**Noise Momentum around the World**

**Abstract**:

We argue that arbitrageurs will strategically limit their initial investment in an arbitrage opportunity in anticipation of further mispricing caused by the deepening of noise traders’ misperceptions. Such ‘noise momentum’ is an important determinant of the overall arbitrage process. We design an empirical strategy to capture noise momentum in a two-period generalized error correction model (GECM). Applying it to a wide range of international spot-futures market pairs, we document pervasive evidence of noise momentum around the world.

*JEL Classification:* C12, C22, G13, G14

*Key Words:* limited arbitrage; noise momentum; initial mispricing correction; futures and spot prices

**1. Introduction**

Existing empirical study of the limits to arbitrage focus on the initial mispricing error correction coefficient in a one-period framework designed to measure the level of arbitrage activity. In contrast, the theoretical work of Shleifer and Vishny (SV 1997) models the arbitrage process in a multi-period setup. In their paper, over and above the error correction component, we identify a theoretical concept (hitherto ignored in the literature) that forms a potentially important part of the determinants of the arbitrage process. Specifically, in SV’s model one of the critical considerations for the arbitrageur in deciding how much effort to apply to correcting initial mispricing, is the probability of persistence in mispricing. We label this persistence of the uncorrected pricing errors (after the next period of trading) as “noise momentum”. Further, we design an empirical strategy to capture noise momentum in a two-period generalized error correction model (GECM). We execute an empirical application to test the importance of noise momentum across global markets. In so doing, we extend the existing body of knowledge by showing that noise momentum together with the initial unarbitraged pricing error affects price movements and the path to equilibrium. In short, we document empirical evidence consistent with the existence of noise momentum around the world.

The uncertainty regarding the level of noise trading and the uncertainty of other arbitrageurs’ actions poses a nontrivial risk to rational arbitrageurs’ activities (see, e.g., Abreu and Brunnermeier (2002), Kondor (2004), and Stein (2009)). These models suggest that knowing the aggregate level of the arbitrage activity present in the market is important to participants. In contrast to the rich insights offered in existing theoretical models on the limits of arbitrage (see Gromb and Vayanos (2010) for a survey of the theoretical studies), the short-term dynamics in these models is under-researched empirically. Our study seeks to redress this situation.

To test our hypotheses, we examine the impact of limited arbitrage and noise momentum in the index futures market, motivated by the ongoing interest in spot and futures price dynamics (e.g., Kurov and Lasser (2004)). Derived from the theory of no-arbitrage, we use a cost of carry framework which predicts that spot and futures prices are cointegrated. Market equilibrium is achieved by active trading of arbitrageurs in both markets (e.g., Garbade and Silber (1983), Stoll and Whaley (1990)). Similar to other equilibrium models, the cause of any disequilibrium and the path followed to reach an equilibrium state are not explicitly described in theory. Our modeling approach provides an ideal tool to unlock this “black box”.[[1]](#footnote-1) Using daily index futures and spot data from 29 international markets over the maximum period 1984 – 2012, we find pervasive evidence of limited arbitrage linked to noise momentum.

Our core hypothesis concerning the two-period adjustment process is strongly supported by the empirical analysis. In particular, we document a continuation of unarbitraged pricing error i.e. that there is ‘noise momentum’ in the price adjustment. Including the potential for noise momentum as an extra dimension in the short-term adjustment process enhances our understanding of the price discovery process. Previous empirical literature and the standard one-period arbitrage models show that the effect of arbitrage is limited when arbitrageurs face various types of risk, financial constraints, and transaction costs.[[2]](#footnote-2) We further demonstrate that the trading behavior conditional on the initial level of arbitrage also plays a significant role in determining the speed of adjustment or the duration of the pricing errors. In particular, we find that the overall speed of adjustment depends not only on the initial error correction coefficient but also on the noise momentum coefficient which captures the market response to the prior period’s unarbitraged error. Overall, our results highlight the importance of taking into consideration partial correction when modeling the short-term dynamics of the price-fundamentals relationship.

Our study contributes to the literature in three fundamental ways. First, we revisit SV’s analysis and demonstrate the importance of a two-period model for investigating the full effects arbitrage behavior. Specifically, the concept of noise momentum delivers an extra, rich dimension into understanding the price discovery process. It is an important determinant of the overall mispricing duration. Second, our empirical application illustrates that the generalized error correction model we develop for the purpose, provides a powerful tool for analyzing the dynamics of the price-fundamentals relationship. Finally, our empirical study makes a direct contribution to the spot-futures literature by documenting new insights into the price dynamics evident between these two markets.

The remainder of our paper is organized as follows. In Section 2, extending the foundation provided by Shleifer and Vishny (1997), our hypotheses are outlined. In Section 3 we develop a general empirical setup designed to best test our hypotheses. In Section 4 we outline a specific empirical application based on the linkage between index futures and spot markets. In Section 5 we present and discuss our empirical results, focusing on a large international dataset. A conclusion is offered in the final section.

**2. Hypothesis Development**

The limits of arbitrage and its consequences in financial markets have been highlighted in prior empirical analysis and incorporated in a growing body of theoretical work (see, for example, DeLong et al. (1990a, b), Shleifer and Vishny (1997), Abreu and Brunnermeier (2002, 2003), Liu and Longstaff (2004), Kondor (2004, 2009), Stein (2009), Hombert and Thesmar (2009), and Oehmke (2009), Moreira (2012), Makarov and Plantiny (2012), Buraschi et al. (2013), Ljungqvist and Qian (2014), Edmans et al. (2014)). See also Gromb and Vayanos (2010) for an excellent survey. These theoretical studies provide important and useful models regarding the equilibrium price-fundamentals relationship and the dynamic interactions between rational and, sometimes, ‘behaviorally-biased’ agents. However, in general the models are characterized as a one-step correction to equilibrium and arbitrageurs are either unable to learn about market-wide arbitrage capacity or the poor timing of this knowledge renders it a useless input into their decision making.

Shleifer and Vishny (1997), however, study the impact of equity constraints on the limits of arbitrage in a fully dynamic two-period setting (with three-dates/times: “time 1”; “time 2” and “time 3”). In a nutshell, SV’s model suggests two alternative paths to market equilibrium – the two scenarios depend on whether arbitrageurs engage in a fully-invested or a partially-invested strategy. A critical determinant of the investment strategy and the associated price adjustment path is the probability (denoted by ‘*q*’) that noise traders’ misperceptions deepen at time 2. We refer to the deepening of noise traders’ misperceptions as ‘noise momentum’. In the SV model there is another key parameter, a threshold point for *q,* denoted *q\**. When *q < q\**, i.e., when the probability that noise traders misperceptions deepen is relatively low, arbitrageurs will be more likely to fully invest at time 1. Alternatively, when *q > q\** (i.e. the probability of deepening misperceptions is ‘critically’ high), arbitrageurs will defer some of their investment, expecting that the time 2 price (p2) will be further away from fundamentals.

Arbitrageurs care about the deepening of time 2 mispricing because their funding is constrained by their initial arbitrage performance. SV describe this structure as performance-based-arbitrage (PBA). Essentially, investors in the arbitrage fund would withdraw/augment funds conditional on the performance of the fund between time 1 and time 2. Alternatively, this structure can be interpreted as the funding allocation strategy of a large arbitrage fund among its different fund managers. Such a PBA approach can also be applied to an arbitrage fund in which leverage is used, thus magnifying the predicted effects, and in this case changes in the market price affect margin requirements. When mispricing deepens, the arbitrageurs’ initial investment would require higher margins and, therefore, they would have to liquidate part of their holdings and realize losses to generate sufficient cash to meet the margin calls. The overall effect predicted by this model is a reduction of arbitrage-focused funds in the market. On the other hand, if market conditions improve by time 2, leveraged arbitrageurs can release some funds from their margins and reinvest further into the market.

The SV model focuses on analyzing the effect of funding constraints on the overall efficiency of the price. A key focus of our enhancement to the SV model is to analyze the realized mispricing correction/persistence in time 1 and time 2. In so doing, we are able to characterize the arbitrageurs’ impact on the subsequent price movements and the duration of pricing errors.

