**Effects of Risk-aversion on Competing Shipping Lines’ Pricing Strategies**

**with Uncertain Demands**

(For the published version, please refer to: Zheng, W., Li, B. and Song, D.P. (2017). Effects of risk-aversion on competing shipping lines’ pricing strategies with uncertain demands, Transportation Research Part B. 104, 337-356. DOI: http://dx.doi.org/10.1016/j.trb.2017.08.004.)

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**Abstract**: Container shipping is facing severe overcapacity, fierce price-based competition and high demand uncertainty. It is natural that some shipping lines may adopt a risk-aversion attitude in their pricing strategies. This paper considers the pricing strategies of two competing ocean carriers facing uncertain demand. The first carrier is risk-neutral with sufficient capacity, whereas the second carrier is risk-averse with limited capacity. The conditional value at risk (CVaR) is used to measure the risk-averse attitude of the second carrier. A Nash game model is formulated to model the pricing decisions and the equilibrium solution is obtained. We find that the pricing solution takes two forms, which can be determined by a threshold value of carrier 2’s capacity. Under uniformly distributed demand, we show that as the second carrier becomes more risk-averse, both carriers’ optimal prices are decreasing, and the threshold value that determines the pricing strategy is also decreasing. We also analyze the impact of price sensitivity and competition intensity parameters on two carriers’ price decisions under more specific conditions. A necessary and sufficient condition is established to determine whether two carriers’ optimal prices would be positively or negatively affected by the competition intensity parameter. A range of numerical experiments are provided to illustrate the analytical results and explore their validity in more general cases. Moreover, it is shown that the main analytical results in this paper can carry over to the cases when both carriers are risk-averse.

**Keywords**: container shipping; pricing; risk-averse; uncertainty; conditional value at risk.

**1. Introduction**

Maritime transportation plays a vital role in the development of world economy for there is no effective alternative way to transport large scale of goods over a long distance. According to a research conducted by Lloyds Marine Intelligence Unit (Mandryk 2009), about 75% of the world trade by volume (or 60% of the world trade by value) were carried by sea; among the maritime transport sectors, 52% of cargoes by value were carried by container ships. Container shipping has experienced a rapid growth in the last two decades with the largest containership size increasing from 8,000 TEUs in 2000 to 19,000 TEUs in 2015, where TEU stands for twenty-foot equivalent-unit.

Container liner shipping is a capital-intensive industry with long investment lead times (Fransoo and Lee 2013). A modern containership costs over a hundred million US dollars and takes several years to build. Containerships sail along a published schedule on a regular basis. In order to respect the schedule, a containership may have to depart from a port even though it is under-loaded. Nowadays global shipping lines often offer similar shipping services on a weekly basis, e.g. in the Asia-Europe route, Trans-Pacific route, or Trans-Atlantic route, although individual shipping services may differ in terms of port of calls. The capital-intensive nature associated with the requirement of regular shipping services leads to fierce competition between shipping lines. Low service differentiation in liner shipping implies that the competition is mainly on a cost or price (freight rate) basis (Lee and Song 2017). In addition, the seasonal variations of the shrinking market and the fluctuation of the bunker price drive the volatility of freight rate. For example, Lloyd’s List reported that freight rates slumped 40% within a week in November 2015.

Shipping lines face two types of customer demands: long-term contractual demand and spot market demand. The price for the long-term demand is often fixed and contracted once a year, whereas the price for spot market demand may be agreed between shipping lines and shippers (or freight forwarders) dynamically on daily/weekly basis. Since the financial crisis of 2008, economic recession and declining trade demand have led to overcapacity in liner shipping services. The situation is worsened by the fragmented container shipping market and the shipping lines' persistent pursuit of economies of scale (e.g. purchasing larger and larger vessels). In recent years, shippers tend to go for spot market to search for a lower freight rate (Lee et al. 2015). This intensifies the competition between shipping lines and increases the unpredictability of cargo volume for individual shipping lines. Uncertainty in customer demands has been one of the main challenges that shipping lines have to cope with in order to survive in the current highly competitive environment.

The regularity of liner shipping services indicates that liner services are perishable products, which means unutilized vessel slots will lose the opportunity to generate revenue. Therefore, it is vital for a shipping line to seek an appropriate pricing strategy to maximize its revenue in the competitive and uncertain spot market. Understandingly, given the perishable feature of liner services, some shipping lines may adopt a risk-aversion attitude in their pricing strategies. However, there has been no study investigating the effects of risk aversion on the pricing strategies for competing shipping lines.

This paper aims to fill the above research gap by investigating the competition outcome and pricing strategy of shipping lines facing uncertain spot market demand with risk-averse behavior. We consider a competition between two shipping lines: a larger one and a smaller one. The larger shipping line has sufficient shipping capacity and is risk-neutral, while the smaller shipping line has constrained shipping capacity and is risk-averse. Here the smaller shipping line may be regarded as a freight forwarder or NVOCC (non-vessel operating common carrier) who has purchased a certain number of vessel slots from a shipping line. We would like to address the following questions:

1. What are the optimal pricing strategies and the associated profits of two carriers when the smaller carrier is risk-averse in meeting uncertain spot market demand?
2. What is the difference between two carrier's pricing strategies? How does the degree of the risk-averse attitude influence the pricing strategies and the profits?
3. What is the effect of other parameters, like competition intensity and the spot market price sensitivity, on the decision variables and the profits?

The main contributions of this paper include: (i) to the best of our knowledge, this study is the first to investigate the effects of risk aversion on the pricing strategies for competing shipping lines. We are among the first to introduce the concept of conditional value at risk (CVaR) to measure the risk-averse attitude in the shipping industry. A Nash game model is formulated to model the pricing decisions of two competing carriers (carrier 1 is risk-neutral and carrier 2 is risk-averse) facing uncertain demands; (ii) our model with CVaR takes into account the capacity constraint of the risk-averse carrier. Our model is quite generic and is essentially applicable to other industrial sectors with capacity constraint and perishable products. There has been no similar model in the risk-averse literature. In that sense, our study enriches the literature on CVaR in the broad context; (iii) we solve the Nash game model and obtain the equilibrium solution analytically. We find that the solution takes two forms, which can be determined by a threshold value of carrier 2’s capacity; (iv) when the demand uncertainty follows a uniform distribution, we are able to establish the monotone properties of the carriers’ optimal prices and the threshold value that determines the pricing strategy, with respect to the second carrier’s risk-averse preference; (v) under some specific conditions, we establish analytical results about the impact of price sensitivity and competition intensity on two carriers’ optimal price decisions. An interesting finding is that the competition intensity parameter could have either negative or positive impact on two carriers’ prices, and more importantly we provide a necessary and sufficient condition to determine whether two carriers’ optimal prices would be positively or negatively affected by the competition intensity parameter. Numerical experiments show that some of the above analytical results can carry over to more general cases; (vi) The main analytical results also hold in the cases when both carriers are risk-averse.

The remainder of this paper is organized as follows. In section 2, we review the literature related to competition and pricing in liner shipping, and some studies related to decision-making with risk-averse behavior. In section 3, we describe the problem and formulate a Nash game model. In section 4, we analyze the model, obtain the equilibrium solution, and establish a series of analytical results. In section 5, a range of numerical experiments are performed to illustrate the analytical results and explore their validity in more general situations. In section 6, we extend the model to the cases with both shipping lines are risk-averse. Finally, conclusions and suggestions for further research are drawn in section 7.

**2. Literature Review**

Operations management has had a huge impact on transport and logistics (Song 2017), but its applications in container shipping industries are seldom compared to other transport sectors such as air transport (Lee and Song 2017). Our work is related to two existing research streams. The first concerns competition and pricing strategies in liner shipping. The second is about decision making with risk-averse behavior facing uncertainty in broad contexts.

