Portfolio of Compositions:

Pitch-Class Set Theory in Music and Mathematics

Volume I: Commentary

Thesis submitted in accordance with the requirements of the University of Liverpool for the degree of Doctor in Philosophy

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Declaration

This thesis is my own work, has not been previously submitted for an award at this university of any other institution, and the content is legally allowable under copyright legislation.

Signed ……………M McNeilis……….. Date ……2/3/2017…….

Abstract

This portfolio contains six scores of contemporary works, one electroacoustic piece submitted in concrete audio format and a critical commentary. Three additional scores are included as appendices, which present alternative arrangements of the acoustic works. An accompanying data DVD includes stereo recordings of a concert performance and four workshop performances, as well as stereo and 5.1 versions of the electroacoustic piece.

With existing literature primarily focussing on pitch-class set theory as an analytical tool, this study aims to answer the question: Can set theory be used as the basis of a compositional framework to advance the creative process within the interdisciplinary field of music and mathematics? The pieces in the portfolio consider how set theory can be applied alongside mathematical principles like mutation, symmetry and proportion to inform musical networks and topologies based on material such as the octatonic scale, whole-tone scale and the all-interval tetrachords. A particular emphasis is placed on relationships between pitch-class sets in order to produce coherent compositional designs which link micro-level material to macro-level form and structure. Post-compositional evaluation of each work helps to create subsequent designs in the portfolio, and culminates in the piece *Aggregation*, which assesses the mathematical principles and compositional techniques developed earlier in the portfolio to encapsulate the research within a unified design and large-scale work. The results of the research establish set theory as a viable mathematical language for both the pre-compositional and compositional stages of the creative process, and demonstrate refinement of my own compositional methodology and musical style.

Acknowledgements and Dedication

Firstly, I would like to thank my supervisors Robin Hartwell and Dr Ian Percy for their expertise and guidance throughout my studies.

I would also like to acknowledge Liverpool Hope's Music department for hosting research seminars and providing performance opportunities with RLPO musicians, as well as for its academic and pastoral support.

Thanks to my family and friends, who have encouraged and motivated me throughout my time at Hope.

Finally, I would like to dedicate this PhD to my Grandfather, John Robinson (1931-2016). Without his constant support, I could have not undertaken this research journey.

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Scores (See Volume II)

(Works in italics are additional arrangements of the primary compositions)

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In Equal Measure **(2011-2016)** **ca. 9’15**	Symmetry	Solo piano
Unequal Measures **(2012-2016)** **ca. 10’30**	Asymmetry, Fibonacci series, Golden section	String quartet: violin x 2 viola violoncello
Space Ripple **(2013-2016)** **ca. 8’15**	Pi	Solo electric guitar and tape
Three Minds Fractured **(2013-2016)** **ca. 8’25**	Octatonic scale	Brass sextet: trumpet in Bb x 2 tenor trombone x 2 tuba x 2
One to Another **(2014-2016)** **ca. 14’15**	All-interval tetrachords	Large ensemble: flute oboe clarinet in Bb bassoon French horn in F trumpet in Bb tuba timpani violin x 2 viola violoncello contrabass

Composition Title	Research Subject	Instrumentation
One to Another ***(Octet Arrangement)*** ***(2015)*** ***ca. 4’45*** ***Appendix E***	*All-interval tetrachords*	*Small ensemble:* *flute* *oboe* *clarinet in B♭* *bassoon* *French horn in F* *trumpet in B♭* *trombone* *contrabass*
Aggregation **(2016-2017)** **ca. 8’20**	N/A	Large ensemble: piccolo flute oboe clarinet in B♭ bassoon French horn in F trumpet in B♭ tenor trombone tuba timpani suspended cymbal snare drum bass drum violin x 2 viola violoncello contrabass
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In Three Elements ***(2016)*** ***ca. 4’10*** ***Appendix G***	*N/A* *(workshop medley comprising sections of In Equal Measure, Three Minds Fractured and Aggregation)*	*Mixed quartet:* *violin* *clarinet in B♭* *cello* *piano*

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(The Data DVD also contains a copy of the commentary and scores)

Chapter 1

Introduction

1.1 Background and Context

Pitch-Class Set Theory

As the 20th century witnessed a rise in atonal music, theorists began to devise methods of analysis to understand newly developed musical material. One such method is pitch-class set theory[1], which segments musical works into collections of pitch classes that permit in-depth analysis of micro-level patterns and macro-level structures, with emphasis on pitch, intervals and harmony. In the 1960s, theorists and composers such as Milton Babbitt[2] and George Perle[3] used numerical classifications and the term 'set' within the context of twelve-tone systems, primarily when applied through functions such as inversions, transpositions and retrogrades. Their influence can be seen in Allen Forte's articles[4] and subsequent 1973 text *The Structure of Atonal Music*[5], which laid the foundation for the future of PC set analysis in non-serial atonal music[6]. Although operations such as inversion and transposition remained important to Forte, he moved away from the context of 12 ordered pitch classes in twelve-tone music. Dealing largely with non-serial, unordered sets containing 3 to 9 elements[7], the book focussed on classifying sets in order to define and harmonically analyse relationships between material.

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[1] Abbreviated to PC set theory from this point.
[2] Milton Babbitt, 'Twelve-Tone Invariants as Compositional Determinants', *The Musical Quarterly,* 46 (1960).
Milton Babbitt, 'Set Structure as a Compositional Determinant', *Journal of Music Theory,* 5 (1961).
[3] George Perle, *Serial Composition and Atonality: An Introduction to the Music of Schoenberg, Berg, and Webern*, (Berkeley: University of California Press, 1962).
[4] Allen Forte, 'A Theory of Set-Complexes for Music', *Journal of Music Theory,* 8 (1964).
Allen Forte, 'The Domain and Relations of Set-Complex Theory', *Journal of Music Theory,* 9 (1965).
Allen Forte, 'Sets and Nonsets in Schoenberg's Atonal Music', *Perspectives of New Music,* 11 (1972).
[5] Allen Forte, *The Structure of Atonal Music*, (New Haven; London: Yale University Press, 1973).
[6] For the purpose of consistency, Forte's established method for calculating the prime form of PC sets is used. Forte's prime form is "the most compact form chosen from a set and its inverse" (Larry J. Solomon, 'Solomon's Music Resources', (2002a)). Solomon's usage of 'A' and 'B' to represent pitch class values 10 and 11 within a set is adopted for visual clarity within compositional systems.
[7] Forte (1973), p. 65. Sets of all sizes are classified by other theorists, such as John Rahn, *Basic Atonal Theory*, (New York: Longman, 1980); Solomon (2002a).

As noted by Schuijer, Forte's text has been very influential, and has remained prominent in university teaching into the 21st century, particularly in North America[8]. Part 1 on 'Pitch-Class Sets and Relations' has been especially successful, since many music theorists have adopted the usage of his set names. For example, set 6-20 represented by [0,1,3,5,8,9], is the Ode-to-Napoleon hexachord, whilst set 5-Z17, containing [0,1,3,4,8], can be used to construct Schoenberg's Farben chord[9]. However, the book's second section on 'Pitch-Class Set Complexes' has proved less significant, since it was not integrated into John Rahn's *Basic Atonal Theory*[10] or Joseph N. Straus' *Introduction to Post-Tonal Theory*[11], two key texts on atonal music theory also used widely in North America. Rahn and Straus do discuss the principles outlined in this section, such as relationships between sets, but there is more of a focus on their creative applications rather than any classification system. There is an increased consideration of non-prime form sets, with the texts providing practical exercises for both the analyst and composer.

Although it has offered a viable platform for analytical investigation, PC set theory has been criticised by several reputable theorists for a range of reasons. George Perle has remained adamant in his rejection of Forte's classifications, stating: "I find the system irrelevant to my experience as a composer, to my perceptions as a listener, and to my discoveries as an analyst"[12]. The ability to aurally perceive PC sets and their segmentation is a common complaint of set theory's critics, as highlighted by Schuijer[13] when describing a debate from the 1999 European Music Analysis Conference. After it was suggested that set theory should be used to analyse twentieth-century music, the chairman responded: "We do not talk about pitch-class sets, because we do not hear them!". While Forte provides terminology such as 'primary segment' and 'composite segment'[14], there remains potential ambiguity for structural segmentation since there are no definitive criteria that can be applied universally. The reliance on the analyst to choose this segmentation can result in a variety of contrasting analyses of a single musical section or piece, and can be a particularly troublesome issue for large-scale

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[8] Michiel Schuijer, *Analyzing Atonal Music: Pitch-Set Theory and Its Context*, (Rochester, NY: Univ. of Rochester Press, 2008), pp. 19-20.
[9] Ibid, p. 109.
[10] Rahn (1980).
[11] Joseph Nathan Straus, *Introduction to Post-Tonal Theory*, (Englewood Cliffs, N.J.: Prentice Hall, 1990).
[12] George Perle, 'Pitch-Class Set Analysis: An Evaluation', *The Journal of Musicology,* 8 (1990), p. 152.
[13] Schuijer, pp. 1-4.
[14] Forte (1973).

works with complex arrangements or pieces that consider advanced polytonality. Furthermore, as Schuijer points out, the most important features of musical composition, such as phrases, rests and instrumentation "may not always help to identify the 'relevant' segmentation"[15]. However, these issues are only a concern if one believes that there is a single correct segmentation, and consequentially a single correct analysis and interpretation of such pieces. Ultimately, the intentions of the individual composer or analyst may dictate the potential usefulness of set theory as a standalone tool. Some theorists, such as Perle, do admit that, when paired with a variety of other techniques, PC set theory can be beneficial, since "it is only when one defines *everything* in terms of pitch-class sets that the concept becomes meaningless"[16]. Finding compatibility between PC set theory and other analytical techniques and appropriate modes of listening is crucial to the integration of set theory across a wider range of musical parameters.

Conversely, rather than looking at post-compositional analyses of works by theorists, composers who actively use PC set theory could potentially present their own compositional plans and subsequent segmentation for consideration by analysts, further promoting understanding within this area. Limited usage can be seen within this creative context through composers such as Pierre Boulez and Elliott Carter. Boulez used multiplications of PC sets within total serialism, as seen in works such as *Le Marteau sans Maître* (1955), which makes use of a tone row and multiplies subset combinations to produce related material[17]. Set multiplication was not something that Forte included in *The Structure of Atonal Music*, but as Schuijer[18] outlines, it was adopted by Rahn's *Basic Atonal Theory* due to American composers being interested in its function as a creative rather than analytical tool[19]. Carter's application of pitch-class sets as a central compositional device has also been discussed and analysed within *The Music of Elliott Carter*[20] and particularly *Elliott Carter: Harmony Book*[21], which details his own classification system of sets. His system uses different numerical terminology and visual representations but the fundamental principles can largely be mapped to

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[15] Schuijer, p. 229.
[16] George Perle, *The Listening Composer*, (Berkeley; London: University of California Press, 1996), p. 67.
[17] Lev Koblyakov, *Pierre Boulez: A World of Harmony* (Harwood Academic Publishers, 1990), p. 4.
[18] Schuijer, p. 83.
[19] Although initially considered, PC set multiplication has not been applied within the works of this portfolio as it was felt that many of its operations helped to create new material in a largely arbitrary fashion.
[20] David Schiff, *The Music of Elliott Carter*, (London: Faber and Faber, 1998).
[21] Nicholas Hopkins and John F. Link, 'Elliott Carter: Harmony Book', (New York: Carl Fischer, 2002).

Forte's classification system. Carter maintained a particular interest in sets with special properties, such as the all-interval tetrachord (abbreviated to AIT from this point) and superset/subset relationships of these distinctive sets[22].

A prominent pedagogical study in this area is Robert Morris' *Composition with Pitch-Classes*[23], which focusses on his concept of a compositional design. The designs "denote an abstract, uninterpreted composition of pitch-classes"[24] that help to influence the early stage of composition but not necessarily stylistic or personal choice. They can be created at the micro-level for a local harmonic or melodic passage or at the macro-level to suggest structure. Morris creates visual networks that can be applied by both the composer and the analyst, with a key aim to create "a theory of compositional design that can satisfy structural requirements arising from different functions, uses, and aesthetics of music"[25]. He later introduced his idea of a compositional space as an extra step before the design stage, in which a space acts as an abstract, structure for a set of related musical objects[26]. For example, related PC sets may be extracted into multiple tables or networks based on combinations of transformations such as transposition and inversion, but without instructions on how this information should inform specific temporal events until the design stage. Morris provides the examples of Babbitt and Carter, who created similar trichord, network arrays of the all-trichord hexachord, set 6-17, [0,1,2,4,7,8]. Based on transpositions and inversions of related trichord sets, Morris manufactures complex pre-compositional spaces for their works[27]. These spaces show how trichords can be used to modulate between each other via related hexachords. In turn, this produces multivalent relationships between set material that permits the creation of more complex structures. In combination with his designs, he identifies the spaces as aids to "bridge the gap between material and design"[28].

A further work of note within set theory composition is Ciro Scotto's system, which "integrates generalized tonal concepts with tone rows and arrays"[29]. His designs are

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[22] Discussed in more detail in Chapter 8.
[23] Robert Morris, *Composition with Pitch-Classes: A Theory of Compositional Design*, (New Haven: Yale University Press, 1987).
[24] Ibid. p. 3.
[25] Loc. cit.
[26] Robert Morris, 'Compositional Spaces and Other Territories', *Perspectives of New Music,* 33 (1995), p. 330, p. 336.
[27] Ibid. pp. 340-2.
[28] Ibid. p. 336.
[29] Ciro Scotto, 'A Hybrid Compositional System: Pitch-Class Composition with Tonal Syntax', *Perspectives of New Music,* 38 (2000), p. 78.

particularly noteworthy for their hierarchical networks that produce hexachords based on combinatorial properties of sets [0,3,7], [0,4,7] and related trichord material. This system is not as complex as some of those of Morris, but is less abstract and delivers more detail at the compositional rather than pre-compositional stage.

Music and Mathematics

The long debate as to whether music is an art of unmediated expression or an art of rational calculation stretches back centuries to philosophers such as Pythagoras, Aristoxenus and Boethius, who “imagined music as a branch of mathematics”[30]. The construction of sound waves that produce frequencies which can be classed as ‘pitch’ rather than ‘noise’ underpins the scientific nature of musical tuning systems, such as the one constructed by Pythagoras based on ratios of 3:2 and perfect 5th intervals. It was not until the late 16th and early 17th centuries that music started to be seen as “an art and to be treated pedagogically as language and analysed in expressive terms”[31], before being declared as one of the Arts in the mid-18th century by French philosophers like Charles Batteux[32]. Significant disputes arising over the connections between the two areas include the distinction of ‘truth’ versus ‘beauty’. For example, Rothstein claims that, while music does have perceivable aesthetic qualities, mathematicians do not attach emotions to their theorems[33]. In stark contrast, Henle believes mathematics is about taste, and at its best, it is not simply about calculation and being true, but is, in fact, about communication and being beautiful[34], assertions which can also be applied to music.

In my view, a theorem can be admired for its originality and eloquent proof that faultlessly follows logical processes, yet the potential emotion of mathematics on a wider scale comes through the structures and relationships outside of the theoretical, mathematical context. For example, the most evident, practical connection between music and mathematics is through the usage of numbers and number systems first established by Pythagoras and others. Beyond the direct application of numbers within

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[30] Jim Henle, 'Classical Mathematics', *The American Mathematical Monthly,* 103 (1996), p. 18.
[31] John Fauvel, *Music and Mathematics: From Pythagoras to Fractals*, (Oxford: Oxford University Press, 2003), p. 1.
[32] Charles Batteux, *Les Beaux Arts Reduits a Un Même Principe* (Paris: Chambre Royale des Libraires & Imprimeurs de Paris, 1747).
[33] Edward Rothstein, *Emblems of Mind: The Inner Life of Music and Mathematics*, (Chicago [u.a.]: Univ. of Chicago Press, 2006), p. 6.
[34] Henle, p. 28.

musical parameters such as pitch, harmony and rhythm, patterns can be created which can potentially open up more interesting compositional possibilities. As Stewart notes, ‘‘Patterns possess utility as well as beauty. Once we have learned to recognize a background pattern, exceptions suddenly stand out”[35]. These micro-level patterns and sequences, found in rhythmic motifs for example, provide repetition, consistency and development that can lead to more complex structural models and systems within single pieces of music or across advanced collections of works. The usage of such models intensified from the early years of the 20th century through composers including Bartók, Stravinsky and Webern, the works of whom are considered in detail throughout the thesis.

Having attended both a ‘Music and Numbers’ conference (Canterbury Christ Church University, 2010) and a ‘Music and Mathematics’ study day (University of Leeds, 2014), it was interesting to note the distinctions between the analytical approaches of the speakers. At Canterbury, I was surprised to hear of the high number of theorists who were interested in analysing structure with relation to natural, mathematical proportions and ‘lucky’ numbers, rather than through formal mathematical methods that were present in more depth at Leeds. One particular note of interest from Leeds came from the keynote speaker, Alan Marsden, who stated that we, as analysts and composers, should not believe mathematical significance naturally translates to musical significance[36].

A further philosophical consideration lies in Gestalt theory. Originally established within the field of psychology by key theorists such as Max Wertheimer[37], it considers visual perception with relation to notions such as proximity, regularity, similarity and symmetry. More recently, its principles have been applied to both music and mathematics, with particular attention placed on the perception of musical parameters such as pitch and rhythm. At the micro-level, ‘proximity’ could relate to notes being close together, ‘regularity’ might group notes into similar or comparable note durations, while at the macro structural level, this could consist of more complex, hierarchical pitch systems. Deutsch and Feroe note that listeners prefer regular groupings over

[35] Ian Stewart, *Nature's Numbers: The Unreal Reality of Mathematical Imagination*, (London: Clays Ltd, 1995), p. 3.
[36] Alan Marsden, 'Music, Mathematics, Morality and Motion', in *RMA Study Day: Music and Mathematics* (University of Leeds: 2014).
[37] Max Wertheimer, 'Music Der Wedda (Music of the Vedda)', *Sammelbande der internationalen Musikgesellschaft* 11 (1910), 300-309.

irregular ones, and symmetrical groups over asymmetrical ones[38]. Moreover, tradition dictates that we accept tonal pitch structures based on mathematically proportional relationships as somewhat superior to dissonant tuning systems and atonal harmony. Whether we, as listeners, have a higher appreciation of a piece of music if we are aware of these mathematical relationships is open for consideration, and most likely an individual choice. As a composer who works with mathematical systems, any extra insight I can gain about another composer's creative process furthers my understanding of compositional methods but does not necessarily heighten my enjoyment as a listener.

Beyond these philosophical thoughts, Assayag et al report that in the second half of the 20th century, "the evolution of Music composition has led to a complex configuration of formal reasoning and search for internal consistency, sometimes calling explicitly the power of Mathematics, as in the case of Babbitt, Xenakis and many others"[39]. Xenakis, in *Metastasis* (1953-54) for example, applied complex systems within a large orchestral setting. He would employ glissando to the strings over long, musical phrases in order to plot frequency development over time, with each performer playing independently from one another based on mathematical ratios. It could be debated whether the nature of this evolution suggests improvement or simply adaptation to contemporary thought, but the rise of technology has added an additional dimension to these considerations. The use of computation aiding the application of mathematics within the creative thought process is investigated in modern-day journals such as *The Journal of Mathematics and Music*. However, although PC set theory was originally "devised for computer-aided analysis"[40], the usage of computers and formal mathematical theory is outside the aim of this study.

1.2 Research Aims and Methodology

The aim of this study is to answer the question: Can set theory be used as the basis of a compositional framework to advance the creative process within the interdisciplinary field of music and mathematics? The research considers how mathematical structures inspired by set theory can inform musical structures, and a varied portfolio of acoustic

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38 Diana Deutsch and John Feroe, 'The Internal Representation of Pitch Sequences in Tonal Music', *Psychological Review,* 88 (1981).
39 Gérard Assayag et al, 'Mathematics and Music: A Diderot Mathematical Forum', (Berlin; New York: Springer, 2002), p. vi.
40 Schuijer, p. 237.

and electronic works, with appropriate commentary, has been produced to demonstrate and evaluate this. Within these musical works, set theory is used to construct the basis of a unified pitch language for the varied mathematical settings; it is the constant throughout the portfolio. Lester notes that it can help to construct raw material that enables interaction between musical parameters such as harmony and melody[41]. However, Cross et al are less convinced that structures can be formulated through pitch alone, and additional parameters such as rhythm and timbre are required[42]. Although the portfolio's primary concern relates to pitch organisation within the context of set theory, the study's focus is tightened when considered alongside mathematical properties such as symmetry and proportion, and offers potential to formulate complex networks and topologies. This then permits the deliberation of not only a single parameter in isolation, but also allows us to group stimuli into patterns and relate these patterns to one another[43]. Through the usage of systematic processes and the creation of formal structural designs within the varied mathematical contexts, the resultant compositions move beyond the more abstract applications of set theory within creative settings as outlined by Morris[44], and present original manifestations of this approach within a new context.

Each musical work considers set theory within a distinct mathematical setting, while aspects of musical style and dependence on mathematics at the pre-compositional and compositional stages function as primary distinguishing factors between pieces. Certain restrictions have been put in place at each of these stages to allow for a comparative analysis across a varied portfolio, ranging from their application within formal structural designs on the macro-level to pitch organisation at the micro-level. The works are constructed from pre-compositional designs[45] inspired by set theory within a particular mathematical context (e.g. symmetrical sets within the context of symmetrical structures). In order to create such designs, a variety of approaches have been considered, which focus on analysis and composition in both set theory and mathematics. Through an understanding of selected past musical works and, in

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[41] Joel Lester, *Analytic Approaches to Twentieth-Century Music*, (New York: W.W. Norton, 1989), p. 88.
[42] Ian Cross, Peter Howell and Robert West, 'Structural Relationships in the Perception of Musical Pitch', in *Musical Structure and Cognition,* ed. by Peter Howell, Ian Cross, and Robert West (London: Orlando : Academic Press, 1985), p. 121.
[43] Leonard B. Meyer, *Emotion and Meaning in Music* (University of Chicago Press, 1961), p. 6.
[44] Morris (1987).
[45] The term 'pre-compositional design' is used to describe any form of pre-compositional plan or system, however small or large, which is used as a basis for the compositional stage. It may suggest form, pitch organisation or instrumentation, etc.

particular, a comprehension of compositional models, the resultant creative works are placed within relevant historical, theoretical, critical and design contexts. Essentially, the outcome of the research is validated by its subsequent practical applications.

There are numerous strategies for compositional and pre-compositional designs used by composers. Some purely rely on instinct, some use instinct after an initial set of formal ideas, while others construct whole pieces based on rules and systems. This portfolio covers a combination of the latter two approaches to investigate how set theory and mathematics can apply strategies and mechanisms to initiate the creative process in distinct ways. Even though composers may share similar approaches (by using comparable pitch structures at the macro-level, for example), the resultant musical works naturally maintain a creative individual voice through the varied application of these structures within parameters such as melody, harmony and texture.

The importance of continued research in set theory and mathematics in past compositions and the resultant analytical models cannot be understated. Primarily, an understanding of the technical employment of compositional devices in their progression towards a piece of music is vital, since "the purpose of the analysis is the discovery, not of the fundamental structure, but of the way that leads from this fundamental to the actual work"[46]. The particular connections between PC set theory and mathematics offer vast potential for such theoretical foundations and compositional decisions. Schuijer expresses the argument that "set theory employs a mathematical vocabulary and mathematical models of presentation", which provide "an image of rationality that may confirm people in their rejection of the music concerned"[47]. For example, there are numerous collections of related sets (such as the whole-tone scale, [0,2,4,6,8,A], the diminished 7th chord, [0,3,6,9], and the augmented triad, [0,4,8]) that can inform compositional strategies within mathematical contexts such as symmetry and proportion. This principle is fundamental to the study, and discussed more in the Journey of Research, as well as throughout the portfolio.

Schuijer further notes that PC set theory considers musical coherence with respect to arrangements of musical tones within parameters such as harmony and form[48], and that

[46] Schuijer, p. 14.
[47] Ibid. p. 18.
[48] Ibid. p. 4.

a set's interval content suggests a basic sound[49]. It could even be said that a piece's interval content, whether applied explicitly through the likes of set theory, or appearing more implicitly, *is* its inherent sound. Essentially, this integration between analytical and compositional models allows for the construction and control of a coherent and consistent musical language or grammar that is open to analytical deconstruction, with the capability of informing compositional strategies for future works. The research is intended for an audience of music composers who are interested in mathematical principles, rather than an audience of mathematicians experimenting with musical structures. It is hoped that the final portfolio will offer valuable practical evidence towards the understanding of compositional techniques in an increasingly prominent area of academic interest, as the link between the interdisciplinary topic of music and mathematics is ultimately bridged by the application of PC set theory as a versatile, compositional tool.

[49] Ibid. p. 126.

Chapter 2

Journey of Research

My initial interest in PC set theory stems from investigating its potential as an analytical device during my undergraduate and MA studies. In this time, I also became interested in the numerical links between the two AITs, and my first use of set theory as a creative tool soon followed. It helped me to see musical patterns in closely related number sequences, and provided stimuli to construct sequences of vertical harmonic blocks across multiple instruments. This venture was sufficient in leading me to apply set theory within composition on a larger scale.

The use of hierarchies in structural designs was of particular interest, and research within the context of set theory led to the exploration of symmetrical qualities of sets, with a specific focus on transpositional symmetry. Morgan defines a PC set with transpositional symmetry as one that “divides PC space into equidistant smaller segments, reducing its twelvefold symmetry to a more limited order”, the simplest examples being the whole-tone scale, the diminished 7th chord, the augmented triad and the tritone[50]. Perle provides a similar definition of interval cycles, stating that they are types of scales that have a repeated interval series[51]. I was especially intrigued by his observations on the partitioning of the twelve-tone scale into symmetrical structures that have roots in the tonal music of Liszt, yet can also be applied to the atonal works of Stravinsky[52]. The potential to use the minor 3rd, for example, in an isolated tonal manner, but then provide more dissonance through its stacked usage towards the diminished 7th, is indicative of the consideration of consonance and dissonance throughout the portfolio.

Related to these principles are Olivier Messiaen’s Modes of Limited Transpositions[53], which also encompass notions of symmetry within scales or chords. Examples include

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[50] Robert P. Morgan, 'Symmetrical Form and Common-Practice Tonality', *Music Theory Spectrum,* 20 (1998), p. 8.

[51] George Perle in E. Pearsall, *Twentieth-Century Music Theory and Practice*, (Routledge, 2012), p. 57.

[52] George Perle, *The Listening Composer*, (Berkeley; London: University of California Press, 1996), p. 92.

[53] “Modes are formed of several symmetrical groups, the last note of each group always being common with the first of the following group. At the end of a certain number of chromatic transpositions which

the whole-tone scale, the octatonic scale and the augmented triad, and Messiaen has used combinations of these modes to "elicit a sound which is often rich, exotic, eastern pandiatonic and other-worldly"[54]. Although this project does not apply interval cycles and Modes of Limited Transpositions directly as primary compositional devices, their influences can be seen within the portfolio, particularly through the inclusion of the whole-tone scale, the octatonic scale, diminished 7th chords and augmented triads on both a micro-level and through macro-level structures. Similarly, my interest in Gestalt theory did not act as a formal thread throughout the portfolio, yet its notions such as regularity and similarity presented a series of terms and principles that had potential to be used as mantras within my own compositional framework.