To lay the foundations of our analysis, we define a range of basic concepts (and associated symbolic representations) which, as much as possible, accord with the SV model setup. Let be the fundamental value of the asset at time 3, which is known to the arbitrageurs but not to the fund investors or noise traders. and are the noise traders’ shocks at time 1 and 2, respectively – that is, these “shocks” represent the extent to which noise traders in aggregate under-value the asset relative to its fundamental value (a larger S indicates a greater undervaluation “shock”). and are cumulative resources under management by arbitrageurs at time 1 and 2, respectively. SV assume that is exogenous, while is determined endogenously within the model. is the amount that arbitrageurs invest in the asset at time 1. Parameter captures the sensitivity of arbitrage funds under management at time 2 (i.e. ) to its initial performance, suggesting that these investors would withdraw or increase funds according to performance-based arbitrage.[[3]](#footnote-3) Noise trader demand for the asset is given by: . Arbitrageurs’ demand for the asset at time 1 is given by: . When the market is cleared, and we have the price at time 1 given by , while , derived similarly.

Now define the pricing error correction activity by the ratio , a metric designed to capture the proportion of mispricing correction achieved by arbitrageurs at time 1. At one extreme, , implies that there is no error correction by arbitrageurs. At the other extreme, 1, indicates that full error correction occurs.[[4]](#footnote-4) Our initial hypothesis (*H1*) relates to the basic action of correcting (at least partially) initial mispricing:

***Hypothesis 1 (H1): Initial Mispricing Correction.***

*Arbitrageurs engage in initial mispricing correction i.e.* and the limited arbitrage version of this hypothesis is captured by .

After time 1 trading is complete, the quantity, , measures the pricing error which has not been arbitraged away, where is the price determined by the supply and demand at time 1. This pricing error is observed before the next round of trading. The quantity, , measures the pricing error that remains after time 2 trading. Now we introduce a new parameter, , capturing the degree of error persistence or noise momentum after time 2 trading.[[5]](#footnote-5) At one extreme, when such that none of the error persists, we have = 0. Conversely, if all of the time 1 pricing error persists, then and, thus, there is 100% error persistence, i.e., = 1. It is also possible that the pricing error might even become exacerbated in time 2, such that , in which case > 1.

Our setup shows an important insight in terms of overall arbitrage activity. All existing empirical models are based on the one-period error correction model in which the speed of mispricing correction or arbitrage adjustment is determined solely by. In our extended analysis of the SV model, is an additional key parameter that jointly with dictates the speed of overall arbitrage adjustment in our two-period setting. Hence, the standard one-period error correction model is valid only if is equal to zero which occurs when noise momentum is absent, such that . We therefore have our core testable hypothesis (*H2*):

***Hypothesis 2 (H2): Noise Momentum.***

*Noise momentum affects arbitrageurs’ behavior regarding the mechanism for correcting mispricing i.e.* > 0*.*

Our extension of the SV model argues that the (initial) mispricing correction parameter, , and the noise momentum (or mispricing persistence) parameter after time 2 trading, , are both important in characterizing the overall speed of the arbitrage adjustment process or the duration of pricing errors. The noise momentum coefficient is zero only if noise traders’ misperceptions are corrected completely in the second period.

1. **A Generalized Error Correction Model with Noise Momentum**
   1. *Basic One-period ECM Setup accommodating Mispricing Correction*

To begin the empirical side of our analysis, we setup a basic one-period error correction modeling (ECM) framework, consistent with the majority of the developments in the extant literature. Consider the long-run price-fundamentals relationship given by:[[6]](#footnote-6)

(1)

where is the observed market price, is the fundamental value of the asset, and is the short-term deviation of observed price from its fundamental value. Notice that is the martingale difference sequence such that is stationary but serially correlated. For simplicity we represent as an AR process:

, (2)

where is regarded as the mispricing innovations. Taking the first difference of (1), and using , with κ = ϕ – 1, we obtain the standard ECM:

(3)

The parameter, , measures the impact of arbitrage trading activity in correcting the pricing error towards the long-run equilibrium relationship, and lies between -1 and 0. Next, we suppose that the reduced form data generating process for is given by:

, (4)

which allows for the (possible) feedback trading pattern, where positive (negative) implies positive (negative) feedback trading, and captures the innovations from the fundamental value of the asset,[[7]](#footnote-7) after controlling for feedback trading.[[8]](#footnote-8) This setup is motivated by both empirical and theoretical evidence that market price might potentially induce fundamental changes. It has been documented that the futures market can influence pricing of the underlying index (see, e.g., Chen, 1992).

One important issue is whether or not the pricing error innovation () and the innovation of fundamentals () are independent of each other. If they are not correlated, then the pricing error innovation is random noise. If the pricing errors is linked to fundamental news then these two error innovations will be correlated. If their contemporaneous correlation is significantly different from zero, is weakly endogenous with respect to in Equation (3). To deal with this issue, we consider the following regression:

(5)

where is uncorrelated with by construction. Then, replacing in Equation (3) by Equation (5) and rearranging, we obtain the more efficient ECM as follows:

(6)

where and . Notice that the model (6) accommodates the dynamics of price overreaction or underreaction with respect to fundamental changes through the contemporaneous reaction coefficient, , as well as the short-run momentum effects through the coefficient, . Only if the market is efficient (i.e. is iid, in which case trivially), then we expect that one unit (permanent) change in fundamentals should cause one unit change in the market price, instantaneously.

* 1. *Two-period GECM accommodating Mispricing Correction and Noise Momentum*

The model developed so far, called the standard (one-period) ECM, is a natural starting point for an analysis of hypotheses relating to the limits of arbitrage. However, the ECM suffers from a fundamental weakness as only limited dynamics are covered; namely, the speed of adjustment (or the “reciprocal” concept, duration of mispricing) in Equation (6) is measured solely by the error correction coefficient, .

SV’s model suggests that extending the analysis of arbitrage into a two-period model is important. Recall, we label the continuation of unarbitraged errors the ‘noise momentum’ effect, and measure it by the further pricing impacts of initial unarbitraged pricing error components. We can accommodate this important new dimension most simply by supposing that the pricing errors, follow an AR(2) process of the form:

(7)

where is the unarbitraged error carried over from the previous period and the parameter, , measures the further pricing impact of these (initial) unarbitraged pricing error components, i.e. ‘noise momentum’ effects. The higher is , the higher is the noise momentum in the price.

Combining Equations (1), (4), (5) and (7), we finally obtain the two-period GECM given by:

(8)

The GECM simultaneously captures the (complex) dynamics of the two-period interaction between arbitrageurs and noise traders. The distinguishing feature of the GECM is that we can accommodate ‘noise momentum’ effects through the term , with the parameter measuring the strength of noise momentum and represents the unarbitraged component of the pricing errors from the previous period. It is easily seen that the standard one-period GECM will be biased in the case where the coefficient is non-zero (i.e. *H2*).

We can transform the GECM, Equation (8), into the following autoregressive distributed lag (ARDL) specification:

(9)

where , ,, , and . The overall speed of convergence to equilibrium is now determined jointly by parameters, and ; namely, it is captured by . Since , positive noise momentum would make the pricing errors more persistent. Notably, this is a dynamic issue (of potential importance) which cannot be addressed by the conventional one-period model.

* 1. *Decomposition of the Pricing Errors*

The GECM incorporates several important economic concepts which are most clearly identified and understood by decomposing the pricing error. Specifically, the pricing error, , from the two-period GECM in Equation (8), can be represented as:

(10)

where captures the innovations from the fundamental value of the asset. Equation (10) presents a five-part decomposition of the pricing error, which is discussed below.

The first and the second components provide a natural framework for testing our hypotheses regarding the limits to arbitrage and its impact on the further price dynamics through the two parameters, and .[[9]](#footnote-9) As discussed in Section 2, arbitrage is limited, suggesting that is below unity. In the context of this setup, hypothesis H1 now becomes: 0 < |κ| < 1. Hence, the unarbitraged pricing error component persists into the next trading period and the extent of such persistence is captured by the parameter .