***Competition and pricing in liner shipping***

In terms of the first research stream, competition and pricing in liner shipping has been a long debated topic. Historically, liner conferences have been used as a device by shipping lines to agree a set of tariffs, and terms and conditions of carriage in certain trade routes. Heaver et al. (2000) presented an overview of the competitiveness of the shipping industry under different cooperation agreements including alliances and mergers among shipping lines, conferences, and vertical integration. Panayides and Cullinane (2002) discussed the issues of competitive advantages in liner shipping by focusing on the main themes in the literature including vertical integration, strategic alliances, mergers and acquisition, regulation, pricing, and shipper relationships. From October 2008, liner conference ceases to exist on routes to and from Europe and diminishes in other routes.

Modelling competition and pricing in the liner shipping industry is one of the research areas that have been well understudied (Lee and Song 2017). Only recently, a few studies have started to address this issue. Zhou and Lee (2009) examined the optimal pricing decisions in container transport services by taking into account the empty container repositioning cost. In a duopoly market with symmetric ocean carriers, they showed that there is a unique Bertrand Nash Equilibrium and obtained some analytic properties. Xu et al. (2015) extended the model in Zhou and Lee (2009) to a three-echelon supply chain consisting of one carrier, two forwarders, and shippers. They presented a Stackelberg game model and analyzed the optimal joint pricing policy and the repositioning cost sharing policy from the perspective of the entire service chain.

Gelareh et al. (2010) modelled the competition between a newcomer shipping line and an existing dominating shipping line. They emphasized on hub port locations and service network design for the newcomer company. A mixed-integer programming model is formulated and solved using a Lagrangian relaxation method. But they did not consider the pricing decisions. Shah et al (2012) proposed an analytical model of price and frequency competition among freight carriers considering brand loyalty in the choice of carriers. However, their model is not specifically for liner shipping, because liner shipping normally has a weekly frequency.

Lee et al. (2012) presented a three-level model to capture the interactions among oligopolistic ocean carriers, port terminal operators, and land carriers. A game theoretic approach is used to model these players who compete with each other in their pricing and routing decisions. Alvarez-SanJaime et al. (2013) modelled the competition between a road transport firm and two shipping lines, and investigated the impact of the horizontal integration of two shipping lines. The optimal pricing strategy is obtained by analyzing the equilibrium conditions.

Wang et al. (2014) conducted a game theoretical analysis on the competition between two shipping lines in a new emerging container shipping market consisting of two ports located in different continents. The shipping lines’ decisions include the freight rate, service frequency and ship capacity. The market share of each shipping line is determined by the logit-based discrete choice model. Three types of game models were presented and solved numerically. Kou and Luo (2016) developed a game theory model for the ocean carriers’ capacity expansion decisions in a competitive market. They attempted to explain the phenomenon of persistent low freight rate in the current shipping industry. Chen et al. (2016) studied a shipping market with two shipping lines providing services between two locations, in which the shipments can be classified into two categories: goods and waste. The optimal pricing strategy for normal shipment and for waste shipment at two regions for two carriers is obtained through finding the Nash equilibrium of the game theoretical model.

In the above studies, customer demands are essentially treated as deterministic. In the global container transport chain, customer demands for container transportation are affected by many factors and are unpredictable in nature, especially in the current high competitive spot market. There are a few studies that examined the pricing strategies with uncertain demands in container shipping. For example, Yin and Kim (2012) recognized that container transport services cannot be stored. They discussed the optimal freight tariff that a shipping line can offer to freight forwarders by maximizing the expected profit. The shipping line's freight tariff is characterized by price-break points, discounted freight rates and penalties for unsold space. Liu and Yang (2015) considered the joint slot allocation and dynamic pricing problem in a container sea-rail multimodal transport system with random demands. A two-stage optimal model is presented. The first stage is formulated as a stochastic integer programming model to determine long-term slot allocation based on contract sale and empty container allocation based on O-D control equation. The second stage is formulated as a stochastic nonlinear programming model to determine dynamic pricing and slot allocation in each period of free market by applying a multiproduct joint dynamic pricing and inventory control approach. In Yin and Kim (2012) and Liu and Yang (2015), the demands are not related to the pricing decisions. Lee et al. (2015) designed a pricing matching contract for the shipping line in order to encourage shippers to go for long-term contract instead of spot market. They show that the carrier can generate a higher demand from the shippers for using the fractional price matching contract. However, in this group of studies (i.e. Yin and Kim 2012; Liu and Yang 2015; Lee et al. 2015), they did not consider the competition between shipping lines.

***Decision making with risk-averse behavior facing uncertainty***

In the second research stream, risk-averse behavior is explicitly considered for decision makers in uncertain situations. Risk aversion is the behavior of decision makers to attempt to reduce the negative impact of uncertainty. Owing to the presence of uncertainties on supply, demand, market price and others, a substantial level of risk is faced in normal operations, which is called operation risk (Choi et al. 2016). It is usually embodied in decision-makers’ risk preference. Typical methods to measure this type of risk and optimize the strategies include mean-variance model (Chiu et al. 2013), value at risk (VaR) (Tapiero 2005), conditional value at risk (CVaR) (Li et al. 2014a). Each of the risk measures has its own characteristics. Mean-variance is the first analysis method proposed to estimate risk attitude (Markowitz, 1959). It is often used when decision makers have a concave quadratic utility function. Mean-variance is based on the mathematical concept of variance to measure risk attitude. It is a symmetric measure that treats desirable outcomes as the same as undesirable outcomes in the risk measure (Ma et al., 2012). As a result, it is not suitable for the cases where decision makers are more concerned with undesirable outcomes, e.g. losses. On the other hand, the VaR and CVaR are more suitable for the situations where decision makers are more concerned with undesirable outcomes. The VaR allows the decision makers to set a confidence level *η*, and the VaR can be defined as a threshold such that the probability that the profit below this value is *η* (Artzner et al., 1997). VaR has some limitations, e.g. the tail end of the distribution of profit or loss is not assessed, and suffering from being unstable and difficult to work with numerically when losses are not ‘‘normally’’ distributed (Rockafellar and Uryasev 2002). CVaR is a more recent development of risk measures that can overcome the shortcoming of VaR. It measures the average profit falling below the *η*-quantile level, and ignores the contribution of profit beyond the specified quantile (Rockafellar and Uryasev 2000). This paper will adopt the CVaR model to measure the risk attitude of ocean carriers as they are more concerned with capacity underutilization and profit losses.

The above risk-averse measurement methods have rarely been considered in the maritime industry. In the following, we give a brief review of the literature on risk-averse behavior in broad context of supply chain management and logistic industry.

Agrawal and Seshadri (2000) presented an inventory model to explore the impact of uncertainty and risk attitude on price and order quantity of a retailer in a newsvendor problem. They found that a higher price and a lower order quantity will be developed by a risk-averse retailer if a change in price affects the scale of the distribution. Gan et al. (2005) designed a risk-sharing contract which can coordinate a supply chain composed of a risk-neutral supplier and a risk-averse retailer. Yang et al. (2009) presented a game theoretic model in a supply chain consisting of one supplier and two risk-averse retailers. The retailers compete with each other in price, service and lot size. They found that a retailer will reduce its price and increase its service level if the supplier increases its capacity. The effect of retailer's risk-averse attitude on its rival's price depends on its service cost. Ma et al. (2012) examined a Nash-bargaining problem in which a risk-neutral manufacturer and a risk-averse retailer negotiate about the order quantity and wholesale price. They found that the retailer’s bargaining power for the supply chain profit increases as she becomes more risk averse. Li et al. (2014a) considered the pricing strategy in a dual-channel supply chain consisting of a risk-neutral manufacturer and a risk-averse retailer. They utilized the Nash bargain model and found that the retail price will decrease as the retailer becomes more risk-averse. Li et al. (2014b) studied the promised delivery lead time and price decisions of a risk-averse firm in a situation that the market demand depends on the selling price and the promised delivery lead time. They illustrated that the influence of the decision-makers’ risk-averse attitude on the optimal decision of price and the guaranteed delivery lead time.