Morris' compositional spaces and designs provided more direct potential for my own creative approach. Although the majority of his harmonic and structural plans typically remain within the abstract, pre-compositional stage, my conversion from the abstract to reality is formalised within a similar process that can also be enhanced by the aforementioned elements of regularity, similarity and symmetry provided by mathematical principles. Designs could be considered as equivalent to the results provided by the reductive nature of Schenkerian analysis, but with the intention reversed in order to develop a musical work rather than condense one for post-compositional reflection and analysis. The integration of these designs within set theory is enhanced through its emphasis on categorising set relationships, while their application within a variety of mathematical contexts based on the likes of balance and proportion further tightens links between the initial research areas. For most of the pieces within the portfolio, the designs are often key to the overall structure and harmonic language, and consequently dictate a certain workflow. With an emphasis on set theory, an iterative compositional process is used to add extra harmonic, melodic and textural detail to a global outline, resulting in the increased refinement at various hierarchical levels and ultimately the micro-level material.

These designs and the processes involved in the creation of each work are outlined in full in the following chapters. Each chapter starts with background information on the mathematical context, moves on to talk predominantly abstractly about each pre-compositional design's consideration of pitch structure and similar musical parameters,

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varies with each mode, they are no longer transposable, giving exactly the same notes as the first" (Olivier Messiaen, *The Technique of My Musical Language*, (Paris: A. Leduc, 1944), p. 58).

[54] Jonathan Dimond, 'Theory of Music – Messiaen', (2010), p. 4.

before going into specific detail on the work's application of such considerations. Common themes exist across the collection of works. For example, notions of neutrality are key, and can be seen in pieces that use the invariance of the whole-tone scale, diminished 7th chord and the augmented triad, as well as in the circular nature of pi and uniform nature of the octatonic scale and AITs. Furthermore, the links between the mathematical properties across the overall portfolio also aid the development from each work to the next.

The portfolio begins with *From Zero*, which investigates the mutation of individual PC sets starting from the whole-tone scale. The emphasis here is on very closely related sets and creating material that aids these transitions. *In Equal Measure* then considers further symmetrical sets through the diminished 7th chord and augmented triad within the context of hierarchical symmetry. It uses Morris' notion of compositional spaces and designs to construct a pre-compositional design that uses sets to formally dictate the harmony and structure. *Unequal Measures* is the natural next step from *In Equal Measure*, in that it looks into asymmetry and proportion through sets related to the Fibonacci series and golden section. Although Gestalt principles declare that symmetry is often preferred to asymmetry[55], widespread opinion is that asymmetry inherently offers an extra element of interest and attraction. For example, Xenakis states that "at the beginning of a transformation towards asymmetry, exceptional events are introduced into symmetry and act as aesthetic stimuli. When these exceptional events multiply and become the general case, a jump to a higher level occurs"[56]. Theorist Morgan also argues that "some degree of asymmetry is almost always desirable in music"[57]. It is at this transformation towards asymmetry that my pre-compositional designs become less rigid, as they begin to move from strict rules and systematic outlines into guides that allow more input and flexibility for the composer. This contrasting approach has been adopted in order to demonstrate the versatility of composing through these pre-compositional designs.

Space Ripple, an electroacoustic piece focussing on pi, is the next work in the portfolio. Its intrinsic links to a circle maintain notions of symmetry, asymmetry and balance, but

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55 Robert West, Peter Howell and Ian Cross, 'Modelling Perceived Musical Structure', in *Musical Structure and Cognition*, ed. by Peter Howell, Ian Cross, and Robert West (London: Orlando : Academic Press, 1985), pp. 21-52.
56 Iannis Xenakis, *Formalized Music: Thought and Mathematics in Composition*, (New York: Indiana University Press, 1971), p. 25.
57 Morgan (1998), p. 6.

consider them within an electronic, aesthetic context. From here, the focus shifts slightly away from a direct emphasis on natural mathematical properties, and looks into how these properties appear within musical theory. *Three Minds Fractured* does this through a focus on the octatonic scale, which may be segmented into various symmetrical and asymmetrical partitions, while *One to Another* maintains links to the octatonic scale by a consideration of the AITs and their related sets. The portfolio also contains *Aggregation*, which takes influence from multiple pre-compositional designs and mathematical principles earlier in the portfolio, and aims to further establish the validity of my methodology on a larger scale.

In addition to these primary works, the portfolio also comprises scores and recordings of three further workshop performances. It includes an octet arrangement of *One to Another* (Appendix E), a short orchestral excerpt of the finale of *Aggregation* (Appendix F), and a medley titled *In Three Elements* (Appendix G), which contains passages from *In Equal Measure*, *Three Minds Fractured* and *Aggregation*, arranged for a mixed quartet. Further details of these pieces are provided in the relevant chapters of the primary works.

Chapter 3

From Zero (2011-2016)

The portfolio begins with a solo flute piece titled *From Zero*. The research aim at this stage focusses on the process of converting a pre-compositional design into an experimental, musical work by considering the linear mutation of single PC sets[58], and helps to form the basis for more complex structures in later compositions. The flute was chosen specifically for its varied tones available across its wide pitch register and dynamic range, as well as offering potential for a variety of expressive articulations. Prominent works from the repertoire, such as Debussy's *Syrinx* (1913) and Varèse's *Density 21.5* (1936/1946), were researched to aid the composition of the piece, and are discussed in further detail below.

3.1 Mutation

Mutation comprises notions of development and decay, with a boundless path of deviations conceivable within a musical work. This raises the question of 'choice' for a composer. How much impact do the composer's own choices have in the pre-compositional and compositional processes? Consider composer and theorist John Cage as an example, who "in his search for lack of order, used computer-generated random numbers for composing"[59]. For example, in his solo piano work, *Music of Changes* (1951), Cage used the visual charts of the Chinese divination text *I Ching*, in combination with probability matrices and coin tosses, to create systematic rules that determine pitch, note duration and dynamics. Further variation in each performance of the work also relied on the indeterminate nature of chance to dictate the interpretation of performers[60]. Since my project places central importance on a variety of different methods of restriction across the pre-compositional and compositional stages, the usage of restrictions in computer-aided and algorithmic composition has a considerable history that can be considered when building controlled models. A variety of techniques to create algorithmic music composition have been developed over the years, including stochastic processes, genetic algorithms, rule-based grammars and fractals. One of the

[58] Linear mutation is the progression from one PC set to another over a period of time.
[59] Rothstein, p. xi.
[60] Grant Chu Covell, '1951 and Cage's Music of Changes', (2016).

particularly interesting instances amongst these approaches is the Markov chain, which uses probability to transition from one state to another based on a series of rules. Using stochastic elements within pre-compositional designs can replace certain compositional choices across musical parameters, and directly provide my varied levels of restrictiveness.

Over the second half of the 20th century, composer and theorist Iannis Xenakis developed several ideas and philosophies for musical creation within stochastic music and Markov chains, as seen in *Formalized Music*[61]. His *Analogique A* (1958-1959) for string orchestra and *Analogique B* (1958-1959) for tape both make use of Markov chains and probability matrices to consider density, dynamics and pitch, often through multi-layered systems. With a focus on note onsets, the use of density is particularly interesting. These onsets are mapped onto a time grid, from which a set of rules dictate how density develops within a piece. In combination with further musical parameters, systems develop complexity and become formalised processes, which Xenakis called 'mechanisms'[62]. There are those who doubt the usefulness of these models, however, with Howell providing a warning that they are "unlikely to capture all of the important elements of a listener's musical experience as we are not good at discerning probabilistic dependencies"[63]. Perhaps more closely related to my research, Ames has formed compositional models for linear Markov chains[64]. He considers primitive pitch sequences and uses probability to map towards more complex sequences. This type of melodic construction is becoming more prominent within the game audio industry, in which adaptive music is created in middleware[65] software such as FMOD and WWise to add variation within vast game worlds. While Markov chains within musical composition have typically been applied through formal mathematical structures, middleware permits the composer to see and hear how choices play out depending on various game states, and as the software develops in the future, these adaptive features could be potentially useful outside of the immediate scope of game audio.

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[61] Xenakis (1971).
[62] Ibid.
[63] West, Howell and Cross, p. 31.
[64] Charles Ames, 'The Markov Process as a Compositional Model: A Survey and Tutorial', *Leonardo,* 22 (1989).
[65] Middleware is software that acts as a bridge between two contrasting pieces of software. In the context of game audio, the bridge is between a game engine for programmers and a Digital Audio Workstation for composers or sound designers.

3.2 Pre-Compositional Design

Two designs were initially considered for the construction of *From Zero*: The Primary Design and Alternative Design. The former relied on using sets in prime form, while the latter avoided the dependence on this. Through initial experimentation of both methods, it was decided that the Primary Design would be chosen since non-prime form sets would offer greater flexibility in more complex, later pieces, thus offering more varied employment of PC sets throughout the portfolio[66]. The notion behind the Primary Design was to choose a PC set in prime form, and then mutate through the usage of Markov chains to other prime form sets in a linear manner based on pre-determined rules. By using prime form sets, it permitted the restriction of pitch classes being considered as specific pitches within the piece (i.e. 0 would be C, 2 would be D, etc.) rather than acting as abstract structures. Higher order Markov chains were initially developed, in which the mutation would depend on multiple sequences to determine set mutation rather than just a single sequence or set. However, their integration in experimental sketches of solo and small ensemble pieces proved unsuccessful aesthetically within this context, while the research began to veer into highly technical considerations, more suitable for an audience of mathematicians rather than music composers and theorists.

In discussions with Richard Kostelanetz, Cage talks about the challenge of 'zero' acting as a start and end point in composition, with notions of uniformity and our own memories creating paradoxical issues[67]. *From Zero* aimed to find the most neutral set to use as an axiomatic starting point, effectively attempting to begin mutation 'from zero'. Set 6-35, [0,2,4,6,8,A], the whole-tone scale, was ultimately chosen to carry out this function. It provided initial interest as a symmetrical set and one of Messiaen's Modes of Limited Transpositions. Furthermore, its cardinality[68] of 6 was suitable to incorporate a variety of subsets and supersets, while equidistant pitch content permitted a neutral position from which to mutate. Being tonally unstable in nature and lacking the "fundamental harmonic and melodic relationships of major–minor tonality"[69], it also had the potential to directly mutate towards sets with atonal characteristics. Finally, the

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[66] The Primary Design is discussed in further specifics below, while details of the Alternative Design can be seen in Appendix A.
[67] Richard Kostelanetz and John Cage, 'The Aesthetics of John Cage: A Composite Interview', *The Kenyon Review,* 9 (1987), 102-30.
[68] A set's cardinality is the number of pitch classes it contains.
[69] H.K. Andrews, 'Whole-Tone Scale', in *Grove Music Online*.

whole-tone scale aligned with my choice of the flute since it is also explored in *Syrinx* and *Density 21.5*. This is particularly evident through melodic, stepwise motion of [0,2,4] and [0.2.4,6] in both pieces, while *Density 21.5* ends in a descending phrase of [8,6,4,2,0], seen through B, A, G, F, E♭ and D♭.

Further details of the Primary Design will now be discussed.

Primary Design

From the initial set of 6-35, three possibilities for mutation were constructed:

1. One pitch class within the set could be removed to reduce cardinality by 1
2. One pitch class within the set could be replaced by a uniquely new pitch class, which results in the same cardinality
3. One new pitch class could be added to the set to increase cardinality by 1

Let us consider the first possibility. If we begin with [0,2,4,6,8,A], then it is possible to remove one of the 0, 2, 4, 6, 8 or A. Below are the six possible mutations for a cardinality of 5, which, in prime form, all naturally result in a subset of 6-35 (set 5-33, [0,2,4,6,8])[70].

0,2,4,6,8,A ➔ 0,2,4,6,8 = 0,2,4,6,8 = 5-33
0,2,4,6,8,A ➔ 0,2,4,6,A ➔ 0,2,4,6,8 = 5-33
0,2,4,6,8,A ➔ 0,2,4,8,A ➔ 0,2,4,6,8 = 5-33
0,2,4,6,8,A ➔ 0,2,6,8,A ➔ 0,2,4,6,8 = 5-33
0,2,4,6,8,A ➔ 0,4,6,8,A ➔ 0,2,4,6,8 = 5-33
0,2,4,6,8,A ➔ 2,4,6,8,A ➔ 0,2,4,6,8 = 5-33

Secondly, we can change a pitch class to a uniquely new one. Take pitch class A, for example. In the below cases, it been replaced by the six possible pitch classes that are not already present in set 6-35, namely 1, 3, 5, 7, 9 and B.

[70] For example, the second row has had pitch class 8 removed to give 0,2,4,6,A, which is 0,2,4,6,8 when converted into prime form.

0,2,4,6,8,A → 0,2,4,6,8,1 → 0,1,2,4,6,8 = 6-22
0,2,4,6,8,A → 0,2,4,6,8,3 → 0,2,3,4,6,8 = 6-21
0,2,4,6,8,A → 0,2,4,6,8,5 → 0,2,3,4,6,8 = 6-21
0,2,4,6,8,A → 0,2,4,6,8,7 → 0,1,2,4,6,8 = 6-22
0,2,4,6,8,A → 0,2,4,6,8,9 → 0,1,3,5,7,9 = 6-34
0,2,4,6,8,A → 0,2,4,6,8,B → 0,1,3,5,7,9 = 6-34

Once converted into prime form, the newly formed sets created by replacing pitch class A are 6-21, 6-22 and 6-34, which appear twice each. Because of the evenly spread pitch content of 6-35, when each of the five pitch classes other than A (i.e. 0, 2, 4, 6 and 8) carry out the same six mutations of being replaced by new pitch classes, sets 6-21, 6-22 and 6-34 also appear twice in each of the six possible mutations. This results in 12 occurrences each of 6-21, 6-22 and 6-34, giving 36 overall for replacing a pitch class.

Finally, when one uniquely new pitch class is added to the set 6-35, we have the following six mutations, all of which convert to superset 7-33 when in prime form:

0,2,4,6,8,A → 0,2,4,6,8,A,1 → 0,1,2,4,6,8,A = 7-33
0,2,4,6,8,A → 0,2,4,6,8,A,3 → 0,1,2,4,6,8,A = 7-33
0,2,4,6,8,A → 0,2,4,6,8,A,5 → 0,1,2,4,6,8,A = 7-33
0,2,4,6,8,A → 0,2,4,6,8,A,7 → 0,1,2,4,6,8,A = 7-33
0,2,4,6,8,A → 0,2,4,6,8,A,9 → 0,1,2,4,6,8,A = 7-33
0,2,4,6,8,A → 0,2,4,6,8,A,B → 0,1,2,4,6,8,A = 7-33

Overall, initial set 6-35 has 48 possible mutations (6+36+6), and Table 3.1 highlights these as probabilities towards what will be labelled first iteration sets. Within all of these first iteration sets, there remains a natural emphasis on whole-tone related intervals. However, besides set 5-33, the remaining sets all introduce a semitone step, resulting in the potential to use more material relating to minor 2nd intervals.

	5-33	**6-21**	**6-22**	**6-34**	**7-33**
6-35	$^{6}/_{48}$	$^{12}/_{48}$	$^{12}/_{48}$	$^{12}/_{48}$	$^{6}/_{48}$

Table 3.1: *From Zero – Set 6-35: Initial Set Mutation Probability Matrix*

A probability of $^{1}/_{2}$ was assigned on the basis that a set would mutate, and a probability of $^{1}/_{2}$ that a set would remain the same, as it was felt that mutating too often would reduce the music's harmonic coherence. For the same reason, the number of times a set could be played consecutively was limited to two. Taking into account that half of the time the set will remain unchanged, this was factored together with the previous probabilities to include mutation to set 6-35 in the probability matrix.

6-35	**5-33**	**6-21**	**6-22**	**6-34**	**6-35**	**7-33**
	$^{3}/_{48}$	$^{6}/_{48}$	$^{6}/_{48}$	$^{6}/_{48}$	$^{24}/_{48}$	$^{3}/_{48}$

Table 3.2: *From Zero – Set 6-35: Final Set Mutation Probability Matrix*

After constructing Table 3.2 for the initial set 6-35, similar tables for the first iteration sets (5-33, 6-21, 6-22, 6-34 and 7-33) were also built, shown in Appendix B. These sets actually have a probability of more than $^{1}/_{2}$ of remaining unchanged, since when replacing pitch content by uniquely new pitch classes, on occasion it still results in the same set when converted to prime form[71]. First iteration sets all have the possibility to mutate to the initial set 6-35, to all other first iteration sets within a cardinality of 1, as well as several newly formed second iteration sets. The full breakdown of possible mutations can be seen in Appendix C[72].

When considering the interval classes of the above sets across a variety of cardinalities, there is an abundance of contrasting intervallic qualities. In order to manage the harmonic distance that sets travel from the home content of 6-35, a restriction was put in place so that when the piece mutated to a second iteration set, it had a probability of $^{1}/_{2}$ of remaining the same and a probability of $^{1}/_{2}$ of returning to the previous first iteration set. Since sets can only be played on a maximum of two consecutive occasions, after this point second iteration sets must return to the previous first iteration set.

A random number generator was used to determine how the sets would mutate[73]. For example, when considering set 6-35 and Table 3.2, there are 48 possibilities, and so the random number generator would be set up between 1 and 48. Boundaries were set up so

[71] For example, within set 6-21, [0,2,3,4,6,8], if we replace pitch class 3 by 5 to produce [0,2,5,4,6,8], we still have set 6-21 when converted to prime form. This happens twice with set 6-21, and so its probability is $^{2}/_{96}$ + $^{48}/_{96}$ (the latter representing the arbitrary probability of $^{1}/_{2}$) = $^{50}/_{96}$.

[72] In Appendix C, some first iteration sets also appear in the second iteration set column of other first iteration sets as a natural consequence of the close relationships between these sets.

[73] 'Random Number Generator', (2011).

that if the random number generator produced a number between 1 and 24, the set would remain 6-35, between 25 and 27 it would mutate to set 5-33, between 28 to 33 it would mutate to set 6-21, etc.

3.3 From Zero

After the pre-compositional design was established, the piece was further restricted to unordered PC sets[74]. For the emphasis to remain on mutation within pitch, other musical parameters such as note duration, dynamics, octave placement, articulation and tempo were not restricted within this system. Set mutation was used as a creative starting point, and in the first instance, was split into several small phrases with regular rests, before various post-mutation edits were made to aid musical coherence.

A key purpose of *From Zero* is to highlight the development of pitch content within new sets across small musical phrases. Once the order of the sets has been generated, the only rule is that each pitch class within a set must be played before linearly moving on to the next one. Repeated pitches are permitted within a single set.

The initial set mutation produced from the Primary Design and random number generator is highlighted in Table 3.3.

Set	Set Name	Set Content
1	6-35	0,2,4,6,8,A
2	6-35	0,2,4,6,8,A
3	7-33	0,1,2,4,6,8,A
4	8-21	0,1,2,3,4,6,8,A
5	7-33	0,1,2,4,6,8,A
6	7-24	0,1,2,3,5,7,9
7	7-33	0,1,2,4,6,8,A
8	7-33	0,1,2,4,6,8,A
9	6-34	0,1,3,5,7,9
10	6-34	0,1,3,5,7,9
11	7-26	0,1,3,4,5,7,9
12	6-34	0,1,3,5,7,9

Table 3.3: *From Zero – Set Mutation*

[74] Unordered and ordered PC sets relate to whether there is a pre-determined order of pitch classes within an individual PC set.

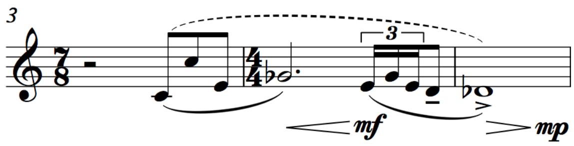
After analysing Table 3.3 for contrasting/complementary set material, two contrasting types of sets were determined:

1. Supersets of the initial set 6-35, [0,2,4,6,8,A], namely 7-33, [0,1,2,4,6,8,A], and 8-21, [0,1,2,3,4,6,8,A]
2. Supersets of the first iteration set 6-34, [0,1,3,5,7,9], namely 7-24, [0,1,2,3,5,7,9], and 7-26, [0,1,3,4,5,7,9]

Although some of these supersets share common pitch classes, when converted to prime form there is a disparity in the upper pitch content of [4,6,8,A] against [3,5,7,9]. The structure of the piece was created based on this distinction and potential for complementation.

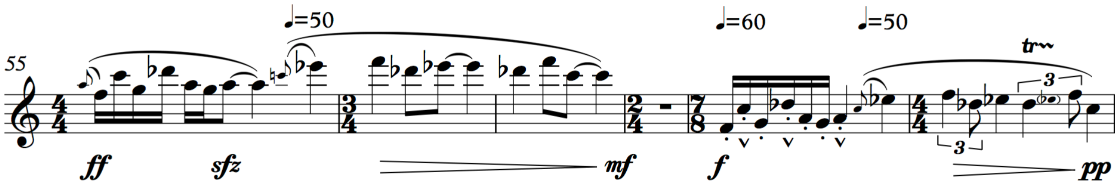
The piece is split into three sections, producing an ABA' format. Section A develops pitch content relating to set 6-35 and its supersets listed above, section A' (from bar 47) relates to set 6-34 and its supersets, while section B (from bar 27 to 46) incorporates material from both. The differences between sections A and A', enforced by the pre-determined set mutation, give the piece a structure which starts from the 'home' of 6-35, develops interrelated content within section B, before moving to an acceptable 'home' of 6-34. On a macro-level, the piece effectively mutates from 6-35 to 6-34.

Since the principal idea within this piece is to use the concept of mutation to develop new pitch content at appropriate phrasing points, the differences between set content become significant. Looking more closely at section A, it begins with set 6-35 but mutates towards 7-33, which naturally introduces the extra pitch class 1 (D♭), and then towards set 8-21 which includes a further pitch class 3 (E♭). For this reason, as well as the fact that these odd-numbered pitch classes introduce material other than a whole-tone emphasis, these new pitch classes were chosen as crucial mutation nodes to transition from one phrase to another. As seen in bar 5 of Example 3.1, pitch class 1 (D♭) is introduced through a semibreve at the end of the phrase to emphasise its introduction as the piece mutates and settles into this new set content. Similarly, pitch class 3 (E♭) is introduced firstly in bar 9 at the end of a phrase, with its presence consolidated in bar 12, as the final note in the passage.



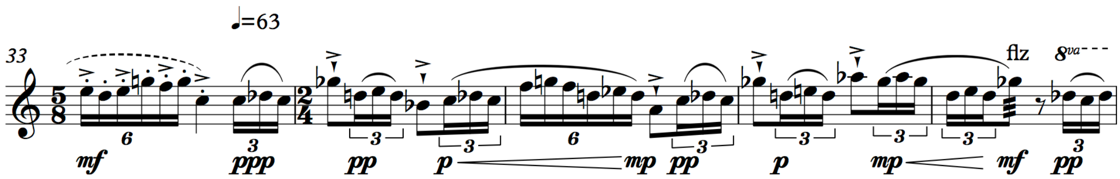
Example 3.1: *From Zero – Pitch Class 1 Introduction*

Equivalent passages from section A’ utilise an identical mutation method to introduce new pitch content. In this section, set 6-34 moves towards 7-26 by introducing the extra pitch class 4 (E). However, unlike section A, in which set 7-33 then adds a further new pitch class to mutate towards 8-21, section A’ material, as determined by the set mutation, returns from 7-26 to 6-34 to move towards the end of the piece. Subsequently, pitch class 4 (E) is lost. In order to support the elimination of the E, bars 56-57 and 60 include a largely descending sequence of [5,1,3,1,5,0] (Example 3.2).



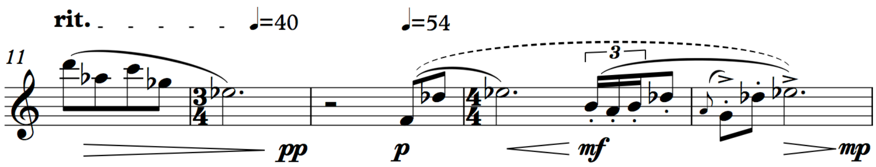
Example 3.2: *From Zero – Pitch Class 4 Removal*

As a stark contrast to sections A and A’, which have a pensive mood, the transitory section B has a lighter disposition, and is more lively and spirited in nature through its quicker tempo and higher pitch register. Adapting the triplet phrase first used in bar 1, section B contains phrases that move hastily and less ambiguously back and forth between 7-33, [0,1,2,4,6,8,A], and 7-24, [0,1,2,3,5,7,9], using shared pitch classes 0 and 1 (C and D♭) to pivot between sets with a C - D♭ - C sequence. This use of pivoting between triplet, neighbour notes was influenced by its integration into *Syrinx*, in which Debussy also uses the pivots as a platform for more expressive runs. In Example 3.3, the dynamics hairpin overlapping bars 34-35 represents set 7-24 while the remainder of the material relates to set 7-33.



Example 3.3: *From Zero – Section B Central Passage*

After working with a flautist in 2015 to produce a studio recording of *From Zero*, it was decided that the piece had potential to be expanded from the initial structure created via Table 3.3. One key expansion can be seen through a modulated repetition of section A, with added melodic and rhythmic variation. Example 3.4 demonstrates this transition, with the first section A statement ending in bar 12, and the transposed repeat beginning in bar 13. Since E♭ ended section A in the original set mutation, it was felt that it should be prominent at the beginning of the repeat so that the modulation would feel more natural. Consequently, a transposition of the original content up a perfect 4th was applied. Additional amendments were carried out by adding further phrases to section B, as well as extending the ending of the piece by interweaving the transposed and non-transposed material of section A and A'.



Example 3.4: *From Zero – Section A Modulation*

As well as these structural changes, there is more expressive usage of articulations, dynamics and phrasing, making the piece more suitable to the idiomatic nature of the flute, as can be heard in the included recording of the concert performance. *Syrinx* was particularly influential in this regard, through its dynamic use of grace notes in fast ascending and descending runs. Debussy's consideration of short, repeated triplets, alongside sustained, legato parts aligned with material in *From Zero*, and further influenced the reconstruction of isolated phrases into longer, coherent passages.

3.4 Summary

Based on the notion of mutating PC sets, *From Zero* illustrated a linear progression of sets through compositional rules linked to Markov chains. The slowly developing journey away from the whole-tone scale was paramount to the melodic and harmonic progression of the piece, and the resultant set content provided a suitable structure, founded on the contrasting nature of sets related to the whole-tone scale and 6-34 ([0,1,3,5,7,9]). In particular, I made use of the inclusion of the minor 2nd, minor 3rd and major 6th interval classes within 6-34, and utilised them to directly contrast with the major 2nd, major 3rd and minor 7th emphasis of 6-35, while the idiomatic nature of the flute helped to highlight this distinction through ornamentation in particularly fast passages. Although the final version of *From Zero* placed less importance on maintaining rigid structural functions, the concept of mutation remained fundamental to the piece's underlying structure and harmonic content. The piece's expansion via harmonic modulation and rhythmic iteration was carried out at key structural points originally devised as mutation nodes, and aided the transformation from an initial experimental sketch into a more substantial solo work.

Chapter 4

In Equal Measure (2011-2016)

In Equal Measure is a two-movement piano piece that aims to move beyond the linear application of PC sets in order to investigate the usage of multi-layered symmetry within set theory as a framework for composition. The choice of piano was influenced by its application of hierarchical symmetry at both structural and local harmonic levels within Bartók's *Slow Melody* as part of his *Petite Suite for Piano* (1936). For instance, the piece opens with five consecutive crotchet notes of set 5-Z36, [0,1,2,4,7]. Harmony is then layered upon the central note, which acts as a melodic axis, from which subset and superset relations of 5-Z36 are formed. This principle expands across independent and related left-hand and right-hand phrases, which sees the development of macro-level, reflective symmetry of individual bars, then pairs of bars and ultimately longer structural phrases. The piano also offered potential for vast harmonic variety and a contrapuntal style not possible with the flute of *From Zero*. The pre-compositional design comprises a harmonic and structural outline based on symmetry, as well as a symmetrical twelve-tone row.