The third component measures the degree of the over- or under-reaction with respect to the contemporaneous fundamental changes. Generally, is likely to be non-zero unless the market is perfectly efficient. The sign of determines the direction of the price reaction to fundamental impact. If is positive (i.e. the pricing error innovation, is positively correlated with the fundamental valuation innovation, ), then the futures price overreacts to the fundamentals’ shock, irrespective of the sign of the innovation. On the other hand, a negative implies that the futures price underreacts to the fundamentals’ shock. The fourth term represents the short-run momentum effect. The sign of ( ) is generally ambiguous since it depends on the product of the correlation coefficient, , and the feedback trading coefficient, . As such this is an empirical issue.

The last component, is an idiosyncratic error term with zero mean and finite variance, . Notice that the total variance of the mispricing innovation, , is obtained simply as the sum of the variance of fundamental innovation, , and the variance of idiosyncratic error, .

Finally, in the context of Equation (10) we see that the magnitude and amplitude of the initial pricing errors are determined mainly by parameters , and **,** while the overall speed of convergence to equilibrium (as already outlined above) is determined jointly by and , namely (). Importantly, positive noise momentum would make the pricing errors more persistent.

1. **Empirical Application to Index Futures**
   1. *Empirical Model*

The cost of carry model is based on the exclusion of arbitrage and assumes that the risk-free rate and dividend yield are given. Specifically, we expect the following relationship to hold in equilibrium:

 (11)

where is “fair value” of a futures contract maturing at time *T*; *St* is the current value of the spot index; *rt* is the risk-free interest rate,  and *qt* is the dividend yield on the index.

Assuming that the risk-free rate and dividend yield are deterministic, and  will share the same stochastic trend. The futures and spot prices are cointegrated under general conditions (Ghosh (1993), Wahab and Lashgari (1993), Brenner and Kroner (1995)). Significant deviations from the prediction of cost of carry can reflect violations of the model’s assumptions.

The key assumption underlying the cost of carry model is that market participants take advantage of arbitrage opportunities as soon as they occur (Hull (2008, Ch. 3)).[[10]](#footnote-10) However, empirically, only partial adjustment is found (e.g., Stoll and Whaley (1986), MacKinlay and Ramaswamy (1988)). The ECM and GECM developed above provides an ideal tool for helping us to understand the rich dynamics behind the pricing error generated and the associated convergence processes. Their empirical counterparts, respectively, are given by:

(12)

(13)

where is the natural log of the futures contract price; is the natural log of the fundamental value implied by the cost of carry model, ;  is the natural log of the spot index price;  is the risk-free rate;  is the dividend yield on the index. The pricing error, , is estimated from the long-run equation:

(14)

Comparing Equation (14) with Equation (1), we allow for both an intercept and a non-unity long-run coefficient for general purposes. According to the cost of carry model, the theoretical value of equals 1.

* 1. *Data*

We collect spot and futures data from 29 international markets to illustrate the existence of noise momentum around the world. We use differential time periods covering the complete lifespan of the daily index spot and futures contracts between January 1982 (earliest available) and December 2013. The markets covered are: (a) North America – Canada and the US (3 alternatives); (b) Asia-Pacific – Australia, China, Hong Kong, Japan, Malaysia, South Korea, Thailand; (c) Europe – Austria, Belgium, France, Germany, Greece, Hungary, Netherlands, Poland, Portugal, Russian Federation, Spain, Switzerland, Turkey and the UK.[[11]](#footnote-11)

These markets are summarized in the Appendix A, Table A1. Proxies for the risk-free interest rate are shown in Table A2. Divided yields on the indices are also collected. The main data are sourced from DataStream and where the dividend yields and interest rate data are missing we supplement with data from Bloomberg. A continuous series of the nearest term futures contracts is constructed by DataStream. The series switch to the next nearest contract on the first day of the expiry month for the nearest term contract. We use a full set of expiry dates for all the contracts to ensure correct matching of the date to maturity in the continued futures price series. Table 1 reports the sample averages for all variables (measured in percentage terms), across the different markets in our full sample.

As expected, the movements of the paired spot and futures prices closely mimic each other. For each market, the average price changes are of the same magnitude while the volatilities are higher in the futures contracts. For example, in the case of the US S&P 500 (ISP), the average daily basis (the log difference between futures and spot prices) is 0.36 percent. After applying the cost of carry model, the difference between the futures price and the fair estimate () is zero on average.[[12]](#footnote-12) Similar findings are documented in other country pairs. The mean pricing error is zero suggesting that on average the markets are in equilibrium.

1. **Empirical Results**
   1. *Initial Mispricing Correction around the World: One-period ECM Results*

The detailed market-by-market estimation results for the one-period ECM in Equation (12) are reported in Table 2, with an accompanying summary provided in Table 3. Our main focus in this baseline estimation is the role of the kappa parameter, reflecting the initial mispricing correction. At a general level, we observe that in all 29 cases the estimated coefficient is negative and significant (at the 1% level). Accordingly, there is pervasive evidence supporting the role of initial mispricing correction in futures-spot markets, in line with *H1*. That is, our analysis in the context of this simple ECM framework supports the view that arbitrageurs engage in initial mispricing correction (i.e. |κ| > 0). Moreover, we see the magnitude of kappa estimates range in the (0,1) interval, thereby indicating that this initial correction phenomena is consistent with limited arbitrage (i.e. |κ| < 1).

Figure 1 plots the initial error correction parameters from highest to lowest (in magnitude) across our sample of markets. We see that the maximum (minimum) magnitude of this effect occurs in Canada (Italy). Other large magnitude cases are evident for the US (except CJD), Japan and Germany. These results are suggestive that a stronger role for initial mispricing correction is more likely to occur in the larger more prominent markets. However, the patterns are quite mixed. While the developing markets of China, Malaysia and Poland exhibit small magnitude estimates, relatively close to zero; so too, do the much bigger developed markets of France and the UK. Thus, while we do observe considerable variation across market settings and we are unable to draw strong conclusions regarding any trends, we do present pervasive evidence of the predicted initial mispricing correction effect.

* 1. *Noise Momentum around the World: Two-period GECM Results*

The detailed market-by-market estimation results for the two-period GECM in Equation (13) are reported in Table 4, with an accompanying summary provided in Table 5. In Table 4, various findings are worthy of special focus – primarily, those linking to our two key hypotheses. We see in the table, that the estimated initial mispricing parameters, κ, are negative and significant in all 29 cases once more, mimicking the outcome of the one-period ECM. Again, this supports the limits to arbitrage version of the initial mispricing correction hypothesis (*H1*).

Based on the range of point estimates produced, the median value (with a magnitude of approximately 41%) suggests that just under half of the prior period pricing error is corrected. It is noteworthy, that this is a much higher correction than the counterpart median observed in the one-period ECM – with a value less than 20% in magnitude. Indeed, comparing the initial mispricing correction coefficients between the ECM and GECM models market-by-market, in most cases it appears that arbitrageurs play a much bigger role in bringing the price back to its fundamental value than suggested by the simpler ECM setup. Only in the case of Spain does the magnitude of the estimate not increase. Such a contrasting result suggests that the one-period ECM gives a biased and unreliable view of the initial error correction forces evident in the data. This is most noticeable for the markets of France and the UK – while both of these in the ECM analysis showed statistically significant correction effects, neither seemed to differ markedly from zero in economic terms. Now both France and the UK show coefficient estimates exceeding 30%, that are much more economically meaningful.

Our primary focus is on the estimated noise momentum coefficient , which links to the “noise momentum” hypothesis, *H2*. Notably, in Tables 4 and 5 we find that the effect of noise momentum is positive and significant in all but one case (the one exception is Spain). This finding provides strong support for our noise momentum hypothesis which highlights that, over and above any correction for initial mispricing in the first period, the convergence to equilibrium displays a degree of momentum in the mispricing or “noise” component. It is noteworthy that all the estimated positive coefficients are less than unity.

