**1. INTRODUCTION**

The main disadvantage of concrete under low confining pressure is its brittleness, i.e., relatively low tensile strength and ductility, and poor resistance to crack opening and propagation. Therefore once cracking is initiated, concrete drastically loses its load carrying capacity in tension. This drawback which limits practical applications of plain concrete, could be overcome by the inclusion of a small amount of discontinuous and short steel fibres which are randomly distributed within the concrete. This composite material is generally known as Fibre Reinforced Concrete (FRC) [[1](#_ENREF_1)].

Among all the common fibre materials, steel is the most suitable for structural purposes [[1](#_ENREF_1)]. In the early days of Steel Fibre Reinforced Concrete (SFRC), the fibres were mostly undeformed (straight) [[2](#_ENREF_2)]. The crack bridging performance of such fibres directly depends on the physical and chemical adhesions between the fibres and the surrounding matrix (physicochemical bond) which are predominantly determined by the properties of the fibre-matrix interface and matrix packing density [[3](#_ENREF_3)]. More recent research has shown that mechanical anchorage in the deformed fibres effectively improves pullout resistance [[2](#_ENREF_2), [4](#_ENREF_4), [5](#_ENREF_5)]. The mechanical bond properties are determined by the physical geometry of the fibre and the transverse tensile stress resistance of the matrix. The mechanical anchorage could be provided by deformation at the fibre ends, such as with hooked-end fibres which locally increase the mechanical bond, or deformation along the fibre length, such as in crimped or twisted fibres which provides a mechanical bond along the fibres [[3](#_ENREF_3)]. Typical steel fibres are shown in **Fig. 1**.

The mechanical behaviour of SFRC is mainly governed by the fibre-matrix interfacial bond characteristics. Although such characteristics are best described by a bond-shear-stress-slip relationship, the direct experimental determination of such a relationship has not yet been possible. However, a load versus slip response which can be obtained from the fibre pullout test, has been employed by researchers to study the characteristics of the fibre-matrix interface [[6-8](#_ENREF_6)]. Numerous experimental works [[2](#_ENREF_2), [3](#_ENREF_3), [5-10](#_ENREF_5)] and analytical studies [[8](#_ENREF_8), [11](#_ENREF_11), [12](#_ENREF_12)] have been conducted to investigate the pullout mechanism of fibres.

The fibre-bridging-pullout process is a highly sophisticated mechanism which consists of cohesion, interfacial debonding, sliding frictional contact, fibre deformation and material plasticity. For most of the cases, such complexities have made the use of analytical methods almost impossible. Therefore, the establishment of a nonlinear FE model to simulate the mechanism of fibre pullout from the concrete matrix has been an instructive solution to tackle such a complex problem.

Two-dimensional (2D) FE models for the fibre pullout have been adopted by several researchers [[13-15](#_ENREF_13)]. In addition, Georgiadi-Stefanidi et al. [[16](#_ENREF_16)] have proposed a three-dimensional (3D) FE model for hooked-end fibre pullout from cementitious matrices. The physicochemical bond of the fibre-matrix interface was taken into account by a number of nonlinear spring elements which transfer the bond stresses through a set of equivalent point loads. However, consideration of point loads rather than interfacial bond stresses is not accurate especially in the case of unsymmetrical and/or coarse mesh pattern at the interface. Introduction of additional elements, i.e. spring elements, would also increase the solution time.

In this paper, a general nonlinear 3D finite element model is proposed to analyse the complex fibre-matrix bond-slip behaviour of fibres with varying physical and geometrical properties. The numerical results have been validated using experimental results obtained from the literature [[3](#_ENREF_3), [5](#_ENREF_5), [8](#_ENREF_8), [17](#_ENREF_17)]. The validated model was then employed to simulate pullout from concrete of a newly designed steel fibre. In this study, ANSYS was employed as a modelling platform.

