement. It benefits the younger-than-average pensioner and creates a risk of relative poverty for the older elderly (Chłón-Domińczak et al. 2012). The indexation of pensions is therefore adjusted to the deviation of the actual notional rate from the 1.6% per year factored in the annuity divisor.

The longevity adjustment compares the observed survival probability p_t with the theoretical survival probability p_t^*. So far, this adjustment is not found in any of the countries which have implemented NDCs. However Alho et al. (2013) argue that this kind of adjustment would be one of the solutions to the systematic miscalculation of the life expectancy in practice.

Finally, the pension expenditures, denoted by P_t, is described by the following expression:

$$P_t = P_t^3 N_t^3 + P_{t-1}^3 \left(1 + \lambda_t\right) A_t N_t^4 = \frac{NPS_t^3}{a_t} + p_t \frac{NPS_{t-1}^3}{a_{t-1}} \left(1 + \lambda_t\right) A_t$$

$$= C_t \underbrace{\left(\frac{K_t^N}{a_t} + \frac{K_{t-1}^N}{a_{t-1}} \underbrace{\frac{p_t \left(1 + \lambda_t\right) A_t}{1 + r_t}}_{a_{t-1}-1}\right)}_{K_t^P} \tag{3.11}$$

The pension expenditures, P_t, is also proportional to the total contribution C_t paid during the same period.

The following proposition shows that the NDC scheme may not be liquid even when the indexation ensures actuarial fairness and/or the notional rate incorporates demographic and salarys changes. Our result extends Valdés-Prieto (2000) when the notional capital is converted to a payment stream paid during retirement.

Proposition 1. *In general, contributions are not equal to pension expenditures in this 4-period OLG unfunded dynamic model, that is $C_t \neq P_t \ \forall t$.*

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[11]Note that this adjustment would be unnecessary if the notional capital at retirement was paid as a lump sum as done in Valdés-Prieto (2000). Furthermore, the adjustment A_t, allows us to develop expressions which are independent of the government's assumptions regarding the annuity.

Proof. This result is proven by counterexample. Let us assume that $n_t = n$=cte and $g_t = g$=cte $\forall t$ and deterministic. Let us assume further that $p_t^* \neq p_s^*$ for $s \neq t$ and that the theoretical indexation and discount interest rate coincide, that is, $\lambda_s^* = r_s^* \ \forall s$. Then the notional capital of the retiring cohort is represented as follows:

$$NPS_t^3 = C_t K_t^N = C_t \frac{S_0^1 \cdot (1+n)}{S_0^1 \cdot (1+n) + S_0^2} + \frac{S_0^2}{S_0^1 \cdot (1+n) + S_0^2} = C_t$$

Under this setting the expression of the annuities are:

$$a_t = 1 + p_{t+1}^*$$
$$a_{t-1} = 1 + p_t^*$$

The pension expenditures are then represented as follows:

$$P_t = C_t \left(\frac{1}{a_t} + \frac{a_{t-1} - 1}{a_{t-1}} \right) = C_t \left(\frac{1 + 2p_t^* + p_t^* p_{t+1}^*}{1 + p_t^* + p_{t+1}^* + p_t^* p_{t+1}^*} \right) \neq C_t$$

The income from contributions does not equal the pension expenditures when the parameters are chosen as above. □

3.2 Solvency ratio

The solvency ratio (also called balance ratio), that emerges from the actuarial balance sheet ("Swedish method") is defined as the relationship between assets and liabilities as follows:

$$SR_t = \frac{CA_t + F_t^-}{V_t} \tag{3.12}$$

where
CA_t corresponds to the contribution asset at time t.
V_t represents the liabilities to all participants in the pension system at time t.
F_t^- is given by (3.2).
Assets are calculated using an accounting measure known as the Contribution Asset. This method of valuing assets was derived for a steady state population. In Sweden, both assets and liabilities are valued based on verifiable cross-sectional facts, meaning that no projections are made. This should not be interpreted as a belief that all the basic parameters determining the items on the balance sheet will remain constant but as a conscious policy to prefer cross-sectional data. Changes are not included until they occur and can be verified. Then, these changes are incorporated into the balance sheet on an annual basis. Swedish authorities note that system solvency does not depend on either assets or liabilities but on the relationship between these two via the solvency ratio. Therefore, valuing assets and liabilities with cross-sectional data is adequate if applied consistently.[12]

3.2.1 The Contribution Asset

The contribution asset was firstly defined by Settergren and Mikula (2005) as the product of the current income from contributions C_t and a function called the 'turnover duration'. The contribution asset, CA_t is represented as follows:

$$CA_t = C_t \times TD_t \tag{3.13}$$

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[12]See Settergren and Mikula (2005) for the notional model, Boado-Penas et al. (2008) and Boado-Penas and Vidal-Meliá (2013) for the defined benefit PAYG system and Bommier and Lee (2003) for a general transfer model under the golden rule.

where

$$TD_t = A_t^R - A_t^C = \frac{3 \cdot P_t^3 N_t^3 + 4 \cdot P_{t-1}^3 \left(1 + \lambda_t\right) A_t N_t^4}{P_t^3 N_t^3 + P_{t-1}^3 \left(1 + \lambda_t\right) A_t N_t^4} - \frac{1 \cdot \pi S_t^1 N_t^1 + 2 \cdot \pi S_t^2 N_t^2}{\pi S_t^1 N_t^1 + \pi S_t^2 N_t^2}$$

$$= 2 + \frac{K_{t-1}^N}{K_t^P} \frac{a_{t-1} - 1}{a_{t-1}} - \frac{S_0^2}{K_t^C}$$

The first component of the turnover duration, A_t^R, is the weighted average age of pensioners and A_t^C is the weighted average age of contributors. The turnover duration represents the 'the average time a unit of money is in the system' as stated in Palmer (2006) assuming that economic, demographic and legal conditions remain constant.[13] In other words, it is the time in years that is expected to elapse before all system liabilities are renewed or rotated.

3.2.2 The Liabilities

Under a NDC framework, liabilities are calculated using the accrual method, also known as retrospective, whose value coincides with the current cohort's notional capital. The value of the liabilities using the forecasted method produces the same results only if the adjustment A_t is used. See Appendix B to see how the adjustment ensures actuarial fairness for all cohorts.

The sum of the liabilities of all participants at time t, V_t, can be split into the liabilities to contributors, V_t^1 and V_t^2, and the liabilities towards the retirees, V_t^3 and V_t^4, that is:

$$V_t^1 = NPS_t^1 = 0$$

$$V_t^2 = NPS_t^2 = \pi S_{t-1}^1 N_{t-1}^1 \left(1 + r_t\right) = C_t \frac{S_0^1(1 + n_{t-1})}{K_{t-1}^C}$$

$$V_t^3 = NPS_t^3 = C_t K_t^N$$

$$V_t^4 = NPS_t^4 = \left(NPS_{t-1}^3 - P_{t-1}^3\right)\left(1 + r_t\right) = C_t K_{t-1}^N \frac{a_{t-1} - 1}{a_{t-1}}$$

Then the total liabilities, V_t, are:

$$V_t = C_t \left(\frac{S_0^1(1 + n_{t-1})}{K_{t-1}^C} + K_t^N + K_{t-1}^N \frac{a_{t-1} - 1}{a_{t-1}} \right)$$

$$= C_t \underbrace{\left(2 + \frac{S_0^1(1 + n_{t-3})}{K_{t-3}^C} - \frac{K_{t-1}^N}{a_{t-1}} \right)}_{K_t^V} \tag{3.14}$$

The liabilities are proportional to the current contributions of the active population.

The following result demonstrates that the pension scheme is not generally solvent in absence of a buffer fund.

Proposition 2. *In general, the contribution asset does not equal liabilities in this four-period OLG unfunded dynamic model, that is $CA_t \neq V_t \ \forall t$.*

Proof. This result is proven by counterexample. Let us assume that $n_t = n$=cte and $g_t = g$=cte $\forall t$ and deterministic. The working population and salarys are then not time dependent and are denoted by $(1 + n)$ and S_0^i for $i = 1, 2$. Let us assume further that $p_t^* \neq p_s^*$ for $t \neq s$ and that the theoretical indexation and discount rate coincide, that is, $\lambda_s^* = r_s^* \ \forall s$. The notional capital of the retiring cohort at time t is then equal to C_t as shown in Proposition 1.

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[13]For more details, see Boado-Penas et al. (2008) and Settergren and Mikula (2005).

The liabilities and contribution asset at time t can be respectively represented as follows:

$$V_t = C_t \left(2 + \frac{S_0^1 \cdot (1+n)}{S_0^1 \cdot (1+n) + S_0^2} - \frac{1}{a_{t-1}} \right)$$

$$CA_t = C_t \left(2 + \frac{1}{K_t^P} \frac{a_{t-1}-1}{a_{t-1}} - \frac{S_0^2}{S_0^1 \cdot (1+n) + S_0^2} \right)$$

The liabilities at time t, V_t, are equal to the contribution asset, CA_t, when the following expression holds:

$$1 - \frac{1}{a_{t-1}} = \frac{1}{K_t^P} \frac{a_{t-1}-1}{a_{t-1}}$$

Rearranging and replacing the annuities by their explicit expression we obtain:

$$1 - \frac{1}{1+p_t^*} = \frac{(1+p_{t+1}^*)(1+p_t^*)}{(1+p_t^*) + p_t^*(1+p_{t+1}^*)} \frac{p_t^*}{1+p_t^*} \tag{3.15}$$

which only holds when $p_{t+1}^* = p_t^*$. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\Box$

4 Design of automatic balancing mechanisms

As demonstrated in the previous section, a notional defined contribution pension system does not guarantee liquidity or solvency by design in a dynamic environment. The desired liquidity and solvency objectives can be reached through automatic balancing mechanisms (ABMs). The purpose of successive application of ABMs is to provide automatic financial stability, which can be defined as 'the capacity of a pension system to adapt to financial turbulence without legislative intervention' (Settergren 2013). It is understood that turbulence can be caused by economic, financial or demographic shocks that affect the system's financial equilibrium. These ABMs are used to depoliticize the management of PAYG systems by adopting measures suited to long-term planning.

The following questions arise when designing an ABM:

- Which type of ABM should be applied?

- Should an ABM be symmetric or asymmetric?