Looking at the country-by-country results, we see the overall maximum case of noise momentum occurs in the US market (CJI) with a value of 0.752, closely shadowed by Canada (0.694). In the Asia-Pacific region, Australia (0.637) and Japan (0.572) produce the highest values, while in Europe Germany (0.725) and Sweden (0.649) are prominent. At the other end of the spectrum, aside from Spain which exhibits the only negative noise momentum coefficient, we have Italy (0.206) and Poland (0.178) standing out with the lowest values. Nevertheless, no strong patterns emerge e.g. in terms of a developed vs. emerging market divide. For a visual appreciation, Figure 2 plots the Kappa and Lambda parameters produced by the GECM across our sample of markets.

For completeness, we make some closing general observations regarding the remaining estimated parameters of the two-period model. First, regarding the intercept we see that in all cases it is insignificant. While not a guarantee, it is suggestive that the specification is one not greatly challenged by mis-specification. In other words, the empirical specification is closely matched with the theoretical model in which this parameter is expected to be zero.

Second, regarding the parameter ω, we observe only 3 instances of insignificance – we have 16 (10) significant negative (positive) cases, at the 10% level or better. Recall, that in the context of Equation (10) which decomposes the pricing errors into its various parts, the coefficient relates to the component that measures the degree of the over- or under-reaction with respect to the contemporaneous fundamental changes. Our findings suggest that the futures price is more likely to underreact than overreact on the fundamentals shocks from the underlying stock index.

Finally, regarding parameter π, there is a preponderance of (few) significant cases with a positive (negative) sign – namely, 15 (4) cases at the 10% level. Recall, that in the context of Equation (4), this coefficient relates to the short-run momentum effect. Specifically, a positive (negative) π implies positive (negative) feedback trading. Accordingly, this GECM analysis suggests a high incidence of positive feedback trading.

* 1. *Speed of Mispricing Correction* and *Duration of Error Convergence*

As indicated earlier, measuring the speed of mispricing correction is a point of contrast between our two models: in the one-period ECM it is given by κ, whereas in the two-period GECM it is a function of both initial mispricing correction, κ, and noise momentum, λ, – namely, κ + λ(1 + κ). To illustrate, we can isolate a few interesting examples. In the case of the US (CRI) the ECM provides a value of -0.559, whereas the GECM provides an overall speed of -0.470 (= -0.652 + 0.524(0.348)). Hence, by ignoring the noise momentum component, the ECM overstates the speed of mispricing correction by 9 percentage points (i.e. 55.9% vs. 47%). As another example, consider Japan. In this case, the ECM provides a value of -0.512, whereas the GECM provides an overall speed of -0.417 (= -0.629 + 0.572(0.371)). Hence, by ignoring the noise momentum component, the ECM overstates the speed of mispricing correction by close to 10 percentage points (i.e. 51% vs. 41%). As final example, consider Italy. In this case, the ECM provides a value of -0.037, whereas the GECM provides an overall speed of -0.0316 (= -0.197 + 0.206(0.803)).

An associated question of interest is how long does it take the price change to converge to its long run value, implied by either of our models? In other words, what is the “duration” (in days) of the mispricing error convergence? The answer to this question is quite simply evaluated by taking the reciprocal of the (overall) adjustment coefficient. To this end, Figure 3 provides a comparative plot of the error duration implied by the ECM versus GECM models.

Generally, it is true that we see somewhat similar values in several cases, but this belies the relative role of the underlying components (i.e. Kappa versus Lambda), as discussed above. Consider a few interesting examples. At the short end of the spectrum, across our sample markets, the US (CRI) exhibits durations of 1.8 days vs. 2.1 days for the ECM vs. GECM models. In contrast, for the longest durations we see Italy with values of 27 days vs. 31.6 days for the ECM vs. GECM models. Thus, regarding the question of the duration of error convergence, while it seems to matter little for the US, the choice of modeling ECM vs. GECM does make an appreciable difference for a market like Italy. Nevertheless, in percentage terms we generally see a nontrivial difference between the alternative paired duration estimates – in the order of 15% (measured relative to the ECM benchmark).

1. **Conclusion**

Building on Shleifer and Vishny’s (1997) theoretical work, we study the dynamics of limited arbitrage. Shleifer and Vishny’s (1997) model shows an important insight into why arbitrageurs might deliberately limit their initial arbitrage, given their concern about further mispricing in the next period. We show that second-period error persistence (labeled “noise momentum”) is an important parameter in characterizing the overall speed of adjustment process, augmenting the initial error correction coefficient as commonly used in the standard one-period ECM.

To test our model predictions, we develop a two-period error correction model. We apply our model to study the dynamics of limited arbitrage in the index futures market, an important area of ongoing research interest in its own right. Using paired daily index futures and spot data from 29 international markets over the maximum period 1984 – 2012, in addition to initial mispricing error we document pervasive evidence of limited arbitrage linked to noise momentum around the world.

Notably, the significance of this noise momentum coefficient suggests a serious misspecification in the standard error correction models used in the literature. Our empirical application illustrates that the generalized error correction model that we develop for the purpose, provides a powerful tool for analyzing the dynamics of the price-fundamentals relationship. The potential applications of this approach go well beyond that developed in the current paper. For example, our approach can be applied to explore the short-term dynamics associated with fundamental long-run cointegrating relationships (e.g., price-dividend relationship) and the pricing dynamics between segmented markets for single assets (for example, cross-listing and commodity contracts in different markets). We commend these and other meaningful applications to future research agendas.

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**Table 1: Sample Means for Base Variables across Markets**

This table reports the sample mean for all of the variables employed across each market. Table A1 in Appendix A provides the full list of the markets covered. The variables included are:() is the first difference of log spot (futures) price;  where ,  is the annualized risk-free interest rate on an investment for the period =T-t;is the annualized dividend yield on the index, while N is the sample size for each market. All numbers are tabulated in percentage point terms.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Panel A. North America | | | | | | | |
| Country | CN | US | | | |  |  |  |
| Futures | CDD | CJD | CJI | CRI | ISP |  |  |  |
| ∆*f* | 4.14 | 3.66 | 4.08 | 7.21 | 8.32 |  |  |  |
| ∆*s* | 4.05 | 3.51 | 4.00 | 7.20 | 8.26 |  |  |  |
| *f –s* | 0.05 | -0.03 | 0.06 | 0.44 | 0.36 |  |  |  |
| *f –f\** | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  |  |  |
| *r* | 2.47 | 2.41 | 2.34 | 3.79 | 4.25 |  |  |  |
| *q* | 2.00 | 2.24 | 2.24 | 1.19 | 2.50 |  |  |  |
| N | 3710 | 4096 | 4211 | 4007 | 8242 |  |  |  |
|  | Panel B. Asia-Pacific | | | | | | | |
| Country | AU | CH | HK | JP | KO | MY | TA | TH |
| Futures | AAP | CIF | HSI | JSX | KKX | KLC | TTX | TST |
| ∆*f* | 3.79 | -7.44 | 8.95 | -2.00 | 5.03 | 3.19 | 0.40 | 7.14 |
| ∆*s* | 3.81 | -7.56 | 8.86 | -2.01 | 5.00 | 3.16 | 0.47 | 7.08 |
| *f –s* | 0.09 | 0.38 | 0.10 | 0.14 | 0.19 | 0.06 | -0.29 | -0.50 |
| *f –f\** | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| *r* | 5.07 | 2.75 | 4.39 | 1.48 | 5.85 | 4.20 | 2.28 | 3.03 |
| *q* | 3.99 | 1.57 | 3.30 | 0.78 | 2.53 | 0.66 | 2.99 | 3.87 |
| N | 3539 | 942 | 7190 | 6581 | 4582 | 4682 | 4005 | 1977 |
|  | Panel C. Europe | | | | | | | |
| Country | BD | BG | ES | FR | GR | HN | IT | NL |
| Futures | GDX | BFX | MBX | FCX | ASI | BUX | MSM | ETI |
| ∆*f* | 7.52 | 0.19 | 5.70 | 0.45 | -13.34 | 13.00 | -3.57 | 4.79 |
| ∆*s* | 7.48 | 0.27 | 5.64 | 0.41 | -13.21 | 12.73 | -3.27 | 4.77 |
| *f –s* | 0.52 | -0.04 | -0.05 | -0.05 | -0.49 | 1.11 | -0.35 | 0.01 |
| *f –f\** | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| *r* | 3.50 | 2.84 | 4.54 | 2.41 | 2.33 | 10.02 | 1.87 | 3.93 |
| *q* | 1.98 | 3.33 | 3.51 | 3.36 | 3.17 | 1.63 | 1.85 | 3.47 |
| N | 5846 | 4601 | 5636 | 3882 | 3716 | 4755 | 2526 | 6544 |
| Country | OE | PO | PT | RS | SD | SW | TK | UK |
| Futures | VTX | WIG | PSX | RTS | OMF | ZMI | TRF | LSX |
| ∆*f* | 1.24 | 3.60 | -6.14 | 6.25 | 5.89 | 7.36 | 9.88 | 5.78 |
| ∆*s* | 0.18 | 3.85 | -5.98 | 6.20 | 5.86 | 7.31 | 11.72 | 5.80 |
| *f –s* | -0.29 | -0.21 | -0.23 | -0.51 | -0.14 | -0.08 | 0.27 | 0.35 |
| *f –f\** | 0.02 | 0.00 | 0.02 | 0.00 | 0.00 | 0.00 | 0.03 | 0.00 |
| *r* | 2.59 | 8.08 | 2.63 | 4.38 | 1.72 | 2.27 | 11.76 | 7.10 |
| *q* | 2.40 | 2.44 | 3.59 | 1.13 | 2.77 | 1.95 | 2.47 | 3.44 |
| N | 3730 | 4137 | 3705 | 2169 | 2290 | 6012 | 1846 | 7713 |

