A new frequency matching technique for FRF-based model updating

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Abstract. Frequency Response Function (FRF) residues have been widely used to update Finite Element models. They are a kind of original measurement information and have the advantages of rich data and no extraction errors, etc. However, like other sensitivity-based methods, an FRF-based identification method also needs to face the ill-conditioning problem which is even more serious since the sensitivity of the FRF in the vicinity of a resonance is much greater than elsewhere. Furthermore, for a given frequency measurement, directly using a theoretical FRF at a frequency may lead to a huge difference between the theoretical FRF and the corresponding experimental FRF which finally results in larger effects of measurement errors and damping. Hence in the solution process, correct selection of the appropriate frequency to get the theoretical FRF in every iteration in the sensitivity-based approach is an effective way to improve the robustness of an FRF-based algorithm. A primary tool for right frequency selection based on the correlation of FRFs is the Frequency Domain Assurance Criterion. This paper presents a new frequency selection method which directly finds the frequency that minimizes the difference of the order of magnitude between the theoretical and experimental FRFs. A simulated truss structure is used to compare the performance of different frequency selection methods. For the sake of reality, it is assumed that not all the degrees of freedom (DoFs) are available for measurement. The minimum number of DoFs required in each approach to correctly update the analytical model is regarded as the right identification standard.

1. Introduction

Damage identification is an important tool that protects people from the loss of property and even life caused by structure failure. Methods based on changes in vibration characteristics of the structure have attracted the attention of many researchers as they provide quantitative global damage assessment to complex structures [1-3]. Among them model updating techniques, which correct the finite element (FE) model by experimental data, have been developed extensively in recent decades [4-9].

The frequency-domain model updating method can be roughly divided into three categories based on different dynamic characteristics used: eigenvalue based, mode shapes based and FRF based methods. One of them, The FRF based model updating method, directly uses the measured FRF data to update the structure's parameters. Imregun, et al. [10] proposed an early version of this method. Lin and Zhu [11] presented a method which employed measured response data under base excitation for model updating. Esfandiari, et al. [12] used the FRF data of the measured displacement, velocity or acceleration of the damaged structure to update the model. Kwon and Lin [13] developed a frequency selection method for efficient FRF based model updating. Garcia-Palencia and Santini-Bell [14] utilized a two-step algorithm to update the unknown mechanical properties and damping ratios of linear elastic damped structures.

The main idea of these methods is to establish the equation of the residuals between analytical and experimental FRF data with the sensitivity matrix of updating parameters and uses the least-square method to solve the over-determined equation. The updated parameters that bring the theoretical response closest to the actual response are considered to be the actual parameters and the reduction in their initial state, if big enough, is considered to be the occurrence of damage. Using FRF data directly does not require thorny modal identification work and hence the results will not be affected by the errors it produces. Moreover, available test data are broadened to the whole frequency domain of interest beyond measured frequencies. The abundant information provided by the FRF data not only allows more parameters to be updated but also can be used to reduce the effects of noise by choosing more precise regions of FRF. Also the reliability of the updating results can be checked by using different combinations of the experimental data.

However, there are some problems which the FRF based model updating methods have to face. The first is the incompleteness of the experimental data of FRF. Nearly all FRF based model updating methods require at least a complete row or column of the FRF matrix while it is inevitable that some DoFs cannot be measured in actual tests. To solve this problem Imregun used the analytically-generated FRFs for the unmeasured responses [10]. The reduction of the theoretical model and the expansion of the measured FRFs by dynamic reduction method [15] were also suggested. Similarly, Gang, et al. [16] proposed a new iteration formulation for the reduced model using pseudo master DoFs to improve the convergence of that technique.

Besides, the magnitude of FRF date changes dramatically in the vicinity of a natural frequency while most FRF based model updating methods use the theoretical and experimental FRF data at the same frequency to establish the sensitivity equation without the control of the magnitude difference between them. If this difference is big, the test and modelling errors will be enlarged and the level of ill-conditioning of the sensitivity matrix will increase. To overcome this situation, Pascual [17, 18] presented the Frequency Domain Assurance Criterion (FDAC) to localize the right special theoretical frequency to be used by determining the frequency shift at all measured frequencies. Asma and Bouazzouni [19] selected the theoretical frequency at which the distance between theoretical and measured FRFs is shortest. Both methods use the residuals of theoretical and experimental FRFs at different frequencies. However, the distance between these relative frequencies is not easy to control. If the whole frequency domain of interest is searched, it is not only time-consuming but also easy to make bad selections when using these methods per se may select inappropriate frequency values (whose FRFs are required) that fit these criterions best.