4.1 Symmetry

The principal aspects of music, whether intentionally or unintentionally on the part of the composer, maintain inherent symmetrical qualities, from micro-level motivic content such as melodic inversion, augmentation and diminution, to macro-level compositional techniques linked to phrase structures and form. The works of Bartók and Webern have been a particular focus of analysis for music theorists of the 20th century. For instance, Berry analyses Bartók's string quartets in relation to symmetrical interval sets and tonality[75], with local patterns of intervals such as the 2-1-2 tetrachord (F-G-A♭-B♭, for example) or 3-2-2-3 pentachord (A-C-D-E-G, for example) forming worthwhile investigation. These types of symmetrical interval sets can extend to interval cycles as well as larger scales such as the octatonic, and provide theoretical links between what would be considered on the surface to be musically unrelated material. Webern's

[75] Wallace Berry, 'Symmetrical Interval Sets and Derivative Pitch Materials in Bartók's String Quartet No. 3', *Perspectives of New Music,* 18 (1979), 287-379.

Symphonie, Opus 21 (1928), has also been extensively examined for its advanced applications of symmetry across twelve-tone rows and form, with Kempf noting that the second movement's structure contains a "complex system of symmetries within symmetries", with emphasis on mirror reflections around the vertical and horizontal axes[76]. As *Symphonie* moves into the second movement, this use of symmetry becomes explicitly audible through symmetrical inversions, canons and repetition.

Larry J. Solomon's PhD dissertation, which has been adapted into a website section titled *Symmetry as a Compositional Determinant*, contains a general yet comprehensive overview of symmetry within musical composition[77]. He defines symmetry as "a congruence resulting from the operations of reflection, rotation, or translation"[78] and includes definitions and examples of each type, with reference to common musical transformations such as transposition and inversion[79]. His visual representations of works such as Webern's *Variations for Piano, Opus 27* (1936) provide harmonic and structural outlines which consider symmetrical and proportional pitch centre relationships at both micro and macro levels, with a further focus on the development of dynamics and instrumentation. These outlines and subsequent analyses have been valuable for the construction of my own designs.

4.2 Pre-Compositional Design

The distinguishing factor between the two movements of *In Equal Measure* is in the level of restrictiveness in their compositional approach. The first movement utilises the harmonic and structural outline within a strictly systematic process that has more of an emphasis on a macro-level structure, while the second movement is more flowing in nature, using micro-level harmonic and motivic gestures as starting points. The use of both methodologies demonstrates the design's flexibility and provides contrasting compositional outcomes.

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[76] Davorin Kempf, 'What Is Symmetry in Music?', *International Review of the Aesthetics and Sociology of Music,* 27 (1996), p. 162.
[77] Larry J. Solomon, 'Symmetry as a Compositional Determinant', (2002b).
[78] Ibid.
[79] For the purpose of this research, the terminology used by Solomon will be used to discuss all aspects relating to symmetry.

Harmonic and Structural Outline

The initial idea for the outline was based on the notion of symmetry within a set, with influence taken from interval cycles, symmetrical interval sets and mirror sets[80]. These areas of interest led to the examination of the symmetrical nature of the augmented triad and diminished 7th chord, in particular. The two movements are an exploration of the manner in which the augmented triad and diminished 7th chord can be entwined into a harmonic progression, which forms the basis of a structural outline. Consequently, the musical character of both movements remains inherently linked to the musical character of the above chords[81], which Schuijer notes "have that relatively invariant sound"[82].

Presented in Figure 4.1 is the first stage of the outline. The background structure was crafted by interweaving an ascending C diminished 7th chord with a descending G augmented triad, separated into the treble clef and bass clef for clarity. This offers 180 degrees rotational symmetry around the bass clef note of F# (pitch class 6 within [0,3,6,9,0]), while the treble shows 180 degrees rotational symmetry between notes D# and B, where C# would lie as pitch class 1 within [7,3,B,7].

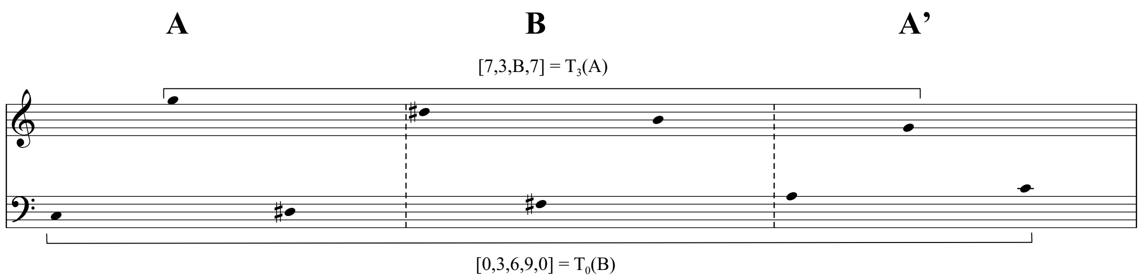


Figure 4.1: *In Equal Measure – Harmonic and Structural Outline: Root Notes*

The structural decisions made at this early stage have a multitude of implications for the subsequent compositional process. Although operations such as transposition could be performed on the outline, in its current state it suggests an overall form of pieces starting and ending on the same pitch centre of C. It contains nine pitches, which permits the outline to be separated into three sections, which could be used as a loose ternary or ABA' form, as highlighted, with F# being used as an axis of symmetry.

[80] As noted by Solomon (2002a), a mirror set is one that has its own inverse.
[81] In prime form, the augmented triad consists of [0,4,8], set 3-12, but for the purpose of this work will be titled set A, and labelled as T_0(A) to distinguish between other transpositions utilised. Similarly, the diminished 7th chord, consisting of [0,3,6,9] and titled 4-28, will be set B, labelled as T_0(B).
[82] Schuijer, p. 126.

Furthermore, utilising nine pitches also allows for nine individual sections, or an additional ABA' form to be created within each A, B and A', thus permitting a dual-layered hierarchical design, as detailed below.

In order to further interlace the links between the two chords, augmented triads were built on each note in the diminished 7th chord of the bass clef, while diminished 7th chords were constructed upon the augmented triad notes in the treble clef. The nine chords shown in Figure 4.2 are the outline's primary chords.

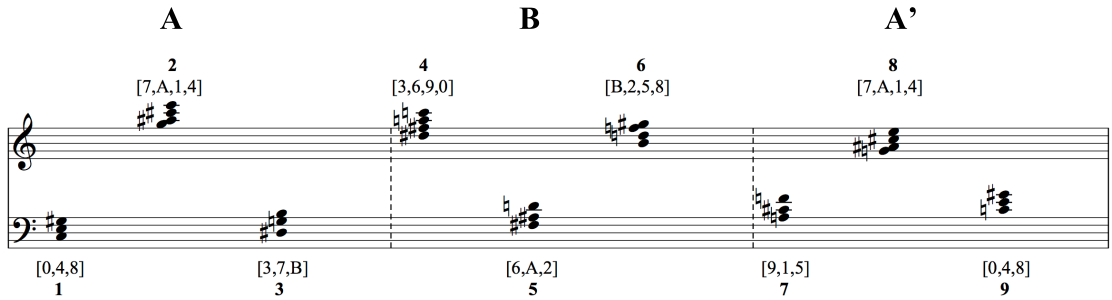


Figure 4.2: *In Equal Measure –Harmonic and Structural Outline: Primary Chords*

Solomon's analytical sketch of the second movement of Webern's *Variations for Piano, Opus 27* (1936) [83] provides a Schenkerian-like reduction of tonal centres and a symmetrical pivot around the pitch A4. This is similar to the central F# within Figure 4.1 and the F# augmented chord in Figure 4.2, which acts as central point around a harmonic axis. To further adopt Webern's model, an extra level of harmony was added to the hierarchy to allow for the potential of more advanced multi-layered symmetry, shown through Figure 4.3. Through a structurally-influenced systematic series of transpositions on the nine primary chords, the outline was expanded to include a total of 27 chords (including a number of secondary chords labelled as 1b and 1c, etc.), which results in the inclusion of all transpositions of sets A and B[84]. Notable new features of the outline include the reflective symmetry of section B and the in-depth similarity of sections A and A'.

[83] Solomon (2002b).
[84] Chords within Figure 4.3 are all labelled as a limited number of transpositions of sets A and B (since the PC set content is repeated under transposition), yet are visually presented with different root notes. For example, chords 2b and 3c of section A are both labelled as T_2(A) but have root notes of A and 6 respectively. This is a natural consequence of the systematic series of transpositions, and these root notes have had a direct influence on the compositional processes within the piece.

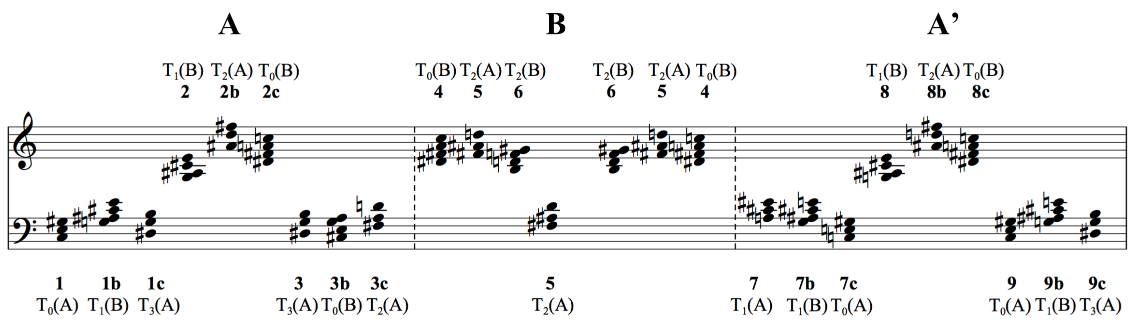


Figure 4.3: *In Equal Measure –Harmonic and Structural Outline: Complete*

This outline is not necessarily restricted to a rigid framework in which linear expositions of sequential chords for fixed durations are fashioned, but is intended for use as a fluid creative platform for underlying harmony across each movement.

Twelve-Tone Row

With the harmonic and structural outline complete, a second compositional tool was sought which could interact in a variety of ways with the augmented triad and diminished 7th chord to allow additional harmonic colour. Motivated by Webern’s integration of a structural plan and twelve-tone row into his *Symphonie, Opus 21*[85], and the similar symmetrical properties of all-interval rows, as discussed by writers such as Morris and Starr[86], a twelve-tone row was created, based on sequential interval order, which contains retrograde-inverse symmetry. It comprises [0,6,2,5,3,4,9,A,8,B,7,1] and is labelled as XYZ. Its notational and visual representations highlight its symmetrical qualities in Figures 4.4 and 4.5.

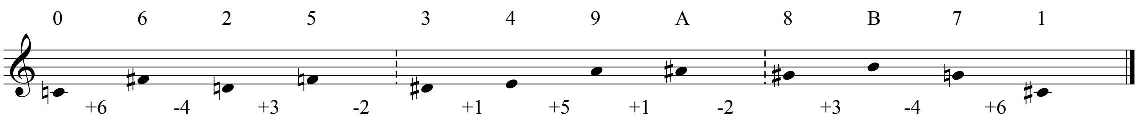


Figure 4.4: *In Equal Measure – Twelve-Tone Row: Notational Representation*[87]

[85] As analysed by Lejaren Hiller and Fuller Ramon, 'Structure and Information in Webern's Symphonie, Op. 21', *Journal of Music Theory,* 11 (1967).
[86] 'The Structure of All-Interval Series', *Journal of Music Theory,* 18 (1974).
[87] Figure 4.4’s notation of +6, -4, +3… +3, -4, +6 demonstrates the construction of the row’s reflective symmetry based on pitch-class intervals, similar to the construction of the interval set discussed previously. Registral placement is not a key factor in its formation.

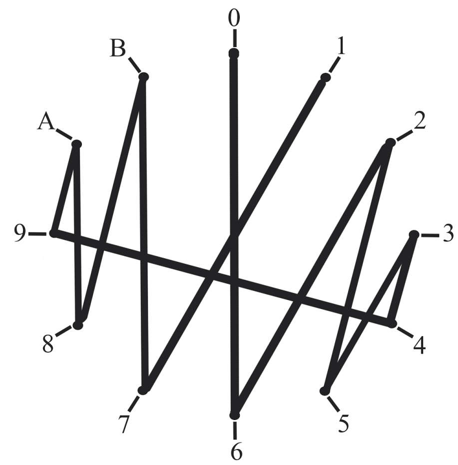


Figure 4.5: *In Equal Measure – Twelve-Tone Row: Visual Representation*[88]

Restrictions were placed on the application of the row within the musical works to allow for consistency and continuity when combining harmonic functions with the harmonic and structural outline. The row could only be used in its original sequence, while it was split into three sets: X, [0,6,2,5], Y, [3,4,9,A] and Z, [8,B,7,1]. It was determined that X would be restricted to only appear in section A of each movement, Y could only appear in section B, and Z could only appear in section A'. As a natural consequence of the row's construction and symmetrical properties, X and Z both form the AIT 4-Z15, [0,1,4,6], and through transposition, Z is a retrograde of X in the same manner that sections A and A' of Figure 4.3 have reflective qualities. Moreover, the central set Y forms the double tritone tetramirror set 4-9, [0,1,6,7], which has only 6 unique transpositions[89], further enhancing the symmetrical qualities of the row. Ultimately, the inclusion of the twelve-tone row has less focus on its surface level ordering of PC sets, but more on how it integrates in a variety of ways with the harmonic content of the augmented triad and diminished 7th chord from the outline.

4.3 In Equal Measure

The usage of the harmonic and structural outline and the twelve-tone row in each of the piano movements will now be discussed.

[88] Michael McNeilis and Jackson Wade, 'In Equal Measure – Twelve-Tone Row: Visual Representation', (2015). To interpret the visual representation, follow the vertical line from 0 to 6. This represents the first two pitch classes in the row. From here, 6 moves 2, and 2 moves to 5, etc. as it makes its way through the sequence of the row [0,6,2,5,3,4,9,A,8,B,7,1].
[89] Solomon (2002a).

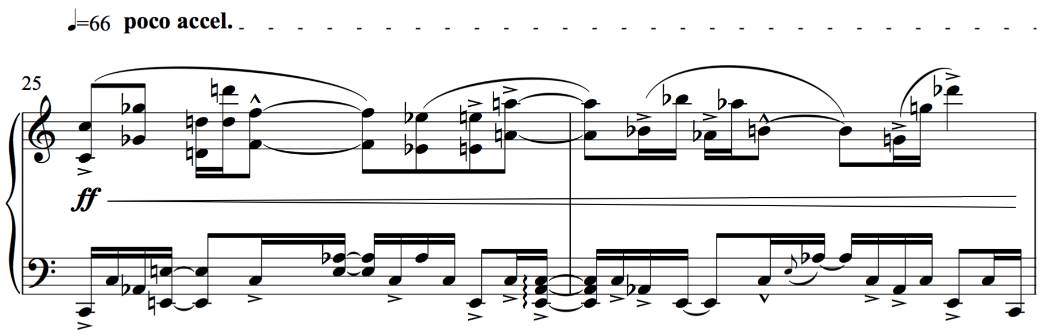
Movement One

As mentioned above, movement one is generally restricted in nature. This is due to its strict usage of hierarchical symmetry on a structural level. With influence from Sergei Prokofiev's use of *moto perpetuo* in the scherzo of his *Piano Concerto No. 2 in G Minor, Opus 16* (1913), which places emphasis on a lack of rests and persistent semiquaver arpeggios, the movement aims to display its own *moto perpetuo* through an ongoing palindromic wave of rhythmic and harmonic progression, primarily seen in the left hand throughout sections A and A'. The texture thickens towards the end of section A, and thins out at the end of section A' and the movement as a whole. All nine primary chords from the outline are fundamental to the symmetrical harmony of the movement, as is the twelve-tone row (XYZ) and sets X, Y and Z. If we first consider section A, it is divided into three segments of 10 bars each, and the left-hand content stems from the outline, while the right-hand content is from the twelve-tone row (see Table 4.1). As a consequence of this segmentation and combination of right and left- hand harmony, a variety of contrasting sonorities are produced. For example, primary chord 1, [0,4,8], in the left hand is combined with set X, [0,6,2,5], in the right hand, producing content [0,2,4,5,6,8], which is the inverted form of set 6-21, [0,2,3,4,6,8].

Temporal reflection and rotational symmetry within and across segments are a natural consequence of this cyclical setup. Rather than continuing this sequence and navigating back to set X in segment 3 as the cyclical pattern would suggest, there is an explicit statement of the twelve-tone row in the right hand (Example 4.1). This highlights the only surface level occurrence of the twelve-tone row, made more apparent by the D♭ crotchet held at the end, which nullifies the rhythmic symmetry over the two bars.

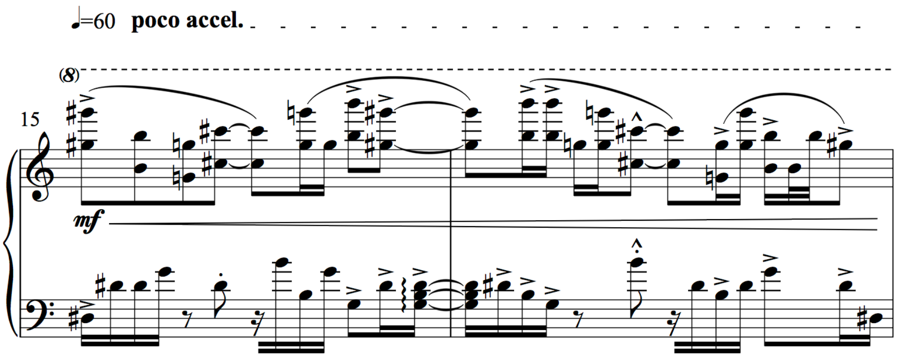
Section A	Segment 1 (10 bars)			Segment 2 (10 bars)			Segment 3 (10 bars)		
Bar Numbers	1-4	5-6	7-10	11-14	15-16	17-20	21-24	25-26	27-30
Primary Chords (left hand)	1	2	1	2	3	2	3	1	3
Twelve-Tone Row Sets (right hand)	X	Y	X	Y	Z	Y	Z	XYZ	Z

Table 4.1: *In Equal Measure*, *Movement One – Section A Segmentation*



Example 4.1: *In Equal Measure*, *Movement One – Twelve-Tone Row*

Symmetry within rhythm is particularly important in section A. The inclusion of tied notes across bar lines in the left hand and at the halfway point in the right hand bring this symmetry more lucidly to the forefront. Example 4.2 shows an approximation to non-retrogradable rhythms (and pitch) in both the bass and treble, and is one of the clearest forms of reflective symmetry within the section[90]. These rhythms have a direct link to Messiaen's Modes of Limited Transpositions, in that the modes "realize in the vertical direction (transposition) what non-retrogradable rhythms realize in the horizontal direction (retrogradation)"[91].



Example 4.2: *In Equal Measure*, *Movement One – Non-Retrogradable Rhythm*

Although embellishments reduce their precise symmetrical qualities, these non-retrogradable rhythms are apparent throughout the rest of the section. For example, if we consider segment 1 of ten bars, it has reflective symmetry within rhythm at its halfway point, but there also exists self-similarity as there is reflective symmetry halfway through each bar, and also at the halfway point between bars 1 and 2, 3 and 4, 5 and 6, 7 and 8, 9 and 10, 1 and 4, and 7 and 10. This results in multi-level symmetry

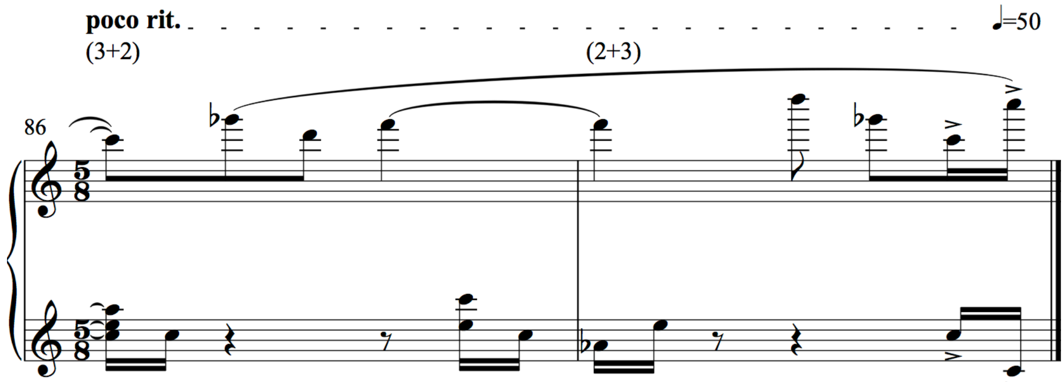
[90] A non-retrogradable rhythm is a series of note values that are read the same both forwards and backwards.
[91] Messiaen, p. 62.

within symmetry, again inspired by Bartók's *Slow Melody* (1936). There is a further unifying link between both rhythms (apparent in Example 4.2), in that the left-hand rhythm is seen augmented by a factor of two and slightly varied to form the syncopated right-hand rhythm. Although a strong, cyclic progression is produced from the consistent rhythmic and harmonic nature, minor variation of rhythm and harmony is present in order to attach a less mechanistic quality. For instance, dotted rhythms become more prominent as the section progresses, while there are also micro-level changes in registral placement and expansions, such as octave doubling of vertical harmony.

If we now turn to section A' of the movement, it is almost identical to section A through reflective symmetry. Besides the alteration in harmony[92] and embellishments, the only other fundamental change can be seen in the reformed bar lengths of section A'. In order to divert the listener from an overly repetitive and predictable element of symmetry, the final four bars of each segment were sequentially shortened from 4/4, so that ultimately the movement ends in 5/8. As a comparison, the rhythmic differences are noticeable within Examples 4.3 and 4.4. Through the construction of the pre-compositional design, they appear as retrogrades of each other (besides minor textural and harmonic changes)[93], and Example 4.4 shows that the final three quavers of bar 86 and the first three of bar 87 have been omitted to form the new rhythm.

Example 4.3: *In Equal Measure*, *Movement One – Section A' Retrograde 1*

[92] Section A begins with primary chord 1 while section A' begins with 7 as a consequence of the harmonic and structural outline.
[93] In actuality, due to the extensive self-similarity of sections A and, in this case, A', the rhythms of Examples 4.3 and 4.4 are not only loose retrogrades, but they actually almost form a repeat also.



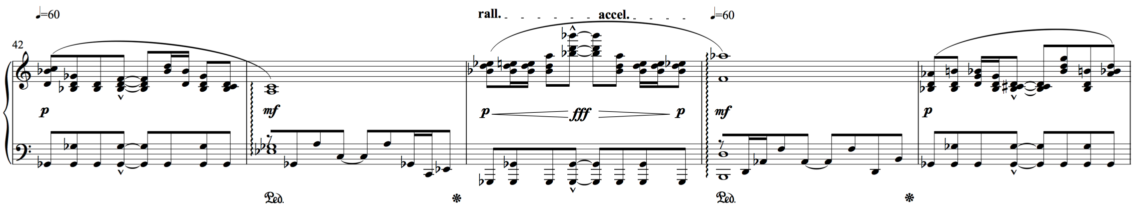
Example 4.4: *In Equal Measure*, *Movement One – Section A' Retrograde 2*

In contrast to sections A and A', section B (bars 31-57) places less emphasis on steady momentum, and more on a measured expansion of rhythm and harmony, while its fundamental pitch content is almost completely taken from primary chords 4, 5 and 6. The section is structurally more fluid than A and A' in that it weaves back and forth between focus on each primary chord. Although reflective symmetry is once again paramount, there is less dependence on its strict usage, and it is consequently somewhat more expressive in nature.

Section B begins with a brief transition, by way of combining similar rhythms of section A with pitch content from chord 4 (and later 6) in the left hand, as well as introducing new pitch content in the treble. Seen in Example 4.5, the left hand of bar 38 shows an instance of melodically interlinking symmetry via an axis of pitch into primary chord harmony. The integration of axes of pitch in this context was motivated by the concept of symmetrical interval sets and the usage of symmetrical axes for melodies and harmony in Webern's *Three Little Pieces for violoncello and piano, Opus 11* (1914). Primary chord 4 (in its derived order within the harmonic and structural outline of [3,6,9,0] rather than converted to a prime form of [0,3,6,9]) has an axis of pitch that falls between 7 and 8. This axis is explicitly played rather than just felt, as demonstrated with the alternating 7 and 8 pitches (G and A♭). Similar axes of pitch were also constructed for chord 6, alongside further variations, and were used as material for the right hand of section B.

Example 4.5: *In Equal Measure*, *Movement One – Axis of Pitch for Primary Chord 4*

After the transition, primary chords 4, 5 and 6 are stated consecutively, and interlaced alongside further transitional material, often by temporally reflective symmetry within pitch and rhythm. The material progresses towards a central point of the whole movement (bar 44 of 87), **e**xpressed in Example 4.6.



Example 4.6: *In Equal Measure*, *Movement One – Central Passage*

Harmony here is formed by a combination of primary chord 5 (the centre point on the harmonic and structural outline) as well as the middle set of the twelve-tone row (Y). The first and last sets of the row (X and Z) also support the preceding and following bars (X with 4, and Z with 6) to emphasise the importance of this central passage, as do the registral expansion and the temporally reflective symmetry within the shifts in dynamics and tempo. The insistent nature of the surrounding sections is at its most distant here, as the movement temporarily becomes less mechanical and more expressive.

Movement Two

Movement two is split into three smaller sections[94], which form the harmonic material for the outline, with additional content taken from the twelve-tone row sets. Although other related pitch content is evident, Table 4.2 displays the underlying harmonic material.

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[94] Initially these three sections were considered as separate works.

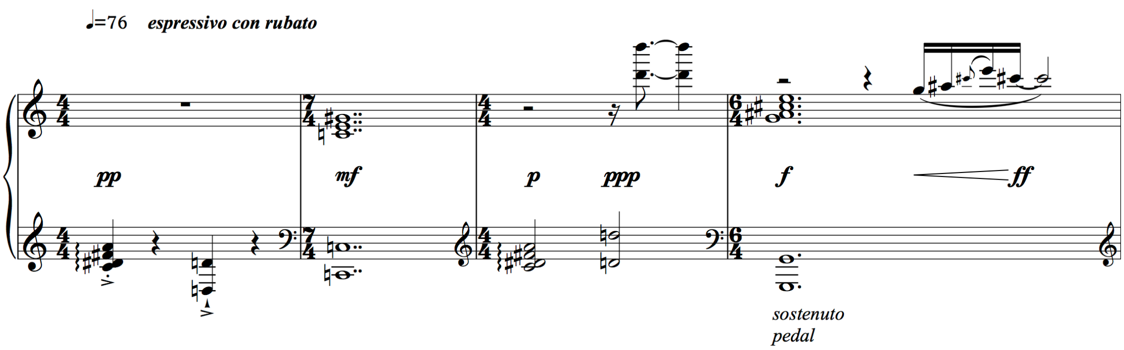
Movement Two	Primary Chords	Twelve-Tone Row Sets
Section 1	1, 2, 3	X
Section 2	4, 5, 6	XYZ
Section 3	7, 8, 9	Z

Table 4.2: *In Equal Measure*, *Movement Two – Harmonic Content*

Since sections 1 and 3 have emphases on sets X and Z respectively, one might expect set Y to be the emphasis of section 2. However, the full twelve-tone row XYZ replaces set Y here in order to make a complete statement of the twelve-tone row at the halfway point of the movement. As discussed earlier, movement two is primarily constructed via initial micro-level ideas, which are expanded in each of the varied sections to form macro-level structural devices.