**Table 2: One-period ECM Estimation Results - Initial Mispricing Correction around the World**

This table reports the estimation results for the one-period Error Correction Model (ECM) for pairs of futures contracts and country stock indices, around the world, as listed in Table A1 in Appendix A. The ECM is specified as follows (Equation (12)):

,

where is the first difference operator; are the parameters and is the error term. The residual, of the long-run model is obtained from the following estimation:

where is the natural log of the actual futures contract price; is the natural log of the fundamental value implied by the cost of carry model, ;  is the natural log of the spot index price;  is the risk-free rate; and  is the dividend yield on the index. \*\*\*, \*\* and \* indicate significance at 1%, 5% and 10% levels, respectively.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Panel A. North America | | | | | | | | | | | | | | | |
|  | CN | | US | | | | | | | |  |  |  |  |  |  |
| Coeff. | CDD | | CJD | | CJI | | CRI | | ISP | |  |  |  |  |  |  |
| 𝛼 | 0.001 |  | 0.001 |  | 0.000 |  | 0.000 |  | -0.002 |  |  |  |  |  |  |  |
| 𝜅 | -0.714 | \*\*\* | -0.039 | \*\*\* | -0.617 | \*\*\* | -0.558 | \*\*\* | -0.394 | \*\*\* | |  |  |  |  |  |
|  | -0.035 | \*\*\* | -0.026 | \*\*\* | -0.002 |  | 0.044 | \*\*\* | 0.045 | \*\*\* | |  |  |  |  |  |
| 𝜋 | -0.370 | \*\*\* | -0.923 | \*\*\* | -4.318 |  | 0.608 | \*\*\* | 0.015 |  |  |  |  |  |  |  |
| N | 3705 |  | 4091 |  | 4206 |  | 4002 |  | 8237 |  |  |  |  |  |  |  |
|  | Panel B. Asia-Pacific | | | | | | | | | | | | | | | |
|  | AU | | CH | | HK | | JP | | KO | | MY | | TA | | TH | |
| Coeff. | AAP | | CIF | | HSI | | JSX | | KKX | | KLC | | TTX | | TST | |
| 𝛼 | 0.001 |  | -0.002 |  | 0.002 |  | 0.000 |  | 0.001 |  | 0.000 |  | 0.001 |  | 0.000 |  |
| 𝜅 | -0.301 | \*\*\* | -0.151 | \*\*\* | -0.268 | \*\*\* | -0.512 | \*\*\* | -0.162 | \*\*\* | -0.102 | \*\*\* | -0.180 | \*\*\* | -0.235 | \*\*\* |
|  | -0.038 | \*\*\* | -0.054 | \*\*\* | -0.028 | \*\*\* | 0.049 | \*\*\* | 0.045 | \*\*\* | 0.125 | \*\*\* | 0.077 | \*\*\* | 0.049 | \*\*\* |
| 𝜋 | -0.085 |  | 0.395 | \*\* | -1.077 | \*\*\* | 1.006 | \*\*\* | 1.932 | \*\*\* | 0.984 | \*\*\* | 0.992 | \*\*\* | 0.998 | \*\*\* |
| N | 3534 |  | 937 |  | 7185 |  | 6576 |  | 4577 |  | 4677 |  | 4000 |  | 1972 |  |
|  | Panel C. Europe | | | | | | | | | | | | | | | |
|  | BD | | BG | | ES | | FR | | GR | | HN | | IT | | NL | |
| Coeff. | GDX | | BFX | | MBX | | FCX | | ASI | | BUX | | MSM | | ETI | |
| 𝛼 | 0.001 |  | 0.005 |  | 0.017 |  | 0.000 |  | -0.002 |  | 0.010 |  | 0.001 |  | 0.000 |  |
| 𝜅 | -0.435 | \*\*\* | -0.186 | \*\*\* | -0.132 | \*\*\* | -0.071 | \*\*\* | -0.222 | \*\*\* | -0.281 | \*\*\* | -0.037 | \*\*\* | -0.123 | \*\*\* |
|  | -0.019 | \*\*\* | -0.029 | \*\*\* | -0.727 | \*\*\* | -0.020 | \*\*\* | -0.027 | \*\*\* | -0.139 | \*\*\* | -0.035 | \*\*\* | -0.008 | \*\*\* |
| 𝜋 | -0.435 | \*\*\* | -0.423 | \*\* | 0.002 |  | 0.057 |  | -1.059 | \*\*\* | -0.330 | \*\*\* | 0.808 | \*\*\* | -0.616 | \* |
| N | 5841 |  | 4464 |  | 5631 |  | 3877 |  | 3711 |  | 4750 |  | 2521 |  | 6539 |  |
|  | OE | | PO | | PT | | RS | | SD | | SW | | TK | | UK | |
| Coeff. | VTX | | WIG | | PSX | | RTS | | OMF | | ZMI | | TRF | | LSX | |
| 𝛼 | 0.001 |  | 0.003 |  | 0.002 |  | -0.001 |  | 0.001 |  | 0.001 |  | 0.039 |  | 0.001 |  |
| 𝜅 | -0.148 | \*\*\* | -0.051 | \*\*\* | -0.082 | \*\*\* | -0.228 | \*\*\* | -0.094 | \*\*\* | -0.137 | \*\*\* | -0.193 | \*\*\* | -0.098 | \*\*\* |
|  | -0.010 | \* | -0.059 | \*\*\* | 0.006 |  | 0.076 | \*\*\* | 0.012 | \*\*\* | -0.007 | \*\* | -0.537 | \*\*\* | 0.009 | \*\*\* |
| 𝜋 | -0.679 |  | -0.722 | \*\*\* | 3.584 |  | 0.959 | \*\*\* | 3.891 | \*\* | -3.249 | \*\* | -0.162 | \*\*\* | 3.062 | \*\*\* |
| N | 3628 |  | 4132 |  | 3606 |  | 2164 |  | 2285 |  | 6007 |  | 1841 |  | 7708 |  |

