Stick-slip vibration of a friction damper for energy dissipation

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**Abstract**: This paper studies energy dissipation of a friction damper (due to stick-slip vibration) in the context of harmonic excitation. There are numerous applications of such friction dampers in engineering. One particular example is a new kind of under-platform dry friction dampers for aero engines. The model consists of a clamped cross-like beam structure and two masses (friction dampers) in contact with the short beam of the cross. The two masses are allowed to slide along two extra short vertical clamped beams. They can exhibit three distinct dynamic regimes: pure slip, pure stick and a mixture of stick-slip relative to the short horizontal beam.

The finite element method is used to obtain the numerical modes of the structure. The friction at the contact interface between the short horizontal beam and the friction dampers is assumed to follow the classical discontinuous Coulomb friction law in which the static coefficient of friction is greater than the kinetic coefficient. Modal Superposition method is applied to solve the dynamic response of the structure with numerical modes. One major finding of this investigation is that there is an intermediate range of the normal contact forces (in stick-slip regime) that provides the best energy dissipation performance.

**Keywords**: stick-slip vibration, discontinuous Coulomb’s friction law, damper, energy dissipation, numerical simulation.

**1. Introduction**

Generally speaking, friction causes energy dissipation and as such acts as a stabilisation mechanism. For instance, various friction dampers have been used in buildings against vibration caused by earthquakes and under-platform dampers have been used to reduce vibration magnitudes and high dynamic stresses of turbine blades in aero engines. The latter is the motivation of this paper. On the other hand, friction can also excite vibration at certain conditions, for example, earthquake motion, sound generated by a bowed string [[1](#_ENREF_1)], door hinges, squeaky chalk on a blackboard, brake squeal [[2](#_ENREF_2),[3](#_ENREF_3)] that happens in the brake system of an automobile and dry friction dampers in aero engines. These problems are outside the scope of this paper.

One interesting form of friction-induced vibration is stick-slip vibration. Popp and Stelter [[4](#_ENREF_4)] studied two discrete and two continuous models of stick-slip vibration which exhibited various kinds of dynamic behaviour, including chaos. A theoretical analysis of stick-slip instability in systems with one degree of freedom was made in [[5](#_ENREF_5)]. Through approximating the discontinuous friction forces by smooth functions, an approximate analysis of dry-friction-induced stick-slip vibrations was introduced in [[6](#_ENREF_6)]. A very interesting review of stich-slip vibration was presented by [Feeny et al. [7]](#_ENREF_7). Stick–slip and impact motion of a single degree-of-freedom oscillator excited by a frictional moving base and restricted by a unilateral rigid or deformable obstacle were studied in [[8](#_ENREF_8)]. A shooting method for calculating limit cycles based on a simple and efficient alternate friction model to simulate stick-slip vibrations was presented in [[9](#_ENREF_9)]. On the basis of the local theory of non-smooth dynamical systems on connectable domains, the force criteria for stick and non-stick motions in harmonically forced, friction-induced oscillators were developed [[10](#_ENREF_10)]. The critical conditions, amplitudes, and base frequencies for the initiation of stick-slip oscillation from pure sliding oscillation were derived by approximate analytical expressions in [[11](#_ENREF_11)]. The effects of the non-smooth Coulomb friction on the stability and bifurcation behaviour of vibrational systems with self-excitation due to negative effective damping were investigated by [Hetzler [12]](#_ENREF_12). In [[13](#_ENREF_13)], the sticking and non-sticking orbits of a two-degree-of-freedom oscillator subjected to dry friction and a harmonic load were obtained in a closed form. [Li et al. [14]](#_ENREF_14) developed a model of an elastic disc in sliding frictional contact with a rotating oscillator. Separation of the moving slider form the disc and its subsequent reattachment to the disc were considered, and various dynamic behaviour was discovered. Several friction force models to deal with different friction phenomena in the background of multibody system dynamics were examined and compared, and a particular emphasis on the pure dry sliding friction, stick–slip effect, viscous friction and Stribeck effect were discussed in [[15](#_ENREF_15)].