This section will focus on two types of ABMs. The first mechanism re-establishes liquidity to the system, whereas the second mechanism restores solvency as defined in the previous section. Both mechanisms will be considered for symmetric and asymmetric designs, although symmetric cases are rarely applied in practice. Palmer (2013) states that under a symmetric ABM any surplus that might arise would be automatically distributed. Alho et al. (2013) state that the mechanism can be symmetric in the sense that these adjust for both positive and negative deviations according to the financial health indicator.

Only Sweden, among the four NDC countries has put in place an ABM to restore the health of the system (Chłón-Domińczak et al. 2012). The ABM is asymmetric, as it is only triggered when the solvency ratio is lower than 1. However, the Swedish ABM allows for recovery. After a period of low returns as a consequence of the mechanism, a period of higher-than-normal returns follows.[14] Auerbach and Lee (2006) examine the stability of eight pension systems with a particular attention to the differences between asymmetric and symmetric cases in the Swedish context. In this sense, the asymmetric Swedish balance mechanism, applied only when the system is underfunded, could lead to the accumulation of surpluses when the system is overfunded.

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[14]For more detailed explanations, see Barr and Diamond (2011).

ABMs adjust the notional factor with a time-dependent variable to obtain liquidity or solvency ratios of at least unity at all times, depending on whether it is symmetric or asymmetric. The adjusted notional factor applied to the system to attain the desired equilibrium will be $I_t^z = (1 + r_t) B_t^z$, where $z = LR$ when the ABM for liquidity equilibrium is used and $z = SR$ when the ABM for solvency equilibrium is used. ABMs affect contributors by adapting the notional factor by which their notional capital is accumulated and retirees through changes in pension indexation.

At time t formulas for the notional capital NPS_t^3 (3.6), pension expenditures P_t (3.11), buffer fund F_t^- (3.2), contribution asset CA_t (3.13), liabilities V_t (3.14) and adjustment factor A_t (3.10) as well as their related factors K_t^N, K_t^P, TD_t^z, and K_t^V correspond to formulas $NPS_t^{3,z}$(C.1), P_t^z (C.2), $F_t^{-,z}$ (C.3), CA_t^z (C.4), V_t^z (C.5), A_t^z (C.6)[15], $K_t^{N,z}$ (C.7), $K_t^{P,z}$ (C.8), TD_t^z (C.9), and $K_t^{V,z}$ (C.10) in the presence of an ABM with z = LR, SR. Formula (3.3) corresponding to the income from contributions C_t remains the same as it is not affected by the notional rate. The detailed formulae are presented in Appendix C.

4.1 The ABM for the liquidity ratio

This ABM is designed to attain a liquidity ratio of 1. The notional factor is therefore affected by B_t^{LR}; therefore, $I_t^{LR} = (1 + r_t) B_t^{LR}$. The automatic mechanism, taking into account expression 13, can be written as follows:

$$B_t^{LR} = \frac{C_t + F_t^{-,LR}}{C_t K_t^{P,LR}} \tag{4.1}$$

If the ABM is symmetric the pension scheme behaves as a pure defined contribution system, providing participants with available funds in both good and bad times.

If the ABM is asymmetric, participants do not benefit from positive economic shocks and funds can accumulate according to formula (C.3) for z=LR.

4.2 The ABM for the solvency ratio

This ABM is designed to restore the system's solvency by reducing the growth of the pension liabilities, that is, pension payments and contributor's notional capital. The adjusted notional factor is then $I_t^{SR} = (1 + r_t) B_t^{SR}$. The contribution asset at time t, CA_t^{SR} does not depend on the ABM B_t^{SR} at time t, but depend on those applied before t, as noted in formula (C.4) and (C.9). Only the liabilities, V_t^{SR}, depend on the ABM factor at time t. The automatic mechanism in this case can be represented as follows:

$$B_t^{SR} = \frac{CA_t^{SR} + F_t^{-,SR}}{V_t^{SR}} \tag{4.2}$$

This ABM might accumulate some funds because a solvency ratio equal to or greater than 1 does not imply a liquidity equal to or greater than 1 (Alonso-García 2015).

5 Numerical illustration

This section presents a numerical example and analyses the behaviour of the notional factor under both ABMs in a stochastic environment. The effects of these ABMs are evaluated in terms of the expected value and variance of the notional factor and the expected value of the ratio of the buffer fund to contributions.[16]

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[15]Note that the results presented in B still hold in presence of an automatic balancing mechanism.

[16]The ratio provides more insights than examining the nominal value of the fund because we can easily compare to values of contributions.

The salary and population processes are given by the following expression:

$$1 + n_s = e^{n - \frac{\sigma_P^2}{2} + \sigma_P(W_s^P - W_{s-1}^P)} \tag{5.1}$$

$$1 + g_s = e^{g - \frac{\sigma_W^2}{2} + \sigma_W(W_s^S - W_{s-1}^S)} \tag{5.2}$$

The stochastic process $1 + n_s$ (resp. $1 + g_s$) is distributed log-normally with mean $n - \sigma_P^2/2$ and standard deviation σ_P (resp. with mean $g - \sigma_W^2/2$ and standard deviation σ_W) $\forall t$. The processes W^P and W^S are correlated Wiener processes, that is, the expected value of $W^P(s)W^S(s)$ is equal to $\rho \cdot s$.

The demographic and the salary processes, $1 + n_s$ and $1 + g_s$, respectively, are stationary stochastic processes, which means that their distributions do not depend on time. This holds in the absence of exogenous shocks, such as a drop in fertility or a baby boom. For instance, if the demographic process at time t is affected by a one-time exogenous shock δ, the demographic process at time t is represented by $1 + n_t^* = (1 + n_t)(1 + \delta)$. This special case will be studied in Subsection 5.2.

The ABM is applied at time t for the first time. Before this moment t, no funds have accumulated. The details of the theoretical framework and recurring formulas used for the calculations as well as further details concerning the joint distribution of the random vectors are presented in Appendix C.

To calculate the expected value and variance of the notional factor and the expected value of the ratio of the buffer fund to the contributions, we use the closed-form expressions developed above in Section 2 and the tools described in Appendix C.

We explore these results under symmetric and asymmetric designs. In the symmetric case, the notional factor increases when the system is liquid and/or solvent and decreases during periods of deficit and/or insolvency. In the asymmetric design, changes in the notional factor occur only during deficit and/or insolvency.

The following are the main assumptions of our numerical model:

- Two generations of contributors and two generations of pensioners coexist at each moment in time.[17]

- At time t=0, the salary is 30,000 for individuals aged y, and 45,000 for those aged y+1.

- The entry population process is log-normally distributed, where the percentage drift and volatility R and σ_P equal 0.25 percent and 5 percent, respectively.

- The salary process follows a log-normal distribution, where the percentage drift and volatility γ and σ_W equal 1.5 percent and 10 percent, respectively.

- The Wiener processes for salarys and population are correlated with $\rho = -0.25$. The negative correlation reflects the fact that cohort size negatively affects earnings (see Brunello (2009)).

- The return of the buffer fund i_t is 0 percent for all periods.

- The probability of dying equals 0.5 for the first generation of pensioners and 1 for the second generation. There is no mortality before retirement. Three different scenarios (base, up and down) for mortality are considered. First, we consider constant mortality rates (base scenario). Second, the survival probability is assumed to increase 0.005 per period (up scenario). Third, the survival probability is assumed to decrease 0.005 over time (down scenario).

- The theoretical indexation rate λ_t^* is equal to the theoretical discounting rate r_t^*.

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[17]Unemployment is not considered in our analysis, therefore the number of contributors at ages 1 and 2 coincides with the number of individuals in the general population at those ages.

In this section two different life tables are studied. First, we study the system when the future longevity patterns are known and incorporated in the annuity calculation (Case 1). This corresponds to $p_t^*=p_t$ in the annuity formula. The Case 2 addresses the mortality risk when the government uses current mortality data in the annuity and does not allow for the longevity adjustment in the pension indexation, which makes the system no longer actuarially fair. This corresponds to $p_t^*=p_{t-1}$ in the annuity formula. Mathematically, the annuity for the Case 1 is $a_t^1 = \mathbb{E}_t\left[1 + p_{t+1}\right]$, whereas the annuity for the Case 2 is $a_t^2 = \mathbb{E}_t\left[1 + p_t\right]$.

The results are obtained through Monte Carlo simulations for 1 million randomly generated paths. The analysis is conducted for 8 periods, which represents two full population renewals.

In the first column of Figure 2, we observe that the value of the expected notional factor under the liquidity ratio ABM is higher than under the solvency ratio ABM. The solvency ratio ABM is not triggered in this scenario; therefore, the notional factor under a non-ABM provides the same results.

However, in a scenario of increasing longevity, the second column of Figure 2, the expected notional factor is higher in the presence of either ABM. This result is expected because the longevity trend is known in advance and the initial pension is reduced to adapt to the new mortality pattern. Consequently, the expected value of the buffer fund with ABMs, Figure 4, is always lower than the non-ABM case due to the increase in the notional factor after applying an ABM. The opposite pattern is observed with the decreasing longevity trend, where the ABMs decrease the expected notional factor while maintaining the value of the buffer fund near zero.

Under an asymmetric design (Figure 3), it is not surprising that the activation of an ABM reduces the expected value of the notional factor in all longevity scenarios because the asymmetric ABMs are designed to be capped at 1. When we calculate the average value of the million scenarios, values less than 1 will have higher relative weights, which significantly decrease the expected value of the notional factor. This reduction is greater for the solvency ratio ABM. In the symmetric design, this ABM also has the lowest notional factor variance (Figure 8 and Table 2).

In all scenarios, the expected value of the notional factor under the liquidity ratio ABM and variance is higher than under a solvency ratio ABM (Figure 8 and 9). In all scenarios, the solvency ratio ABM produces the lowest values of the expected notional factor and its variance, which are depicted in Table 2.

The average, 75, 95 and 99 percentiles over the entire study period indicate that the preferred ABM is the liquidity ratio when the mechanism is symmetric and no ABM when the mechanism is asymmetric.[18] This result is consistent with the expected values provided Figure 2 and 3. In the asymmetric case, the highest expected value occurs with the absence of an ABM.

5.1   Mortality risk

The system is exposed to mortality risk to the extent that we do not know the survival probability of individuals. It is well understood that this risk has an impact on the financial health of pension systems. Hence, Case 2, in which the current historical values of mortality are used, is examined to evaluate to what extent mortality risk affects our results. In this case, mortality rates are based on verifiable facts; therefore, current longevity is used even though it is expected to change. Note that in Case 2 we no longer have a fair pension system.