This paper presents a new frequency selection method which directly finds the theoretical frequency that minimizes the difference of the order of magnitude between the theoretical and experimental frequency responses in a certain range of measured frequencies. Since the frequency searching range is limited, it is not time-consuming to apply this frequency matching technique and the robustness of the algorithm is improved remarkably, as proved by a simulated truss structure.

2. Basic Theory

The essential purpose of the FRF based model updating method is to use the parameterized theoretical FRFs to approximate the measured ones. A brief description of the fundamental theory is given below which begins with a simple equation:

$$\mathbf{H}_{\mathrm{a}}(\omega)-\mathbf{H}_{\mathrm{x}}(\omega)=\mathbf{H}_{\mathrm{a}}(\omega)-\mathbf{H}_{\mathrm{x}}(\omega) \tag{1}$$

where $\mathbf{H}_{\mathrm{a}}(\omega)$ and $\mathbf{H}_{\mathrm{x}}(\omega)$ are $\mathrm{n}\times\mathrm{n}$ theoretical and experimental FRF matrices at frequency ω, respectively. Equation (1) can be rewritten as

$$\mathbf{H}_{\mathrm{a}}(\omega)\left(\mathbf{H}_{\mathrm{x}}^{-1}(\omega)-\mathbf{H}_{\mathrm{a}}^{-1}(\omega)\right)\mathbf{H}_{\mathrm{x}}(\omega)=\mathbf{H}_{\mathrm{a}}(\omega)-\mathbf{H}_{\mathrm{x}}(\omega) \tag{2}$$

Replacing $\mathbf{H}^{-1}(\omega)$ by the dynamic stiffness matrix $\mathbf{Z}(\omega)$

$$\mathbf{H}_{\mathrm{a}}(\omega)\Delta\mathbf{Z}\mathbf{H}_{\mathrm{x}}(\omega)=\mathbf{H}_{\mathrm{a}}(\omega)-\mathbf{H}_{\mathrm{x}}(\omega) \tag{3}$$

where

$$\Delta\mathbf{Z}=\mathbf{Z}_{\mathrm{x}}(\omega)-\mathbf{Z}_{\mathrm{a}}(\omega)=-\omega^2\Delta\mathbf{M}+\mathrm{j}\omega\Delta\mathbf{C}+\Delta\mathbf{K} \tag{4}$$

The basic assumption here is that the damage causes a reduction of the mass, damping and stiffness matrices of the model at element level. So the difference of the 3 global matrices can be expressed as

$$\Delta\mathbf{M}=\sum_{i=1}^{\mathrm{n_e}}p_{\mathrm{m}i}\mathbf{M}_i^{\mathrm{e}},\quad \Delta\mathbf{C}=\sum_{i=1}^{\mathrm{n_e}}p_{\mathrm{c}i}\mathbf{C}_i^{\mathrm{e}},\quad \Delta\mathbf{K}=\sum_{i=1}^{\mathrm{n_e}}p_{\mathrm{k}i}\mathbf{K}_i^{\mathrm{e}} \tag{5}$$

where $\mathrm{n_e}$ is the number of updating elements, $\mathbf{M}_i^{\mathrm{e}}$, $\mathbf{C}_i^{\mathrm{e}}$ and $\mathbf{K}_i^{\mathrm{e}}$ are respective contributions of the *i*th clement to the global mass, damping and stiffness matrices and $p_{\mathrm{m}i}$, $p_{\mathrm{c}i}$ and $p_{\mathrm{k}i}$ are the corresponding scalar multipliers representing the proportional changes from their values in the intact state and are also regarded as updating parameters. Substituting equations (5) and (4) into (3) and assuming only one column vector $\mathbf{h}_{\mathrm{x}}(\omega)$ of the measured FRF matrix $\mathbf{H}_{\mathrm{x}}(\omega)$ is available, one obtains