Section 1

The opening section is based around a rondo form of ABACA. Section A begins with a one-bar statement of [0,3,6,9], which is T_0(B), plus the pitch 2 (the note D)[95]. This phrase (seen in bar 1 of Example 4.7), acts as a static point from which instances of primary and secondary chords (bars 2 and 4) can fluidly pivot back and forth[96].



Example 4.7: *In Equal Measure*, *Movement Two – Opening Pivot*

[95] The pitch 2 (the note D) was taken from set X, while the remaining pitches within X (0,6 and 5) are used as decoration within the left hand of bars 12, 14 and 16.
[96] Recurrences of this pivot device were transposed to better suit the harmony of recurrences of section A (i.e. in bars 13 and 15, it is transposed up 7 semitones to better suit the G diminished 7th chord in bar 12, while in bars 22 and 24, it is transposed up 3 semitones to better suit the D# augmented chord in bar 21).

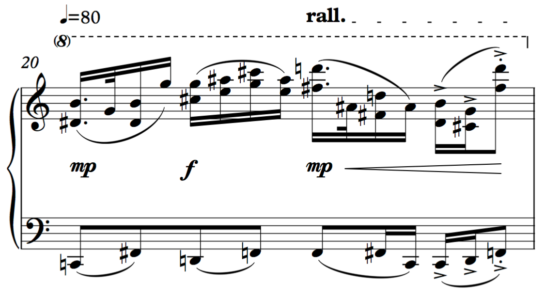
The primary and secondary chords in bars 2 and 4 (and similar) are seen in the order stated within the harmonic and structural outline (i.e. 1, 1b, 1c, 2, 2b, 2c, 3, 3b), while 3c is omitted to allow for the progression to section B. As these chords are played out, the right hand in bar 4 of Example 4.7 shows the first in a series of decorations that accompany the primary and secondary chords. These decorations were created from the retrograde of the middle seven primary and secondary chords, so appear in the order of 3b, 3, 2c, 2b, 2, 1c and 1b. For example, bar 4 shows a combination of a 1b chord and 3b decoration. As a natural consequence of the limited number of transpositions of the augmented triad and diminished 7th chord, at times harmony coincides, as in the case of 1b and 3b.

The seven decorations display loosely reflective symmetry when considered as a collection[97], and mutate to and from a central point of reflection in both rhythm and harmony, as seen within chord 2b (the right-hand decoration in Example 4.8).

Example 4.8: *In Equal Measure*, *Movement Two – Symmetrical Decoration*

Sections B and C are two similar four-bar expansions of primary and secondary chord harmony (seen in bars 8-11 and 17-20, with bar 20 seen in Example 4.9). These sections show sequential arpeggios of varied harmony, which are considered rudimentary instances of translational symmetry. The particular pitch registers in these sections were chosen to create loosely symmetrical ascending and descending wave contours.

[97] The first decoration maps to the seventh, the second to the sixth, and the third to the fifth. The decorations have more ascending pitch sequences at the beginning of the movement, and descend more by the end.



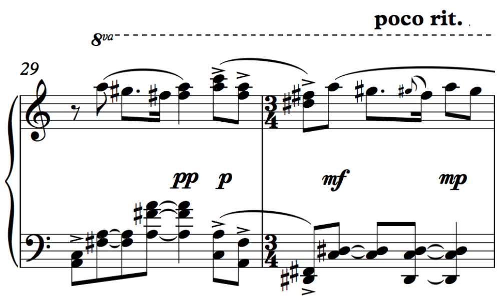
Example 4.9: *In Equal Measure*, *Movement Two – Translational Symmetry*

Section 2

This section, in an ABA’ form, is a straightforward exhibit of symmetry via melodic repetition and chordal progression, with underlying harmonic advances. Like movement one’s middle section, the central part of movement two is gentler and more tranquil in nature compared to its surrounding parts. In section A, the right-hand melody derives its pitch content of [0,3,4,6,8,9] from the addition of sets A, [0,4,8], and B, [0,3,6,9], while the left-hand accompaniment consists of primary chord 4. As the section progresses, the melodic content develops with minor decorations, repetition, expansion and variation in rhythm and pitch. In order to emphasise the almost tonal quality of the melody, the left-hand accompaniment is set into consonant dyad pairs of minor 3rd intervals that build into more dissonant diminished 7th harmony only when considered jointly. The section contains surface level examples of temporal reflective symmetry with respect to rhythm, an illustration of which can be viewed in the left-hand accompaniment in Example 4.10.

Example 4.10: *In Equal Measure*, *Movement Two – Section 2 Reflective Symmetry*

The twelve-tone row can be seen woven into the pedal accompaniment in each bar between 29 and 40, acting as both the section and movement’s central point. Example 4.11 displays the first two bars of this sequence with pitch 0 (C) added to bar 29, and pitch 6 (F#) added to bar 30. The presence of the twelve-tone row at this point augments harmonic variety to an otherwise simple progression of primarily minor 3rd dyads.

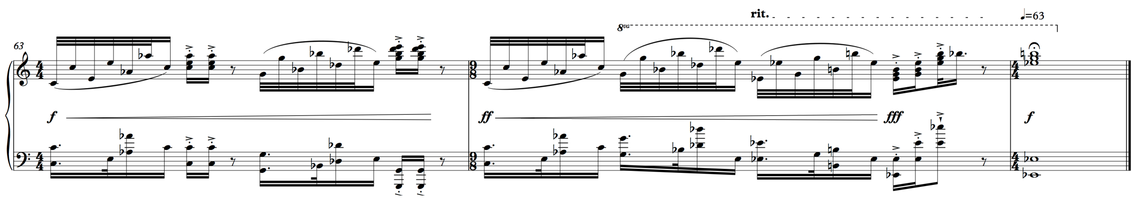


Example 4.11: *In Equal Measure*, *Movement Two – Twelve-Tone Row Entrance*

Section 3

Although composed with the same general approach as the first two sections, this section displays glimpses of strict systematic processes, reminiscent of the first movement, particularly through its use of *moto perpetuo*. Despite flowing quite freely, the changeable harmony, combined with the quickly-paced rhythm, suggests a mechanical process.

The section is in a standard form of ABABAB. Section A contains examples of harmonic complexity as pitch content from primary chords 7 and secondary chords 7b and 7c is interlaced in a flowing semiquaver-based rhythm, which places emphasis on rhythmic and harmonic diminution and augmentation. The second occurrence of section A develops with chords 8, 8b and 8c before its final occurrence utilises 9, 9b and 9c. Section B material is an expansion of rhythmic and harmonic progression as each primary chord 7, 8, and 9 and their secondary chords build to a climax of demisemiquaver octaves, with set Z acting as melodic decoration. The progression of ascending harmonic arpeggios in this passage (Example 4.12) uses primary chord 9 and displays compound translational symmetry akin to those seen in section 1.



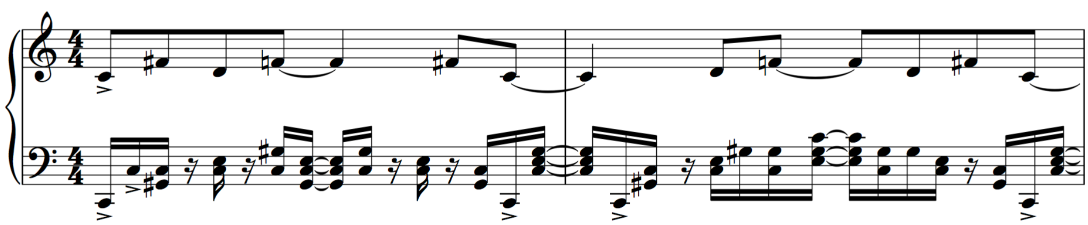
Example 4.12: *In Equal Measure*, *Movement Two – Ending*

With respect to the dynamic characteristics, the two movements contrast with each other. Movement two has regular noticeable shifts in dynamics, while movement one, through its gradual developmental motion of sections A and A’ in particular, has slower and more iterative dynamic shifts. As can be seen from Table 4.3, movement one also has its dynamic crescendo during the middle, while the beginning and end of movement two are its loudest points. This roughly symmetrical inversion was purposely made to further contrast the two movements.

Movement One	ppp < fff > ppp		
Movement Two	fff >	pp < f > pp	< fff

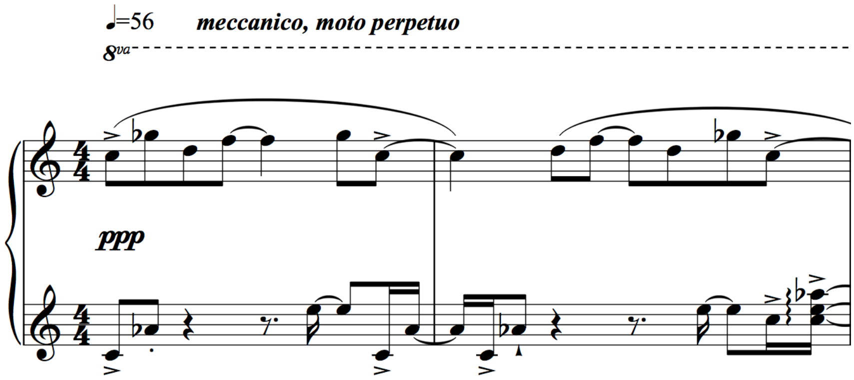
Table 4.3: *In Equal Measure – Dynamic Comparison of Movements*

Although the first draft of the piece was successful in its explicit usage of symmetry, the final version aimed to address the playability of the piece, with the wide-ranging leaps in the first movement proving particularly difficult. Ligeti's *18 Études Pour Piano* (1985 to 2001) were studied to take influence from their depth of texture and registral spacing. In order to address this issue without compromising the symmetrical qualities of the piece, textures were made thinner and sparser at the beginning and end of the piece, before developing in complexity closer to section B in the middle**.** Examples 4.13 and 4.14 show the distinction between the opening section A material from the initial symmetrical sketch[98] and the final piece.



Example 4.13: *In Equal Measure*, *Movement One – Sketch Texture*

[98] As with *From Zero*, in which set mutation produced a starting point for further edits, the initial raw notation of section A can be seen in full in Appendix D. It contains the clearest demonstration of symmetry, before embellishments and further variation were added to lessen the movement's rigidity.



Example 4.14: *In Equal Measure*, *Movement One – Final Texture*

Further influence in textural layering was taken from Webern's *Variations for Piano, Opus 27* (1936), through its spatial development of interplay between the left and right hand. Although the first movement of *In Equal Measure* places emphasis on mechanical forward motion, further amendments were made to add variation in tempo, dynamics and register to provide more subtlety to this approach.

In Three Elements (Appendix G), a mixed quartet of violin, clarinet, cello and piano, was performed at a workshop in 2016. It incorporates three sections from three distinct works within the portfolio, and uses the opening of the second movement of *In Equal Measure* to begin the piece. The piano remains the primary focus of this section, but melodic gestures, such as those in bar 4 of Example 4.7, are rearranged for the clarinet and violin. Although the clarinet's gestures were somewhat overpowering dynamically in the performance, having these instruments as a lead voice to the piano's rhythmic accompaniment highlighted potential for *In Equal Measure* beyond the scope of a solo piano work.

4.4 Summary

In Equal Measure, for solo piano, focussed on the concept of symmetry, moving beyond the linear expression of PC sets demonstrated in *From Zero*. The pre-compositional design consisted of a hierarchical structure based on symmetrical sets, namely the augmented triad and diminished 7th chord, as well as a symmetrical twelve-tone row. The strict application of the pre-compositional design within movement one offered an explicit presentation of various types of symmetry through a driving *moto perpetuo* feel, while the second movement exhibited more implicit uses of symmetry, and applied the pre-compositional design as a guide rather than a strict template.

Bernard discusses the importance of symmetry with relation to Bartók, highlighting how symmetrical systems can create unity by operating on a micro-level through single chords or motives, and how hierarchical relationships can form from smaller symmetries which feed into larger ones[99]. It is this level of multivalent symmetry that influenced the pre-compositional design within this piece, and also informed works later in the portfolio. The flexibility of the harmonic and structural outline was particularly successful in creating the contrasting types of symmetry and proportion across the musical form of both movements, and provided illustrations into how formal structures can link directly to PC sets through hierarchical formations of self-similarity. I felt that the design had further potential to be used on a larger scale, and so adapted the root note outline of Figure 4.1 for the large ensemble work, *Aggregation*.

[99] Jonathan W. Bernard, 'Space and Symmetry in Bartók', *Journal of Music Theory,* 30 (1986), p. 192.

Chapter 5

Unequal Measures (2012-2016)

Unequal Measures is a two-movement string quartet based on the Fibonacci series and golden section. Having previously listened to the fugue in Bartók's *Music for Strings, Percussion and Celesta, Sz. 106* (1936), I was intrigued by its dramatic, textural climax. Subsequent research led me to Solomon's analytical model of the work[100], which visually demonstrates local proportions of harmonic and melodic structures. These structures build up alongside the unified development of dynamics, instrumentation, tempo and articulations as the movement reaches a peak that aligns with ratios based on the golden section. In contrast to symmetry, which creates balance and static proportion, asymmetry seen within the Fibonacci series and the golden section exhibits dynamic proportion[101], and the varied segmentation of musical material based on these contrasting divisions presented potential for the portfolio to contain a comparison between symmetry and proportion. Like *In Equal Measure*, there are two movements in *Unequal Measures* that contrast in restrictiveness, while the homogenous nature of the string family is a suitable comparison to the inherently homogenous nature of the piano in *In Equal Measure*. Additionally, the pieces have been named based on the potential for direct comparison.

5.1 Proportion, The Fibonacci Series and the Golden Section

The golden section, also known as the golden mean, golden ratio or divine proportion, is an irrational mathematical constant[102] denoted by the Greek letter φ (phi) and can be written as $(1+\sqrt{5})/2$ (≈ 1.61803399). The reciprocal of this number is 0.618 (1/1.618), which is a unique property of the golden section. The Fibonacci series is a series of numbers in which the next number is produced from the addition of the previous two numbers (with starting values of 0 and 1 resulting in 0,1,1,2,3,5,8, etc.), and the ratio between consecutive numbers in the Fibonacci series converges to an approximation of

[100] Solomon (2002b).
[101] Ernö Lendvai, *Béla Bartók: An Analysis of His Music*, (London: Kahn & Averill, 1971), p. 71.
[102] An irrational number is a real number that cannot be denoted by a ratio of integers or repeating decimals, and a mathematical constant is a non-changing value, typically a real number that contains special properties.

the golden section as values become larger (i.e. 5/3 = 1.67, 8/5 = 1.6, 144/89 = 1.618). For centuries, there has been widespread interest in naturally occurring proportional phenomena, such as the Fibonacci-related patterns spotted in the sunflower, which has twenty-one clockwise and thirty-four anticlockwise spirals[103]. These patterns and resulting proportions have been examined significantly within the arts, from the paintings of Mondrian, who explored asymmetry and proportion for their structural and geometrical properties[104], to more contemporary architectural uses, as seen in Eden's Education Centre in south-west England, which has been built on Fibonacci numbers and the golden section[105].

The Fibonacci series and the golden section have been utilised extensively by composers. For example, Stockhausen used symmetrical groupings of Fibonacci-related time signatures such as 5/8, 8/8 and 13/8 in pieces such as *Klavierstück IX* (1961). While these time signatures provided the theoretical foundation for the work's structural proportions, the emphasis on combining long, sustained notes with extended pauses resulted in these proportions being largely indiscernible to the casual listener. Stockhausen did later confirm his widespread usage of the numbers to create section durations in addition to (for example) density[106].

Roy Howat is a prominent analyst within proportion and the golden section, focussing primarily on Debussy's works. He claims that composers such as Debussy often took inspiration from these proportions to create structures, but that their application never became formulaic, and always presented distinct forms from piece to piece[107]. Theorist Ernö Lendvai also examines how Bartók applied the Fibonacci series to converge towards the golden section through ratios such as 2:3, 3:5 and 5:8 in *Sonata for Two Pianos and Percussion, Sz. 110* (1937), as well as using Fibonacci intervals (2 = major 2nd, 3 = minor 3rd, etc.) to create chords and harmony[108]. However, Howat, amongst others, has long debated the accuracy of Lendvai's calculations, claiming his approximations to the golden section points are too varied. For example, there are only

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[103] Tibor Bachmann and Peter J. Bachmann, 'An Analysis of Béla Bartók's Music through Fibonaccian Numbers and the Golden Mean', *The Musical Quarterly,* 65 (1979), p. 73.
[104] Anthony Hill, 'Art and Mathesis: Mondrian's Structures', *Leonardo,* 1 (1968).
[105] 'Eden Project', (2013).
[106] Piergiorgio Odifreddi, 'Intervista a Karlheinz Stockhausen', (2004).
[107] Roy Howat, *Debussy in Proportion: A Musical Analysis*, (Cambridge: Cambridge University Press, 1983b), p. 136.
[108] Lendvai (1971).

88 bars rather than the Fibonacci number 89 in Bartók's fugue, an issue that Howat has raised on multiple occasions[109,110].

Analysis and composition drawing on these ideas continues, and the debates remain as to whether the presence in a work of the Fibonacci series and the golden section is intentional on the part of the composer, or whether the analyst is overemphasising the importance of numerical patterns. It could be instinctual that composers place crescendos close to golden section points, particularly in works developed before the widespread rise in appeal of the Fibonacci series and golden section. At the very least, Howat believes that if Debussy was subconsciously using the golden section in his earlier works, then the apparent uses of proportion in his later pieces suggest a more conscious effort on his part[111]. As was the case with my listening experience of *Music for Strings, Percussion and Celesta,* golden section points are most likely heard as structurally significant moments within a piece. Whether the necessity for this moment to be so mathematically precise is open to discussion. One could argue that a piece with structural focus points and *any* type of asymmetrical form could be used as an analytical comparison to golden section works, with potentially no significant distinctions judged to be perceivable, or at least favourable.

5.2 Unequal Measures

The two movements of *Unequal Measures* do not share a unified pre-compositional design, but were formed through linked structural and motivic ideas. Movement one uses the Fibonacci series as a basis for melodic content in particular, whereas movement two makes more explicit and strict usages of Fibonacci-related set material for both melodic development and harmonic structures. The eventful, fast, triplet nature of movement one also directly contrasts with the sparseness and slower, sustained character of movement two, resulting in opposing musical characters, ultimately demonstrating distinct approaches and outcomes when composing through the Fibonacci series and the golden section.

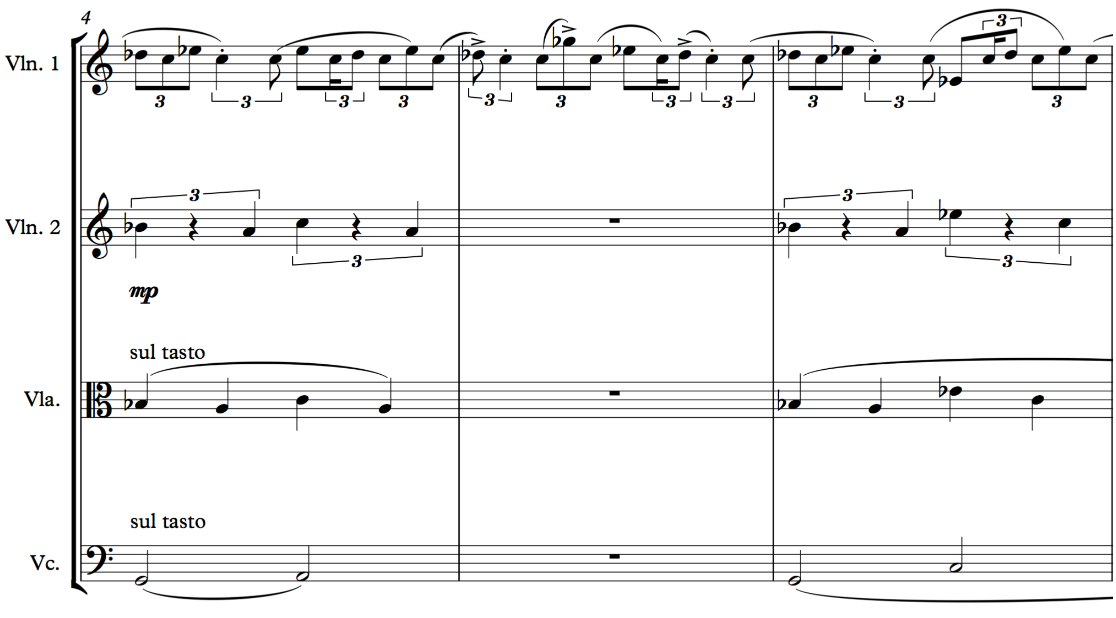
[109] Roy Howat, 'Debussy, Ravel and Bartók: Towards Some New Concepts of Form', *Music & Letters,* 58 (1977), p. 286.
[110] Roy Howat, 'Bartók, Lendvai and the Principles of Proportional Analysis', *Music Analysis,* 2 (1983a), pp. 78-80.

[111] Howat (1983b), p. 162.

Movement One

Movement one is in sonata form. However, it considers sonata form primarily as a structural tool, avoiding an emphasis on tonal functions based on the tonic and dominant. The structural aspect lends itself to golden section and Fibonacci proportions through its potential for asymmetrical divisions of the exposition, development, recapitulation and further sub-sections. There are 144 bars in the movement, with the recapitulation beginning after 89 bars. Since both 89 and 144 are Fibonacci numbers, this creates the golden section point of the piece (89/144 = 0.618). Each section was broken down into smaller Fibonacci-related phrase lengths of 2, 3, 5 or 8 bars, and the structural outline for the movement can be seen in Table 5.1.

As seen in bars 4 and 6 of Example 5.1, the first subject is based around triplet quavers in the violin 1 working against the viola crotchets, suggesting a 3:2 hemiola effect. A percussive, rhythmic emphasis is provided from the use of local slurs with irregular spacing of accents, supported by occasional staccato. The pitch content in the violin 1 part was taken from the Fibonacci numbers 2, 3, 5 and 8 (set 4-13, [0,1,3,6] in prime form), and transposed down a major 2nd from D, E♭, F and A♭ to form C, D♭, E♭ and G♭. Set 4-13 appears extensively within the work, and as a superset of the augmented triad, it also places strong emphasis on the minor 3rd and tritone intervals, further tightening the links between *In Equal Measure* and *Unequal Measures*. It can be seen again in the violin 2 and viola parts of bars 4 and 6 through notes A, B♭, C and E♭.



Example 5.1: *Unequal Measures, Movement One – Opening Hemiola*

144 Bars							
89				55			
34	13	21	21	21	8	13	13
Exposition			**Development**	**Recapitulation**			**Coda**
First Subject	*Trans.*	*Second Subject*	*Development*	*First Subject*	*Trans.*	*Second Subject*	*Coda*

Table 5.1: *Unequal Measures, Movement One – Structural Outline*

The transition between first and second subject (from bar 35) sees the triplet quaver rhythms move to the cello and viola, with less emphasis on a triplet nature and 3:2 ratio, allowing an ascending sequence in the cello to drive the rhythm forward more strongly. The cello here introduces the set [0,1,4,7], represented by Fibonacci numbers 1, 2, 5 and 8, seen in bar 43 of Example 5.2 through the pitches D, E♭, G♭ and A. It is used as a direct contrast to set [0,1,3,6], still seen in bar 47 through pitches C#, D, E, G. The section moves back and forth between these sets as the second subject approaches.

Example 5.2: *Unequal Measures, Movement One – Change in Rhythmic Emphasis*

Up until the second subject (which starts from bar 48), the movement predominantly focusses on rhythmic counterpoint, whereas the main emphasis of the second subject is the melodic line of violin 1, with strong rhythmic support of the other instruments. The driving force of the rhythm changes the emphasis further into a simple rather than compound stress. Fibonacci-related sets [0,1,5] and [0,1,5,8] appear prominently in the viola and cello, moving away from the [0,1,3,6] featured abundantly earlier in the movement, while the violin 1 melody takes part of its pitch content from these supporting lines.

The development section (from bar 69) utilises the same rhythmic emphasis as the second subject in the accompanying cello line. However, certain notes expected within the arpeggio have been omitted at the start of the section, which adds a somewhat stop-start nature to the movement at this point. As the section progresses, these notes gradually re-enter and help to regularise the rhythm to build momentum towards its climax. The accented cello notes here follow a repeating sequence of [1,0,3], which initially begin quite sparsely, but incrementally become more regular toward the end of the section, again to aid the climax[112]. The remaining instruments form chordal developments based on recurring [0,1,3] melodic set content, taken from Fibonacci numbers 2, 3 and 5. Disregarding embellishment notes, violin 1, for example, cycles around pitches of note length 5.333 in four-note phrases of [0,1,3,0], [1,3,0,1], [3,0,1,3] and [0,1,3], rather than just [0,1,3] x 5. This creates distinct variation across rhythm, while harmonically the violins and viola work together to produce alternating diminished and augmented triads[113]. After the first cycle of [0,1,3,0], [1,3,0,1], [3,0,1,3] and [0,1,3], diminution occurs to add urgency to the end of the section, and the cycle repeats with note lengths halved to $2^{2/3}$ beats. See Example 5.3, where further rhythmic diminution takes place towards the golden section point at the end of bar 89. The recapitulation utilises the same pitch organisation as before the golden section climax, but with further variation observed through changes such as instrument register, articulation and part swapping.

Example 5.3: *Unequal Measures, Movement One – Golden Section Point*

Overall, dynamics within the movement remain relatively steady between mp and mf, with a greater increase in dynamics included towards the golden section point to offer urgency and further heighten its importance[114]. The inclusion of *sul tasto* during the

[112] This sequence of 1,0,3 is also reminiscent of the opening passage of the piece.
[113] Bars 69 up to the first third of bar 74 display [0,1,3,0] (transposed up to 7,8,A,7 and seen as G, G#, B♭ and G) in the violin 1 part, with an additional A note included at the start of bar 71 for embellishment.
[114] The coda also uses the chord progression of the violins and viola but without the cello's rhythmic emphasis.

legato passages of the development, and occasionally throughout the movement elsewhere, helps to add contrast while also acting as a precursor to the extended use of harmonics in movement two.

Movement Two

Movement two is in an ABCDB'A' form, with its structure modelled on Bartók's *Music for Strings, Percussion and Celesta*, particularly through the build-up of the first movement golden section climax, as well as the arch form seen in the third movement. It has a focus on Fibonacci-related phrase lengths, with 89 5/8 bars in total, splitting into 55 and 34 (both Fibonacci numbers) to create a golden section point after 55 bars (55/89 = 0.618). The movement also contains a reverse of the golden section point after 34 bars (34/89 = 0.382, which is 1 – 0.618). As with movement one, each section was also broken down into further Fibonacci-related phrases of 2, 3, 5 or 8 bars. Table 5.2 demonstrates that the harmonic content also forms part of the structural outline. Each subsection of bar length 8, 8, 5, 8, 5, etc. was assigned a pitch centre and, like the fugal first movement of *Music for Strings, Percussion and Celesta*, cycles harmonically through circle of fifth transpositions (0, 7, 2, 9, 4, etc.) before arriving once again at its starting point. It reaches its furthest point from the start harmonically at the climax of 55 bars, where the pitch centre is 6. The dynamics also reach a climax after 55 bars, linking further with the first movement of Bartók's work.