As indicated in Figures 2, 8 and 4 the results vary if current mortality rates are used (Case 2) rather than prospective mortality rates (Case 1). In both symmetric and non-symmetric designs, under increasing longevity, the value of the expected notional factor is lower after applying an ABM because the ABMs correct errors introduced by using current mortality rates, which resulted in higher initial pensions. Note that the notional factor decreases further using a non-symmetric ABM. The contrary occurs in Figure 2 under decreasing longevity for a symmetric mechanism. For the asymmetric case, Figures 3, 9 and 5, the explanation is the same as that for Case 1 for the up scenario. The ABMs are capped at 1, which produces a lower global average by assigning lower

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[18]The numerical results associated with the percentiles are not depicted to limit the length of the paper.

values higher relative weights. As in case 1, the expected value of the notional factor and its variance under the liquidity ratio ABM is higher than under the solvency ratio ABM for both symmetric and asymmetric designs (Table 2).

The effect of mortality on the expected value of the fund is straightforward for the symmetric case without ABMs, depicted in Figure 4. The expected value in the Up (Down) scenario enters a state of systematic debt (surplus), whereas with the introduction of an ABM, the expected value of the fund approaches zero. In Case 1, the opposite pattern can be clearly observed through the expected value of the buffer fund depicted in Figure 4.

In the asymmetric case, Figure 5, ABMs increase the expected value of the fund. If longevity increases but current mortality rates are used in the pension calculation, the amount of the initial pension increases. This results in higher pension expenditures and a corresponding decrease in the value of the fund. The ABMs consequently decrease the notional factor to establish liquidity or solvency. Finally, we observe that asymmetric mechanisms for Cases 1 and 2, the expected value of the fund increases more than that of its symmetric counterpart because the much lower average notional factor systematically increases the size of the buffer fund. Note that the final values of the expected value of the fund in the asymmetric design approach 2, which means that the value of the fund is two times greater than the value of contributions at the same time. This value implies that asymmetric ABM may be too conservative and lead to excessive capital accumulation under our assumptions.

5.2   Baby boom

To evaluate a demographic shock, we examine the impact of one-time positive exogenous shock that affects the entrant population at time t. This shock is denoted by δ and is assumed to have a value of 10.5 percent. The effects of this shock are illustrated by a so-called baby boom case.

This model is based on a stationary stochastic process, which means that one-time shocks can only occur exogenously. We included a positive shock, which replicates the effect of a fertility boost as indicated by the phenomenon's name, baby boom. We evaluate the financial health of the pension system after a baby boom and observe the ABM reaction. Figure 6 indicates the expected value of the notional factor and Figure 7 indicates the ratio of the fund to the contribution level for the symmetric design.[19]

We observe that the notional factor increases substantially from the average value of approximately 1.018 due to the sudden entrant population increase, which increases the total covered salary bill substantially. Once the baby boom generation retires, there is no longer an effect on the notional factor because the entry population returns to the relative stationary level after the exogenous shock. Note that the notional factor in absence of ABM does not depend on the retired population but on the contributors, which explains that the level of the notional factor goes back to the pre-shock level once that the shocked generation retires.

The liquidity ratio balance mechanism increases the notional rate during the first two periods due to the surplus created by the higher relative contribution level. However, once the baby boom generation retires, this factor decreases significantly to avoid debt.

The solvency ratio balance mechanism responds differently to this shock due to its construction. When the baby boom generation enters the pension system, turnover duration increases because it assumes that the future population will have the same composition, that is, it behaves as if the current shock is permanent. This anticipation increases the level of the notional factor affected by the solvency ratio during the first period, which affects both contributors and retirees. This first leads to debt as the expenditures increase above its sustainable levels. This increase affects the solvency balance mechanism during the second period, which decreases the notional factor. The decrease is sufficient to restore a surplus level, which will positively affect the notional factor. The notional rate affected by both balance mechanisms approaches the level in absence of adjustments once the shock generation exits the pension system.

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[19]The asymmetric case results are omitted because the explanation is similar to the non-baby boom case.

In absence of ABMs, the system only partially regulates itself after the shock population exits the system, as indicated by the triangle in Figure 7. For instance, in Case 1 of the longevity base scenario, we observe that the triangle occurs because pension expenditures equal contributions as observed in the expression (3.11). The pre-contribution fund equals 0 during the second period because we assume an initial fund of 0. The level of the notional rate, although higher than its normal levels, is insufficient to attain fund neutrality. Therefore, a surplus occurs. However, this surplus will be used to pay the pensions to the higher number of baby boom pensioners until they leave the pension system, decreasing the fund to a level approaching but not reaching zero. This interpretation is independent of the market interest rate and is due to the simple nature of the shock.

Finally, our results are robust to alternative assumptions.[20] For simplicity, we provide only some of the representative results.

6   Conclusion

This paper describes how liquidity and solvency indicators in notional defined contribution accounts are affected by fluctuations in economic and demographic conditions in a stochastic environment. The analysis uses a four-generation model in which two generations are contributors and two generations are pensioners. The consideration of four cohorts introduces heterogeneity to the pension system while maintaining a tractable dependence structure. Furthermore, we can draw interesting conclusions without the drawback of heavier computation.

Under this scenario, the notional rate, defined as the rate of increase of the covered salary bill (Börsch-Supan 2006), that indexes the accumulated notional capital is affected by both salary and population processes. The pension is revalued by an adjusted indexation process that guarantees longitudinal equilibrium among the cohorts inspired by Alho et al. (2013). However, this equilibrium, also referred as actuarial fairness, does not guarantee liquidity or solvency of the pension system in a dynamic framework. Consequently, some ABMs should be triggered.

An ABM is a predetermined measure established by law to be applied immediately according to an indicator, such as solvency or liquidity (Vidal-Meliá et al. 2010). The purpose of an ABM is to allow the pension system to adapt to financial turbulence without legislative intervention. This paper considers two ABMs. The first mechanism makes income from contributions equal pension expenditures (liquidity ratio ABM), and the second mechanism, based on the current Swedish adjustment, makes the assets of the pension system equal to liabilities (solvency ratio ABM). The results are evaluated in terms of the expected values and variances of the notional factor and buffer fund and are calculated under both symmetric and asymmetric designs. In most of cases, the introduction of an ABM reduces the volatility of the fund. The solvency ratio ABM yields the lowest value of the notional factor's variance and the lowest expected value.

The paper also indicates how longevity risk affects our analysis. It might be beneficial to consider longevity trends to calculate annuities. We find that there are clear advantages of introducing a symmetric ABM in the case of unanticipated longevity increase. This adjustment would avoid debt accumulation in absence of ABMs. Furthermore, note that asymmetric ABMs can lead to significant capital accumulation after two population renewals. This is consistent with the fact that an 'NDC scheme does little to prevent significant asset accumulation (...) on average' as argued by Auerbach and Lee (2006).

These conclusions are robust to a one-time exogenous demographic shock. Additionally, the notional system that uses the rate of increase of the contribution base as the compounding factor can regulate itself almost completely in presence of such a shock, that is, once the shock generation exists the pension system, the level of debt or surplus approaches the level in absence of a shock. The choice of automatic mechanism depends on the preferences of the government in terms of expected value, variance of the notional rate and level of the buffer fund.

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[20]We also addressed the possibility that contributors aged 2 earn less than contributors aged 1 do. Furthermore, we considered the possibility of a negative notional rate with a negative sum of g and n. Finally, we examined a case with negative parameter R as well as a case with a negative exogenous shock δ.

Finally, based on the two ABMs presented in this paper, at least three important directions for future research can be identified:

- To identify the ABM preferences of pension system participants and legislators as well as of contributors and pensioners. According to Barr & Diamond (2011), the current Swedish balancing mechanism has the undesirable consequence of favouring workers over retirees.

- To present an ABM that smooths volatility over the period and partially transfers to future system participants, and to compare participant and legislator preferences towards this ABM.

- To extend the four OLG model to a real society and assess the impact of economic, financial and demographic conditions on the ABM.

References

J. Alho, J. Bravo, and E. Palmer. Annuities and life expectancy in ndc. In R. Holzmann, E. Palmer, and D. Robalino, editors, *Nonfinancial Defined Contribution Pension Schemes in a Changing Pension World - Volume 2: Gender, Politics, and Financial Stability*, chapter 22, pages 395–443. The World Bank, Washington, D.C., 2013. ISBN 978-0-8213-8852-5.

J. Alonso-García. *Notional (non-financial) defined contributions: essays on sustainability and pension adequacy*. PhD thesis, UCL, 2015.

A. Auerbach, L. Kueng, and R. Lee. Propagation and smoothing of shocks in alternative social security systems. Technical report, National Bureau of Economic Research, 2013.

A. J. Auerbach and R. Lee. Notional defined contribution pension systems in a stochastic context: design and stability. In J. R. Brown, J. B. Liebman, and D. A. Wise, editors, *Social Security Policy in a Changing Environment*, page 38. University of Chicago Press, Chicago, 2006. ISBN 9780226076485.

A. J. Auerbach and R. Lee. Welfare and generational equity in sustainable unfunded pension systems. *Journal of public economics*, 95(1):16–27, 2011.

N. Barr and P. A. Diamond. Improving sweden's automatic pension adjustment mechanism. *Center for Retirement Research Brief*, 11(2), 2011.

M. Belloni and C. Maccheroni. Actuarial fairness when longevity increases: An evaluation of the italian pension system. *Geneva Papers on Risk and Insurance-Issues and Practice*, 38(4):638–674, 2013. ISSN 1018-5895; 1468-0440. doi: 10.1057/gpp.2013.27.

J. Bhattacharya and S. Russell. Two-period cycles in a three-period overlapping generations model. *Journal of Economic Theory*, 109(2):378–401, 2003.

M. d. C. Boado-Penas and C. Vidal-Meliá. The actuarial balance of the payg pension system: the swedish ndc model versus the db-type model. In R. Holzmann, E. Palmer, and D. Robalino, editors, *Nonfinancial Defined Contribution Pension Schemes in a Changing Pension World - Volume 2: Gender, Politics, and Financial Stability*, chapter 23, pages 443–485. The World Bank, Washington, D.C., 2013. ISBN 978-0-8213-9478-6.

M. d. C. Boado-Penas and C. Vidal-Meliá. Nonfinancial defined contribution pension schemes: is a survivor dividend necessary to make the system balanced? *Applied Economics Letters*, 21(4): 242–247, 2014. ISSN 1350-4851.