$$\begin{bmatrix}\mathbf{S}_\mathrm{m}(\omega) & \mathbf{S}_\mathrm{c}(\omega) & \mathbf{S}_\mathrm{k}(\omega)\end{bmatrix}\begin{Bmatrix}\mathbf{P}_\mathrm{m}\\ \mathbf{P}_\mathrm{c}\\ \mathbf{P}_\mathrm{k}\end{Bmatrix}=\mathbf{h}_\mathrm{a}(\omega)-\mathbf{h}_\mathrm{x}(\omega) \tag{6}$$

where $\mathbf{S}_\mathrm{m}(\omega)$, $\mathbf{S}_\mathrm{c}(\omega)$ and $\mathbf{S}_\mathrm{k}(\omega)$ are the sensitivity matrices with n_e submatrices arranged in a row and $\mathbf{P}_\mathrm{m}$, $\mathbf{P}_\mathrm{c}$ and $\mathbf{P}_\mathrm{d}$ are the column vectors of the corresponding updating parameters. The *i*th submatrix of the 3 sensitivity matrices is calculated as

$$\begin{aligned}\mathbf{S}_\mathrm{m}^{i}(\omega)&=-\omega^2\mathbf{H}_\mathrm{a}(\omega)\mathbf{M}_i^\mathrm{e}\mathbf{h}_\mathrm{x}(\omega)\\ \mathbf{S}_\mathrm{c}^{i}(\omega)&=\mathrm{j}\omega\mathbf{H}_\mathrm{a}(\omega)\mathbf{C}_i^\mathrm{e}\mathbf{h}_\mathrm{x}(\omega)\\ \mathbf{S}_\mathrm{k}^{i}(\omega)&=\mathbf{H}_\mathrm{a}(\omega)\mathbf{K}_i^\mathrm{e}\mathbf{h}_\mathrm{x}(\omega)\end{aligned} \tag{7}$$

Equation (6) is the final form of most FRF based methods. However, as mentioned before, choosing good theoretical frequencies different from their counterparts can improve the robustness of the method. To achieve this, equation (6) and (7) are rebuilt where two different frequencies are considered: ω_a as the analytical frequency and ω_x as the experimental frequency. They turn into

$$\begin{bmatrix}\mathbf{S}_\mathrm{m}(\omega_\mathrm{a},\omega_\mathrm{x}) & \mathbf{S}_\mathrm{c}(\omega_\mathrm{a},\omega_\mathrm{x}) & \mathbf{S}_\mathrm{k}(\omega_\mathrm{a},\omega_\mathrm{x})\end{bmatrix}\begin{Bmatrix}\mathbf{P}_\mathrm{m}\\ \mathbf{P}_\mathrm{c}\\ \mathbf{P}_\mathrm{k}\end{Bmatrix}=\mathbf{h}_\mathrm{a}(\omega_\mathrm{a})-\mathbf{h}_\mathrm{x}(\omega_\mathrm{x})-\mathbf{e}_\omega(\omega_\mathrm{a},\omega_\mathrm{x}) \tag{8}$$

and

$$\begin{aligned}\mathbf{S}_\mathrm{m}^{i}(\omega_\mathrm{a},\omega_\mathrm{x})&=-\omega_\mathrm{x}^2\mathbf{H}_\mathrm{a}(\omega_\mathrm{a})\mathbf{M}_i^\mathrm{e}\mathbf{h}_\mathrm{x}(\omega_\mathrm{x})\\ \mathbf{S}_\mathrm{c}^{i}(\omega_\mathrm{a},\omega_\mathrm{x})&=\mathrm{j}\omega_\mathrm{x}\mathbf{H}_\mathrm{a}(\omega_\mathrm{a})\mathbf{C}_i^\mathrm{e}\mathbf{h}_\mathrm{x}(\omega_\mathrm{x})\\ \mathbf{S}_\mathrm{k}^{i}(\omega_\mathrm{a},\omega_\mathrm{x})&=\mathbf{H}_\mathrm{a}(\omega_\mathrm{a})\mathbf{K}_i^\mathrm{e}\mathbf{h}_\mathrm{x}(\omega_\mathrm{x})\end{aligned} \tag{9}$$

where the additional item $\mathbf{e}_\omega(\omega_\mathrm{a},\omega_\mathrm{x})$ comes from equation (4) representing the difference of theoretical dynamic stiffness matrices between the two frequencies