The movement is based on a single theme, directly taken from early violin 2 material of movement one, converted into a 5/8 time signature with a largely 2+3 emphasis. Due to its fugal nature and the emphasis in the harmonic and structural outline on transformations based on the perfect 5th, the movement has a neo-classicist feel. There are similarities to the opening material in the first movement of Bartók's *String Quartet No. 1, Sz. 40* (1909), through the emphasis on developing a single theme that builds up across all parts via thicker textures of overlapping, legato phrasing.

	89 Bars					
	34		55			
	55			34		
	21 - A	13 - B	21 - C	13 - D	13 – B’	8 – A’
	First Theme	Second Theme	Merge and Climax after 55	Develop-ment	Second Theme	First Theme
Further Bar Breakdown	8 8 5	8 5	8 5 8	5 8	5 8	3 5
Transpositions	0 7 2	9 4	B 6 6	1 8	3 A	5 0
Dynamics	pp p mp	mf f	ff < fff	ff f	mf mp	p pp

Table 5.2: *Unequal Measures, Movement Two – Harmonic and Structural Outline*

The theme in *Unequal Measures* can be seen in its simplest form in Example 5.4, as the movement begins in the cello. The phrases are predominantly structured across two bars, based on Fibonacci sets ([0,1,3] for bars 1-2, [0,1,3,6] for bars 3-4, etc.) and expanded through repetition and variation. The longer C notes in bars 1-2 act as the tonal centre here, and there are links to the octatonic scale through the development of two entwined diminished 7th chords[115].

Example 5.4: *Unequal Measures, Movement Two – Opening Theme*

The eight-bar phrase ends a perfect 5th higher than the pitch centre C (on the note G), and from bar 9 the movement shifts its pitch centre to this G, the theme moves to violin 2, and the cello now provides supporting harmony. The harmony thickens further from bar 17, as the theme moves to the viola, and violin 2 joins the cello in supporting harmony. Table 5.3 highlights the harmony between bars 17 and 21, in which Fibonacci-related sets are present, in addition to [0,1,4,5] in bar 19, which is closely related to these sets through common pitch classes. The aim was to advance set content and harmonic development in the same linear manner as seen in *From Zero*, and to also preserve micro-level symmetry between sets**.** Similar tables were set up for each bar within the movement in order to develop the idea, and this type of related harmony can be seen as the movement progresses through thicker textures. The serialised nature of this harmonic construction links to the pre-compositional restrictions of previous pieces but without automated or fully systematic processes applied.

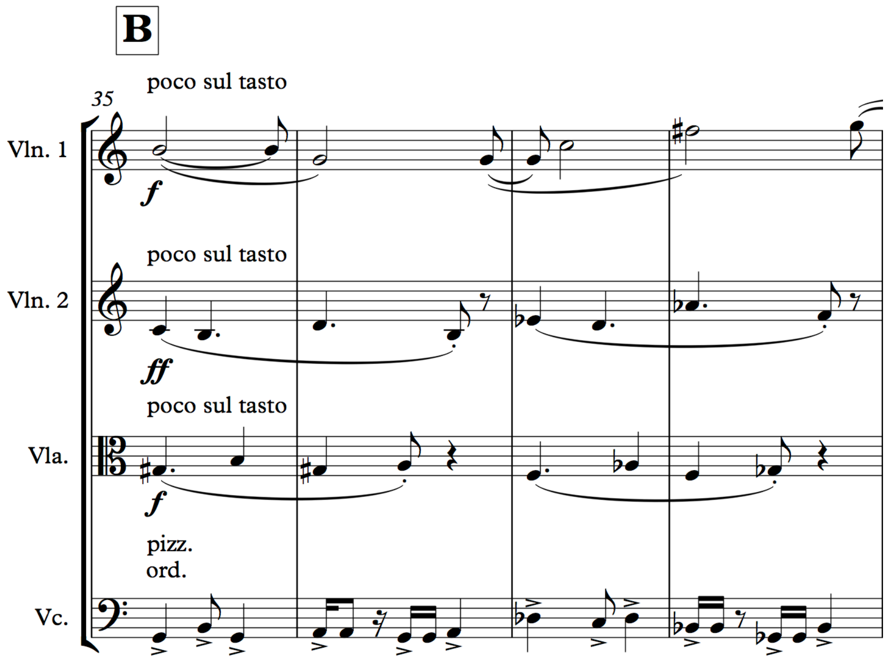
Bar No.	Bar 17	Bar 18	Bar 19	Bar 20	Bar 21
Pitches	5,3,2,7	7,5,2,5	1,6,5,2	6,B,8,B	B,9,8,1
Prime Form	[0,1,3,5]	[0,2,5]	[0,1,4,5]	[0,2,5]	[0,1,3,5]

Table 5.3: *Unequal Measures, Movement Two – Bars 17-21 Pitch Content*

Section B begins at bar 22, and continues to use the same harmonic and rhythmic devices, with the second theme almost identical to the first, except for the somewhat inverted shaping of the melodic contours in the lead melodic line. The melody is now in

[115] [1,4,7,A] as the first note in each of bars 1, 3, 5 and 7, and [0,3,6,9] as the second note in these bars.

violin 1, and the cello drops out of the movement for the first time. Although there are formally distinct sections and timbral variation through the use of *sul tasto* and *sul ponticello*, the contrast between most sections is minimal, particularly compared to the varied nature of movement one. This is intentional due to the gradually shifting nature of the movement, with its focus on developing harmony through subtle melodic and rhythmic variations of the starting motif.



Example 5.5: *Unequal Measures, Movement Two – 2+3 and 3+2 Metres*

At bar 35, the greatest single shift in texture takes place as Section C begins. The section introduces more textural complexity through two melodic lines playing against each other as melody and countermelody in an alternating 2+3 and 3+2 separation of the 5/8 bar. This can be as seen above in Example 5.5 in bars 35-38 of the violin 2 and the viola. Further rhythmic and textural variation is seen here within the violin 1 and the cello, through the introduction of pizzicato and a more staccato nature, both influenced from movement one.

Movement two's golden section point at the end of bar 55 (see Example 5.6) is more prominent than the climax seen in movement one. The dynamics increase, the pizzicato is more driving, the registral range is at its greatest at any point of the movement, while the harmonic statement of set [0,1,3,6] is derived from Fibonacci numbers 2, 3, 5 and 8. For the remainder of the movement after the climax, it reduces in dynamic intensity and simplifies texturally. Harmonic variation still exists, as seen in Section D, in which all-interval tetrachords [0,1,4,6] and [0,1,3,7] are present. The tetrachords are closely linked to several key Fibonacci-related sets, such as [0,1,3,6], due to common subsets like [0,1,6].

Example 5.6: *Unequal Measures, Movement Two – Golden Section Point*

Unequal Measures was performed at a string quartet workshop in 2015, and a number of improvements were carried out as a result. Most notably, the persistent, staccato nature throughout movement one was lessened, and replaced with a combination of local slurs which combine legato, staccato and accents that become more prominent as the movement develops. The second movement was deemed more successful, and it was interesting to hear the harmonics layer numerous levels of textural phasing, linked to the varied dynamics throughout the movement. Perhaps in the future, I would prefer the application of *sul ponticello* and *sul tasto* to be slightly less extreme.

5.3 Summary

Unequal Measures, a string quartet influenced by the Fibonacci series and golden section, was a natural next step from the research on symmetry developed through *In Equal Measure*. It maintained a similar aim, in that it intended to establish two contrasting movements from comparable compositional plans, but with a focus on asymmetry and proportion rather than symmetry. Throughout both movements, there was particular emphasis on the Fibonacci series for micro motivic material, with a reliance on the golden section for macro-level structures. The harmonic and structural outlines focused on emphasising the key golden section points within each movement, and provided crucial detail with relation to dynamic peaks and the development of harmony. Movement two is perhaps more successful in its build-up to this golden section point, through more incremental and unified transformations in texture and dynamics that play out via intentionally sparse, legato layers of a single motivic idea. Although a large number of variables renders it difficult to fully assess the static nature of symmetry within *In Equal Measure* against the dynamic nature of asymmetry in *Unequal Measures*, one future experiment could be to align the climaxes in both works in order to test how listeners perceive these comparable structural proportions. This would enhance the importance of local material and micro-level proportion seen through hierarchical symmetry and Fibonnaci-related phrase lengths.

Chapter 6

Space Ripple (2013-2016)

Space Ripple is a single-source, single-movement electroacoustic piece for solo electric guitar and tape, influenced by the properties of pi. The choice of electric guitar was made due to its potential to create an array of contrasting sounds, pitting the e-bow's capacity for creating sustained notes in the underlying concrete audio against a quickly decaying lead guitar line. The previous pieces on symmetry and proportion required the construction of pre-compositional designs with explicit emphases on similarity, continuity and balance. *Space Ripple* aims to consider these same principles by establishing pi as an effective compositional stimulus within an electroacoustic setting, distinct from the usage of classical instrumentation in previous works. The included score is intended to be used as more of a performance guide than a strict score. The guitarist may wish to work with an engineer who has access to the original audio stems for the purpose of real-time interaction, which may require alternative cue points.

6.1 Pi

Pi is formed from the ratio of a circle's circumference to its diameter, can be approximated to 3.14159, and is often represented by the Greek letter π. Similar to phi, it is an irrational mathematical constant, and its fundamental properties have also created widespread appeal within philosophy and mathematics, among other areas. Berggren et al presents a varied collection of articles on pi from around the world, including excerpts as far back as circa 1650 B.C., covering mathematical proofs, cultural perspectives, as well as the computation of pi's values to boundless decimal places[116]. In the present day, pi can be seen abundantly within popular culture. Every year on March 14th, pi enthusiasts celebrate Pi Day through a series of educational events, hosting a variety of pi-related puzzles[117], while the film *Pi* (1998)[118] sees

[116] J. L. Berggren, Jonathan M. Borwein and Peter B. Borwein, *Pi, a Source Book*, (New York: Springer, 2004).
[117] David H. Bailey and Jonathan Borwein, 'Pi Day Is Upon Us Again and We Still Do Not Know If Pi Is Normal', (2014), p. 2.
[118] Darren Aronofsky, 'Pi', (Live Film & Mediaworks Inc., 1998).

characters testing each other with pi-related problems, and also discussing its similarity to the Fibonacci series.

The history of pi as a compositional device is relatively limited. One notable classical piece is Lars Erickson's *Pi Symphony* (ca. 1992), whose goal was to use pi to create a melody that "imparts the emotions associated with scientific discovery"[119]. The piece's core idea stems from mapping the digits of pi onto musical pitches based on a pre-determined scale to produce a linear, melodic sequence. However, the most intriguing property occurs at the macro-level. Erikson creates the harmonic progression of the second movement by aligning the journey around a circle with the journey across the circle of fifths[120]. This idea works effectively since opposing pitch centres (C and F#, for example) are mapped at opposing structural points within the movement, linking both the harmonic and thematic content. A similar device was carried out in movement two of *Unequal Measures* (see Table 5.2), but with golden section proportions providing these structural functions, rather than a circle. Michael Blake has more recently constructed a popular music song titled *What Pi Sounds Like* (2012), which uses similar methods to construct local pitch material. Erickson deemed that the likeness between the two works was too similar to be coincidental, and unsuccessfully attempted to sue Blake for copyright infringement in the USA, with the judge declaring that the transcription of pi to music could not be copyrighted[121].

Other applications of pi largely relate to further use of its digits to create melodies. A web-based computer program has been created by Felix Jung in which a user can map whatever 10 notes he/she desires onto pi's digits, and then the program plays out the sequence for 10,000 digits[122]. The websites Teach Pi[123] and Pi 314[124] also contain collections of linear, melodic pieces based on pi, but these online sources serve a primitive function compositionally, and are more suited to non-composers intrigued by contextual potential for pi. In general, pi has become an area of increasing interest within both the sciences and arts, yet besides Erickson's work, its application within music has rarely been developed beyond the usage of linear, melodic phrases. The

[119] Lars Erickson, 'Pi Symphony', (No date).
[120] Lars Erickson, 'The Pi Symphony: Numeric Sequences in Music', (2002).
[121] Stephen Ornes, 'US Judge Rules That You Can't Copyright Pi', (2012).
[122] Felix Jung, 'Pi10k', (2012).
[123] Luke Anderson, 'Teach Pi', (2006).
[124] Boris Gourévitch, 'Pi 314', (1999).

influence of Erickson can be seen within *Space Ripple*, primarily in the development of the work's structure based on harmonic opposites.

6.2 Pre-Compositional Design

For the pre-compositional design, pi was converted from base 10 to base 12 in order to fit in with the 12 notes of an octave[125]. So, 3.1415926535, etc. in base 10 now becomes 3.184809493B, etc. in base 12, with A representing 10 and B representing 11[126]. Following this base conversion, ordered collections of specific pitches were constructed and aligned into 12 cells[127], seen in Table 6.1[128]. The decision to include 12 cells was made due to the multiple divisors of the number 12, which offers flexibility for layering various hierarchies of cells. The number 12 also matches with the nature of the typical circular clock face, which contains numbers 1 to 12, offering 12 equal 30° increments. The numbers in the resultant cell segmentation may be considered arbitrary since pi is an irrational number[129], yet the segmentation was solely used as a basis to find noteworthy pitch material that maintained a thematic link to a circle, and thus pi.

These 12 cells align with 12 subsections within the piece (four overall sections, and three subsections within each one). Table 6.1 also contains cell content for each subsection, as well as Forte's prime form equivalent when duplicate pitches are discarded. Rows titled 'Cell Size 3', 'Cell Size 4', etc. contain the pitch content mapped from the digits of pi, with the 12 cells for each row separated according to a restricted hierarchical structure, discussed below. Bold lines in the table represent distinct separation between cell content, while dotted lines represent freedom to overlap material.

[125] Converted through Rod Pierce, 'Math Is Fun', (2000).
[126] Since the first draft of *Space Ripple* in 2013, there are others who have composed with pi in base 12, such as Jim Zamerski who applied it within linear, melodic expressions ('The Ancient Melodies', (No Date)).
[127] The term 'cell' is used in this chapter to avoid any confusion with the term 'set', which is used to directly relate to Forte's classification system.
[128] The first 36 pitches of pi were split up into 12 cells of 3 pitches, the first 48 pitches became 12 cells of 4, the first 60 were 12 cells of 5, while the first 72 became 12 cells of 6.
[129] For example, the 5th cell of size 3 derived by partitioning pi has no direct pitch relation to the 5th cell of size 5.

Section	1			2			3			4		
Subsection	1	2	3	4	5	6	7	8	9	10	11	12
Cell Size 3	3,1,8 [0,2,7]	4,8,0 [0,4,8]	9,4,9 [0,5]	3,B,9 [0,2,6]	1,8,6 [0,2,7]	6,4,5 [0,1,2]	7,3,A [0,3,7]	6,2,1 [0,1,5]	1,B,B [0,2]	1,5,1 [0,4]	5,5,1 [0,4]	A,0,5 [0,2,7]
Cell Size 4	3,1,8,4 [0,2,3,7]	8,0,9,4 [0,1,4,8]	9,3,B,9 [0,2,6]	1,8,6,6 [0,2,7]	4,5,7,3 [0,1,2,4]	A,6,2,1 [0,1,4,8]	1,B,B,1 [0,2]	5,1,5,5 [0,4]	1,A,0,5 [0,2,3,7]	7,2,9,2 [0,2,7]	9,0,A,7 [0,2,3,5]	8,0,9,A [0,1,2,4]
Cell Size 5	3,1,8,4,8 [0,2,3,7]	0,9,4,9,3 [0,1,4,7]	B,9,1,8,6 [0,2,3,5,7]	6,4,5,7,3 [0,1,2,3,4]	A,6,2,1,1 [0,1,4,8]	B,B,1,5,1 [0,2,6]	5,5,1,A,0 [0,2,3,7]	5,7,2,9,2 [0,2,4,7]	9,0,A,7,8 [0,1,2,3,5]	0,9,A,4,9 [0,1,3,7]	2,7,4,2,1 [0,1,3,6]	4,0,A,6,0 [0,2,6,8]
Cell Size 6	3,1,8, 4,8,0 [0,1,3,4,8]	9,4,9, 3,B,9 [0,1,5,7]	1,8,6, 6,4,5 [0,2,3,4,7]	7,3,A, 6,2,1 [0,1,2,5,6,9]	1,B,B, 1,5,1 [0,2,6]	5,5,1, A,0,5 [0,2,3,7]	7,2,9, 2,9,0 [0,2,5,7]	A,7,8, 0,9,A [0,1,2,3,5]	4,9,2, 7,4,2 [0,2,5,7]	1,4,0, A,6,0 [0,2,3,6,8]	A,5,5, 2,5,6 [0,1,4,8]	A,0,6, 6,1,A (0,1,3,7]

Table 6.1: *Space Ripple – Pre-Compositional Design Space*

In Table 6.1:

- Material for cells of size 6 appears only in the specific subsections; they act as underlying background material only
- Material for cells of size 5 appears in subsection pairs of 1-2, 3-4, 5-6, 7-8, 9-10, 11-12. Since they overlap each section, they act as transitional material between each of the four sections
- Material for cells of size 4 appears in subsection groups of 1-3, 4-6, 7-9, 10-12, and act as the distinguishing factors to identify each of the four sections
- Material for cells of size 3 can appear freely throughout the piece as recurrent melodic and gestural content

Figure 6.1 displays the segmentation of a circle into 12 equal parts. The piece could be considered as starting from 0° and cycling through each 30° increment (which represents a single subsection) until the circle's circumference and all 12 subsections have been covered. The piece, hence, cycles through four quadrants of the circle, each quadrant representing one of the four sections within the piece.

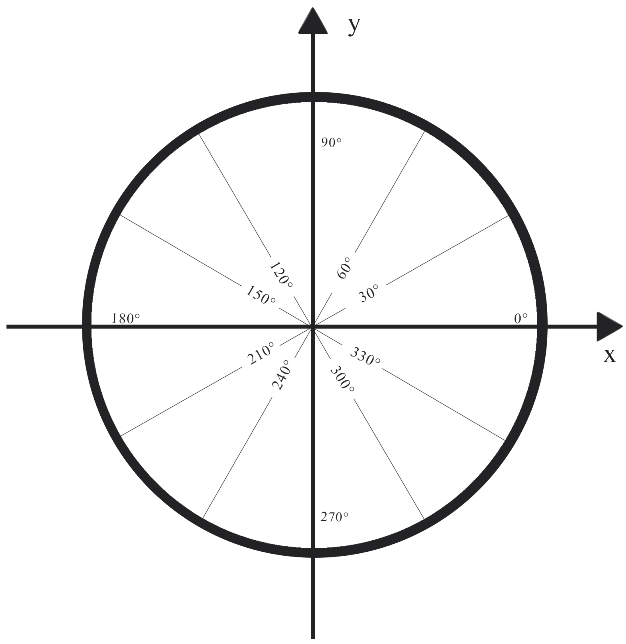


Figure 6.1: *Space Ripple – Circular Segmentation*[130]

[130] Michael McNeilis and Jackson Wade, 'Space Ripple – Circular Segmentation', (2015).

6.3 Space Ripple

A variety of short audio samples were recorded from an electric guitar, ranging from traditional guitar picking, to glissando and the use of an e-bow[131]. These samples were processed through an array of effects, from which a multi-layered concrete sound world was created. On top of this concrete audio lies a solo guitar part, which takes initial influence from the linear melody techniques of other composers mentioned above. Due to its role between a circle's diameter and its circumference, pi could be considered as a means of travel and movement, and the piece uses Table 6.1 to take an almost literal journey around a circle. The inclusion of the concrete audio aids this idea through its potential to create a free-flowing, spatial construct that offers multiple layers of depth and timbre.

Due to the extra-musical nature of a circle, the theme of the piece is that of a sun in deep space surrounded by orbs in different orbits. The sun is represented by a stable guitar part, while the cell sizes from Table 6.1 denote the different orbits surrounding it, each one containing distinct collections of composite sounds[132] that signify the orbs in the concrete audio. The notion of being frozen in time versus a sense of motion is created, with varied orbits forming an overall dense, spherical character.

The four sections of the piece each offer a distinct character, although the characters themselves are relatively similar in order to display natural development and gradation throughout the piece. The original samples used to record the guitar and resultant concrete audio remain relatively static throughout, and the distinguishing features of each section stem from the underlying effects applied to these samples. Varying effects with a range of intensity are introduced and layered gradually within each section, altering the texture and slowly developing the piece's character. Although reverb and delay can be heard from the outset, the reverbs become thicker and longer from section 2, while the delays begin to provide advanced rhythmic variation. This progression develops into section 3, which has the most extreme use of effects. Synthesis plays a huge part in varying these effects to produce distinct characteristics, while modulation

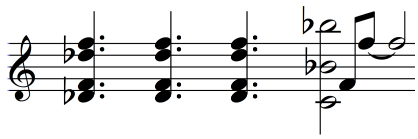
[131] An e-bow is a performance device (typically battery-powered) that provides string resonance and sustain effects.
[132] A composite sound is a combination of single sounds that often form multiple layers and textures. In this context, the combined characteristics of the individual sounds help to define the overall characteristic for that layer.

from low frequency oscillators with different types of waveforms gives effects like tremolo and vibrato distinct pulses from section to section. In addition, dynamics increase towards a peak at the end of section 3, after which they diminish. This is the most prominent peak of the piece, and aims to display how sections that are polar opposites on the visual representation of the circle ('section 1 versus 3', and 'section 2 versus 4') provide the structure for contrasting effects, levels of intensity and timbre.

The Guitar Part

A melodic sequence was derived from the first 36 digits of pi, heard through a largely linear expression of single notes or harmonic dyads. By analysing the sequence, and identifying adjacent pitches that provided potential for consonant interval classes, these dyads were constructed from intervals such as thirds and fifths to bring uniformity to a non-repeating sequence of numbers (and hence pitches). The sequence is separated into several short isometric phrases, and maintains a regular pulse but no formal metre. There is somewhat of a downbeat rhythmic emphasis present towards the end of most phrases, while dynamics are intentionally kept constant throughout, and peak within these phrases towards the downbeat. In the first section, the melodic sequence can be heard in full, while for the second section it is reconstructed and played largely in reverse pitch order but with similar rhythmic devices. Sections 3 and 4 repeat this pattern with slight variation to produce symmetrical sequencing of the phrases.

A recurring guitar motif (Example 6.1 with pitches D♭, B♭, C and F) represents digits 33-36 of pi (1, A, 0, 5). It is played at the end of each section (at 01:45 for section 1, for example) in order to signal transitions between sections. It is also present at the start and end of the piece in a form drenched in heavy reverb to encapsulate the piece and highlight the current location on the circumference of the circle. In this guise, it is labelled as 'MOT' in the score.



Example 6.1: *Space Ripple – Recurring Motif*

Emphasising the downbeat of the guitar part is a direct duplicate of the melodic content, encompassing only the notes on the downbeats towards the end of each phrase. This duplicate track contains effects such as reverb, tremolo and flanger, which develop through each section as the effects start to peak in intensity. Furthermore, there is a third track containing a rolling crescendo of notes (adapted from the guitar part) leading up to first melodic phrase of each section. These tracks are integrated as 'GS' in the score to represent guitar support.

Concrete Audio

There are multiple layers to the concrete audio, with each layer derived from the various cell sizes in Table 6.1. Within each layer, composite sounds were created from the pitch material in the corresponding box of the table, and a variety of sound design and processing took place. In order to add more continuity and specific harmonic unity to the concrete audio, Table 6.1 was analysed to search for recurring, related pitch content.

Considering these cells in relation to Forte's sets, it was noticed that sets 4-14, [0,2,3,7], and 4-19, [0,1,4,8], appear most often throughout. For example, both sets appear in section 1, with 4-14 particularly prominent as it occurs twice in the opening statement, together with closely related supersets, [0,2,3,4,7] and [0,2,3,5,7]. Sections 2 and 4 contain more of 4-19, while 4-14 appears more in section 3. Moreover, set 5-Z17[133], [0,1,3,4,8], is related to 4-14 and 4-19 as they are the only two tetrachords in the piece that are subsets of 5-Z17. These three sets initiate the piece[134], so there is further harmonic coherence established from an early stage based on the initial values of pi. Furthermore, the only trichord subset of both of these tetrachords is set 3-11, [0,3,7], and so there is an occasional tonal quality that bursts through the otherwise atonal concrete audio, and this complements the tonal nature of the guitar melody's harmonic dyads. These harmonic expressions are labelled as 'HAR' in the score, often appearing as aural performance cues. In addition, an arpeggiator effect ('ARP' in the score) created from opening pitch content [3,1,8,4,8,0], linked to set 5-Z17, is included once at the start of each section to build up the beginning of the guitar content for that section. It consists of a guitar melody, is split into three [3,1,8] + three [4,8,0] expressions, and is first heard at 00:11 and 00:15 in the piece.

[133] Schoenberg's 'Farben' chord.
[134] 5-Z17 is mapped from the cell of size 6, [3,1,8,4,8,0], 4-14 is taken from the cell of size 5, [3,1,8,4,8], while 4-19 is taken from the cell of size 4, [8,0,9,4].

The Panoptic Space

Two versions of the piece are present on the accompanying DVD (Appendix H). The first version is in stereo, and uses binaural panning and the Doppler effect to represent a 3D nature, particularly present in the cells of size 6 content. For cells of size 5, panning is more towards the back, for cells of size 4 it is towards the front, for cells of size 3, it is spread to the sides, while the guitar part remains in the typical stereo field. To fully appreciate this panning, headphones are recommended for listening. However, the second version, as a surround mix, replicates this 3D space more realistically through advanced 3D panning automation. A 5.1 surround system is required for playback.

6.4 Summary

Space Ripple's electroacoustic nature provided extra-musical considerations through the theme of orbits in space, further deliberating character, gesture and texture in a new context. The usage of pi as a creative device aided the construction of a design that resulted in the exclusion of sets related to each other at the pre-compositional stage, and presented compositional challenges that needed to be overcome to achieve harmonic coherence. Combined with the fundamental premise of using pi in base 12 to generate melodies, the spatial design of Table 6.1 helped me to create cells of gestural and textural material that would complement each other based on pitch relationships and mathematical proportion linked to divisors of the number 12. Using repetition of pitch sequences within this context limited the issue of pi being irrational, a problem that remained in other works based on pi that focussed on linear melodies only. As with the contrast between *In Equal Measure* and *Unequal Measures*, attention was paid to the static nature of the guitar against the multi-layered movement of the concrete audio. The subsequent surround mix of the piece resulted in PC sets being used beyond the vertical and horizontal aspects of harmony, melody and structure, and instead permitted their placement within a quasi-3D space, supported by incremental applications of reverb and delay to represent depth and width.