M. d. C. Boado-Penas, S. Valdés-Prieto, and C. Vidal-Meliá. The actuarial balance sheet for pay-as-you-go finance: Solvency indicators for spain and sweden. *Fiscal Studies*, 29(1):89–134, Mar. 2008. ISSN 0143-5671. doi: 10.1111/j.1475-5890.2008.00070.x.

Boards of Trustees of the Federal Old-Age and Survivors Insurance and Disability Insurance Trust Funds. The 2015 annual report. Technical report, U.S. Government Printing Office, Washington, D.C., 2015.

A. Bommier and R. D. Lee. Overlapping generations models with realistic demography. *Journal of Population Economics*, 16(1):135–160, Feb. 2003. ISSN 0933-1433. doi: 10.1007/s001480100102.

A. Börsch-Supan. What are ndc systems? what do they bring to reform strategies? In R. Holzmann and E. Palmer, editors, *Pension Reform: Issues and Prospects for Notional Defined Contribution (NDC) Schemes*, chapter 3, pages 35–57. The World Bank, Washington, D.C., 2006. ISBN 0-8213-6038-8.

A. Börsch-Supan, A. Reil-Held, and C. B. Wilke. How to make a defined benefit system sustainable: the" sustainable factor" in the german benefit indexation formula. *MEA Discussion Papers*, 37, 2004.

G. Brunello. The effects of cohort size on european earnings. *Journal of Population Economics*, 23 (1):273–290, 2009. ISSN 09331433. doi: 10.1007/s00148-009-0250-y.

A. Chłón-Domińczak, D. Franco, and E. Palmer. The first wave of ndc reforms: The experiences of italy, latvia, poland, and sweden. In R. Holzmann, E. Palmer, and D. Robalino, editors, *Nonfinancial Defined Contribution Pension Schemes in a Changing Pension World - Volume 1: Progress, Lessons, and Implementation*, chapter 2, pages 31–85. The World Bank, Washington, D.C., 2012. ISBN 978-0-8213-8852-5.

A. C. D'Addio and E. R. Whitehouse. Towards financial sustainability of pensions systems: the role of the automatic-adjustment mechanisms in oecd and eu countries. Technical Report 8, Federal Social Insurance Office, Switzerland, 2012.

P. A. Diamond. A framework for social security analysis. *Journal of Public Economics*, 8(3):275–298, 1977. ISSN 00472727. doi: 10.1016/0047-2727(77)90002-0.

P. Fajnzylber and D. A. Robalino. Assessing fiscal costs and pension distribution in transitions to defined contribution systems: A retrospective analysis for chile. In R. Holzmann, E. Palmer, and D. A. Robalino, editors, *Nonfinancial Defined Contribution Pension Schemes in a Changing Pension World - Volume 1: Progress, Lessons, and Implementation*, chapter 9, pages 281–316. The World Bank, Washington, D.C., 2012. ISBN 978-0-8213-8848-8.

S. Gronchi and S. Nisticò. Implementing the ndc theoretical model: A comparison of italy and sweden. In R. Holzmann and E. Palmer, editors, *Pension Reform: Issues and Prospects for Notional Defined Contribution (NDC) Schemes*, chapter 19, pages 493–519. The World Bank, Washington, D.C., 2006. ISBN 0-8213-6038-8.

R. Holzmann. Towards a coordinated pension system in europe: Rationale and potential structure. In R. Holzmann and E. Palmer, editors, *Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*, chapter 11, pages 225–292. The World Bank, Washington, D.C., 2006. ISBN 0-8213-6038-8.

R. Holzmann, E. Palmer, and D. A. Robalino, editors. *Nonfinancial Defined Contribution Pension Schemes in a Changing Pension World - Volume 1: Progress, Lessons, and Implementation*. The World Bank, Washington, D.C., 2012. ISBN 978-0-8213-8848-8.

M. Knell. How automatic adjustment factors affect the internal rate of return of payg pension systems. *Journal of Pension Economics and Finance*, 9(01):1–23, 2010. ISSN 1474-7472. doi: 10.1017/S1474747209990035.

Office of the Superintendent of Financial Institutions Canada (OSFIC). Optimal funding of the canada pension plan. actuarial study n. 6. Technical report, Office of the Chief Actuary, 2007.

Office of the Superintendent of Financial Institutions Canada (OSFIC). Actuarial report (23rd) on the canada pension plan. Technical report, Office of the Chief Actuary, 2008.

E. Palmer. What's ndc? In R. Holzmann and E. Palmer, editors, *Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*, chapter 2, pages 17–35. The World Bank, Washington, D.C., 2006. ISBN 0-8213-6038-8.

E. Palmer. Generic ndc: Equilibrium, valuation, and risk sharing with and without ndc bonds. In R. Holzmann, E. Palmer, and D. Robalino, editors, *Nonfinancial Defined Contribution Pension Schemes in a Changing Pension World - Volume 2: Gender, Politics, and Financial Stability*, chapter 10, pages 309–343. The World Bank, Washington, D.C., 2013. ISBN 978-0-8213-8852-5.

A. Papoulis. *Probability, Random Variables, and Stochastic Processes*. McGraw-Hill International Editions, New York, 1991. ISBN 0-07-100870-5.

M. Queisser and E. R. Whitehouse. Neutral or fair?: actuarial concepts and pension-system design. OECD Social, Employment and Migration Working Papers, no. 40, 2006.

J. Sakamoto. Japan's pension reform. The World Bank - Social Protection Discussion Paper, 2005.

P. Samuelson. An exact consumption loan model of interest with or without the social contrivance of money. *Journal of Political Economy*, 66(6):467–482, 1958.

O. Settergren. A decade of actuarial accounting for the ndc scheme in sweden: Quantifying change in the financial position of a payg pension plan. In R. Holzmann, E. Palmer, and D. A. Robalino, editors, *Non-Financial Defined Contribution (NDC) Pension Systems: Progress and New Frontiers in a Changing Pension World: Gender, Politics, and Financial Stability*, chapter 2, pages 361–395. The World Bank, Washington, D.C., 2013. ISBN 978-0-8213-9478-6.

O. Settergren and B. D. Mikula. The rate of return of pay-as-you-go pension systems: a more exact consumption-loan model of interest. *Journal of Pension Economics and Finance*, 4(2):115–138, July 2005. ISSN 1474-7472. doi: 10.1017/S1474747205002064.

Swedish Pension Agency. Orange report - annual report of the swedish pension system 2014. Technical report, Swedish Social Insurance Agency - Försäkringskassan, Stockholm, 2015.

S. Valdés-Prieto. The financial stability of notional account pensions. *Scandinavian Journal of Economics*, 102(3):395–417, Sept. 2000. ISSN 0347-0520. doi: 10.1111/1467-9442.03205.

I. Vanovska. Pension reform in latvia. In E. Fultz, editor, *Pension reform in the Baltic States*, chapter 2, pages 143–266. International Labour Office, Budapest, 2006. ISBN 92-2-118385-8.

C. Vidal-Meliá and M. d. C. Boado-Penas. Compiling the actuarial balance for pay-as-you-go pension systems. is it better to use the hidden asset or the contribution asset? *Applied Economics*, 45(10): 1303–1320, 2013.

C. Vidal-Meliá, M. d. C. Boado-Penas, and O. Settergren. Automatic balance mechanisms in pay-as-you-go pension systems. *The Geneva Papers on Risk and Insurance Issues and Practice*, 34(2): 287–317, Apr. 2009. ISSN 1018-5895. doi: 10.1057/gpp.2009.2.

C. Vidal-Meliá, M. d. C. Boado-Penas, and O. Settergren. Instruments for improving the equity, transparency and solvency of payg pension systems. ndcs, abs and abms. In M. Micocci, G. N. Gregoriou, and G. B. Masala, editors, *Pension Fund Risk Management. Financial and Actuarial Modelling*, chapter 18, pages 419–473. Chapman and Hall, New York, 2010. ISBN 1439817520.

C. Vidal-Meliá, M. d. C. Boado-Penas, and F. Navarro-Cabo. Notional defined contribution pension schemes: why does only sweden distribute the survivor dividend? *Journal of Economic Policy Reform*, (ahead-of-print):1–21, 2015.

Table 1: Population evolution in terms of the entry age $x = 1$

Age[a]\ Time	t	$t+1$	$t+2$	$t+3$
1	N_t^1	$N_t^1(1+n_{t+1})$	$N_{t+1}^1(1+n_{t+2})$	$N_{t+2}^1(1+n_{t+3})$
2	N_{t-1}^1	N_t^1	$N_t^1(1+n_{t+1})$	$N_{t+1}^1(1+n_{t+2})$
3	N_{t-2}^1	N_{t-1}^1	N_t^1	$N_t^1(1+n_{t+1})$
4	$p_t N_{t-3}^1$	$p_{t+1} N_{t-2}^1$	$p_{t+2} N_{t-1}^1$	$p_{t+3} N_t^1$

[a]*Source:* the authors.
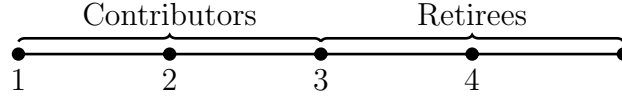
Figure 1: A four OLG framework at time t. The figure illustrates the four-period OLG dynamics at time t when two generations of contributors and two generations of pensioners coexist.