**2. MODELLING OF THE STEEL FIBRE AND CEMENTITIOUS MATRIX**

The 8-noded solid element with three degrees of freedom at each node (i.e. translations in the nodal x, y, and z directions) is used to model the steel fibres. The element has plasticity, stress stiffening, large deflection, and large strain capabilities. Cementitious matrices are modelled using the 8-noded solid brick elements with cracking, crushing and plastic deformation capabilities [[18](#_ENREF_18)].

The von Mises yielding criterion is adopted to simulate the plasticity of the steel fibre. The von Mises yielding criterion was combined with the modified Willam and Warnke failure criterion [[19](#_ENREF_19)] to model the cementitious matrices.

The yield and ultimate stresses (*fy* and *fu*) and the yield and ultimate strains (*ɛy* and *ɛu*) for fibres material were taken from the actual experimental results. The modulus of elasticity and the Poisson’s ratio of the steel material (*Es* and *υs*) were taken as 210 GPa and 0.3 respectively. The uniaxial tensile and compressive strengths (*fctm* and *fck*) and the modulus of elasticity (*Ec*) of cementitious matrix were specified according to those experimentally obtained. However, in some cases where the actual tensile strength and the modulus of elasticity were not available, they were estimated by using **Eqs. 1** and **2** in accordance with Eurocode 2 [[20](#_ENREF_20)]. The Poisson’s ratio of the cementitious material (*υc*) was taken as 0.2.





**Eqs. 3** to **5** [[21](#_ENREF_21)] were used to define the uniaxial stress-strain relationship of the cementitious matrices in compression.







where *σ* and *ɛ* are the stress and strain at any point of the curve. The stress-strain curve requires a first point to define the linear part of the relation which must satisfy the Hooke’s law (**Eq. 5**). In this research, the stress equivalent to 0.3*fck* and its corresponding strain (0.3*fck*/*Ec*) were considered as the elastic limit.

**3. THE FIBRE-MATRIX INTERFACE**

The surface-to-surface contact is used to represent the fibre and matrix surfaces which are in contact (the fibre-matrix interface) where both contact and target surfaces would make up a “Contact Pair”. The contact and target elements have the same geometric characteristics as the inter-connecting fibre and cementitious matrix which are capable of simulating the deformable contact interface.

The physicochemical bond of the fibre-matrix interface is defined by the Coulomb friction model where two contacting surfaces carry shear stresses up to a certain magnitude across their interface before they start sliding relative to each other. An equivalent shear stress (*τ*) was defined as a fraction of the normal contact pressure (*ρ*) at which sliding on the surface begins, as seen in **Eq. 6** and **Fig. 2**.



where *µ* is the coefficient of friction and *c* is the contact cohesion which provides sliding resistance even with zero normal pressure [[22](#_ENREF_22)].

To consider interfacial debonding between the fibre and the surrounding matrix, the friction decay was specified based on **Eq. 7** [[23](#_ENREF_23)].



where *µ* is the frictional coefficient, *µd* and *µs* are the dynamic and static coefficients of friction, respectively, *DC* is the decay coefficient, and *S* represents slip rate (see **Fig. 3**). After debonding, the interface friction is determined by the dynamic frictional coefficient, *µd*, and the decay coefficient (*DC*) determines the transitional coefficients from the static to the dynamic states.

During pullout, certain regions of the fibre-matrix interface might undergo high stress, depending on the interface material properties, could result in the failure of the regions and consequent slippage. Therefore, the maximum contact friction, *τmax*, is introduced in the friction model as seen in **Fig. 2**, so that, irrespective of the magnitude of the normal contact pressure, sliding will occur once the frictional stress reaches this value [[24](#_ENREF_24)]. The sticking/sliding calculations determine when a point transitions from sticking to sliding or vice versa.

The pure penalty method was employed to establish a relationship between the contact and target elements to prevent or limit penetration where the contact compatibility is achieved using a fictitious spring. When contact is detected, the spring would deflect, thus creating an action (contact force) to resist the penetration. The spring stiffness is known as contact or penalty stiffness. The elements can transmit compressive normal and tangential forces but not tensile normal forces and they are free to separate and move away from each other [[24-27](#_ENREF_24)].