In the logistics industry, Liu and Wang (2015) investigated the effect of different risk attitudes in a logistics service supply chain. They found that the integrator prefers a risk-seeking provider. Cui et al. (2016) studied the optimal decision of a risk-averse retailer by introducing her store brand product and using the mean-variance formulation in a two-echelon logistics system. Xiao et al. (2013) developed an adaptive navigation approach for risk-averse travelers in a stochastic network. They applied the prospect theory to measure the travelers’ utility. Kang et al. (2014) proposed a model to generate the optimal route decision for a hazmat shipment with Value-at-Risk method. They obtained different paths and concluded that the route choice is a function of the level of risk tolerance of the decision makers. Soleimani and Govindan (2014) applied a risk-averse two-stage stochastic programming approach to design a reverse supply chain network with CVaR method.

To the best of our knowledge, no research has been published on the shipping lines’ competition taking into account risk-averse behavior. The current container shipping marketplace is experiencing a high degree of uncertainty and fierce cost-based competition. The drastic fluctuations of freight rates since 2015 and the bankruptcy of the seventh largest shipping line (i.e. Hanjin Shipping) in early September 2016 reflected the intensity of the competition. Shipping service overcapacity has put a huge pressure on the shipping lines, especially on the smaller ones. It is reasonable that smaller shipping lines would adopt risk-averse behavior in order to mitigate negative impact of uncertainty. This paper will analyze the impact of risk-averse behavior of a smaller shipping line on the pricing strategies of both larger and smaller shipping lines when facing uncertain and competitive spot market.

**3. Model Description and Notations**

We consider a transport system consisting of two liner shipping companies, denoted by carrier 1 and carrier 2, who compete with each other for the spot market demands in a specific shipping service route. Assume that carrier 1 is a large company with sufficient carrying capacity, while carrier 2 is a small one with constrained carrying capacity. In practice, carrier 1 can represent a major shipping line who may operate multiple strings in the given service route, whereas carrier 2 can represent a smaller shipping line or a NVOCC who has pre-purchased a fixed number of slots from major shipping lines in the given service route. In the spot market, customer demands are uncertain and the selection of shipping lines is mainly based on freight rate (i.e. shipping price). This represents the fluctuation of trade volume and the volatility of shipping price in liner shipping industry in the last few years.

In this paper, we limit our service route into a two-location structure, under which two carriers essentially offer the same service product. The selection of this type of service structure can be justified as follows. Firstly, our problem setting is particularly suitable for two-port shuttle services, which are not unusual in the liner shipping industry. For example, Song (2007) stated that based on the data in 2002, among a total of 1521 regular shipping services, 253 of them are two-port shuttle services. Specific examples of two-port shuttle services include: Wuhan-Shanghai shuttle service operated by Jihai Shipping (a subsidiary of Shanghai International Port Group); Wuhan - Shanghai shuttle service operated by Cosco-owned Shanghai Fanya; and Fremantle - Singapore shuttle service operated by Pacific Asia Express. In the long-haul liner services, Cosco Container Lines once launched a weekly trans-Pacific shuttle service between Shanghai and Long Beach in 2004. Secondly, the two-location one product service may be regarded as an approximation to a service route that connects two continents, e.g. a trans-Pacific service route that calls a few ports in China and a few port in USA, or a trans-Atlantic service that calls a few port in North Europe and a few ports in USA. In such cases, the ports in the same continents may be clustered together as a virtual port because the cargos on the vessels are predominately between two continents and the freight rates are similar between two ports that are located in two continents. Song and Dong (2013) reported that among 154 long-haul service routes in three major trade lanes (Asia–North America, Asia–Europe, Europe–North America) in 2008, 128 service routes connect only two continents. Thirdly, for our problem setting, it is analytically tractable to characterize the pricing strategies and the impacts of system parameters that lead to managerial insights. In fact, two-location transportation systems were often used as a starting point for similar types of research in the literature, e.g. Zhou & Lee (2009); Xu, et al. (2015); Chen, et al. (2016).

This paper will focus on the pricing decision-making problem for carrier 1 and carrier 2 facing uncertain spot market demand. Because of the intensive competition in the spot market, the smaller carrier is normally in a disadvantageous status and cares more about loss and business risk owing to its limited capacity. Therefore, it is rational to assume that carrier 2 may have risk-averse behaviors in its decision making. It should be noted that a risk-averse company means the decision maker (or planner) of the company is risk-averse. In that sense, a company’s risk attitude is similar to a person’s risk attitude. In the general supply chain context, risk-averse companies/players are common. For example, risk-averse retailers are studied in the literature such as Vipul and Sridhar (2000), Gan et al. (2005), and Li et al. (2014a); risk-averse manufacturers/suppliers are considered in the literature such as Tsay et al. (2002), Wu (2010) and Shen et al. (2013). In addition, companies are also assumed to be risk-averse players in many other research fields like finance and economics. For example, Jullien et al. (2007) considered the risk-averse agents under moral hazard in insurance market. Bruno et al. (2016) proposed a model to obtain the strategies for investing in new projects by considering risk attitude.

In the shipping industry, Freight Forwarders or NVOCCs may purchase large bundles of slots from ocean carriers (shipping lines) and then sell the slots to shippers. In that sense, Freight Forwarders and NVOCCs can be regarded as a small shipping line. The risk-averse behavior of freight forwarders is more evident, e.g. Dominik Tichelkamp (the global head of ocean freight of CEVA, which is the 14th largest ocean Freight Forwarder) explicitly stated that “CEVA is rather risk-averse” when they sell slots to shippers (Lennane 2013). In addition, Lee et al. (2015) investigated whether ocean carriers should bear some of the “price risk” by offering a “fractional” price matching contract in which the shipper pays a constant contracted freight rate in advance. If the realized spot price is below the regular price, the ocean carrier will refund the shipper a “fraction” of the difference between the regular price and the realized spot price.

Due to severe overcapacity in the current shipping environment, large ocean carriers are often not constrained with shipping capacity. Small ocean carriers (e.g. Freight Forwarders or NVOCCs) are more vulnerable to capacity utilization, because they are small, and/or have to pay the charges for the purchased slots in advance regardless whether these slots are utilized. Therefore, the risk-averse attitude and the capacity constraint are important characteristics for smaller carriers in the container shipping spot market. In addition, the benchmark price (which represents the long-term contracted freight rate) should be considered when making pricing decisions for spot market.

The notations and parameters in the paper are defined as follows:

|  |  |
| --- | --- |
|  | , where  represents the larger carrier and  represents the smaller carrier. |
|  | the forecasted market demand, which is a random variable with the probability density function and the cumulative distribution function , respectively. |
|  | the fraction of the forecasted market demand occupied by carrier 2, then  represents the forecasted market demand to be met by carrier 1, where *θ*∈(0, 1). |
|  | the price sensitivity for carrier , . |
|  | the shipping price per unit as a benchmark, which can be regarded as the reference freight rate or the long-term contracted price; |
|  | the shipping price per unit for carrier , which is a decision variable, . |
|  | the competition intensity of the price for carrier , . |
|  | the shipping cost per unit for carrier ,. |
|  | the random demand for carrier ,. |
|  | the shipping capacity of carrier 2. |
|  | the profit of carrier , . |
|  | the risk-averse indicator of carrier 2, where . |

We assume that the demand for one carrier’ service depends linearly on its own price and the rival’s price as shown in the following equation:

, (1)

 . (2)

The linear demand models for two or more players in the market have been commonly used in the supply chain and logistics literature (e.g. Charnes et al. 1972; Choi 1991; Tsay et al. 2000; Majumder and Groenevelt 2001; Xiao et al. 2008; Kurata et al. 2007; Zhou et al. 2009; Li et al. 2014a; Chen et al. 2015; Liu et al. 2016). In particular, the linear random demand function was adopted in Li et al. (2014a) and Liu et al. (2016). In this paper, we made the similar assumption.

