PhD in Music

Gödel, Plato, Xenakis: Eternal Forms, Numbers and Rules–
A Portfolio of Compositions

A commentary to support a portfolio of compositions submitted in accordance with the requirements of the University of Liverpool for the degree of PhD in Music in the Faculty of Humanities

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List of Scores and Recordings

***Guitar Sonata I* for classical guitar** 2008-2015 Circa 10′

Recorded at Ness, Wirral, April 2017 by Matthew Sallis

CD1 Track 1,2,3

***Two Explorations of a Chord Sequence* for solo electric guitar** 2010-2015 Circa 7′

Recorded at Ness, Wirral, May 2017 by Matthew Sallis

CD1 Track 4,5

***String Quartet I First and Second Movements* for violin I, violin II, viola, cello** 2008-2015 Circa 8′

Recorded at Liverpool Hope University Creative Campus, May 2015 and May 2017 by RLPO[1] string players

CD1 Track 6,7

***Flumina* for chamber orchestra** 2012-2015 Circa 5′

Recorded at Herne Hill, London, December 2014 by LCCO[2]

CD1 Track 8

***Concertino* for chamber orchestra** 2012-2015 Circa 5′

Recorded at Herne Hill, London, December 2014 by LCCO

CD1 Track 9

***Mutation of Mozart's Jupiter Symphony* for double quintet** 2010-2015 Circa 12′

Recorded at Liverpool Hope University Creative Campus, May 2015 by players of RLPO

CD1 Track 10

***Mutation of Dowland's Midnight* for clarinet, violin, cello, piano** 2014 Circa 4′

Recorded at The Friary, Liverpool, May 2014 by RLPO Messiaen Quartet

CD1 Track 11

This Day Dawns for solo voice SATB 2015 Circa 5′

Recorded at Liverpool Hope University Creative Campus, May 2015 by The Sixteen

CD1 Track 12

[1] Royal Liverpool Philharmonic Orchestra
[2] London Contemporary Chamber Orchestra

***Mutation of Kyrie* for flute, clarinet doubling bass clarinet, vibraphone, violin, cello, piano** 2014-2015 Circa 7'

Recorded at Victoria Rooms, Bristol University, January 2015 by The Bristol Ensemble

CD2 Track 1

***Mutation of Gloria* for flute, oboe, clarinet, bassoon, trumpet, horn, trombone, double-bass** 2015 Circa 7'

Recorded at Liverpool Hope University Creative Campus, May 2015 by players of RLPO

CD2 Track 2

Mutation of Sanctus for Chamber Orchestra 2016 Circa 5'

Recorded at Herne Hill, London, December 2016 by LCCO

CD2 Track 3

***Mutation of Benedictus* for violin I, violin II, viola, cello** 2015-2015 Circa 5'

Recorded at Liverpool Hope University Creative Campus, March 2016 by RLPO string players

CD2 Track 4

***Mutation of Agnus Dei* for flute, clarinet, horn, violin, cello, piano** 2016 Circa 6'

Recorded at Liverpool Hope University Creative Campus, June 2016 by players of RLPO

CD2 Track 5

***The Universe Began (Arrangement of Extracts)* for flute, clarinet, horn, violin, cello, piano** 2016-2017 Circa 6'

Recorded at Liverpool Hope University Creative Campus, May 2017 by players of RLPO

CD2 Track 6

***The Laws of Science (Arrangement of Extracts)* for clarinet, violin, cello, piano** 2016 Circa 7'30"

Recorded at Liverpool Hope University Creative Campus, December 2016 by RLPO Messiaen Quartet

CD2 Track 7

List of Papers

Outside Time Structures in Music

Short paper at Composers Forum at Liverpool Hope University Creative Campus February 2009.

Rhythm and Absolute Time

20-minute paper with questions and answers at Theory and Analysis Graduate Students Conference 2010 Bangor University Music Department April 2010.

Wind Piece I Tonality and Form

Short Paper to Composers Forum at Liverpool Hope University Creative Campus December 2010.

Compositional Techniques

Short Paper to PGR[3] Music Forum at Liverpool Hope University Creative Campus March 2012.

Towards a Universal Theory of Music from ideas of Xenakis' Symbolic Music

20-minute paper with questions and answers at Northwest Music Exchange Conference at Bangor Music Department March 2013 and Postgraduate Symposium at Liverpool Hope University Creative Campus April 2013.

Composing Post-Modern, Post-Tonal Music

20-minute paper with questions and answers at Liverpool Hope University PGR Conference, Hope Park December 2013.

Gödel, Plato, Xenakis, Symbolic Music and Numbers

20-minute paper with questions and answers at Royal Musical Association Music and Mathematics Study Day, Leeds University April 2014 and Postgraduate Symposium at Liverpool Hope University Creative Campus April 2014.

Abstract Musical Theory, Practical Application

20-minute paper with questions and answers at Liverpool Hope University PGR Conference, Hope Park December 2014.

[3] Postgraduate research

Abstract

This portfolio contains eight compositions for solo guitar, string quartet, small ensemble and chamber orchestra. There are ten scores in total, two additional pieces are submitted as appendices. Most of the music has been realised acoustically through workshops and performances, the recordings of these are included on CD.

There is a bound document explaining the processes, concepts and philosophies for each piece. This written commentary describes how the pieces were composed from individual strands of music, independent parts combined in a polyphonic style. A study of the rules of species counterpoint in Fux's *Gradus Ad Parnassum* inspired the first compositions; a modified set of rules, developed from the writings of Fux controlled initial pitch-organisation. This set of rules underwent modifications informed by innovations introduced in subsequent compositions.

The theories discussed in Xenakis' *Symbolic Music* were utilised to generate pre-compositional pitch-sets and duration-sets. The pitch-sets were organised vertically to produce chords. These developments led to considerations of form, proportion and structure in the research. Changes in tonality with an associated contrast of chords articulated formal sections, bearing similarity to the use of the contrast of key found in traditional tonality. The pre-composed duration-sets gave more impetus and rhythmic momentum to the music and expanded the overall rhythmic content.

Speculation on the ideas of musical Platonism raised two questions: Which musical elements might be considered universal? Is the act of composition invention or discovery? Furthermore, does a composer discover a copy of an eternal Platonic Form? Contemplation upon the metaphysics of Plato and Platonic Forms inspired an extension of Xenakis' *Symbolic Music* using numbers, specifically integers, similar in construction to Gödel Numbers. Once music has been encoded symbolically as an integer the fundamental elements of the music may be manipulated, to produce an alternative copy of an eternal Platonic Form. Compositions by John Dowland and Palestrina were encoded symbolically and the encoding was used to control the rhythm, pitch and harmonic structure of new compositions.

Declaration

No portion of the work referred to within this commentary and portfolio has been submitted in support of an application for another degree or qualification of this or any other university or institute of learning.

Copyright Statement

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I am grateful for the financial and moral support of my parents John and Pauline Sallis; and Julie Hine.

The players and singers of the RLPO, LCCO, The Sixteen and The Bristol Ensemble deserve praise and gratitude for their rehearsing, performing and recording of the music on the audio CDs.

Likewise, the conducting skills of Professor Stephen Pratt from Liverpool Hope University, Alan Taylor of the LCCO and Neil Farwell from The Bristol Ensemble are much appreciated.

Finally, I would like to thank the staff and students at Liverpool Hope University for all their help and encouragement. The composition workshops organised by Liverpool Hope University were particularly useful.

Chapter 1. Introduction

The expression: "The whole is greater than the sum of its parts" applies particularly well to the art of polyphony. A monody offers linear melodic formulae usually outlining a mode; however, if it combines with one or more well-constructed contrapuntal parts, the music acquires vertical pitch-organisation; a sharper focused rhythm; greater motivic interest.

The music submitted in the portfolio and appendix adopts this approach to composition, the combination of two or more horizontal pitch-structures to achieve linear and vertical pitch-organisation, a dynamic interplay of rhythm and motif.

In this way, the compositions submitted may be viewed as absolute music, with no extra-musical or narrative elements to them.

This commentary outlines the reproducible compositional techniques used to combine two or more independent horizontal pitch-structures to produce coherent, post-tonal music[4].

[4] A term used by the music theorist Josef Straus and others to describe pitch-organisation not based on traditional tonality. Joseph Straus, *Introduction to Post-Tonal Theory,* Upper Saddle River, N.J. 1990.

Chapter 2. Fundamental Philosophy, Question of Research

The initial, fundamental question for this research portfolio was: What would modern music informed by the polyphonic style of Palestrina sound like[5] ? Is it possible to rework the rules of species counterpoint found in Fux's treatise *Gradus Ad Parnassum*[6] to inform modern, post-tonal music? Furthermore, the intention from the beginning was to subdivide rhythms and metre using a denominator of 2; and a continuously changing metre would be a feature of the musical style. Would it be possible to modify the rules that apply to the metric structure of Palestrina's polyphony, to a metric scheme that continuously changes and expands or contracts in relation to the number 2?

A significant portion of Fux's rules of species counterpoint deals with the treatment of dissonant intervals and the rhythmic treatment of dissonance. What relevance could such rules have to post-tonal music in which dissonance is no longer an absolute concept and a regular metre cannot be assumed?

Thus, the philosophy informing all the pieces submitted in the portfolio and appendix is the search for a set of rules of counterpoint that will generate C21st, post-tonal music. These rules must have a clear definition of what is dissonant, the dissonance must be controlled rhythmically; and the subdivision of metre should be considered regarding this rhythmic control.

The question of research becomes: Is it possible to find a coherent set of rules to control the rhythmic placement of dissonance in C21st polyphonic music with a continuously changing metre?

[5] This question is regarding the style of Palestrina outlined in Fux's *Gradus* rather than a general consideration of the influence of polyphonic composition in the Western musical tradition.
[6] Fux, *Gradus Ad Parnassum,* 1725. Trans. Steps to Parnassus, a region in Ancient Greece sacred to Apollo and the Muses.

Chapter 3. Journey of Research

Initially the rules of species counterpoint in Fux's *Gradus* and other treatises were studied. These rules were adapted to control the rhythmic treatment of dissonant intervals in the pitch-organisation. Dissatisfaction with a rule-based interpretation of Palestrina's style led to a harmonic analysis of his motet *Stabat Mater* (c.1590) to examine the use of triads in an actual composition by Palestrina. A notable stylistic deviation from the rule-based style described in Fux's *Gradus* is Palestrina's use of expressive, triadic harmony. Further analysis of Palestrina's music was undertaken after the research had examined ways of symbolically encoding music. Extracts of Palestrina's motets *Stabat Mater* and *Super Flumina Babylonis,* were encoded in sequences labelled as musical DNA by the author.

Guitar Sonata I and *String Quartet I,* submitted in the portfolio, used the rules derived from Fux's *Gradus* to inform a C21st polyphonic style. Reflections upon this polyphonic style revealed a necessity to achieve greater clarity and consistency in the pitch-organisation. The chapter *Symbolic Music* from Xenakis' book *Formalized Music*[7] provided a model from which to produce pre-compositional pitch and duration sets to address the issues arising from the composition of *Guitar Sonata I* and *String Quartet I*. Regarding pitch-sets, four modes were generated from considerations of the classification of consonant and dissonant intervals described in Fux's *Gradus.* Musical decisions regarding durations were informed by Xenakis' concept of *temporal structures* relying on musical time rather than absolute, metric time[8]. Ideas on musical time, found in the writings of Husserl[9], gave further insight. *Mutation of Mozart's Jupiter Symphony* and *Two Explorations of a Chord Sequence,* submitted in the portfolio and appendix, utilised the modes derived from the classification of consonant and dissonant intervals and a new approach to *temporal structures.*

Using pre-existing music to inform new compositions is a strategy that has been employed by several composers. The American theorist Joseph Straus describes the reworking of previous music as an anxiety of influence and anxiety of style in his book *Remaking the Past*[10]. Straus' book describes how neo-Classical composers such as Stravinsky and Bartók remake tonality, triads and sonata form in their music. The concept of *remaking the past* regarding triads, tonality, sonata form and the use of pre-existing music to inform new compositions became of central importance to the

[7] Iannis Xenakis, *Formalized Music*, Stuyvesant NY, 1992, pg. 155
[8] Absolute time measured by a clock as opposed to a variable, psychological time.
[9] German philosopher, 1859-1938, who developed the philosophy of Phenomenology.
[10] Joseph Straus, *Remaking the Past,* Cambridge: Harvard Uni. Press, 1990

research. The modes used in *Mutation of Mozart's Jupiter Symphony* and *Two Explorations of a Chord Sequence* were used to generate a pitch-organisation based on tri-note chords[11]. This pitch-organisation informed the composition of *Flumina* and *Concertino* submitted in the portfolio. These two compositions *remake* tonality, triads and *Flumina remakes* sonata form.

Xenakis' reduction of pitch and duration sets to abstract entities existing independently to metric time implies these pre-compositional sets might be regarded as eternal, Platonic Forms[12]; in particular, Xenakis' use of numbers to model musical processes seemed Platonic in its intentions. A critique of Xenakis' use of group theory and vectors suggested an ideal, eternal Platonic model in which all music is built from quanta of pitch and duration. This led to ideas of how to encode pitch-organisation symbolically.

Xenakis' composition *Herma*[13] applied Boolean functions to control the large scale formal structures in the music. An extension of Xenakis' use of Boolean functions[14] produced symbolical encodings for medium[15] and large[16] scale musical structures. Musical Platonism was extended beyond the idea of an external Form of pitch and duration quanta, into a universal music theory in which all music was regarded as pitch and duration ratio sets unfolding in musical time. This extension of musical Platonism is clearly derived from Xenakis' *Symbolic Music* which utilises the concept of time-ordered sets which may be expressed as vectors; and thus, provides a model to encode any piece of music symbolically. The technique of Gödel Numbering was utilised to encode time-ordered pitch and duration sets as an integer, named as a *musical integer* by the author.

Musical integers informed new compositions from past music. *Mutation of Dowland's Midnight, Mutation of Missa Papae Marcelli* and *Evolution of the Cosmos,* submitted in the portfolio, used *musical integers* to re-compose music by John Dowland and Palestrina. The symbolic encoding in a *musical integer* was regarded as being in a direct relationship to the eternal, Platonic Form of the music. The *musical integer* may be manipulated mathematically into a copy of the Form. The Form controls all elements of pitch-organisation, duration and medium and large scale musical structures in the copy. The model and process of encoding music as an integer unites all the disparate theories and philosophies studied in the research. The rule-based polyphony of Palestrina; the *Symbolic Music* of Xenakis; musical Platonism; and remaking the past.

[11] Tri-note chords simply means a chord built from three distinct pitches and does not imply the chord contains a tri-tone.
[12] In philosophical writings Forms usually begins with a capital letter
[13] Iannis Xenakis, *Herma,* n.p. B&H, 1962
[14] Propositions in logic, built from set operations union, intersection and complementation.
[15] i.e. Chord sequences, musical phrases
[16] I.e. Formal sections, the form (musical not Platonic) of the entire composition

Chapter 4. Portfolio in Context

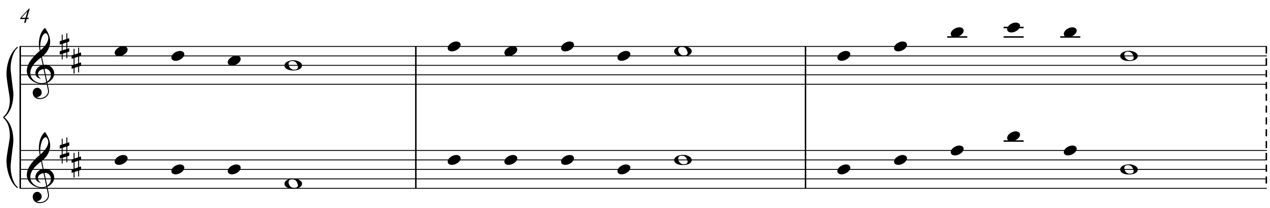
The defining techniques in the compositional methodology outlined in this commentary may be revealed by a comparison of similar approaches to writing music by other modern composers. Three composers have been chosen that utilise compositional techniques similar to those outlined in this commentary, Pärt, Schnittke, and Maxwell Davies. In addition, the techniques of Messiaen and Berio are briefly discussed.

The Estonian Composer Arvo Pärt (b. 1935) studied techniques from the Medieval and Renaissance era; his *tintinnabuli* style, in part, emerged from this study.

> "I work with very few elements - with one voice, with two voices. I build with the most primitive materials - with the triad, with one specific tonality."
> (Rodda RE, liner notes for *Fratres*, 1995 Telarc CD-80387)

Pärt's style utilises a polyphony in which one part presents the notes of a triad while another part moves melodically against this part.

Ex.1 Part: *Für Alina* b.4-7



Ex.1 illustrates the *tintinnabuli* style with an extract from the solo piano piece *Für Alina*. The left-hand of the piano plays notes from a B minor triad, the right-hand weaves a melodic line against the left-hand part.

To arrive at his style Pärt studied the music of Gregorian chant, the Notre Dame School, Machaut, Obrecht, Ockeghem, Josquin, Palestrina and Victoria. (McCarthy J, Pärt A, An interview with Arvo Pärt, *The Musical Times*, Vol. 130, No. 1753, (Mar. 1989)). The simple style shown in Example 1 is extended in pieces such as *Arbos*, *Stabat Mater*, *Festina lente*, *Cantus in Memorium Benjamin Britten;* with the use of iso-rhythms and mensuration canon (Clarke D, Parting Glances, *The Musical Times*, Vol. 134, No. 1810, (Dec. 1993)).

There is common ground between the style of Pärt and the music in the portfolio and appendix:

1. The *remaking* of polyphony in C20th and C21st music.
2. The use of triads.
3. The study of the music of Palestrina to inform new music.

However, there are fundamental differences of approach:

1. Pärt's *tintinnabuli* style uses one triad or a limited number of triads to produce either a consonant sound or a dissonant *tonal* sound. The music in the portfolio and appendix makes use of *harmo-rhythmic structures*[17] to control pitch-organisation.
2. Pärt's music does not seem to make systematic use of suspensions and passing notes, however, in pieces such as *Cantus in Memorium Benjamin Britten* he follows a strict set of rules in regard to iso-rhythms and canon. Rules to regulate dissonance is a fundamental feature of the *harmo-rhythmic structures* used in the music in the portfolio and appendix.
3. Pärt uses static, triadic harmony in his *tintinnabuli* style meaning that a sense of tonality is used for its sonority rather than a means to articulate formal structures, for example, *Cantus in Memorium Benjamin Britten* remains in A minor throughout. This commentary outlines the strategies used to generate an *artificial tonality* that provides contrast between formal structures.

Alfred Schnittke (1934-1998), a Soviet and Russian composer, used quotations of past music and *remade* past styles in his compositions. In his First Symphony:

> "Music by Beethoven, Haydn, Grieg, Chopin, Tchaikovsky, J. Strauss is quoted and brutally interrupted and transmuted."
>
> (Moody I, The Music of Alfred Schnittke, *Tempo,* CUP, No. 168, Mar. 1989)

Schnittke's *String Quartet 3* quotes Orlando di Lasso *Stabat Mater;* Beethoven's *String Quartet 16* (op. 135); and a monogram DSCH from the name: Dmitri Shostakovich (Schnittke, A. *String Quartet 3*, n.p. UE, 1983, b. 1-8).
Schnittke's style is often referred to as *polystylistic* or to exhibit *polystylism*:

> "More recent works have proved "polystylism" to be an efficient generator of that kind of alienation, expressed in ironic manipulation of various stylistic elements, which Schnittke has taken over from Shostakovich."
>
> (Moody I, The Music of Alfred Schnittke, *Tempo,* CUP, No. 168, Mar. 1989)

Schnittke's music moves from one identifiable style to another, often in an abrupt, juxtaposed fashion. Regarding the use of several musical elements within one work the score preface to his *Concerto Grosso 1* writes of:

> "formulas and forms of Baroque music; free chromaticism and micro-intervals; and banal popular music which enters as it were from the outside with a disruptive effect."
>
> (Schnittke, A. *Concerto Grosso 1*, n.p. B&H, 1977)

[17] A term invented by the author to describe musical structures where fixed rules rhythmically control pitch-organisation.

The defining feature of the use of *polystylism* is to offer past music in a largely unaltered fashion such that it is heard as post-modern irony in contrast to the modern styles.

> "It is only in the C20th that polystylism has become a pronounced feature… Before this time, a composer would usually ensure that a foreign style was seamlessly integrated into his own."
>
> (Webb J, Schnittke in Context, *Tempo,* CUP, No. 182, Sep. 1992)

Schnittke's First and Third Symphonies, Third Violin Concerto and his *Cello Sonata* are works that employ polystylism (Moody I, The Music of Alfred Schnittke, *Tempo*, CUP, No. 168, Mar. 1989. Kleinmann, J. *Polystylistic Features of Schnittke's Cello Sonata (1978)*, Doctoral dissertation, UNT, 2010).

The compositions in the portfolio and appendix share certain characteristics with the style of Schnittke:

1. The use of direct or manipulated quotations in *Mutation of Dowland's Midnight*, *Flumina* and *Mutation of Mozart's Jupiter Symphony*.
2. A combination of distinct styles, the compositions from *Flumina* onwards combined the polyphony of Palestrina with a tonality informed by Mozart.

The differences of approach to that of Schnittke are as follows:

1. Schnittke's use of quotation and polystylism could be viewed as post-modern irony, past music is presented in a sarcastic, mocking fashion. (Moody, I. *The Music of Alfred Schnittke*, Tempo CUP, No. 168, Mar. 1989). The *remaking* of past music described in this commentary is reverential, past music is viewed as an eternal Form to be *mutated* into a new *copy* or *imitation*.
2. This commentary outlines how the styles of Palestrina and Mozart were combined in new music, the intention is not to contrast the styles but to integrate them.

The British composer Peter Maxwell Davies (1934-2016) used mathematics to transform pre-existing music. This transformation was achieved using a device called the magic square:

> "A square array of numbers where each row, column and main diagonal sums up to the same amount … An n x n matrix is a square array of numbers with n rows and n columns."
>
> (Roberts, G.E. *Composing with Numbers*, Math, Music and Identity, Monserrat Seminar, 2015)

Maxwell Davies' composition *A Mirror of Whitening Light* (1977) uses an 8 x 8 magic square of Mercury, for an n = 8 magic square the rows, columns and main diagonals sum to a value of 260 (Ibid.). Eight notes were taken from the plainchant *Veni Sancte Spiritus*:

G E F D F♯ A G♯ C – sequence VSS

Sequence VSS is transposed to a starting note of each of pitches in the sequence to give an 8 x 8 array called a *transposition square*:

Table1 Transposition Square

	P0	P9	Pt	P7	Pe	P2	P1	P5
P0	1 **G**	2 **E**	3 **F**	4 **D**	5 **F♯**	6 **A**	7 **G♯**	8 **C**
P9	9 **E**	10 **C♯**	11 **D**	12 **B**	13 **D♯**	14 **F♯**	15 **F**	16 **A**
Pt	17 **F**	18 **D**	19 **E♭**	20 **C**	21 **E**	22 **G**	23 **G♭**	24 **B♭**
P7	25 **D**	26 **B**	27 **C**	28 **A**	29 **C♯**	30 **E**	31 **E♭**	32 **G**
Pe	33 **F♯**	34 **D♯**	35 **E**	36 **C♯**	37 **F**	38 **G♯**	39 **G**	40 **B**
P2	41 **A**	42 **F♯**	43 **G**	44 **E**	45 **G♯**	46 **B**	47 **B♭**	48 **D**
Pe	49 **G♯**	50 **F**	51 **F♯**	52 **D♯**	53 **G**	54 **A♯**	55 **A**	56 **C♯**
P5	57 **C**	58 **A**	59 **A♯**	60 **G**	61 **B**	62 **D**	63 **C♯**	64 **F**

Thus, every row and column contains a prime or transposed version of sequence VSS. However, the main diagonals do not contain a version of the sequence VSS. The pitches in the *transposition square* are numbered from 1-64, this numbering allows the pitches of the transposition square to be inserted into the 8 x 8 square of Mercury. For instance, the numbers 1, 22, 32, 39, 43, 53 and 60 correspond to the pitch **G**. The first row of the square of Mercury has the numbers that sum to 260:

8 58 59 5 4 62 63 1

Substituting the corresponding pitches from the *transposition square* gives a first row of:

C A A♯ F♯ D D C♯ G

Proceeding in this fashion for the whole square of Mercury gives Maxwell Davies a pre-compositional pitch resource to work from (Ibid.).

Maxwell Davies used a similar approach to compose his Third Symphony using a fragment of plainchant *Sante Michael Archangele* transformed by a *transposition square* and the square of Mercury (Jones, N. The Precompositional Process in Maxwell-Davies Third Symphony, *Tempo,* CUP, No. 204, Apr. 1998).

The use of integers to *remake* past music, outlined in this commentary, has advantages over other uses of mathematics to transform or generate musical elements:

1. The process is essentially a musical process and allows the composer a mixture of flexibility and control over the outcome of the transformation.
2. Integers encoding music have the property of behaving simultaneously as numbers and as a one-to-one encoding of music. Due to this property, any mathematical manipulation of the integer will have a direct musical outcome.
3. The use of integers allows for a symbolic rather than literal transformation (see 24xii. pg.172).
4. The use of integers to *remake* music has a meta-level aesthetic: the integer encodes the Platonic *Form* of a well-regarded pre-existing composition. The *remaking* process is viewed as producing an alternative *copy* of the *Form* of the pre-existing music.

The French composer Olivier Messiaen (1908-1992) used his *modes of limited transposition* and the addition or subtraction of durations to modify pre-existing music. In addition to plainchant and birdsong, Messiaen modified music by Debussy, Dukas, Mussorgsky and Wagner amongst others. Page 13-14 of Messiaen's *The Technique of My Musical Language*[18] illustrates with musical examples how he modified a fragment of Mussorgsky's *Boris Godounow,* Grieg's *La Chanson de Solvieg* and Debussy's *Image* for use in his own compositions. This bears similarity with the use of *musical integers* to *mutate* the music of Dowland and Palestrina in the compositions *Mutation of Dowland's Midnight, Mutation of Missa Papae Marcelli* and *Evolution of the Cosmos,* submitted in the portfolio.

Rendering[19] by the Italian composer Luciano Berio (1925-2003) is a re-composition and orchestration of Schubert's sketches for an unfinished Tenth Symphony. This work juxtaposes Berio's orchestration of the sketches by Schubert with newly composed material by Berio.

[18] Olivier Messiaen, *The Technique of my Musical Language,* trans. Satterfield J, Paris, 1956

[19] Luciano Berio, Schubert F, *Rendering,* n.p. UE, 1990

"*Rendering* with its dual authorship is intended as a restoration of these sketches, it is not a completion nor a reconstruction. This restoration is made along the lines of the modern restoration of frescoes that aims at reviving the old colours without however trying to disguise the damage that time has caused, often leaving inevitable empty patches in the composition (for instance as in the case of Giotto in Assisi)."

(Berio L, *Rendering Preface,* n.p. UE, 1989)

Berio creates a feel of gaps, an empty space using sparse, non-descript music.

"Alluding once again to the frescoes, he has referred to these interpolations as "cement work". "Cement" captures the neutral quality of his contribution, which comes across as grey and lifeless next to the revived Schubert, an effect achieved by this music, often consisting of thin static sonorities, being played without expression …"

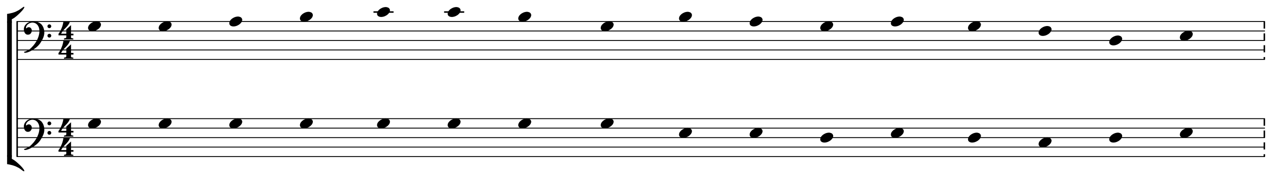
(Metzer D, Musical Decay*, RMA Journal* Vol. 125 No.1, 2000)

Berio's music is a dialogue between past and present in which a sense of loss and emptiness, a gap in an ancient fresco, is a stylistic feature. The *mutations* submitted in the portfolio view the music of Dowland and Palestrina as an eternal, Platonic Form. The *mutation* presents an alternative copy of this eternal Form by altering the pitch-organisation and durations from that of the original.

Chapter 5. Polyphony

5i. The Development of Polyphonic Music

Ex.2 Parallel Organum



Musica Enchiriadis, c.859, brittanica.com
https://www.britannica.com/topic/Musica-enchiriadis
Ex.2 shows the characteristics typical of early polyphony. The upper voice is a fragment of plainchant and the lower voice moves in parallel with the first voice, mostly employing the interval of a fourth.
The development of *free organum* and *melismatic organum* allowed for greater independence of the vocal parts. The plainchant melody was elongated rhythmically and the second voice moved freely against the long notes of the plainchant, employing mostly the intervals of 4th, 5th and the unison or octave (Ohl J, and Parrish C, *Masterpieces of Music*, London, 1980, pg.18-21).
The Notre Dame School, featuring Leonin (1150s-1201) and Perotin (c.1200), standardised four-part vocal harmony based around a cantus firmus; introduced the motet as a musical form; and its practice of writing polyphony in score allowed the composer a visual aid in the setting of one part against another. The use of rhythmic modes in rhythmic notation was a major advance in the art of polyphony (Everist M, The C13th, *Cambridge Companion to Medieval Music*, ed. Mark Everist (Accessed 18 April 2017). The *Ars Nova* of de Vitry (1291-1361) and Mauchaut (1300-1377) built upon the achievements of the Notre Dame School. Their innovations included: more fluid rhythms in comparison to the rhythmic modes; iso-rhythmic motets; the setting of the complete ordinary of the Mass. (Ohl, J. and Parrish, C. *Masterpieces of Music*, London, 1980, pg. 36-39).
John Dunstable (c.1390-1453) added harmonic variety to the art of polyphony with his use of the intervals thirds and sixths, often as part of a root position chord:

> "Dunstable practiced … the harmony of the English descant with its sensuous full chords. However, ... he liked to write the triad in its root position… It is a new ideal of the blending of sound, of chords which Dunstable brought to the Continent."
> (Bukofzer M, John Dunstable and the Music of His Time, *RMA*, Taylor&Francis 65th Sess. 1938-9).

The music of Palestrina (c.1525-1594) may be viewed as the summation of the polyphonic style with the use of multi-voiced polyphony controlled by triadic harmony. Palestrina's legacy was passed on to subsequent generations of composers through the treatise *Gradus Ad Parnassum* by Fux (c.1660-1741).
A diverse polyphonic tradition continued into the C20th and C21st in the music of, amongst others, Schoenberg (1874-1951), Vaughan Williams (1872-1958), Tavener (1944-2013) and Pärt (b.1935).

5ii. *Gradus Ad Parnassum*

Palestrina's influence on music has lasted several centuries after his death, due in part to the study of his style in Fux's *Gradus Ad Parnassum:*

> "By safeguarding the polyphonic tradition in an age in which its appreciation was fast waning, the Viennese master (Fux) laid the foundation for a classical Viennese style. His foremost disciples were Haydn, Mozart and Beethoven… Haydn took infinite pains to assimilate the theory of Fux…"
>
> "It is likely that Mozart studied Fux's work first under the influence of his father…"
>
> "Beethoven turned first to Johann Schenk and later to Johann Georg Albrechtsberger for tutelage. Each of the teachers based the course of studies upon the *Gradus Ad Parnassum*..."
>
> (Mann A, *The Study of Counterpoint from Fux's GAP,* New York, London, 1971, xi-xv).

Mann cites Schubert, Bruckner, Brahms, Strauss and Hindemith as other composers influenced by Fux's work.
Fux makes it clear that his treatise on counterpoint relates to the style of Palestrina:

> "I have used the form of dialogue. By Aloysius … I refer to Palestrina, the celebrated light of music…"
>
> (Ibid. pg.18).

5iii. Mensural Rhythms, Palestrina's Compositional Method

Palestrina's use of mensuration to notate rhythms has the consequence that in modern rhythmic notation the music looks more regular in its metric pulse than perhaps Palestrina intended it to be. The approach taken was that the modern editions of Palestrina's music used for analysis were a sufficiently accurate representation of the pitch-organisation intended by Palestrina. The vertical pitch-organisation would be the same in either mensural or modern notation.
The treatises studied for this commentary have the disadvantage that they either produce a summation of the rules governing Palestrina's compositional style which when applied, produce music only superficially like that of Palestrina; or they show examples of his music and that of other Renaissance composers but fail to pin down

the methodology used to compose the music. *Gradus Ad Parnassum* by Fux would fall into the first category along with *Polyphonic Composition by* Owen Swindale and *The Art of Counterpoint* by Gioseffo Zarlino. The second category includes the exhaustive study of the use of dissonance by Palestrina in *The Style of Palestrina and the Dissonance* by Knud Jeppeson; and *The Art of Counterpoint by* Tinctoris.

5iv. The Rules of Species Counterpoint

Metre in the style of Palestrina relates to the note value of a modern minim:

> "In music written for the Church in the sixteenth century, the minim beat is almost universal."
>
> (Swindale O, *Polyphonic Composition,* London, 1972, pg. 125).

The minim is the unit of duration which regulates the harmony in terms of consonant and dissonant intervals.

The classification of intervals outlined in Fux *Gradus* are:

> "Unison, third, fifth, sixth … and the octave are consonances… The unison, fifth and octave are perfect. The sixth and third are imperfect. The remaining intervals, like the second, fourth, diminished fifth, tritone, seventh … are dissonances."
>
> (Mann A, *The Study of Counterpoint from Fux's GAP,* New York, London, 1971, pg. 20).

Owen Swindale lists the following rules of harmony:

1. With very few exceptions … there must be a consonance on every minim beat.
2. Adjacent parallel fifths and octaves are forbidden…
3. Passing dissonances should only occur on unaccented beats…
4. Passing dissonances must be approached and left by step. Leaps may only be made to and from consonances.
5. Suspensions are the only discords allowed on the accented beat. They must resolve downward one step.

(Swindale O, *Polyphonic Composition,* London, 1972, pg. 124).