One important application that involves friction in a vibration environment is aero engine blades equipped with under-platform frictional dampers. Various types of friction models and methods for blade vibration analysis have been investigated. [Griffin [16]](#_ENREF_16) presented a macroslip model to investigate the resonant stresses of a blade with a dry friction damper. Menq et al. [[17](#_ENREF_17)] developed a microslip model for analysing the dynamic response of frictionally damped structures in which the friction interface was subjected to high normal loads. A microslip model derived by Menq et al. [[17](#_ENREF_17)] was improved by [Csaba [18]](#_ENREF_18). A new two-dimensional model for point friction contacts was introduced by Sanliturk and Ewins [[19](#_ENREF_19)]. [Cigeroglu et al. [20]](#_ENREF_20) adopted a one-dimensional dynamic microslip friction model, including the damper inertia. This microslip friction model was further developed for a two-dimensional distributed parameter model with normal load variation induced by normal motion [[21](#_ENREF_21)], in which they explored a harmonic balance method (HBM) for frictionally constrained structures with a varying normal load. It has been found that there is a normal load range that provides effective damping on the steady-state vibration of an aero engine blade under a certain condition [[22](#_ENREF_22)], and a bilinear hysteretic model was adopted to calculate the optimum normal loads under different external excitations [[23](#_ENREF_23)]. However, the blade was modelled as a lumped mass and hence only one mode could be included in these investigations [22, 23]. [Allara [24]](#_ENREF_24) proposed a model to characterize friction contact of non-spherical contact geometries obeying the Coulomb friction law with constant friction coefficient and constant normal load. From this model, the effect of the main contact parameters (contact geometry, material properties, loads) on the contact behaviour could be effectively estimated. In [[25](#_ENREF_25)] the dampers were modelled with the finite elements. [Ostachowicz [26]](#_ENREF_26) established an HBM for forced vibration analysis of dynamic systems damped by dry friction forces. Guillen and Pierre [[27](#_ENREF_27)] introduced a Hybrid Frequency-Time (HFT) method for analysing the steady-state response of the large-scale dry-friction damped structural systems. The correlation of the static/dynamic coupling of the under-platform dampers was investigated by [Firrone et al. [28]](#_ENREF_28). A new microslip friction model with an elastoplastic shear layer was put forward to analyse dynamic response of a system with under-platform dry friction dampers under high normal force [[29](#_ENREF_29)]. Mathematical relationships of dry friction force versus relative velocity in friction contact of two bodies were studied by [Půst et al. [30]](#_ENREF_30). Schwingshackl et al. [[31](#_ENREF_31)] focused their research on contact interface parameters in a nonlinear dynamic analysis of assembled structures. Among all the available friction models in the above literature review, friction dissipates energy and thus reduces blade vibration; however, friction excited stick-slip motion of the contact interface has been neglected.

This paper studies forced vibration of a cross-like structure in frictional contact with two masses. Dry friction forces at the contact interfaces dissipate energy due to the relative rubbing motion at the interfaces and can also cause stick-slip vibration of the surfaces in contact. This structure is meant to represent a simplified turbine blade model with a new kind of under-platform dry friction dampers. Each damper is located at the end of a short cantilever beam and thus is different from those dampers reported in the above-mentioned papers. The effectiveness of these dampers in dissipating vibration energy is assessed through numerical simulation. A numerical approach for determining the dynamic response of the structure with nonsmooth friction contacts is put forward and three distinct types of vibration, including stick-slip vibration between the dampers and the blade platform surface, are studied. This numerical approach can also take any requisite modes of the structure into account.