**Fig. 4** shows a contact scenario in which a contact element is penetrated into a target element where the contact element consists of a slave node and the target element is represented by a master line connecting nodes 1 and 2. *So* and *S* represent the slave node before and after penetration.

The penetration is resisted by the contact force which is resolved into two components in the normal and tangential directions, i.e. *fn* and *ft*, and are defined by **Eqs. 8** and **9** [[24](#_ENREF_24), [25](#_ENREF_25)].





where *gn* and *gt* are penetrations along the normal and tangential directions and *kn* and *kt* are penalty terms which respectively express the relationship between the contact force and the penetration along the normal and tangential directions.

Once the tangential contact force exceeds the static friction force, sliding will occur, as below.



As seen in **Fig. 5** and mentioned above, the contact and target elements overlay the surfaces of contacting bodies (the steel fibre and matrix), and have the same geometric characteristics as the underlying elements. Therefore, the compatibility is applied to the fibre and matrix surfaces which respectively underlay the contact and target elements.

To derive the contact stiffness matrix, the contact surface is divided into a set of contact elements. The element represents the interaction between the surface nodes of the contact body with the respective element face of the target body. The contact stiffness matrix, *Kc*, is assembled into the stiffness matrix of the contacting bodies (*Kb*) which can be expressed as:



where *u* and *F* are displacement and force vectors respectively [[25](#_ENREF_25), [28](#_ENREF_28)].

The contact stiffness matrix depends on the contact status, whether surfaces are touching or separated, therefore the global stiffness matrix is a nonlinear term [[27](#_ENREF_27)]. In this study, the contact stiffness matrix will be updated at each iteration. Ideally, zero penetration is only possible with an infinite contact stiffness which is numerically impossible with penalty-based methods. However, as long as the penetration value is in the allowable interpenetration range, the results are deemed to be valid. If the penetration is larger than the allowable values, the global solution would be invalid, even though the residual forces and displacements have met the convergence criteria [[24](#_ENREF_24)].

In this research, asymmetric contact pair was selected for the contact modelling where the fibre and matrix surfaces were specified as contact and target surfaces respectively [[24](#_ENREF_24)].

Contact elements are constrained against penetration into the target surface at Gauss integration points as they generally provide more accurate results compared to nodal points (**Fig. 6**) [[24](#_ENREF_24)].

A spherical zone around each detection point of contact elements is used to determine far field open and near field open status known as the “Pinball Region”. If a node on the target surface is within this sphere, it is considered to be in near contact and its relationship to the contact detection point will be monitored more closely. The computational cost of searching for contact depends on the size of the pinball region [[24](#_ENREF_24), [29](#_ENREF_29)].

An allowable penetration value is calculated based on the average depth of each individual contact element in the contact pair. Moreover, normal penalty stiffness, *kn*, is determined based on the Young’s modulus and the size of the underlying elements of the contact surface and tangential penalty stiffness, *kt*, is proportional to the frictional coefficient (*µ*) and normal penalty stiffness (*kn*). However, these default values can be respectively adjusted by penetration tolerance factor (*FTOLN*), and normal and tangential penalty stiffness factors (*FKN* and *FKS*). The pinball region as a sphere of radius 2 times depth of the underlying elements of the contact surface is considered for all FE models.

**4. DEBONDING OF THE FIBRE-MATRIX INTERFACE**

As mentioned in **Section 3**, in order to model debonding at the interface, two frictional coefficients, i.e. static and dynamic coefficients of friction, are introduced in the models. Therefore, before any slip occurs, the frictional force is calculated using the static coefficient (*µs*). After debonding, the interface friction is determined by using the dynamic frictional coefficient (*µd*). However according to **Eq. 6**, the frictional stress, *µρ*, is available just by the presence of normal contact pressure. Since at the beginning of the analysis, there is no normal pressure to activate friction, an initial normal pressure on contact elements (the fibre surface at the interface) is required, before any slip occurs. To this end, the tunnel of the matrix surrounding the fibre is modelled with a diameter smaller than that of the fibre wire. Then the model is analysed with the presence of initial penetration. Since the interpenetration is detected, the contact pressure will be applied on the contact surface, i.e. fibre surface at the interface (see **Fig. 7**), to enforce the contact compatibility. Afterwards, incremental load is applied at the free end of the fibre in a displacement-controlled manner to capture bond-slip response. In this respect, dynamic coefficient of friction (*µd*) and parameter *FACT*, which is the ratio of static to dynamic coefficients of friction (*µs*/*µd*) are introduced into the models.