The Renaissance music theorist and contemporary of Palestrina, Gioseffo Zarlino, gives an interesting insight into the reasons to avoid parallel fifths and octaves in his treatise *The Art of Counterpoint*:

> "the consecutive use of two consonances with similar proportions was merely a change of pitch which did not produce a good harmony ... Thus, they did not wish that two or more perfect consonances having the same ratio should be used consecutively with the parts ascending or descending together, without an intervening interval…
>
> Numbers and proportions invite us to variety, for in them we do not find two similar proportions following one another, such as 1:1:1, 2:2:2, etc. which

would form two unisons: or 1:2:4:8, which is a geometric rather than a harmonic progression and gives three consecutive octaves; or 4:6:9, which gives two consecutive fifths. So we must not write consecutive unisons, octaves or fifths, for the natural cause of consonance – the harmonic number – does not contain in its progression or natural order two consecutive proportions…”

(Zarlino G, *The Art of Counterpoint,*1558, trans. GA Marco, CV Paliska, New Haven and London YUP, 1968, pg. 60-61).

Zarlino states the use of consecutive perfect consonances should be avoided not only for the reason it produces poor counterpoint with no independence of the parts; the music should follow the proportions of the harmonic proportion, not the geometric proportion.

The harmonic proportion is of the form 6:4:3 in which:

$$\frac{1}{c} - \frac{1}{b} = \frac{1}{b} - \frac{1}{a}$$

(Mann A, *The Study of Counterpoint from Fux’s GAP*, New York, London, 1971, pg. 141).

Thus, a=6; b=4; c=3

$$\frac{1}{3} - \frac{1}{4} = \frac{1}{4} - \frac{1}{6}$$

With a common denominator of 12:

$$\frac{4}{12} - \frac{3}{12} = \frac{3}{12} - \frac{2}{12}$$

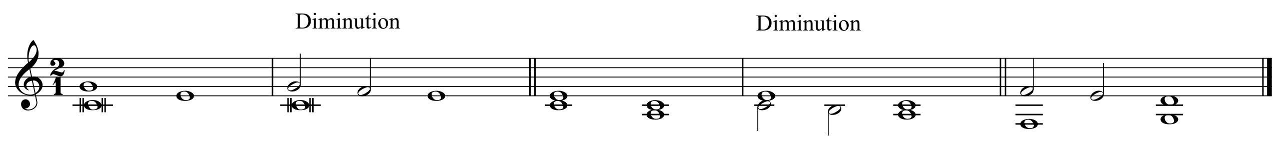
Thus:

$$\frac{1}{12} = \frac{1}{12}$$

This idea of number controlling musical proportion became important regarding musical Platonism (see 21v. pg.134).

The passing note is uncommon for minims and is usually a crotchet or quaver. Fux states a passing minim must only be a diminution which fills in the gap of a consonant skip. This is shown in Figure 1:

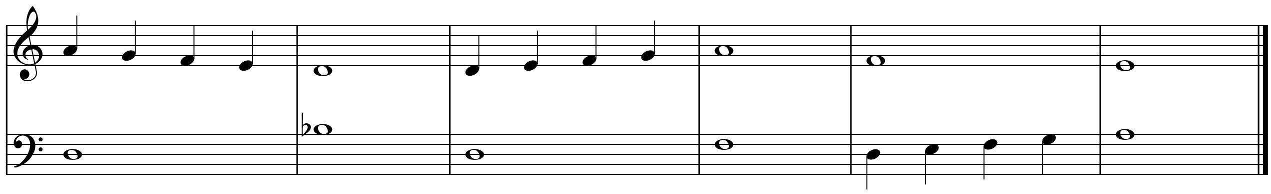
Fig.1 Passing Note Minims



(Ibid. pg. 41).

Figure 2 shows the use of passing note crotchets:

Fig.2 Passing Note Crotchets

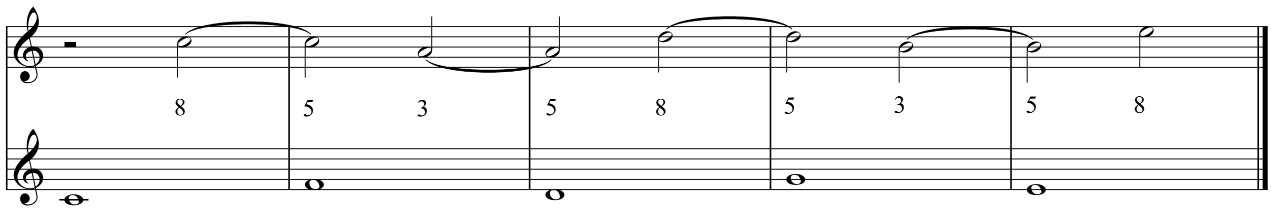


(Ibid. pg.51)

Accented crotchets must be consonant while unaccented crotchets may be dissonant if they pass by step between consonances. The rules for quavers are the same.

The suspension is the only dissonance allowed on a strong metric beat and it must obey the rules that it is suspended from a consonance and resolves into a consonance, usually by a downward step. Fux calls the suspension a *ligature* which may be consonant or dissonant. Figure 3 shows consonant suspensions:

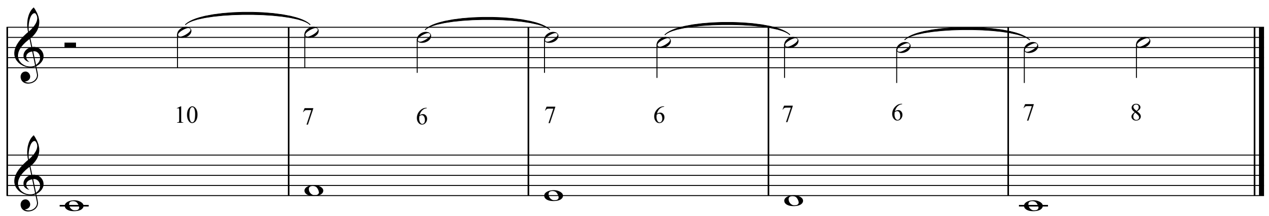
Fig.3 Consonant Suspensions. Numbers show the interval between notes.



(Ibid pg. 55)

The suspensions in Figure 3 suspend a consonant interval into the strong beat, dissonant suspensions suspend a dissonant interval into the strong beat. This is shown in Figure 4:

Fig.4 Dissonant Suspensions. Numbers show the interval between notes.

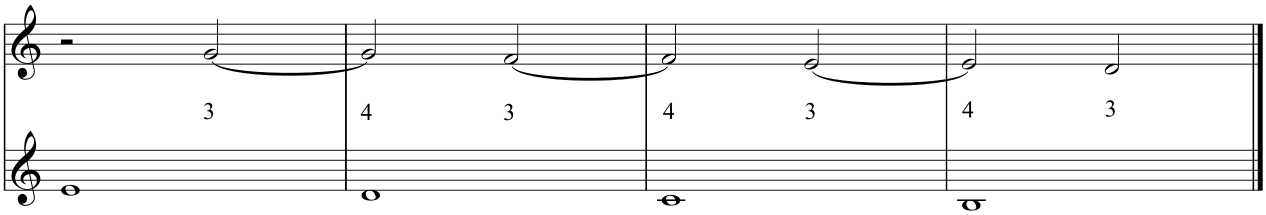


(Ibid. pg. 55)

Fux gives the following explanation of the resolution of dissonances:

> "Before I proceed to explain the manner in which dissonances are to be resolved, you should know that the notes held over and, as it were, bound with fetters, are nothing but retardations of the notes following, and thereafter proceed as if brought from servitude into freedom. On this account dissonances should always resolve descending stepwise to the next consonances"

Fig.5 Resolution of Dissonant Suspensions. Numbers show the interval between notes.



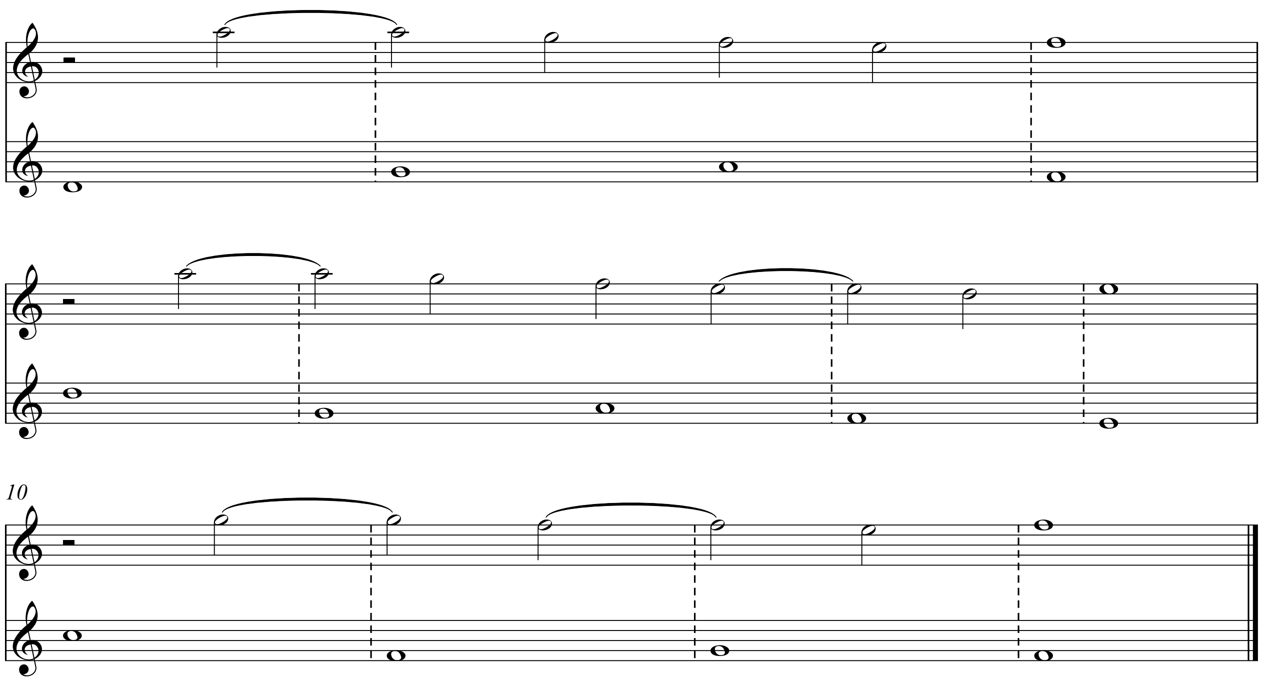
(Ibid. pg. 56)

Zarlino regards the suspension as a syncopated semibreve:

> "It is possible to have a dissonance on the first part of the measure, when this beat is occupied by the second minim of a syncopated semibreve in the counterpoint… the first part of such a note is placed on the upbeat of the preceding measure, and the second part of the dissonance falls on the down-beat of the measure."
>
> (Zarlino G, *The Art of Counterpoint,*1558, trans. GA Marco, CV Paliska, New Haven and London YUP, 1968, pg. 96-97)

Zarlino gives the suspensions in Figure 6 as well resolved.

Fig.6 Suspensions Well Resolved

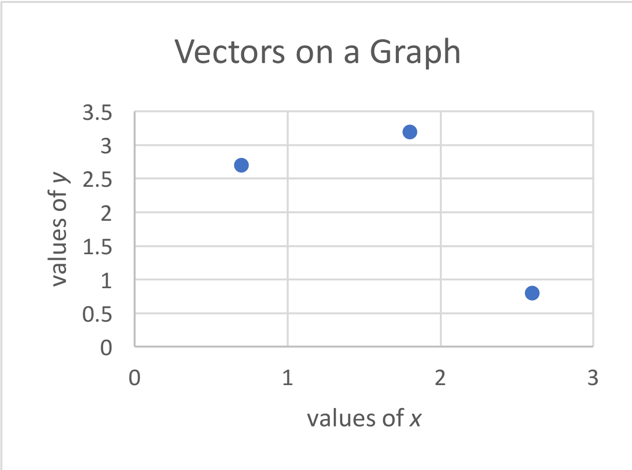


(Ibid. pg. 99)

5v. *Harmo-rhythmic Structures*

To investigate the relationship between harmony and rhythm, music can be analysed and expressed as a *harmo-rhythmic vector* with an x-value representing the harmonic content and the y-value the rhythmic displacement. In this sense vector implies a line which may be plotted on a Cartesian Graph and thus, has a

value and a direction.
A two-dimensional vector may be written as: (x,y) where "x" represents the value and direction along the horizontal axis of a graph and "y" the vertical axis, this is shown in Figure 7.
Fig.7 Vectors plotted on a graph



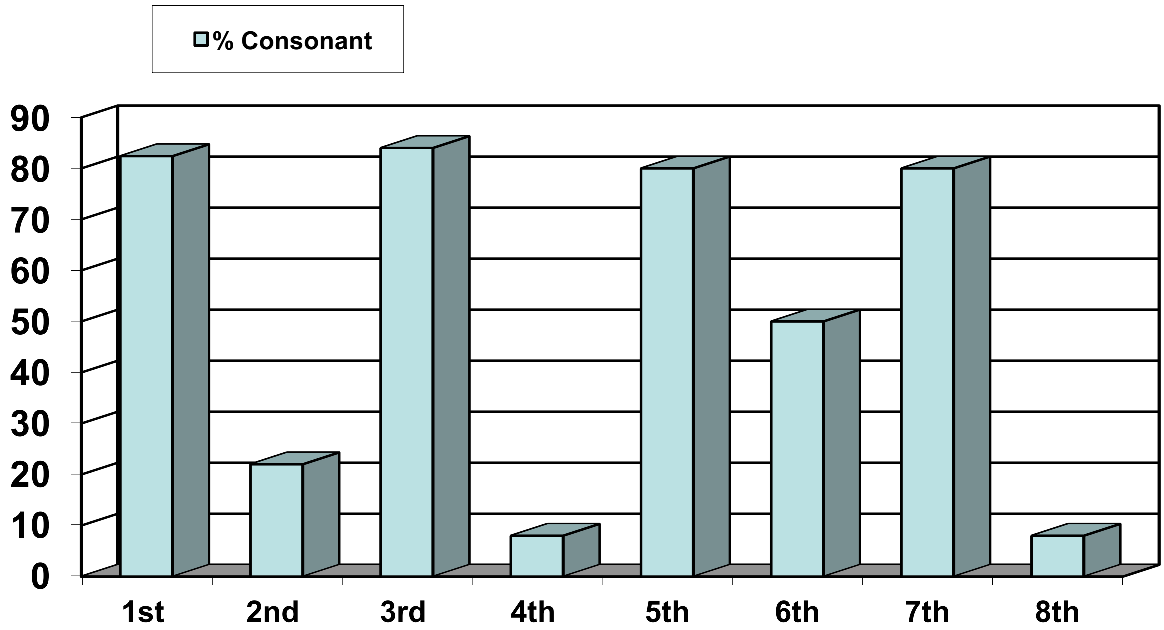
The *harmo-rhythmic vector*s can be plotted on a graph and a two-dimensional space of the *harmo-rhythmic structure*[20] emerges. This procedure has been carried out for the Angus Dei (I) from Palestrina's mass *Veni Sponsa Christi* (Orr JF, C Parrish C, *Masterpieces of Music before 1750*, London, 1980, pg.88-89).
Rather than plot the value for each note in the piece, the percentage of occurrences of consonant harmony for the 1st quaver, 2nd quaver etc. was calculated. To find these values the number of occurrences of the 1st, 2nd quaver etc. in each bar were worked out and it was noted if these positions had a consonant or dissonant interval or chord, these values were used to calculate the percentage level of consonance. The intervals of major and minor 3rd, 5th, major and minor 6th and octave were taken to be consonant; and major and minor 2nd, 4th and augmented 4th, and major and minor 7th as dissonant. Harmonic intervals were measured note against note except for four-note chords where the interval of a fourth between upper parts was not classed as dissonant.
Taking the relative value of 1 to measure the quaver displacement from the start of the bar and the percentage level of occurrences of consonant harmony, Table 2 shows the resulting *harmo-rhythmic vectors*:
Table2 *Harmo-Rhythmic Vectors*

V1 = (82.4, 1)	V2 = (22.0, 2)	V3 = (84.0, 3)	V4 = (8.0, 4)
V5 = (80.0, 5)	V6 = (50.0, 6)	V7 = (80.0, 7)	V8 = (8.0, 8)

[20] A term invented by the author to describe musical structures where fixed rules rhythmically control pitch-organisation

In V1: the value of x shows that 82.4% of percussed intervals at this point were consonant; the value of y = 1st quaver beat of the bar. These *harmo-rhythmic vectors* are plotted on the graph in Figure 8.

Fig.8 Graph of *harmo-rhythmic vectors*



The graph clearly shows the consonance spikes occurring on metric strong beats. After several bars the listener associates consonant intervals and chords, or suspensions; with metric strong beats. This generates a sense of pulse and metre using the rhythmic control of dissonance, the rhythmic shape of the music comes from within its own workings, it does not need to be superimposed in the form of rhythmic modes or dance rhythms. The harmony and rhythm co-operate in a dynamical and interdependent relationship to produce *harmo-rhythmic structures*.

Chapter 6. Analysis of Stabat Mater

To investigate Palestrina's use of triads, an analysis of the chord progressions of *Stabat Mater* was undertaken. Two editions were used in this analysis, the first is from the Choral Domain Library and the second is Novello's Original Octavo Edition. Debussy is attributed the quote: "Great Art makes rules, rules do not make great art." Fux *Gradus* sets out the rules of Palestrina's style yet the examples and music resulting from applying the rules sound only superficially like the music of Palestrina. A way to apprehend the use of triads in Palestrina's style is to look at the source of the great art, not to try to replicate his art through following rules. The poem used in *Stabat Mater* consists of stanzas of three lines each. Each of these stanzas was analysed regarding its chord progressions. The analysis uses rhythm-guitar notation, a capital letter gives the chord name such as **E** for an E major chord, **Em** for a minor chord and **E5** for a chord with no third. A full stop indicates where a chord lasts for more than a beat. E.g. **E . . .** would mean an E major chord lasting four crochet beats. Non-root note chords are given by a slash and the bass note. E.g. **Am/C** is an A minor chord with a C in the bass, a first inversion chord.

Chords labelled as **S/D** are suspension chords, they are written in this way because the strong metric beat contains a dissonant interval. Chords occurring on quaver beats are written in lower case. The passing note chords labelled **s/c** are lower case because they occur on quaver beats, **s/c** indicates the strong metric beat is consonant.

Table3 Analysis of Chord Progressions in *Stabat Mater*

1st Stanza

Stabat mater dolorosa,	juxta Crucem lacrimosa	dum pendebat Filius.
A . G . F . . C . F Gm . S/D A	**A . G . F . . C . F Gm . S/D A**	**A5 . Dm Am Em G Dm s/c S/D E A5 .**

2nd Stanza

Cuius animam gementem,	contristatam et dolentem,	pertransivit gladius.
A . . Dm . s/c Dm Am/C Bb . . . F . . C .	**F . C F C/E Dm . s/c Dm Am/c Bb F/A s/c Bb6-5 Bb S/D Gm6 F**	**F Dm/F Gm6 S/D s/c F/A Bb Gm6-5 A Dm S/D A D .**

Only the first two stanzas of the poem are shown in Table 3 since the detail of the harmony is only of passing interest. The main purpose of the exercise was to increase familiarity with Palestrina's use of triads in his music.

The analysis of the chord progressions clearly shows the correlation between rhythm and harmony, the *harmo-rhythmic structure*. The only chords allowable on strong minim beats are consonant chords and prepared suspensions, there are no counter-examples in *Stabat Mater*. Passing note dissonances are permissible if they proceed by stepwise motion on crotchet or quaver weak beats. Suspensions in Renaissance music vary the rhythm by syncopation, delaying the consonant sound of the strong beat to a weak beat:

> "The only consistently used dissonance of this type is the suspension. Its characteristics are: It is a syncopation, and it resolves…"
>
> (Swindale A, *Polyphonic Composition*, London, 1972).

A Komar writes in his *Theory of Suspensions*:

> "Finally, a suspension is normally metrically accented with respect to resolution at the level at which the suspension is generated."
>
> (Komar A, "*Theory of Suspensions",* Princeton N.J. 1971, pg. 69).

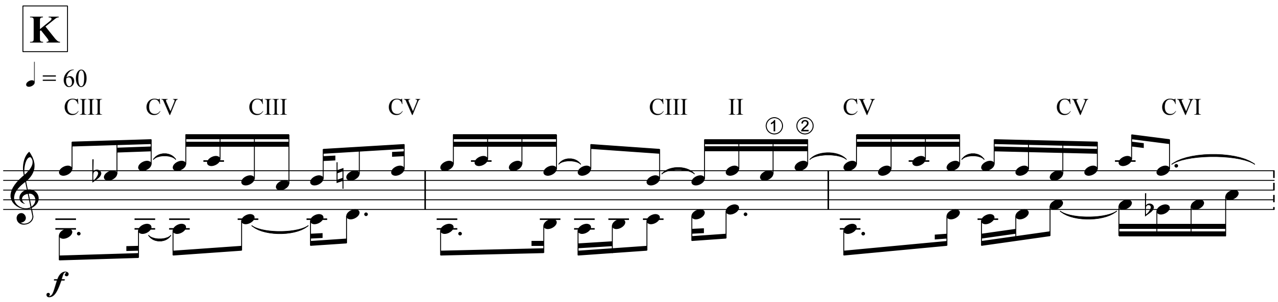
This analysis of *Stabat Mater* informed the pitch-organisation used in *Flumina, Concertino, Mutation of Dowland's Midnight, Mutation of Missa Papae Marcelli, The Evolution of the Cosmos* and *This Day Dawns.*

Chapter 7. Synopsis of the Portfolio and Appendices I

Guitar Sonata I for solo classical guitar was the first composition in the research to use a rule-based, polyphonic approach to writing music, developed from the species counterpoint described in Fux *Gradus. Guitar Sonata I* has extensive passages in two-part counterpoint.

Ex.3 *Guitar Sonata I* first movement. Plainchant-like cantus firmus in the bass with a countersubject in the other part.

Ex.4 *Guitar Sonata I* second movement. Imitation between lower and upper parts.



String Quartet I extended this rule-based compositional strategy in music of up to four parts.

Ex.5 *String Quartet I* third movement. Imitative entries between parts.

Two Explorations of a Chord Sequence and *Mutation of Mozart's Jupiter Symphony* used pre-compositional pitch and duration sets inspired by Xenakis' Symbolic Music. (See 9v. pg.53). The rules of polyphony used in earlier compositions were modified in response to the use of a pre-compositional mode, the pitch-organisation used chords[21] as a fundamental entity. These chords were regarded as a relative consonance and alterations of the chords by passing notes or suspensions were considered as dissonant.

Ex.6 *Mutation of MJS* strings only. Mode cd in violin I part.

Flumina for chamber orchestra used the motet *Super Flumina Babylonis* (c. 1584) by Palestrina as inspiration. The extension of Xenakis' *Symbolic Music,* in particular the use of vectors to symbolically encode pitch-organisation, gave insight into the use of tri-note[22] chord pitch-organisation.

[21] Chords used in this sense means any collection of pitches sounded simultaneously or as an arpeggio, rather than traditional, diatonic chord types.

[22] Tri-note chords simply means a chord built from three distinct pitches and does not imply the chord contains a tri-tone.

Ex.7 *Flumina* brass, piano, strings*.* Chords derived from mode cd.

Concertino for chamber orchestra further modified the rules of polyphony used in *Flumina* to give a polyphonic style like the fifth species counterpoint described by Fux in *Gradus Ad Parnassum*. This allowed for greater rhythmic flexibility in the music.

Chapter 8. Intervallic Compositions

8i. Guitar Sonata I

This piece uses *harmo-rhythmic structures (HRS)* to control pitch-organisation. The sonata uses polyphonic two-part writing contrasted with extended percussive techniques found in Flamenco and contemporary guitar music. The third movement uses a $\frac{7}{8}$ rhythm inspired by Bulgarian folk music. The sonata blends ancient and modern music, classical and folk traditions; and explores the extensive tone colours of the instrument.

The rules for *Guitar Sonata No. 1* are as follows:

1. Intervals between notes are classified as consonant or dissonant. Consonant intervals are: minor third, major third, perfect fifth, minor sixth, major sixth and all the equivalent consonant compound intervals. Dissonant intervals are: minor second, major second, perfect fourth, augmented fourth, minor seventh, major seventh and all the equivalent dissonant compound intervals.
2. Dissonant intervals are allowed on metric strong beats, with consonant intervals occurring on metric weak beats. The first beat of the metre is classed as the metric strong beat, all other beats are weak.
3. Rhythms and metric schemes subdivide with the number 2 as a denominator. Thus, in the first movement the $\frac{5}{4}$metre subdivides into $\frac{5}{8}$ which further divides into $\frac{5}{16}$. In the second movement $\frac{3}{4}$ subdivides into $\frac{3}{8}$ which further divides into $\frac{3}{16}$, while in the third movement $\frac{7}{8}$ subdivides into $\frac{7}{16}$.
4. When the metre subdivides, the metric strong beat changes position. The new position of the strong metric beat affects the placement of consonant and dissonant intervals and rules 1 and 2 must be reconfigured accordingly.

Regarding the control of pitch-organisation, the ruled-based compositional methodology explores fundamental relationships between two notes sounded simultaneously. Intervals are classed as consonant or dissonant in the manner described in Fux *Gradus*. Whilst the definition of consonant and dissonant harmony is a contentious issue, the classification of intervals in Fux *Gradus* is a historical precedent which may be applied to decide which intervals are consonant and which dissonant. This gives a consistent approach in the composition of *Guitar Sonata I,* leading to a clear contrast of consonant and dissonant intervals on different metric beats.

In *Guitar Sonata I* rhythms and metre subdivide with the number 2 as a denominator. Thus, in the first movement $\frac{5}{4}$ subdivides into $\frac{5}{8}$; in the second movement $\frac{3}{4}$ subdivides to $\frac{3}{8}$ etc. In his paper on the use of oscillators to model human perceptions of rhythms Large writes:

> "1:1 phase-locks are more stable than 2:1 phase-locks[23], which are more stable than 3:2 phase-locks and so forth …
> The model I have described here is consistent with results in human rhythm perception, which show that temporal pattern structure affects human abilities to perceive, remember and reproduce rhythmic sequences."
> (Large, E.G. *Models of Metrical Structure in Music,* Ohio State University, 1994).

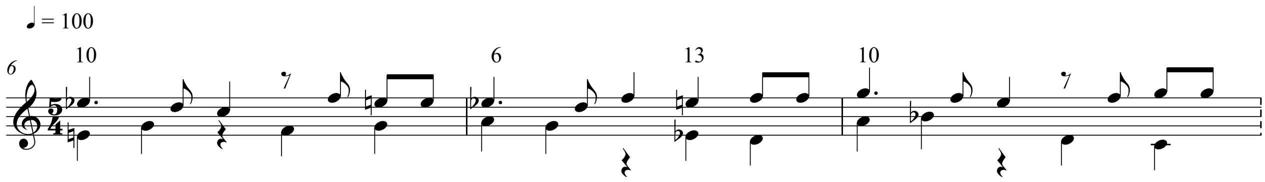
The implication of Large's model is that humans will perceive subdivisions in proportion to the number 2 more easily than for instance, patterns involving subdivisions in proportion to 5 or 7 etc.

The use of subdivisions in proportion to the number 2 influenced the choice of prime number metres for the three movements i.e. $\frac{5}{4}$, $\frac{3}{4}$ and $\frac{7}{8}$ respectively.

Diminutions or augmentations of prime number rhythmic groupings in terms of the number 2, give more interesting results than even number groupings; even number groupings merely replicate the same patterns at twice or half the speed.

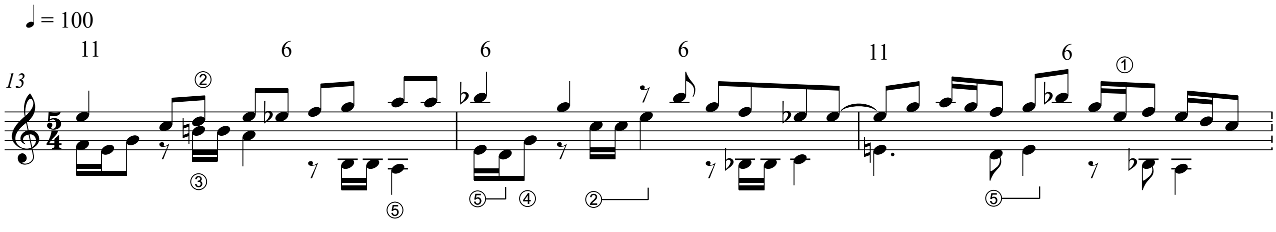
Example 8 shows bars 6-8 of the first movement, dissonant intervals occur on crotchet beats, 1 and 4.

Ex.8 *1. Guitar Sonata I* b.6. Numbers show intervals between parts in semitones.



Example 9 shows bar 13 of the first movement. The metric scheme subdivides into $\frac{5}{8}$ at bar 11, Example 9 shows how the change of metric strong beat affects the *HRS;* dissonant intervals now occur on quaver beats 1 and 6.

Ex.9 *1. Guitar Sonata I* b.13. Numbers show intervals between parts in semitones.



[23] A sympathetic resonance of two oscillators in a stable ratio

8ii. Form of Guitar Sonata I

Form Charts 1, 2 and 3 show the forms of the three movements of *Guitar Sonata I*, indicating where the *harmo-rhythmic structure* changes.

Form Chart 1 *1 Guitar Sonata I*

	A	B	C	D	E	F	G	H
Bars 1-10 5/4 Pitched	Bars 11-20 5/8 Pitched	Bars 21-10 5/16 Pitched	Bars 26-30 5/4 *Tambora*	Bars 31-35 5/8 Pitched	Bars 36-40 5/16 *Snare Drum*	Bars 41-50 5/4 Pitched	Bars 51-60 5/8 Pitched	Bars 61-65 5/16 Pitched
Ai	A ii	A iii	Bi	B ii	B iii	A′ i	A′ ii	A′ iii

Form Chart 2 *2 Guitar Sonata I*

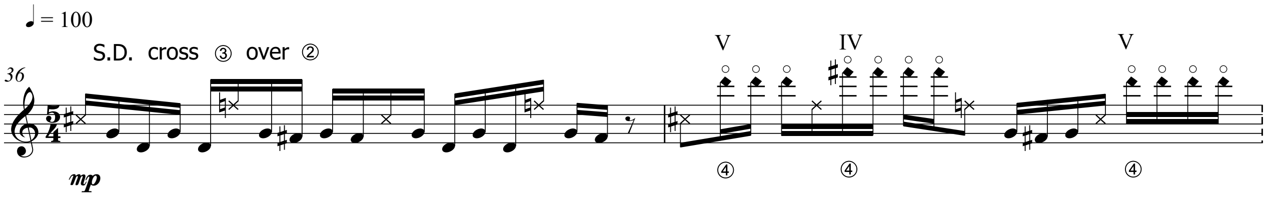
	I	J	K	L	M
Bars 1-9 3/4 Pitched	Bars 10-15 3/8 Pitched	Bars 16-29 3/8 then 3/16 Pitched, brief *Tambora* on bridge	Bars 30-40 3/16 Pitched	Bars 41-45 3/4 *Snare drum*	Bars 46-54 3/4 Pitched
Ai	A ii	Bi	B ii	A iii	A′ i

Form Chart 3 *3 Guitar Sonata I*

	N	O	P	Q	R	S	T
Bars 1-21 7/8 Pitched						Bars 126-135 7/8 Pitched	
	Bars 22-77 7/16 Pitched				Bars 113-125 7/16 Pitched		Bars 136-148 7/16 then 7/8 Pitched
		Bars 78-84 7/8 *Tambora* on bridge and body	Bars 85-98 7/16 Pitched with *pont.*	Bars 99-112 7/16 *Snare Drum*			
A	B	Ci	C ii	C iii	B ii	A′	B′ ii

Each of the movements uses a cantus firmus against which is written a second contrapuntal part, the cantus firmus in the second and third movements is a variant of that in the first movement. In addition to the contrapuntal sections, there are passages which use extended percussive techniques with minimal pitch content. These passages explore the contrast of metre through rhythmic means alone. Example 10 shows how the *snare drum* effect is used to convey a $\frac{5}{16}$ stress pattern at bar 36. The *snare drum* is sounded on semiquaver beats 1, 6, 11 and 16.

Ex.10 *1. Guitar Sonata I* b.36



8iii. *String Quartet I*

The title refers to the use of the traditional four solo instruments: two violins, viola and cello.

String Quartet I extends the counterpoint of *Guitar Sonata I* by using contrapuntal writing of up to four-parts. The rules for this composition are similar to *Guitar Sonata I,* however, in this piece consonant intervals are placed on the metric strong beats, dissonant intervals occur on metric weak beats. The rules for *String Quartet I* are as follows:

1. Intervals between notes are classified as consonant or dissonant. Consonant intervals are: minor third, major third, perfect fifth, minor sixth, major sixth and all the equivalent consonant compound intervals. Dissonant intervals are: minor second, major second, perfect fourth, augmented fourth, minor seventh, major seventh and all the equivalent dissonant compound intervals.

2. Consonant intervals are allowed on metric strong beats, with dissonant intervals occurring on metric weak beats. The first beat of the metre is classed as the metric strong beat, all other beats are weak.

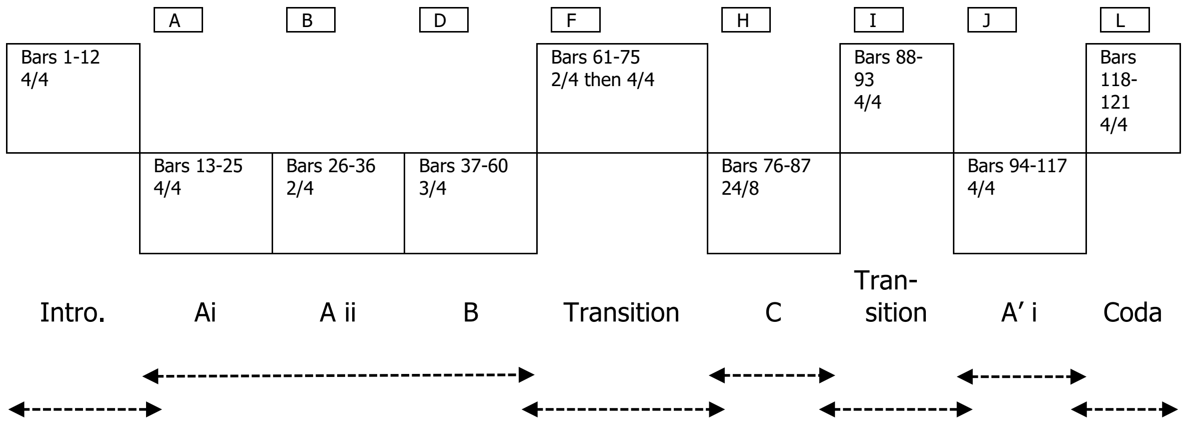
3. The metre changes constantly throughout the three movements, when the metric strong beat changes position this affects the placement of consonant and dissonant intervals and rules 1 and 2 must be reconfigured accordingly.