$$\mathbf{e}_\omega(\omega_\mathrm{a},\omega_\mathrm{x})=(-\omega_\mathrm{x}^2+\omega_\mathrm{a}^2)\mathbf{H}_\mathrm{a}(\omega_\mathrm{a})\mathbf{M}\mathbf{h}_\mathrm{x}(\omega_\mathrm{x})+\mathrm{j}(\omega_\mathrm{x}-\omega_\mathrm{a})\mathbf{H}_\mathrm{a}(\omega_\mathrm{a})\mathbf{C}\mathbf{h}_\mathrm{x}(\omega_\mathrm{x}) \tag{10}$$

By now the relationship between the difference of the FRFs and the updating parameters has been established. The next task is to find the best corresponding theoretical frequencies for every experimental frequency. A well-known method for doing this is the FDAC method [17] which matches the measured frequency with the theoretical frequency whose FDAC value is the greatest. The FDAC value can be regarded as the correlation of the two frequencies and its expression is

$$FDAC(\omega_\mathrm{a},\omega_\mathrm{x})=\frac{\{\mathbf{h}_\mathrm{a}(\omega_\mathrm{a})\}^\mathrm{T}\{\mathbf{h}_\mathrm{x}(\omega_\mathrm{x})\}}{\left|\{\mathbf{h}_\mathrm{a}(\omega_\mathrm{a})\}_i\right|\left|\{\mathbf{h}_\mathrm{x}(\omega_\mathrm{x})\}_i\right|} \tag{11}$$

The experimental FRF can be considered as the shift of theoretical FRF caused by the change of the updating parameters. The key point of FDAC method is to find the corresponding theoretical frequency to the experimental one. However, this method requires searching for the whole frequency range, which is very time-consuming.

Since the main idea of this method is to avoid a big difference of the theoretical and experimental FRFs, a new index is proposed to find the theoretical frequency with a minimum order of magnitude difference (MOMD) from the measured frequency and is expressed as:

$$MOMD(\omega_{\mathrm{a}},\omega_{\mathrm{x}})=\sum_{i=1}^{\mathrm{n}}\left|\log_{10}(\mathbf{momd}_i)\right|$$
$$\mathbf{momd}_i=\begin{cases}\left|\dfrac{\{\mathbf{h}_{\mathrm{a}}(\omega_{\mathrm{a}})\}_i}{\{\mathbf{h}_{\mathrm{x}}(\omega_{\mathrm{x}})\}_i}\right|, \{\mathbf{h}_{\mathrm{a}}(\omega_{\mathrm{a}})\}_i\cdot\{\mathbf{h}_{\mathrm{x}}(\omega_{\mathrm{x}})\}_i\geq 0\\ \left|\dfrac{\{\mathbf{h}_{\mathrm{a}}(\omega_{\mathrm{a}})\}_i-\{\mathbf{h}_{\mathrm{x}}(\omega_{\mathrm{x}})\}_i}{\min\left(\left|\{\mathbf{h}_{\mathrm{a}}(\omega_{\mathrm{a}})\}_i\right|,\left|\{\mathbf{h}_{\mathrm{x}}(\omega_{\mathrm{x}})\}_i\right|\right)}\right|, \{\mathbf{h}_{\mathrm{a}}(\omega_{\mathrm{a}})\}_i\cdot\{\mathbf{h}_{\mathrm{x}}(\omega_{\mathrm{x}})\}_i< 0\end{cases} \tag{12}$$

where $\mathbf{momd}_i$ represents the degree of closeness of every pair of elements in the two vectors.

$\mathbf{momd}_i$ has different expressions to reduce the influence of anti-phase vector with the same modulus value. The closer its value is to 1, the smaller *MOMD* is. After calculating all the *MOMD* values in a certain interval around the measured frequencies, the one with smallest *MOMD* value is chosen as the theoretical frequency. By using the MOMD index, the robustness of the algorithm has been improved significantly, which is verified in the next section. For simplicity the FRF based model updating methods without and with different frequency matching techniques under comparison are referred to as the original method, the MOMD method and the FDAC method, respectively.