Chapter 7

Three Minds Fractured (2013-2016)

Three Minds Fractured is a brass sextet (trumpet in B♭ x 2, tenor trombone x 2 and tuba x 2) that aims to demonstrate the potential of the octatonic scale within PC set theory. The inherent symmetrical properties of the octatonic scale are similar to the scales and sets explored in previous works involving mutation, symmetry and proportion. In particular, its interval set pattern of 1-2-1-2-1-2-1 or 2-1-2-1-2-1-2 can be compared to the whole-tone scale's pattern of 2-2-2-2-2, and although they are harmonically distinct, both scales are also Modes of Limited Transpositions[135]. As an expansion from the four parts of a string quartet, a brass sextet allows for more sophisticated combinations of pitch collections, while also remaining relatively homogenous as part of the same instrument family. While researching potential instrumentation, I was intrigued by the wide variety of articulations in a brass quintet performance of Liszt's *Hungarian Rhapsody No. 2* (1847)[136]. An expressive, solo trumpet would play fast passages of repeated notes and act as a trigger for harmonic blocks of the full quintet based on staccato and sforzando statements. Alongside the richness of the tuba within slower and quieter passages, this dynamic, registral and timbral contrast particularly appealed to me.

7.1 The Octatonic Scale

The octatonic scale, labelled as set 8-28 by Forte with prime form [0,1,3,4,6,7,9,A], is a symmetrical, mirror set. It has been widely used by a number of historically influential composers, with Blechner tracing its usage as far back as Beethoven's *Piano Sonata No. 29 in B-Flat Major* (1818). The sonata's final movement places emphasis on modulation between major keys, while an expression of the octatonic appears late on within a single bar to function as a "turbulent loosening of the harmonic structure"[137].

[135] Messiaen (1944).
[136] Love Tuba, 'Liszt - Hungarian Rhapsody No. 2 - Brass Quintet', (2014).
[137] Mark J. Blechner, 'The Octatonic Mode', *The Musical Times,* 130 (1989), p. 322.

There is particular appeal in its flexibility to create material through a variety of subset combinations. For example, it contains four different minor triads, namely [0,3,7], [3,6,A], [6,9,1] and [9,0,4], and also four different major triads [0,4,7], [3,7,A], [6,A,1] and [9,1,4], while it can also be broken down into two discrete diminished 7th chords ([0,3,6,9] and [1,4,7,A]). This latter segmentation places central importance on the minor 3rd and tritone intervals, further linking to *In Equal Measure*, while composers such as Liszt and Rimsky-Korsakov have applied this segmentation to create harmonic motion and melodic sequences[138]. In addition, a number of 7th chords, such as the dominant [0,4,7,A] and half-diminished [0,3,6,A], can also be constructed from the octatonic scale. If the scale is reformed to begin with a major 2nd interval ([0,2,3,5,6,8,9,B]) rather than a minor 2nd, it provides potential for contrasting material, with Baur noting that the scale's emphasis is now on Dorian tetrachords (i.e. a tone, semitone, tone sequence, as seen in [0,2,3,5])[139].

Three forms of the scale exist, and are identified by Forte as collections[140]:

Collection 1 – [1,2,4,5,7,8,A,B]
Collection 2 – [2,3,5,6,8,9,B,0]
Collection 3 – [3,4,6,7,9,A,0,1]

Forte[141] analyses how Webern infuses these three collections within a complex structural topology for *Six Bagatelles for String Quartet, Opus 9, No. 1* (1913). He presents a skeletal structure in a similar vein to Schenkerian analysis but via prominent pitch centres and chordal strands linked to combinations of the three collections. This visual representation is similar to the hierarchical designs constructed for *In Equal Measure*, which also place emphasis on diminished 7th chords. Further insight into Webern's micro-level octatonic material is also presented by Forte through subset relations of local harmonic and melodic material. It is precisely this combination of macro-level structural space and micro-level deconstruction of pitch material that interests me as a composer, and has influenced the structural design of *Three Minds*

[138] Steven Baur, 'Ravel's "Russian" Period: Octatonicism in His Early Works, 1893-1908', *Journal of the American Musicological Society,* 52 (1999), 533.
[139] Ibid.
[140] Allen Forte, 'An Octatonic Essay by Webern: No. 1 of the "Six Bagatelles for String Quartet," Op. 9', *Music Theory Spectrum,* 16 (1994). Forte adopted the term 'collection' and its usage in this context from van den Toorn (1983).
[141] Forte (1994), p. 187.

Fractured, outlined in further detail below.

7.2 Pre-Compositional Design

Despite the vast array of possible segmentation highlighted previously, the scope of *Three Minds Fractured* was restricted in order to focus on internal segmentation. A pre-compositional design was constructed, modelled on the three octatonic collections discussed above, and it is the interaction within and between these collections that moves beyond the compositional strategies seen so far within the portfolio. Each octatonic collection was assigned its own instrument (Collection 1 – trumpets, Collection 2 – trombones, Collection 3 – tubas), and was split down the middle into two adjacent tetrachords, resulting in each one of the six instruments being assigned its own 'primary tetrachord'. For example, Collection 1 contains the two tetrachords [1,2,4,5] and [7,8,A,B], hence [1,2,4,5] was assigned to trumpet 1, while [7,8,A,B] was assigned to trumpet 2. The full breakdown can be seen in Table 7.1.

Instrument	Collection 1	Collection 2	Collection 3	Primary Tetrachord
Trumpet 1	X			[1,2,4,5]
Trumpet 2	X			[7,8,A,B]
Trombone 1		X		[2,3,5,6]
Trombone 2		X		[8,9,B,0]
Tuba 1			X	[3,4,6,7]
Tuba 2			X	[9,B,0,1]

Table 7.1: *Three Minds Fractured – Primary Tetrachord Breakdown*

This particularly simple segmentation was chosen for its resultant emphasis on the mirror set 4-3 (prime form [0,1,3,4]), with Berger noting that starting octatonic-related material with a minor 2nd interval sounds fresher and more intriguing [142]. Each instrument was purposely restricted to the pitches within its primary tetrachord,

[142] Arthur Berger, *Reflections of an American Composer*, (Berkeley: University of California Press, 2002), p. 188.

although this rule was relaxed occasionally to allow an instrument to use closely related pitch material, or to octave/unison double other parts for effect. This restrained approach forms the central function of the piece, and adopts similar methods seen previously through *In Equal Measure*, in which material was largely limited to the augmented triad and diminished 7th chord. By maintaining this restriction, there arises the necessity to build a fluid network of interaction between tetrachords, collections and instrument parts. A structural design was created from primary tetrachord material to develop this approach (Table 7.2).

The design is split into seven sections, with each section containing a particular emphasis on certain primary tetrachords. For example, section 1 has a strong emphasis on octatonic collection 1 and the trumpets, and this forms part of the primary material for this section. Octatonic collection 2 accounts for secondary supporting material through the trombones, while less important tertiary material is expressed through the tubas and collection 3. Sections 3 and 5 are similarly constructed, but the emphasis on each octatonic collection cycles throughout the piece, so that collection 2 is highlighted within primary material of section 3, and collection 3 is most prominent in section 5.

In the design, sections 2, 4 and 6 are in direct contrast to 1, 3 and 5. These even-numbered sections contain only primary and secondary material, but there is less importance placed on the strict dissection of material and the necessity to keep collections in a hierarchical weighting system. For example, section 2 sees trumpet 1 link with trombone 2 and tuba 2 as primary material, whereas trumpet 2 joins the trombone 1 and tuba 1 as secondary material. Consequently, using trios instead of duos allows for textural variation between each section.

Finally, section 7 encapsulates the piece by taking material from sections 1, 3 and 5, and bringing all set content together. By structuring the piece through this multi-layered design, it allows for varied timbres, melodies and rhythms across all instrument parts.

	Section 1	Section 2	Section 3	Section 4	Section 5	Section 6	Section 7
Primary Material	1 [1,2,4,5] 2 [7,8,A,B] **Trumpets** **1**	1 (1,2,4,5] 4 [8,9,B,0] 6 [9,A,0,1]	3 [2,3,5,6] 4 [8,9,B,0] **Trombones** **2**	2 [7,8,A,B] 3 [2,3,5,6] 5 [3,4,6,7]	5 [3,4,6,7] 6 [9,A,0,1] **Tubas** **3**	1 [1,2,4,5] 2 [7,8,A,B] 3 [2,3,5,6] 4 [8,9,B,0] 5 [3,4,6,7] 6 [9,A,0,1]	1 [1,2,4,5] 2 [7,8,A,B] 3 [2,3,5,6] 4 [8,9,B,0] 5 [3,4,6,7] 6 [9,A,0,1]
Secondary Material	3 [2,3,5,6] 4 [8,9,B,0] **Trombones** **2**	2 [7,8,A,B] 3 [2,3,5,6] 5 [3,4,6,7]	5 [3,4,6,7] 6 [9,A,0,1] **Tubas** **3**	1 (1,2,4,5] 4 [8,9,B,0] 6 [9,A,0,1]	1 [1,2,4,5] 2 [7,8,A,B] **Trumpets** **1**	**X**	**X**
Tertiary Material	5 [3,4,6,7] 6 [9,A,0,1] **Tubas** **3**	**X**	1 [1,2,4,5] 2 [7,8,A,B] **Trumpets** **1**	**X**	3 [2,3,5,6] 4 [8,9,B,0] **Trombones** **2**	**X**	**X**

Normal font = Primary tetrachord

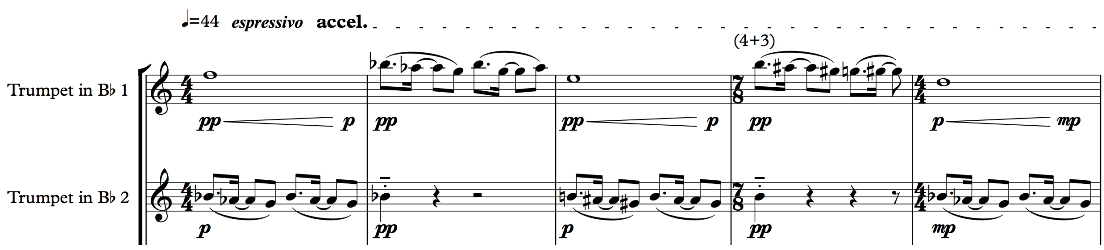
Bold font = Prominent instrument and octatonic collection

Table 7.2: *Three Minds Fractured – Structural Design*

7.3 Three Minds Fractured

Since sections 1, 3 and 5 were constructed through the same method, they were chosen to be linked musically too; they offer a generally fast, percussive nature through regular use of staccato, and often share the same motivic material. Sections 2, 4 and 6 are also linked musically; they are slower in nature, and more emphasis is placed on linear, melodic content. Although each section is short, the contrasting musical character when combined with the primary, secondary and tertiary layers within the structural design of Table 7.2, compares to the complex topology of Webern's *Six Bagatelles* but with the goal to gradually add momentum as the piece develops through each section.

The piece begins with an explicit declaration of the primary tetrachord of trumpet 2, [7,8,A,B]. Through dovetailing, both trumpets take turns to cycle through trumpet 2's tetrachord via an arpeggiated sequence of pitches 7,8,A for two bars and 8,A,B for a further two bars. The final bar in the four-bar phrase is cut to a time signature of 7/8 to disrupt the regularity of the dotted rhythm, while providing a natural return to the opening B♭ (Example 7.1). The slow pacing and soft dynamics at this point help to present a melancholy start to the piece, while the sequence is repeated throughout the section, with a gradual increase in dynamics and slight variations in rhythmic emphasis to add variety and urgency.



Example 7.1: *Three Minds Fractured – Opening Passage*

In addition to supporting trumpet 2's primary tetrachord, trumpet 1 plays its own primary tetrachord, [1,2,4,5], in a descending semibreve sequence of F, E, D and C# spread over eight bars. The first three notes of the primary tetrachord can be seen above in Example 7.1 in bars 1, 3 and 5, while further embellishments of supporting trumpet 2 material appear later in the section in between these semibreves. The trombones enter from bar 9 and work together on a rhythmic countermelody to the trumpets. Trombone 1, [2,3,5,6], and trombone 2, [8,9,B,0], highlight an additional example of overlapping tetrachord material, as trombone 1 borrows pitches 8 and 9 from trombone 2, while trombone 2 uses pitches 5 and 6 from trombone 1.

Section 2, from bar 21, introduces new material related to the primary tetrachords. Mirror sets 6-Z13, [0,1,3,4,6,7], and 6-Z23, [0,2,3,5,6,8], are both subsets of the octatonic scale, with 6-Z13 linked directly to the 4-3 tetrachord of [0,1,3,4]. Set 6-Z23 is more directly associated with the alternative form of the octatonic scale that starts with a major 2nd, and its closely related tetrachord is set 4-10, [0,2,3,5]. The section moves between harmonic statements of the hexachords spread across all six instruments, alternating loud and softer dynamics throughout. Bar 21 (Example 7.2) contains notes C#, D#, C, F#, G and E, directly taken from the prime form of 6-Z13, [0,1,3,4,6,7], while bar 22 transposes 6-Z23's prime form up a semitone to give [1,2,4,5,7,B]. This enables tuba 2 to keep a consistent bass note of C#, while each instrument still plays pitches from its primary tetrachord. In between these chordal statements, tuba 2 takes [3,4,6,7] from tuba 1's tetrachord to create a repeated, melodic phrase.

Example 7.2: *Three Minds Fractured – 6-Z23 Emphasis*

The usage of set 6-Z23 continues in section 3 (from bar 40) through a melody and countermelody within the primary material of the trombones. Trombone 1 uses [2,3,5,6,8,0] as an extension of its primary tetrachord [2,3,5,6], while trombone 2 does likewise by expanding its primary tetrachord of [8,9,B,0] into [8,9,B,0,2,6]. This melody also expands into the trumpets using the same pitch collections but at louder dynamics and higher registers, producing a call and response effect. The supporting material of the tubas provides accompaniment to these melodies through the same rhythmic device and overlapping tetrachord material used by the trombones of section 1 (Example 7.3).

Example 7.3: *Three Minds Fractured – Call and Response 1*

Section 4, from bar 60, contains the only explicit usage of the octatonic scale in its full form. Tuba 1 incorporates all of tuba 2's primary tetrachord material, to reform collection 3 of the octatonic scale, [3,4,6,7,9,A,0,1]. A largely solo melody line with relatively soft and consistent dynamics is created from this collection in the tuba 1 part, with direct influence taken from tuba 2 material in section 2, as seen in bars 60-61 of Example 7.4.

Example 7.4: *Three Minds Fractured – Octatonic in Full Form*

This section has a strong emphasis on the minor 3rd interval, with bars 64 and 65 of Example 7.4 displaying a diminished triad and diminished 7th chord respectively, built from combining the lead melody with supporting trombone and trumpet parts. The section transposes up a minor 3rd from bar 71 onwards to take advantage of the invariant nature of the scale, resulting in the same pitch collection but with different chord inversions and melodic contours.

Like the third section, Section 5 from bar 83 is based on an animated call and response, largely between the primary material of the tubas and secondary material of the trumpets (Example 7.5). Tuba 1 and trumpet 1 play the dominant role early in the section, with tuba 2 and trumpet 2 becoming more prominent as the section progresses through a unification of these four parts. Using a rhythmic variation of the motivic material from the start of section 3, a staccato-centred, semiquaver pattern using varied time signatures was created through primary tetrachords. The section starts sparsely, with a focus on the interplay between each individual part, before the texture thickens with the introduction of the supporting trombones.

Example 7.5: *Three Minds Fractured – Call and Response 2*

The crescendo of a lively section 5 leads into a much slower and more passive section 6. Building on the hexachord idea first seen within section 2, this section, from bar 105, has a central focus on four hexachords. A single pitch was taken from the primary tetrachord for that instrument in order to form four distinct hexachords (Table 7.3). For example, within trumpet 1's primary tetrachord of [1,2,4,5], pitch 5 appears in the first hexachord, pitch 4 appears in the second, pitch 2 in the third and pitch 1 in the fourth. Set 6-Z23 was taken as a basis to construct these formations, and its content of [1,4,B,2,7,5] can be first heard in bar 22 of section 2, resulting in bars 22 (Example 7.2) and 105 (Example 7.6) being very similar in nature.

Primary Tetrachord	**Hexachord 1** 6-Z23	**Hexachord 2** 6-Z41	**Hexachord 3** 6-34	**Hexachord 4** 6-21
[1,2,4,5]	5	4	2	1
[7,8,A,B]	7	A	8	B
[2,3,5,6]	2	6	5	3
[8,9,B,0]	B	8	0	9
[3,4,6,7]	4	3	6	7
[9,A,0,1]	1	9	A	0

Table 7.3: *Three Minds Fractured – Hexachord Construction*

By using this method to form hexachords, material directly related to the octatonic scale (i.e. the primary tetrachords) was recycled to create new material, most of which is not immediately related to the octatonic. In this way, the linear melodic element links directly to the octatonic, while the vertical harmonic content has a more implicit connection[143]. Example 7.6 shows the opening statement of these four hexachords, which alternate between static and dynamic expressions of the harmony in a relatively open harmonic formation. Across these four hexachords, there is a gradual softening in the dynamics, while the section ends through the retrograde of this sequence, culminating with a crescendo and the same 6-Z23 hexachord that begins the section.

Example 7.6: *Three Minds Fractured – Hexachords Passage*

Section 7 of the piece, from bar 141, is formed from three subsections, each of which has a central focus on primary tetrachords, and takes particular influence from either section 1, 3 or 5 in order to end the piece via the same harmonic function with which it begins. The final two bars of the piece (Example 7.7) show all six instruments working rhythmically in tandem towards the finale, making use of [A,B,1,2], resulting in the principal set 4-3 ending the piece across all parts.

[143] Set 6-Z23 is the only one of these hexachords that is a subset of the octatonic scale, and so was used at the start and end of the section to bridge the gap between octatonic and non-octatonic material.

Example 7.7: *Three Minds Fractured – Finale*

At a workshop in 2016, section 4 of the piece was rearranged to form part of the mixed quartet, *In Three Elements* (Appendix G). The contrast and interplay brought about via the clarinet as lead solo instrument, with the violin and cello as primary melodic support, created a complementary texture that worked effectively in the context of octatonic harmony, with bursts of tonality particularly intriguing.

7.4 Summary

Three Minds Fractured, a brass sextet that explored the octatonic scale, maintained a heightened focus and restriction on pairs of the set, 4-3 ([0,1,3,4]). The piece demonstrated a method of building a pre-compositional design linked to the octatonic scale, but with foreground emphasis on a subset and the subset's musical character. The primary tetrachords based on set 4-3 created from the three octatonic collections were utilised across multiple combinations of the six instruments to encompass all 12 pitch classes equivalently. By using combinations of duos and trios determined by the structural design, wide-ranging contrast between sections and a suitable textural balance were achieved without persistent focus on one particular instrument.

Additional areas of note include the construction of distinct hexachords from the primary tetrachords (Table 7.3). This method worked well theoretically, and offers potential to be used on a larger scale in the future. With Section 7.1 outlining extensive potential for segmentation of the octatonic scale, an alternative approach could have been adopted to segment and foreground chords such as the major, minor and diminished triads, as well as the dominant, diminished and half-diminished 7^{th} chords, which would have subsequently resulted in contrasting compositional choices. Breaking down the scale into two diminished 7^{th} chords, for example, would have hugely affected the melodic and harmonic nature of the piece due to the opposing sonority of each of the mirror sets, and would have perhaps exhibited similar harmonic qualities to *In Equal Measure*.

Chapter 8

One to Another (2014-2016)

One to Another, a three-movement work for large ensemble based on the two AITs, aims to move beyond the smaller ensembles of earlier pieces to investigate the compositional potential for PC set theory on a larger scale across multiple instrument families, with 13 different instruments in total[144]. There are 5 string, 4 woodwind and 3 brass instruments, as well as the timpani for percussive support. This 5 – 4 – 3 balance was chosen specifically to permit composition through heterogeneous timbres and distinct harmonic content, with the strings emphasising pentachords, the woodwinds focussing on tetrachords and the brass centring on trichords. Through their harmonic links to the octatonic scale and symmetrical sets/intervals, the AITs allow for further consideration of related mathematical principles.

8.1 The All-Interval Tetrachords

An AIT is a tetrachord with four pitch classes that comprises all six distinct interval classes. There are two such tetrachords: set 4-Z15 with prime form [0,1,4,6] and set 4-Z29 with prime form [0,1,3,7]. These two sets have the same intervallic content, and since they are not transpositionally or inversionally equivalent, they are defined by Forte as a Z-related pair[145]. Figure 8.1 provides a visual representation of the sets, with each interval class between 1 and 6 highlighted:

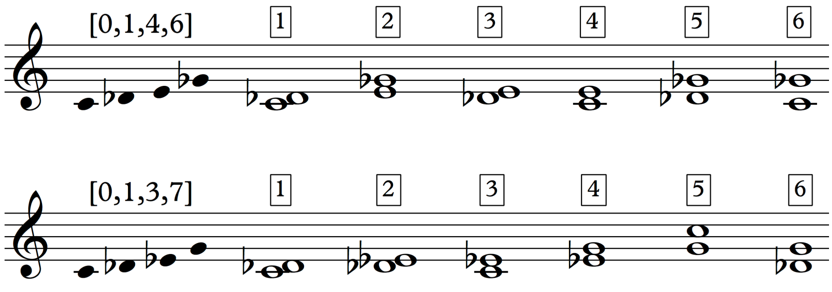


Figure 8.1: *One to Another – AIT Interval Class Breakdown*

[144] Flute, oboe, clarinet in B♭, bassoon, French horn in F, trumpet in B♭, tuba, timpani, violin 1, violin 2, viola, violoncello and contrabass.
[145] Forte (1973), p. 21.

Like symmetry, proportion and the octatonic scale examined thus far, composers of the 20th century have utilised the tetrachords as a compositional device on both the micro and macro levels. Their “limitless intervallic flexibility while maintaining harmonic uniformity” [146] has fascinated composers such as Elliott Carter, who increasingly focussed on the tetrachords (and similar all-trichord hexachord) after 1990[147]. Carter’s *First String Quartet* (1951) contains numerous instances of the AITs, beginning in its opening cello passage, which fuses a fragmented melody alongside consecutive, quadruple-stop AIT statements of 4-Z15 T_A and 4-Z29I T_7 [(A,B,2,4) to (7,B,1,2)]. Schiff has noted further usage in which 4-Z15 can be seen more expansively across both melody and harmony in “various spacings, inversions and transpositions”[148]. Carter himself confirms its intentional usage, declaring that it “functions as a harmonic ‘frame’ for the work” and helps to “constitute a convincing and unified musical continuity”[149].

Carter also examines the relationship between the two tetrachords through sets related by pitch classes in common, including the [0,3] and [0,6] dyad subsets. Theorists such as Goyette[150] and Capuzzo[151] discuss his emphasis on these dyads in his solo violin pieces *Statement* (1999) and *Fantasy* (1999) respectively, where AIT dyad pairs often alternate or repeat in sequence, while also forming the basis for harmonic blocks linked to AIT-related trichords. Similar applications of the dyads [0,3] and [0,6] were explored within *In Equal Measure* and *Three Minds Fractured* through the consideration of the tritone and diminished triad, in particular. Additional links to the octatonic scale are present through sets 5-19 and 5-28, since they are the only two pentachord supersets shared by the AITs[152], and are also both subsets of the octatonic scale. Naturally, all forms of the AITs are thus subsets of the octatonic scale.

As with the octatonic scale, the AITs have been used by composers as the basis to harmonically and structurally unify more complex material. Hanninen[153] notes the links

146 Alan Theisen, 'A Multifaceted Approach to Analyzing Form in Elliott Carter's Boston Concerto', (PhD Thesis, 2010), p. 31.
147 Hopkins and Link, p. ix.
148 Schiff, p. 56.
149 Allen Edwards and Elliott Carter, *Flawed Words and Stubborn Sounds: A Conversation with Elliott Carter* (New York : W.W. Norton, 1971), pp. 106-107.
150 Jeremiah Goyette, 'The Z-Relation in Theory and Practice', (PhD Thesis, 2012).
151 Guy Capuzzo, 'The Complement Union Property in the Music of Elliott Carter', *Journal of Music Theory,* 48 (2004).
152 i.e. 4-Z15 and 4-Z29I share the supersets 5-19I and 5-28, while 4-Z15I and 4-Z29 share the supersets 5-19 and 5-28I.
153 Dora A. Hanninen, 'Associative Sets, Categories, and Music Analysis', *Journal of Music Theory,* 48 (2004).

between Carter's visual representation of two AITs forming octachords and the compositional spaces of Morris discussed previously. This approach demonstrates similarities to my hexachord construction for *Three Minds Fractured* in Table 7.3, where, conversely, the octatonic scale breaks down into related subset hexachords. Ultimately, the flexibility of the AITs to link related sets across varied systems provides similar potential to the versatile nature of the octatonic scale.

8.2 Pre-Compositional Design

The overall form of the piece was constructed in a similar manner to *In Equal Measure* and *Unequal Measures*; pitches within the AITs were used as pitch centres for overall phrases or longer sections. The first AIT, 4-Z15, was used as a starting point for the design, with material constructed from the second tetrachord, 4-Z29, used to complete it. The prime form of 4-Z15, [0,1,4,6], was paired with its inverted form, 4-Z15I ([0,2,5,6]), so that when the latter is considered in reverse, a sequence of 0,1,4,6,6,5,2,0 could be produced once attached together. Similarly, for the prime form of 4-Z29 ([0,1,3,7]) and its inverted form 4-Z29I, [0,4,6,7], a sequence of 0,1,3,7,7,6,4,0 was created. Linking these two sequences together linearly, we can create the following sequence, where overlaps exist at 6, 0 and 7:

0	1	4	**6**									
			6	5	2	**0**						
						0	1	3	**7**			
									7	6	4	0

Furthermore, when harmonically inverted, 4-Z15, 4-Z15I, 4-Z29 and 4-Z29I were used to create a similar sequence (0,1,4,6 directly inverts its pitch classes through 0 to form 0,B,8,6, etc.):

0	B	8	**6**									
			6	7	A	**0**						
						0	B	9	**5**			
									5	6	8	0

Table 8.1 shows the integration of this data into each movement of the piece. The table is separated into primary and secondary harmonic material, similar in construct to the design of Table 7.2 for *Three Minds Fractured*. Primary material was derived from the sets in their prime form, while secondary material was taken from harmonic inversions through 0. The middle two pitches from each four-note set were reversed to allow for a more flowing harmonic progression, rather than a strictly ascending and descending pattern throughout.

	Movement One				**Movement Two**								**Movement Three**			
Primary Material	0	4	1	**6**	**6**	2	5	**0**	**0**	3	1	**7**	**7**	4	6	0
Secondary Material	0	8	B	**6**	**6**	A	7	**0**	**0**	9	B	**5**	**5**	8	6	0

Table 8.1: *One to Another – Structural Design: Pitch Centres*

Since each pitch class within the above table represents the pitch centre for a portion of that movement, movement one has an emphasis on set 4-Z15, movement three focusses on set 4-Z29, while movement two acts as a transition between the two tetrachords. The overlapping pitches identified in the construction of the design now act to harmonically unify the transitions between each movement.

In order to devise compositional strategies based on the relationship between the two tetrachords, Table 8.2 was constructed, which highlights key subsets and supersets of each tetrachord, as well as content related to both sets[154].

[154] Set relationships have been included based on either the set's prime or inverted form. For example, 4-Z15 and 4-Z29I share the superset 5-19I, hence both forms of these three sets has been included.