The rule-based compositional methodology of *String Quartet* explores new possibilities in comparison with the note against note pitch-organisation of *Guitar Sonata I. String Quartet I* uses vertical interval collections of up to four notes. On metric strong beats, each interval of the vertical collection will be consonant, for metric weak beats the intervals will be dissonant. Thus, there is a clear contrast of sound in the rhythmic placement of consonance and dissonance vertical interval collections that reinforces the metre of the music. The placement of contrasting interval collections on metric strong and weak beats regulates the pitch-organisation.

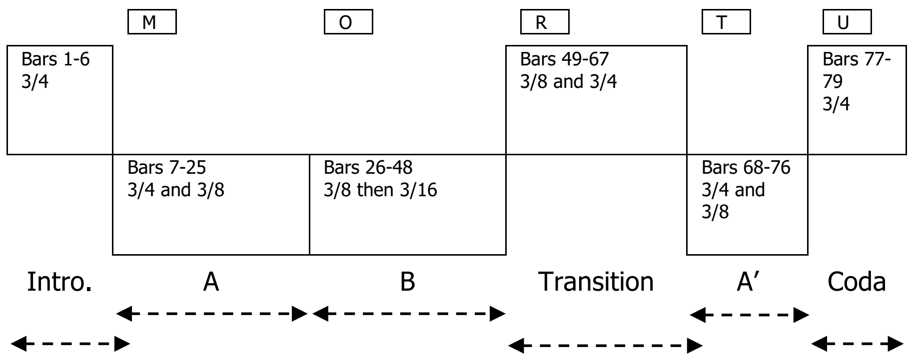
8iv. Form of String Quartet I

Form Charts 4, 5 and 6 show the form of the three movements and where the metre changes in the formal sections. Each of the three movements uses a cantus firmus that freely moves between the instruments. The cantus firmus used in the second and third movement is a variant of that used in the first movement.

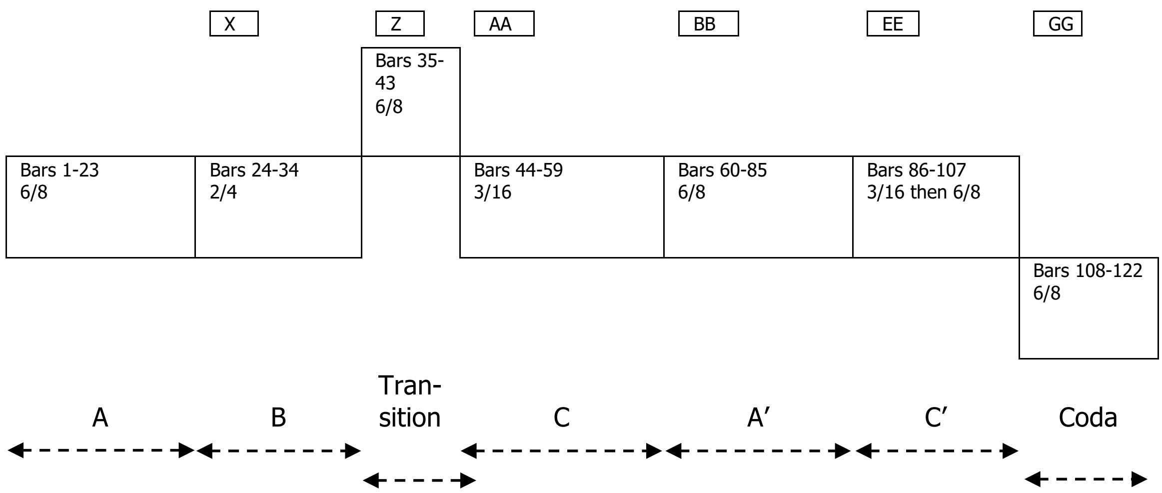
Form Chart 4 *1 String Quartet I*



Form Chart 5 *2 String Quartet I*



Form Chart 6 *3 String Quartet I*



The formal structure of the three movements of *String Quartet I* makes use of changes of *harmo-rhythmic structure (HRS)* to articulate the form. This method of using a contrast of *HRS* as a means of differentiating sections of a form is found in all subsequent compositions in the portfolio and appendix. The contrast of *HRS* is achieved through changes of metre, with an associated change in the rhythmic placement of consonant and dissonant interval collections.

Example 11 shows how at the start of Ai (shown in Form Chart 4) in $\frac{4}{4}$ metre, consonant interval collections occur on beat 1. The numbers above the cello part show the vertical interval collections on the first beat of the bar. These are either consonant intervals or suspended intervals of a 2nd or 4th.

Ex.11 *1. String Quartet I* Ai. Numbers above cello part show intervals in semitones.

Example 12 illustrates how the *HRS* changes at the start of B (shown in Form Chart 4) in $\frac{3}{4}$ metre, the new metre has consonant interval collections occurring every 4 beats.

Ex.12 *1. String Quartet I* B. Numbers above cello part show intervals in semitones.

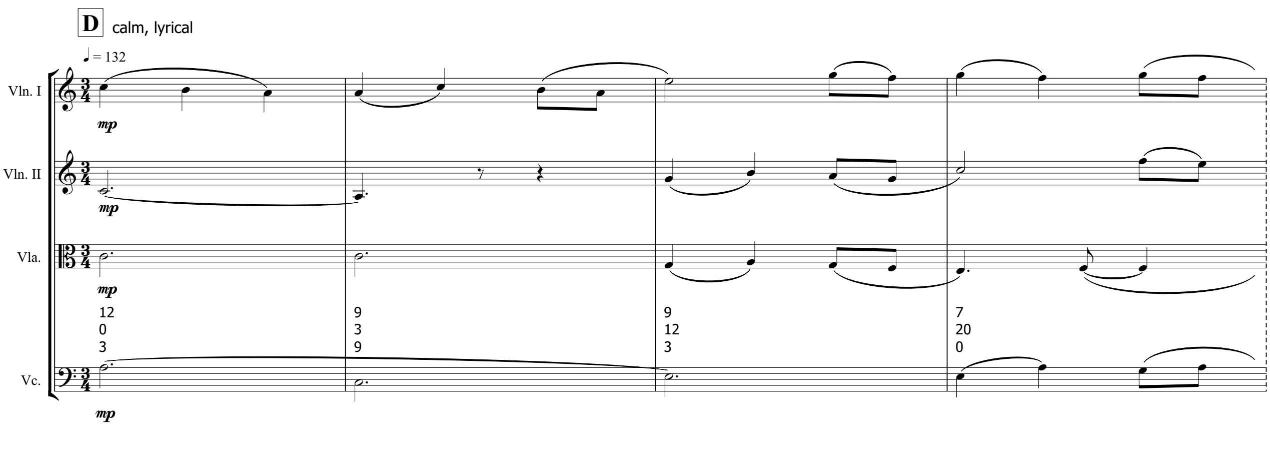
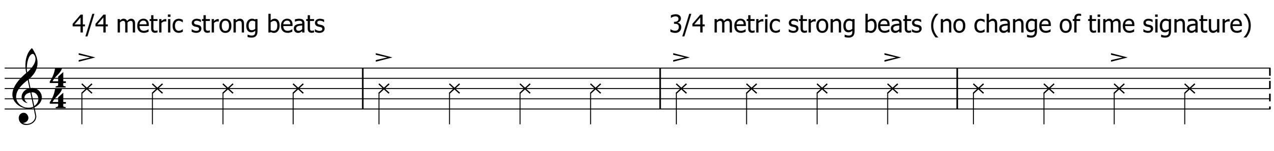


Figure 9 shows a schematic outline of this change of metric strong beat.

Fig.9 Change of Metric Strong Beat



8v. Pitch Organisation of String Quartet I.

The use of *HRS* in *String Quartet I* controls the counterpoint in regard to the rhythmic treatment of dissonant intervals. The pitch-organisation has similarities with traditional, functional harmony, a vertical interval collection in *String Quartet I* consists of either consonant intervals or dissonant interval with the consequence that major or minor chords (and occasional augmented fifth chords) occur on strong beats whilst on weak beats the vertical interval collections are dissonant. Over a period of several bars this produces unexpected harmonic progressions which are perhaps compared unfavourably by the ear against traditional harmony.

Example 6 shows *Ai* (shown in Form Chart 4) from the first movement. The first four bars have the following chords on the first beat:

b.1: **E** unison; b.2: first inversion **D** major chord with a suspended **B**; b.3: first inversion **D** major chord; b.4: **C**♯ minor chord with a suspended **G**♯ bass. On metric weak beats, there are a variety of dissonant chord types, this pitch-organisation produces harmonic progressions that do not occur in functional harmony.

8vi. Evaluation of the Intervallic Compositional Methodology

The compositions described in this chapter introduced several stylistic features that occur in all the subsequent compositions:

1. The use of *HRS* controlled by rules relating to the treatment of dissonance.
2. The subdivision of rhythms and metres with the number 2 as a denominator.
3. A change in *HRS* with each change of metre. When the position where the strong metric beat occurs is altered, the *HRS* changes accordingly.

The use of rules in the Intervallic Compositions sets up certain restrictions that the composer must follow. However, this begs the question if there is strict control over some of the musical elements, why are aspects of the music unrestricted, allowing the composer freedom of choice? For instance, the opening of *1. Guitar Sonata I* consists of a cantus firmus based around the linear interval of a major second which is a dissonant interval. If there are intervallic restrictions in place why is a dissonant interval allowed in the cantus firmus?

In subsequent compositions, the areas of freedom open to the composer were reduced regarding pitch-organisation and which pitches may be used at any point. However, in all the compositions in the portfolio and appendix the composer is allowed certain freedoms regarding register, dynamic, articulation, instrumentation and orchestration, allowing for free artistic expression at every point in the music within the strict boundaries of the rules controlling pitch-organisation and duration.

Chapter 9. Modality

9i. Introduction

Modality is seen as distinct to tonality since the former implies the use of a collection of pitches while the latter relies on hierarchies of triads derived from scales or modes.

9ii. The application of Xenakis' Ideas

Ideas found in Xenakis' *Symbolic Music* gave rise to a more systematic pitch-organisation and a reconsideration of duration, tempo and musical time. The impact on the compositional methodology was the development of strategies to control medium and large scale formal structures through the contrast of *harmo-rhythmic structures.* In addition, new ideas on the remaking of past music became apparent. Xenakis wrote of the need to reconsider musical fundamentals in response to the total serial technique of mid-Twentieth Century music, pitch and duration are initially considered to be abstract and are organised algebraically or formally into coherent structures without regard to temporal considerations. Once pitch and duration structures are realised *in-time* these abstract structures inform tangible, temporally ordered music.

In *Formalized Music* Xenakis describes how mathematics was employed to control musical parameters in his compositions. The musical decisions taken are controlled to a greater or lesser extent by mathematical outcomes. However, the compositions in the portfolio and appendix use little or no mathematics regarding the control of musical parameters.

Xenakis employed stochastic theory to create musical structures, but none of the music in the portfolio or appendix is stochastic.

In contrast to Xenakis, the compositions in the portfolio and appendix embrace polyphonic technique rather than renounce it.

9iii. *Symbolic Music*

In the chapter, *Symbolic Music* (Xenakis I, *Formalized Music*, Stuyvesant, NY, 1992, pg. 155) Xenakis applies ideas from symbolic logic to music and identifies three levels of structure in compositions: *outside time structures*, *temporal structures* and *in-side time structures*. He shows how algebra distinguishes one level from another. He imagines two sounds a and b emitted one after the other:

> "If no account is taken of the temporal element, then the two elements a and b are considered as a pair. Consequently emitting first a then b, or first b then a, gives us no more information about these distinct events than when they are heard in isolation after a long period of silence."
> (Ibid. pg. 156)

Xenakis concludes that if no account is taken of temporal relations then:
for $a \neq b$

$$avb = bva \text{ - commutative law}$$ [24]

where $\neq$ means *not equal to* and v means *put next to*
(Ibid. pg. 157).
If a third element c is introduced then:

$$(avb)vc = av(bvc) \text{ - associative law}$$ [25]

Thus, for *outside time structures* a commutative and an associative law exists.
When time is taken into consideration the commutative law will no longer hold true:

$$aTb \neq bTa$$

where T is a new operator meaning *anterior to* (ibid pg. 157).
If time is considered by itself, and no account is taken of the events which are dividing time into recognizable units then:

$$aTb = bTa \text{ - commutative}$$

$$(aTb)Tc = aT(bTc) \text{ - associative.}$$

Xenakis calls this the temporal level or structure.
He reaches the following conclusion:

> "most musical analysis and construction may be based on:
> 1. The study of an entity, the sonic event, which ... possesses a structure outside-time;
> 2. ... time which possesses a temporal structure; and
> 3. The correspondence between the structure outside-time and the temporal structure: the structure in-time"
>
> (Ibid pg. 160).

9iv. *Symbolic Music* and Compositional Technique

There are three areas in which Xenakis' *Symbolic Music* informed the compositional techniques found in the portfolio and appendix:

1. The separation of musicals elements into those constructed *outside time* i.e. pitch structures; *temporal structures* i.e. metre and rhythmic cycles; the correspondence between these elements *in-time* i.e. tri-note chord pitch-organisation, formal structures.
2. The idea that music may be expressed symbolically and musical entities may be considered as abstract sets, these sets are members of the set **R** of real numbers. Each element of an abstract set may be expressed as a vector.

[24] A mathematical law stating that the ordering of entities in an equation does not alter the outcome of the overall numerical value. E.g. 2+4 = 4+2
[25] A mathematical law stating that the groupings of entities in an equation does not alter the outcome of the overall numerical value. E.g. (2+4)+3 = (2+3)+4

3. The abstract sets representing musical entities may be viewed as modelling *imitations* or *copies* of a *paradigm* Platonic *Form.* Xenakis did not state this idea in any of his writings, however, the author believes this to be a valid extension of *Symbolic Music*.

9v. Tetrachordal Structure of Scales

Xenakis describes how ancient Greek and Byzantium culture produced scales and modes through the combination of tetrachords:

> "The disjunct juxtaposition of two tetra-chords one tone apart form the diapason scale spanning a perfect octave. The conjunct juxtaposition of several of these perfect octave diapason leads to the scales and modes with which we are familiar."
>
> (Xenakis I, *Formalized Music*, Stuyvesant, NY, 1992, pg. 188).

In regard to the C major scale Xenakis writes:

> "The diatonic scale is derived from a disjunct system of two diatonic tetrachords separated by a whole tone and is represented by the white notes on the piano."
>
> (Ibid, pg. 189).

The diatonic tetrachord of Ancient Greek music corresponds to the modern tone – tone - semi-tone. When two of these diatonic tetrachords are joined together separated by a tone the familiar T T S - T - T T S or major scale: **C D E F G A B C** is produced. Thus, the major scale has a structure derived from the combination of two tetrachords.

The combination of two pitch sets generated four modes that mimic the tetrachordal structure of traditional scales. The pitch sets are:

Dissonant intervals from a common origin: (**0 1 2 5**) = set **D**

Consonant intervals from a common origin: (**0 3 4**) = set **C**

The sets **C** and **D** combine to give four modes shown in Table 4, the set with an asterisk begins from a common origin of the note **F♯**:

Table4 Four Modes

	sets		pitches	name
Consonant with consonant:	**C** + **C***	**=**	**C E♭ E F♯ A A♯**	**mode cc**
Dissonant with dissonant:	**D** + **D***	**=**	**C C♯ D F F♯ G G♯ B**	**mode dd**
Consonant with dissonant:	**C** + **D***	**=**	**C E♭ E F♯ G G♯ B**	**mode cd**
Dissonant with consonant:	**D** + **C***	**=**	**C C♯ D F♯ A A♯**	**mode dc**

Mode cd was used in *Two Explorations of a Chord Sequence, Mutation of Mozart's Jupiter Symphony* and *Flumina.* Mode cc was used in *Concertino,* mode dd was used in *Mutation of Mozart's Jupiter Symphony.*

Chapter 10. Husserl and Musical Time

10i. Introduction.

Music in performance, or listened to by electronic means, relies on the passage of time. However, the perception of musical time is not reliant upon a sense of absolute, chronometric time:

> "Time in composition can be thought of in intervals: a starting point, an ending point and the interval or space in between. The intervals, while having a chronometric time, can seem shorter or longer depending on how that interval is filled."
>
> (Stockhausen K, How Time Passes*, Die Reihe*, 1959 3, pg. 10-40).

10ii. Xenakis' Symbolic Logic and Husserl's Existentialism

In his explanation of the composing of *Herma* Xenakis describes the temporal classes in relation to time in the following way:

> "The role of time is again defined in a new way. It serves as a crucible, mould or space in which are inscribed the classes whose relations one must decipher. Time is in some ways equivalent to the area of a sheet of paper or a blackboard."
>
> (Xenakis I, *Formalized Music*, Stuyvesant, NY, 1992, pg. 173)

This space-like view of time is like that of modern physics and has interesting parallels with the writings of Husserl in his: *The Phenomenology of Internal Time Consciousness*.

With the theories of Special and General Relativity and Quantum Mechanics, a radically different theory of time replaced the Newtonian concept of absolute time. The modern view of time is one in which time is flexible, relative and observer dependent.

> "The way in which time is treated in modern physics is not essentially different from the way in which space is treated and the "time" of physical descriptions does not really flow at all; we just have a static-looking fixed space-time in which the events of our universe are laid out."
>
> (Penrose R, *The Emperor's New Mind,* Oxford, New York, 1999, pg. 574)

This modern view of time has similarities to the time perception of consciousness, not least because conscious minds seem able move freely through time. The now is experienced, the past remembered and the future anticipated.

> "There is something very odd about the way that time enters into our conscious perception ... it is possible that a very different conception may be required when we try to place conscious perceptions into a conventionally time-ordered framework."
>
> (Ibid. pg. 154)

Husserl sets out a theory of *time consciousness* like the one just expressed. These ideas from Husserl's lectures are especially relevant, he explores the processes involved when a consciousness assimilates a melody.

Husserl writes the following regarding the origin of time consciousness:

> "When a melody sounds … the individual tone (note in common English usage, author's parenthesis) does not utterly disappear with the cessation of the stimulus or of the neural movement it excites. When the new tone is sounding, the preceding tone has not disappeared without leaving a trace. If it had, we would be quite incapable of noticing the relations among the successive tones; in each moment we would have a tone … but never the representation of a melody. On the other hand, the abiding of the tone- representations does not settle the matter. If they were to remain unmodified, then instead of a melody we would have a chord of simultaneous tones, or rather a disharmonious tangle of sound, as if we had struck simultaneously all the notes that had previously sounded. Only because the that peculiar modification occurs, only because every tone-sensation, after the stimulus that produced it has disappeared, awakens from out of itself a representation that is similar and furnished with a temporal determination, and only because this temporal determination changes, can a melody come to be represented in which individual tones have their definite place and definite tempos."
>
> (Husserl E, *On the Phenomenology of ITC,* trans. Brough JB, Dordrecht, Kluwer Publishers, 1991, pg. 11-12).

Husserl suggests that a consciousness may *intend* or apprehend a time dependent event like a melody with a space-like time consciousness in which the past events are retained and future events anticipated. The melody is *intended* as a whole temporal event rather than a punctiform[26] succession of separate entities; however, the melody is not *intended* as a simultaneous impression of its constituent parts, the notes of the melody retain a temporal ordering and a consciousness may move freely through this temporal ordering in a space-like fashion.

> "That several successive tones yield a melody is possible only because the succession of psychic events is united "at once" into a total formation. They are in consciousness successively, but they fall within one and the same act. We obviously do not have the tones all at once, and we do not hear the melody by virtue of the circumstance that the earlier tones continue to endure while the last tone is heard. The tones rather form a successive unity with a common effect, the apprehension-form. Naturally, the latter is only consummated with the last tone."
>
> (Ibid. pg. 22-23)

[26] Having the form of a point i.e. not continuous

Xenakis view of time as a “crucible or mold, space or blackboard upon which are inscribed the outside time and temporal classes” resonates with Husserl’s space-like time consciousness. In addition, an internal perception of time plays a crucial role in the perception of *in-time structures* in which temporal ordering and succession *operate* upon the unordered *outside time* and *temporal* sets into coherent musical structures, chords, melodies, formal structures. For this reason, the perception of musical time and temporal ordering was of fundamental importance to the compositions in the portfolio and appendix.

Chapter 11. Extending Symbolic Music

11i. Vector Space

Xenakis' *Symbolic Music* shows how music may be expressed symbolically as vectors for pitch, duration and intensity corresponding to the set **R** of real numbers and the functions that may be performed on that set. (Xenakis I, *Formalized Music*, Stuyvesant, NY, 1992, pg. 161 - 169). He imagines a set **E3**[27] which contains all possible values for sets **H** (melodic intervals) **G** (intensity intervals) **U** (duration intervals). A vector X has values x1, x2, x3 which are elements of the sets **H**, **G** and **U**. Thus, X = (x1, x2, x3) gives a three dimensional vector in the space **E3.** Every element of **E3** is a member of the set **R** of real numbers.

If an origin on an absolute scale is established and a unit vector for **H**, **G** and **U:** h, g and u respectively then the vector X can be written as:

$$X = x1h + x2g + x3u.$$

Xenakis uses an origin, an arbitrary number chosen on an absolute scale, of:

h is 0 at the note **C**3;

g is 0 at the intensity 50 db

and u is 0 at the duration of 10 seconds.

The base unit for h is a semitone; for g it is 10 decibels and for u a second. Thus, the vector:

$$X = 5h - 3g + 5u$$

corresponds to the note F3 at a dynamic of pp (20 db) lasting for a duration of 5 seconds. (Ibid. pg. 163).

Musical structures like a melody can be expressed as a set of vectors, for Xenakis this set of vectors corresponds to an algebra *outside time*. These vectors can be plotted on a three-dimensional graph in the space **E3**. When the temporal set **T** is considered the sonic statement of the vectors X1 *T* X2 *T* X3 … etc. divide metric time into recognizable units. (*T* is the operator from pg.60 meaning anterior to). This division of metric time is independent of the vector space **E3** and shifts the origin along a time axis. (Ibid. pg. 166) This corresponds to a temporal algebra. The algebra *in-time* is the correspondences and functional relations between the vectors in space **E3** and the set of metric time **T** (Ibid. pg. 170).

Xenakis attempts a universal theory of music using the ideas set out above. He writes of:

> "a formalization of many aspects of the various kinds of music of our planet."
>
> (Ibid. Pg. 183)

[27] **E3** is an abstract set and does not refer to the pitch E3.

He describes the process of music making in the following manner:

> "I propose to make a distinction in musical architectures or categories between *outside-time, in-time* and *temporal*. A given pitch scale, for example, is an outside-time architecture for no horizontal or vertical combination of its elements can alter it. The event in itself, that is, its actual occurrence, belongs to the temporal category. Finally, a melody or a chord on a given scale is produced by relating the outside-time category to the temporal category. Both are realizations *in-time* of *outside-time* constructions"
>
> (Ibid. pg. 183).

To understand the ideas of Xenakis it is instructive to look at Xenakis' composition of the solo piano piece *Herma.*

11ii. Composition of Herma

Xenakis describes his solo piano piece *Herma* as:

> "… an example of musical composition constructed with the aid of the algebra of classes."
>
> (Ibid. pg. 173).

These classes are the members of a temporal set, an outside-time set and the operations and functions performed on the classes represent the in-time architecture of the music. Xenakis writes of three kinds of algebra:

> "1. The algebra of a sonic event, with its vector language, independent of the procession of time, therefore an algebra out-side time.
>
> 2. A temporal algebra, which the sonic events create on the axis of metric time, and which is independent of the vector space.
>
> 3. An algebra in-time issuing from the correspondences and functional relations between the elements of the set of vectors X and of the set of metric time T, independent of the set X."
>
> (Ibid. pg.170).

At this point a basic discussion of set theory will prove useful:

A Referential Set contains all elements or members of interest. For example:

Let referential set **R** = 0 1 2 3 4 5 6

Set **A** is a subset of **R** if every member of **A** is a member of **R**

Set **A** = 0 1 3 5. Set **B** is also a subset of **R**, **B** = 0 2 4 6

The operation of **union** combines the members of **A** and **B**: **A+B** = 0 1 2 3 4 5 6

The operation of **intersection** produces a set which contains member that occur in both **A** and **B**: **AB** = 0

The operation of **complementation** gives a set of members found in **R** but not in **A**: **-A** = 2 4 6

[A.4] The set operations … can be applied to three or more sets. Figure 8 illustrates how the universal set, *R*, may be *partitioned* into eight disjoint sets equal to intersections between three sets *A*, *B*, *C* and their complements.

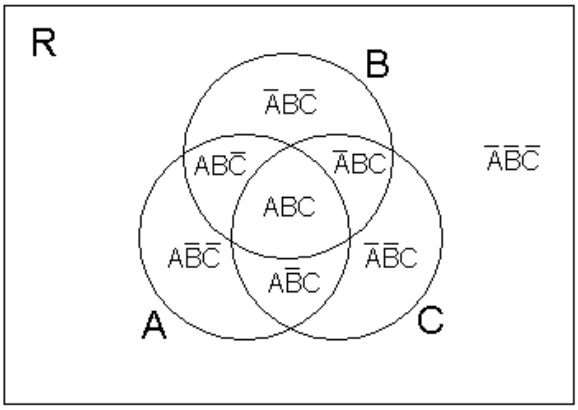


Figure 8.

[A.5] Observe that any set formed from unions, intersections, and/or complementations of *A*, *B*, and *C* can be expressed as a union of the disjoint sets which are illustrated. For instance, the complex looking set, *F*, of Figure 9

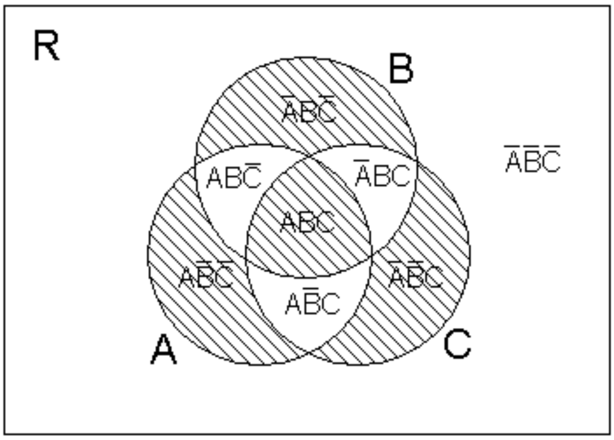


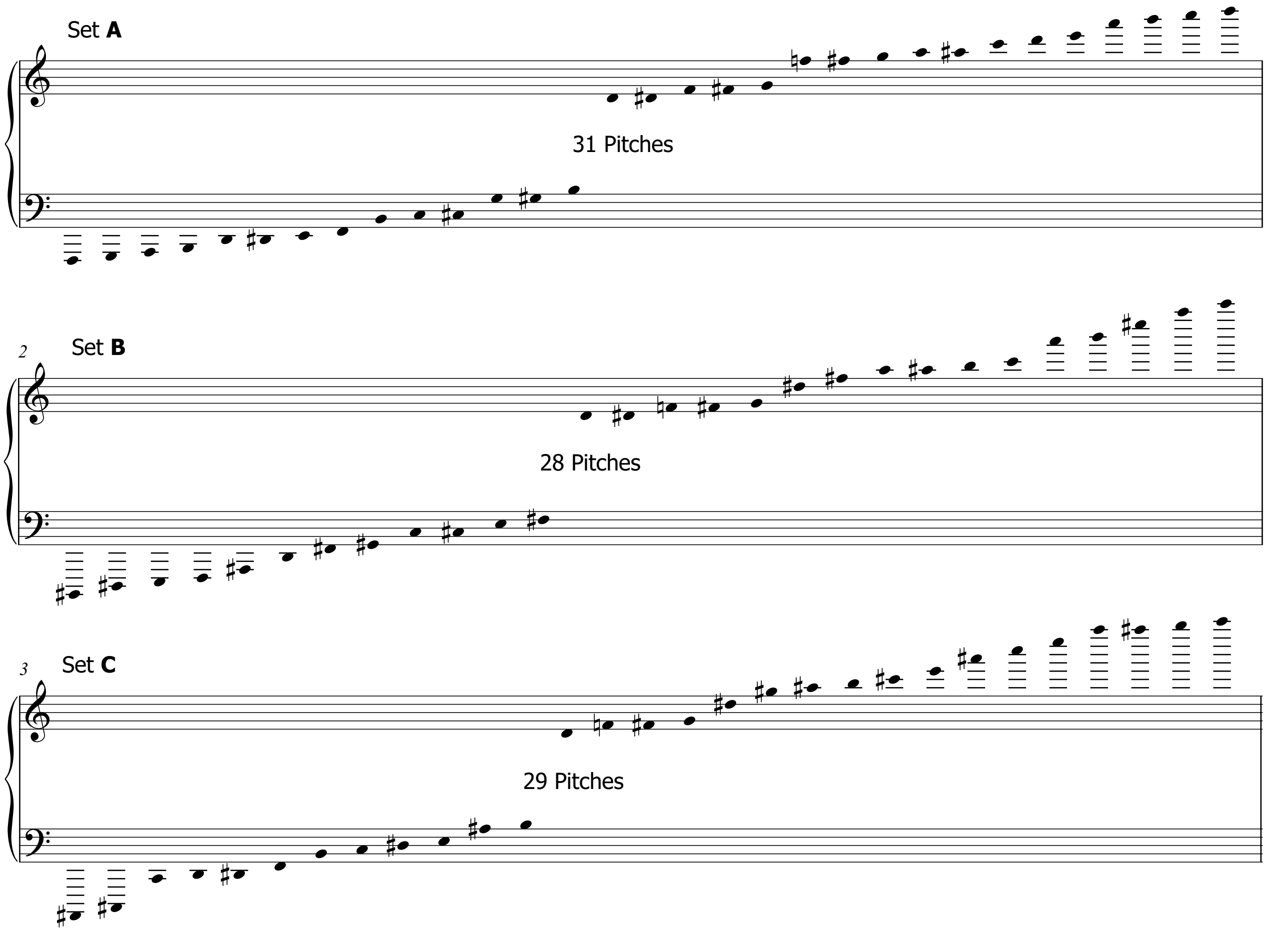
Figure 9.

may be expressed as: *F = ABC + A-B-C + -AB-C + -A-BC.*

(Wannamaker RA, Structure and Perception in Herma by Iannis Xenakis, *Music Theory Online* 7.3 (May 2001))

Xenakis takes the 88 keys of the piano as the referential set **R** in the composition of *Herma***.** He generates three other sets **A B** and **C** shown in Figure 10.

Fig.10 Sets **A**,**B** and **C** *Herma*



Sets **A B C** have some notes in common and other notes that only appear in either **A B** or **C.** Xenakis states that a listener will become aware of the operations of union, intersection and negation applied to sets **A B C** if they hear the sets themselves in comparison with pitch sets derived by applying these operations to the sets.

> "If the observer, having heard ***A*** and ***B***, hears a mixture of all the elements of ***A*** and ***B***, he will deduce that a new class is being considered, and that a logical summation has been performed on the first two classes . . . If class ***A*** has been symbolized or played to him and he is made to hear all the sounds of ***R*** except those of ***A***, he will deduce that the complement of ***A*** with respect to ***R*** has been chosen."
>
> (Xenakis I, *Formalized Music*, Stuyvesant, NY, 1992, pg. 171)

Xenakis uses the disjunctive canonic expression as a target set to which the music will gravitate. This function **F** is the union of disjoint sets from figure 9 above and has the form: **F** = ABC + A-B-C + -AB-C + -A-BC

F can also be expressed as: **F** = (AB + -A-B)C + -(AB + -A-B)-C

Xenakis writes in the score preface to *Herma*:

> "Starting from these four classes, others can be formed *outside if time,* as a

result of complementary relationship (negation) e.g. group **A**, the negation of **A** is written $\overline{A}$; also as a result of the operations of union … and of intersection …

The relationships and the operations *outside of time* (these are abstract operations) defined above are materialised in linear time (lexicographically) with the aid of the operations *in-time* (a) simultaneously and (b) successively.

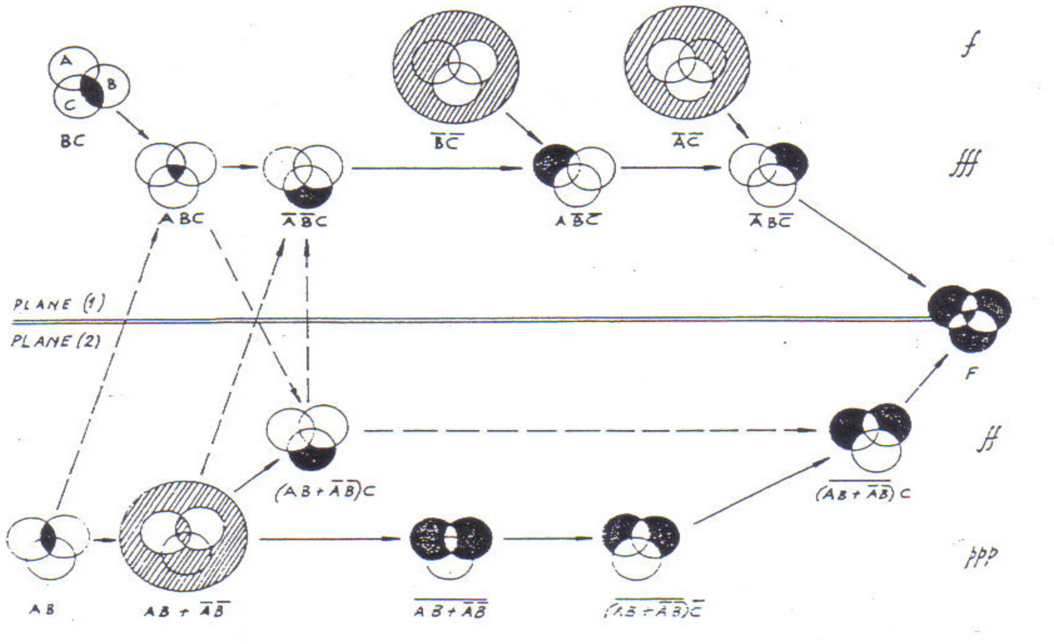
The classes in this piece are defined solely within the realm of pitch.