The parameter *θ* is introduced to represent market shares of two shipping lines in the spot market, which may be determined by the factors such as service, size and reputation. The cases of *θ*<0.5 represent the situations that carrier 1 has a larger share of the spot market, whereas *θ*>0.5 indicates that carrier 2 has a larger share of the spot market. The case of *θ* =0.5 is equivalent to the situation that the parameter *θ* is removed and two carriers are competing for the whole demand. In addition, the parameter *θ* can also be regarded as customer’s preference, or the degree of customer’s loyalty to the carriers (Liu et al. 2016, Feng et al. 2009). Note that the preference of customers is often not very susceptible to the prices. In container shipping, each shipping line often has its own customer base due to its supply chain relationship and network. In the general supply chain literature, this setting of demand split has been adopted in Tsay et al. (2000), Li et al. (2014a), Liu et al. (2016) and Feng et al. (2009). The parameter  represents the demand that carrier *i* attracts from its competitor by cutting one unit of price, whereas *αi* + *γi* indicates the total increased demand when carrier *i* decreases its shipping price by one unit. To simplify the narrative, let ,. Then, the total demand is:

=.

Here, some constraints must be satisfied, that is, ,, and , . It should be noted that the constraint  may be not needed if we allow shipping lines to make a loss, e.g. when a shipping line operates a global shipping network, it is acceptable to make a loss for some service products.

Since carrier 2 has the finite shipping capacity , he can only generate the revenue from the fulfilled demands constrained by his shipping capacity. The unmet demands are assumed to be lost. Thus, the profits for two carriers can be given by:

, (3)

.

Note that both *D*1 and *D*2 are random variables. The expression min(*C*, *D2*) can be explained in three cases. If the value of *C* is smaller than the whole support of the distribution of *D*2, then the result of min(*C*, *D*2) will always be the scalar value *C*. If the value of *C* is greater than the whole support of the distribution of *D*2, then the results of min(*C*, *D*2) will be the random variable *D*2 itself. If the value *C* lies within the support of distribution of *D*2, then the result will be a mixed random variable. More specifically, the continuous part of the result will be in the range (-∞, *C*), and the density function in that range will be the same as that of *D*2, while its area will be integrated up to *F*(*C*), where *F*(.) is the cumulative distribution function of random variable *D*2. The remaining 1 – *F*(*C*) area is actually vaporised to the probability of the value *C*. Clearly, *π*1 and *π*2 are also random variables because of the random demand. Denote , . The carrier 2’s profit function can be rewritten as:

. (4)

To model the risk-averse behavior of carrier 2, we first introduce the concepts of value at risk (VaR) and conditional value at risk (CVaR). Let  be the of the profit , i.e. Prob. Let *Fπ*(*.*) denote the cumulative distribution function of *π*. We have: . The above formula defines the value at risk (Artzner et al., 1997), . Statistically,  represents the confidence level that profit *π* is greater than. In that sense, the VaR criterion allows managers to limit the likelihood of incurring profits (or losses) caused by uncertain events, which represents the risk-averse behaviour. More specifically,  can be interpreted as a risk-averse indicator of the carrier taking a value in the interval (0, 1]. When, the carrier is risk-neutral; otherwise it is risk-averse. Moreover, as  decreases towards 0, the carrier tends to be more risk-averse, i.e. tends to prefer a higher confidence level to generate at least a specific amount of profit.

Note that VaR has a limitation that the tail end of the distribution of profit (or loss) is not assessed. Therefore, if profits (or losses) are incurred at the tail end, the amount of profits (losses) could be substantially small (or large) in value. Conditional Value at Risk (CVaR) was created as an extension of VaR to overcome this shortcoming. Mathematically (Rockafellar and Uryasev 2000，2002; Sarykalin et al. 2008),

,

where = 1, if  ; , if. It should be noted that the above definition of CVaR is slightly different from the one in Sarykalin et al. (2008), because their definition is based on the objective function representing the loss (that means the losses exceeding VaR should be accounted for). In our context, the objective function is the profit, which means the profits less than VaR should be accounted for.

Intuitively, for a random variable *π* with continuous distribution function, CVaR*η*(*π*) equals the conditional expectation of *π* subject to *π* < VaR*η*(*π*). Namely, CVaR is derived by taking a weighted average between the value at risk and profits less than the value at risk.

In this paper, we use the CVaR as the criterion to evaluate the risk-averse behavior. CVaR can be expressed by a minimization formula. Following the arguments in (Artzner et al., 1997; Rockafellar and Uryasev 2000), carrier 2’s utility under criterion can be defined as follows:

. (5)

It is easy to show that the expression within {} on the right-hand-side of (5) is a convex function with respect to *v* in the interval [0, +∞); and it reaches its maximum when *v* takes *vη*(*π*2), i.e. the value of VaR. Thus, we can set the domain of *v* as [0, +∞). Now the problem can be formulated as follows:

 and . (6)

**4. Model Analysis**

In this section, we aim to derive the optimal pricing strategy for risk-neutral carrier 1 and risk-averse carrier 2 at the Nash equilibrium state. We will also analyze how the risk-averse indicator and other parameters influence two carriers’ pricing decisions.

**4.1 Equilibrium Solutions**

The equilibrium solution can be found by using the first order method. First, we need to analyze CVaR*η*(*π*2). From (2), (4) and (5), the utility of carrier 2 through the method can be given:

.

To further simplify the narrative, let , and . Then, , and  can be rewritten as follows:



.

To discuss the above expression, we consider the following two cases:

(1) If , then



.

Note that , which indicates that  decreases in , thus  reaches its maximum at the lower boundary point, i.e. .

(2) If , then





.

Obviously,  and . Meanwhile, note that

.

It follows that and  is concave. Define . Thus,  reaches its maximum at , if ; otherwise, its maximum is reached at . That is

.

Note that





,



.

Thus, we have

 (7)

With regard to the constraints in the above equations, the amount  can be regarded as the perceived demand of carrier 2 with the risk-averse indicator *η*. Thus, the above equation can be interpreted as follows: when the perceived demand of carrier 2 is less than its carrying capacity, CVaR*η*(*π*2) takes the first form, otherwise, it takes the second form.

It is known that in a strategic form game with infinite actions, a pure strategy Nash equilibrium exists if the following conditions are satisfied: (i) the action sets of each player are non-empty, compact and convex, (ii) the payoff functions are continuous with respect to the actions, (iii) each player’s payoff function is concave in its own action (Debreu 1952; Fan 1952; Glicksberg 1952).

In our model, the actions of two carriers are price decisions (i.e. *p*1 and *p*2), which take values within closed intervals in the practical context. From (6) and (7), it is easy to see that the payoff functions of two carriers, *E*(*π*1) and *CVaRη*(*π*2), are continuous in both *p*1 and *p*2. In addition, we will show that the payoff functions are concave in the player’s own price decision in the following.

To further simplify the narrative, let ,. Now we can obtain the equilibrium solutions by solving the above game model under two cases as follows：

1. When , that is, the capacity of carrier 2 is greater than its perceived market demand, we have





and







It follows that *E*(*π*1) is concave in *p*1 and *CVaR*(*π*2) is concave in *p*2, when . Note that  is a linear equation with respect to *p*2. Therefore represents an interval for *p*2.

1. When , that is, the capacity of carrier 2 is less than its perceived market demand, then we have



,

.