Figure 2: The expected value of the notional factor after the introduction of an ABM for the symmetric case: without ABM (dotted black line), liquidity ratio ABM (dark gray squares) and solvency ratio ABM (light gray rhombuses). *Note:* The first row of the graphic corresponds to Case 1 and the second row to Case 2. *Source:* the authors.
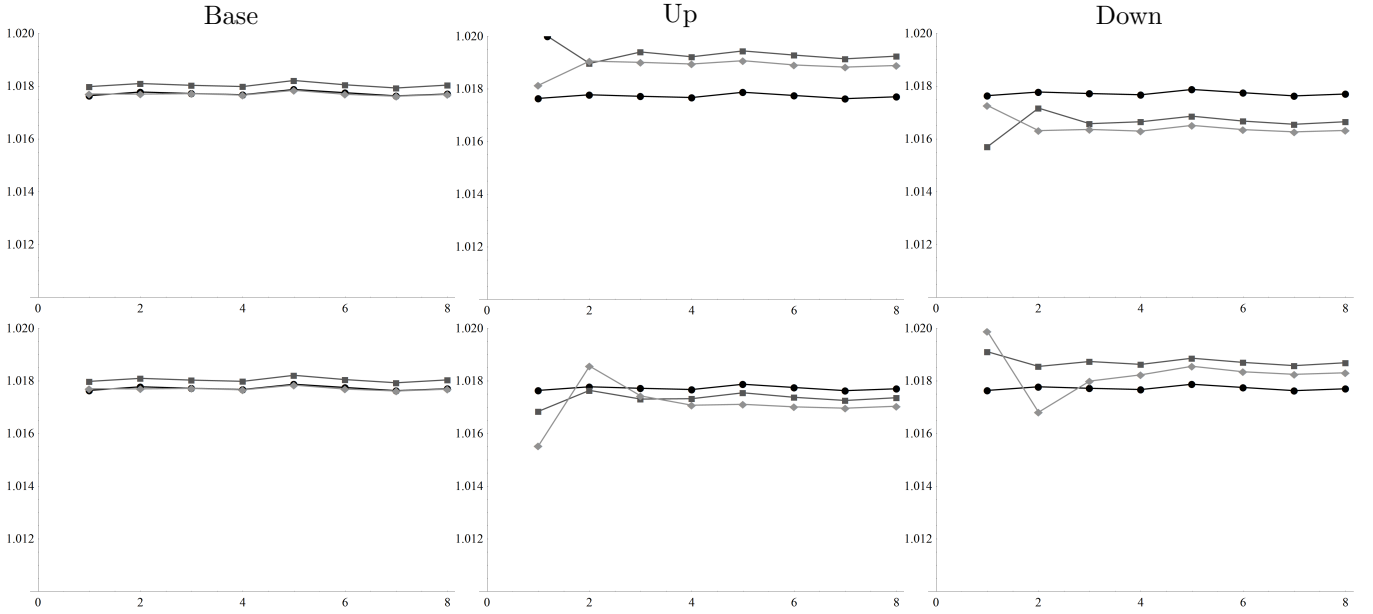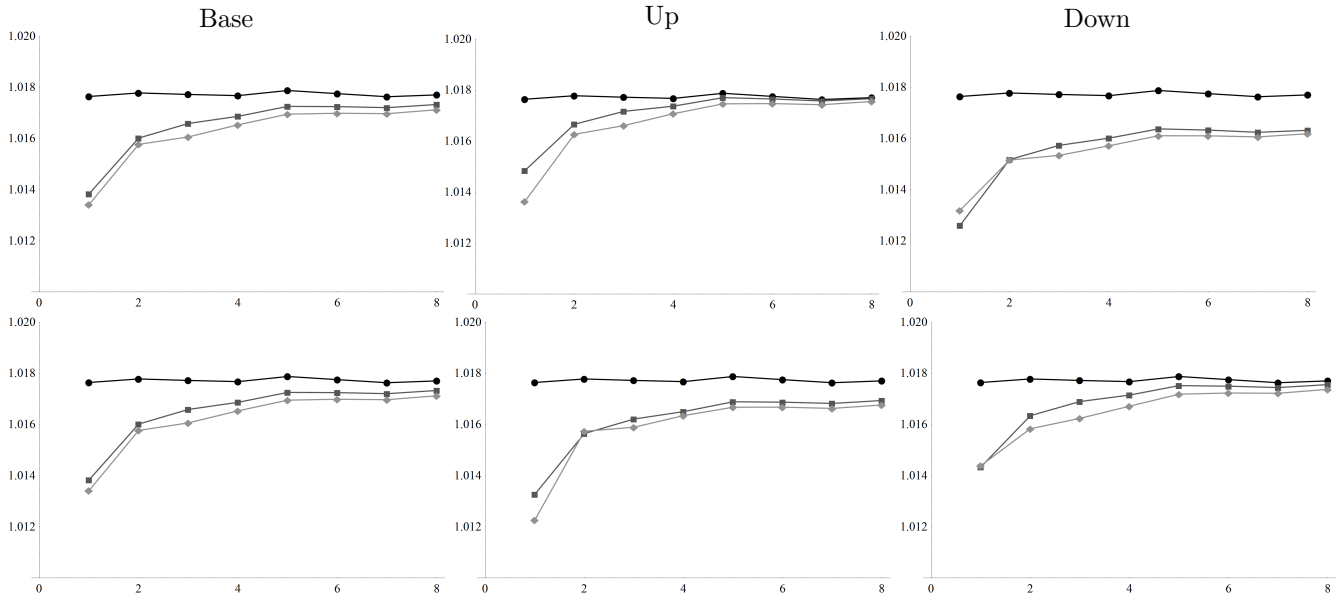
Figure 3: The expected value of the notional factor after the introduction of an ABM for the non-symmetric case: without ABM (dotted black line), liquidity ratio ABM (dark gray squares) and solvency ratio ABM (light gray rhombuses). *Note:* The first row of the graphic corresponds to Case 1 and the second row to Case 2. *Source:* the authors.
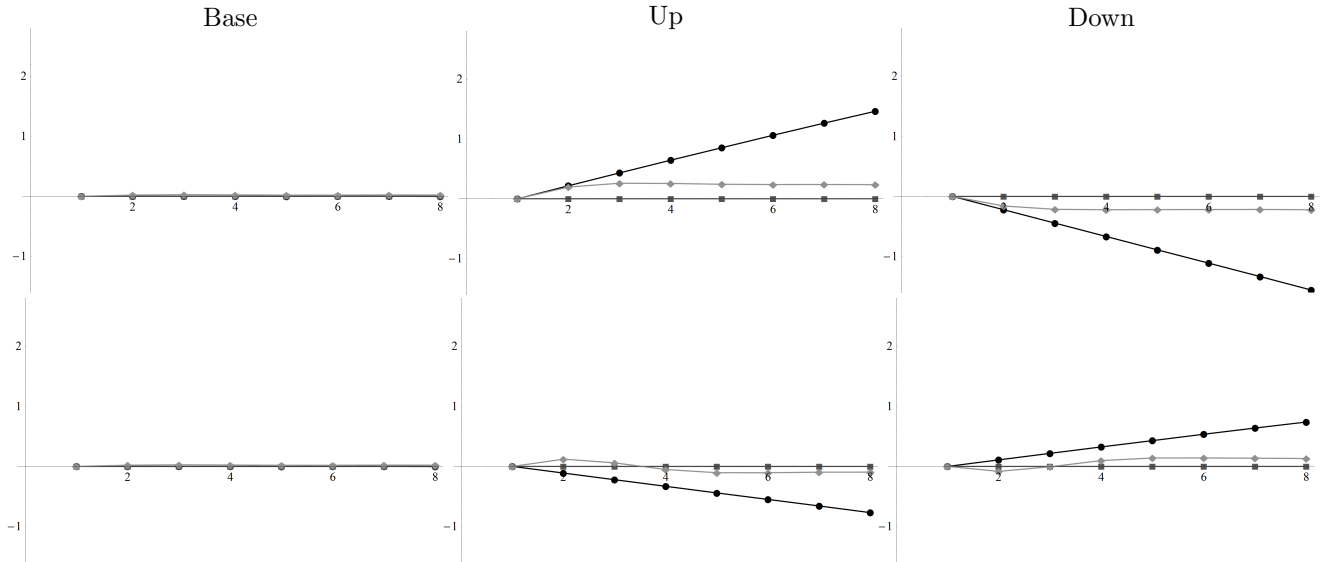

Figure 4: Expected value of the ratio between the fund and contributions after the introduction of an ABM for the symmetric case: without ABM (dotted black line), liquidity ratio ABM (dark gray squares) and solvency ratio ABM (light gray rhombuses). *Note:* The first row of the graphic corresponds to Case 1 and the second row to Case 2. *Source:* the authors.
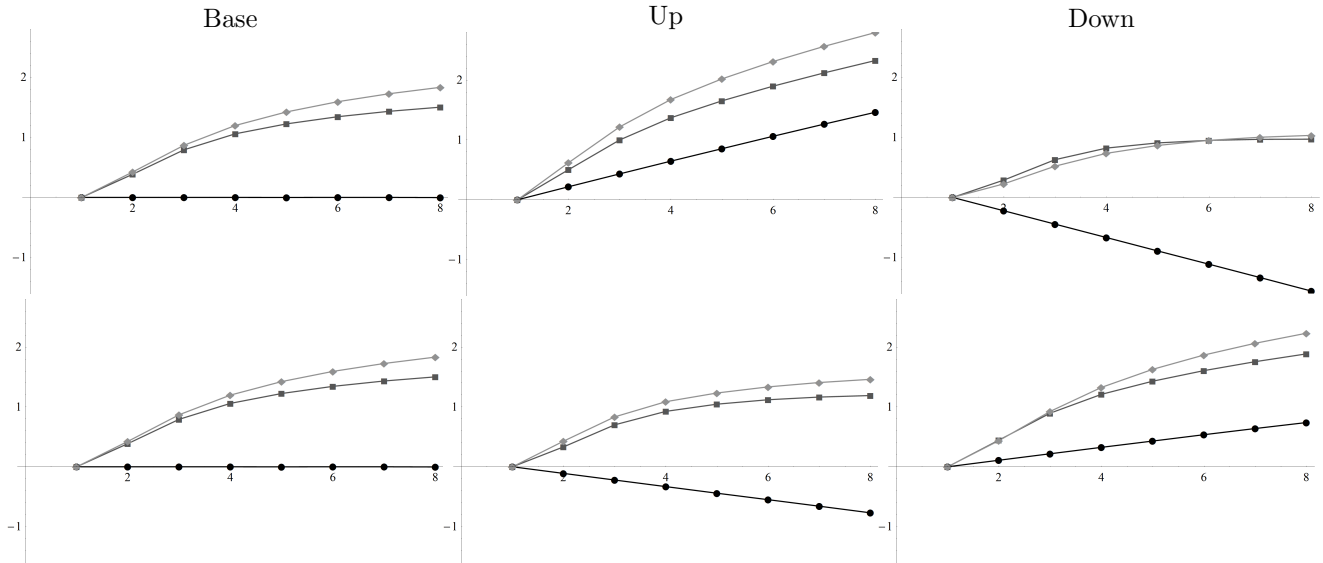
Figure 5: Expected value of the ratio between the fund and contributions after the introduction of an ABM for the non-symmetric case: without ABM (dotted black line), liquidity ratio ABM (dark gray squares) and solvency ratio ABM (light gray rhombuses). *Note:* The first row of the graphic corresponds to Case 1 and the second row to Case 2. *Source:* the authors.