The elements of each class are presented stochastically, that is unrestrictedly …"

(Xenakis I, *Herma,* n.p. B&H,1962).

Having established the identity of the sets **R, A, B** and **C** used in *Herma*, Xenakis presents the expressions of the two differing functions for **F** shown above until the target set expressed in **F** is reached. Figure 11 illustrates how this is done: Plane I uses the first expression for **F** and plane II the second.

Fig.11 Form of *Herma*



(Xenakis I, *Formalized Music*, Stuyvesant, NY, 1992, pg. 171)

11iii. Universal Theory of Music

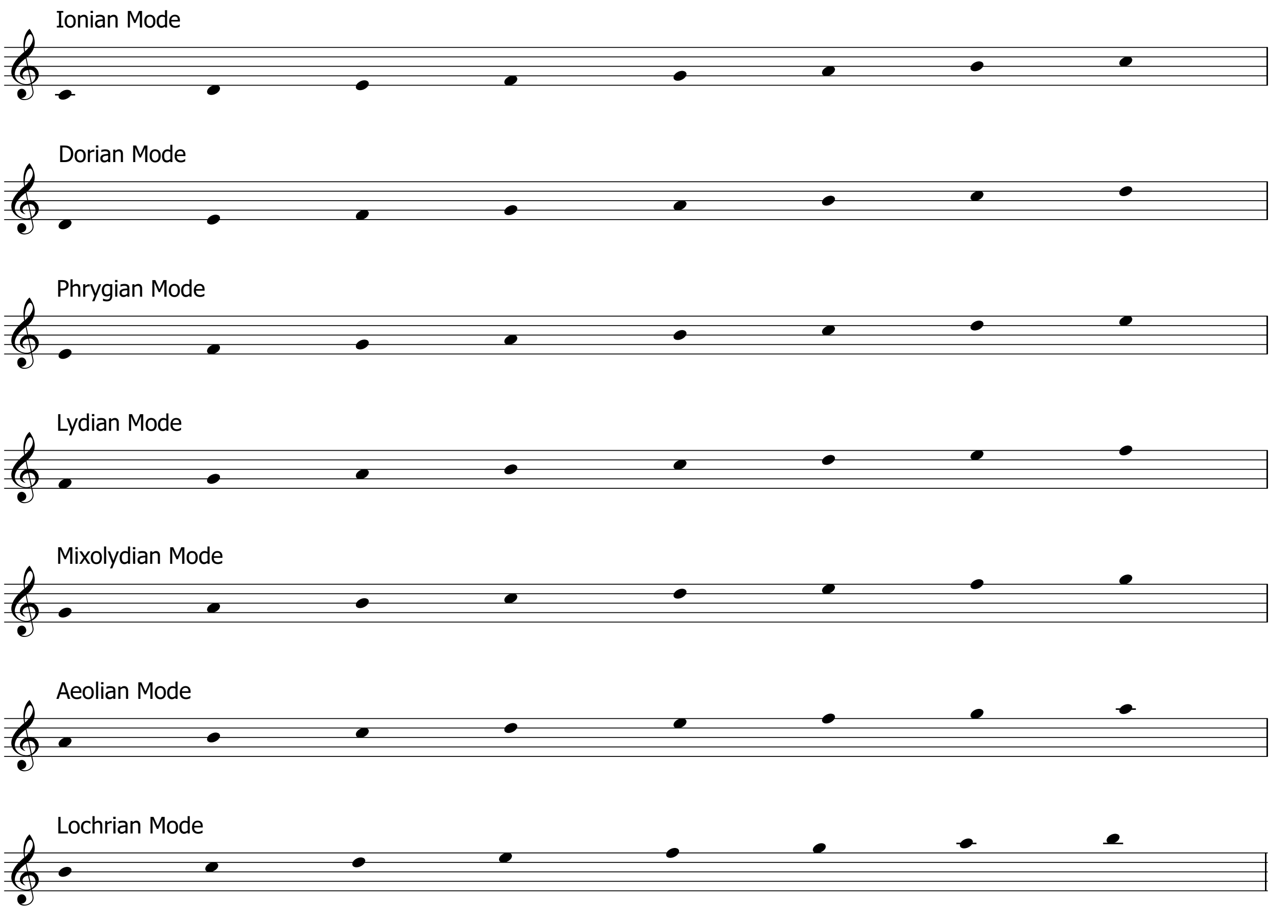
Xenakis' *Symbolic Music* applied in the piano piece *Herma* requires that a listener is able to comprehend and appreciate the logical structure of an *outside time* construct. This comprehension must apply when the *outside time* construct is presented *in-side time*. A listener to *Herma* is unlikely to be able to differentiate between sets **A** and **B** let alone an expression such as **AB** or **BC**. However, Xenakis does suggest an interesting possibility that an *outside time* construct such as his set **A** may be regarded as having an identity or logic that exists independently of

the music *in-time.* In the case of *Herma,* set **A, B** and **C** are constructed by Xenakis and are found only in this composition but other *outside time* constructs are universal.
Xenakis hints that stochastic methods were used to generate the rhythmic structures or *TS* employed in *Herma;* without giving any detail how he did this. A more traditional composer might use a rhythmic pattern or motif to realise the pitch sets in *Herma in-time.* Once the music materialises *in-time* through temporal operations on the pitch sets, Boolean functions may be used to represent the *ITS* symbolically. This can be seen in *Herma* where the Boolean functions shown in Figure 19 represent the *ITS* of *Herma.* Thus, a universal method to encode *ITS* symbolically arises from Xenakis' composition of *Herma.*

11iv. Expressing Music Symbolically

The following explanation shows how the modern *church modes* can be constructed using the Boolean operators of union, intersection and negation, outlined in the description of the composition of *Herma*. This is similar in approach to Xenakis' *sieve theory* (Ibid. pg. 194 - 200). Figure 12 below shows the *church modes* in notation.

Fig.12 Church Modes



In the composition of *Herma,* Xenakis used the 88 keys of the piano as a referential set **R.** The pitch sets **A, B,** and **C** are subsets of referential set **R.** The referential set used to express the *church modes* is the intervals that span an augmented fourth:

R = 0 1 2 3 4 5 6

Where 0-6 is a vector showing the number of semitones travelled up from a common origin.

In terms of pitch starting from the note **C** this would be:

R = C C♯ D D♯ E F F♯

Following the methodology of Aristoxenos (Ibid. pg. 184 - 187) tetrachords can be constructed from **R**:

Table5 Tetrachords

H	=	0	1	2	5
I	=	0	1	3	5
J	=	0	1	4	5
K	=	0	2	3	5
L	=	0	2	4	5
M	=	0	3	4	5
N	=	0	2	4	6

Using a first origin of the note **C**, no asterisk; and a secondary origin of the note **G**, with an asterisk the *church modes* shown in Figure 20 can be constructed as follows:

Table6 Symbolic Representation of Modes

Ionian	=	**L + L***	-	0	2	4	5	+	7*	9*	11*	12*
Dorian	=	**K + K***	-	0	2	3	5	+	7*	9*	11*	12*
Phrygian	=	**I + I***	-	0	1	3	5	+	7*	8*	10*	12*
Lydian	=	**N+L***	-	0	2	4	6	+	7*	9*	11*	12*
Mixolydian	=	**L+K***	-	0	2	4	5	+	7*	9*	10*	12*
Aeolian	=	**K+I***	-	0	2	3	5	+	7*	8*	10*	12*
Lochrian	=	**I+K***	-	0	1	3	5	+	7*	9*	10*	12*

This approach has the disadvantage of being an inelegant means of expressing the *church modes*, however, its strength lies in the fact that this method would produce any known scale or mode from any period or culture.

> "Moreover this method can unify the expression of fundamental structures of all Asian, African and European music."
>
> (Ibid. Pg. 200)

Xenakis used Boolean functions in *Herma* to produce an *ITS* (11ii. pg.60).

Operations of union, intersection and negation may be used to analyse musical structures. For *OTS* and *TS* the sets are un-ordered, the order of the members of a set does not affect the underlying logic of the set:

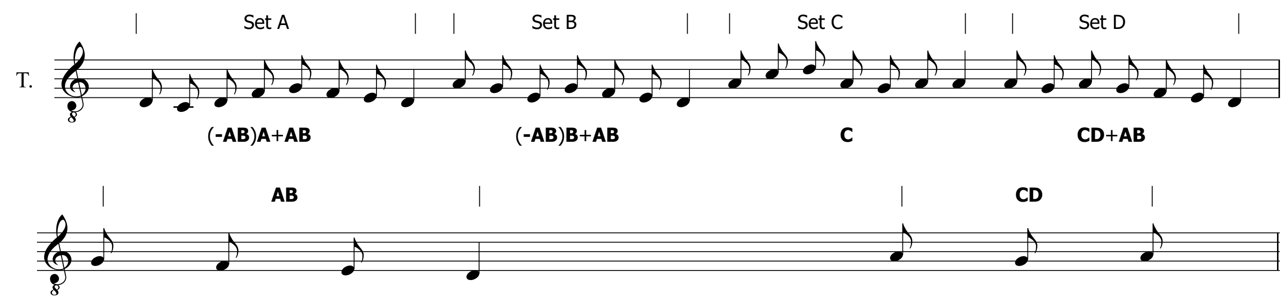
$$ATB = BTA$$

However, once time is considered and the *temporal functions* operate upon the *OTS* then:

$$ATB \neq BTA.$$

Thus, when sets are considered *in-side time* they are ordered sets and the members of a set must be ordered in the same way for one set to be regarded as equal to another. These ideas were applied to symbolically encode a monody. Example 13 shows the opening of a sequence *Victimae Paschali* attributed to the monk Wipo of Burgundy. The musical extract has been modified in its layout to show the ordered sets more clearly:

Ex.13 *Victimae Paschali*



(Ohl JF, Parrish C, *Masterpieces of Music before 1750*, London, 1980, pg.8)

The referential set **R** for this composition is the Dorian mode; this is also the *OTS*. When this set is considered *in-side time* it is regarded as containing every possible permutation of the Dorian mode.

Table7 Sets for *Victimae Paschali*

The set	**A**	=	D	C	D	F	G	F	E	D
	B	=	A	G	E	G	F	E	D	
	C	=	A	C	D	A	G	A	A	
	D	=	A	G	A	G	F	E	D	

These sets are shown in Example 13. The ordered sets of **A, B, C** and **D** give rise to a Boolean function which describes the unfolding of the music *in-time*:

F = (-**AB**)**A**+**AB**+(-**AB**)**B**+**AB**+**C**+**CD**+**AB**

AB and **CD** are shown in Example 13. This is not the only way in which the music may be expressed symbolically, for instance a trivial function would be **A+B+C+D**; Boolean function **F** was chosen to outline the reoccurrence of the ordered set **AB** and the use of the motifs **AB** and **CD.**

The strength of this method of analysis is its universality, it would apply equally well to all styles and genres of music. It shows the compositional choices made by the composer, in the case of the fragment of music above each segment ends with the same melodic formula **AB.**

The other advantage is the consideration of time is at the forefront of the analytical method. Boolean functions may be used to produce the un-ordered sets of the *OTS* and the time ordered functions representing the *ITS* built from ordered sets. It is the use of temporal operations that differentiate the unordered *outside time* sets and the ordered sets of the *ITS.*

Chapter 12. Xenakis' Attack on Polyphony

Xenakis is highly critical of polyphonic and linear principles in music, he says:

> "In 1954 I denounced linear thought (polyphony), and demonstrated the contradictions of serial music."
>
> (Xenakis I, *Formalized Music*, Stuyvesant, NY, 1992, pg. 182.)
>
> "Linear polyphony destroys itself by its very complexity; what one hears is in reality nothing but a mass of notes in various registers."
>
> (Xenakis I*, The crisis in serial music, Gravesaner Blatter,* 1 July 1956).

His antipathy extends to all polyphonic music not just serial or integral serial music. He writes about the:

> "Progressive Degradation of Outside-Time Structures."
>
> (Xenakis I, *Formalized Music*, Stuyvesant, NY, 1992, pg. 193).

For him, polyphony is purely *temporal*, with no *outside time* architecture. Only the Hellenic and Byzantine traditions described by theorists such as Aristoxenos and Ptolemy; the whole tone scale of Debussy; and the *modes of limited transpositions* described by Messiaen have a structure *outside time* (Ibid. pg. 208).

It will be shown that polyphonic and harmonic music have *outside time* entities which inform the logic and coherence of *ITS*. *Outside time sets* will be constructed for triads and *ITS* for functional harmony. The method used relates to the ideas of vectors to define pitch class sets. It follows Xenakis' set **E** which contains all possible values for pitch, duration and intensity (see 11i. pg.59). In this instance, only pitch values from a common origin will be considered. The intervals will be *normalised* into one octave only.

The following method expresses symbolically the triads of C major. The referential set is the C major scale; this is expressed as vectors from the common origin C:

R = 0 2 4 5 7 9 11

It is useful to represent this as letters rather than the pitch names:

R = h i j k l m n

Vectors representing major, minor and the chord on the seventh degree of the scale are as follows:

M	=	0	4	7	-	Major chord
N	=	2	5	9	-	Minor chord
O	=	11	2	5	-	Diminished chord

If the vectors in a set that represent a triad are altered by the addition of a scalar, an integer that is added to each member of the set, the triads of C major maybe written as follows:

Table8 Triads Expressed Symbolically

I	= **M**	=	0	4	7	=	**C E G**
II	= **N**	=	2	5	9	=	**D F A**
III	= **N** +2	=	4	7	11	=	**E G B**
IV	= **M** +5	=	5	9	0	=	**F A C**
V	= **M** +7	=	7	11	2	=	**G B D**
VI	= **N** +7	=	9	0	4	=	**A C E**
VII	= **O**	=	11	2	5	=	**B D F**

This approach shows the fundamental structure at the basis of harmony and tonality. **M** is the low energy state of the system. Chords **IV** and **V** have a higher potential energy with the addition of the scalar. They have an inbuilt tendency to reach **M.** If chord **IV** is written as: **M** – 7 then it behaves like chord **V** but in reverse with chord **I** in the higher energy state. Thus, chord **V** to **I** is the stronger relationship, with chord **IV** to **I** a subsidiary relationship which mirrors **V** to **I**. Chords **II**, **III** and **VI** form an alternative substructure in which **VI** to **II** is similar to **V** to **I** and **VI** to **II** pertains to **IV** to **I**. **N** is nearly equal to (**M** + 7) + 7 and thus behaves like a substitute chord **V** to approach **M** + 7 , the original chord **V**. This view of tonality concurs with the writings of Hauptman:

> "In *Die Natur der Harmonik und der Metrik* (1853), Hauptmann represented the harmonic infrastructure of C major as F–a–C–e–G–b–D, where large letters designate dominant-related perfect 5ths and small letters their mediant major (or minor) 3rds. In this arrangement of intervals and pitch classes, each string of three consecutive letters forms a diatonic triad: the tonic C–e–G, dominant G–b–D, and subdominant F–a–C, of course, but also the mediant e–G–b and submediant a–C–e. In this case, E minor mediates between the tonic and dominant above, while A minor mediates between the tonic and subdominant below; the submediant is a mediant below the tonic."
> (Tonality, *Oxford Music Online,* ed. Brian Hyer (Accessed 18 April 2017).

If the triads of C major are written in an array using letters the interrelationship of the scale to the chords is revealed:

Table9 Array of C Major Triads

Array of letters			**Array of pitches**		
h	j	l	**C**	**E**	**G**
i	k	m	**D**	**F**	**A**
j	l	n	**E**	**G**	**B**
k	m	h	**F**	**A**	**C**
l	n	i	**G**	**B**	**D**
m	h	j	**A**	**C**	**E**
n	i	k	**B**	**D**	**F**

If just one note of the scale was changed it would upset the whole scheme, for instance if D♭ was used instead of D this would impact on chords **II**, **V** and **VII** and the vector relationships shown above would collapse. The way the scale interacts with the harmony means that only a certain number of scale types will produce a vector hierarchy like that found in the C major scale.

The triadic hierarchy of C major is an *OTS* since the ordering of the sets does not alter the logic of the system and:

(0 4 7) = (4 0 7) = (7 4 0) etc.

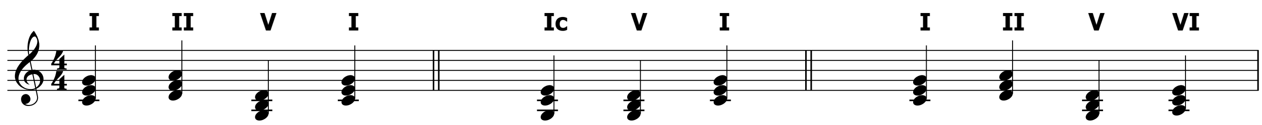
MT**N** = **N**T**M** etc.

Once time is taken into consideration the ordering of the triads *in-time* becomes significant, resulting in chord progressions. Following a similar procedure for *Victimae Paschali,* the chord progressions may be written as functions. A further operator **s** is included which shuffles the order of the vector:

Ms1 = (4 7 0) ; **Ms2** =(7 0 4)

In convectional notation these are chords **Ib** and **Ic**. Figure 13 below shows some common chord progressions:

Fig.13 Common Chord Progressions



These maybe be written as functions in the following way:

I II V I: **F** = **M** + **N** + (**M** + 7) + **M**

Ic V I: **F** = **Ms2** + (**M** + 7) + **M**

I II V VI: **F** = **M** + **N** + (**M** + 7) + (**N** + 7)

This approach could be used to analyse the harmonic structure of any piece of music. It gives ideas on how to construct tonal systems which may be used as the basis for composition. The symbolic encoding of music and hierarchies of chords, outlined in this chapter, was utilised in the composing of *Flumina, Concertino, Mutation of Dowland's Midnight, Mutation of Missa Papae Marcelli, Evolution of the Cosmos* and *This Day Dawns.*

Throughout the explanation of the composing of *Herma,* and in the use of vectors to describe chord progressions, the "+" sign has taken on a new meaning to that found in mathematics. It usually joins the members of one set to those of another set to form a new, larger set. Whilst this is true to a certain extent in the symbolical encoding for *Victimae Paschali* and the common chord progressions, in the musical usage, "+" partitions metric time into discreet, recognisable musical entities. A listener hears the vectors that represent chords as discreet entities, not simultaneously. Thus, the "+" sign represents the start-point of a new entity entering the metric set **T**, the temporal space in which the music unfolds *in-time.* This new musical meaning for the "+" sign becomes significant when music is encoded as an integer. (20i. pg.125).

Chapter 13. Modal Compositions

13i. Relative Consonance and Dissonance

For the modal compositions, the notion of dissonance and consonance is conceptually different to the historical classification of interval-types (see 5iv. pg.31). Consonant intervals in music have been explained in terms of Pythagorean simple ratios of: 2:1, 3:2, 4:3 etc. corresponding to an octave, fifth, fourth. Dissonance is the antonym of consonance and regarded as "rough" or "unpleasant" in contrast to consonant sounds. (Consonance, *Oxford Music Online,* ed. Moore BCJ and Palisca CV (Accessed 18 April 2017).

A distinction of ratio maybe used as a criterion to classify intervals, with consonant intervals having simple ratios in regard to the division of a string length, while dissonant intervals have complex ratios. However, once harmony and chord types are considered in relation to consonance and dissonance the situation becomes ever more complex. Perhaps only that which is learnt culturally ultimately decides whether a listener finds a chord smooth and pleasant or rough and unpleasant (Ibid.). This concept of relative consonance replaces the historical classification of intervals.

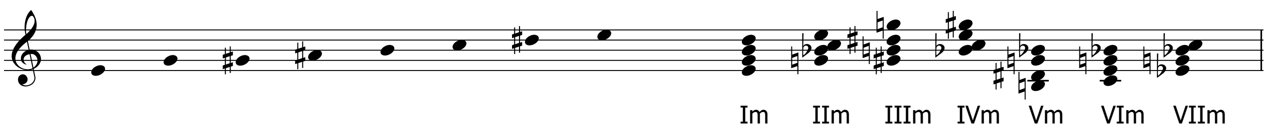
13ii. *Two Explorations of a Chord Sequence*

This composition for solo electric guitar is submitted as an appendix. The title refers to the formal structure of the music which is a repeated chord sequence where the harmonic rhythm either increases or decreases in proportion to the number 2 upon each repetition of the sequence.

This composition used mode cd shown in Figure 14 to expand the pitch-organisation in comparison to that found in *Guitar Sonata I* and *String Quartet I,* in which only the interval type between two or more notes was considered.

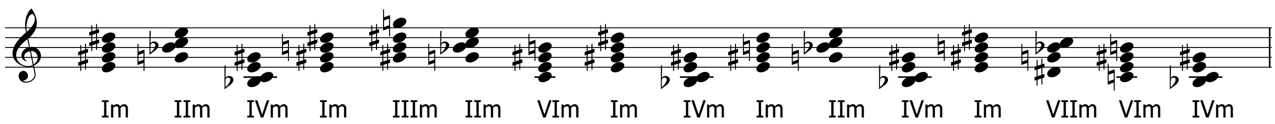
Chords were generated from mode cd by following the traditional practice of taking alternate notes from the mode. Figure 14 shows the mode and the chords produced. The chords are labelled as Im, IIm etc. to differentiate them from traditional chords I, II etc.

Fig.14 Mode, Chords: *Two Explorations of a Chord Sequence*



This mode and associated chords was used to generate the chord sequence referred to in the title. Figure 15 shows this chord sequence:

Fig.15 Chord Sequence: *Two Explorations of a Chord Sequence*



13iii. Mutation of Mozart's Jupiter Symphony

This is the first of in a series of pieces to use pre-existing music as a model to control medium and large scale structures in a new composition. The process used to achieve this has been labelled as a *mutation. Mutation of Mozart's Jupiter Symphony (MJS) mutates* the first movement of Mozart's *Symphony 41* in C major, K.551*.* This movement was chosen for its rhythmic energy and thematic diversity. The constant changes of tempo in the *Mutation of MJS* were included after hearing performances of K.551 in which the tempo varied from one section to another.

This composition uses mode cd from a starting note of **C**. The chords are generated by taking alternative notes from mode cd. Figure 16 shows the mode and associated chords. The development section of the sonata form used mode dd. Figure 17 shows this mode and chords, the recapitulation returns to the original mode.

Fig.16 Modes, Chords: *Mutation of Mozart's Jupiter Symphony*

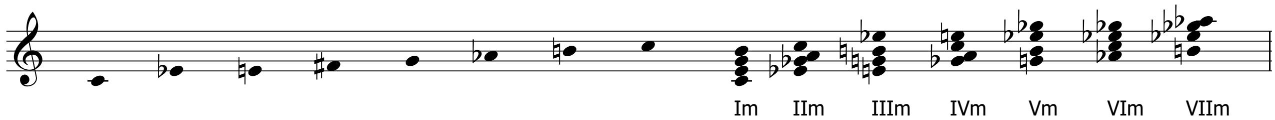
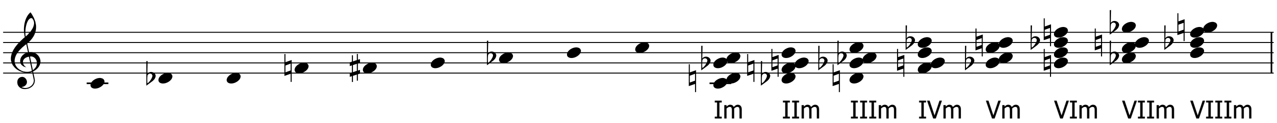


Fig.17 Modes, Chords: *Mutation of MJS* Development



Mutation of MJS extends the technique first used in *Two Explorations of a Chord Sequence,* of a repeated chord sequence to familiarise the listener with the consonant sounds of this standalone composition. After several repetitions of the chord sequence the chords become a relatively consonant sound to the listener. If a passing note or suspension alters a chord this becomes a momentary, unsettling dissonance which is resolved once the familiar chord sequence resumes. Figures 18, 19 and 20 show the chords sequences used for the exposition, development and recapitulation.

Fig.18 Chord sequence: *Mutation of MJS* exposition

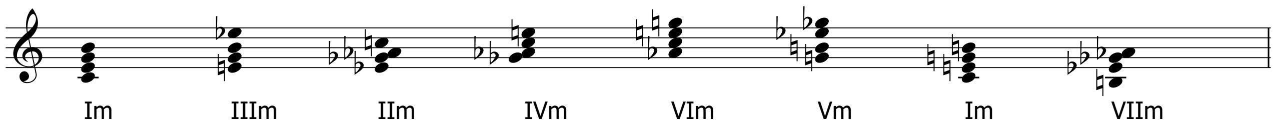


Fig.19 Chord sequence: *Mutation of MJS* development

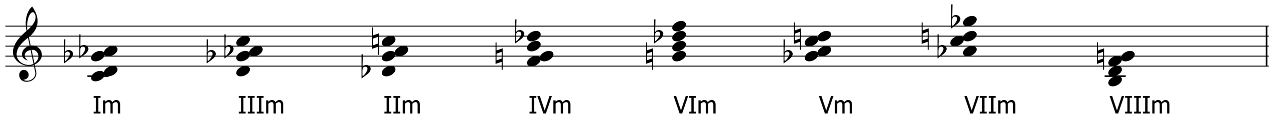
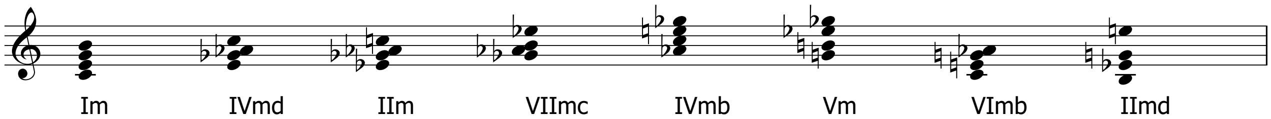


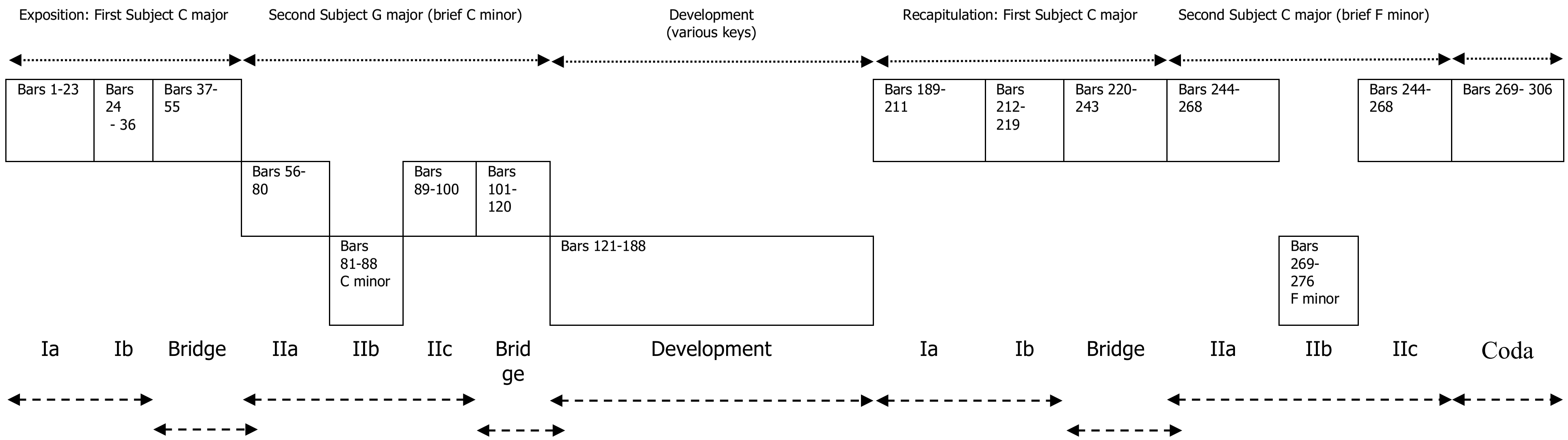
Fig.20 Chord sequence: *Mutation of MJS* recapitulation



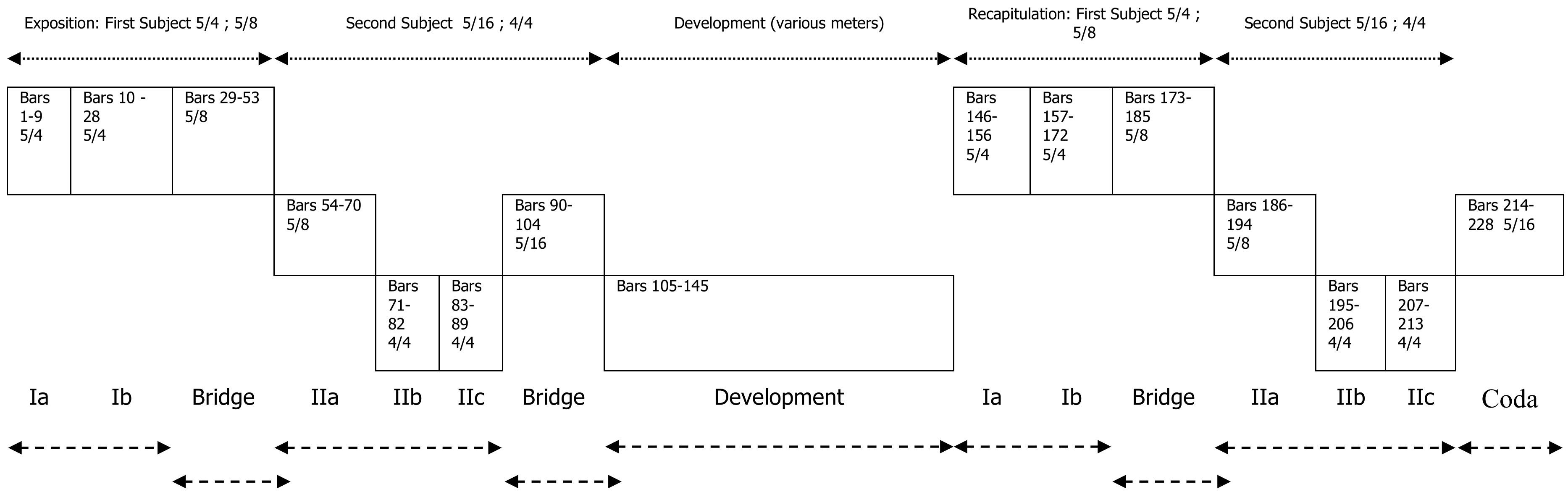
13iv. Form of *Mutation of Mozart's Jupiter Symphony (MJS)*

Form Charts 7 and 8 shows the forms of Mozart's *Symphony 41* first movement and the *Mutation of MJS.*

Form Chart 7 *Mozart's Symphony 41, K551*



Form Chart 8 *Mutation of Mozart's Jupiter Symphony*



Mutation of MJS uses a subdivision of metre in proportion to the number 2, to articulate a sonata form. The subdivision of the metre produces changes in *harmo-rhythmic structure (HRS),* a contrast of *HRS* articulates the formal sections.
Mozart's original music uses a contrast of key to delineate sections of the sonata form, in the *mutation* a contrast of *HRS* articulates the form.
The *mutation* of Mozart's music regards the *in-time structure (ITS)* of the first movement of Symphony 41, K.551, as a *copy* of an eternal Platonic *Form*. For the mutation, the *ITS* of Mozart is *mutated* using an alternative *outside time structure (OTS)* and *temporal structure (TS)* (see 9iii. pg.51).
Mozart's *OTS* is a C major scale and its associated triads. The music modulates to the related key of G major and the remote keys of C minor and F minor, the development section utilises several keys. The *TS* is a 4/4 metre throughout.
Mozart's *ITS* uses the triads of C major and the other keys to produce functional harmony, the large-scale contrast of key results in a sonata form.
The following analysis of the 1st movement was undertaken after reference to Sisman E, *The Jupiter Symphony,* Cambridge, UK, 1993; the preface to Mozart W, *Jupiter Symphony,* Harmondsworth, England, 1951 by Jacob G; and Rosen C, *The Classical Style*, London, 1976.
Table10 Form of Symphony 41, K551 1st Movement. (Shown in Form Chart 7).
"Jupiter Symphony" I: Allegro vivace, C major, C time, 313 bars

EXPOSITION, bars 1-120

First group C major		
	Ia	1 loud orchestral tutti in octaves contrasted with piano vln I melody
		9 fanfare with inversion of tutti figure in strings
First group / bridge passage		
	Ib	24 restatement of Ia with countermelody in the woodwind
	Br.	37 Ia with vln I melody extended over dominant pedal.
		49 fanfare with tutti figure in the strings
Second group G major		
	IIa	56 Ia vln I melody rhythm in augmentation and diminution. Imitation between vln I and vla, vlc and cb.
	IIb	81 abrupt modulation to remote key of C minor.
	IIc	89 rhythmic development of b. 37-48 over tonic pedal.
Closing G major		
		101 aria theme in vln and woodwind.
		111 chromatic figure in vln I. Dominant pedal in vln II and fl.
		117 fanfare codetta with tutti figure in vln I.

DEVELOPMENT, bars 121-188

121 woodwind passage cadence in E♭ cf. Sym 40 1st Mov.
124 b. 101 aria theme in E♭.
133 cadential phrase of aria theme in E♭. Imitation between upper and low strings. B.9 fanfare in woodwind.
155 false retransition in subdominant key of F.
161 false recapitulation in F.
171 development of b. 49 fanfare.
181 proper retransition in C.

RECAPITULATION, bars 189-313

First group C major		
	Ia	189
		197 fanfare
bridge passage		
	Ib	212
	Br.	220
		225 fanfare
		237 fanfare at b. 49 reworked in C.
Second group C major		
	IIa	244 IIa reworked in C.
	IIb	269 IIb reworked in remote key of F minor
	IIc	277 b. 89-100 reworked in C.
Closing (tonic)		
		289 b. 101 aria theme in C.
		299 b. 111 chromatic theme reworked. Dominant pedal in timp.s
		306 b. 117 fanfare reworked and extended.