**Table 3: Summary of Estimation Results for One-period ECM**

This table reports the summary estimation results for the one-period Error Correction Model (ECM) applied to pairs of futures contracts and country stock indices, around the world, as reported in Table 2. It reports the mean, minimum (Min), first quartile (Q1), median, third quartile (Q3) and maximum of each parameter estimated across the 29 pairs. It also reports the number of parameters that are insignificant (Insig), and significant at 1%, 5% and 10% levels with negative and positive signs, respectively.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Distribution of Estimates | | | | | |  | Negative Sign | | | |  | Positive Sign | | |  |
| Parameter | Mean | Min | Q1 | Median | Q3 | Max |  | Sig1% | Sig5% | Sig10% | Insig |  | Sig1% | Sig5% | Insig | N |
| 𝛼 | 0.0028 | -0.0023 | 0.0001 | 0.0008 | 0.0013 | 0.0385 |  | . | . | . | 6 |  | . | . | 23 | 29 |
| 𝜅 | -0.2328 | -0.7142 | -0.2807 | -0.1795 | -0.1024 | -0.0374 |  | 29 | . | . | . |  | . | . | . | 29 |
|  | -0.0437 | -0.7269 | -0.0347 | -0.0103 | 0.0436 | 0.1249 |  | 15 | 1 | 1 | 1 |  | 10 | . | 1 | 29 |
| 𝜋 | 0.1671 | -4.3177 | -0.6156 | 0.0016 | 0.9840 | 3.8911 |  | 8 | 2 | 1 | 3 |  | 9 | 2 | 4 | 29 |

**Table 4: Two-period GECM Estimation Results - Noise Momentum around the World**

This table reports the estimation results of the two-period Generalised Error Correction Model (GECM) for pairs of futures contracts and country stock indices, around the world, as listed in Table A1 in Appendix A. The GECM is specified as follows (Equation (13)):

where is the first difference operator; are the parameters and is the error term. The residual, of the long-run model is obtained from the following estimation:

where is the natural log of the actual futures contract price; is the natural log of the fundamental value implied by the cost of carry model, ;  is the natural log of the spot index price;  is the risk-free rate; and  is the dividend yield on the index. \*\*\*, \*\* and \* indicate significance at 1%, 5% and 10% levels, respectively.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Panel A. North America | | | | | | | | | | | | | | | |
|  | CN | | US | | | | | | | |  |  |  |  |  |  |
| Coeff. | CDD | | CJD | | CJI | | CRI | | ISP | |  |  |  |  |  |  |
| 𝛼 | 0.001 |  | 0.000 |  | 0.000 |  | -0.001 |  | -0.002 |  |  |  |  |  |  |  |
| 𝜅 | -0.761 | \*\*\* | -0.188 | \*\*\* | -0.705 | \*\*\* | -0.652 | \*\*\* | -0.540 | \*\*\* | |  |  |  |  |  |
| 𝜆 | 0.694 | \*\*\* | 0.190 | \*\*\* | 0.752 | \*\*\* | 0.523 | \*\*\* | 0.466 | \*\*\* | |  |  |  |  |  |
|  | -0.033 | \*\*\* | -0.023 | \*\*\* | 0.000 |  | 0.047 | \*\*\* | 0.048 | \*\*\* | |  |  |  |  |  |
| 𝜋 | -0.199 |  | -0.573 | \*\* | 19.672 |  | 0.300 | \*\*\* | -0.390 | \*\*\* | |  |  |  |  |  |
| N | 3705 |  | 4091 |  | 4206 |  | 4002 |  | 8237 |  |  |  |  |  |  |  |
|  | Panel B. Asia-Pacific | | | | | | | | | | | | | | | |
|  | AU | | CH | | HK | | JP | | KO | | MY | | TA | | TH | |
| Coeff. | AAP | | CIF | | HSI | | JSX | | KKX | | KLC | | TTX | | TST | |
| 𝛼 | 0.001 |  | -0.001 |  | 0.000 |  | 0.001 |  | 0.000 |  | -0.001 |  | 0.001 |  | -0.001 |  |
| 𝜅 | -0.518 | \*\*\* | -0.365 | \*\*\* | -0.410 | \*\*\* | -0.629 | \*\*\* | -0.349 | \*\*\* | -0.325 | \*\*\* | -0.365 | \*\*\* | -0.462 | \*\*\* |
| 𝜆 | 0.636 | \*\*\* | 0.399 | \*\*\* | 0.316 | \*\*\* | 0.572 | \*\*\* | 0.328 | \*\*\* | 0.343 | \*\*\* | 0.342 | \*\*\* | 0.524 | \*\*\* |
|  | -0.033 | \*\*\* | -0.052 | \*\*\* | -0.020 | \*\*\* | 0.051 | \*\*\* | 0.050 | \*\*\* | 0.125 | \*\*\* | 0.079 | \*\*\* | 0.053 | \*\*\* |
| 𝜋 | 0.467 | \*\*\* | 0.492 | \*\*\* | -0.103 |  | 0.540 | \*\*\* | 0.998 | \*\*\* | 0.401 | \*\*\* | 0.563 | \*\*\* | 0.259 | \* |
| N | 3534 |  | 937 |  | 7185 |  | 6576 |  | 4577 |  | 4677 |  | 4000 |  | 1972 |  |
|  | Panel C. Europe | | | | | | | | | | | | | | | |
|  | BD | | BG | | ES | | FR | | GR | | HN | | IT | | NL | |
| Coeff. | GDX | | BFX | | MBX | | FCX | | ASI | | BUX | | MSM | | ETI | |
| 𝛼 | 0.001 |  | 0.004 |  | 0.016 |  | 0.000 |  | 0.000 |  | 0.007 |  | 0.002 |  | 0.000 |  |
| 𝜅 | -0.599 | \*\*\* | -0.462 | \*\*\* | -0.101 | \*\*\* | -0.324 | \*\*\* | -0.474 | \*\*\* | -0.454 | \*\*\* | -0.197 | \*\*\* | -0.404 | \*\*\* |
| 𝜆 | 0.725 | \*\*\* | 0.610 | \*\*\* | -0.088 | \*\*\* | 0.404 | \*\*\* | 0.602 | \*\*\* | 0.436 | \*\*\* | 0.206 | \*\*\* | 0.536 | \*\*\* |
|  | -0.016 | \*\*\* | -0.032 | \*\*\* | -0.707 | \*\*\* | -0.020 | \*\*\* | -0.030 | \*\*\* | -0.138 | \*\*\* | -0.034 | \*\*\* | -0.008 | \*\*\* |
| 𝜋 | -0.115 |  | 0.363 | \*\* | -0.001 |  | 0.204 | \* | 0.357 |  | 0.004 |  | 0.858 | \*\*\* | 0.514 |  |
| N | 5841 |  | 4464 |  | 5631 |  | 3877 |  | 3711 |  | 4750 |  | 2521 |  | 6539 |  |
|  | OE | | PO | | PT | | RS | | SD | | SW | | TK | | UK | |
| Coeff. | VTX | | WIG | | PSX | | RTS | | OMF | | ZMI | | TRF | | LSX | |
| 𝛼 | -0.002 |  | 0.003 |  | 0.003 |  | -0.002 |  | 0.000 |  | 0.001 |  | 0.033 |  | 0.000 |  |
| 𝜅 | -0.450 | \*\*\* | -0.188 | \*\*\* | -0.364 | \*\*\* | -0.415 | \*\*\* | -0.357 | \*\*\* | -0.447 | \*\*\* | -0.431 | \*\*\* | -0.349 | \*\*\* |
| 𝜆 | 0.636 | \*\*\* | 0.178 | \*\*\* | 0.475 | \*\*\* | 0.395 | \*\*\* | 0.450 | \*\*\* | 0.648 | \*\*\* | 0.494 | \*\*\* | 0.423 | \*\*\* |
|  | -0.009 |  | -0.055 | \*\*\* | 0.005 |  | 0.082 | \*\*\* | 0.015 | \*\*\* | -0.006 | \*\* | -0.547 | \*\*\* | 0.014 | \*\*\* |
| 𝜋 | 3.207 |  | -0.563 | \*\*\* | -2.982 |  | 0.435 | \*\*\* | 1.862 | \*\*\* | -1.613 | \*\* | 0.171 | \*\*\* | 0.555 | \*\* |
| N | 3628 |  | 4132 |  | 3606 |  | 2164 |  | 2285 |  | 6007 |  | 1841 |  | 7708 |  |