**5. VALIDATION OF THE NUMERICAL MODEL**

Experimental pullout data of various steel fibres including straight, hooked-end, crimped, and twisted which were tested in the single-sided manner are employed to validate the proposed numerical model. Except for matrices dimensions, all the geometrical and material properties of the FE models are the same as those of the experimental specimens. Regarding the matrices, irrespective of the specimens dimensions, cylinders with a diameter 20 to 30 times of the wires diameter, depending on the matrices strength, with heights which are equal to the fibres embedded lengths, are modelled as the cementitious matrix surrounding the fibres. In order to consider debonding in the nonlinear analyses, the tunnel of matrices surrounding the fibres is modelled 0.001 times of wire diameter smaller (see **Section 4**).

Taking advantage of symmetry in geometry and loading of some specimens, a fraction of the specimens with consideration of proper boundary conditions were modelled. The contact parameters including *µd*, *FACT*, *c*, *FKN*, *FKS*, *FTOLN*, and *τmax* were adjusted for each model in accordance with the respective experimental data.

Considering the frictional contact problem produces non-symmetric stiffnesses, the unsymmetric solution is incorporated in the Newton-Raphson method to improve the computational efficiency [[30](#_ENREF_30)].

**5.1. STRAIGHT STEEL FIBRE**

Pullout testing data of a straight fibre given by Naaman and Najm [[5](#_ENREF_5)] are employed. The geometrical and material properties of the fibre are provided in **Table 1**. The cylinder compressive strength of the cementitious matrix is 60 MPa. Due to symmetry in geometry and loading, a quarter of the specimen is modelled, as shown in **Fig. 8**. The contact parameters used for this model are presented in **Table 2**.

**Fig. 9** shows the comparison between the experimental and numerical bond-slip responses. From the figure, agreement between the experimental and numerical results is good and the model predicts the peak load as well as the pre-peak, debonding, and post-peak responses well.

Average interfacial shear stresses at various values of displacement, derived from the pullout load-slip relationship along with the respective frictional contact stress (shear stress at the interface) contours are illustrated in **Fig. 10**. Agreement between the results is good which verifies a relatively constant shear stress at the interface after debonding [[9](#_ENREF_9)].

**Figs. 11(a)** and **(b)** show the von Mises stress contours of the fibre at slips equal 0.02 mm, corresponding to the maximum pullout load, and 2 mm. The normalized von Mises stress at three levels along the fibre length, bottom, middle, and top of the fibre, versus slip are illustrated in **Fig. 12**. The results indicate that the entire length of the fibre responds similarly to the pullout process by suffering considerable stress decay at the displacement corresponding to the debonding.

It is well known that straight steel fibres are normally characterized by bond-slip softening behaviour after debonding which implies a constant shear stress at the fibre-matrix interface. The bond strength of such fibres embedded in concrete is generally small and mostly frictional in nature [[10](#_ENREF_10)]. However, by tailoring the matrix composition and surface coating of the fibres, it is possible to achieve slip-hardening behaviour. The key feature of such performance is that when the fibre is pulled out from a cementitious matrix, its resistance increases with increase in slip (the more it pulls out, the harder it resists). This can be attributed to the intensive damage of the fibre surface by matrix particles during the pullout process which leads to an increase in the interface friction and consequent improvement in bond behaviour [[3](#_ENREF_3), [9](#_ENREF_9)]. Therefore, during pullout depending on the interface properties, matrix packing density and fibre configuration, the coefficient of friction between the fibre and matrix could increase.