For the *mutation,* the *OTS* are the modes and associated chords shown in Figures 16 and 17. The *TS* is a $\frac{5}{4}$ metre with subdivision to $\frac{5}{8}$ and $\frac{5}{16}$ and an additional section in $\frac{4}{4}$ metre. The *ITS* of the *mutation* uses Mozart's original *ITS* as a model, however, the chord sequences shown in Figures 18, 19 and 20 replace the functional harmony of Mozart, the contrast of key, outlining the formal sections of Mozart's music, are replaced by a contrast of *HRS.*

Theme Ia (shown in Form Chart 8) in the Exposition of the *mutation* is clearly derived from the Ia (shown in Form Chart 7) in Mozart's exposition, Example 14 shows Ia of the *mutation.* This theme uses the chord sequence shown in Figure 18, with a chord change occurring mostly on beat one of a $\frac{5}{4}$ bar.

Ex.14 *Mutation of Mozart's Jupiter Symphony* Ia, string quartet only. Chord change shown above violin I.

At bar 29 the metre subdivides to $\frac{5}{8}$ and the chords change mostly on quavers 1 and 6, Example 15 shows the start of the Bridge (shown in Form Chart 8).

Ex.15 *Mutation of MJS* Bridge. Chord change shown above oboe and clarinet.

The second subject begins at bar 54 in $\frac{5}{8}$ metre with a chord change mostly on semiquavers 1, 6, 11 and 16, Example 16 shows the beginning of theme IIa (shown in Form Chart 8), the viola part in particular outlines the change in *HRS.*

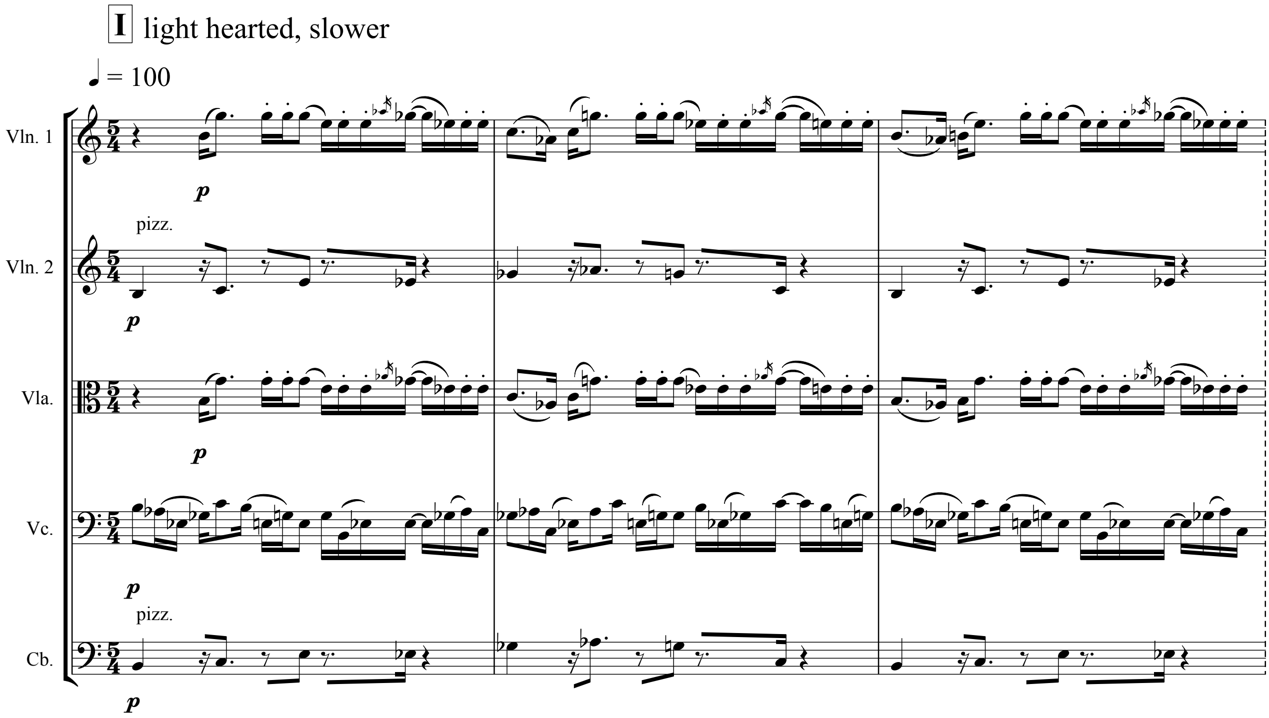
Ex.16 *Mutation of MJS* IIa. Chord changes shown above viola.

IIb (shown in Form Chart 8) at bar 71 abruptly switches to a $\frac{4}{4}$ metre and the chord sequence shown in Figure 18 briefly modulates up a major third, this mirrors the shock modulation in Mozart's music to the unexpected key of C minor, Example 17 shows the start of IIb. In IIc (shown in Form Chart 8) at bar 83 the chord sequence shown in Figure 18 modulates down a major third.

Ex.17 *Mutation of MJS* IIb, string section only. Contrabass outlines modulated chord sequence.

Example 18 shows the Bridge (shown in Form Chart 8) which leads into the development at bar 90. The music is in $\frac{5}{16}$ metre, with chord change every 5 semiquavers.

Ex.18 *Mutation of MJS* Bridge, string section only. Contrabass outlines chord sequence starting from VIIm.



The development section uses mode dd and the chords shown in Figure 17, the chord sequence for the development section is shown in Figure 19. This change of chord sequence is a parallel to Mozart’s development section which explores a variety of contrasting keys. The development section of the *mutation* uses a number of differing metres with associated *HRS.*

In Mozart’s original music the recapitulation returns to the home key of C major and remains within this key area throughout the second subject apart from a brief modulation to F minor at bar 269.

In the *mutation,* the contrast of *HRS* articulates the form of the music. The consequence of this is the second subject will retain the same *HRS* in the recapitulation and the resolution of key that occurs in Mozart’s original music is lacking in the mutation since the *HRS* remain unaltered.

To address this problem the recapitulation uses an altered chord sequence shown in Figure 20 until the return of IIb. This has the effect that the second subject feels like a resolution since the chord sequence of the exposition is finally heard in its entirety once more. In the recapitulation theme IIb briefly modulates to a starting note of **A**, down a 5th from the exposition, for the chord sequence shown in Figure 18, in order to mirror the return of this theme in F minor in Mozart’s music.

13v. Evaluation of The Modal Compositions

Two Explorations of a Chord Sequence and Mutation of MJS introduced several innovations which featured in all the subsequent compositions:

1. The use of relatively consonant or dissonant pitch-organisation. In particular, the use of repeated chord sequences to familiarise the listener with a relatively consonant sound.
2. The use of pre-compositional *OTS* to regulate the pitch-organisation of the music.
3. The use of the contrast of *HRS* to articulate musical forms.
4. Remaking the past in regard to pitch structures, durations, tonality and sonata form.
5. The *mutation* of the *ITS* of pre-existing compositions by altering the *OTS* and *TS* of the original music.

The sonata form of *Mutation of MJS* used an altered chord sequence to differentiate the recapitulation from the exposition. In subsequent compositions, a contrast of tonality was used to articulate the form of the music. Thus, the recapitulation in the sonata forms of subsequent pieces, followed the traditional practise of rewriting all the themes in the recapitulation within the home tonality (see 15iii. pg.91).

Chapter 14. Tonality

14i. Introduction

Mutation of Mozart's Jupiter Symphony remakes the opening movement of Mozart's *Symphony 41,* K.551*.* This *remaking* of the past is a strategy found not only in the music of neo-Classicists such as Stravinsky, Bartók, Britten, Copland but also Schoenberg, Webern and Berg who extended tonal practices in new ways in their atonal and twelve tone works. Joseph Straus describes this *reworking* of previous music as an anxiety of influence and anxiety of style in his book *Remaking the Past*. (Straus J, *Remaking the Past,* Cambridge: Harvard Uni. Press, 1990).

14ii. Straus' discussion of Tonality, Scales, Triads and Sonata Form.

Straus describes the anxiety of style in the following way:

> "This anxiety has manifested itself in the twentieth century in a number of musical ways. When composers use triads, the central sonority of traditional tonal music, they are responding to a widely shared musical element, not to some specific work or individual composer."
>
> (Straus, J. *Remaking the Past,* Cambridge: Harvard Uni. Press, 1990, pg. 18).

Straus applies the theories of Harold Bloom, who developed his ideas regarding poetry, to the music of the twentieth century. For Straus and Bloom this anxiety of style leads later composers/poets to wilfully misread the stylistic practice of older composers/poets. This misreading is a rejection of a parent in a Freudian sense:

> "The later poet seeks artistic freedom by symbolically killing the precursor-parent. This symbolic murder takes the form of misreading."
>
> (Ibid. pg. 14)

Thus, for Straus a composer such as Stravinsky strives to retrospectively re-invent traditional musical practise in a post-tonal setting. The parent composer's characteristic musical traits now exist only in relation to the new post-tonal musical language. The traditional elements of the older composer are done away with and are remade in the image of the younger composer.

Straus devotes a whole chapter in *Remaking the Past* to a discussion of the use and implications of triads in post-tonal music. He writes:

> "Twentieth century composers enmesh the most characteristic and fundamental sonority of common practice music in a new network of structural relations. They misread the triad, striving to neutralize its tonal implications and to redefine it within a post-tonal context."
>
> (Ibid. pg. 74).

Straus and Charles Rosen outline how sonata form has historically been regarded as either a binary form based on contrast of key or a ternary form utilising contrast of theme. For Straus:

"There have been two principle ways of interpreting and writing sonata forms…. For eighteenth century theorists, the sonata form… was essentially a two-part structure shaped by contrasting harmonic areas.

In the nineteenth century the sonata form changed as the nature of the musical language changed. With the rise of a less sharply focused tonal language … the generating force of the harmony became dulled… For nineteenth century theorists, the essence of the form was its themes; the form was determined by thematic contrast and thematic repetition. This view… considers the sonata a three-part form.

(Ibid. pg. 96-97).

Rosen states:

"A description of the sonata in fundamentally melodic terms was as unsuited to the eighteenth century's more dramatic structures as the long-breathed melodies of the nineteenth century were inapt for late eighteenth century forms."

(Rosen, C. *The Classical Style,* London, 1976, pg. 32)

The works Straus analyses in his discussion of sonata form in a post tonal setting are: Stravinsky's *Symphony in C* and his *Octet;* Bartok's *Piano sonata* and *String Quartet No. 2, 1st movement*; Schoenberg's *String Quartet No. 3.* (Straus J, *Remaking the Past,* Cambridge: Harvard Uni. Press, 1990, pg. 98-132). Stravinsky's *Symphony in C* employs a contrast of tonal areas while Schoenberg's *String Quartet 3* follows a more thematic approach, the other works contain both elements.

14iii. Remaking *In-time Structures*

In-time structures (ITS) may be regarded as containing the choices a composer has made *in-side time*. Informing these choices lie deeper, more abstract structures such as scales, triadic hierarchies, metres, rhythmic cycles etc. *OTS* and *TS* are described as modelling *copies* or *imitations* of an eternal Platonic Form built from quanta of pitch and duration in 18v. pg.117. *ITS* may be modelled as *copies* or *imitations* of an eternal Platonic *Form*. An acknowledged masterpiece such as Mozart's *Symphony 41,* K.551, has an *ITS* that may be moulded into another *copy* or *imitation* of a Platonic Form by altering the *OTS* and *TS* from that of the original (see 18vi. pg.121).

Straus describes several works by composers who have employed a methodology approaching this compositional strategy[28]. He gives detailed descriptions of the following remaking of previous compositions by post tonal composers: Bartók's *Piano Concerto No. 3* reworks Beethoven's *String Quartet in A minor*, op. 132; Berg's *Violin Concerto* is based on Bach's harmonization of *Es ist Genug*; Berg's

[28] However, Straus makes no mention of Platonic Forms or Xenakis' *Symbolic Music*

Lyric Suite uses elements from *Tristan and Isolde* by Wagner; Stravinsky's *Serenade in A* responds to Chopin's *Ballade No. 2*; scene's from *The Rake's Progress* by Stravinsky reworks music from *Don Giovanni* by Mozart; Schoenberg's *String Quartet No. 3* reworks Schubert's *String Quartet in A minor*, op. 29. (Straus J, *Remaking the Past,* Cambridge: Harvard Uni. Press, 1990, pg. 133-168).
The *mutation* of Mozart's *Symphony 41,* K.551, first movement (see 13iii. pg.74) is an overt use of an existing *ITS* to inform a new composition. The approach is notably different to that described by Straus since the mutation of the music is piece specific rather than a response to an anxiety of style and influence. Mozart's music is viewed as a *copy* or *imitation* of a Platonic *Form*. The *ITS* of Mozart's first movement is *mutated* by changing the *OTS* and *TS* of the original. These is no need for a symbolic murder of the parent composer, the choices made by Mozart remain untouched in the *ITS* of the new music.

Chapter 15. Chordal, Tonal Compositions

15i. Overview

The pitch-organisation in *Flumina* and *Concertino* expands upon the intervallic approach used in *Guitar Sonata I* and *String Quartet I* and the four-note chords employed in *Two Explorations of a Chord Sequence* and *Mutation of Mozart's Jupiter* by using a contrast of consonant and dissonant intervals to construct di-note and tri-note chords, built either entirely from dissonant intervals or entirely consonant intervals. This mirrors the pitch-organisation of Palestrina where chords are either major or minor.

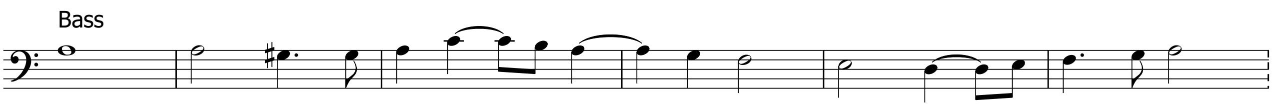
15ii. Flumina

Flumina is a single movement work for chamber orchestra based on the motet *Super Flumina Babylonis* by Palestrina. The title means river in Latin and refers to the Latin word in the title of the motet. This motet was chosen since the opening bass melody may be modified to outline chords built from mode cd.

The musical style in *Flumina* blends stylistic elements from both Palestrina and Mozart. The use of tonality and sonata form is a *remaking* of the style of Mozart. The introduction of *Flumina,* from bars 1-8 is reminiscent of the opening of *Symphony 41,* K.551 first movement with block orchestration on the tonic and dominant notes in Mozart's music replaced with the notes **C** and **F♯** in *Flumina*. The first and second subject of Flumina make use of imitative entries, characteristic of Palestrina's music.

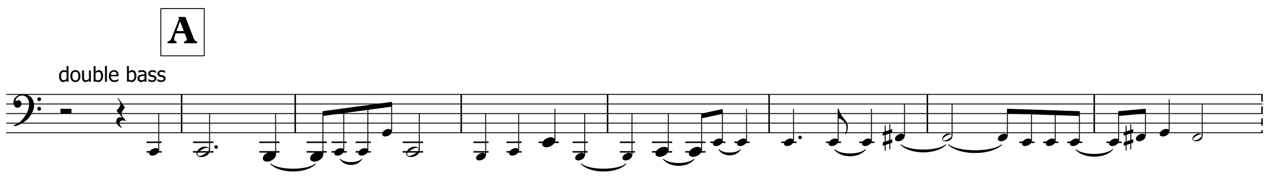
The themes employed throughout *Flumina* are a variant of the opening melodic phrase in the Bass of *Super Flumina Babylonis,* shown in Example 19.

Ex.19 Bass melody from *SFB* by Palestrina

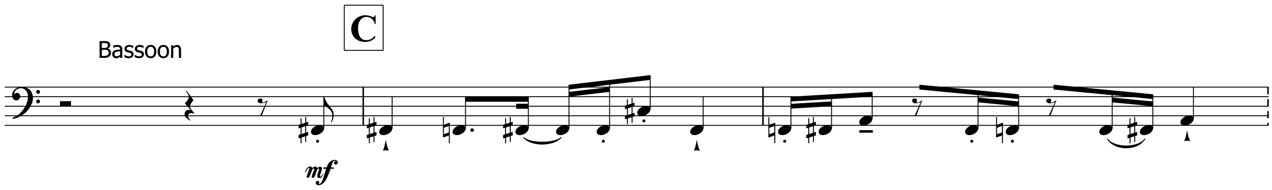


The larger notes in Example 20 show how the notes in Ia (shown in Form Chart 9) from *Flumina* outline the melodic phrase from the Bass of *Super Flumina Babylonis.* IIa (shown in Form Chart 9) in Example 21 is clearly derived from the Ia.

Ex.20 *Flumina* Ia



Ex.21 *Flumina* IIa

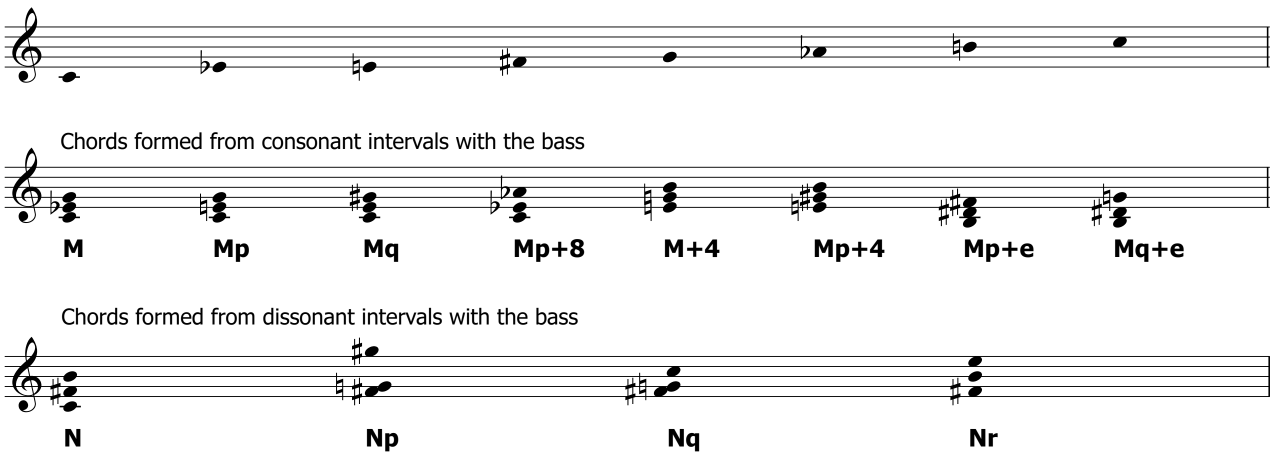


Flumina uses the mode cd starting on the note **C**. The following rules were employed in the generation of tri-note chords from mode cd shown in Table 4. This replaces the previous method of taking alternate notes from the mode to form chords (see 13ii. pg.73):

1. For pitch class set **C E♭ E**: no chord or melodic phrase can contain both **E♭** and **E** unless **E♭** or **E** are used as passing notes between chords.
2. **C E** may form chords with **G** and **A♭** from the pitch set **F♯ G A♭**. **C E♭** may form chords with **G** or **A♭** if **A♭** appears below **E♭**.
3. For pitch set **F♯ G G♯ B**: no chord or melodic phrase can contain both **G** and **B** or **G♯** and **B** unless they are used as passing notes between chords.
4. **F♯ G B** can form chords with **C** from the first pitch set. **G** must appear below **C**. **F♯, B** can form chords with **E. B** must appear below **E**.

These rules extend the idea of classifying intervals to produce modes (see 9v. pg.53) to the chords produced from the modes, since the chords consist of notes either consonant or dissonant with the bass note but not a mixture of consonant and dissonant notes. Figure 21 shows the chords produced when these rules are applied to the mode. This approach to pitch-organisation encapsulates the ideas of producing a hierarchy of chords, like those shown in Table 10.
Two aspects of this approach to pitch-organisation relate to Palestrina's use of triadic harmony in *Stabat Mater* (see 6. pg.37). Firstly, the actual use of the tri-note chords shown in Figure 21 rather than the four note chords used in the composition of *Two Explorations of a Chord Sequence* and *Mutation of Mozart's Jupiter Symphony* (see 13ii. pg.73). Secondly, there are two basic harmonic sounds in the chords shown in Figure 21 dissonant chord types and consonant chord types, this mirrors the pitch-organisation of Palestrina where chord types are either major or minor.

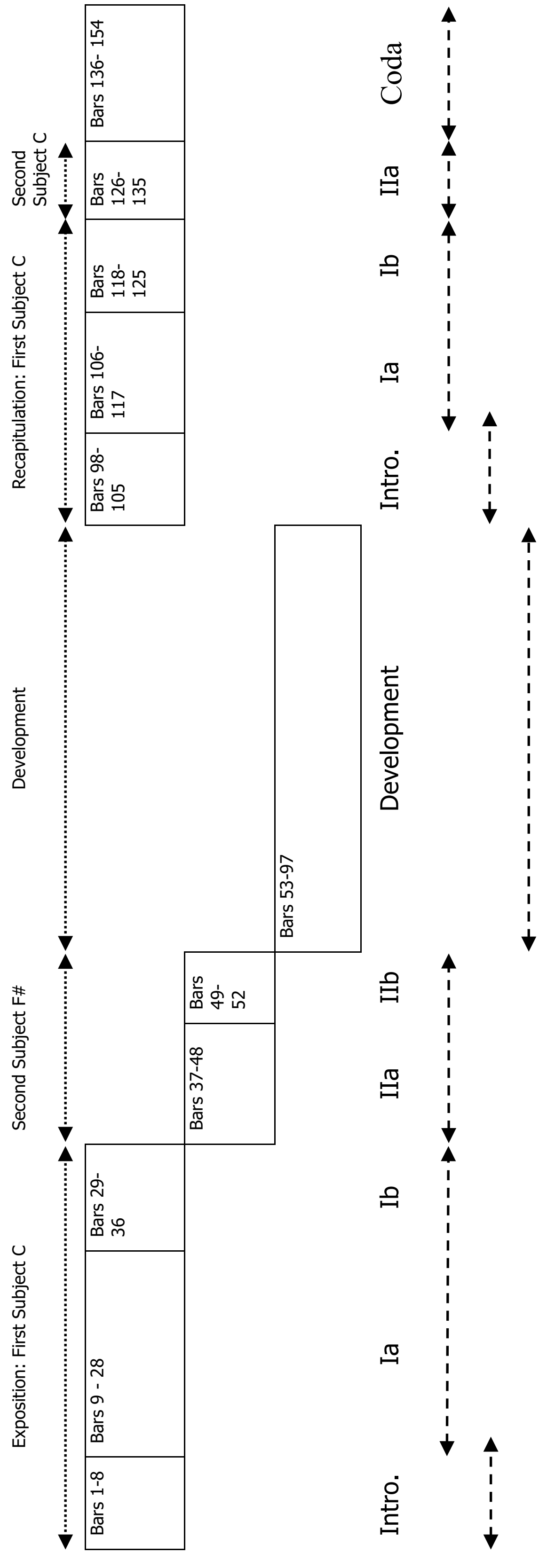
Fig.21 Mode, Chords: *Flumina*



15iii. Form of Flumina

Form Chart 9 shows the form of *Flumina*.

Form Chart 9 *Flumina*



Like *Mutation of Mozart's Jupiter Symphony, Flumina* uses the subdivision of metre with the number 2 as a denominator, to articulate a sonata form. In addition to using a contrast of metre and associated *harmo-rhythmic structure (HRS), Flumina* uses a modulation to the mode shown in Figure 21 starting from the note **F♯** , to delineate the second subject group. Following classical procedure, in the recapitulation the second subject group does not modulate and remains in the mode shown in Figure 21 starting from the note **C**. In a traditional sonata form a contrast of key outlines the sections of the music; in *Flumina* this is replaced by a contrast of *harmo-rhythmic structure,* reinforced with a modulation in the second subject.

Writing about tonality and the musical language of the late eighteenth century Charles Rosen states:

> "The scales by themselves would imply a system which is not tonal but "modal": one in which the centre of is a note, each work is restricted to the notes of its mode, and the final cadences are conceived as melodic, rather than harmonic, formulas."
>
> (Rosen, C. *The Classical Style,* London, 1976, pg. 25).

He defines tonality as:

> "Tonality is a hierarchical arrangement of the triads based on the natural harmonics or overtones of a note."
>
> (Ibid. pg. 23).
>
> "… the centre of a tonal work is not a single note but a triad."
>
> (Ibid. pg. 25).

In his book *Theory of Suspensions,* Komar demonstrates that triads inform all the structural levels of a tonal work:

> "Following Schenker, I conceive of a tonal piece as a hierarchy of structural levels progressing from relatively simple higher levels through middle levels of greater complexity to still more elaborate levels at the bottom of the hierarchy. The highest level of a tonal piece consists of a root-position major or minor triad, and the lowest level consists of the music itself."
>
> (Komar, A. *Theory of Suspensions,* Princeton N.J. 1971, pg. 11).

The tonality used in *Flumina* is described as an *artificial tonality* since the hierarchy of the chords shown in Figure 21 is not related to the natural overtone series.

The *artificial tonality* in *Flumina* differs from that described by Rosen, Komar and Schenker since the centre of the music does not involve a root position major or minor triad. However, where there is common ground is the alteration of previously sounded chords by transposition. For example, in Mozart's *Symphony 41,* K.551 first movement the dominant seventh chord on **G** found in the first subject is transposed to a dominant seventh on **D** in the second subject. In *Flumina* the chords shown in Figure 21 are transposed by an augmented fourth at the start of the second subject, introducing a new set of chords. In a similar procedure to that of Mozart, in the recapitulation of *Flumina* the second subject remains in the home tonality, in

contrast to the exposition, the chords remain unaltered.
In *Flumina* a rhythmic cycle is employed as the *temporal structure* (see 9iii. pg.51). The introduction and first subject are based on a rhythmic cycle of thirty-two quavers; this is subdivided into:

9 + 9 + 8 + 6 quavers.

This is further subdivided into:

6 + 3 +3 + 6 + 4 + 4 + 3 + 3 quavers.

The harmony mostly changes on quavers 1, 10, 19 and 27 in bars 1 – 28. Example 22 shows Ia (shown in Form Chart 9) at bar 16.

Ex.22 *Flumina* Ia. Numbers over violin II and cello show beat number in rhythmic cycle. Chord changes are shown above piano and flute.

The modulation at bar 29 to a starting note of **F♯** for mode cd and chords shown in Figure 21 is reinforced by a new *HRS* in which the rhythmic cycle subdivides into:

9 + 9 + 8 + 6 semiquavers.

This subdivision leads to a change of *HRS* and the chords change on semiquavers 1, 10, 19 and 27. Thus, in comparison to the first subject, the second subject has a contrast of *harmo-rhythmic structure* reinforced with a modulation. Example 23 shows IIa (shown in Form Chart 9) at bar 42.

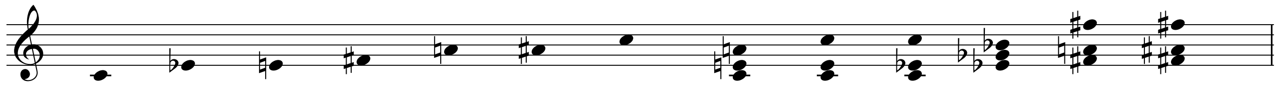
Ex.23 IIa *Flumina.* Numbers above flute and horn show beat number in rhythmic cycle. Chord changes are shown above oboe.

Examples 22 and 23 illustrate the change in *artificial tonality* from the first to the second subject. Ia, Example 22, has a chord **Nr**, **F♯ B E**, on beat 1 of bar 17. In IIa, Example 23, has a similar chord transposed to **C F B♭**, on beat 1 of bar 45. This chord represents a new sound in the music, the chords heard in the first 36 bars are now transposed by an augmented fourth.
In the recapitulation, the second subject remains in the home tonality throughout. Example 24 shows IIa in the recapitulation at bar 133. Beat 1 of bar 134 uses a chord of **F♯ B E.** This is chord **Nr** from the exposition transposed by an augmented fourth and thus, returns to the pitches of the original chords shown Figure 21.
Ex.24 *Flumina* IIa Recapitulation. Numbers above flute show the beat number in rhythmic cycle. Chord changes are shown above oboe.

15iv. Concertino

The title implies a small-scale Concerto for chamber orchestra. *Concertino* uses mode cc shown in Figure 22:

Fig.22 Mode, Chords: *Concertino*



For this piece, the rules stated previously (see 8i. pg.43 and 8iii. pg.46) were modified further to allow for greater rhythmic flexibility when the metre of the music changes:

1. Only consonant intervals can form chords or melodic phrases. **E** cannot appear with **E**♭, **A** cannot appear with **A**♯ etc.
2. Notes forming dissonant intervals may be employed as passing notes on weak beats or as suspensions on strong beats.
3. At the minim level crotchets are the largest duration that may form passing notes and resolution notes. Likewise, at the crotchet level quavers are the largest allowable duration etc.
4. At the crotchet level, e.g. $\frac{3}{4}$ time, minims or longer duration values may form chords with shorter notes but cannot alter chords. If a pitch lasting for a minim appears in $\frac{3}{4}$ time the chord must remain unchanged for this duration or the pitch must be a consonant interval of any new chord formed in this duration. Likewise, for crotchets at the quaver level, e.g. $\frac{6}{8}$ time; quavers at the semiquaver level, e.g. $\frac{3}{16}$ time; and for all other rhythmic values and levels.
5. At the crotchet level quavers, semiquavers etc. cannot alter chords; likewise, for other levels. If quavers appear at the crotchet level, e.g. $\frac{3}{4}$ time, quavers, semiquavers must be notes of the chord or passing notes. The same applies to all other rhythmic values and levels.

These new rules extend the pitch-organisation used in *Flumina* and bring the polyphonic writing closer to *species counterpoint.* In *Gradus Ad Parnassum* Fux labels the species of counterpoint as follows:

1. First Species: note against note.
2. Second Species: two half notes set against a whole note. The interval on the downbeat must be consonant. On the offbeat the note must be consonant if it proceeds with a skip but may be dissonant if it proceeds by step between two consonances.
3. Third Species: four quarter notes against a whole note. If four quarter notes follow one another by step the first and third must be consonant but the third and fourth may be dissonant. If the second and fourth note are consonant the third

note may be dissonant in it proceeds by step. Any note that skips must arrive on a consonance.

4. Fourth Species: allows the use of the ligature or suspension into the strong beat. The ligature may be consonant, if it is dissonant it must be prepared as a consonance and resolve by step to a consonance.
5. Five Species: summation and application of the previous four species.

(Mann A, *The Study of Counterpoint from Fux's GAP,* New York, London, 1971).

Example 25 illustrates both aspects of the relationship between the rhythmic strata and the dissonance allowable within that strata. Long notes define the chord throughout their duration while passing notes momentarily sound a dissonant note against the chord without fundamentally altering it.

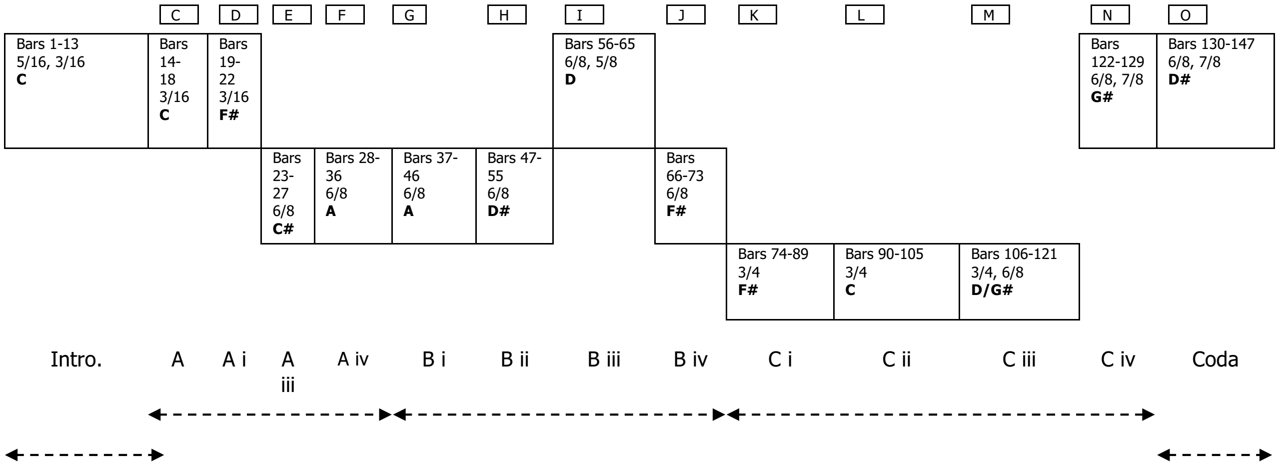
Ex.25 *Concertino* b.80

In bar 83 of Example 25 the bassoon sounds a note for a duration of 10 crotchet beats. Throughout this duration, the note in the Bassoon either defines the chord or becomes a consonance within a new chord. The violin sounds a dissonant **E**♭ for a crotchet beat in bar 81 beat 2. This is a momentary passing note against a background chord, the passing note in the violin sounds an un-percussed dissonance without fundamentally changing the chord.