It follows that *CVaR*(*π2*) is concave in *p*2 within the interval specified by .

We can easily verify that at the boundary point of *p*2 satisfying , the first partial derivative of *CVaR*(*π*2) with respect to *p*2 takes the same value in the above two cases. Namely, ∂*CVaR*(*π*2)/∂*p*2 = (–*A*2 + *H*2) – (*p*2 – *c*2)(*α*2 + *γ*2) when . Hence, ∂*CVaR*(*π*2)/∂*p*2 is continuous in *p*2, and *CVaR*(*π*2) is differentiable with respect to *p*2. Together with the analysis of the above two cases, it follows that ∂*CVaR*(*π*2)/∂*p*2 is monotonically decreasing, and thus the payoff function *CVaR*(*π*2) is concave in *p*2. Note that carrier 1’s profit takes the same form in both cases. Thus, the payoff function *E*(*π*1) is concave in *p*1. In addition, the action sets of two carriers are characterized by closed intervals, which are non-empty, compact and convex. Therefore, there exists a pure strategy Nash equilibrium in our game model.

The equilibrium solution can be obtained by solving two first order differential equations as follows.

If , we have,

, . (8)

If , we have,

. (9)

 is given implicitly by . (10)

Where .

**Proposition 1.** In the Nash bargaining model, there exists a threshold value  for carrier 2’s capacity, and the equilibrium solutions are given as follows:

, (11)

. (12)

**Proof**: Equations (8)~(10) define two sets of pricing strategies corresponding to the cases  and  respectively. That means the pricing strategies are separated by a specific capacity level of carrier 2. This capacity level is termed as the capacity threshold value, denoted by *C*\*. Because of the continuity of CVaR(π2) and E(*π*1), we should have:  and  at the boundary point *C* = *C*\*. Note that the expressions of  and  are independent of carrier 2’s capacity *C*. Hence, we can use  and  to express the capacity threshold  as given in (12). This completes the proof.

The optimal profit and utility of two carriers can be obtained by plugging the optimal pricing solutions into equation (6). An interesting point can be observed from Proposition 1. There exists a threshold value for carrier 2’ shipping capacity, which characterizes the optimal pricing strategy into two forms. When carrier 2’s capacity is greater than , two carriers are facing a buyer’s market and the competition of two carriers is intense. In this situation, they will reduce their prices to grab more customers’ orders until reaching an equilibrium by choosing the optimal prices in the form of , . We call this situation as High Capacity (HC) condition.

When carrier 2’s capacity is less than , the market situations are different for two carriers. Carrier 1 is still confronting the buyer’s market, whereas carrier 2 is facing a seller’s market. Hence, carrier 2 may increase its price to gain more profits. However, because of its risk-averse attitude, even though its capacity is limited, it will not always raise its price. The equilibrium will be reached when two carrier choose the optimal prices in the form of  and . We called this situation as Low Capacity (LC) condition.

A special case of Proposition 1 is that both carriers have sufficient shipping capacity; then the optimal pricing strategy at Nash equilibrium can only be given by Eq. (8), i.e. the form under the HC condition.

It should be pointed out that the existence of two pricing strategies is the result of the capacity constraint. Without the capacity constraint, we would have only one equilibrium pricing strategy that continuously changes with the risk-averse attitude. However, the combination of two effects (capacity constraint and risk-averse attitude) is more interesting, because the risk-averse attitude could influence two carriers’ decisions on which pricing strategy to adopt. Although the existence of the threshold value *C*\* is the result of the capacity constraint, it is a function of the risk-averse indicator *η*. On the one hand, the threshold value *C*\* characterizes the optimal choice of two pricing strategy according to carrier 2’s current shipping capacity; on the other hand, the formula of the threshold value *C*\* determines how the risk-averse attitude influences such choice at the equilibrium.

**Proposition 2**. Under the HC condition, (i) the optimal prices of two carriers are symmetric and given in Eq. (8); (ii) the optimal prices for two carriers are increasing in the benchmark shipping price and the cost of each carrier; (iii) the optimal price decisions under the HC condition is independent of the capacity constraint *C*.

**Proof**: these assertions can be easily observed from equation (8).

In the current shipping environment, overcapacity is severe. Drewry Shipping Consultancy reported that the number of idle containerships reached 435 in total with aggregating 1.7 million TEUs in early November 2016, which accounts for 9% of the global containership fleet (Drewry 2016). This implies that major shipping lines are having sufficient carrying capacity in the current shipping market. The competition of two major shipping lines with one being risk-neutral and the other being risk-averse would fit the scenario of Proposition 2. Hence, the results in Proposition 2 can provide managerial insights into the current shipping environment.

In the above analysis, the probability distribution of the market demand is general. Namely, no specific type of demand distribution is assumed. This leads to general but complicated forms of the optimal prices and even an implicit expression for , which makes the theoretical analysis difficult. To ease the theoretical analysis, we assume that the random demand follows a uniform distribution.

**Proposition 3.** Assuming that the random demand  follows a uniform distribution U(*a*, *b*), the equilibrium solutions  and the threshold value  can be simplified as follows:

(i) If , then , and  is given implicitly by the solution of to , where

, , ,

and , .

(ii) If , then

, (13)

; (14)

(iii) . (15)

**Proof**: Since the random demand  follows a uniform distribution U(*a*, *b*), we have:  ,. It follows that , . From Proposition 1, we can obtain the results in assertions (i)~(iii). This completes the proof.

**Corollary 1.** Assuming that the carrier 2is risk neural, i.e. , and the random demand  follows a uniform distribution U(*a*, *b*), then the equilibrium solution  and the threshold value  are given as follows:

1. If , then , and  is given implicitly by the solution of ,

where , , ,

and , .

(ii) If , then

,

;

where .

From Corollary 1, we can see that the equilibrium solution for the case with a risk-neutral carrier 2 can also be characterized by two regimes based on the capacity of the second carrier. However, in the case with a risk-averse carrier 2, an important feature is that the threshold value for carrier 2’ shipping capacity is varying in carrier 2’s risk preference. We will analyze this point in more detail in next section.

**4.2 Analysis**

Firstly, from Proposition 3, the properties of  with  and  are given as follows:

**Proposition 4.** Assuming that the random demand  follows a uniform distribution U(*a*, *b*) and other parameters are constant. Then two carriers’ optimal prices  and  are increasing as carrier 2’s risk-averse indicator  increases in .

**Proof**: If , we can get

; .

If , we can get

,

Note that . It follows, , .

Proposition 4 can be explained as follows. When carrier 2 becomes more risk-averse, it tends to decrease the price to attract more customers to make better utilization of its shipping capacity. Carrier 1 will then respond to decrease its price as well because of the competition between two carriers, even though carrier 1 is risk-neutral. On the other hand, the highest possible prices of two carriers would be achieved if carrier 2 becomes risk-neutral. The profits of two carriers will subsequently be affected by risk-averse indicator . However, due to the complicated forms of the profit functions, it is difficult to analyze the impact analytically. We will examine the impact of *η* on two carriers’ profits numerically in the numerical experiment section.

**Proposition 5.** Assuming that the random demand  follows a uniform distribution U(*a*, *b*) and other parameters are constant.

1. Let *C*\*(*η*) denote the threshold value, defined in Proposition 3, as a function of carrier 2’s risk-averse indicator *η*. Then the threshold value (*η*) is increasing as  increases in the interval (0, 1].
2. If the carrier 2’s capacity *C* is in the range of , then as carrier 2’s risk preference *η* increases to a certain level, it will trigger the switch of two carriers’ price strategies from one regime to the other. That is, there is a *boundary jump* for pricing strategies in response to carrier 2’s risk-averse attitude.
3. *C*\*(1) – *C*\*(0) = is increasing in *γ*1 and *γ*2 (competition intensity parameters), but decreasing in *α*1 and *α*2 (price sensitivity parameters).