To deal with the problem that in practice usually not all DoFs are measurable and incomplete measured data cannot be directly substituted into equations (8), (9) and (10), the dynamic reduction method is considered which generates the slave DoFs by the master DoFs if a harmonic excitation is applied in the master DoFs. The transformation is given as

$$\begin{bmatrix}\mathbf{x}_{\mathrm{m}}\\ \mathbf{x}_{\mathrm{s}}\end{bmatrix}=\begin{bmatrix}\mathrm{I}\\ -\left(-\omega^2\mathbf{M}_{\mathrm{ss}}+\mathrm{j}\omega\mathbf{C}_{\mathrm{ss}}+\mathbf{K}_{\mathrm{ss}}\right)^{-1}\left(-\omega^2\mathbf{M}_{\mathrm{sm}}+\mathrm{j}\omega\mathbf{C}_{\mathrm{sm}}+\mathbf{K}_{\mathrm{sm}}\right)\end{bmatrix}\left[\mathbf{x}_{\mathrm{m}}\right]=\mathbf{T}_{\mathrm{d}}\mathbf{x}_{\mathrm{m}} \tag{13}$$

where subscripts m and s stand for the master and slave DoFs, respectively, and displacement vector $\mathbf{x}$ and mass, damping and stiffness matrices $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are divided into sub vectors and sub matrices relating to these DoFs. By suitably dividing the measured DoFs into the master DoFs and the slave DoFs, the theoretical model is reduced by

$$\mathbf{M}_{\mathrm{r}}=\mathbf{T}_{\mathrm{d}}^{\mathrm{T}}\mathbf{M}\mathbf{T}_{\mathrm{d}},\quad \mathbf{C}_{\mathrm{r}}=\mathbf{T}_{\mathrm{d}}^{\mathrm{T}}\mathbf{C}\mathbf{T}_{\mathrm{d}},\quad \mathbf{K}_{\mathrm{r}}=\mathbf{T}_{\mathrm{d}}^{\mathrm{T}}\mathbf{K}\mathbf{T}_{\mathrm{d}} \tag{14}$$

where $\mathbf{M}_{\mathrm{r}}$, $\mathbf{C}_{\mathrm{r}}$ and $\mathbf{K}_{\mathrm{r}}$ are the reduced mass, damping and stiffness matrices.

It should be noted that the fewer the measured DoFs there are, the less information the experimental FRF data contains and the more model errors the theoretical model produces, which causes the updating procedure to diverge. Thus, the method which updates parameters correctly with fewer measured DoFs is considered to be more robust.

3. Case study using simulated FRF data

A six-bay truss structure with 25 rods of identical cross-section and 21 DoFs as shown in figure 1, adapted from [12], is used in simulation. The structure is modelled with 2D truss elements. The properties of the structure are Young's modulus of 200GPa, mass density of 7800kg/m^3 and cross section area of 1800mm^2. The DoFs are arranged in the order of the node numbers and for each node the horizontal displacement is placed ahead of the vertical displacement.

In this study, two damage scenarios listed in Table 1 are considered to gauge the performance of the proposed method. Both damage scenarios are represented by the reduction factor of the mass and stiffness matrices of some elements and for the sake of simplicity, the 'experimental' FRF data from only the first column of the FRF matrices directly obtained by the finite element simulation, are used to update the original intact structure.

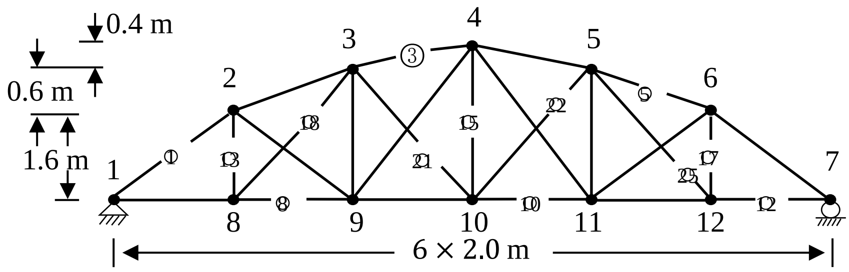

Figure 1. Geometry of a truss model

Table 1. Damage scenarios of the truss structure

Scenario 1			Scenario 2		
Element no.	M (%)	K (%)	Element no.	M (%)	K (%)
7	15	20	7	50	70
12	30	40	12	60	80
21	25	32	21	40	60

First, scenario 1 which represents a moderate damage case is used to get experimental FRF data without noise. Figure 2 shows the results of damage identification using both methods when all DoFs are measurable. It is clear that the FRF based model updating method is able to update so many parameters at the same time if enough and accurate test data are available.