15v. Form of Concertino

Form Chart 10 shows the form of *Concertino.*

Form Chart 10 *Concertino*

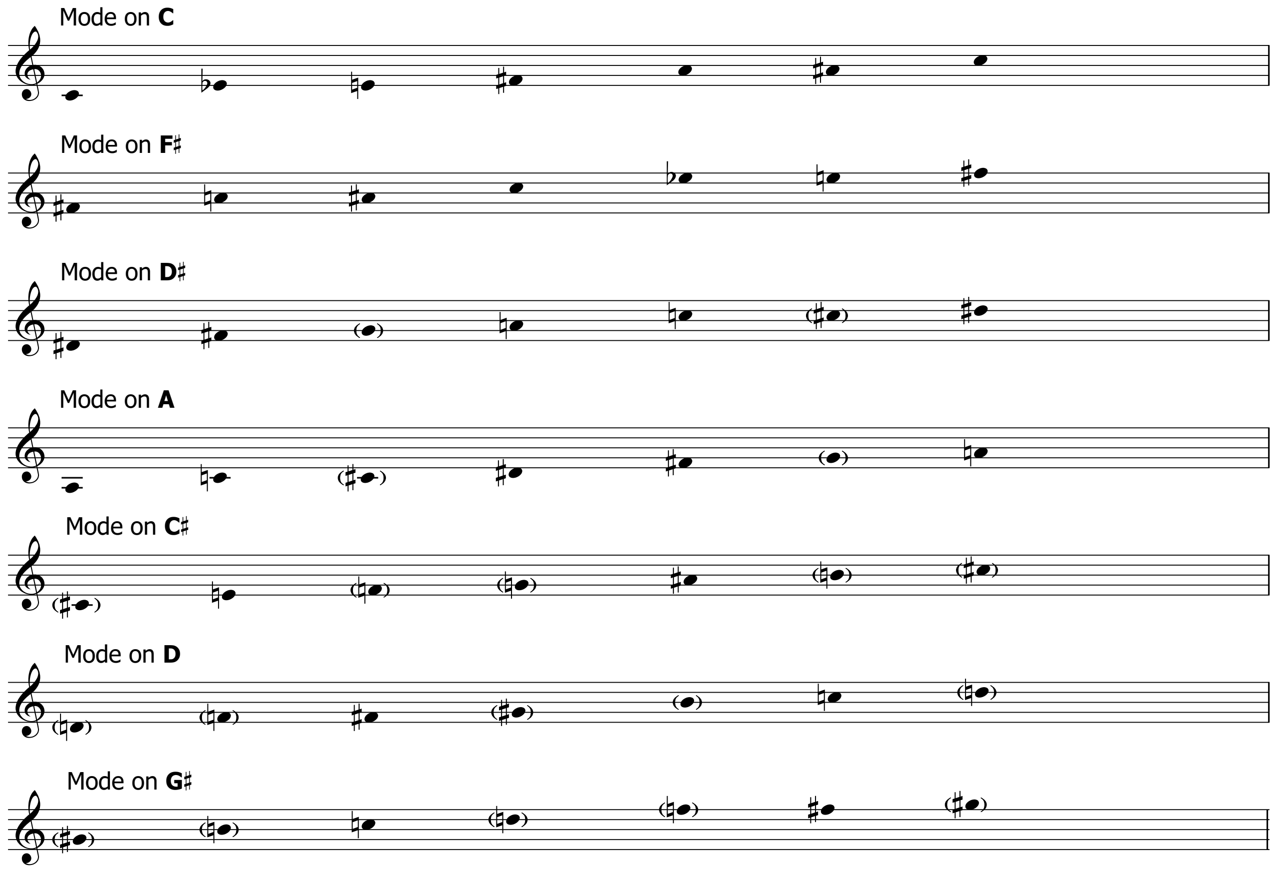


Concertino is in a three-section form ABC, in which there is no return to the original mode in the C section; this contrasts with a traditional ternary structure in which the C section normally returns to the tonic key area and repeats or re-works the A section:

> "It is perhaps the most fundamental of musical forms, based on the natural principles of departure and return, and of thematic contrast then repetition. The term is most commonly associated with the so-called composite ternary form, as found in the da capo aria or the minuet and trio, but is also applied to the 'small ternary' form, where the *ABA* shaping governs a single structure."
>
> (Sutcliffe DW, Ternary Form*, Oxford Music Online,* ed. Sutcliffe DW (Accessed 18 April 2017).

In *Concertino* the mode in Figure 22 was written out from each of the twelve possible starting notes. The differences in pitches between each version of the mode were compared to find those with the most pitches in common and the least. *Concertino* begins using the mode shown in Figure 22 from a starting note of **C**. The mode modulates to a starting note of **F**♯ which is identical in pitch content to the mode based on **C**. Further modulations to the mode with a starting note of **D**♯ and **A** have four notes in common; the remote modulations to a starting note of **C**♯**, D** and **G**♯ have two notes in common. Figure 23 shows the mode and alternative starting notes:

Fig.23 Mode and alternative starting notes: *Concertino.* Notes in brackets are not found in mode on **C**.



Form Chart 10 shows the form of *Concertino* indicating the points at which the mode shown in Figure 22 modulates to a new starting note. In addition, each formal section has a contrast of metre with associated change in *harmo-rhythmic structure.* The metre subdivides or contracts with the number 2 as a denominator, $\frac{3}{4}$ divides into $\frac{3}{8}$ which further divides into $\frac{3}{16}$, this is shown in Form Chart 10.

15vi. Evaluation of the Chordal, Tonal Compositions

The compositions featuring in this chapter introduced several innovations in the compositional methodology:

1. Chords: Chapter 12, pg.69, describes how traditional triads may be represented symbolically in a hierarchical fashion. The chords shown in Figures 22 and 23 have a hierarchical structure in that certain chords with the root note of the chord in the bass, define the mode more strongly than other chords.
2. Tonality: An *artificial tonality* based on the contrast of transposed chords is used to articulate the form.
3. Modulations: The twelve possible modulations are considered in regard the closeness of pitch content. The modulations may be close to the home tonality or remote from it depending on the circumstances of the music.

4. Sonata form: With the addition of an *artificial tonality* greater contrast between sections is possible. In the recapitulation, the second subject remains within the home tonality, following tonal practise found in traditional sonata forms.

15vii. Reflections on Triadic/Tonal Compositions

The *artificial tonality* described in this chapter is an advance on the modal approach of previous compositions to the extent that a contrast of chords articulates the formal sections in the music.

Traditional triads have an inbuilt hierarchy arising from the harmonic series. The chords described in this chapter have a weak hierarchy, subsequent compositions employed a more systematic approach to the hierarchy of the tri-note chords used, the hierarchy emerges from the compositional methodology used.

Mutation of MJS uses Mozart's *ITS* as a model to generate a new composition while *Flumina* and *Concertino* remake the style and influence of Palestrina and Mozart. However, the *mutation* of pre-existing *HRS* in these compositions relied upon analysis and subjective choices. In subsequent compositions, an extension of Xenakis' *Symbolic Music* allowed for a precise encoding and *mutation* of pre-existing compositions.

Chapter 16. Synopsis of the Portfolio and Appendices II

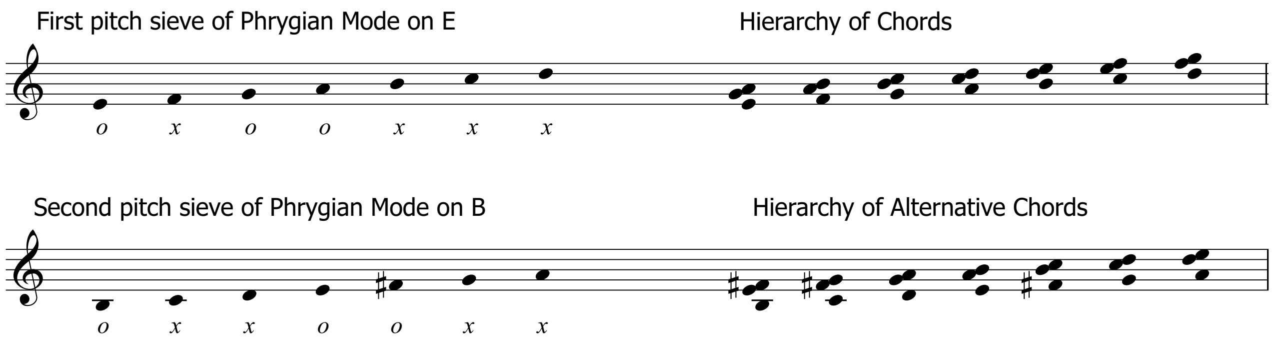
The final pieces in the portfolio and appendix utilised a different approach to the use of modes and tri-note chords. The modes and tri-note chords used in previous compositions did not have the organic derivation found in traditional harmony, where a hierarchy of triads results from the mode/scale used. The concept of a *triadic hierarchy mode*[29] was introduced in which a hierarchy of tri-note chords results from the mode employed in the music.

An additional concept following on from *triadic hierarchy modes* is an *artificial tonality.* This concept, introduced by the author, contrasts with traditional tonality in which the triads, in theory at least, follow the outline of the harmonic series.

> "Tonality is a hierarchical arrangement of the triads based on the natural harmonics or overtones of a note."
>
> (Rosen C, *The Classical Style*, London, 1976, pg. 23).

In addition, an *alternative* set of tri-note chords using a different pitch sieve[30] of the *triadic hierarchy mode* may be employed to heighten the contrast between sections. Figure 24 shows two different pitch sieves of a Phrygian Mode.

Fig.24 Modes, Chords: *This Day Dawns*



The first piece to *mutate* a pre-existing composition by encoding the music as an integer was *Mutation of Dowland's Midnight* (portfolio), using John Dowland's lute dance *Mr Dowland's Midnight.* Dowland's original music controlled the pitch, rhythm and structure of the *mutation.* The *mutation* process used theories derived from Xenakis' *Symbolic Music* extended by *musical integers* and the philosophy of musical Platonism.

Mutation of Missa Papae Marcelli and *Evolution of the Cosmos* submitted in the portfolio, use the same techniques employed in *Mutation of Dowland's Midnight.* These pieces extend the methodology by regarding the symbolic encodings in the *musical integer* as a source of musical DNA which is mutated into new music.

---

[29] A term invented by the author to describe scales and modes that generate a hierarchy of triads/tri-note chords.

[30] A term used in musical analysis meaning certain notes of a scale/mode are chosen and other notes left out.

The compositional methodology used in the final pieces consolidates all the aspects of the research undertaken: The rule-based polyphony of Palestrina; the use of *musical integers* to encode and *mutate* the musical DNA of Palestrina; the philosophy of musical Platonism; the symbolic encoding of music described in Xenakis' *Symbolic Music;* the use of Gödel Numbering to produce *musical integers;* the *mutation* of past music into new compositions.

Chapter 17. Recomposing Music

17i. Introduction

The use of an existing *in-time structure* to inform a new composition may be extended by encoding the pre-existing music symbolically. The encoding is *mutated* into a new composition by altering the *outside time* and *temporal structures* from that of the original. This section describes how ideas from Platonism and Xenakis' *Symbolical Music* inspired the use of *musical integers* to re-compose music.

The philosophy of Platonism, in a modern interpretation, regards abstract, ideal entities as pre-existing their realisation as tokens or kinds of the ideal. Xenakis' *Symbolic Music* expresses music in terms of abstract vectors for pitch, duration and intensity. Are Xenakis' *outside time structures* and *temporal structures* ideal in the Platonic sense?

17ii. Modern Platonism

A prevalent modern interpretation of the philosophy of Plato is that of Platonism. This involves a limited acceptance of the meta-physics of Plato but rejects his epistemology[31] and ontology[32] in favour of empiricism.

> "Platonism must be distinguished from the view of the historical Plato. Few parties to the contemporary debate about platonism make strong exegetical claims about Plato's view, much less defend it. Although the view which we are calling 'platonism' is inspired by Plato's famous theory of abstract and eternal Forms …, platonism is now defined and debated independently of its original historical inspiration.
>
> Not only is the platonism under discussion not Plato's, platonism as characterized above is a purely metaphysical view: it should be distinguished from other views that have substantive epistemological content. Many older characterizations of platonism add strong epistemological claims to the effect that we have some immediate grasp of, or insight into, the realm of abstract objects …
>
> Many philosophers who defend platonism in this purely metaphysical sense would reject the additional epistemological claims."
>
> (Øystein L, "Platonism in the Philosophy of Mathematics", *The Stanford Encyclopedia of Philosophy* (Winter 2013 Edition))

Thus, musical Platonism in this sense would imply universals in music such as a score, of which a musical performance is a *token* or *kind* of this universal.

[31] Theory of knowledge.
[32] Theory of being.

"Platonism, the view that musical works are abstract objects, is currently the most popular view … Platonism has been tenacious, with much of the debate centering around what variety of abstract object musical works are. What we might call 'simple Platonism', is that works are eternal existents, existing in neither space nor time …"
(Andrew K, "The Philosophy of Music", *The Stanford Encyclopedia of Philosophy* (Spring 2014 Edition))

Peter Kivy addresses the concept of Musical Platonism in his collection of essays *The Fine Art of Repetition*:

"If musical works are Platonic objects then they are agreed on all hands to be eternal objects: they do not, cannot come into being; they do not, cannot pass away. They pre-exist the act of composition… if that is the case, then composition cannot be an act of creation … it must be an act of discovery instead."

(Kivy P, *The Fine Art of Repetition*, Cambridge, 1993, pg. 66-67)

Peter Kivy responds to objections raised against musical Platonism. One such objection, which has relevance to the reworking of music from the past, is found in the writings of Jerry Levinson:

"A piece of music is some sort of sound structure … if two distinct composers determine the same sound structure; they necessarily compose the same work."

(Levinson J, What a Musical Work Is, *Journal of Philosophy 77*, pg. 6)

Levinson argues this is a logical necessity if musical Platonism is true yet it cannot be true since if two composers produce the same sound structure:

"the musical works must be non-identical."

(ibid. pg. 10)

He provides two examples:

"if Richard Strauss had composed Schoenberg's *Peirrot Lunaire* in 1897 it would be aesthetically different from Schoenberg's work. Stamitz symphonies employed novel devices such as the Mannheim rocket. If a Stamitz symphony was reproduced by a modern composer note for note devices such as the Manheim Rocket would not sound novel, they would sound funny."

(ibid. pg. 11)

Another consideration identified by Kivy is that no individual composers have ever reproduced even a small amount of identical music independently of each other, unlike in mathematics and science, where for instance Newton and Leibniz discovered calculus separately and unknown to each other. (Kivy P, *The Fine Art of Repetition*, Cambridge, 1993, pg. 69).

17iii. Xenakis' *Symbolic Music* and Musical Platonism

An investigation into Xenakis' application of group theory to musical entities gives an insight into a universal musical Platonism. In mathematics, a set forms a group if the following criteria are met:

1. Every product of two elements and every square of each element are elements of the set.
2. The associative law applies such that a(bc)=(ab)c.
3. There exists a neutral element I, such that IA=AI=A.
4. Each element of the set has an inverse A' belonging to the set such that AA'=A'A=I.

"Product" is the rule of combination used, it could be multiplication, addition etc. An example of a set forming a group is all integers, positive, negative and zero which form a group under addition. (Stephenson G, *Mathematical Methods*, Canada, 1996, pg. 324-325)

Xenakis attempts to show that *OTS* are sets that form groups. He imagines a set H which consists of all pitch intervals. h_a, h_b and h_c are elements of set H such that the intervallic displacement of h_c is given by combining h_a and h_b. He rewrites the rules above in the following way:

1. $h_a + h_b = h_c$ - (i.e. the rule of combination is addition)
2. $h_a + (h_b + h_c) = (h_a + h_b) + h_c$ – (associative law applies when combining intervals of pitch)
3. There exists a neutral element h_o such that $h_o + h_a = h_o + h_a = h_a$ – (h_o is a vector of zero pitch or unison)
4. For every h_a there exists an inverse element h'_a such that $h'_a + h_a = h_a + h'_a = h_o = 0$ – (the inverse element returns h'_a returns the displacement h_a to the origin of zero pitch or h_o).

(Xenakis, *Formalized Music*, Stuyvesant, NY, 1992, pg. 159).

He applies similar reasoning to sets G for intensities and U for durations and writes:

"For pitch the neutral element has a name, unison, or the zero interval; for intensity the zero interval is nameless; and for duration it is simultaneity."
"Corresponding to an ascending interval h_a there may be a descending interval h'_a which returns to the unison ... corresponding to a positive time interval there may be a negative time duration, such that the sum of the two is zero, or simultaneity."
(Ibid. pg. 159).

Xenakis' attempt to show that *OTS* are sets that form groups under addition is not convincing. The first problem is with the postulate 4 from above regarding the inverse element, since there is no convincing argument that there exists an *OTS*

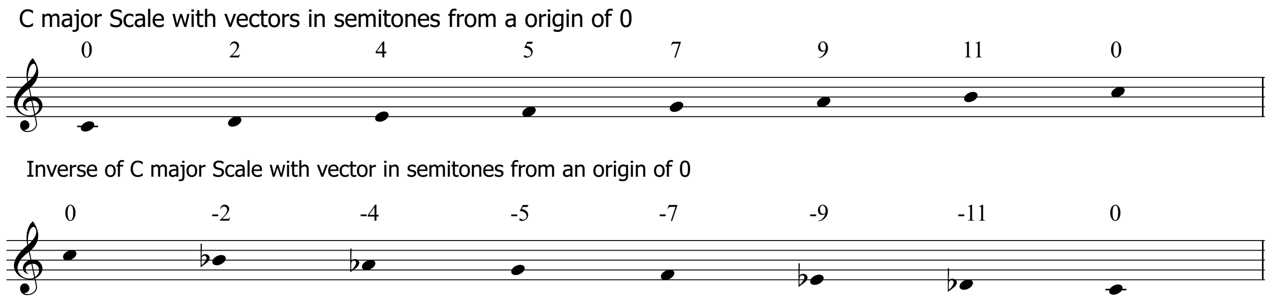
that contains an inverse element for every element within it.
In a major scale **C D E F G A B C**, the first interval in the scale is a major second. Every music student who has written inverse counterpoint knows the inverse of a major second is a minor seventh not a major second reversed. The crucial point is that intervals must be centred around the tonic note for them to function the same way melodically. Thus, the inverse of **C** to **D** is **C** to **B♭** not **D** to **C** as Xenakis seems to suggest, he wants to shift the tonic note from **C** to **D** for his scheme to work. If the C major scale is written as vectors from a common origin of 0 this point may be illustrated:

C major scale: 0 2 4 5 7 9 11 12

C major with inverse intervals: -12 -11 -9 -7 -5 -4 -2 0 2 4 5 7 9 11 12

Following the usual convention for vectors, -2 would imply a jump in the opposite direction to a vector of 2. Thus, 2 would indicate an upwards jump of 2 semitones, -2 a descent of 2 semitones etc. Figure 25 illustrates this:

Fig.25 C major Scale and Inverse

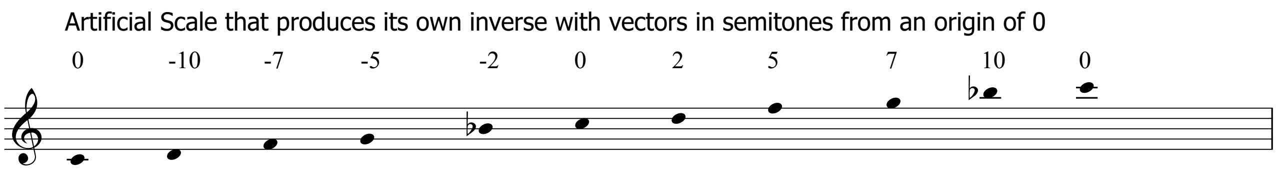


If pitches are applied to the inverse intervals the scale is clearly not a C major scale. Thus, a C major scale does not form a group under addition.
An artificial scale may be constructed which does produce its own inverse, for instance:

-12 -10 -7 -5 -2 0 2 5 7 10 12

If the pitches are substituted starting from -12 the scale would be:
C D F G B♭ C D F G B♭ C. Figure 26 shows this scale:

Fig.26 Scale That Produces Its Own Inverse



However, this scale would not form a group under addition since postulate 1 from above states: Every product of two elements and every square of each element are elements of the set. When the scale is shown as vectors it may be seen that:
2 + 7 = 9 yet the vector 9 does not occur in the scale. This same problem applies

to the C major scale since 2 + 4 = 6 ; the vector 6 does not occur in the C major scale.
Mathematically, the whole tone scale and the chromatic scale are symmetrical around a tonic note but musically they are not since the inverse of a minor second up is a major seventh down and this does not meet the requirements of postulate 4 from above.
Likewise, for duration intervals, there doesn't seem to exist a rhythmical construct in which each element is matched with an inverse element. If we take a simple crotchet beat what is the meaning of a negative crotchet? It would have to involve traveling backwards through time, to a point prior to where the music begins.
An interesting approach would be to go back a level to a pre-musical level where sets that form groups exist and could be considered Platonic. Ancient musical theorists such as Aristoxenos recognised there was a quantum amount of pitch which was not divided up further. Aristoxenos used the twelfth tone, a whole tone divided into twelve equal segments of pitch, as his pitch quanta (see 18v. pg.117). The size of the pitch quanta may change from one period or culture, but the principle is the same that all pitch structures are built on combinations of these quanta. The quanta obey the mathematical properties of all integers and zero and therefore, would form a group under addition and could be considered Platonic[33].
For rhythm, the situation is not as complex:

> "The primary time is theoretically indivisible; it is described as the smallest time-division that is perceptible to the senses… a long syllable occupies two or more primary times."
> (Williams CFA, *The Aristoxenian Theory of Musical Rhythm,* Cambridge, 1911, pg. 28)

The crucial point being that the long note lasts precisely twice the length of the short note. Thus, the pre-musical level of rhythm only needs the following numbers from which all other rhythms could be produced:

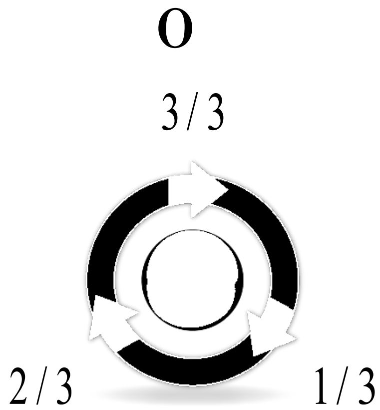
-3, -2, -1, 0, 1, 2, 3

These numbers form a group under additive rotation. Figure 27 shows this sequence:

[33] There is no general agreement that numbers themselves are Platonic, however, in this case integers are used to model of the idea of pitch quanta. It is the idea of a pitch quanta that is Platonic

Fig.27 Additive Rotation

-1, -2/3, -1/3, 0, 1/3, 2/3, 1



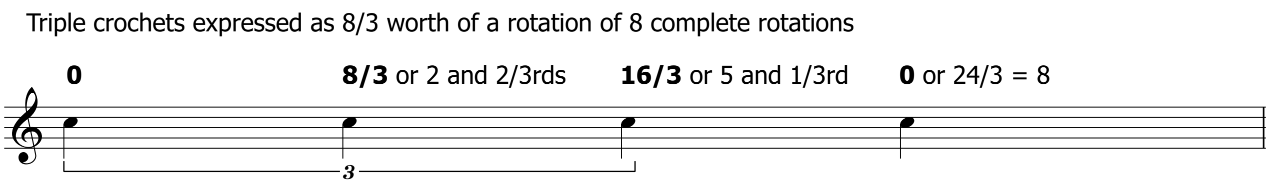
A rotation of 1/3 (clockwise) followed by a rotation of -1/3 (anti-clockwise would take us back to 0. This scheme has the advantage of being periodic therefore, if we rotated by 1 + 1 we would be back at the origin, like metres built on cyclic rhythms.

The way to apply the rhythmic quanta is relativistic to the requirements of the music. For a steady crotchet beat in 4/4 time at metronome mark 60, the speed of rotation would be sixty revolutions a minute. In this case 4/4 time would correspond to a rotation of: 0 1 1 1.

If the music required a steady semiquaver rhythm in 4/4 at metronome mark 60 the speed of rotation would be increased to two hundred and forty revolutions per minute. Now the crotchet beat would be every four revolutions and 4/4 time would correspond to 0 4 4 4. The semiquaver rhythm would be equal to one revolution and four semiquavers would correspond to 0 1 1 1.

A more exotic rhythm like three triple crotchets may be achieved by eight rotations of a 1/3 with 24 complete rotations in total, giving a scheme of: 0 8/3 8/3 for a minim’s worth of triple crotchets, Figure 28 illustrates this.

Fig.28 Triplet Crotchets as a Rotation



Using ideas from Xenakis suggests that a Platonic pre-musical level exists for pitch and rhythm. This is a universal musical Platonism that would apply to all time periods and genres.

Chapter 18. *Transcendental Musical Platonism.*

18i. Introduction

The ideas of *Transcendental Musical Platonism (TMP)*[34] differ from modern platonism in that the epistemology[35] and ontology[36] of Plato is preserved in *TMP.* An epistemology regarding how we have knowledge of music and how we use this knowledge is a feature of *TMP.* Hence, the use of the term *transcendental* to imply knowledge about music exists in a separate realm to that in which music is practised. The ontology suggested in the philosophy of Plato applies to music in *TMP*, music is regarded as an imperfect copy of a perfect, eternal music.

18ii. Historical Plato

Discussing the philosophy of Plato is a problematic undertaking for the following reasons:

1. Plato writes in a dialogue form and a decision must be made as to whether the views expressed are those of the character in the dialogue or Plato.
2. There is no systematic writing on for instance, the theory of Forms. The ideas are collected from various dialogues and an attempt made to form a coherent theory.
3. Plato wrote of his contempt for written philosophy and the use of rhetoric to win an argument by artifice rather than by logic and reason. However, only the writings of Plato survive and a theory must be constructed from this alone.

(Cambridge Companion to Plato, *Cambridge Companions Online,* ed. Kraut R (Accessed 18 April 2017)

Nonetheless, the following passage attempts to outline Plato's *metaphysical epistemology.* This explanation follows that of the chapter *Plato's Metaphysical Epistemology* by NP White from the *Cambridge Companion to Plato.* (White NP, Plato's Metaphysical Epistemology, Cambridge Companion to Plato, *Cambridge Companions Online,* ed. Kraut R (Accessed 18 April 2017).

18iii. Plato's *Forms*

Plato describes the distinction between *perceptibles* and Forms in a passage in the *Phaedo* where he discusses the fact that equality cannot be established through opinion alone, it is knowable only through an understanding of the concept of equality itself:

---

[34] A term invented by the author.
[35] Theory of knowledge.
[36] Theory of being.

> "… in the case of equal sticks and the other equal objects we just mentioned? Do they seem to us to be equal in the same sense as what is Equal itself? Is there some deficiency in their being such as the Equal, or is there not?
> A considerable deficiency, he said…
> We must then possess knowledge of the Equal before that time when we first saw the equal objects and realized that all these objects strive to be like the Equal but are deficient in this."
> (Phaedo 74d-75)

Two sticks may appear equal to one observer but to another observer at an oblique angle the sticks are not equal. Thus, equality and inequality in the case of *perceptibles* is a matter of opinion and circumstance whereas the concept or Form of Equality and In-Equality are always and invariably Equal or un-Equal. Our knowledge of equal cannot be derived from *perceptibles* it comes from knowledge of the Form of Equal.

In addition, Plato suggests that a *perceptible* is no more like a Form that it is not. *Perceptibles* have a quality or an absence of this quality but they are not judged in a relational way. A *perceptible* is beautiful or ugly but not beautiful in comparison to one thing but ugly regarding another:

> "My dear fellow … of all the many beautiful things, is there one that will not also appear ugly? Or is there one of those just things that will not also appear unjust …
> There isn't one, for it is necessary that they appear to be beautiful in a way and also ugly in a way …
> So, with the many bigs and smalls and lights and heavies, is any one of them anymore what we say it is than its opposite?
> No, each of them always participates in both opposites …
> they are like the ambiguities one is entertained with at dinner parties … for they are ambiguous, and one cannot understand them as fixedly being or fixedly not being as both are neither."
> (*Republic* 479-479c)

Plato is arguing that *perceptibles* are deficiently a representation of a Form such as Beauty or Justice. They are as much like the Forms as not like them, only the Forms allow knowledge, *perceptibles* are merely opinion. Regarding a *perceptible* we might opine that it is beautiful or big however, due to the deficient nature of *perceptibles* we cannot say for certain if they are beautiful or not beautiful, we end up in an ambiguous position based on opinion in a similar way that something perceived as big in a certain context might be small in another. Certain knowledge is only possible if we know the Form.

In the *Greater Hippias* a passage states that the finest of pots is ugly put together with the class of girls while the class of girls is ugly put next to the class of gods (*Greater Hippias* 289a-b). In both cases ugly is seen as an intrinsic quality not a relational quality. In a similar way the *Symposium* describes the Form of Beauty as:

> "… it is not beautiful this way and ugly that way, nor beautiful at one time and ugly at another, nor beautiful in relation to one thing and ugly in relation to another…"
>
> (*Symposium* 211a)

Plato would seem to regard a perception as pertaining to a non-relational quality. If the quality pertains to a *perceptible* the quality is deficient by its nature it is no more like the Form it copies that it is not. A *perceptible* therefore, might be beautiful in one context, ugly in another since it is a defective copy of a Form. In contrast a Form is a non-relational quality that is unchanging regardless of the context it appears in. In the *Phaedo* Socrates describes *perceptibles* as that which are perceived by the senses but Forms are grasped by the reasoning power of the mind. *Perceptibles* never remain the same within themselves or in relation to others while the Forms always remain the same (*Phaedo* 78d-79a).

In the *Timaeus* Plato gives an account of the idea of a visible realm of *perceptibles* and an invisible realm of the Forms that is grasped by the intellect and reasoning alone. A creation myth defines the relationship between the realm of the *perceptibles* and the World of Forms in which a craftsman produces an imperfect copy of the Forms to produce the visible universe. The universe is a work of craft modelled after that which is changeless and grasped by rational account and wisdom (*Timaeus* 28a-29a).

The *Timaeus* describes the Forms as eternal and outside of time:

> " … the model was itself an everlasting Living Thing, he set himself to bringing this universe to completion in such a way that it, too, would have that character to the extent that was possible. Now it was the Living Thing's nature to be external, but it isn't possible to bestow eternity fully upon anything that is begotten. And so he began to think of making a moving image of eternity: at the same time as he brought order to the universe, he would make an eternal image, moving according to number, of eternity remaining in unity. This number, of course, is what we now call time."
>
> (*Timaeus* 37c-d)

Thus, the realm of the Forms from which the Universe was copied is an unchanging unity. In contrast the universe itself is a moving image of this eternal unity, the movement and diversity coming about due to time.

Plato's writings establish two distinct realms, the unchanging, eternal unity of the Forms and the changing, time driven diversity of the visible world. This gives two categories: the perishable *perceptibles* which are knowable through the senses and opinion and change according to circumstance; the Forms which are eternal, unchanging and may be grasped purely through intellect and form the basis of true knowledge.

This philosophical position is set out in the famous analogy of *The Cave* found in the *Republic.* Plato describes human beings, dwelling in a cave, bound in fetters that prevents them from turning their heads around. They see only shadows from the outside world behind them, cast on to a wall in front of them. These humans are living purely in the realm of *perceptibles.* Only if they cast off their shackles and turn around to confront the outside world will they realise that what they have apprehended up to this point is a pale imitation of true reality (*Republic* 514a-517c). This confronting of *true* reality is analogous to a philosopher discovering the world of Forms through intellect and reason.

18iv. Plato's Epistemology

Plato's *Theory of Forms* is not a theory of aesthetics[37], which is a common misunderstanding, rather it is a metaphysics that includes an epistemology and ontology within its workings.

In *TMP,* Plato's epistemology and ontology have a mathematical interpretation, the Forms are viewed as a perfect mathematical model of the imperfect nature of reality. A classic example of a mathematical interpretation of Plato's epistemology is that of the triangle. We have full knowledge of all the properties of triangles including the size of internal angles and ratios of the lengths of the three sides yet a triangle when measured would be defective in these regards and would only approximately agree with the abstract properties.

The Forms are not duplicates of *perceptibles*, rather they are mathematical ratios and proportions:

> "Plato never tells us that there is a Form corresponding to every general idea that we can think of … It is far nearer the mark to think of a mathematical ratio than to think of nonsensible duplicates as Forms…
>
> A Form is analogous to a mathematic ratio of function. It is not tangible or visible but something grasped by the intellect. It is itself not a particular but applies to many particulars; particulars participate in it although it is not physically present in them or they in it …
>
> Plato does not say that the Forms are mathematical ratios or functions or numbers. At the end of his life he did think of them as being capable of being expressed by a mathematical formula or number…"
>
> (Allen D & Springsted E, *Philosophy for Understanding Theology,* Louisville,

[37] Philosophy that is concerning with what might be considered beautiful

KY, 2007, pg. 5)

The perfect mathematical model is not the Form but it allows the Form to be expressed in a way that models reality, albeit in an imperfect fashion since reality will always contain imperfections.

18v. Plato's Epistemology Applied to Universal Music

A universal musical Platonism built upon pitch and duration quanta is described in 18v. pg. 117. The *OTS* and *TS* of Xenakis are considered to model the *imitations* or *copies* of an eternal *Form*. This *Form* may be modelled as a set of pitch or duration ratios built from pitch and duration quanta.

To illustrate this with an example it is useful to consider the Form of a bed which crops up in the writings of Plato. The Form could be considered the ratio, proportions and geometric configuration which allows a bed to function successfully. A property of numbers which is difficult to conceive is that of infinity. However, the fact that numbers are infinite would imply that although only a certain number of ratios, proportions and geometric shapes would function successfully as a bed; there must be an infinite number of these successful beds. In practise a bed four miles long, for instance; would be neither practical nor useful, however, such a bed is conceivable by the intellect.

In the case of *OTS* and *TS* there are conceivably an infinite number of possible sets of pitch or duration ratios. However, only a very limited number of these would be practical. For instance, a pitch scale built upon a ratio set with each successive member four octaves apart would not be of much use since the pitches would quickly pass beyond the range of human hearing. Likewise, a duration set based on a base duration of six months would have no practical application in music.

Xenakis describes in *Formalized Music* how ratio sets have been used to produce *OTS.* He identifies two languages used to describe ratio sets, the additive, logarithmic language used by Aristoxenos; the geometric, multiplicative language used by the Pythagoreans (Xenakis, *Formalized Music,* Stuyvesant, NY, 1992, pg. 185). He describes the diatonic, syntonon of Aristoxenos as a pitch ratio set of (2, 2, 1) where the numbers represent a logarithm expression for the number of segments for the 30th division of the interval of a 4th (Ibid. pg. 184). Thus, this ratio set as a *copy* would be (12, 12, 6). If this ratio set is joined to an identical set separated by a logarithm of 2 or 12 segments the ratio set of the modern major scale emerges: (2, 2, 1, 2, 2, 2, 1).

The previous statement from pg.111: "The size of the pitch quanta may change from one period or culture, but the principle is the same that all pitch structures are built on combinations of these quanta" may now be demonstrated in practise. The *Form* of the major scale may be modelled by the pitch ratio set:

(2, 2, 1, 2, 2, 2, 1). At various points in musical history different pitch quanta have been used to give *copies* of this Form. Pythagorean tuning is given by ratios of string lengths corresponding to intervals. In practical usage, the smallest interval would be given as a ratio of: 256/243; a diminished second or limma (Lindley M, Pythagorean Intonation and the Rise of the Triad, *Royal Musical Association Research Chronicle* 16:4-61, 1980a).

In Aristoxenos' system the smallest interval is a 12^{th} of a tone. In modern tuning the semitone is the smallest interval.

Figure 29 gives a comparison of the pitches derived from Aristoxenos' tuning, Pythagorean tuning and modern tuning. A base pitch of 100 Hz is used for ease of calculation. For the calculations $\sqrt[12]{2}$ = 1.059 and $\sqrt[72]{2}$ = 1.0097[38].