**2. Theoretical development of nonsmooth vibration**

As shown in Fig. 1 below, the structural model consists of two rigidly connected elastic beams in the form of a cross that is clamped to the ground; the vertical beam and the horizontal beam can be considered to represent the blade and the platform of an aero engine, respectively; underneath the horizontal beam, there is a mass connected to a thin and short, vertical elastic cantilever beam on each side. The mass is in frictional contact with the underside of the horizontal beam in the model. Unlike a conventional under-platform damper in an aero engine which is a floating mass actuated by centrifugal force, the friction damper studied here is mounted on the short vertical cantilever beam which can slide along the beam and is forced into contact with the underside of the horizontal beam by a constant force (which can represent the centrifugal force due to rotation in an aero engine).



**Fig. 1** The model of an exemplary structure with friction dampers

The mechanical model of the structure is illustrated in Fig. 2. In the *x-y* coordinate system, *x* and *y* represent the tangential and radial directions, respectively. The plane Cartesian coordinate system *x*1-*y*1 (called the left damper coordinate system) is a local coordinate system whose origin is at the left initial contact point, and the plane Cartesian coordinate system *x*2-*y*2 (called the right damper coordinate system) is defined similarly. *F* is the external excitation, which is taken as a harmonic function of time *t* in this paper. and are the normal contact forces between the horizontal beam and the dampers, and are the friction forces on the contact interfaces, and and are the constant forces acting on the dampers. is a spring constant (the equivalent lateral stiffness at the tip of the short vertical cantilever beam). In the context of an aero engine,，, where and are the masses of the dampers, is the distance between the dampers and the centre of the turbine disc, and is its rotating speed. By selecting different damper mass values, contact forces and can be regulated.

In this preliminary investigation, the dampers are assumed to undergo horizontal and vertical vibrations, but no rotation. They are in point contact with the horizontal beam and do not separate with the horizontal beam during vibration excited by *.* Therefore displacements and of the dampers are equal to the local lateral deflections of the horizontal beam; but they can slide along the underside of the horizontal beam in the global *x* direction. The horizontal and vertical beams of the cross-like structure are both assumed to behave as an Euler-Bernoulli beam.



**Fig. 2** The mechanical model of the cross-like structure

A classical discontinuous Coulomb friction model [[32](#_ENREF_32)] is adopted in this paper: before sliding occurs, the friction force balances the applied external force (together with the inertia force) and acts to resist the initiation of sliding; during sliding the friction force is proportional to the normal force at the contact interface and acts in the direction opposite to the relative sliding. A further assumption is that the static coefficient of friction is greater than the kinetic coefficient of friction (so that stick-slip vibration of the damper is possible). Fig. 3 describes the discontinuous Coulomb friction model. Although this friction model looks simple, due to its discontinuous nature, it can cause numerical difficulties and distinct dynamic behaviour of the structure under study.



**Fig. 3** The classical Coulomb friction model

Accordingly, the relationship between the friction force and the relative velocity is

(1)

The equations of motion in the local transverse and longitudinal directions of the vertical beam modelled as a rotating Euler-Bernoulli beam [[33](#_ENREF_33)] (denoted by v) in its local coordinate system are

(2)

Similarly, the equations of motion of the horizontal beam modelled as an Euler-Bernoulli beam (denoted by h) in its local coordinate systems are

(3)

where and are the local transverse displacements, and are the local longitudinal displacements, all in their local coordinate systems; and are Young’s moduli of these two beams, and are their mass densities, and are the cross-sectional areas, and and are the second moment of the areas. The -component and -component of the vertical beam’s *n*-th analytical mode can be denoted as and , in its local coordinate (which is *y* for the vertical beam); the -component and -component of the horizontal beam’s *n*-th analytical mode can be denoted as and , in its local coordinate (which is *x* for the horizontal beam).