**Proof**: From Proposition 3(iii), we have

 .

It follows that the threshold value (*η*) is increasing as  increases in (0, 1]. Hence, assertion (i) is true.

Note that *C*\*(*η*) characterizes two carriers’ price strategies into two regimes. From assertion (i) and Proposition 3, there exists a unique point *η*0∈(0, 1) such that *C*\*(*η*0) = *C*. When *η*<*η*0, we have *C*\*(*η*) < *C*; two carriers will take the pricing strategy in Proposition 3(ii). On the other hand, when *η*>*η*0, we have *C*\*(*η*) > *C*; two carriers will take the pricing strategy in Proposition 3(i). Therefore, *η*0 can be regarded as the boundary where two carriers switch their pricing strategies. Assertion (ii) holds.

For Assertion (iii), from Proposition 3(iii), we have: *C*\*(1) – *C*\*(0) =. Due to the symmetry of the expression with respect to the parameters, we only need to consider its sensitivity to *α*2 and *γ*2.

,

.

Thus, assertion (iii) is true. This completes the proof.

Proposition 5 shows how the threshold value *C*\* responds to the change of carrier 2’s risk-averse attitude . Physically, the threshold value *C*\*(*η*)can be interpreted as carrier 2’s perceived demand when carrier 2 takes the risk-averse preference at a level *η*. It is possible that as  increases in , the corresponding threshold value (*η*) exceeds the carrier 2’s shipping actual capacity *C*, which means when carrier 2 becomes less risk-averse, its perception about the market demand becomes more optimistic and even over-optimistic. As a result, the operation environment may change from HC condition to LC condition. This implies that carriers’ pricing strategies may take different forms (e.g. the price strategies in HC and LC conditions given in Proposition 3) as the carrier 2’s risk-averse attitude changes. We use a numerical example to illustrate this point. Fig. 1 shows the curve of *C*\*(*η*) with varying *η* for a specific example. It divides the pricing strategies into two regimes: HC Condition and LC Condition. When carrier 2’s capacity is at a moderate level (i.e. in the range of ), its price strategy will transform from the HC regime to the LC regime as the risk-averse indicator  reaches a certain level (i.e. when *C*\*(*η*) intersects with *C*). This result will be further illustrated in the numerical experiments (see Fig. 2).



Fig. 1. *C*\*(*η*) with varying *η*

Proposition 5(iii) gives the range of carrier 2’s capacity, under which the switch of the pricing strategies is possible when *η* changes. This range of the switch boundary is increasing as the competition intensity parameter (*γ*1 or *γ*2) increases. The implication is that in a more competitive market carrier 2’s perceived demand *C*\*(*η*) tends to be more sensitive to the risk-averse indicator *η*, which results in a larger range of capacity for carrier 2, under which the pricing strategy will switch from one regime to the other when the risk-averse indicator changes. On the other hand, the price sensitivity parameter (*α*1 or *α*2) has the opposite impact on the range of the switching boundary. This may be interpreted as: when a carrier’s demand becomes more sensitive to its own price, carrier 2’s perceived demand *C*\*(*η*) tends to be less sensitive to the risk-averse indicator.

Next, we consider the impacts of price sensitivity and competition intensity parameters on the optimal prices. To simplify the analysis, we let the benchmark price. We also assume that two carriers have the same price sensitivity in the spot market. Let =, then we can obtain more specific results as follows:

**Proposition 6.** Assuming that the random demand  follows a uniform distribution U(*a*, *b*), and , =. Suppose both carriers have sufficient shipping capacity, then

(i) The prices of two carriers  and  are decreasing in .

(ii) The prices of two carriers  and  are decreasing in  if , and increasing in  otherwise; where.

The prices of two carriers  and  are decreasing in  if , and increasing in  otherwise; where .

(iii) Under *α* ≠ 0, two carriers’ prices are increasing in  and  if carriers’ shipping cost  and/or  is sufficiently large.

(iv) Under *α* = 0, two carriers’ prices are decreasing in  and  regardless the values of  and .

**Proof**: Since , , and both carriers have sufficient shipping capacity, by Proposition 1 and Proposition 3, we have , and



On the right-hand-side of the above equation, we select some terms from the numerators including all positive terms, we have







It follows that . Note that the price strategies of two carriers are symmetric when the capacity of carrier 2 is sufficiently large. Moreover, ,  are both positive,  and  take values between 0 and 1. Thus, we can get  as well. Proposition 6(i) is proved.

Now let’s examine how the competition intensity influences on two carriers’ optimal pricing decisions. We focus on the competition intensity γ1 and γ2 by assuming other system parameters are fixed. The impact of the competition intensity γ1 on the optimal pricing decision variables can be given as follows,

 and 

Where 

It can be observed that  is equivalent to , which is further equivalent to the condition: . That is, both carriers’ prices are decreasing as the competition intensity γ1 increases, if and only if the condition  holds. This yields the first part of Proposition 6(ii).

Similarly, the impact of the competition intensity γ2 on the optimal pricing decision variables can be given as follows,

, and 

Where 

Both carriers’ prices are decreasing as the competition intensity γ2 increases, if and only if the condition  holds. This yields the second part of Proposition 6(ii).

Moreover, from the expression of *I* and *J*, clearly it is independent of the shipping cost parameters  and . If and/or  increases to a certain level, we must have and  under the conditions *α* ≠ 0. From Proposition 6(ii), we know the assertion (iii) is true. Finally, Proposition 6(iv) can be similarly derived from Proposition 6(ii). This completes the proof.

Proposition 6(i) may be interpreted as follows. Since the price sensitivity parameter  represents the degree of demand changes in response to the change of price, increasing price sensitivity parameter from a Nash equilibrium would imply the decrease of demand for both carriers (but in different scales because their optimal prices are different). This causes two carriers to reduce prices in order to reach a new Nash equilibrium. Proposition 6(ii) is more interesting and counter-intuitive to some extent. It states that the competition intensity could have either negative impact on two carriers’ prices, or positive impact on two carriers’ prices. More importantly, Proposition 6(ii) provides a necessary and sufficient condition to characterize whether two carriers’ optimal prices would be positively or negatively affected by the competition intensity parameter  and . More specifically, two carriers’ optimal prices  and  are decreasing in  if and only if ; and the optimal prices  and  are decreasing in  if and only if . Note that *J* is decreasing in *θ* with *J*(*θ*=0) = (*a*+*b*)(4*α*+3*γ*2) and *J*(*θ*=1) = -2*α* [(*b*-*a*)*η* + 2*α*], where *θ* represents the market share of carrier 2 and 1 – *θ* represents the market share of carrier 1. From the above necessary and sufficient conditions, we can observe that when carrier 1 has a large market share (i.e. *θ* → 0), it is likely that the condition  holds. This implies that carrier 1’s competitive intensity *γ*1 could have a negative effect on both carriers’ prices when *θ* is small. On the other hand, as carrier 1’s market share decreases to a certain point (i.e. *θ*  is increasing), we will certainly know that  cannot be satisfied because *J*(*θ*=1) is negative. Under such circumstance, carrier 1’s competitive intensity *γ*1 would have a positive effect on both carriers’ prices. From Proposition 6(ii), the impact of carrier 2’s competitive intensity *γ*2 on the optimal prices can be similarly interpreted in relation to carrier 2’s market share. In addition, Proposition 6(iii) reveals that when shipping cost parameter  and/or  increases to a certain level under the conditions *α* ≠ 0, the competition intensity  and  would have positive impact on both carriers’ prices. Proposition 6(iv) considers a scenario that the price sensitivity is negligible (i.e. *α* = 0) whereas the competition intensity is significant (i.e. *γ*1 ≠ 0 and *γ*2 ≠ 0). In such scenario the competition intensity  and  would always have negative impact on both carriers’ prices.