Figure 6: Expected Value of the notional factor after a baby boom with an ABM: no ABM (dotted black line), liquidity ratio ABM (dark gray squares) and solvency ratio ABM (light gray rhombuses). *Note:* The first row of the graphic corresponds to Case 1 and the second row to Case 2. *Source:* the authors.
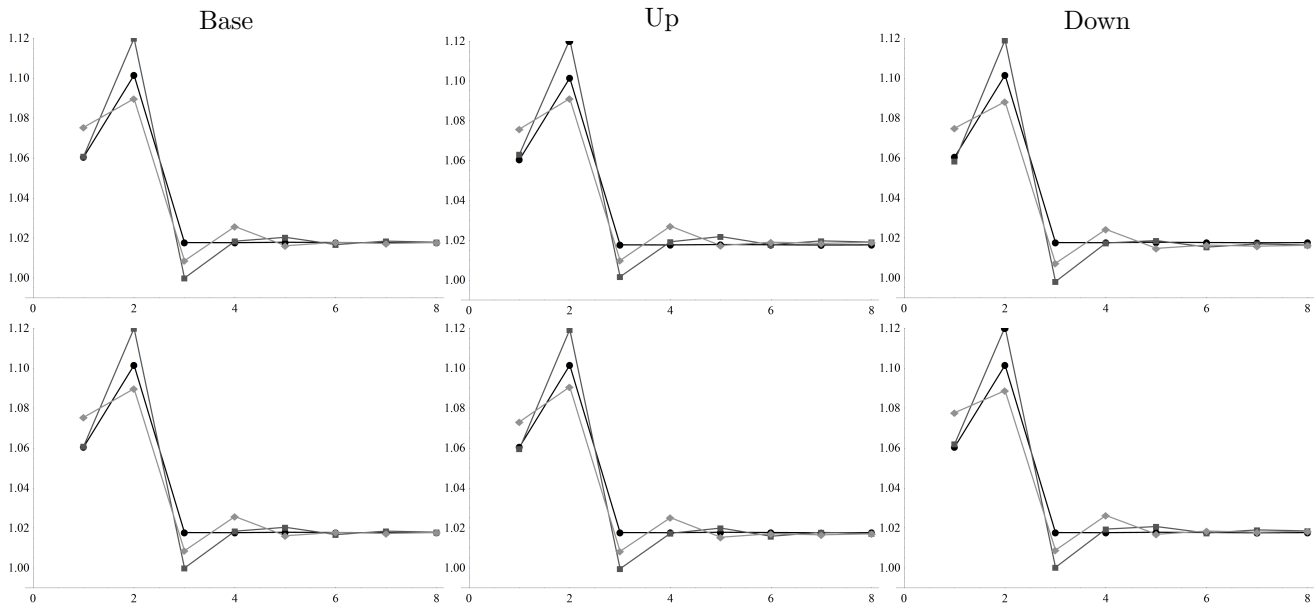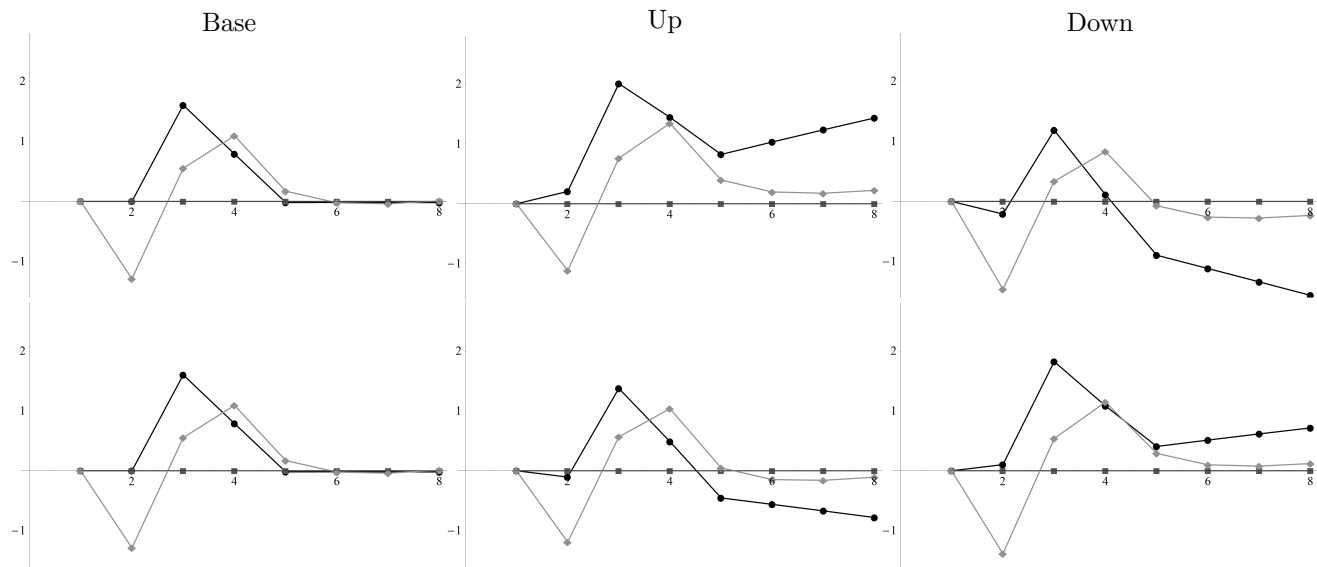
Figure 7: Expected Value the ratio between the fund and contributions after a baby boom with an ABM: no ABM (dotted black line), liquidity ratio ABM (dark gray squares) and solvency ratio ABM (light gray rhombuses). *Note:* The first row of the graphic corresponds to Case 1 and the second row to Case 2. *Source:* the authors.

A   Derivation of the longitudinal adjustment factor

The notional capital is transformed into an annuity at retirement. Therefore, some assumptions regarding the future must be made to calculate annuities. However, observed values rarely coincide with the expected values. In this case, longitudinal equilibrium is not reached in general, that is, individuals receive less or more than what they contributed. Forecasted liabilities (calculated by considering the present value of future cash flows) are then not equal to accrual liabilities (calculated by considering the compounded value of past cash flows).

This appendix shows that adjustments to the pension indexation can be made in order to attain this longitudinal equilibrium. However, it implies that indexation of pensions in practice differs among retired cohorts, which may not be desirable when various generations coexist. The processes r_t^*, p_t^* and λ_t^* represent respectively the theoretical notional rate, mortality rate and indexation of pensions for period t. The processes r_t, p_t and λ_t denote, respectively, the observed notional factor, which could be affected by some ABM B_t^z, mortality rate and indexation rate at time t. In this framework, the annuity is given by (3.8).

The aim is to produce equivalence between accrual and forecasted liabilities for all cohorts, in particular, for the cohort aged 3 at time t. The accrual liability $V_t^{3,A}$ is denoted by (A.1) and the forecasted liability $V_t^{3,F}$ is denoted by (A.2). The adjustment factor at time $t+1$ by A_{t+1} is represented as follows:

$$V_t^{3,A} = NPS_t^3 \tag{A.1}$$

$$V_t^{3,F} = P_t^3 N_t^3 + P_t^3 N_{t+1}^4 \frac{1+\lambda_{t+1}}{1+r_{t+1}} A_{t+1}$$

$$= NPS_t^3 \frac{1}{a_t} \left(1 + p_{t+1} \frac{1+\lambda_{t+1}}{1+r_{t+1}} A_{t+1} \right) \tag{A.2}$$

The following adjustment factor makes both liabilities equivalent:

$$A_{t+1} = \left(\frac{1+r_{t+1}}{1+\lambda_{t+1}} \right) \frac{1}{p_{t+1}} \mathbb{E}_t \left[p_{t+1}^* \left(\frac{1+\lambda_{t+1}^*}{1+r_{t+1}^*} \right) \right] \tag{A.3}$$

When the mortality, demographic and economic processes are independent, the adjustment factor A_{t+1} (A.3) can be rewritten as follows:

$$A_{t+1} = \underbrace{\frac{\mathbb{E}_t \left[\frac{1+\lambda_{t+1}^*}{1+r_{t+1}^*} \right]}{\frac{1+\lambda_{t+1}}{1+r_{t+1}}}}_{\text{Economic adjustment}} \underbrace{\frac{\mathbb{E}_t \left[p_{t+1}^* \right]}{p_{t+1}}}_{\text{Longevity adjustment}} \tag{A.4}$$

The economic adjustment affects all retirees because the calculation only considers the difference between the theoretical and the observed notional and indexation rates. However, the second part, which corresponds to the longevity adjustment, would differ by retired cohort because it depends on the theoretical mortality at the time of annuity calculation and the experienced mortality of the same cohort during retirement.

B   Derivation of the accrual and forecasted liabilities

In this section, the accrual and forecasted liabilities are calculated for each cohort at time t. The accrual (forecasted) liabilities for the cohort aged x at time t will be denoted by $V_t^{x,A}$ ($V_t^{x,F}$). These liabilities are calculated before any payment is made.

The following are the liabilities for the contributing cohort aged 1:

$$V_t^{1,A} = 0 \times N_t^1 = 0 \tag{B.1}$$

$$V_t^{1,F} = \frac{P_{t+2}^3 N_{t+2}^3}{(1+r_{t+1})(1+r_{t+2})} + \frac{P_{t+2}^3 p_{t+3} N_{t+2}^3 (1+\lambda_{t+3}) A_{t+3}}{(1+r_{t+1})(1+r_{t+2})(1+r_{t+3})}$$

$$- \pi S_t^1 N_t^1 - \frac{\pi S_{t+1}^2 N_{t+1}^2}{1+r_{t+1}}$$

$$= \frac{NPS_{t+2}^3}{(1+r_{t+1})(1+r_{t+2})} \frac{1 + p_{t+3}\frac{1+\lambda_{t+3}}{1+r_{t+3}} A_{t+3}}{a_{t+2}} - \frac{NPS_{t+2}^3}{(1+r_{t+1})(1+r_{t+2})} = 0 \tag{B.2}$$