For a complicated structure, analytical modes are difficult to obtain. In general, the finite element (FE) method provides a simple and effective approach to obtain the frequencies and modes of a structure of arbitrary configuration and hence is used. The cross-like structure is discretized with a number of 2-node Euler-Bernoulli beam elements. Each node has 3 degrees-of-freedom: one translation along the longitudinal axis of the beam element, denoted by ; one translation lateral to the beam axis, denoted by ; and a rotation around the axis normal to this plane, denoted by [[34](#_ENREF_34)]. ANSYS is used to get the frequencies and numerical modes of the cross-like structure considering the pre-stress effects (produced by the centrifugal force due to rotation of turbine discs in an aero engine). First, the pre-stress of the structure under rotating condition is acquired, and then, the frequencies and numerical modes of the structure including pre-stress effects are obtained.

Using the FE method, the equations of motion can be written in general as

(4)

where **M**, **C** and **K** are respectively the mass, damping and stiffness matrices of the cross-like structure (without the dampers and their associated short vertical cantilevers); **x** is the nodal displacement vector and **f** is the nodal force vector which includes the external excitation force acting on the vertical beam, and the normal and friction forces acting at the contact points. Damping in the structure is very small and thus **C** is taken to be zero in this investigation. As **x** can be expressed in the mode superposition (MS) method as

(5)

equation (4) can be transformed into the equation below in the modal coordinate vector **q**:

(6)

where contain the first *j* number of modes in columns and **q** has *j* number of elements; ‘diag’ stands for a diagonal matrix that contains the first *j* number of natural frequencies squared of the cross-like structure. Once **q**(*t*) at discrete time instants is determined through numerical integration (Runge-Kutta algorithm), displacements , and , of the horizontal beam at the contact points can be found from equation (5). Regardless of relative sticking or slipping of the dampers to the horizontal beam, the equations of motion of the dampers are:

(7)

(8)

For simplicity in this preliminary investigation, the normal forces at the contact points are assumed to be a known constant, which satisfy the equations below

(9)

(10)

where and are the local transverse displacements of the two dampers; the in the above equations is due to the equal share of a damper by 2 adjacent ‘blades’ of an aero engine turbine disc.

It is assumed that due to the symmetry of the structure and because normal load is equal to and is a constant, the two dampers undergo identical motion. Subsequently the motion of only the left damper is presented. If the left damper slips relative to the horizontal beam, the friction force is known as

(11)

From equation (7), and can be found. The latter can be compared with to disclose whether slip really occurs or would continue.

One the other hand, if the damper sticks to the horizontal beam, then

(12)

According to equation (7), the friction force can be found from

(13)

It must be checked whether

(14)

If this is true, then sticking is taking place; otherwise, slipping is taking place and equation (11) should be used. In either case, the friction force can be determined and hence energy dissipation can subsequently be calculated.

During the time integration of equation (6), the normal force and the friction force at a contact point must be known. For simplicity, the normal force is assumed to be a known constant, as given in equations. (9) and (10); however, the friction force is not known a priori, which depends on whether the damper sticks or slips relative to the horizontal beam. So the vibration states of the damper must be monitored and the precise time instant at which stick switches to slip or slip switches to stick must be captured. This is not a trivial matter. Fig. 4 illustrates the basic idea of this matter and how it is solved, using stick to slip as an example.



**Fig. 4** Close look-up of transition from stick to slip

In Fig. 4, is a very small positive number that controls accuracy, and represent the last time instant within the stick regime and the first time instant within the slip regime, respectively, so that the time instant of the transition point is in between and needs to be found. Once it is realised in the numerical integration that at the end of a time step a change of vibration regime occurs, the bisection method is used: a new time step of half of its previous size is now used to recalculate the various quantities concerned at the middle point and equation (14) is checked. Through the iteration of the bisection method, the precise time instant for transition can be determined within the set tolerance of error . Similarly, the precise time instant for transition from slip to stick can be captured within an acceptable error too.