Scenario 1 is also used to compare the identification performance of the three methods. The minimum number of the required measurable DoFs (MD) is considered a criterion. For the sake of fair comparison, for both methods the measurable DoFs are removed gradually from the end of the DoF sequence and the experimental frequencies (whose FRFs are measured) are selected at a regular interval in the range of 0 to 900Hz controlled by the total number of measured frequency points, denoted by NF, which is from 20 to 60. Table 3 lists the results of this comparison which show that the MOMD method can get acceptable updating results using far fewer measured DoFs than the FDAC and the original method. The FDAC method is slightly better than the Original method as it has more acceptable NFs with the same MD. But it takes the longest time to get the final solution of updating parameters.

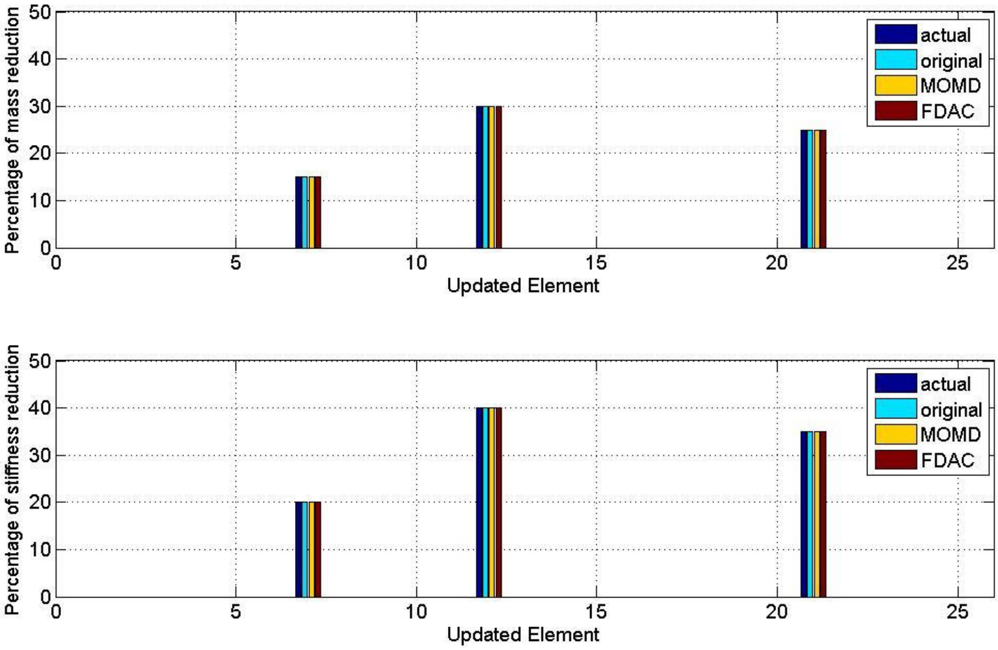

Figure 2. Actual and updating results for damage scenario 1 (top graph for mass and bottom graph for stiffness)

Table 3. Updating limits of the three methods

Method	minimum of MD	Acceptable NF	Single operation time(s) (MD=15,NF=30)
Original	8	21, 24, 32, 38, 40, 58	1.75
MOMD	4	33, 59	4.26
FDAC	8	20, 22, 28, 37, 40, 41, 42, 44, 47, 48, 49, 57, 59	83.86

Secondly, scenario 2 which represents a more severe damage case than scenario 1 is used to compare the performance of the three methods in extreme circumstances. This time the experimental FRF data is contaminated by 1% white Gaussian noise which is introduced though function awgn in MATLAB. Other parameter values are the same as before. The numbers of measured DoFs are from 12 to 18; and the corresponding numbers of acceptable NF, for which an acceptable result can be obtained by the three FRF-based methods are illustrated in figure 3. It is clear that for each number of measured DoFs the MOMD method is more likely to achieve convergence than the original method.

This time the FDAC method performs poorly. A reasonable explanation of this phenomenon is that the severe locations of damage cause a large frequency shift and the big difference between theoretical and experimental frequencies may overwhelm the FRF residuals. That is why the MOMD method targets the theoretical frequency in the vicinity of the experimental frequency and thus has a better performance.