The following are liabilities for the contributing cohort aged 2:

$$V_t^{2,A} = \pi S_{t-1}^1 N_{t-1}^1 (1+r_t) \tag{B.3}$$

$$V_t^{2,F} = \frac{P_{t+1}^3 N_{t+1}^3}{1+r_{t+1}} + \frac{P_{t+1}^3 p_{t+2} N_{t+1}^3 (1+\lambda_{t+2}) A_{t+2}}{(1+r_{t+1})(1+r_{t+2})} - \pi S_t^2 N_t^2$$

$$= \frac{NPS_{t+1}^3}{1+r_{t+1}} \frac{1 + p_{t+2}\frac{1+\lambda_{t+2}}{1+r_{t+2}} A_{t+2}}{a_{t+1}} - \pi S_t^2 N_t^2$$

$$= \pi S_{t-1}^1 N_{t-1}^1 (1+r_t) \tag{B.4}$$

The following are liabilities for the retired cohort aged 3:

$$V_t^{3,A} = NPS_t^3 \tag{B.5}$$

$$V_t^{3,F} = P_t^3 N_t^3 + P_t^3 (1+\lambda_{t+1}) A_{t+1} N_{t+1}^4 \frac{1}{1+r_{t+1}}$$

$$= NPS_t^3 \frac{1 + p_{t+1}\frac{1+\lambda_{t+1}}{1+r_{t+1}} A_{t+1}}{a_t} = NPS_t^3 \tag{B.6}$$

Finally, the following are the liabilities for the retired cohort aged 4:

$$V_t^{4,A} = \left(NPS_{t-1}^3 - P_{t-1}^3 N_{t-1}^3\right)(1+r_t) = NPS_{t-1}^3 \frac{a_{t-1}-1}{a_{t-1}} \tag{B.7}$$

$$V_t^{4,F} = P_t^4 N_t^4 = P_{t-1}^3 p_t N_{t-1}^3 (1+\lambda_t) A_t = NPS_{t-1}^3 \frac{a_{t-1}-1}{a_{t-1}} \tag{B.8}$$

C   Theoretical framework

Introducing ABM in the pension scheme changes the formulae presented in Section 3. The superscript z denotes the kind of ABM that is implemented, with $z = LR$ representing the liquidity ratio and $z = SR$ representing the solvency ratio. Note that the notional factor and the income from contributions are not affected by the ABMs.

$$NPS_t^{3,z} = C_t B_t^z K_t^{N,z} \tag{C.1}$$

$$P_t^z = C_t B_t^z K_t^{P,z} \tag{C.2}$$

$$F_t^{-,z} = F_{t-1}^{+,z}(1+i_t) = F_0^- \prod_{j=1}^{t}(1+i_j) + \sum_{j=0}^{t-1}(C_j - P_j^z)\prod_{k=j+1}^{t}(1+i_k) \tag{C.3}$$

$$CA_t^z = C_t TD_t^z \tag{C.4}$$

$$V_t^z = C_t B_t^z K_t^{V,z} \tag{C.5}$$

$$A_t^z = \frac{\mathbb{E}_{t-1}\left[p_t^* \frac{1+\lambda_t^*}{1+r_t^*}\right]}{p_t \frac{1+\lambda_t}{1+r_t}} B_t^z \tag{C.6}$$

where

$$K_t^{N,z} = \frac{S_0^1(1+n_{t-2})}{K_{t-2}^C}B_{t-1}^z + \frac{S_0^2}{K_{t-1}^C} \tag{C.7}$$

$$K_t^{P,z} = \frac{K_t^{N,z}}{a_t} + B_{t-1}^z K_{t-1}^{N,z}\frac{a_{t-1}-1}{a_{t-1}} \tag{C.8}$$

$$TD_t^z = 2 + B_{t-1}^z \frac{K_{t-1}^{N,z}}{K_t^{P,z}}\frac{a_{t-1}-1}{a_{t-1}} - \frac{S_0^2}{K_t^C} \tag{C.9}$$

$$K_t^{V,z} = 1 + B_{t-1}^z\left\{\frac{S_0^1(1+n_{t-2})}{K_{t-2}^C} + K_{t-1}^{N,z}\frac{a_{t-1}-1}{a_{t-1}}\right\} \tag{C.10}$$

Due to the structure of the ABMs B_t^{LR} (4.1) and B_t^{SR} (4.2), it is not possible to specify their probability distributions. Therefore, the expected value and variance can't be calculated through analytical formulas. However, given two random variables X and Y and a function $h(X,Y)$, the expected value of this function can be expressed directly in terms of the transformation function $h(X,Y)$ and the joint density $f_{X,Y}(x,y)$ of X and Y: $\mathrm{E}[h(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(x,y)f_{X,Y}(x,y)dxdy$ (see Papoulis (1991)). Furthermore, if the two random variables X and Y are independent, the joint density is $f_{X,Y}(x,y) = f_X(x)f_Y(y)$. This property can be applied to our problem. In the remainder of the appendix we will present the recursive formulas used in our framework.

Denote the transformation of the multivariate random vector X_s at time $s > t$ for the ABM $z = LR, SR$ as $h_{t,s}^z(x_s)$. Note that t indicates the time when the ABM is first implemented. The entries' and salarys' rate of increase, $1+n_s$ and $1+g_s$ respectively, will be rewritten in the remainder of this section in terms of D_s and G_s respectively in order to shorten the lengthy notation. The following formulae presents the explicit dependence of (C.7-C.9) to the different demographic and salary processes D_s and G_s:

$$K_s^C = f_{K_s^C}(D_s) \tag{C.11}$$

$$K_s^{N,z} = f_{K_s^{N,z}}(D_{s-1}, D_{s-2}, h_{t,s-1}^z(x_{s-1})) \tag{C.12}$$

$$K_s^{P,z} = f_{K_s^{P,z}}(D_{s-1}, D_{s-2}, D_{s-3}, h_{t,s-1}^z(x_{s-1}), h_{t,s-2}^z(x_{s-2})) \tag{C.13}$$

$$TD_s^z = f_{TD_s^z}(D_s, D_{s-1}, D_{s-2}, D_{s-3}, h_{t,s-1}^z(x_{s-1}), h_{t,s-2}^z(x_{s-2})) \tag{C.14}$$

$$K_s^{V,z} = f_{K_s^{V,z}}(D_{s-2}, D_{s-3}, h_{t,s-1}^z(x_{s-1}), h_{t,s-2}^z(x_{s-2})) \tag{C.15}$$

$$I_s^z = f_{I_s^z}(D_s, D_{s-1}, G_s) \tag{C.16}$$

$$\frac{F_s^{-,z}}{C_s} = f_{t,s}^z = f_{f_{t,s}^z}(D_{t-1}, .., D_s, G_{t+1}, .., G_s) \tag{C.17}$$

Furthermore, the δ affecting the demographic process at time t represents a one-time exogenous shock affecting the population, as presented in Section 5. This shock will vary the notation of the notional factor during the first two periods t, $t+1$ as follows:

$$1 + r_t = G_t D_{t-1} \frac{S_0^1 D_t (1+\delta) + S_0^2}{S_0^1 D_{t-1} + S_0^2} \tag{C.18}$$

$$1 + r_{t+1} = G_{t+1} D_t (1+\delta) \frac{S_0^1 D_{t+1} + S_0^2}{S_0^1 D_t + S_0^2} \tag{C.19}$$

The notional factor for $j \geq t+2$ will be given by:

$$1 + r_j = G_j D_{j-1} \frac{S_0^1 D_j + S_0^2}{S_0^1 D_{j-1} + S_0^2} \tag{C.20}$$

Furthermore, we assume that the individual increments have zero expected value, i.e., $E[B_x(j) - B_x(k)] = 0$ for for $x = P, W$ and $j \neq k$, and, finally, that the increments of both processes are independent, that is:

$$E[(W^P(j) - W^P(k))(W^S(j) - W^S(k))] = 0 \text{ for } j \neq k.$$

These assumptions lead to $Cov(D_j, D_k) = 0$ and $Cov(D_j, G_k) = 0$ for $j \neq k$ and to a non-zero covariance when the processes interact during the same period $Cov(G_s, D_s) = e^{R+\gamma+\frac{\sigma_P^2+\sigma_W^2}{2}} (e^{\rho\sigma_P\sigma_W} - 1)$.

The ABM at time $s = t$ depends on the random variable vector $X_t = (D_{t-3}, D_{t-2}, D_{t-1})$ for $z = LR$ and $X_t = (D_{t-3}, D_{t-2}, D_{t-1}, D_t)$ for $z = SR$, and on the random variable vector $X_s = (D_{t-3}, ..., D_t, G_t, ..., G_s)$ for $s \geq t+1$ for both $z = LR, SR$ with $n = 2(s-t)+4$ length of the vector.

The following represents the joint distribution of a random vector X_s:

$$f_{X_s}(x) = \prod_{j=t-3}^{t} f_{D_j}(d_j) \prod_{j=t+1}^{s} f_{G_j, D_j}(g_j, d_j) \tag{C.21}$$

for $s \geq t+1$ where the distribution of the product of the demographic process D_s and salary process G_s $\forall s$ is denoted by $G_s D_s \sim logN(R + \gamma - \frac{\sigma_P^2+\sigma_W^2}{2}, \sigma_{P,W}^2)$ with $\sigma_{P,W}^2 = \sigma_P^2 + \sigma_W^2 + 2\rho\sigma_P\sigma_W$. Then the joint density function of (G_s, D_s) is represented as follows:

$$f_{G_s, D_s}(x, y) = \frac{1}{xy\sqrt{|\Sigma|}} e^{-\frac{1}{2|\Sigma|}\left((\log z - \mu)' \Sigma^{-1} (\log z - \mu)\right)} \text{ for } xy > 0 \tag{C.22}$$

with:

$$\log z = \begin{pmatrix} \log x \\ \log y \end{pmatrix}, \mu = \begin{pmatrix} R - \frac{\sigma_P^2}{2} \\ \gamma - \frac{\sigma_W^2}{2} \end{pmatrix} \tag{C.23}$$

$$\Sigma = \begin{pmatrix} \sigma_W & \rho\sigma_W\sigma_P \\ \rho\sigma_W\sigma_P & \sigma_P \end{pmatrix} \tag{C.24}$$

$$|\Sigma| = \text{determinant of variance-covariance matrix } \Sigma \tag{C.25}$$

Finally, the kth raw moment of the the ABM for $z = LR, SR$ is given by the following:

$$E[(B_s^z)^k] = E[(h_{t,s}^z(x_s))^k]$$
$$= \int_0^\infty ... \int_0^\infty (h_{t,s}^z(x_1, ..., x_n))^k f_X(x_1, ..., x_n) dx_1...dx_n \tag{C.26}$$