It should be noted that the results in Proposition 6 are based on the assumptions that , =, and both carriers have sufficient shipping capacity. In more general cases, analytical results are difficult to obtain. Numerical methods will be used to explore whether the results can be generalized in the next section.

**5. Numerical Experiments**

This section serves two purposes. First, some of the analytical results established in the previous sections will be verified numerically. Second, we further investigate the effects of some system parameters on the price decisions and the profits (or the utility) in more general cases. More specifically, we will examine the impacts of carrier 2’s risk-averse indicator, the impacts of the price parameters, and the impacts of the capacity constraint in three sub-sections respectively.

The following system parameters are set up largely based on Lee et al. (2015) and Chen et al. (2016). Let , , , . Since carrier 2 is a smaller company, we suppose its operating cost is higher than carrier 1’s. That is, , . The uncertain market demand *ξ* is assumed to follow a uniform distribution U(*a*, *b*) with *a* = 80000 and *b* = 150000.

From proposition 5(i), when  changes from 0 to 1, the threshold value *C*\*(*η*) increases from 39806 to 55726. Three cases can be categorized. First, when the capacity of carrier 2 is lower than 39806, no matter what value  takes, *C* is always lower than . This case belongs to the *LC condition only*. Second, when the capacity of carrier 2 is higher than 55726, we always have *C* > *C*\* regardless the value of *η*. This case belongs to the *HC condition only*. Third, when carrier 2’s capacity is in the range , whether this case belongs to HC or LC condition will depend on the value of . We can term this case as the *medium capacity (MC) condition*. To compare the influence of carrier 2’s capacity on price decisions and profits, we take three levels of carrier 2’s capacity as follows: *CH* = 80000 in the HC condition only category, *CL* = 20000 in the LC condition only category, and *CM* = 50000 in the MC condition category.

*5.1. Impacts of the carrier 2’ risk-averse indicator*

The results of how carrier 2’s risk-averse indicator influences the pricing strategies and the profit or utility are displayed in Fig. 2 and Fig. 3 respectively. To save space, we put the results of the HC condition and the LC condition cases on the left side of the figure and the results of the MC condition case on the right side of the figure.



|  |  |
| --- | --- |
| (a) HC & LC condition | (b) MC condition |

**Fig. 2.** The effects of  on the prices of two companies

Fig. 2(a) shows the optimal prices of two carriers in the HC condition and LC condition cases with varying *η*. Fig. 2(b) shows the optimal prices in the MC condition case. Clearly carrier 2’s prices are increasing as carrier 2 becomes less risk-averse in all three cases. Due to the competition of two carriers, carrier 1’s prices also change in the same direction as carrier 2’ prices when the risk-averse indicator changes, but in much less significant mode. These observations confirm Proposition 4. In addition, it is observed that carrier 2’s price is lower than carrier 1’s in all three cases, which may be caused by carrier 2’ weak market position (e.g. lower market share, higher operating cost). Furthermore, the price gap between two carriers is significantly higher in the HC or MC condition case than in the LC condition case, and this gap is decreasing as the risk-averse indicator increases. The former can be explained by the fact that under the LC condition it is possible that customer demands may exceed the capacity constraint due to the demand uncertainty. Since the excessive demands will be lost, carrier 2 tends not to quote the price as low as that in the HC or MC condition case. The latter can be explained by the risk-averse behaviour of carrier 2.

From Fig. 2(b), we can see the *boundary jump* phenomenon described in Proposition 5(ii). Note that the threshold value *C*\*(*η*) is increasing in . When  increases to 0.6403, denoted by , we have *C*\*(*η*0) = *CM* = 50000. Thus, when *η* < *η*0, we would have *CM* > *C*\*(*η*), which represents the HC condition case. On the other hand, *η* > *η*0 represents the LC condition case. Two carriers will transform their pricing strategies when carrier 2’s risk-averse indicator reaches *η*0.



|  |  |
| --- | --- |
| (a) HC & LC condition | (b) MC condition |

**Fig. 3.** The effects of  on the utilities of two carriers

Define E(*π*1) as carrier 1’s utility and CVaR(*π*2) as carrier 2’s utility. Fig. 3 shows the utilities of two carriers with varying *η*. In general, the utilities in all three cases are increasing as carrier 2 becomes less risk-averse.

Now let us examine how the carrier 2’ risk averse indicator influences on carrier 2’s expected profit, E(*π*2). We take the MC condition case as an example. The results are shown in Fig. 4(a). The change rate of carrier 2’s profit could be quite different for different risk-averse indicators, e.g. there exist at least two points O and Q in Fig. 4(a), where the rate changes suddenly. More importantly, it can be seen that carrier 2’s expected profit is not necessarily always increasing in *η*, although its price is indeed increasing in *η* as shown in Fig. 2(b). This point is intriguing. The implication is that appropriately selecting risk-averse preference would maximize the expected profit.

When there is a single carrier who monopolies the market, its risk-averse attitude will lead to lower expected profits, which is result to trade off for downside protection against possible losses. When there is a competition in the market, the customers are not only affected by the price of one carrier, but also affected by the price gap between two carriers. Under the assumption of risk-neutral attitude, two carriers would achieve the Pareto optimality at the equilibrium point, and they will not unilaterally change their pricing decisions. However, when one carrier has risk-averse attitude, it tends to choose a lower price, whereas the other risk-neutral carrier will not reduce its price at the same degree (because of its risk-neutral attitude). As a result, the price gap between two carriers increases. The risk-averse carrier can then snatch more demands at the new equilibrium point. In that case, the risk-averse carrier will gain higher profits than before (i.e. when it is risk-neutral). This may be explained by the demand function , which will be increasing as the price gap increases particularly in a market with high competitive intensity γ2. In our research context, with the increase of risk-averse attitude (i.e. the decrease of *η*), the decrease in price will generate additional demand for carrier 2 that could be sufficient to offset the loss caused by the price decrease at the beginning stage. This implies that carrier 2’s expected profit could be increasing as it changes from risk-neutral to be risk-averse. However, when carrier 2’s risk-averse indicator reaches a certain point, its expected profit will be decreasing because the loss due to the price decrease will become larger than the gain from the additional demand. In Fig.4, with the change of carrier 2’s risk-averse attitude, carrier 2’s pricing decision crosses the LC condition and the HC condition. It should be noted that the competition intensity in the two conditions is the same, and the price gap in the LC condition increases more rapidly than in the HC condition. Evidently, appropriately choosing risk-averse preference can help carrier 2 to maximize its expected profit.

To further illustrate the combined effect of the competition intensity (assuming γ2≥γ1) and the risk-averse indicator on carrier 2’s expected profit, a few additional examples are shown in Fig. 4(b), in which  takes 40, 60 and 80, γ1=40, and carrier 2 takes different risk attitudes. It should be pointed out that the profit curve corresponding to γ2=60 in Fig. 4(b) is the same as that in Fig. 4(a). The points denoted by N1, N2 and N3 in Fig. 4(b) represent the maximum expected profit of carrier 2 for the cases γ2=80, 60, 40, respectively. It can be observed as  increases, carrier 2’s maximum expected profit will be achieved at a more risk-averse indicator (i.e. smaller*η*). This observation is interesting. However, we are not able to establish this relationship analytically due to the complexity of the solutions. More research is required in this direction.



|  |  |
| --- | --- |
| (a) the impact of risk attitude | (b) the impact of competition intensity |

**Fig. 4.** The effects of  on the expected profit of carrier 2

*5.2. Impacts of the price sensitivity and the competition intensity*

This sub-section shows the effects of the price-related [coefficients](http://www.baidu.com/link?url=0A0JeJag7pMqiEnsjPXb90bPNFKa48PaXkFhU97X_beGVlz_g0RBXnsK3BDOqkCR2o5J01Xn8WUM0mAumdMvlBnvUKPhQQdQdhp97sYF3u2XYBDZufnt_3qjTmXr2s0a) on the decision variables and the utilities of two carriers. To simplify the discussion, we fix the risk-averse preference at 0.5, and consider carrier 2’s capacity at two levels: 20000 and 80000. Note that 20000 < *C*\*(*η*=0.5) < 80000. It is clear that *C* = 20000 is a LC condition case, and *C* = 80000 is a HC condition case.