From the information shown in Figure 29 the claims of *TMP* have occurred in practise. The ratio set (2, 2, 1, 2, 2, 2, 1) is a model of the *Form* that gives rise to the three different *copies*. In the Aristoxenos and modern tuning the *Form,* modelled as a pitch ratio set, gives the logarithmic ratio; in the Pythagorean tuning the pitch ratio set describes the relationship of the pitch ratios, 2 would imply the ratio of 9/8 or the whole tone and 1 the difference between the octave and the 7^{th}, the diatonic semiquaver. (Ibid.).

The numbers in Figure 29 illustrate the idea of the *copy* being a defective realisation of the paradigm *Form*, since, even before the difficulty of achieving pitches upon a real instrument is considered; virtually all the numbers in the Aristoxenian and modern tuning are approximations of the true value and the values for the 4^{th} and 6^{th} in the Pythagorean tuning are approximate. Even the most accurate instrument in terms of pitch production could not achieve the values necessary for a true realisation of the pitch ratio set (2, 2, 1, 2, 2, 2, 1).

[38] For ½ tones: x^{12}=2 therefore, x= $\sqrt[12]{2}$ = 1.059. For $^{1}/_{12}$ tones x^{72}=2 therefore, x= $\sqrt[72]{2}$ = 1.0097

Fig.29 Comparison of Different Tuning Systems

Calculations to find pitch of interval

	unison	2nd	3rd	4th	5th	6th	7th	octave
Aristoxenos	100	$100x1.0097^{12}$	$100x1.0097^{24}$	$100x1.0097^{30}$	$100x1.0097^{42}$	$100x1.0097^{54}$	$100x1.0097^{66}$	$100x1.0097^{72}$
Pythagorean	100	100x9/8	100x5/4	100x4/3	100x3/2	100x5/3	100x15/8	100x2/1
Modern	100	$100x1.0595^{2}$	$100x1.0595^{4}$	$100x1.0595^{5}$	$100x1.0595^{7}$	$100x1.0595^{9}$	$100x1.0595^{11}$	$100x1.0595^{12}$

Value of intervals

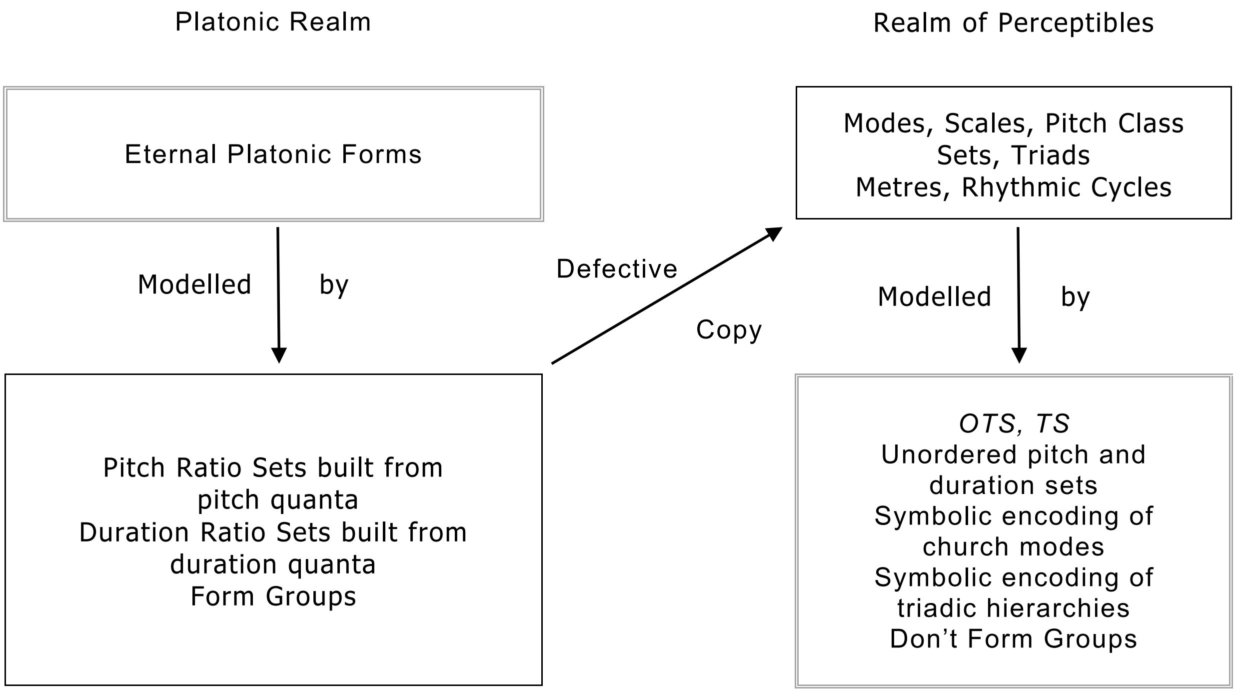
	unison	2nd	3rd	4th	5th	6th	7th	octave
Aristoxenos	100	112.282	126.071	133.589	149.996	168.418	189.102	200
Pythagorean	100	112.5	125	133.333	150	166.667	187.5	200
Modern	100	112.254	126.01	133.507	149.867	168.232	188.847	200

For *temporal structures* the mathematics is easier since durations are measured on a linear scale rather than a logarithmic scale. If a piece of music consists of *n* bars an unchanging metre would have a ratio set given by: (for X_1 to X_n) X=1. X could be divided into any subdivision to give the rhythms usually found in music. Therefore, to produce any theoretical duration ratio set requires that there are an infinite number of real numbers and an infinite number of arrangements of these numbers. The duration quanta used to realise the duration ratio sets would be more flexible than for pitch, it would depend on the nature of the music and the instruments used.

Duration ratio sets are more difficult philosophically and psychologically than pitch ratio sets. To begin with it is a matter of debate as to whether the perception of time in music relies on a perception of metric time as measured by a clock; Husserl addresses these issues (see 10ii. pg.55). Another consideration is that tempo in music is usually variable across a performance of a piece of music. This means that a duration ratio set would not give an absolute ratio between durations, rather it would be a relative duration depending on the tempo of the music at any given point. Thus, any paradigm duration ratio set would be realised imperfectly in reality, due to the perception of the listener and performer and the continuously changing tempo.

Figure 30 shows the ideas of musical Platonism applied to *OTS* and *TS,* in a schematic diagram:

Fig.30 Platonism Applied to *OTS* and *TS*

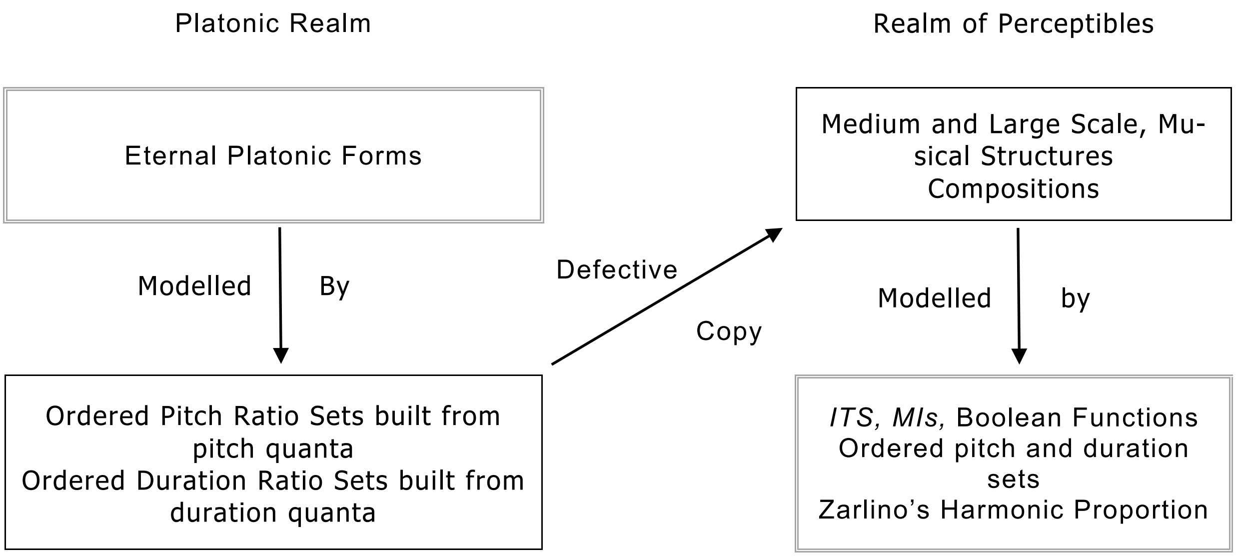


18vi. Plato's Epistemology Applied to In-time Structures

Xenakis writes that an *ITS* is "the correspondence between the *OTS* and the *TS*: the structure *in-time*" (see 9iii. pg.51). In *TMP* the *ITS* are a model of the duration ratio sets performing operations upon pitch ratio sets. Thus, most music in the Western tradition could be expressed as an unfolding, ordered set describing the duration ratios of successive pitch structures. This approach would affirm Xenakis' opinion that his *Symbolic Music* would result in a universal theory that could describe most music. Furthermore, this vindicates Xenakis' idea that music arises from temporal operations performed upon *OTS* (see 11i. pg.59).

Plato's metaphysics applied to music in *TMP* allows for a reinterpretation of Xenakis' *ITS*. The ordered sets model a deficient *imitation* or *copy* of an eternal *Form*. The *Form* may be modelled as ordered pitch and duration sets, built on pitch and duration quanta. These ordered pitch and duration sets are infinite in number and pre-exist the act of composition. In the view of *TMP,* the composer *discovers* the ordered sets that model the *Form* of the composition. The composition itself is a deficient *imitation* or *copy* of the paradigm *Form*. Figure 31 shows the ideas of *TMP* applied to *ITS* in a schematic diagram:

Fig.31 *TMP* Applied to *ITS*



The theories of *TMP* are put forward as a relativistic model that is used purely for the results it produces, it does not matter if the model if counter-intuitive and abstract. It is like complex numbers in mathematics which are built from the square root of -1. Theoretically, the square root of -1 cannot exist and is merely a mathematical construction. However, complex numbers are widely used in mathematics and produce many useful results.

Chapter 19. *Transcendental Musical Platonism* in Practice

Plato writes about contemplation of the Forms as being the true pursuit of knowledge. The analogy of the cave shows that people who only view the world in a sensory fashion are watching shadows dancing on a wall, to gain true knowledge they must throw off their fetters and turn to face true reality that is apprehended through the intellect grasping the nature of the Forms.

If this applied to the view of music expressed in *TMP,* true knowledge of music is to be gained in grasping *Forms* in a long sequence of numbers that model the pitch and duration ratios sets of a composition. It is hard to conceive that contemplating the *Forms* of *TMP* could ever be a more aesthetically satisfying experience than listening to a live performance of music.

Plato perhaps recognised that his metaphysics became difficult to quantify regarding abstract, value judgement nouns such as beauty or justice. To address this Plato introduces his *Theory of Recollection* in the dialogue *The Meno* (Gail Fine, Inquiry in the Meno, The Cambridge Guide to Plato, *Cambridge Companions Online,* ed. Kraut R (Accessed 18 April 2017)

In this dialogue, Plato uses the character Meno to outline Meno's Paradox in regard to Socrates method of inquiry into virtue. This states that if someone knows something there is no need to inquire into it further, yet if they don't know it then they cannot inquire into it since they don't know what it is. (*Meno* 80d5-8).

Socrates answer to Meno's Paradox is the *Theory of Recollection*:

> "Since the soul is immortal and has been born many times … there is nothing it has not learned. Hence, it is no wonder if it can recall virtue and other things it knew previously… For inquiring and learning are just recollection." (*Meno* 81c5-d5).

Socrates proceeds to show that a slave boy can solve a geometric problem despite knowing nothing whatsoever about geometry (*Meno* 82c-e). Thus, for Plato knowledge about abstract entities is recollection about what we know already. The ability to aesthetically appreciate music might be viewed as an *innate* ability in a modern interpretation of the *Theory of Recollection.*

Chapter 20. *Musical Integers*

20i. Extending Symbolic Music using Gödel Numbering

The theories set out in Xenakis' *Symbolic Music* were used to generate Boolean functions that represent *in-time structures (ITS)* (See 11iv. pg.64). A drawback to this technique is the absence of a calibrated representation of durations in music. The "+" sign was used to mark the beginning of a new sonic event. To address this issue, techniques suggested by Gödel Numbering were introduced to add calibrated duration ratio sets into the symbolic encoding.

Gödel states in the introduction to his 1931 paper*:*

> "The most comprehensive current systems are the systems of Principia Mathematica on the one hand, the Zermelo-Fraenkelian axiom-system of set theory on the other hand. These two systems are so far developed that you can formalise in them all proof methods that are currently in use in mathematics, i.e. you can reduce these proof methods to a few axioms and deduction rules. Therefore, the conclusion seems plausible that these deduction rules are sufficient to decide all mathematical questions expressible in those systems. We will show that this is not true, but that there are even relatively easy problem(s) in the theory of whole numbers that cannot be decided from the axioms."
>
> (Gödel K, *On Formally Undecidable Propositions of Principia Mathematica and Related Systems,* 1931; trans. Martin Hirzel, 2000)[39]

To demonstrate his proof of the *incompleteness theorem* Gödel proposed that the functions and proofs of formal systems were nothing more than a finite sequence of formulae and these formulae are expressible as natural numbers:

> "… proofs are formally nothing else than finite sequences of formulae … it is irrelevant for meta-mathematical observations what signs are taken for basic symbols, and so we will choose natural numbers for them. Hence, a formula is a finite sequence of natural numbers, and a proof schema is a finite sequence of finite sequences of natural numbers."
>
> (Ibid.)

To encode formulae as natural numbers in the *Incompleteness Theorem* proof, Gödel introduced the use of Gödel Numbers:

> "We will now uniquely associate the primitive signs of system P with natural numbers as follows:
>
> "0"…1 "succ"…3 "¬"…5 "V"…7 "∀"…9
>
> "("…11 ")"…13

[39] Quoted with translator's permission at the author's own risk. The translator takes no responsibly for incorrect translation.

Furthermore, we will uniquely associate each variable of type *n* with a number of the form p^n (where *p* is a prime >13). Thus, there is a one-to-one correspondence between every finite string of basic signs and a sequence of natural numbers. We now map the sequence of natural numbers (again in one-to-one correspondence) to natural numbers by having the sequence n_1 , n_2 , … , n_k correspond to the number $2^{n1}.3^{n2}. \ldots .P_k^{nk}$ where P_k is the *kth* prime by magnitude. Thus, there is not only a uniquely associated natural number for every basic sign but also for every sequence of basic signs"

(Ibid.)

This would mean that a string of symbols such as *x* V*y* would be encoded as a natural number in the following way:

1. "*x*"…15 , "V"…7 , "*y*"…17
2. $2^{15}.3^{7}.5^{17}$
3. 131072.2187.762939453125=2.187e+20

This technique of Gödel numbering was adapted in order to allow for the formation of *Musical Integers.*

20ii. Constructing Musical Integers

Musical Integers (*MIs*) are integers constructed from the pitch ratio sets and duration ratio sets of an *ITS*. The following criteria was considered necessary in order for *MIs* to be a useful method for encoding *ITS*:

1. *MIs* must be producible from *primitive recursive functions*.
2. *MIs* must preserve the time ordered sets of *in-time structures.*
3. *MIs* should be constructed in a fashion that allows all the encoded musical information to be retrievable.

A *primitive recursive function* is a function that produces an outcome on every input of a variable, put simply it is a function that may be calculated in a mechanical fashion by a computer. *MIs* are constructed in a similar way to Gödel numbers. Hence, the functions to encode *MIs* rely upon *primeness, multiplication* and *exponential* all of which are *primitive recursive* (Ibid).

The formation of Gödel numbers takes the form: $P_1^{x1}.P_2^{x2} . \ldots P_k^{xk}$

where $P_{1 \ldots k}$ is a successive prime number for 1 … k expressions and $x_1 \ldots x_k$ is the information to be encoded. Thus, for an *MI* constructed in a similar way to a Gödel number the prime number P_k acts as a placeholder in the sequence which preserves the time ordered sets of the *in-time structures,* musical information x_k will occur at position *k* in the sequence encoding the *MI*.

The useful property of Gödel numbers is that fact that all the information contained in the large integer encoding a string of symbols may be retrieved by prime

factorisation. Furthermore, prime factorisation is *primitive recursive* (Ibid.).
Prime factors of a number are the prime numbers that multiple together to give the number.
E.g. The prime factors of 147 are:

$$147/3=49$$

$$49/7=7$$

$$7/7=1$$

Thus, the prime factors of 147 are $3.7.7 = 3.7^2$

If an integer undergoes prime factorisation the exponential of each prime number factor of the integer is revealed. In the case of Gödel numbers or *MIs* the original coding is thus restored through prime factorisation.
The meeting of the three criteria listed, indicates why the technique of Gödel numbering may be utilised to encode a piece of music as a single integer which is constructed from the product of successive prime numbers. This number will be in a one to one correspondence with the music it encodes. The number may be manipulated in such a way that it speaks about the music it is encoding and in thus becomes a tool to symbolically express music in the manner employed by Xenakis in the composing of *Herma* (see 11ii. pg.60).
Gödel numbering techniques to encode music require four basic means of expressing musical entities:

1. A number to show the pitch vector from a common origin
2. A number for the duration vector between events
3. A way to show any pitch vectors sustained into a new sonic event.
4. A way of showing bar lines

Pitches will be normalised into one octave since the ultimate purpose of *MIs* is to encode *HRS*. The method could be modified to show pitch vectors in greater detail at the cost of a more complex and lengthier encoding. Starting from the note **C** Table 11 shows how numbers were assigned to pitches. The number 28 will be used to show no pitch or every part is resting during this duration.
Table11 Numbers Assigned to Pitches

C	**C♯**	**D**	**D♯**	**E**	**F**	**F♯**	**G**	**G♯**	**A**	**A♯**	**B**
1	3	5	7	9	12	14	16	18	22	24	26

The reason it is done in this way is to avoid any confusion between sequences like: 112. Using the numbering scheme in Table 11 means 112 splits into 1, 12 not 11, 2 or 1, 1, 2.

If more than one note is sounding at any given time the pitch vector will consist of a number for each of the pitches. Thus, a triad of **C** major is encoded from the bass up by the number: 1916 or **C E G**.

When encoding a single line of music or a point where all parts are resting, the number 28 would encode a rest. For music of more than one part if a pitch vector associated with a part rests for a duration it will drop out of the coding. This is the most efficient way to encode rests and it would be obvious where this occurs in the music.

For duration vectors a single number will show the quantity of the smallest rhythmic unit used in the piece of music to be encoded. Thus, if the smallest unit was a semi-quaver:

1 = semi-quaver

2 = quaver

4 = crotchet

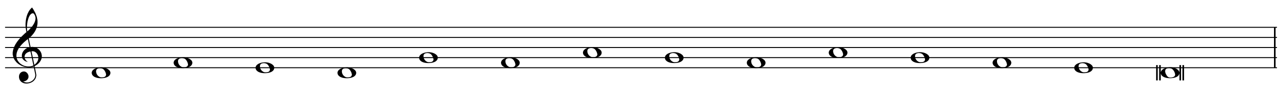
8 = minim etc.

Bar lines are encoded by the number 212. If the end of the piece is reached, 212 would appear successively to show a double bar line.

If a pitch vector sustains into a new sonic event the vector will appear after a 0 in the new pitch vector. Thus, the number 19016 would represent the note **G** sustaining into the new duration of the notes **C** and **E**.

Two primes will be paired for each new sonic event, the first will show the pitch vector from the bass upwards; the second prime will indicate the duration the sonic event lasts for.

Fig.32 Figure 4



(Mann A, *The Study of Counterpoint from Fux's GAP,* New York, London, 1971, pg. 17)

To construct a *MI* for Figure 32 the follow procedure is undertaken:

Pitch vectors: these are labelled in the manner shown in Table 11.

Duration vectors: 1 = semi-breve; 2 = breve.

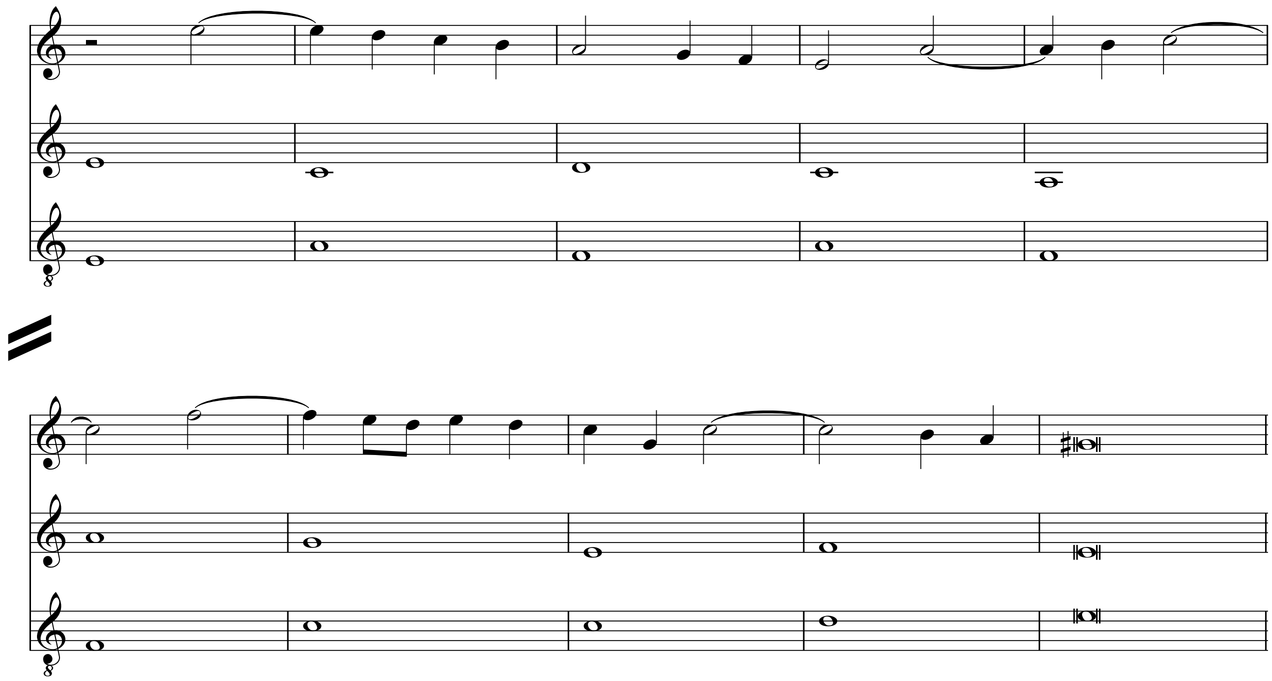
Each note in the plainchant melody is given a pitch vector with associated duration vector:

MI=

$$2^5.3^1.5^{12}.7^1.11^9.13^1.17^5.19^1.23^{16}.29^1.31^{12}.37^1.41^{22}.43^1.47^{16}.53^1.59^{12}.$$
$$61^1.67^9.71^1.73^5.79^2.83^{212}.89^{212}$$

This integer, although it is incredibly huge in length, would uniquely encode the plainchant melody in Figure 32. If the coding system for the pitch and duration vectors is known the integer could be deciphered into the original melody using prime factorisation.

Fig.33 Figure157



(Ibid, pg.57)

Figure 33 will be used to illustrate how to generate a *MI* for music with more than one part.

Pitch vectors: as before written from the bass upwards.

Duration vectors: 1 = quaver, 2 = crotchet, 4 = minim, 8 = semi-breve, 16 = breve.

Encoding the music in the same manner as before gives the following unimaginably huge integer:

MI=

$$2^{99}3^{4}5^{9099}7^{4}11^{212}13^{1109}17^{2}19^{5011}23^{2}29^{1011}31^{2}37^{26011}41^{2}43^{212}47^{12522}53^{4}59^{160125}$$
$$61^{2}67^{120125}71^{2}73^{212}79^{2219}83^{4}89^{220221}97^{4}101^{212}103^{1222022}107^{2}109^{2601222}113^{2}$$
$$127^{101222}131^{4}137^{212}139^{122201}149^{4}151^{1201222}163^{4}167^{212}173^{116012}179^{2}181^{90116}191^{1}$$
$$193^{50116}197^{1}199^{90116}211^{2}223^{50116}227^{2}229^{212}233^{191}239^{2}241^{16019}251^{2}257^{1019}$$
$$263^{4}269^{212}271^{51201}277^{4}281^{260512}283^{2}293^{220512}307^{2}311^{212}313^{9918}317^{16}331^{212}337^{212}$$

The interest comes not from the number itself, it is the fact that a unique *MI* may be constructed for every piece of music. This *MI* encodes the music in a one to one mapping and the music could be reconstructed from the *MI* if the code is known. The *MI* encodes the pitch ratio sets and the duration ratio sets for a piece of music. The major difference between the pitch ratio sets and duration ratio sets of *OTS, TS* and *ITS;* is that the sets for *ITS* are time ordered (see 11iv. pg.64), the *MI*

encoding the *ITS* preserves the time ordered sets. This transcendental view of music is proposed as a relativistic model in which the paradigm *Form,* modelled as ordered pitch and duration ratio sets*,* is in a direct relationship to the *ITS* (see Figure 32). The *ITS* may be mutated to give another *copy* of the *Form.*
MIs were used in the composition of *Mutation of Dowland's Midnight, Mutation of Missa Papae Marcelli* and *Evolution of the Cosmos.*

Chapter 21. *Musical Integers* and *Symbolic Music*

21i. *Musical Integers* and Logarithms

The use of Gödel numbers to generate *MIs* may be seen a generalisation of Xenakis' *Symbolic Music*, allowing for a general theory of music and analytical method vindicating Xenakis' insight that this would be a possibility (see 11i. pg.59). The analytical method relies upon the ability of the *MI* to directly refer to the music it encodes whilst at the same time the *MI* may undergo mathematical operations. If mathematical operations manipulate the *MI* the reformulation of the number will represent a symbolic analysis of the music in the manner of Xenakis' *Symbolic Music*.

Using logarithms, it is possible to express the *MI* in a similar way to the Boolean functions used in the composing of *Herma.* The numbers may be substituted for symbols to reveal the time ordered sets of the *ITS.*

Logarithms (or log in its abbreviated form), reduce exponentials to a single number. For example:

$$\text{For } 10^2,\ \log 10^2 = 2$$

If there are four prime numbers P_1, P_2, P_3 and P_4 they must all be expressed in the same base for the laws of logs to apply. This can be done in the following way:

$$c_1 = \frac{\log P_2}{\log P_1};\ c_2 = \frac{\log P_3}{\log P_1};\ c_3 = \frac{\log P_4}{\log P_1}$$

$$P_2 = P_1^{C_1}\ ;\ P_3 = P_1^{C_2}\ ;\ P_4 = P_1^{C_3}$$

Thus, all the four primes are expressed in the base of P_1. If these prime numbers were in an equation to generate a *MI* the laws of logarithms are that the exponentials are added together if all the numbers are in the same base:

$$P_1^a P_2^b P_3^c P_4^d = P_1^a (P_1^{c_1})^b (P_1^{c_2})^c (P_1^{c_3})^d$$

The laws of logarithms are that exponentials raised to another exponential is the product of the two exponentials. Therefore:

$$P_1^a (P_1^{c_1})^b (P_1^{c_2})^c (P_1^{c_3})^d = P_1^a P_1^{c_1 b} P_1^{c_2 c} P_1^{c_3 d}$$

The exponentials are now all in the same base and they can be added together:

$$P_1^a P_1^{c_1 b} P_1^{c_2 c} P_1^{c_3 d} = P_1^{a+c_1 b+c_2 c+c_3 d}$$

The laws of logarithms allow for a meta-analysis of the *MI* in which the pitch vectors are substituted for sets in a similar way to the composition of *Herma* (See 17iii. pg.89). The meta-analysis of the *MI* allows the music to be expressed symbolically with the addition of a calibrated expression for the duration vector. The following example will illustrate these ideas:

$$2^2.3^2.5^2=900$$

This formula could be the beginnings of a Gödel number or *MI*. Like before:

$$c_1 = \frac{logP_2}{\log P_1};\ c_2 = \frac{logP_3}{\log P_1}$$

$$2^2.2^{2.c_1}.2^{2c_2} = 2^{2+2c_1+2c_1 2} \approx 2^{2+3.17+4.644} \approx 2^{9.814} \approx 900$$

$$c_1 \approx 1.585; c_2 \approx 2.322$$

This example shows how a *MI* could be expressed as the number 2 raised to a large number, this number would be a meta-analysis of the *MI,* showing the sets for the pitch and duration vectors. The two ways of expressing the *MI* would yield exactly the same value; and both methods are *primitive recursive*. This allows the calculation of the original *MI* and from this the music could be reconstructed. Thus, there would be a one to one mapping between the meta-analysis of the *MI* and the music encoded in the *MI*.

These new concepts may be combined with ideas from Chapter 12, pg.69 to give an alternative form to the *MI* for *Fig.157* in Figure 33. The chords for the Phrygian mode on **E** may be expressed as follows:

Table12 Symbolic Coding for Triads of Phrygian Mode

I	=	**N**	=	9 16 26
II	=	**M**	=	12 22 1
III	=	**M**+2	=	16 26 5
IV	=	**N**+5	=	22 1 9
V	=	**O**	=	26 5 12
VI	=	**M**+7	=	1 9 16
VII	=	**N**+t	=	5 12 22

If a scalar is applied to the pitch vectors using the *MI* numbering scheme the scalar operates upon absolute pitch not the numerical value coding for that pitch. The set **N** uses the numbering scheme (9 16 26) to encode the pitches **E G B**. **N** + 5 thus becomes (22 1 9) encoding the pitches **A C E** rather than (14 21 31) which has no meaning in the coding system used. The numbers act as a coding symbol for pitch classes, not as a numerical value.

In addition to the shuffle operator **s** which expresses the chords in different inversions, an operator **p** will be used for partial chords and a **sus** operator when a note or notes are sustained into a new chord:

Np1	=	9
Np2	=	16
Np3	=	26
Np4	=	9 16
Np5	=	9 26
Np6	=	16 26

A similar scheme for **M** and **O** would allow the symbolic expression of any pitch vector.

Np1sus(**Np1+Np1**) would indicate the sounding of a new note **E** into which are sustained the notes **E E.** This encodes the pitch vectors for the second note of bar 1 in *Fig.157* and would have the equivalent numerical value 9099.

The laws of addition allow for a sum to be expressed in any order. This law may be used to separate out the pitch vectors from the duration vectors in the *MI*. The above term is written out again with the letter p showing where the pitch vectors are to be found and d the duration vectors:

$$\underset{p}{a} + \underset{d}{b} + \underset{p}{c} + \underset{d}{c} + \underset{p}{e} + \underset{d}{f} = \underset{p}{(a+c+e)} + \underset{d}{(b+d+f)}$$

For each bar an expression may be written for the pitch vectors with a corresponding expression for the duration vectors and the bar lines. Since the constants c_1, c_2, c_3 … etc. are only required to balance up the equation they may be ignored for the meta-analysis, they would always have the same value if the prime numbers for encoding the *MI* followed the standard procedure: 2, 3, 5, 7 ,11 … etc. Thus:

$$a+c_1b+c_2c+c_3d+c_4e+c_5f$$

becomes

$$(a+c_2c+c_4e)+(c_1b+c_3d+c_5f)$$

and if the constants are ignored:

$$(a+c+e)+(b+d+f)$$

The following expression gives the encoding for the meta-analysis of the *MI* (See 23ii. pg.145) which refers to *Fig.157*:

2^((**Np1**+**Np1**)+**Np1sus**(**Np1**+**Np1**))+(4+4+212)+

((**Mp3**+**Mp3**)**susNp1**+**Op2sus**(**Mp3**+**Mp3**)+**Mp3sus**(**Mp3**+**Mp3**)+ **Np3sus**(**Mp3**+**Mp3**))+(2+2+2+2+212)+

((**Np2**+t+**Np5**+t)+**Np2sus**(**Np2**+t+**Np1**+t)+**Mp1sus**(**Np2**+t+**Np1**+t))+ (4+2+2+212)+

((**Mp1**+5+**Mp4**+5)+**Mp2sus**(**Mp1**+5+**Mp1**+5))+(4+4+212)+

((**Np6**+t)**susMp2**+**Np3sus**(**Np6**+t)+**Mp3sus**(**Np6**+t))+(2+2+4+212)+

((**Np6**+t)**susMp3**+**Mp1sus**(**Np6**+t))+(4+4+212)+

((**Mp5**+7)**susMp1**+**Np1sus**(**Mp5**+7)+(**Mp3**+2)**sus**(**Mp5**+7)+**Np1sus**(**Mp5**+7)+(**Mp 3**+2)**sus**(**Mp5**+7))+(2+1+1+2+2+212)+

((**Mp4**+7)+**Mp1**+7+(**Mp1**+2)**sus**(**Mp4**+7)+**Mp3sus**(**Mp4**+7))+(2+2+4+212)+

((**Np4**+t)**susMp3**+**Np3sus**(**Np4**+t)+**Mp2sus**(**Np4**+t))+(4+2+2+212)+

((**Mp1**+e+**Mp4+e**)+(16+212+212)

This analysis of the *MI* expresses all pitch vectors in terms of triads. Whilst this is cumbersome in some aspects it has the advantage of consistency meaning there is no mixing of vectors encoded by a number representing a single pitch with other vectors written as symbols encoding for triads.

The meta-analysis of the *MI* of *Fig.157* becomes a symbolic means of representing Xenakis' idea of music *in-time* consisting of temporal functions performed upon *OTS* and *temporal sets*. The bracketed term for pitch vectors show the time-ordered pitch ratio sets; the bracketed term for duration vectors shows the temporal functions performed by the duration ratio sets; whilst the entire *MI* represents:

> "The correspondence between the structure outside-time and the temporal structure: the structure in-time."
>
> (Xenakis I, *Formalized Music*, Stuyvesant, NY, 1992, pg. 161).