The transformation $h_{t,s}^z(x_s)$ when the ABM sought is symmetric is given by the following expressions:

$$h_{t,s}^{LR}(x_s) = \frac{1 + f_{t,s}^{LR}}{K_s^{P,LR}} \tag{C.27}$$

$$h_{t,s}^{SR}(x_s) = \frac{TD_s^{SR} + f_{t,s}^{SR}}{K_s^{V,LR}} \tag{C.28}$$

However, if the ABM is asymmetric, the transformation is given by the following:

$$h_{t,s}^{LR}(x_s) = \text{Min}\left[\frac{1 + f_{t,s}^{LR}}{K_s^{P,LR}}, 1\right] \tag{C.29}$$

$$h_{t,s}^{SR}(x_s) = \text{Min}\left[\frac{TD_s^{SR} + f_{t,s}^{SR}}{K_s^{V,LR}}, 1\right] \tag{C.30}$$

As denoted in the section 2, the formulas were developed for cases in which longitudinal equilibrium, also known as actuarial fairness, is a constraint. This constraint implies that indexation differs between retired cohorts according to the realized mortality and economic outcomes. Nevertheless, in practice, this solution is not politically viable. Therefore, we consider a second case without the actuarial fairness constraint by using current mortality data rather than projected mortality values. Throughout the paper, this is called Case 2. In this case, the expressions representing the outcome P_t^z (C.2), turnover duration TD_t^z (C.9) and liabilities V_t^z (C.5) slightly change as follows:

$$K_t^{P,z} = \frac{K_t^{N,z}}{a_t} + B_{t-1}^z K_{t-1}^{N,z} \frac{a_t - 1}{a_{t-1}} \tag{C.31}$$

$$TD_t^z = 2 + B_{t-1}^z \frac{K_{t-1}^{N,z}}{K_t^{P,z}} \frac{a_t - 1}{a_{t-1}} - \frac{S_0^2}{K_t^C} \tag{C.32}$$

$$K_t^{V,z} = 1 + B_{t-1}^z \left\{ \frac{S_0^1(1 + n_{t-2})}{K_{t-2}^C} + K_{t-1}^{N,z} \frac{a_t - 1}{a_{t-1}} \right\} \tag{C.33}$$

where $K_t^{N,z}$ is the same as in (C.7). Note that the main difference between the Case 1 and Case 2 is that the ratio $\frac{a_{t-1}-1}{a_{t-1}}$ becomes $\frac{a_t-1}{a_{t-1}}$. The calculation of the variances and expected values participants parallels Case 1.

D   Numerical results for the variance

This appendix provides the graphics representing the variance of the notional factor after the introduction of a symmetric ABM (Figure 8) and an asymmetric ABM (Figure 9) for Cases 1 and 2. The tables presenting the aggregate variance values for the notional factor (Table 2) and the ratio between the fund and income from contributions (Figure 3) are also presented.

Figure 8: Variance of the notional factor after the introduction of an ABM for the symmetric case: without ABM (dotted black line), liquidity ratio ABM (dark gray squares) and solvency ratio ABM (light gray rhombuses). *Note:* The first row of the graphic corresponds to Case 1 and the second row to Case 2. *Source:* the authors.
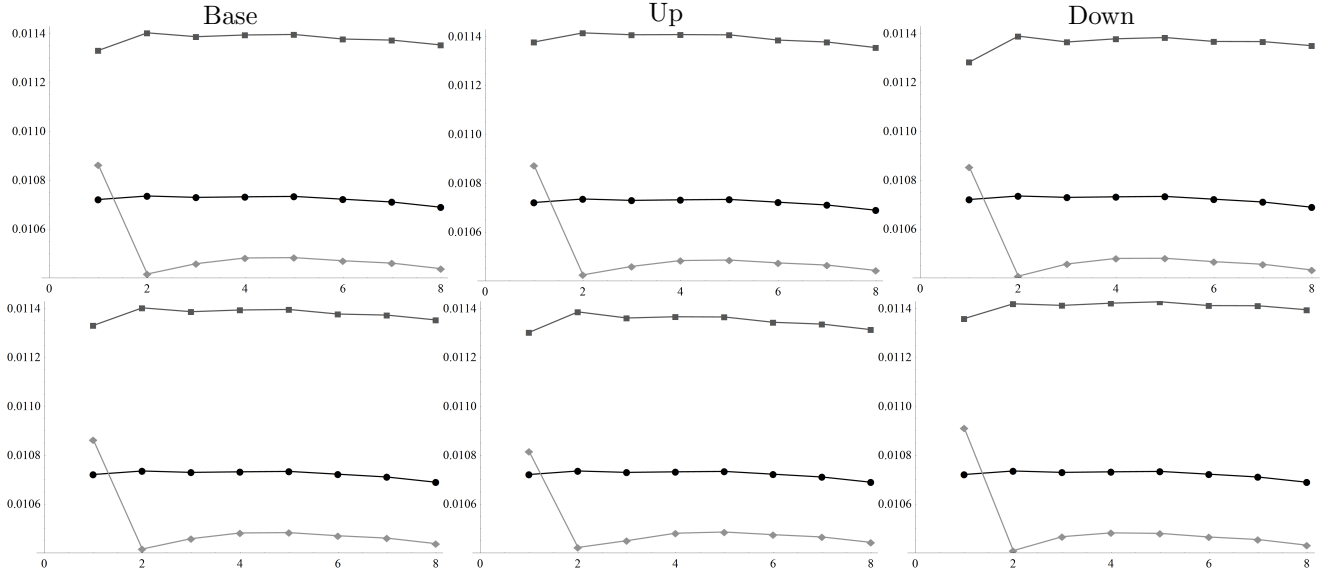

Figure 9: Variance of the notional factor after the introduction of an ABM for the non-symmetric case: without ABM (dotted black line), liquidity ratio ABM (dark gray squares) and solvency ratio ABM (light gray rhombuses). *Note:* The first row of the graphic corresponds to Case 1 and the second row to Case 2. *Source:* the authors.
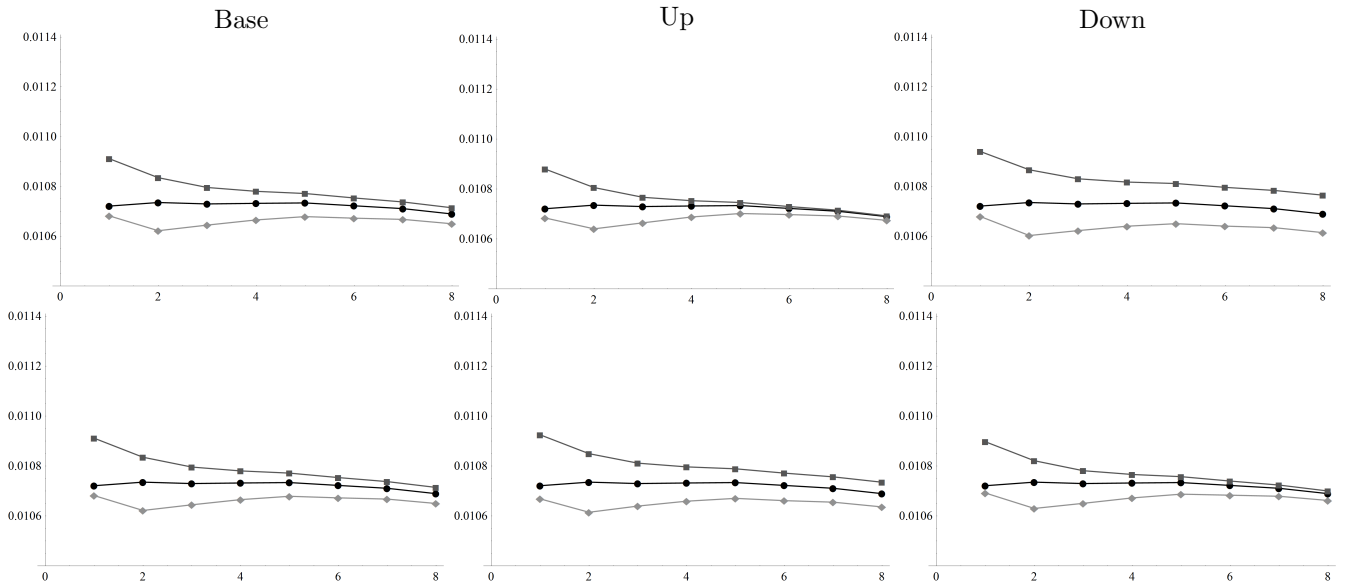
Table 2: The variance of the Notional Factor: Aggregated Values

	Case 1			Case 2		
	Base	Up	Down	Base	Up	Down
Symmetric - No baby boom						
No ABM	0.08578	0.08578	0.08578	0.08578	0.08578	0.08578
LR	0.09102	0.09113	0.09089	0.09102	0.09078	0.09126
BR	0.08408	0.08411	0.08404	0.08408	0.08404	0.08410
Symmetric- Baby boom						
No ABM	0.08843	0.08843	0.08843	0.08843	0.08843	0.08843
LR	0.09391	0.09403	0.09378	0.09391	0.09367	0.09416
BR	0.08674	0.08678	0.08670	0.08674	0.08670	0.08677
No Symmetric - No baby boom						
No ABM	0.08578	0.08578	0.08578	0.08578	0.08578	0.08578
LR	0.08630	0.08609	0.08661	0.08630	0.08644	0.08619
BR	0.08529	0.08545	0.08509	0.08529	0.08521	0.08536
No Symmetric - Baby boom						
No ABM	0.08843	0.08843	0.08843	0.08843	0.08843	0.08843
LR	0.08890	0.08869	0.08920	0.08890	0.08903	0.08879
BR	0.08776	0.08792	0.08757	0.08776	0.08768	0.08784

Table 3: The variance of the ratio between the fund and contributions: Aggregated Values

	Case 1			Case 2		
	Base	Up	Down	Base	Up	Down
Symmetric - No baby boom						
No ABM	0.00106	0.00104	0.00111	0.00106	0.00107	0.00105
LR	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
BR	0.00057	0.00057	0.00057	0.00057	0.00057	0.00057
Symmetric- Baby boom						
No ABM	0.00106	0.00104	0.00111	0.00106	0.00107	0.00105
LR	0.00065	0.00076	0.00052	0.00065	0.00058	0.00072
BR	0.00059	0.00068	0.00052	0.00059	0.00057	0.00063
No Symmetric - No baby boom						
No ABM	0.00102	0.00101	0.00106	0.00102	0.00103	0.00102
LR	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
BR	0.00056	0.00057	0.00056	0.00056	0.00056	0.00056
No Symmetric - Baby boom						
No ABM	0.00102	0.00101	0.00106	0.00102	0.00103	0.00102
LR	0.00070	0.00083	0.00058	0.00070	0.00063	0.00078
BR	0.00073	0.00080	0.00066	0.00073	0.00068	0.00077