A computing strategy is devised as follows:

1. It is assumed at the start of the numerical integration that the damper slips relative to the platform initially.
2. The friction force is assumed to be .
3. Compute **q** at an arbitrary time instant by solving equation (6), which gives .
4. Solving equation (7) yields .
5. At the end of each time step, ( is a very small positive number which controls accuracy,) is checked.
6. If the above inequality is satisfied, slips continues and go back to step 2 with time stepping forward.
7. If and, sticking takes place and then go to step 8. If and, slips continues, and the direction of the friction force is reversed, go back to step2.
8. Solving equation (13) to get friction force . At the end of each time step, check whether ( is a very small positive number that controls accuracy, ) is satisfied. If it is, go to step 9; if not, go to step 10.
9. Sticking continues and compute **q** at the next time instant by solving equation (6), which gives , afterwards, go back to step 8.
10. Slipping takes place, the exact time instant when slipping takes place must be determined. Go back to step 8: restart at the previous time instant with a smaller time step, until (considered to be the time instant of transition from stick to slip), and then go back to step 2.

The above iteration repeats until the end of simulated time period, during which the damper can keep switching between stick and slip. This is a nonsmooth dynamic process and can take much computing time.

**3. Numerical simulation**

In the subsequent numerical simulation, the whole system has the following material and geometric properties: Young modulus ; mass density ; width of the horizontal beam ; thickness of the horizontal beam ; width of the vertical beam ; thickness of the vertical beam ; the length of the horizontal beam ; the length of the vertical beam ; ;;; ; ; ; ; ; . Only the first two vibration modes of the cross-like structure are found to be needed, that is, . The first two natural frequencies of the cross-like structure alone are 44.4 Hz and 240.3 Hz; while the two natural frequencies of the cross-like structure and the two dampers with their cantilevers together are 45.7 Hz and 241.5Hz. 186 20-node solid elements are used in the FE analysis.

3.1 Dynamic analysis and discussion

The responses of the cross-like structure under different normal forces and external sinusoidal excitations are calculated numerically by the Runge-Kutta method. In these numerical examples, the external excitation frequency is fixed as 240 rad/s (38.2 Hz). The time step used in the numerical integrations is 0.0001 second. The simulation results are visualized by means of time histories and frequency spectra. Only some typical results are reported and given in Figs. 5-13.

3.1.1 Gross slip mode

Figs. 5-7 show the results when the normal contact force *N*1 is 2.85N and the external excitation force amplitude is *f* =20N. The steady-state response and frequency spectrum of the ‘blade’ tip are illustrated in Fig. 5, and friction force versus time and relative velocity curves are given in Fig. 6, and the results for the motion and frequency spectrum of the damper with dry friction are given in Fig. 7. It can be seen that the forced vibration of the ‘blade’ tip is periodic and has the same frequency as the excitation force (Fig. 5). Fig. 6 demonstrates that the friction force versus relative velocity curve looks like a rectangular wave and its amplitude is invariably equal to, which indicates that the damper is always slipping relative to the horizontal beam. Fig. 7 shows that the motion of the damper is not harmonic. Through analysing the frequency spectrum, it is found that there are five peaks, and the frequency of the first peak is the same as the excitation frequency, the frequency of the second peak is three times of the excitation frequency, and the frequency of the third, fourth and fifth peaks are five, seven and nine times of the excitation frequency, respectively. Thus it can be concluded that the damper’s vibration is periodic whose period is the period of the excitation. The involvement of the superharmonic frequencies in the damper vibration may seem odd at first glance, in contrast with the blade vibration which is simply harmonic. The explanation is that during gross slip, the friction force (see Fig. 6) keep switching between two constant values at regular time intervals and behaves like a train of rectangular pulses at the same frequency as the driving frequency which contain superharmonics of only odd integer multiples of the driving frequency. Effectively, the pulse-like friction force acts as an excitation to the damper and equation (7) suggests that the damper motion should contain the same frequency components as the friction force. On the other hand, the amplitude of the friction force is fairly small in comparison with the amplitude of excitation and it acts at the contact point far away from the ‘blade’ tip, and therefore its effect on ‘blade’ tip vibration is small so that the ‘blade’ tip vibration only contains one frequency component (the driving frequency).