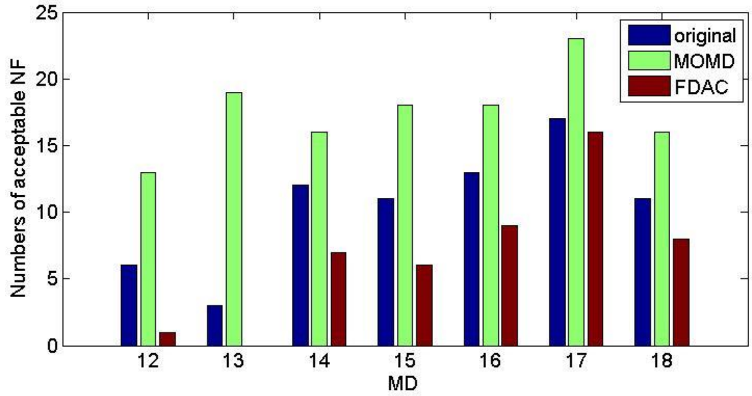

Figure 3. Numbers of acceptable NF for the three methods with different numbers of measured DoFs

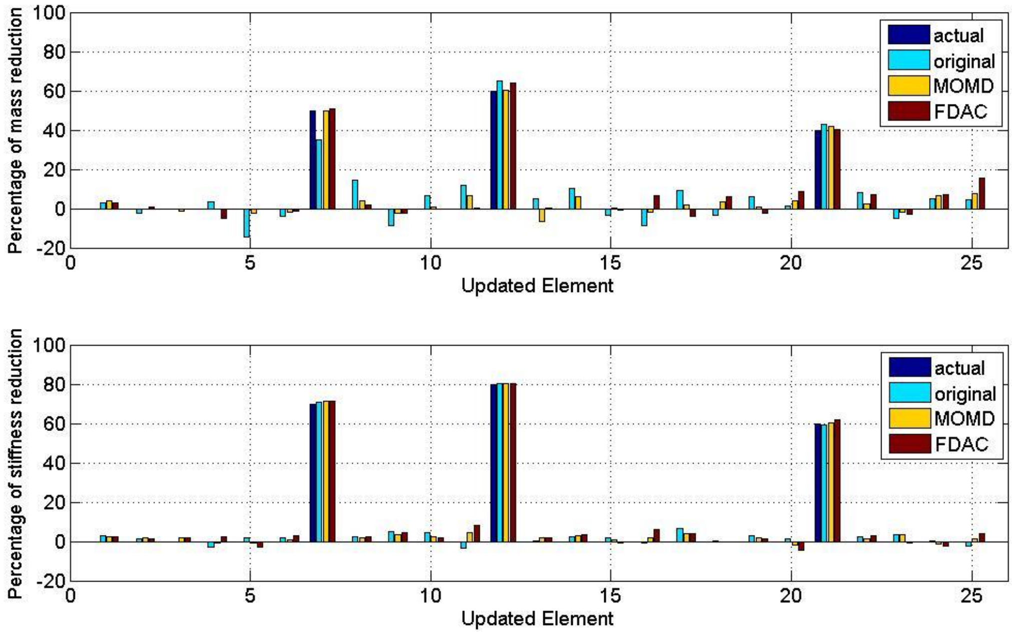

Figure 4. Actual and updating results for damage scenario 2 with 3% white Gaussian noise polluted data (top graph for mass and bottom graph for stiffness)

Then, the proportion of white Gaussian noise is increased to 3% to compare the noise robustness of the three methods. The MD and NF are set to 17 and 30 respectively, by which an acceptable updating results can be obtained by all methods. After enough iterations, the results are considered convergent solutions as shown in figure 4 which illustrates that although all methods are able to identify the damage, the MOMD method makes the best predictions.

4. Conclusions

In this paper, a new frequency matching technique is proposed which can be used for the FRF based model updating method. With a small additional amount of computational time, a best theoretical frequency whose FRF has the minimum order of magnitude difference (MOMD) from the experimental one is selected in the vicinity of the related experimental frequency. The results of the simulated truss structure show that the MOMD method makes a big improvement in the performance of convergence over conventional FRF-based identification method.

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