**5.2. HOOKED-END STEEL FIBRE**

In this section, a hooked-end steel fibre pullout, tested by Jamee and Alaee [[17](#_ENREF_17)] is investigated. The geometrical properties of the fibre are shown in **Fig. 13**. The yield and ultimate stresses of the fibre material are 650 MPa and 800 MPa, and the cylindrical compressive strength and the elastic modulus of the cementitious matrix are 60 MPa and 41 GPa respectively. **Fig. 14** shows FE model of the hooked-end steel fibre embedded in the matrix. The contact parameters used for this model are presented in **Table 3**.

The bond-slip curves of the numerical model along with the experimental specimen are shown in **Fig. 15** in which good agreement is observed; in particular, the model properly captures the two expected load drops during the pullout process.

In **Fig. 16**, the von Mises stress contour of the fibre at slip equals 2 mm which corresponds to the initiation of the first drop in the load-slip curve (see **Fig. 15**) is illustrated. As can be seen in the figure, as a result of the fibre tendency to change its shape during passing through the two bent regions, i.e. fibre straightening, plastic hinges localized in the parts are formed.

Average interfacial shear stress, i.e. pullout load divided by the respective fibre-matrix interface value, versus slip is shown in **Fig. 17**. The graph consists of three distinctive parts, a steep initial ascending portion, followed by a smoothly descending branch up to the slip equals 12.5 mm and then an ascending branch up to the complete pullout. As observed in **Fig. 15**, after slip around 12.5 mm, even though the interface is decreasing, there is only marginal load decay which leads to the continuous increase of the average interfacial shear stress after this slip, as seen in **Fig. 17**. It might be mainly attributed to the existing permanent deformation at the fibre end after straightening, where the fibre experiences higher levels of frictional contact stress compared to the other interface areas (see **Figs. 18(a)** and **(b)**).

The normalized von Mises stress at three cross sections located at bottom and top of the fibre as well as middle of the end hook, are plotted against displacement in **Fig. 19**. Regarding the bottom level, as it is observed, there are two peak points, each one followed by a trough point, as a result of the straightening, and then relatively high levels of stress compared to the two other regions. These levels of stress are indicative of the dominant contribution of the fibre end at the pullout load (see **Fig. 18(b)**).

**5.3. CRIMPED STEEL FIBRE**

Pullout testing data of a crimped steel fibre with an embedded length of 15 mm [[8](#_ENREF_8)] are employed for further validation. The geometrical properties of the fibre are shown in **Fig. 20**. The yield and ultimate stresses of the fibre material are 1450 MPa and 1800 MPa, respectively and the cylinder compressive strength of the cementitious matrix is 39 MPa. The considered contact parameters for the model are provided in **Table 2**.

**Fig. 21** shows the comparison of the experimental and numerical bond-slip curves where good agreement is found and the model can reasonably predicts the bond-slip behaviour of the fibre throughout the pullout process.

**Fig. 22** shows the von Mises stress contour of the fibre at a slip equal to 0.3 mm, which corresponds to the complete interface debonding. The normalized von Mises stress at three levels, bottom, middle and top of the fibre versus displacement is shown in **Fig. 23**. The results indicate that during pullout the embedded length of the fibre is subjected to repetitive bending and straightening, which result in improving the interface friction and consequently an increase of the pullout load.

**5.4. TWISTED STEEL FIBRE**

The pullout performance of steel fibres directly depend on the physicochemical bond and mechanical anchorage. While mechanical anchorage just depends on the fibre geometry, physicochemical bond relies upon the collaborative contribution of the fibre-matrix interface, matrix packing density, and fibre geometry [[3](#_ENREF_3)]. Just after full debonding and initiation of slippage, depending on the fibre-matrix interface, matrix packing density and the fibre shape, the fibre surface experiences continuous damage, scratching, and delamination which leads to gradual increase in the interfacial friction [[3](#_ENREF_3), [9](#_ENREF_9)]. However, in most cases the combined effect of the aforementioned factors just results in a marginal increase of the interfacial friction. The commercially-available twisted fibres are engineered in terms of shape, size, and mechanical properties, as well as compatibility with a given matrix to achieve the gradual increase of the interface friction, i.e. strain hardening behaviour [[10](#_ENREF_10)].