**Table 5: Summary of Estimation Results for Two-period GECM**

This table reports the summary estimation results for the two-period Generalised Error Correction Model (GECM) applied to pairs of futures contracts and country stock indices, around the world, as reported in Table 4. It reports the mean, minimum (Min), first quartile (Q1), median, third quartile (Q3) and maximum of each parameter estimated across the 29 pairs. It also reports the number of parameters that are insignificant (Insig), and significant at 1%, 5% and 10% levels with negative and positive signs respectively.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Distribution of Estimates | | | | | |  | Negative Sign | | | |  | Positive Sign | | |  |  |  |
| Parameter | Mean | Min | Q1 | Median | Q3 | Max |  | Sig1% | Sig5% | Sig10% | Insig |  | Sig1% | Sig5% | Sig10% | Insig |  | N |
| 𝛼 | 0.0022 | -0.0023 | -0.0003 | 0.0003 | 0.0010 | 0.0331 |  | . | . | . | 10 |  | . | . | . | 19 |  | 29 |
| 𝜅 | -0.4236 | -0.7614 | -0.4740 | -0.4148 | -0.3493 | -0.1009 |  | 29 | . | . | . |  | . | . | . | . |  | 29 |
| 𝜆 | 0.4556 | -0.0882 | 0.3432 | 0.4662 | 0.6023 | 0.7521 |  | 1 | . | . | . |  | 28 | . | . | . |  | 29 |
|  | -0.0411 | -0.7071 | -0.0331 | -0.0088 | 0.0466 | 0.1254 |  | 15 | 1 | . | 1 |  | 10 | . | . | 2 |  | 29 |
| 𝜋 | 0.8856 | -2.9820 | -0.1031 | 0.3571 | 0.5396 | 19.6723 |  | 2 | 2 | . | 5 |  | 11 | 2 | 2 | 5 |  | 29 |

**Figure 1. Initial Mispricing Correction implied by the One-period ECM**

This figure plots the 𝜅 coefficients, capturing the initial mispricing correction, in the ECM of Equation (12) as reported in Table 2 across 26 international markets around the world From the four indices used for the US, only the S&P 500 index result is reported here.

**Figure 2. Initial Mispricing Correction and Noise Momentum implied by the Two-period GECM**

This figure plots the 𝜅 coefficients (capturing the initial mispricing correction) and 𝜆 (capturing noise momentum), in the GECM of Equation (13) as reported in Table 4 across 26 international markets around the world. From the four indices used for the US, only the S&P 500 index result is reported here.

**Figure 3. Duration of Mispricing in ECM and GECM models**

This figure plots the mispricing duration estimated by the ECM versus the GECM models. For ECM, duration is calculated as 1/𝜅, where 𝜅 is the ECM estimator as reported in Table 2. For GECM, duration is calculated as 1/ [κ + λ(1 + κ)], where κ and λ are the GECM estimates as reported in Table 4. The sample covers 26 international markets around the world. From the four indices used for the US, only the S&P 500 index result is reported here.

**Appendix A Futures and Spot Market Coverage Tables**

**Table A1: Global Sample – Futures Market Coverage**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Country | Country Code | Futures | Futures Code Code | Start Date | End Date | Number of Contracts\* |
| Panel A. North America | | | | | | |
| CANADA | CN | ME-S&P CANADA 60 INDEX | CDD | 16/12/1999 | 19/12/2013 | 57 |
| UNITED STATES | US | CBOT-DJ INDUSTRIALS COMP | CJD | 19/03/1998 | 20/12/2013 | 54 |
| UNITED STATES | US | CBT-DJ INDUSTRIAL AVG | CJI | 18/12/1997 | 20/12/2013 | 65 |
| UNITED STATES | US | CME-RUSSELL 2000 INDEX | CRI | 18/03/1993 | 18/09/2008 | 63 |
| UNITED STATES | US | CME-S&P 500 INDEX | ISP | 17/06/1982 | 19/12/2013 | 127 |
| Panel B. Asia-Pacific | | | | | | |
| AUSTRALIA | AU | SFE-SPI 200 INDEX | AAP | 30/06/2000 | 19/12/2013 | 55 |
| CHINA | CH | CFFEX-CSI 300 INDEX | CIF | 18/06/2010 | 20/12/2013 | 15 |
| HONG KONG | HK | HKFE-HANG SENG INDEX | HSI | 27/06/1986 | 30/12/2013 | 107 |
| JAPAN | JP | TSE-TOPIX INDEX | JSX | 07/12/1988 | 13/12/2013 | 101 |
| MALAYSIA | MY | KLSE-KLCI | KLC | 29/12/1995 | 31/12/2013 | 73 |
| SOUTH KOREA | KO | KSE-KOSPI 200 INDEX | KKX | 13/06/1996 | 12/12/2013 | 71 |
| TAIWAN | TA | TAIFEX-TAIEX WEIGHTD INDEX | TTX | 16/09/1998 | 18/12/2013 | 62 |
| THAILAND | TH | TFEX-SET50 INDEX | TST | 29/06/2006 | 26/12/2013 | 31 |
| Panel C. Europe | | | | | | |
| AUSTRIA | OE | OTOB-ATX INDEX | VTX | 18/09/1992 | 20/12/2013 | 86 |
| BELGIUM | BG | BELFOX-BEL20 INDEX | BFX | 17/12/1993 | 20/12/2013 | 81 |
| FRANCE | FR | MONEP-CAC 40 INDEX | FCX | 21/12/1998 | 20/12/2013 | 61 |
| GERMANY | BD | EUREX-DAX INDEX | GDX | 20/12/1990 | 20/12/2013 | 93 |
| GREECE | GR | ADEX-FTSE/ASE-20 | ASI | 17/12/1999 | 20/12/2013 | 57 |
| HUNGARY | HN | BSE-BUX INDEX | BUX | 15/06/1995 | 20/12/2013 | 75 |
| ITALY | IT | IDEM-FTSE MIB | MSM | 17/09/2004 | 20/12/2013 | 38 |
| NETHERLANDS | NL | AEX-AEX INDEX | ETI | 16/06/1989 | 20/12/2013 | 99 |
| POLAND | PO | WSE-WIG 20 | WIG | 20/03/1998 | 20/12/2013 | 64 |
| PORTUGAL | PT | BDP-PSI 20 INDEX | PSX | 20/09/1996 | 20/12/2013 | 70 |
| RUSSIAN FEDERATION | RS | RTS-RTS INDEX | RTS | 14/09/2005 | 16/12/2013 | 34 |
| SPAIN | ES | MEFF-IBEX 35 PLUS INDEX | MBX | 24/02/1992 | 20/12/2013 | 88 |
| SWEDEN | SD | OMX-OMXS30 INDEX | OMF | 23/03/2005 | 20/12/2013 | 36 |
| SWITZERLAND | SW | EUREX-SMI | ZMI | 21/12/1990 | 20/12/2013 | 93 |
| TURKEY | TK | TURKDEX-ISE 100 | TRF | 30/12/2005 | 02/08/2013 | 22 |
| UNITED KINGDOM | UK | LIFFE-FTSE 100 INDEX | LSX | 29/06/1984 | 20/12/2013 | 119 |

\*The number of future contracts used in constructing the continued series.