Let the price sensitivity of carrier 2, *α*2, varies from 60 to 100; and other system parameters are the same as in Section 5.1. The results are shown in Fig. 5. From Fig. 5(a), It can be seen that both carriers’ prices are decreasing as the price sensitivity *α*2 increases. However, the impact on carrier 2’s price is more significant than on carrier 1’s price in both cases. Note that we did not assume *α*1 = *α*2 and *p*0 = 0. That means Proposition 6(i) appears to be true in more general situations. In addition, we can see that as the price sensitivity of carrier 2 increases, the price gap between two carriers is increasing. This is due to the fact that α2 has a larger impact on carrier 2 than on carrier 1. Fig. 5(b) shows that the utilities of two carriers are decreasing as the price sensitivity  increases. This implies that both carriers won’t gain benefit if the market becomes more price-sensitive.



|  |  |
| --- | --- |
| (a) Prices | (b) Utilities |

**Fig. 5.** the effect of the price sensitivity of carrier 2

Now we fix , and let  vary from 40 to 80. Fig. 6 shows how the optimal prices and the utilities respond to the varying competition intensity . It can be observed that in the HC condition case the prices of both carriers are decreasing as  increases. On the other hand, in the LC condition case, the prices of both carriers are increasing as  increases. This result is interesting. It indicates that the competition intensity parameters could have negative or positive impact on two carriers’ prices in more general cases (because here we did not impose the assumptions of Proposition 6). From Fig. 6(b), we can observe that in the HC condition case carrier 1’s utility is decreasing whereas carrier 2’s utility is increasing as  increases; in the LC condition case the utilities of two carriers are both increasing as  increases.



|  |  |
| --- | --- |
| (a) Prices | (b) Utilities |

**Fig. 6.** the effects of the competitive intensity

*5.3 Impacts of the capacity constraint*

This sub-section investigates the influence of carrier 2’s capacity on price decisions and utilities. Let *C* vary from 0 to 80000, = 0.5 and other parameters are the same as in Section 5.1. The results are shown in Fig. 7. We can see from Fig. 7(a) that the threshold value, *C*\*(*η*=0.5) = 47766, characterizes the impact of carrier 2’s capacity. Namely, both carriers’ prices are decreasing as the capacity of carrier 2 increases in the LC condition case (i.e. the capacity is less than the threshold level *C*\*); when carrier 2’s capacity exceeds the threshold level (i.e. in the HC condition case), there is no influence on both carriers’ pricing decisions. The latter phenomenon may be explained by the fact that when the capacity of carrier 2 is sufficiently large, the capacity constraint is essentially relaxed and would not influence the pricing strategy. Interestingly, it can be seen that carrier 2’s price is actually greater than carrier 1’s price when carrier 2’s capacity is sufficiently smaller. This may be explained by the fact that carrier 2’s market demand determined by *θ* may be large enough without the need to attract more demands via lowering the price.

Fig. 7(b) shows the utilities with varying capacity of carrier 2. In the LC condition case, it can be seen that carrier 2’s utility is increasing in *C* although its price is decreasing. As for carrier 1, its utility is decreasing in *C*; this is because carrier 2 has attracted more demands from carrier 1 by lowering its price much lower than carrier 1’s. Finally, In HC condition case, *C* has no effect on both carriers’ utility (because it has no influence on pricing decisions).



|  |  |
| --- | --- |
| (a) Prices | (b) Utilities |

**Fig. 7.** the effects of carrier 2’s capacity

**6. An extension to both risk-averse carriers**

In practice it is possible that both shipping lines adopt risk-averse behavior. In this section, we extend the model to such cases. Let *η*1 and *η*2 denote the risk-averse indicators for carriers 1 and 2 respectively. Similar to carrier 2’s CVaR in (5), carrier 1’s CVaR can be defined as follows:

.

Then, the problem is to:  and . With the similar arguments to Section 4.1, we can establish the following proposition:

**Proposition 7.** In the Nash bargaining model, there exists a threshold value  for carrier 2’s capacity, and the equilibrium solutions are given as follows:

,

.

Where







 satisfies ,







In addition, similar results to Propositions 3~6 can be established. We omitted the details of the descriptions and mathematical proofs due to their similarity to Section 4. The interesting point is that the main results in this paper can carry over to the cases when both larger carrier and smaller carrier are risk-averse.

**7. Conclusion**

The current liner shipping market is prominently characterized by severe overcapacity, fierce price-based competition and high level of demand uncertainty. It is natural that some shipping lines may adopt a risk-aversion attitude in their pricing strategies. In this paper, we consider the pricing strategies of two competing ocean carriers facing uncertain demand. The first carrier is risk-neutral with sufficient capacity, whereas the second carrier is risk-averse with limited capacity. The conditional value at risk (CVaR) is used to measure the risk-averse attitude of the second carrier. A Nash game model is formulated to model the pricing decisions. We solve the Nash game model and obtain the equilibrium solution. It is found that the solution takes two forms, which can be determined by a threshold value of carrier 2’s capacity. With the random demand following a uniform distribution, we show that as the second carrier becomes more risk-averse, both carriers’ optimal prices are decreasing, and the threshold value that determines the pricing strategy is also decreasing. Under some specific conditions (e.g. zero benchmark price, same price sensitivity and sufficient capacity for both carriers), we are able to show that the price sensitivity parameter has negative impact on both carriers’ prices; while the competition intensity parameter could have either negative or positive impact on two carriers’ prices. More importantly, we provide a necessary and sufficient condition to determine whether two carriers’ optimal prices would be positively or negatively affected by the competition intensity parameter. In addition, we extend the model to the cases when both carries are risk-averse and find that the main analytical results in this paper still hold. A range of numerical experiments are provided to verify the results, and illustrate that some analytic results can carry over to more general cases.

Our model is quite generic and is essentially applicable to other industrial sectors with capacity constraint and perishable products. In that sense, our study enriches the risk-averse literature in a broader context; It should be noted that this paper has a few limitations. The model is relatively simple that only considers the two-location one product service with two shipping lines. In practice, a shipping service route often covers multiple ports. Naturally, a shipping line may offer customers multiple itineraries (container paths) that share the ship capacity in the shipping network (Wang et al. 2015). It is possible to take into account the carrier’s risk-averse behavior (with respect to the revenue) in a more general shipping network. Such problem may be formulated into stochastic multi-commodity network flow models (in which the expected revenue may be replaced by a CVaR). A few studies have formulated stochastic programming models to maximize the expected profit under uncertain demand or capacity in freight transportation considering network effects (e.g. Wang 2016; Wang et al. 2017). These models may be extendable to include risk-averse behaviors. However, one challenging issue is how to determine and split the vessel capacity among different service products when facing uncertain demands. It appears to be difficult to obtain the pricing strategies and managerial insights analytically due to the complexity of the models. More research is required in this direction. Another limitation is that the demand function is linear and only the effect of prices is considered. Nevertheless, this paper is the first attempt to bring the risk-aversion concept into the context of shipping line competition. Further research could be done by extending the model to more general context or relaxing some assumptions or other industrial sectors.

Acknowledgements

We thank two anonymous reviewers for their constructive comments that have helped to improve the presentation of the paper significantly. This work is supported in part by the National Nature Science Foundation of China under Grant No.71472133.

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