In this section, as the final step for validation of the proposed numerical model, a rectangular twisted fibre embedded in ultra-high performance concrete tested by Wille and Naaman [[3](#_ENREF_3)] is employed in which the fibre shows strain hardening behaviour. The total and embedded lengths of the fibre are 13 mm and 6.5 mm, respectively and the cross-sectional dimensions are 0.24 mm by 0.3 mm. Moreover, the fibre pitch, i.e., the length of one full (360-degree) twist around the fibre axis, is 8 mm. The yield and ultimate stresses of the fibre material are 3100 MPa and 3400 MPa, respectively and the cylinder compressive strength of the cementitious matrix is 100 MPa.

The adjusted contact parameters for the model are provided in **Table 2**. For coefficient of friction, a range which is assumed to be linearly correlated to the fibre slip is considered, as seen in **Fig. 24**.

In **Fig. 25** and **Fig. 26**, experimental and numerical pullout load and interfacial shear stress curves, both versus slip are provided where reasonable correlations are achieved. **Fig. 27** shows the von Mises stress contours of the fibre at slips equal 0.015 mm, 1.35 mm, and 6 mm which respectively correspond to the debonding load, the peak load, and the complete pullout from the matrix. In addition, the normalized von Mises stress at five levels along the fibre length, 0.5 mm, 2 mm, 3 mm, 4 mm, and 6 mm from the bottom of the fibre is plotted against displacement in **Fig. 28**. The results are indicative of the effectiveness of the twist along the fibre as the mechanical anchorage which leads to higher performance and favourable stress distribution along the fibre.

**6. PULLOUT SIMULATION OF A STEEL FIBRE WITH A NEW SHAPE**

As a practical application of the validated numerical model, the pullout mechanism of a steel fibre with a new shape in concrete is simulated. The hooked-end fibre validated in **Section 5.2**, is considered as the basis of the new fibre geometry and material properties. A new fibre with two hooks at each end has been developed. The dimensions of the new fibre shape are shown in **Fig. 29**. The considered contact parameter for this fibre are the same as those of the hooked-end fibre (see **Table 2**).

Since the new fibre benefits from a stronger mechanical anchorage compared to the hooked-end fibre, as it is expected that the fibre will fracture during the pullout process. A few simulations were performed to optimise the fibre material, so that the complete fibre withdrawal was achieved. The optimised yield and ultimate stress of the fibre material are found to be 1600 MPa and 2000 MPa, respectively.

The pullout load-slip curves of the newly-designed fibre and the hooked-end fibre are illustrated in **Fig. 30**. From the graph, it is evident that the double hooks enhances the bond-slip response of the fibre. The maximum pullout load (*Pmax*) and total dissipated pullout energy (*Et*) of the fibres are provided in **Table 3**. The results are indicative of the superior performance of the newly designed steel fibre compared to the hooked-end fibre.

**7. CONCLUSION**

This paper presents a general nonlinear finite element model for steel fibre pullout from concrete. The following conclusions can be drawn:

1) In the numerical model, contact elements were employed to simulate the fibre-matrix interaction where cohesion and frictional coefficients were introduced to consider physicochemical bond and the fibre-matrix interface friction. To activate the interface debonding, a technique is proposed where the tunnel of the matrix surrounding the fibre is modelled with a diameter smaller than that of the fibre wire. In addition, geometric and material nonlinearity are considered in the model. The findings of this research verify the accuracy of the numerical model to simulate the steel fibre pullout mechanism, where good agreement between the experimental and numerical results was achieved.

2) The numerical model delivers a higher level of information about fibre pullout mechanism, e.g. bond-shear-stress-slip relationship and stress distribution over the fibre, which at present is not possible from experimental tests.

3) The conventional fibre development approach is a repetitive process including fibre design, manufacturing and experimental testing which are time consuming and not cost effective. In the future, designing and optimisation of new types of steel fibre could be done with the help of the proposed numerical model which will considerably decrease the number of experiments.

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