<table>
<tr><th colspan="2">4-Z15 Relations</th><th rowspan="3">4-Z15
[0,1,4,6]

4-Z15I
[0,2,5,6]</th><th colspan="3">Shared Relations</th><th rowspan="3">4-Z29
[0,1,3,7]

4-Z29I
[0,4,6,7]</th><th colspan="2">4-Z29 Relations</th></tr>
<tr><td>5-6
[0,1,2,5,6]
5-6I
[0,1,4,5,6]
5-9
[0,1,2,4,6]
5-9I
[0,2,4,5,6]
5-10
[0,1,3,4,6]
5-10I
[0,2,3,5,6]</td><td>5-14
[0,1,2,5,7]
5-14I
[0,2,5,6,7]
5-30
[0,1,4,6,8]
5-30I
[0,2,4,7,8]
5-32
[0,1,4,6,9]
5-32I
[0,2,5,6,9]</td><td colspan="3">5-19
[0,1,3,6,7]
5-19I
[0,1,4,6,7]
5-28
[0,2,3,6,8]
5-28I
[0,2,5,6,8]</td><td>5-5
[0,1,2,3,7]
5-5I
[0,4,5,6,7]
5-13
[0,1,2,4,8]
5-13I
[0,2,3,4,8]
5-16
[0,1,3,4,7]
5-16I
[0,3,4,6,7]</td><td>5-20
[0,1,5,6,8]
5-20I
[0,2,3,7,8]
5-24
[0,1,3,5,7]
5-24I
[0,2,4,6,7]
5-25
[0,2,3,5,8]
5-25I
[0,3,5,6,8]</td></tr>
<tr><td colspan="2">N/A</td><td>4-4
[0,1,2,5]
4-4I
[0,3,4,5]
4-5
[0,1,2,6]
4-5I
[0,4,5,6]
4-11
[0,1,3,5]
4-11I
[0,2,4,5]</td><td>4-12
[0,2,3,6]
4-12I
[0,3,4,6]
4-13
[0,1,3,6]
4-13I
[0,3,5,6]
4-14
[0,2,3,7]
4-14I
[0,4,5,7]</td><td>4-16
[0,1,5,7]
4-16I
[0,2,6,7]
4-18
[0,1,4,7]
4-18I
[0,3,6,7]
4-27
[0,2,5,8]
4-27I
[0,3,6,8]</td><td colspan="2">N/A</td></tr>
</table>

Table 8.2: *One to Another – Set Relationships*

8.3 One to Another

The music of Stravinsky was a central influence on the piece's style and character, particularly through the melodic phrasing within the winds in slower passages of *The Firebird* (1910). The fourth movement of *Symphony in C* (1940) also contains darker melodic phrasing of AITs in the bassoon, while the more dramatic, staccato string passages influence the block harmonic statements within the piece. *One to Another* also contains hints of tonal, film music that link back to the Romantic period, primarily seen within the evolution of rhythm and motivic contours in the second and, particularly, third movement.

Movement One

The first movement of the piece is in a repeating AB form. There are 7 sections in total, resulting in an ABABABA arrangement. Since the primary material from Table 8.1 for movement one is [0,4,1,6], this was combined with data related to set 4-Z15 from Table 8.2 to form the harmonic and structural outline for the movement, seen in Table 8.3:

Section	A	B	A	B	A	B	A
Emphasised Set	4-Z15	5-9	4-Z15	5-10	4-Z15	5-32	4-Z15
Pitch Centres	[0,6,B,8]	[0,5,2,6]	[4,A,3,0]	[4,9,6,A]	[1,7,0,9]	[1,6,3,7]	[6,0,5,2]

Table 8.3: *One to Another, Movement One – Harmonic and Structural Outline*

The data in this table was derived by alternating an emphasis on the AIT of 4-Z15 for section A with three of its supersets for section B. Pitch centres were defined by taking a pitch-class sequence from set 4-Z15, namely [0,6,1,4], and mapping it inversionally through 0. For example, [0,6,1,4] mapped through 0 results is [0,6,B,8], highlighted in the first section A above. For each subsequent occurrence of section B, transpositions of T_4, T_1 and T_6 (based on set 4-Z15's pitch classes of 0,4,1,6) result in [4,A,3,0], [1,7,0,9] and [6,0,5,2]. As with section A, section B maps around a sequence from the inverted form of 4-Z15, i.e. set 4-Z15I with prime form [0,2,5,6], using the same transpositional steps as section A but only reaching T_0, T_4, T_1, before the movement ends in T_6 of section A.

Each section A contains 16 bars, with an additional four-bar finale attached at the end of the concluding one. Within each section A, the 16 bars are broken down into four lots of four, where each four-bar sequence has its own pitch centre, taken from Table 8.3[155]. Section B follows a similar format to assign pitch centres, except each section B occurrence does not break down precisely into a 4 x 4 = 16 structure. At the conclusion of each section B, there is an additional chromatic transition back into section A material.

Musically, section A separates the woodwinds, brass and string sections, with each family having its own 16-bar theme. All instrument families play out their own theme concurrently, with a particular instrument family prominent in each recurrence of section A. Example 8.1 displays a four-bar sequence for each of these themes, taken from the final repeat of section A. There is an emphasis on rhythmic counterpoint and block-like cycling to these themes, as they interact and develop with each other through pitch centre progression to produce the most dynamic and powerful passages within the movement. The final harmonic expression of bar 133 (beat 4) also highlights a technique used throughout the piece to combine harmony, which is similar to methods devised by Carter in his combinatorial pairs of prime form AITs[156]. As such, the strings play [6,7,A,0], which is 4-Z15 T_6, while the brass and woodwinds form [6,5,2,0], which is 4-Z15I T_6. These tetrachords combine to form hexachord 6-Z48 with prime form [0,1,2,5,7,9], which consequently appears regularly within the movement.

Section B directly contrasts section A as the three instrument families do not operate separately through their own theme. From Table 8.3, each repeat of section B was assigned an emphasis based on a superset of 4-Z15. However, this superset acts only as starting harmonic material as it also makes use of the other two supersets. For example, the first occurrence of section B, from bar 18, has a focus on set 5-9I in the strings, while also cycling through sets 5-10I and 5-32I. The second section B makes use of a melodic phrase within the woodwinds and has a focus on set 5-10, while the brass family is most prominent in the third and final repeat of section B, maintaining a central focus on set 5-32. An example of the chromatic transitions between section B and A is highlighted in Example 8.2, which provides a rare instance of secondary material taken

[155] For example, the first four bars of the piece have a pitch centre of 0 before moving on to 6 for the next four bars, B for the next four, and then 8 for the final four within section A.
[156] (Hopkins and Link, 2002).

from Table 8.1. For example, in bars 37-38 the brass instruments use set 5-9I T_A while the strings and woodwinds use set 5-9I T_2.

Example 8.1: *One to Another, Movement One – Section A Themes*

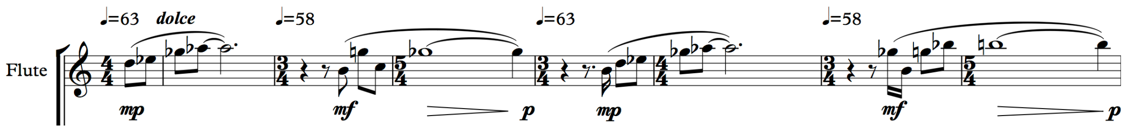
Example 8.2: *One to Another, Movement One – Chromatic Transition*

Usage of the timpani throughout the movement can be seen primarily as rhythmic support in the more dynamic portions of the piece. It uses notes C and F# only with this movement, to highlight the [0,6] dyad subset within the [0,1,4,6] primary tetrachord.

Movement Two

Movement two of the piece is in an ABCB'A' form with five sections in total. Its primary and secondary material from Table 8.1 was drawn from both 4-Z15 and 4-Z29 and subsequently expanded into Table 8.4's harmonic and structural outline, with further information included on the arrangement and dynamics. The fundamental goal within this movement was to transition between the tetrachords through gradual harmonic development. Sections A and B focus on sets related to 4-Z15, sections B' and A' focus on sets related to 4-Z29, while section C unites the two sets through the utilisation of pentachord supersets related to both sets.

Section A begins with a gentle flute melody, supported by chordal harmony within the rest of the woodwinds and a rudimentary countermelody in the cello. The flute moves between supersets of 4-Z15 (5-6 and 5-6I in T_6, with sprinklings of 4-Z15 T_2), while the cello likewise uses 5-6 and 5-6I in T_6. There is limited usage of sets 5-14 and 5-30 at this point too. By incorporating sets 5-6, 5-14 and 5-30, section A uses the supersets of 4-Z15 from Table 8.2 that were not prioritised in movement one. Example 8.3 shows the opening flute melody:

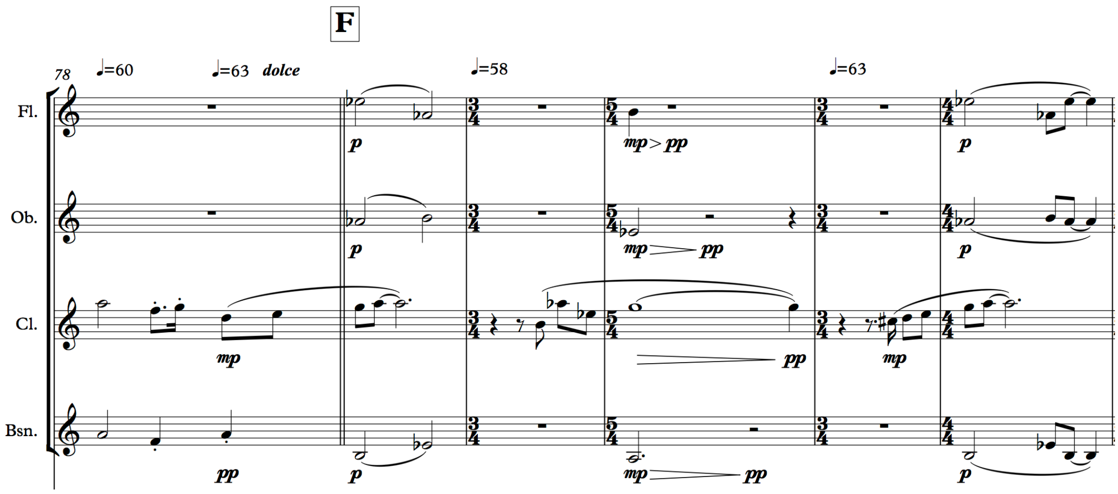


Example 8.3: *One to Another, Movement Two – Section A Opening*

Section	A		B	C		B’	A’	
Primary Pitch Centre	6	2	5	0	0	3	1	7
Secondary Pitch Centre	6	A	7	0	0	9	B	5
Arrangement	Solo and accompaniment		Solo, accompaniment and further backing	All instruments		Solo, accompaniment and further backing	Solo and accompaniment	
Dynamics	p - mp		mf - fff	p - f		fff - mf	p - mp	
Description	Emphasis on [0,1,4,6]		Transition into peak	Textural peak point Supersets of both tetrachords		Transition out of peak	Emphasis on [0,1,3,7]	

Table 8.4: *One to Another, Movement Two – Harmonic and Structural Outline*

Section A', from bar 79, follows a very similar structure to section A, except that the flute melody has been replaced by the clarinet, and the elementary countermelody has been taken on by the horn and tuba. Harmonically, there is now an emphasis on 4-Z29-related sets, with the clarinet making use of superset 5-13 and 5-13I in T_7, with sprinklings of 4-Z29 T_1. The horn and tuba support with 5-13 and 5-13I in T_7, while sets 5-20 and 5-25 are also prominent. Like movement one, sets 5-13, 5-20 and 5-25 are the supersets of 4-Z29 from Table 8.2 not prioritised in movement three. In Example 8.4, the usage of sets related to 4-Z29 in section A' rather than 4-Z15 in section A subtly alters the melodic content and resultant perception.



Example 8.4: *One to Another, Movement Two – Section A' Opening*

Sections B and B' are connected to A and A' in a similar manner. Table 8.5 highlights the gradual harmonic development across sections B and B', with a principal focus on T_5 and T_7 in section B, and T_3 and T_9 from section B', taken from Table 8.4.

Section B	**4-Z15** [0,1,4,6] ↓ **4-18** [0,1,4,7] ↓ **5-19I** [0,1,4,6,7] ↓ **6-Z13** [0,1,3,4,6,7]
(Section C)	
Section B'	**6-Z13** [0,1,3,4,6,7] ↓ **5-19** [0,1,3,6,7] ↓ **4-18** [[0,1,4,7] ↓ **4-Z29** [0,1,3,7]

Table 8.5: *One to Another, Movement Two – Section B and B' Harmonic Outline*

Section B starts from bar 12, and there is an explicit declaration of an F major triad in bar 15 as the movement begins to display occasional hints of tonality through a natural emphasis on sets such as 5-19, 5-19I and 6-Z13 and their [0,3,7] and [0,4,7] subsets. From bar 19, the brass section becomes more prominent through dovetailing of ascending and descending melodic phrases, culminating in 6-Z13 in T_5 and T_7 statements across multiple instrument families. Section B', from bar 53, also utilises set 6-Z13 but through T_3 and T_9. The section contrasts section B via the prominence of tremolo strings instead of brass, while the rhythm is more disjointed through irregular bar lengths, such as 5/8 and 7/8.

Section C, from bar 37, with its primary and secondary material from Table 8.4 all related to pitch centre 0, is the only section in the whole piece that makes use of all 12 pitch classes simultaneously. The harmony is strikingly different to other sections within the movement (Table 8.6). It was created by taking 4-Z15, 4-Z15I, 4-Z29 and 4-Z29I as woodwind material, adding common subset 3-5 (prime form [0,1,6]) as a constant within the brass, and filling out the remainder of the 12 pitch classes with related pentachords in the strings. In the first eight bars, sets 5-30I, 5-9I, 5-32 and 5-10 are related to 4-Z15 (as highlighted in Table 8.1), while in the final eight bars, sets 5-24, 5-13, 5-25I and 5-16 are related to 4-Z29.

	Bars 1-4		Bars 5-8		Bars 9-12		Bars 13-16	
Brass	***3-5*** *[2,8,9]*	***3-5*** *[2,3,8]*	***3-5*** *[3,9,A]*	***3-5*** *[3,4,9]*	***3-5*** *[2,8,9]*	***3-5*** *[2,3,8]*	**3-5** [4,5,A]	**3-5** [4,A.B]
Woodwinds	***4-Z15*** *[0,1,4,6]*	***4-Z15*** *[0,1,4,6]*	***4-Z15I*** *[0,2,5,6]*	***4-Z15I*** *[0,2,5,6]*	**4-Z29I** [0,4,6,7]	**4-Z29I** [0,4,6,7]	**4-Z29** [0,1,3,7]	**4-Z29** [0,1,3,7]
Strings	**5-30I** [3,5,7,A,B]	**5-9I** [5,7,9,A,B]	**5-32** [1,4,7,8,B]	**5-10** [1,7,8,A,B]	**5-24** [1,3,5,A,B]	**5-13** [1,5,9,A,B]	**5-25I** [2,6,8,9,B]	**5-16I** [2,5,6,8,9]

Italics indicate that these sets do not appear in the actual piece but are included to clarify the method of construction.

Table 8.6: *One to Another, Movement Two – Section C Harmonic Outline*

The section is built around a triplet-based rhythmic motif in the viola, and begins with a tonal emphasis. With each additional four bars, the rhythm becomes more complex, while additional harmonic layers are added that enhance the atonal nature of the section. The climax to section C can be seen within Example 8.5, and contains all 12 pitch classes concurrently. The intention here was to use all pitches to help neutralise the harmonic content at this central point in the piece, from which point the transition to the second tetrachord could begin.

Example 8.5: *One to Another, Movement Two – Section C Climax*

The timpani in movement two carries out a similar function to that seen within movement one, with the addition of the note G resulting in pitch classes [0,6,7] being used. This allows for the usage of [0,6] when there is an emphasis on set 4-Z15, with [0,7] used when the transition to set 4-Z29 has occurred.

Movement Three

The final movement was designed as a single climax to the piece, and as such, maintains a consistent yet conflicting musical character, which merges a sorrowful, atonal nature with increasing bursts of heroic tonality. It contains five sections: four main sections plus a finale (labelled A-E). Its set content is taken equally from the primary and secondary material of Table 8.1, while retaining a focus on set 4-Z29. The movement is based on the following three motivic ideas, which become more complex and progressively interweaved as the work progresses:

1. A Stravinsky-esque, melodic motif in the woodwinds, reminiscent of *The Firebird* (1910) opening
2. A staccato-based rhythmic response throughout all instruments
3. A 3+3+2 rhythmic motif driven by the strings

In direct contrast to the first movement in particular, which contains numerous sets of varied harmony spread across instrument families, here all instruments work together in a unified manner, cycling around 4-Z29 and its supersets not used within movement two. Harmonically, Section A leads on from the end of the second movement, making the same use of T_7 and T_5, alternating between 4-Z29 and its supersets. Table 8.7 shows the breakdown of the harmony for the section, which is more advanced than the similar methods used within *Unequal Measures* (Table 5.3) and the linear set mutation of *From Zero*.

Bar Number	1-2	3-4	5	6-7	8-9	10-11	12
Set	4-Z29	5-25I	4-Z29I	5-20	4-Z29I	5-13I	4-Z29
Prime Form/Inversion	[0,1,3,7]	[0,3,5,6,8]	[0,4,6,7]	[0,1,5,6,8]	[0,4,6,7]	[0,2,3,4,8]	[0,1,3,7]
Transposition	T_7	T_5	T_7	T_5	T_7	T_5	T_7
Pitch Classes	7,8,A,2	5,8,A,B,1	7,B,1,2	5,6,A,B,1	7,B,1,2	5,7,8,9,1	7,8,A,2

Table 8.7: *One to Another, Movement Three – Section A Harmonic Outline*

Each instrument was assigned a specific pitch class from each set, dependent on the motif and transposition. For example, the horn was assigned the 2nd highest pitch class in motif 1 but the 5th highest in motif 2, hence its opening two supporting harmony notes in the movement are A♭ (pitch class 8, seen in bar 1) and D♭ (pitch class 1, seen in bar 2). The position of each instrument within a set occasionally changes, depending on its importance within each motif and any harmonic or melodic decorations. This method purposely generates restrictions and boundaries on an instrument's range of pitches (according to the number of different sets and transpositions used), reminiscent of the primary tetrachord constraints placed on each instrument within *Three Minds Fractured*.

Section A alternates between motifs 1 and 2 in one or two-bar phrases, with the two motifs unified by a walking bass line. A conclusive harmonic statement is heard in bar 12 in all instrumentals of 4-Z29 T_7 to signal the end of the section and transition to the next harmonic material. Section B, from bar 14, follows a similar structure to section A, yet its harmony has moved on from a direct usage of 4-Z29 and instead places focus on its supersets. Harmonic cycles for sections B-D were created from the last three columns of Table 8.1, allowing for cyclic transpositions of T_0, T_4, T_6 and T_8. Table 8.8 shows section B's harmonic breakdown, while similar harmonic cycles were derived for sections C (from bar 27) and D (from bar 48), created via different sequences and transpositions. For sections C and D, the harmonic statement at the end of each section is repeated with louder dynamics to emphasise its importance.

Sections A and B place prominence on motifs 1 and 2, while sections C and D see motif 3 becoming more important to the movement, through it gradually being woven into the motivic sequences with incrementally more complex textures. Example 8.6 shows all three motifs in sequence at the end of section C. Motif 1 appears in bars 38-39 of the bassoon, motif 2 in bars 40-41 across all instrument families, and motif 3 in bars 42-43 of the strings.

Bar Number	14-15	16-17	18	19-20	21-22	23-24	25
Set	5-24	5-16	5-5	5-24I	5-16I	5-5I	5-24
Prime Form/Inversion	[0,1,3,5,7]	[0,1,3,4,7]	[0,1,2,3,7]	[0,2,4,6,7]	[0,3,4,6,7]	[0,4,5,6,7]	[0,1,3,5,7]
Transposition	T_4	T_6	T_0	T_8	T_6	T_0	T_4
Pitch Classes	4,5,7,9,B	6,7,9,A,1	0,1,2,3,7	8,A,0,2,3	6,9,A,0,1	0,4,5,6,7	4,5,7,9,B

Table 8.8: *One to Another, Movement Three – Section B Harmonic Outline*

Example 8.6: *One to Another, Movement Three – Motivic Statements*

Section E, from bar 75, contains only motif 3 material, portraying the final displacement of motif 1 and 2 and the complete morphing towards motif 3. Since motif 3 is the most tonal in nature, a major tonality progressively bursts through the movement. The section transitions into an alternating sequence of block chords through 4-Z29 T_0 and 4-Z15 T_6, before the piece concludes on a final fff statement of 4-Z29 T_0. It is the only time in the whole piece that the two AITs directly interact with each other (Example 8.7).

Example 8.7: *One to Another, Movement Three – 4-Z15 and 4-Z29 Link*

Across all three movements, tempo and dynamics play key roles in shaping the piece and signalling transitions in the harmonic set content. Within movements one and three, the tempo increases incrementally to add impetus towards the end of each movement. In movement two, it peaks at the central point of the movement, and hence the central point of the piece, which acts at the point of transition to the second tetrachord. At the micro-level, a systematic drop in tempo in the middle of a section or subsection is occasionally combined with an increase in dynamics, which helps to prolong and build the tension. This effect is reminiscent of some of the uses of tempo and dynamics seen within *From Zero*.

In 2015, an octet arrangement (Appendix E) combining aspects from each movement of *One to Another* was performed at a workshop, and provided particular insight into how the various instrument families worked together texturally and dynamically. It enabled further consideration of these textures, and highlighted additional potential for articulation variation across instrument families.

8.4 Summary

One to Another investigated the relationship between the two AITs as they transitioned from one to the other over three movements. Through the research conducted up to this point, my interest had grown in expressing harmonic devices within minimalist, cyclical structures, and the mathematical properties of the AITs enabled me to construct a pre-compositional design based on the circle of fifths, repetition, variation and evolution. By using macro-level pitch centres at the structural level to represent overall sections within each movement (Tables 8.1 and 8.2) to more micro-level pitch centres to act as local harmony from bar to bar (Tables 8.7 and 8.8), the design presented potential for hierarchical relationships of self-similarity while remaining flexible with relation to melody and the complexity of harmonic textures. The tonal and atonal properties of each tetrachord were studied, with an emphasis on [0,6] and [0,7] dyads within the context of related set [0,1,6], which is itself a subset of both tetrachords. This idea is reminiscent of *From Zero*, which highlighted contrast between interval classes within the piece's structure, but here it was expressed on a grander scale. Finally, although the separation of the instrument families into 5 – 4 – 3 set segmentation was not always aurally evident within the piece, it provided sufficient compositional material for harmonic development of these related subsets.

Chapter 9

Aggregation (2016-2017)

9.1 Introduction

Aggregation is a single-movement piece composed for a large ensemble of 18 players[157]. It aims to take theoretical and practical influence from earlier pieces in the portfolio, and build on their pre-compositional designs in order to encapsulate the research within a single, large-form work. The previous pieces have investigated links between the research subjects of mutation, symmetry, proportion, the octatonic scale and the AITs, and also used set theory to highlight connections between their musical material. Following on from *One to Another*'s application of using three movements to transition from one AIT to another, *Aggregation* extends this principle and demonstrates this method across extensive harmonic material from the portfolio. This includes several of Messiaen's Modes of Limited Transpositions, such as the whole-tone and octatonic scales, since their versatility allows them to "slip easily from one tonality to another without any real sense of modulation"[158]. Overall, the piece uses the whole-tone scale, the diminished 7th chord, the augmented triad, Fibonacci-related sets, the octatonic scale and the AITs, and plots transitions between these via a unified harmonic and structural outline.

9.2 Pre-Compositional Design

Perhaps the most interesting harmonic and structural outline of the portfolio is from *In Equal Measure.* Its fundamental construction links an ascending C diminished 7th chord with a descending G augmented triad, and expands them into hierarchical layers to produce a symmetrical structure that moves between tonic and dominant pitch centres. I was interested in applying the premise behind this design to a more fluid compositional structure, and so used the initial outline based on root notes (Figure 4.1) as a basis for

[157] Piccolo, flute, oboe, clarinet in B♭, bassoon, French horn in F, trumpet in B♭, tenor trombone, tuba, timpani, suspended cymbal, snare drum, bass drum, violin 1, violin 2, viola, violoncello and contrabass.
[158] Robert Sherlaw Johnson, *Messiaen* (Berkeley: University of California Press, 1975), pp. 16-17.

Aggregation's pre-compositional design. Table 9.1 shows this concept expanded with set content of five of the portfolio's research subjects across five sections of the piece.

Section	**1**			**2 and 3**			**4 and 5**		
Pitch Centres	0	7	3	3	6	B	9	7	0
Research Subject (*Musical Work*)	**Mutation** (*From Zero*) **Symmetry** (*In Equal Measure*)			**Fibonacci Series** (*Unequal Measures*) **AITs** (*One to Another*)			**Octatonic Scale** (*Three Minds Fractured*)		
Related Set Content	[0,2,4,6,8,A] [0,3,6,9] [0,4,8]			[0,1,3,6] [0,1,4,7] [0,1,4,6] [0,1,3,7] [0,1,3,4,6,7]			[0,1,3,4] [0,1,3,4,6,7,9,A]		

Table 9.1: *Aggregation – Harmonic and Structural Outline*[159]

The transitions from section to section have been influenced by the similar application in *From Zero*, which applied mutation via related set content to move from the whole-tone scale to connected material. For *Aggregation*, set content across all musical works in the portfolio was analysed in order to find the most natural progression that could encompass material from the majority of these works. Ultimately, set 6-Z13, [0,1,3,4,6,7], was determined to be particularly useful in linking a variety of set content as it maintained the following properties:

1. It is a subset of the octatonic scale, [0,1,3,4,6,7,9,A], and a superset of the octatonic's primary tetrachord, [0,1,3,4], used to compose *Three Minds Fractured*
2. It is a superset of both AITs, [0,1,4,6] and [0,1,3,7] (thus, the AITs are transitively subsets of the octatonic for further links between material)
3. Similarly, it is a superset of Fibonacci-related material, [0,1,3,6] and [0,1,4,7][160] (which are also subsets of the octatonic and closely related to AIT set content)

[159] As the compositional stage unfolded, it was felt that the diminished 7th would be more appropriately applied in section 5 rather than section 1 since it is a direct subset of the octatonic scale, while the augmented triad can be observed more in section 1 as a direct subset of the whole-tone scale.

4. It can be constructed from the addition of two diminished triads, namely [0,3,6] and [1,4,7], and is hence related to Perle's interval cycles[161], applied within *In Equal Measure*

Overall, the set material allowed for a consistent sound world that harmonically unified the majority of research paths in the portfolio into a single, culminating piece.

9.3 Aggregation

The design's harmonic journey, bookended by transitions to and from the tonic and dominant pitch centres, aligned with my interest in applying the concept of momentum towards a final piece with a clearly defined finale. Three of my favourite crescendos/finales include Copland's use of the 19th century work song *Simple Gifts* in *Appalachian Spring* (1944), the fourth movement of Shostakovich's *Symphony No. 5 in D minor, Opus 47* (1937) and Stravinsky's *The Firebird* (1910). I am particularly fond of the ways in which each composer drives momentum by using harmonic and rhythmic development through repetition and variation of motivic content via combinations of solos, duos, instrument families and the full ensemble. Having witnessed extreme variation in tempo and pacing in performances of Shostakovich's work, in particular, the rate of change in textural variety and the importance of pacing towards these climaxes is emphasised. As is clear throughout the portfolio, the harmonic content of Stravinsky has also been a key influence in many of my pieces, while the use of rhythmic repetition by Shostakovich and melodic repetition of *Simple Gifts* in *Appalachian Spring* have also influenced *Aggregation*.