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**Fig. 5** The dynamic response and frequency spectrum of the ‘blade’ tip



**Fig. 6** Friction force versus time and relative velocity



**Fig. 7** The motion and frequency spectrum of the damper

3.1.2 Stick-slip mode

Figs. 8-10 show the results when the normal contact force *N*1 is 17.1N and the external excitation force amplitude is 20N. Fig. 8 shows the steady-state response and frequency spectrum of the ‘blade’ tip, and friction force versus time and relative velocity curves are provided in Fig. 9, and the results for the motion and frequency spectrum of the damper with dry friction are given in Fig. 10. As Fig. 8 shows, the forced vibration of the ‘blade’ tip remains periodic and it also has the same frequency as the excitation force. As illustrated in Fig. 9, on the other hand, the friction force is now very interesting. It fluctuates between ± and is periodic. At times, it equals ±, which is when the damper slips to the horizontal beam. At any other time, the damper sticks relative to the horizontal beam. Therefore, the damper is sometimes slipping and sometimes sticking relative to the horizontal beam.

It turns out that the motion of the damper remains periodic (Fig. 10). From the frequency spectrum plot, it can be found that there are two main harmonic components of the motion of the damper, one is the excitation frequency, and the other is the third-order superharmonic component of the excitation frequency, just like the previous gross slip case. What is different though is the disappearance of three frequency components (191 Hz, 267.4 Hz and 343.8 Hz), which are the fifth-order, seventh-order, and ninth-order superharmonic components of the driving frequency. During this vibration regime, the damper switches between stick and slip states. When in slip state, the friction force cycles between two constant values in the form of a rectangular wave and thus leads to the presence of the driving frequency and its third-order superharmonic component. But when in the stick state, the friction force is no longer in the form of a rectangular wave and must be found from equation (13). In fact, the proportion of stick state increases with the increase of the normal load, which leads to the disappearance of the higher order superharmonics of the driving frequency. This should be clearer in the next case — pure stick regime.



**Fig. 8** The dynamic response and frequency spectrum of the ‘blade’ tip



**Fig. 9** Friction force versus time and relative velocity



**Fig. 10** The motion and frequency spectrum of the damper

3.1.3 Pure stick mode

Figs. 11-13 show the results when the normal contact force *N*1 is 22.8N and the external excitation force amplitude is 20N. Fig. 11 displays the steady-state response and frequency spectrum of the ‘blade’ tip, friction force versus time and relative velocity curves are given in Fig. 12, and the results for the motion and frequency spectrum of the damper with dry friction are illustrated in Fig. 13. Fig. 11 shows that the forced vibration of the ‘blade’ tip with friction is now quasi-periodic and it has two kinds of harmonic components (excitation frequency and the first-order bending frequency of the cross-like structure with the two dampers and the short cantilever beams as a whole). Fig. 12 indicates that the friction force exhibits a quasi-periodic variation and the relative velocity of the contact point of the damper and the blade platform is always zero, hence, the damper is always sticking to the horizontal beam. As a result, the cross-like structure and the dampers form one new system. As shown in Fig. 13, the motion of the damper is non-periodic. There are two peaks in the frequency spectrum plot: the frequency of the first peak is the same as the excitation frequency, but the frequency of the second peak is now the first-order bending frequency of the whole new system.