**Table A2: Global Sample – Underlying Stock Index Coverage**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Country | Country Code | Underling Index | Interest Rate | Currency |
| Panel A. North America | | | | |
| CANADA | CN | S&P/TSX 60 INDEX | CANADA TREASURY BILL 3 MTH. (BOC) | C$ |
| UNITED STATES | US | DOW JONES INDUSTRIALS | US T-BILL SEC MARKET 3 MONTH (D) | U$ |
| UNITED STATES | US | DOW JONES INDUSTRIALS | US T-BILL SEC MARKET 3 MONTH (D) | U$ |
| UNITED STATES | US | RUSSELL 2000 | US T-BILL SEC MARKET 3 MONTH (D) | U$ |
| UNITED STATES | US | S&P 500 COMPOSITE | US T-BILL SEC MARKET 3 MONTH (D) | U$ |
| Panel B. Asia-Pacific | | | | |
| AUSTRALIA | AU | S&P/ASX 200 | AUSTRALIAN $ DEPO 3 MTH (ICAP/TR)Rate | A$ |
| CHINA | CH | CHINA SECURITIES 300 | CHINA REPO 3 MONTH | CH |
| HONG KONG | HK | HANG SENG | TR HONG KONG DOLLAR 3M DEPOSIT | K$ |
| JAPAN | JP | TOPIX | JP CD RATES FIN INS 30 - 59 D, AVG | Y |
| MALAYSIA | MY | FTSE BURSA MALAYSIA KLCI | MALAYSIA INTERBANK 3 MONTH | M$ |
| SOUTH KOREA | KO | KOREA SE KOSPI 200 | KOREA NCD 91 DAYS | KW |
| TAIWAN | TA | TAIWAN SE WEIGHED TAIEX | TAIWAN MONEY MARKET 90 DAYS | TW |
| THAILAND | TH | BANGKOK S.E.T. 50 | BANGKOK INTERBANK 3 MONTH | TB |
| Panel C. Europe | | | | |
| AUSTRIA | OE | ATX - AUSTRIAN TRADED INDEX | Brussels Interbank Offered Rate. | E |
| BELGIUM | BG | BEL 20 | BD EU-MARK 3M DEPOSIT (FT/TR) | E |
| FRANCE | FR | FRANCE CAC 40 | EURO SHORT TERM REPO (ECB) | E |
| GERMANY | BD | DAX 30 PERFORMANCE (XETRA) | EURO SHORT TERM REPO (ECB) | E |
| GREECE | GR | FTSE/ATHEX LARGE CAP | EURO SHORT TERM REPO (ECB) | E |
| HUNGARY | HN | BUDAPEST (BUX) | HUNGARY INTERBANK 3 MONTH | HF |
| ITALY | IT | FTSE MIB INDEX | EURO SHORT TERM REPO (ECB) | E |
| NETHERLANDS | NL | AEX INDEX (AEX) | NETHERLAND INTERBANK 3 MTH | E |
| POLAND | PO | WARSAW GENERAL INDEX 20 | WARSAW INTERBANK 3 MONTH | PZ |
| PORTUGAL | PT | PORTUGAL PSI-20 | OECD Portugal Interest Rates 3 Month VIBOR | E |
| RUSSIAN FEDERATION | RS | RUSSIA RTS INDEX | RUSSIA INTERBANK 31 TO 90 DAY | U$ |
| SPAIN | ES | IBEX 35 | SPAIN INTERBANK W/A 3M(DISC) | E |
| SWEDEN | SD | OMX STOCKHOLM 30 (OMXS30) | SWEDEN TREASURY BILL 90 DAY | SK |
| SWITZERLAND | SW | SWISS MARKET (SMI) | SWISS 3 MONTH LIBOR (SNB) | SF |
| TURKEY | TK | BIST NATIONAL 100 | TURKISH INTERBANK 3 MONTH | TL |
| UNITED KINGDOM | UK | FTSE 100 | UK INTERBANK 3 MONTH | £ |

1. While the long-run spot and futures price relationship is governed by the cost of carry theory; in the short-run, price synchronization is less than perfect due to the uncertainty in inputs (i.e., interest rates and dividend yields) to the cost of carry model. The heterogeneity in futures market pricing is mainly driven by the difference in market participants’ expectations with respect to these input variables. [↑](#footnote-ref-1)
2. Indeed, previous literature predominantly argues that transaction costs cause the slow adjustment to a small mispricing (e.g., Sercu et al. (1995), Panos et al. (1997), Roll et al. (2007), and Oehmke (2009)). [↑](#footnote-ref-2)
3. SV specify the supply of time 2 funding to arbitrageurs as follows:

   , with .

   The arbitrageurs’ funding is determined by the performance of their investment between time 1 and time 2. Notice that when , investors play no role in affecting their available funding. Alternatively, *a* can be regarded as a parameter reflecting the degree to which an arbitrageur chooses to increase or decrease capital investment in an arbitrage strategy. [↑](#footnote-ref-3)
4. In SV’s analysis, it is assumed that arbitrage resources are not sufficient to bring prices all the way to fundamental

   values, i.e. . This implies that . [↑](#footnote-ref-4)
5. Note that that . A simple re-arrangement produces: , and, thus, demonstrates equivalence of the denominators in the two alternative definitions of the noise momentum parameter defined in the text. [↑](#footnote-ref-5)
6. Without loss of generality we assume that the equilibrium coefficient on the fundamentals is unity. [↑](#footnote-ref-6)
7. Consider as an example, the cost of carry model which we investigate later in the empirical section. In this case captures the innovations related to fundamental changes in the valuation of the stocks, the discount rate and the dividend yield. If these three inputs change, then the fundamental values also change, making the futures price react accordingly. [↑](#footnote-ref-7)
8. This is the simplest specification allowing for feedback trading. Following Hasbrouck (1991), we can readily extend Equation (4) by adding the higher lagged terms of and . [↑](#footnote-ref-8)
9. The parameter K (defined in Section 2 as “pricing error correction”) and parameter are closely allied, both relating to the level of initial error correction in the theoretical and empirical models, respectively. Specifically, or i.e. conceptually, in absolute value they are identical. In the theoretical model, we conceptualize the degree of error correction as a positive quantity. In contrast, in the empirical model, given the basic structure of the specification in Equation (10) (i.e. *zt = f(zt-1, zt-2, …)*), the error correction parameter, should be negative if the short-run price dynamics are indeed error correcting, because this parameter is devised to measure the extent to which the impact of the past error reduces relative to today’s error. At one extreme, which reflects no error correction by arbitrageurs at time 1, while at the other extreme thereby capturing the case in which there is 100% reduction in the impact of the past error i.e. the full error-correction case. An intermediate scenario would be i.e. , which captures the case of a 50% reduction in the impact of the past error. As such, in our empirical discussion, we refer to the ‘magnitude of ’ to draw appropriate comparisons with its theoretical counterpart, . Furthermore, the definition of and its theoretical counterpart , are perfectly matched. Both measure the same concept of noise momentum i.e. the percentage of uncorrected error from time 1 which persists after the next period of trading. [↑](#footnote-ref-9)
10. In prior studies, transaction costs and market liquidity have been proposed to explain the temporary deviation from the cost of carry model. Generally, it is documented that liquidity enhances the efficiency of the futures-cash pricing system (e.g. Stoll and Whaley (1986), MacKinlay and Ramaswamy (1988), Roll, et al. (2007)). Temporary deviations from cost of carry also motivate several studies to employ threshold error correction models (e.g. Yadav, Pope and Paudyal (1994), Martens, Kofman and Vorst (1995)). [↑](#footnote-ref-10)
11. The power of our empirical tests is potentially weakened by the lower liquidity evident in some of the individual markets included. While we clearly have a wide variation of liquidity across our sample, in unreported analysis, we find evidence of reasonable activity even in the emerging/developing market sub-sample. In any case, given that an illiquidity effect would tend to induce noise and make it harder to find the predicted relationships, the fact that our results are uniformly strong allays any major concerns around our research design in this regard. [↑](#footnote-ref-11)
12. As expected, the point estimate of long-run coefficient, θ, across all markets is very close to (and statistically indistinguishable from) unity. While full details of these analyses are suppressed here to conserve space, they are available from the authors upon request. [↑](#footnote-ref-12)