As evidenced in *One to Another*, the use of short-term harmonic cycles based on similar set content was something that worked effectively, and a key musical aim of *Aggregation* was to develop this method within the context of momentum. Each section within the piece is based around key motifs with relatively short call and response melodies and rhythms. This motivic content builds over the course of the work, with phrases becoming more direct in later sections to drive waves of momentum more candidly. As with *One to Another*, these cycles and use of repetition are perhaps more commercial and align with popular music aesthetics than most other works within the

[160] [0,1,3,6] derives from Fibonacci numbers 2,3,5,8 while [0,1,4,7] stems from 1,2,5,8.
[161] Perle (1996), p. 92.

portfolio. Besides the opening section, which applies all instruments evenly to advance a unified motif, all four instrument families are used for diverse musical devices. The prominent use of staccato strings continues from previous portfolio pieces such as *Unequal Measures* and *One to Another* to drive rhythmic accompaniment throughout. The winds, particularly the flute, are often used to expand motivic phrases like in *One to Another*, while the brass primarily exchanges between legato, harmonic support and acting as a secondary rhythmic function, as evidenced particularly in the trombones and tubas of *Three Minds Fractured*. The percussion adds further rhythmic support, acting quite expressively early in the piece before becoming more military in nature as it develops the drive of the finale from section 4 onwards.

In alignment with Table 9.1, Section 1 (bars 1 to 49) has a central harmonic focus on the whole-tone scale, with further occasional support from the augmented triad. It is primarily based on a 1-bar whole-tone motif, to be called motif 1, and can be seen in its most basic form in the flute of bar 1 and the trumpet in bar 3 (Example 9.1). The motif works in 2 and 4-bar unit call and response phrases to cycle around transpositions linked to the pitch centres of Table 9.1, developing melodic, rhythmic and textural variation through symmetry and retrogrades as it progresses across all instrument families. Depending on which instrument family is playing motif 1 at any particular point, the accompaniment is filled out texturally by the remaining families. From bar 30, the strings predominantly act as rhythmic support, as the motif expands further in the winds and brass. Usage of irregular bar lengths in combination with motivic anacruses aims to blur the bar lines and disrupt the regularity of the phrasing, and the section culminates in varied repetition of motif 1, before augmented triad statements crown a crescendo.

Since links between augmented and diminished harmony are well-established through music history (and further evidenced within *In Equal Measure*), this harmonic material is used to support the transition from section 1 to 2. The whole-tone scale of section 1 was broken down into two augmented triads ([0,4,8] and [2,6,A]), with a similar segmentation of section 2 material, [0,1,3,4,6,7], forming two diminished triads ([0,3,6] and [1,4,7]). Table 9.2 highlights this harmonic transition.

Example 9.1: *Aggregation – Section 1 Motif 1*

Section	**Section 1**	**Section 2**
Section Set Content	[0,2,4,6,8,A]	[0,1,3,4,6,7]
Section Set Content	[0,4,8], [2,6,A]	[0,3,6], [1,4,7]

Table 9.2: *Aggregation – Section 1 to 2 Transition*

Immediately prior to this, a second motif (motif 2), also based on the whole-tone scale, becomes infused in the texture. It appears in the winds in bar 41, in both the winds and brass in 43, and just brass in 44 (Example 9.2). It has a more direct, downbeat emphasis and serves to counter the more expressive nature of motif 1 as momentum is built towards the section's crescendo.

Section 2 (from bar 50) maintains a slower tempo compared to the opening, yet makes use of similar short-term repetition of call and response phrases to develop material. The key distinctions are in its harmonic content and structure, where the former applies [0,1,3,4,6,7] within a series of transpositions based on the pre-compositional design, and the latter peaks at the central passage rather than at its end. Section 2 also peaks dynamically and texturally at this point, with the winds forming primary motivic fragments that are countered rhythmically by the brass and percussion. The section builds up in small blocks either side of this central point, and is constructed symmetrically on a bar-by-bar basis, influenced by the first movement of *In Equal Measure*. For instance, bars 64 and 67 are built from largely the same material (Example 9.3).

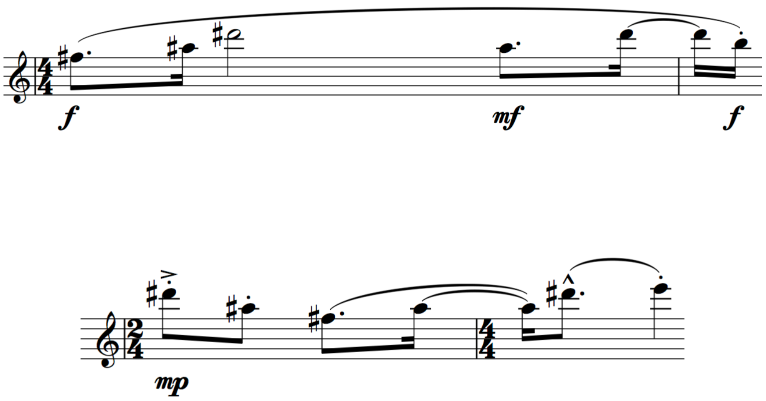
Example 9.2: *Aggregation – Section 1 Motif 2*

Example 9.3: *Aggregation – Section 2 Central Passage*

Section 3, from bar 86, also uses symmetry to inform its structure but in a more intricate manner. There is an absolute central point of the section (quaver beat 4 of bar 97, Example 9.4), and the material is presented through a palindrome of pitch and note duration after this point. Following the construction of the palindrome, minor amendments were made to melodic phrases to maintain momentum into the second half of the section. As with section 2, the winds primarily take on the melodic content, while dynamics, tempo, texture and rhythmic complexity similarly peak at the central point. The harmonic material is from the Fibonacci and AIT set content of Table 9.1 ([0,1,3,6], [0,1,4,6], [0,1,4,7] and [0,1,3,7]), while irregular phrase lengths follow one another at the micro level, based on the design's transpositions of these sets. This is like the technique seen in Table 8.5 and applied in *One to Another*, while these four sets have also been considered previously as a harmonic progression in *Unequal Measures*.

Example 9.4: *Aggregation – Section 3 Central Passage*

Besides symmetry, **s**ections 2 and 3 have further micro-level similarities, the most notable of which can be seen through motivic passages in the flute. The top phrase of Example 9.5 below is bar 60 of section 2, while the bottom phrase is notably after the central point of section 3 (bars 105-106), and part of the retrograde.



Example 9.5: *Aggregation – Sections 2 and 3 Motivic Link*

As the texture of section 3 thins out towards the end, a solo bassoon (assisted by clarinet and oboe) transitions from the end of bar 108 [0,1,4,6] to the octatonic material of bar 112 (Section 4). The section mainly uses general octatonic content but occasionally breaks down to use combinations of its subset tetrachords, akin to the use of primary tetrachords in *Three Minds Fractured.* The momentum towards a single crescendo in section 4 begins from its outset. Short phrase lengths, typically of around 4 bars with the winds once again acting as primary melodic content, become more complex rhythmically, texturally and dynamically as they cycle through pitch centre transpositions. The snare drum, influenced somewhat by its inclusion in Shostakovich's crescendo of his *Symphony No. 5*, adds a military feel as the piece begins to move towards its overall finale. Example 9.6 highlights the approach towards the peak of an accelerated crescendo, which acts as a false finale that pauses the momentum and strips down the texture, ready for the actual finale of section 5.

Example 9.6: *Aggregation – Section 4 Crescendo*

Section 5, from bar 134, is a slow-building crescendo that concludes the overall piece. The previous four sections often made use of anacruses to blur the bar lines. However, in its aims to form a unified and direct crescendo, section 5 provides a block-like harmonic progression, with clear emphasis on the downbeat. The section develops texturally in waves of three smaller sections, initiated predominantly by the strings each time, with consistent forward motion as each wave becomes texturally and rhythmically more complex.

On a local harmonic level, the octatonic is prominent in subset tetrachords, primarily seen through alternating 1-bar units of dominant 7^{th} ([0,4,7,A]) and half-diminished 7^{th} ([0,3,6,A]) material that develops rhythmically over time and provides tonal instability. The use of the strings in this context is an ode to the 'Dances of the Young Girls' opening in Stravinsky's *Rite of Spring* (1913), in which the strings are central to an accented, homophonic passage that makes use of the octatonic scale and its subset divisions. Example 9.7 shows the most surface level instance of this in *Aggregation*, through the ascending melodic content of the dominant 7^{th} in bar 172 of the violin 1. During this bar, the winds borrow the content from the violin, as the dispersion of material across instrument families develops throughout the section. There is no principal melody in section 5, but the flute and clarinet lead the rhythmic counterpoint of the winds against the strings, with the brass and percussion working to complement and contrast rhythmic functions of both instrument families.

As the piece approaches its crescendo, a brass-based motif similar to motif 1 from section 1 is permeated into the texture, but within an octatonic rather than whole-tone context (Example 9.8). It helps to deliver the direct, downbeat nature of the crescendo, with all instruments at their dynamic peaks as the rhythm eventually simplifies towards the final octatonic chord. The crescendo takes influence from the straight quaver feel driven by the percussion in Shostakovich's *Symphony No. 5* finale, while the tremolo and sforzando expression of the final chords is adopted from Stravinsky's *Firebird* finale.

Example 9.7: *Aggregation – Section 5 Harmonic and Melodic Development*

Example 9.8: *Aggregation – Section 5 Finale*

9.4 Workshops

A simplified arrangement of section 3 of *Aggregation* was used as the third and final section of *In Three Elements* (Appendix G). Its 2016 workshop performance demonstrated success in counterpoint between parts, but perhaps highlighted the difficulty in rearranging a complex score for a quartet, and might have benefitted from thinner textures for extra clarity.

In order to hear how the finale of *Aggregation* may sound with a larger ensemble, a modified score without percussion was submitted to a 31-piece orchestra for a 2017 workshop performance in Lisbon (Appendix F). The multi-part nature of the strings was particularly effective, yet the recording rendered the balance between the brass and winds difficult to discern due to the multi-player horns and trumpets overpowering the single-player wind parts.

9.5 Summary

Aggregation is the portfolio's largest work and incorporates several compositional techniques developed earlier in the portfolio. By applying these techniques within a less restrictive pre-compositional design, the piece is one of the most successful works of the portfolio. Although formal macro-level structures were present in the pre-compositional designs of *In Equal Measure* and *Unequal Measures*, my future interest in minimalist phrase structures and development via repetition of short-term cycles expanded through *One to Another* and is particularly prevalent in *Aggregation*. The experience gained through the micro-level usage of symmetry in my earlier pieces allowed me to prepare and construct more suitable non-retrogradable rhythmic phrases in *Aggregation*. Section 3 contains perhaps the most successful application of this (and symmetry in general) within the portfolio, and only required minimal edits to ensure that the palindromic phrases were as natural and expressive as their forward counterparts. Although the pitch content transforms throughout, the use of [0,1,3,4,6,7] as a key set allowed for a unified harmonic identity across all sections, and development of pitch content over time from the whole-tone scale to the octatonic can be heard but is not disjoint. Overall, the piece stands as a suitable exemplar of my portfolio's harmonic content, and establishes a potential new structural path for me to follow.

Chapter 10

Conclusion

This chapter highlights some key findings of my research, before outlining potential for future work and offering some final thoughts for consideration.

10.1 Overall Findings

The focus on sets related to symmetry, proportion and balance within PC set theory was central to the creation of a variety of original pre-compositional designs in a number of related mathematical contexts. As the research progressed, the distinct mathematical properties and restrictions of each design were evaluated and used to create subsequent designs within the portfolio; their conception helped me to elucidate and advance my creative process. For example, the symmetrical design of *In Equal Measure* influenced the asymmetrical form of *Unequal Measures*, while also acting as a suitable harmonic and structural foundation for the all-encompassing nature of *Aggregation*.

With the portfolio placing emphasis on pitch, it was utilised on the micro-level to develop melodic textures and harmonic gestures, as well as on a macro-level to advise harmonic form. The structural elements of the pre-compositional designs were instrumental in my creative approach, lending themselves to a top-down process of composition in which I constructed macro-level harmonic topologies and iteratively refined them to increase detail on a micro-level. Beginning with a skeleton structural outline, often based on mathematical proportions, allowed for the isolation of single musical parameters such as harmony, and it is possible to hear harmonic transitions when cycling through related set content within and between sections of the works.

This is most evident throughout *One to Another*, which demonstrated the most explicit instance of my influence of Stravinsky, particularly through motifs transposed via harmonic cycles, building in intensity through dynamic, rhythmic and textural transformations. *Aggregation* developed these techniques within a more complex arrangement, with elements of film music aesthetics and the influence of Michael Nyman's style present. Morgan notes that Nyman's use of static tonal structures, additive rhythms, textural consistency and constant thematic repetition translates well into popular cultural aesthetics[162]. Although early works within the portfolio, such as the first movement of *In Equal Measure* and the second movement of *Unequal Measures*, do hint at these principles, they are more apparent in my later pieces, but through a harmonic language more closely linked to Stravinsky and Bartók.

The application of prime form sets in my designs was particularly useful in the abstract, pre-compositional stage, while non-prime form sets offered much more flexibility and potential for musical coherence at the compositional stage. As the research progressed, I was able to refine my process and achieve a more successful balance between both types of material. Overall, the use of complex networks of sets and a top-down approach, when considered alongside my own compositional interests within minimalist pitch structures, appeared to suit the larger, later works in the portfolio more so than the earlier solo and small ensemble pieces.

Throughout the research process, I frequently considered how my pre-compositional designs would be interpreted and potentially used as compositional models by other composers. The methodologies adopted and explained throughout each chapter provide insight into the original nature of my approaches, and highlight the benefits that mathematical principles have on their creation. Other composers may find the methods used for my harmonic and structural designs useful starting points for their own pre-compositional plans, and gain insight into the type of music that works effectively alongside distinct structural models.

Although my primary focus has been on pitch, there is further potential for set theory-related designs to be constructed with increased emphasis on other parameters. For example, Forte has underlined rhythm as having an important part to play in PC set theory, since it is "considered inseparable from pitch, even if one or the other is understood analytically as predominant in a particular context"[163]. Compositional devices such as non-retrogradable rhythms (present at the micro-level in *In Equal Measure*) and rhythmic canons could be foregrounded on both local and structural levels, and incorporated directly into pre-compositional designs[164]. At a fundamental

[162] Robert P. Morgan, *Anthology of Twentieth-Century Music*, (New York: W. W. Norton, 1991), p. 423.
[163] Allen Forte, 'Foreground Rhythm in Early Twentieth-Century Music', in *Early Twentieth-Century Music,* ed. by Jonathan Dunsby (Oxford: Blackwell, 1993), p. 145.
[164] A rhythmic canon is a canon that maintains a focus on the same rhythm but allows distinct pitches.

level, set theory gives composers a compositional framework and mathematical language with which they can express both micro and macro-level systems and patterns across numerous musical parameters, and has presented me with further platforms for my own future compositions in a variety of different contexts, discussed in more detail below.

10.2 Future Research

This research has considered the compositional potential of various sets related to mathematical contexts, and is not necessarily an exhaustive list of the techniques possible within PC set theory. The varied pre-compositional designs demonstrate coherence and potential for further invention, outside of the immediate scope of this study. Other aspects related to set theory I would like to incorporate into pre-compositional designs include the full range of Messiaen's Modes of Limited Transpositions, which were closely linked to *From Zero*, *In Equal Measure*, *Three Minds Fractured* and *Aggregation*. Furthermore, Hanninen looks at associate sets, which are unordered collections of related segments [165]. Segmentation of sets is considered within *Space Ripple* and *Three Minds Fractured*, the latter with a focus on set 4-3, [0,1,3,4], and its links to the octatonic scale, but this is something that could be explored further as a distinct area of research. Finally, I have been following the research of Nolan Stolz and his interest in fractional set theory, which breaks down PC sets into a microtonal context[166], and I would be interested in using varied tuning systems to produce xenharmonic pre-compositional designs and electroacoustic works based on this premise.

Having taught music theory to both music and audio students, I have noted an inherent difference in the approach taken by students from creative, non-musical backgrounds when learning harmonic principles. By adopting basic trichord and tetrachords as starting points to represent triads and 7th chords, the more numerical approach of set theory has suited students with a technical understanding of audio, and avoided the necessity to study harmony in depth through traditional methods. Consequently, I am interested in researching the application of set theory as an educational tool for non-

[165] Hanninen (2004).
[166] Nolan Stolz, 'Fractional Set Theory: A System for the Analysis of Microtonal Music', (2012).

music students in creative media subjects, perhaps through the creation of computer programs that support analysis and composition.

Outside of this immediate PC set context, the links between music composition and computer programming offer much potential. Prusinkiewicz[167] details a technique to produce musical scores from curves generated via L-systems[168], and I have investigated bringing its application into the context of Kolams[169] in order to produce melodic material based on its iterative and self-similar elements. This provides potential links to Markov chains, and having recently started to explore programming software such as Max/MSP and Pure Data, I would be interested in developing this towards a fully automated compositional design (akin to those of Xenakis[170]) through the integration of L-systems, Kolams and Markov chains. The visual aspect of programming languages would directly complement my hierarchical structural designs, as seen in *In Equal Measure* for example, and offer vast potential for further interest.

10.3 Closing Thoughts

The research undertaken to produce this portfolio has enabled me to formalise my compositional process within a theoretical context. The research journey, starting at *From Zero* and culminating in *Aggregation*, exhibits development from my initial interest in restrictive pre-compositional designs of smaller works to more expansive systems within larger ensembles, primarily restricted at the compositional stage through pitch organisation. I aim to continue this development after my PhD and look forward to uncovering new avenues to explore creatively.

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[167] Prusinkiewicz 'Score Generation with L-Systems', in *1986 International Computer Music Conference,* (Royal Conservatory of Music, Den Haag, Netherlands, 1986), 455-57.
[168] An L-system is a formal grammar, from which rules are defined and iterative processes can develop, relating to self-similarity (Jon McCormack, 'Grammar Based Music Composition', (1996)).
[169] Kolams are visually decorative Indian designs.
[170] Xenakis (1971).

Appendix A

***From Zero* – Alternative Design**

Within this design, the dependence on prime form sets was disregarded. Furthermore, rather than using a single-stage probability matrix that equally considered all permutations of removing, replacing or adding a pitch class, extra steps were added to the process that affected the fundamental mutation probabilities. An additional step stated that there was a probability of $^1/_3$ that the set would have a pitch class removed, a probability of $^1/_3$ that the set would have a pitch class replaced, and a probability of $^1/_3$ that a pitch class would be added. A practical outline into this multi-step process is demonstrated below[171].

There are three steps to consider for this process:

Step 1: Is the set going to remain the same or mutate? There is a probability of $^1/_2$ that the set will remain the same, and a probability of $^1/_2$ that it will mutate. Like the Primary Design, there is also an imposed limit that a set can only be played twice consecutively before it must mutate.

Step 2: If the set is going to mutate, is it going to have a pitch class removed, replaced or added? There are probabilities of $^1/_3$ for each of these possibilities.

Step 3: n = the number of pitch classes

Step 3a: If a pitch class is to be removed, there is a probability of $^1/_n$ to determine which pitch class is removed.

Step 3b: If a pitch class is to be replaced, there is a probability of $^1/_n$ to determine which pitch class is initially removed, followed by a probability of $^1/_{(12-n)}$ to determine which pitch class replaces it.

Step 3c: If a pitch class is to be added, there is a probability of $^1/_{(12-n)}$ to determine which pitch class is added.

[171] Due to the vast number of non-prime form PC sets, a single stage matrix is not feasible in this instance.

This multi-step process is demonstrated below through an example, using set [0,2,4,6,8].

Step 1: The random number generator chooses either value 1 or 2 to represent the set remaining the same or mutating. On this occasion, the number 2 is generated and the set mutates.

Step 2: Since the set mutates, the random number generator (for values 1-3) now determines whether a pitch class is removed, replaced or added. On this occasion, a pitch class is replaced since number 2 is generated (Move to Step 3b for replacing a pitch).

Step 3b: Since there are 5 pitch classes in the current set, we first remove a pitch class by a probability of $^{1}/_{n}$ ($^{1}/_{5}$ in this case, and so the random number generator uses values 1-5). On this occasion, number 3 is generated, and so pitch class 4 is removed since it is the 3rd highest pitch in the set. This leaves [0,2,6,8]. At this point, a replacement pitch class is added by using the probability $^{1}/_{(12-n)}$ ($^{1}/_{7}$ in this case, and so the random number generator uses values 1-7). The 7 replacement options are non-set pitch classes 1, 3, 5, 7, 9, A and B, and so on this occasion number 5 is generated, resulting in the 5th highest pitch class, 9, being added to the set, ultimately forming the new set [0,2,4,6,8,9][172].

These deviations from the Primary Design would result in less restrictiveness, since mutation could occur across a wider number of PC sets, and not just those restricted to prime form. Step 2, in particular, would result in a divergence in the type and speed of mutation possible, since it brings about an increased probability that the set would have a pitch class removed or added, compared to the dominant nature of replacing a pitch class within the previous system.

[172] A set may mutate to cardinalities between 4 and 8. Once a set reaches these lower or upper limits, it must either remain the same via stage 1, or if it reaches mutation via stage 2, the stage 2 probabilities will change. Thus, if the mutation reaches a set of cardinality 4, then if it is to mutate following stage 1, there is a probability of $^{1}/_{2}$ that a pitch class will be replaced, and a probability of $^{1}/_{2}$ that a pitch class will be added. Conversely, for a cardinality of 8, there is a probability of $^{1}/_{2}$ that a pitch class will be removed, and a probability of $^{1}/_{2}$ that a pitch class will be replaced.

Appendix B

From Zero **– Matrices for First Iteration Sets (Primary Design)**

5-33	**4-21**	**4-24**	**4-25**	**5-8**	**5-9**	**5-13**	**5-15**	**5-24**	**5-26**
	2/94	2/94	1/94	2/94	4/94	4/94	2/94	4/94	4/94
	5-28	**5-30**	**5-33**	**5-34**	**6-21**	**6-22**	**6-34**	**6-35**	
	4/94	4/94	52/94	2/94	2/94	2/94	2/94	1/94	

6-21	**5-8**	**5-9**	**5-13**	**5-26**	**5-28**	**5-33**	**6-2**	**6-Z4**	**6-Z37**
	1/96	1/96	1/96	1/96	1/96	1/96	3/96	1/96	1/96
	6-9	**6-Z10**	**6-Z39**	**6-Z12**	**6-Z41**	**6-15**	**6-16**	**6-Z17**	**6-Z43**
	1/96	2/96	2/96	1/96	1/96	2/96	1/96	1/96	1/96
	6-21	**6-22**	**6-Z23**	**6-Z45**	**6-Z24**	**6-Z46**	**6-Z28**	**6-Z49**	**6-30**
	50/96	4/96	1/96	1/96	1/96	1/96	1/96	1/96	1/96
	6-31	**6-34**	**6-35**	**7-8**	**7-9**	**7-13**	**7-26**	**7-28**	**7-33**
	1/96	4/96	1/96	1/96	1/96	1/96	1/96	1/96	1/96

6-22	**5-9**	**5-13**	**5-15**	**5-24**	**5-30**	**5-33**	**6-2**	**6-Z4**	**6-Z37**
	1/96	1/96	1/96	1/96	1/96	1/96	1/96	1/96	1/96
	6-7	**6-9**	**6-Z10**	**6-Z39**	**6-Z12**	**6-Z41**	**6-15**	**6-16**	**6-Z17**
	1/96	2/96	1/96	1/96	2/96	2/96	1/96	2/96	2/96
	6-Z43	**6-21**	**6-22**	**6-Z24**	**6-Z46**	**6-Z26**	**6-Z48**	**6-31**	**6-33**
	2/96	4/96	50/96	1/96	1/96	1/96	1/96	1/96	1/96
	6-34	**6-35**	**7-9**	**7-13**	**7-15**	**7-24**	**7-30**	**7-33**	
	4/96	1/96	1/96	1/96	1/96	1/96	1/96	1/96	

6-34	**5-24**	**5-26**	**5-28**	**5-30**	**5-33**	**5-34**	**6-9**	**6-Z10**	**6-Z39**
	1/96	1/96	1/96	1/96	1/96	1/96	1/96	1/96	1/96
	6-Z12	**6-Z41**	**6-15**	**6-16**	**6-Z17**	**6-Z43**	**6-21**	**6-22**	**6-Z23**
	1/96	1/96	1/96	1/96	1/96	1/96	4/96	4/96	1/96
	6-Z45	**6-Z24**	**6-Z46**	**6-Z26**	**6-Z48**	**6-Z28**	**6-Z49**	**6-30**	**6-31**
	1/96	2/96	2/96	1/96	1/96	1/96	1/96	1/96	2/96
	6-33	**6-34**	**6-35**	**7-24**	**7-26**	**7-28**	**7-30**	**7-33**	**7-34**
	3/96	50/96	1/96	1/96	1/96	1/96	1/96	1/96	1/96

7-33	**6-21**	**6-22**	**6-34**	**6-35**	**7-8**	**7-9**	**7-13**	**7-15**	**7-24**
	2/94	2/94	2/94	1/94	2/94	4/94	4/94	2/94	4/94
	7-26	**7-28**	**7-30**	**7-33**	**7-34**	**8-21**	**8-24**	**8-25**	
	4/94	4/94	4/94	52/94	2/94	2/94	2/94	1/94	

Appendix C

***From Zero* – Possible Set Mutation (Primary Design)**

Initial Set	**6-35**				
First Iteration Sets	5-33	6-21	6-22	6-34	7-33
Second Iteration Sets Cardinality of 4	4-21, 4-24, 4-25				
Second Iteration Sets Cardinality of 5	5-8, 5-9, 5-13, 5-15, 5-24, 5-26, 5-28, 5-30, 05-33, 5-34	5-8, 5-9, 5-13, 5-26, 5-28, 5-33	5-9, 5-13, 5-15, 5-24, 5-30, 5-33	5-24, 5-26, 5-28, 5-30, 5-33, 5-34	
Second Iteration Sets Cardinality of 6	6-21, 6-22, 6-34, 6-35	6-2, 6-Z4, 6-Z37, 6-9, 6-Z10, 6-Z39, 6-Z12, 6-Z41, 6-15, 6-16, 6-Z17, 6-Z43, 6-21, 6-22, 6-Z23, 6-Z45, 6-Z24, 6-Z46, 6-Z28, 6-Z49, 6-30, 6-31, 6-34, 6-35	6-2, 6-Z4, 6-Z37, 6-7, 6-9, 6-Z10, 6-Z39, 6-Z12, 6-Z41, 6-15, 6-16, 6-Z17, 6-Z43, 6-21, 6-22, 6-Z24, 6-Z46, 6-Z26, 6-Z48, 6-31, 6-33, 6-34, 6-35	6-9, 6-Z10, 6-Z39, 6-Z12, 6-Z41, 6-15, 6-16, 6-Z17, 6-Z43, 6-21, 6-22, 6-Z23, 6-Z45, 6-Z24, 6-Z46, 6-Z26, 6-Z48, 6-Z28, 6-Z49, 6-30, 6-31, 6-33, 6-34, 6-35	6-21, 6-22, 6-34, 6-35
Second Iteration Sets Cardinality of 7		7-8, 7-9, 7-13, 7-26, 7-28, 7-33	7-9, 7-13, 7-15, 7-24, 7-30, 7-33	7-24, 7-26, 7-28, 7-30, 7-33, 7-34	7-8, 7-9, 7-13, 7-15, 7-24, 7-26, 7-28, 7-30, 7-33, 7-34
Second Iteration Sets Cardinality of 8					8-21, 8-24, 8-25

Appendix D

***In Equal Measure* – Initial Sketch of Movement One, Section A**


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