**Fig. 11** The dynamic response and power spectral density of the ‘blade’ tip



**Fig. 12** Friction force versus time and relative velocity



**Fig. 13** The motion and frequency spectrum of the damper

3.1.4 Discussion

The results under different normal contact forces show that the damper could experience three different schemes of motion: gross slip scheme, stick-slip scheme, and pure stick scheme, as the normal contact force increases (relatively to the amplitude of excitation). The ‘blade’ tip could manifest periodic motion when the normal contact force is small, but when the normal contact force is high enough, the motion of the ‘blade’ tip could change to quasi-periodic vibration. Similarly, the motion of the damper can be periodic or quasi-periodic like the ‘blade’. Frequency spectra of the motion of the damper always show a predominant peak at the driving frequency during different schemes of motion. The odd-order superharmonic components of the driving frequency emerges during slip and stick-slip schemes, which seems like the phenomenon illustrated in [Ouyang et al. [35]](#_ENREF_35). However, during the pure stick scheme, the first-order bending frequency of the whole system including the dampers and their cantilevers emerges, this is because they behave like one structure when the dampers stick to the horizontal beam.In the gross slip regime, the friction force alternates between two constant values of (see Fig. 6), given the particular law of friction adopted in this paper. However, in the stick-slip regime, the friction force fluctuates between (see Fig. 9), at this greater level of friction force, it is likely that the blade tip vibration in the stick-slip regime would be smaller than that in the pure slip regime. While in the pure stick regime, the cross-like structure and the dampers with the small cantilevers form a new structure and its first natural bending frequency becomes 45.7 Hz, which is shifted away from the frequency of excitation as compared with the first natural frequency of the cross-like structure in the pure slip mode. As a result, the blade tip vibration would also be smaller than that in the pure slip regime.

3.2 The normal load effects on vibration reduction

Because blade tip vibration is the most concerned quantity in aero engine vibration, it is often used as an indicator of vibration level in industry. The aim of this study is to study the effect of changing the normal contact force on the ‘blade’ tip vibration reduction. A normalised energy density () is defined below as a measure of vibration response so that the vibration reduction effect of the damper in various conditions can be assessed..; is used in this paper. In order to evaluate the vibration of the whole cross-like structure, it is suggested to use , in which **x**(t) is the displacement vector of the cross-like structure, as defined in Eq. (4). The smallest means the best vibration reduction.

As illustrated in Figs. 14-16, with the increase of the normal contact force, the normalised energy density () decreases continuously at first, then increases and finally stays the same. It is found that there is an optimal value of the normal contact force which can make the biggest vibration reduction, which happens in the stick-slip regime. This optimal value of normal contact force increases with the increasing amplitude *f* of the excitation force.



**Fig. 14** The normal load versus Normalised energy density curve (*f* =10N)



**Fig. 15** The normal load versus Normalised energy density curve (*f* =15N)



**Fig. 16** The normal load versus Normalised energy density curve (*f* =20N)

This result is consistent with the previous finding that there exists an optimal mass of the under-platform damper in the aero engine that gives the best energy dissipation. It is can be exploited in the design of aero engines and other structures involving friction dampers.

**4. Conclusions**

This paper studies the forced vibration response of a cross-like beam structure with friction dampers. This structure may crudely represent a turbine blade with a new kind of under-platform dry friction dampers. It presents a numerical approach which uses numerical modes of the structure and is capable of accommodating the discontinuous classical Coulomb friction law.

Parametric analysis demonstrates that the damper will experience three motion regimes: gross slip regime, stick-slip regime, and pure stick regime, as the normal contact force at the contact point between the damper and ‘blade platform’ increases. When the normal contact force is small, the motion of the ‘blade’ tip is periodic; however, when the normal contact force is too high, the motion of the ‘blade’ tip could change to quasi-periodic vibration. The motion of the damper is not harmonic but periodic whose period is the period of the excitation in gross slip regime and stick-slip regime, but is quasi-periodic in pure stick regime. During slip and stick-slip schemes, the odd-order superharmonic components of the driving frequency emerge because the friction force is periodic at the frequency of excitation in these two regimes. However, during pure stick scheme, the cross-like structure and the dampers form one new system, so the first-order bending frequency of the whole system takes part.

A major finding is that there is an intermediate range of the normal contact force at the damper and the ‘platform’ contact interface that results in the best vibration reduction performance under a certain condition, which confirms the similar finding from very simple structural models by a few other researchers in the past. Hence, the damper mass can be optimised in order to reduce the vibration and thus dynamic stresses of an aero engine blade to the maximum extent. The idea of this kind of dampers is believed to be applicable to other structures.

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