In 17v. page 94, a sequence *Victimae Paschali* attributed to the monk Wipo of Burgundy, was analysed in terms of a Boolean function:

F = (-**AB**)**A**+**AB**+(-**AB**)**B**+ **AB**+**C**+**CD**+**AB**

The *MI* for this *Victimae Paschali* may be expressed using the numbers assigned to pitches shown in Table 11 with the number 1 representing a quaver for the duration vector:

MI=

$$2^{5}3^{1}5^{1}7^{1}11^{5}13^{1}17^{12}19^{1}23^{16}29^{1}31^{12}37^{1}41^{9}43^{1}47^{5}53^{2}59^{22}61^{1}67^{16}71^{1}73^{9}79^{1}83^{16}89^{1}97^{12}101^{1}$$
$$103^{9}107^{1}109^{5}113^{2}127^{22}131^{1}137^{1}149^{1}151^{5}163^{1}167^{22}173^{1}179^{16}181^{1}191^{22}193^{1}197^{22}199^{2}211^{22}$$
$$223^{1}227^{16}229^{1}233^{22}239^{1}241^{16}251^{1}257^{12}263^{1}269^{9}271^{1}277^{5}281^{2}$$

If a meta-analysis is performed upon the *MI* from above ignoring constants, the following expression is an alternate statement of the number:

2^(5 + 1 + 5 + 12 + 16 + 12 + 9 + 5) +
(1 + 1 + 1 + 1 + 1 + 1 + 1 + 2) +
(22 + 16 + 9 + 16 + 12 + 9 + 5) +
(1 + 1 + 1 + 1 + 1 + 1 + 2) +
(22 + 1 + 5 + 22 + 16 + 22 + 22) +
(1 + 1 + 1 + 1 + 1 + 1 + 2) +
(22 + 16 + 22 + 16 + 12 + 9 + 5) +
(1 + 1 + 1 + 1 + 1 + 1 + 2)

If the numbers are replaced with Boolean functions using the sets in Table 7, the following expression arises:

2^(-(**AB**)**A) +** (**AB) +** (1 + 1 + 1 + 1 + 1 + 1 + 1 + 2) +
-(**AB**)**B** **+** (**AB) +** (1 + 1 + 1 + 1 + 1 + 1 + 2) +
(**C) +** (1 + 1 + 1 + 1 + 1 + 1 + 2) +
(**CD** + **AB) +** (1 + 1 + 1 + 1 + 1 + 1 + 2)

This is the Boolean function from page? with an added duration vector. If the duration vector is reduced to:

(1 + 1 + 1 + 1 + 1 + 1 + 2) = **E**

The expression becomes:

F = ((-**AB**)**A**+**AB**)**+**1**+E+**((-**AB**)**B**+**AB**)**+E**+**C**+**E**+(**CD**+**AB**)**+E**

The improvement from the earlier procedure is there is now a statement for the duration vector. If all the coding were put back into numerical form the original *MI* could be calculated. From this number, the music the *MI* encodes may be reconstructed.

The symbolic encodings outlined in this chapter were used in the composition of *Mutation of Dowland's Midnight, Mutation of Missa Papae Marcelli* and *Evolution of the Cosmos.*

21ii. Extending *MIs*

The *MIs* used so far utilised a single power expression for each prime number. However, a useful extension of Gödel numbering is to raise a prime number to the power of another Gödel number. The Gödel numbering system becomes a nested loop in which more and more coding may be embedded into the final integer:

> "… since large integers can be exponents just as easily as small ones can – it allows for recursive coding. In other words, strings can contain the integer codes for other strings, and this can go on indefinitely."
>
> (Hofstadter DR, *Metamagical Themas*, London, 1985).

To illustrate this idea the string *(x+y)* will be expressed as a Godel number:

(… 2;)…3; x…4; y…5; +…6

The expression x+y would be encoded as: 4 6 5.

This coding can be used as an exponent to give a Gödel number of:

$$2^2 3^{2^4.3^6.5^5} 5^3$$

Where 2^4.3^6.5^5 encodes the expression *x+y.*

If the exponents were found for the overall Gödel number, the exponent of the prime number 3 would be the Gödel number encoding *x+y.* In this way, several layers of encoding may be set in place inside one Gödel number.

This embedding of a lower level Gödel number inside a higher-level Gödel number may be used to extend the technique of generating *MIs* to encode musical forms. The way this can be done is to raise prime numbers to expressions representing various sections of a piece. This allows for the form of a piece of music to be encoded in a short expression which, in line with previous *MIs*; has a one to one mapping with the actual music itself.

Each note in the music is expressed as a pitch vector raised to the power of a prime number. This prime number is chosen in such a way that it encodes the duration vector. If a bar of music contains four entities, the first four prime numbers are each raised to the exponent of the pitch and duration vector; this would give an expression:

$$P_1^{a^b} P_2^{c^d} P_3^{e^f} P_4^{g^h}$$

where a = 1st duration vector; b = 1st pitch vector etc.

Using the same method from before (24. pg.151) to give an alternative expression for this *MI* the expression from above may be written as:

$$2 \wedge a^b + c_1 c^d + c_2 e^f + c_3 g^h$$

where $c_1 = \frac{logP_2}{\log P_1}$; $c_2 = \frac{logP_3}{\log P_1}$; $c_3 = \frac{logP_4}{\log P_1}$

To give a compact expression the prime numbers to encode for durations are chosen to represent the durations occurring in the music. For example:

2 = semiquaver

3 = quaver

5 = crotchet etc.

This method allows for a quick reading of the pitch and duration vector. In the de-coding process the number encoding these vectors may be expressed as a prime power by dividing it by each prime number in turn until a whole number exponent of a prime number is found. For example, if the pitch vector was 5 and the duration vector was 2:

$$2^5 = 32$$

$$32/2 = 16 \ , \ 16/2 = 8 \ , \ 8/2 = 4 \ , \ 4/2 = 2 \ , \ 2/2 = 1$$

Finding the prime power would reveal that the exponent is 5 and base is 2, no other prime number base would have a whole number exponent for 32.

A prime power is only expressible by one prime number. A number expressible as a power will have whole number exponents for two distinct numbers only if one number is expressible as a simple, whole number ratio power of the other. For example, in the case of 2 and 8:

2^3 = 8

8^2 = 64

substitute 8 for 2^3

(2^3)^2 = 64

multiply exponents ^3 and ^2

2^(3.2) = 2^6 = 64

In this case the numbers 2, 2^1; and 8, 2^3 have a simple ratio of 1:3

this would not be true for 6 and 8 (2^3) since 8 cannot be expressed as a whole number power of 6 since the ratio is not a whole number.

Prime numbers are only divisible by 1 and themselves meaning they cannot be expressed as a power of another number. This means a prime power will have only one prime solution. However, a prime power may have another non-prime solution:

81 = 9^2 = 3^4

Thus, the encoding shown above allows each bar in the music to be expressed as a unique sequence encoding pitches and durations. Since the number arrived at for each bar is not the final, top level *MI* the constants to balance the equation must be left in place.

The unique sequence for each bar may then be raised to a prime number. This process is repeated for each bar in the section using as many prime numbers as there are bars. If there were four bars in the section, this would give an expression of:

$$P_{b1}{}^{i} P_{b2}{}^{j} P_{b3}{}^{k} P_{b4}{}^{l}$$

where b1 = bar 1 etc. and i, j, k, l are the *MI* sequence for each bar.

The *MI* obtained in this way may then be raised to a further prime number to represent the encoding for the section; this may be repeated until every section in the music has been encoded.

If this process is undertaken for a movement in sonata form the *MI* for each section may be substituted for an expression such as **Ia** , **IIb** etc. In this way, a final *MI* encoding the entire form of the music is possible.

For a generalised, standard sonata form this would be:

$$A^{Q_1} B^{Q_2} C^{Q_3}$$

where: Q1 = 2^**Ia** + c1**Ib** + c2**IIa* + c3IIb*;**

Q2 = development section;

Q3 = 2^**Ia** + c_1**Ib** + c_2**IIa + c_3IIb**

Ia = *MI* for first subject first idea etc.

c_1, c_2, c_3 are constants to balance the equation

The asterisk for **IIa* and IIb*** indicates that these numbers are comprised of sequences multiplied by a constant since they are transposed to another key. In the recapitulation, the asterisk is removed to show the music has returned to the tonic key. A sophisticated sonata form will be more complex than the formulae outlined above since it is unlikely that during the recapitulation, the second subject will return unaltered in the tonic key.

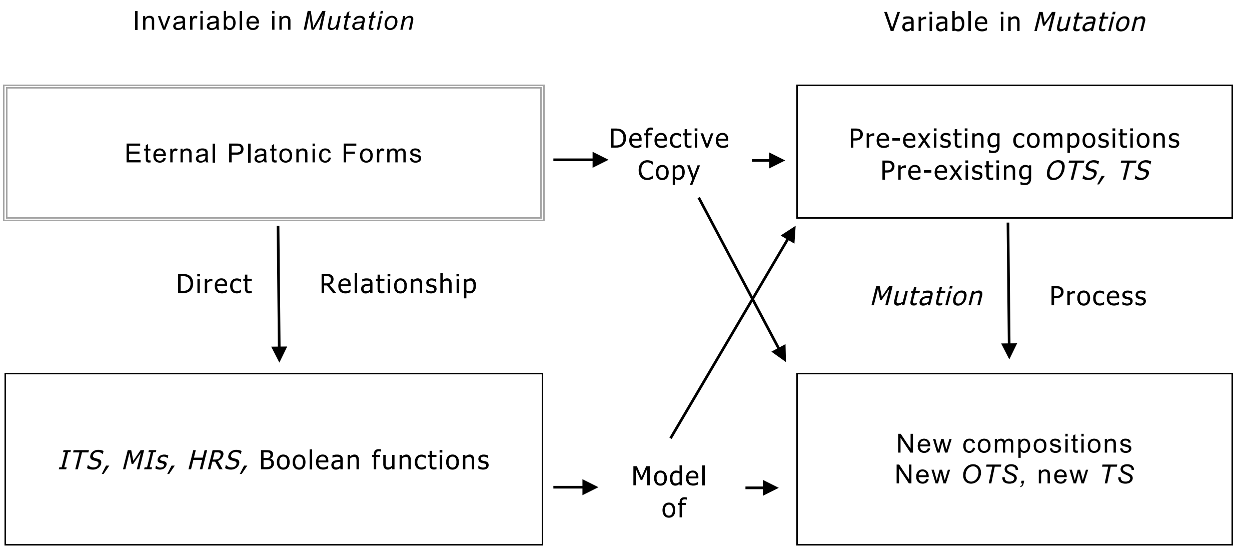
21iii. Mozart's *Symphony 41* Encoding

Appendix I shows how the preceding method was used to encode the sonata form of Mozart's *Symphony 41,* K.551 first movement.

21iv. *MIs* and Composition in Context

MIs offer a conceptual link between Xenakis' *Symbolic Music* and *TMP.* They allow for a symbolic representation of the *Form* of the composition. The *MI* encodes the *ITS* of a musical work, this allows the precise *mutation* of pre-existing music since the *ITS* may be *mutated* into a new composition by altering the *OTS* and *TS* of the pre-existing music. The new composition and the pre-existing music share the same *Form,* modelled by the pitch and duration ratio sets, the new composition is another defective *copy* of the *Form.* Figure 34 shows a schematic diagram of the *mutation* of pre-existing music into new compositions, showing the variable and invariable structures.

Fig.34 *Mutation* Process



Several questions arise from this new compositional methodology:

1. Why choose the *Form* of a piece of music in preference to another?
2. Why is one *Form* considered to be superior to another?
3. Who owns the *mutated* composition, the original or the new composer?
4. Why use a pre-existing *ITS* rather than compose a new one?
5. What is gained from altering the *OTS* and *TS* of a pre-existing composition?
6. Is the use of *MIs* any better than a more traditional, musical approach to *remaking* past music?

The following are suggested as answers to these questions:

1. *TMP* is not an aesthetic philosophy. Therefore, the choice of a Form over another is subjective, composer dependent, cultural decision.
2. *TMP* and the use of *MIs* does not in itself differentiate between the qualities of one *Form* over another, this is a composer dependent, cultural decision.
3. *TMP* views a *Form* as a paradigm sequence of pitch and duration ratio sets discovered by the composer but could not be regarded as belonging to the composer. The *mutated* composition is viewed as another defective *copy* of the *Form*. In this way, the original composer does not own the remade music since they do not own the *Form* from which the *copy* was made.
4. There is a long tradition of using pre-existing music to inform new music, the use of *MIs* follows in this tradition. The advantage of using a pre-existing *ITS* is the original music has stood the test and become a classic of its genre, meaning a composer uses a tried and tested *ITS* rather than trying to find a new one.

5. One interpretation of *MIs* is that they contain musical *DNA*. The *genes* of the musical *DNA* of an existing piece may be *mutated* into a composition bearing the hallmarks of the new composer, using the musical *DNA* found in the music of another composer. The altering of the *OTS* and *TS mutates* the original music into a new composition.

The technique of using *MIs* to mutate music could be achieved using other methods. The advantage of the use of *MIs* is the encoding is symbolic rather than literal (see 24xii. pg.172). The fact that the encoding is symbolic arguably makes the technique easier than more traditional methods. For instance, the *mutation* technique decides which triad/chord in the new composition symbolises the corresponding triad/chord of the pre-existing music.

Chapter 22. Mutation of Dowland's Midnight

22i. Introduction

Mutation of Dowland's Midnight employs an *artificial tonality* to control pitch-organisation. The use of a *triadic hierarchy mode (THM)* and *artificial tonality* extends the previous tri-note chord pitch-organisation described in Chapter 15 *Chordal, Tonal Compositions,* since a hierarchy of chords emerges from system used. In addition, the *mutation* of pre-existing music is more systematic, the chords used in the *artificial tonality* have a symbolic counterpart in traditional tonality.

22ii. *Mutation of Dowland's Midnight*

A single movement work for the clarinet, violin, cello and piano, the quartet of instruments used by Messiaen in his *Quartet for the End of Time* (Messiaen O, *The Quartet for the End of Time.* Paris, Durand S.A. Editions Musicales, 1940)*.*

This composition is a *remaking* of a lute dance *Mr Dowland's Midnight* by the composer John Dowland. *Mutation of Dowland's Midnight* was the first composition to use *artificial tonality* and a *musical integer (MI),* to *remake* pre-existing music.

The score used for the *mutation* was an arrangement by Matthew Sallis using a lute tablature from the *Margaret Board Lute Book (1625,* Public Domain).

The original music of *Mr Dowland's Midnight* was encoded into a *MI.* The procedure to do this used the techniques outline in 26. page 159 in which a bar of *x* number of entities uses 1 to *x* successive prime numbers as a base with an expression for each pitch and duration vector within the bar, as an exponent.

Using the following values for duration vectors Table 13 give the *MI* for *Mr Dowland's* Midnight: Quaver = 2; crotchet = 3; dotted crotchet = 5; minim = 7

Table13 Symbolic Encoding of *Mr Dowland’s Midnight*

Encoding for Each Bar	Bar No.
$2{}^\wedge 5^{22922}+c_1 2^{260229}+c_2 3^{10229}+c_3 3^{220229}$	1
$2{}^\wedge 3^{2291}+c_1 3^{50229}+c_2 3^{926}+c_3 3^{18026}$	2
$2{}^\wedge 5^{22922}+c_1 2^{260229}+c_2 3^{10229}+c_3 3^{1626}$	3
$2{}^\wedge 3^{121}+c_1 3^{5012}+c_2 7^{918269}$	4
$2{}^\wedge 3^{922}+c_1 2^{22022}+c_2 3^{26022}+c_3 3^{1022}+c_4 3^{22022}$	5
$2{}^\wedge 3^{2291}+c_1 3^{50229}+c_2 3^{926}+c_3 3^{18026}$	6
$2{}^\wedge 3^{922}+c_1 2^{22022}+c_2 3^{26022}+c_3 3^{1022}+c_4 3^{1626}$	7
$2{}^\wedge 2^{1222}+c_1 2^{26012}+c_2 2^{1012}+c_3 2^{5012}+c_4 7^{918269}$	8
$2{}^\wedge 3^{926}+c_1 3^{1609}+c_2 3^{165}+c_3 3^{9016}$	9
$2{}^\wedge 3^{52212}+c_1 2^{90522}+c_2 2^{50522}+c_3 3^{12221}+c_4 3^{501222}$	10
$2{}^\wedge 3^{19}+c_1 2^{501}+c_2 2^{901}+c_3 3^{55}+c_4 3^{105}$	11
$2{}^\wedge 3^{926}+c_1 3^{2609}+c_2 3^{22922}+c_3 2^{90229}+c_4 2^{120229}$	12
$2{}^\wedge 3^{916}+c_1 2^{1209}+c_2 2^{909}+c_3 2^{165}+c_4 2^{1016}+c_5 2^{5016}+c_6 2^{9016}$	13
$2{}^\wedge 3^{52212}+c_1 2^{90522}+c_2 2^{50522}+c_3 2^{121}+c_4 2^{26012}+c_5 2^{1012}+c_6 2^{5012}$	14
$2{}^\wedge 3^{19}+c_1 2^{501}+c_2 2^{901}+c_3 2^{512}+c_4 2^{905}+c_5 2^{505}+c_6 2^{105}$	15
$2{}^\wedge 2^{926}+c_1 2^{2209}+c_2 3^{2609}+c_3 7^\wedge{}^{22922}$	16

Using: **M** = 22 1 9 ; **N** = 1 9 16 ; **O** = 26 5 12
and the **sus**; **p** and **s** operators described on page 153, ignoring constants the *MIs* above may be re-written in the follow way:

Table14 Alternative Symbolic Encoding of *Mr Dowland's Midnight*

Symbolic Coding for Each Bar	Bar No.
2^5^(**Mp5**+**Mp1**)+2^(**Op1susMp5**)+3^(**Mp2susMp5**)+3^(**Mp1susMp5**)	1
2^3^(**Mp5**+**Mp2**)+3^(**Op2susMp5**)+3^(**Mp5+7)** +3^(**Np2+4**)**susMp3+5**)	2
2^5^(**Mp5+Mp1**)+2^(**Op1susMp5**) +3^(**Mp2susMp5**) +3^(**Np4+7**)	3
2^3^(**Np4+5**)+3^(**Op2susNp1**+**5**)+7^(**N+4+Np1+4**)	4
2^3^(**Mp3**+**Mp1**)+2^(**Mp1susMp1**)+3^(**Op1susMp1**)+3^(**Mp2susMp1**)+ 3^(**Mp1susMp1**)	5
2^3^(**Mp5**+**Mp2**)+3^(**Op2susMp5**) +3^(**Mp5+7)**+3^(**Np2+4susMp1+7**)	6
2^3^(**Mp3**+**Mp1**)+2^(**Mp1susMp1**)+3^(**Op1susMp1**)+3^(**Mp2susMp1**)+ 3^(**Np4+7**)	7
2^2^(**Np4+5**)+2^(**Op1susNp1+5**)+2^(**Mp2susNp1+5**)+2^(**Op2susNp1+5**)+ 7^(**N+4+Np1+4**)	8
2^3^(**Mp5+7**)+3^(**Mp2+7susMp1+7**)+3^(**Np5+7**)+3^(**Np2susNp1+7**)	9
2^3^(**Mp5+5+Mp2+5**)+2^(**Mp3susMp5+5**)+2^(**Op2susMp5+5**)+3^(**N+5**) +3^(**Op2susNp4+5**)	10
2^3^(**Np4**)+2^(**Op2susNp1**)+2^(**Np2+7susNp1**)+3^(**Op2+Op2**)+ 3^(**Np1susOp2**)	11
2^3^(**Mp5+7**)+3^(**Mp3+7susMp1+7**) +3^(**Mp5**+**Mp1**)+2^(**MpsusMp5**)+ 2^(**Op3susMp5**)	12
2^3^(**Mp4+7)**+2^(**Op3susMp1+7**)+2^(**Mp3susMp1+7**)+2^(**Np5+7**)+ 2^(**Np1susNp1+7**)+2^(**Op2susNp1+7**)+2^(**Np2susNp1+7**)	13
2^3^(**Mp5+5+Mp2+5**)+2^(**Np2susMp5+5**)+2^(**Op2susMp5+5**)+2^(**Np5+5**) +2^(**Op1susNp1+5**)+2^**(Np1susNp1+5**)+2^(**Op2susNp1+5**)	14
2^3^(**Np4**)+2^(**Op2susNp1**)+2^(**Np2susNp1**)+2^(**Mp4+5)**+2^(**Np2susMp1+5**) +2^(**Op2susMp1+5**)+2^(**Mp2susMp1+5**)	15
2^2^(**Mp5+7**)+2^(**Mp1susMp1+7**)+3^(**Op1susMp1+7**)+7^(**Mp5+Mp1**)	16

The *mutation* process uses the *MI* as an encoding of the *in-time structure (ITS)* of Dowland's composition. The *harmo-rhythmic structure (HRS)* of the original music is preserved in the *mutation*, however, the pitch-organisation and rhythmic placement is altered by changing the *outside time structure (OTS)* and *temporal structure (TS)* from that of the original.

In Dowland's original music the *OTS* is an Aeolian mode on **A**; the *TS* is a 4/4 metre. The Aeolian mode is a *THM*, the following hierarchy emerges by taking alternative notes of the mode:

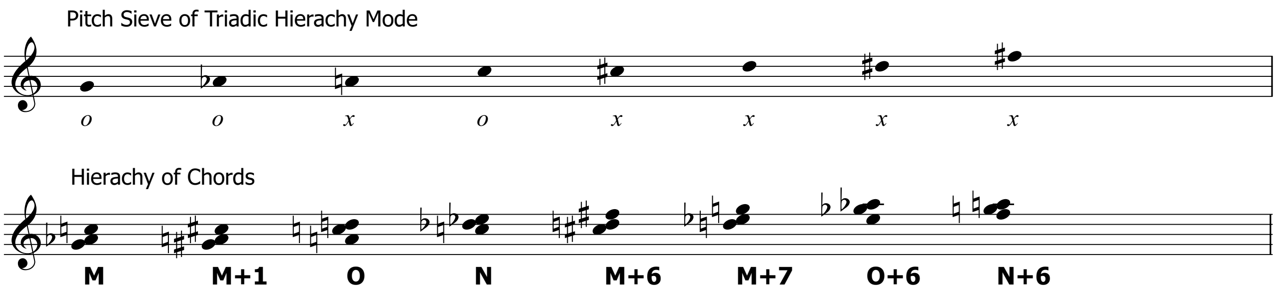
Table15 Symbolic Encoding of Triads of Aeolian Mode

A	**C**	**E**	-	**M**		
B	**D**	**F**	-	**O**		
C	**E**	**G**	-	**N**		
D	**F**	**A**	-	**M**	+	5
E	**G**	**B**	-	**M**	+	7
F	**A**	**C**	-	**N**	+	5
G	**B**	**D**	-	**N**	+	7
Notes of Triad				Symbol for Triad		

Table 15 shows there are three triad types: **M**, **N** and **O**. The triads of **M** and **N** have transposed versions of + 5 or + 7 semitones above the original triad.

For the mutation, the mode and chords shown in Figure 35 are used:

Fig.35 Mode, Chords: *Mutation of DM*



Thus, in a similar manner to before:

Table16 Symbolic Encoding of Chords of *Triadic Hierarchy Mode*

G	**A♭**	**C**	-	**M**		
G♯	**A**	**C♯**	-	**M**	+	1
A	**C**	**D**	-	**O**		
C	**D♭**	**E♭**	-	**N**		
C♯	**D**	**F♯**	-	**M**	+	6
D	**E♭**	**G**	-	**M**	+	7
E♭	**G♭**	**A♭**	-	**O**	+	6
F♯	**G**	**A**	-	**N**	+	6

For the *mutation* Dowland's original triad was substituted using the chords of the *Triadic Harmony Mode* shown in Table 16. The following additional chords, not found in Table 16, were used:

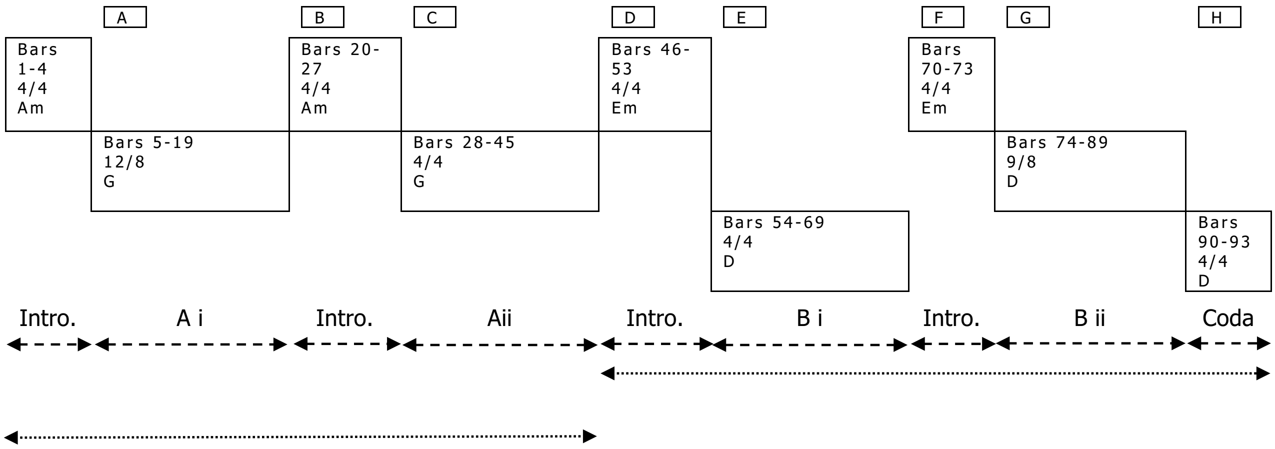
C♯ D F♯	-	**M** + 5
F♯ G A	-	**N** + 5
F F♯ G♯	-	**N** + 4
G G♯ A	-	**N** + 7

The encoding for the triads and durations from Dowland's original music in the *MI,* are substituted with the chords and duration vectors used for the *mutation*.

22iii. Form of Mutation of Dowland's Midnight

Form Chart 11 shows the overall form of the music.

Form Chart 11 *Mutation of Dowland's Midnight*



This composition is a manifesto composition showing how *MIs* and *artificial tonality* could be used in a compositional methodology. For this reason, Dowland's music is heard next to the mutated version to allow the listener an immediate comparison between the original music and its *mutation*.

An innovation for this *mutation* was the use of a scale of timbre. The mode shown in Figure 35 from a starting note of **D** is arranged into chords with associated timbre:

Table17 Chords and Timbre

D D♯ G	**=**	low tremolo
E G♯ A	**=**	middle register staccato
B♭ C D♭	**=**	high harmonic

Thus, when a note of the mode is used in the music, it is assigned the timbre shown in Table 17.

The form of *Mutation of Dowland's Midnight* is AA'BB' binary form which mirrors the form used by Dowland. Each new section of the form is introduced by Dowland's original music. Thus, in bars 1-4 the piano plays the original music with the clarinet taking up the theme in bar 3. Ai (shown in Form Chart 11) uses the chords shown in Figure 35. The duration vectors from Dowland's music are replaced with the following vectors:

Crotchet = 2 ; dotted crotchet = 3 ; dotted crotchet tied to a quaver = 5 ; dotted crotchet tied to a crotchet = 7

The introduction to Aii begins with the piano at bar 20. The theme is taken up by pizzicato violin and cello at bar 22 and modulates to **E**m. Aii (shown in Form Chart

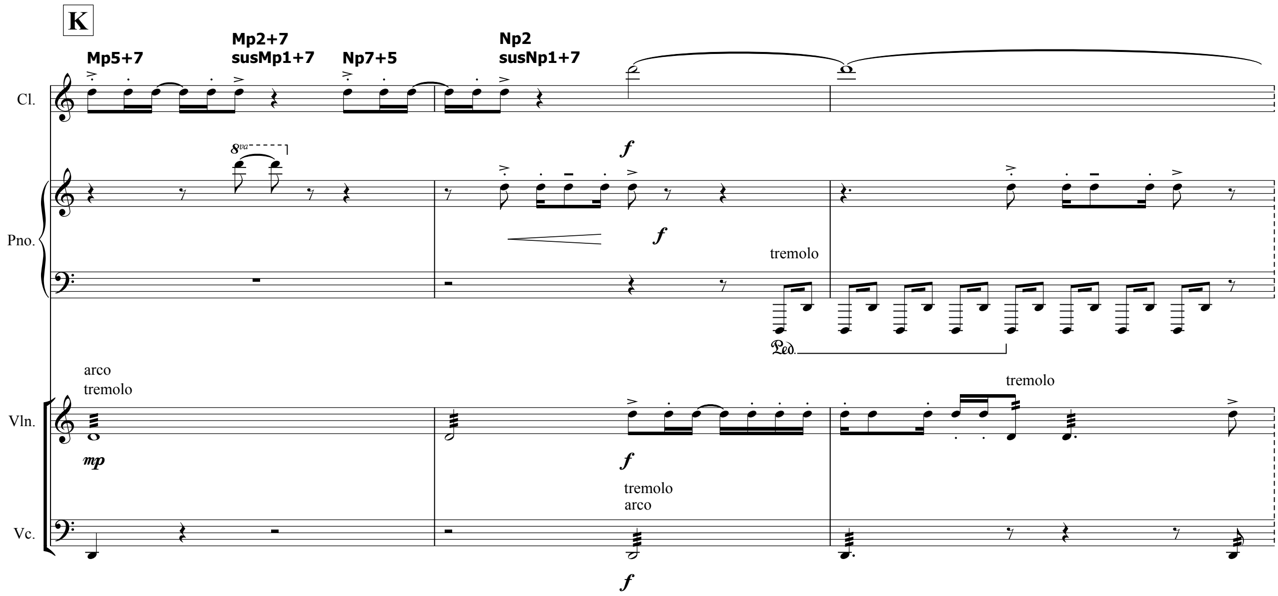
11) uses the chords shown in Figure 35 and uses the following duration vectors: semiquaver = 2 ; quaver = 3 ; dotted quaver = 5 ; crotchet = 7

Bi (shown in Form Chart 11) is introduced with pizzicato violin and cello at bar 46. The piano takes up the theme at the last beat of bar 49 and is doubled by the cello and clarinet.

Bi modulates to a starting note of **D** and uses the timbre scale shown in Table 17. Each pitch is sounded using one the three timbres for the associated chord, this is illustrated in Example 26.

For section Bi the following duration vectors are used this is a rhythmic inversion with the shortest rhythms of the original music replaced with the longest in the mutation: minim = 2 ; dotted crotchet = 3

Ex.26 *Mutation of Dowland's Midnight* Bi b.54. Encoding shown above clarinet.



Thus, the coding for the start of Bi is:

2^3^(**Mp5+7**)+3^(**Mp2+7susMp1+7**)+3^(**Np5+7**)+3^(**Np2susNp1+7**)

The first chord consists of the notes **A D** lasting for a dotted crotchet. This chord is *mutated* using the scale of timbre shown in Table 17 meaning the note **A** implies a mid-range staccato and the note **D** a low-range tremolo. The clarinet plays a mid-range staccato figure for a dotted crotchet on the note **D** while the violin plays a low-range tremolo on the note **D**. The scale is of timbre rather than pitch meaning most of the pitches in Bi are the note **D** sounding in differing octaves with associated timbre. The other chords *mutated* in Bi follow a similar procedure.

The piano plays the introduction to Bii at b.70 with the clarinet joining in at b.72. Bii (shown in Form Chart 11) returns to a scale of pitch scale rather than timbre and uses the following duration vectors: quaver = 2 ; dotted crotchet = 3 ; dotted crotchet tied to a quaver = 7

There is a brief coda at bar 90 (shown in Form Chart 11) which is based on the music material found in Bi.

22iv. *Mutation of Dowland's Midnight* in Context

This composition introduced several innovations into the compositional methodology. The use of *artificial tonality* and *MIs* gives coherence to the pitch-organisation regarding *mutating* pre-existing music. The chords of the *artificial tonality* have a symbolic correspondence with the triads of the original music. A chord sequence in the original music will have a corresponding *mutated* sequence in the *mutated* music. This extends the pitch-organisation used in *Mutation of Mozart's Jupiter Symphony.*

The form of *Mutation of Dowland's Midnight* uses a contrast of *HRS* to articulate the sections in the music. However, the relationship between the sections is no longer the subdivision in proportion to the number 2 found in early compositions; the *mutation* of the *TS* of the original music now gives rise to a new metric structure in the new section. Each new formal section in the music uses different values for the duration vectors. Thus, the *mutation* of the *TS* gives rise to a new *HRS* for each section.

This is illustrated in Examples 27 and 28. The coding for the start of Ai and Aii is:

Ai: 2^5^(**Mp5**+**Mp1**)+2^(**Op1susMp5**)+3^(**Mp2susMp5**)+3^(**Mp1susMp5**)

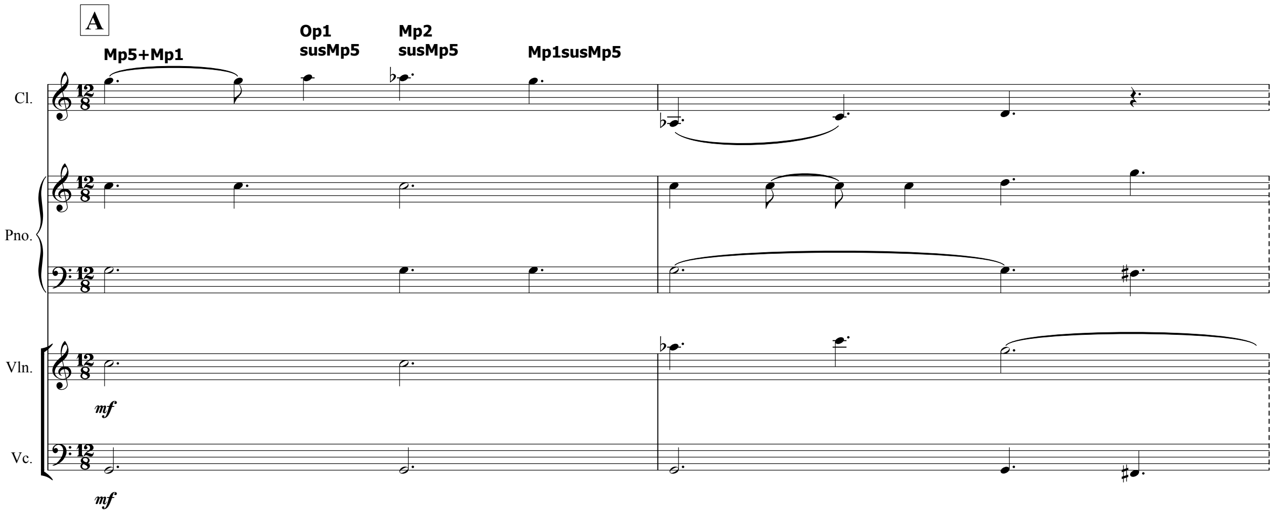
Aii: 2^3^(**Mp3**+**Mp1**)+2^(**Mp1susMp1**)+3^(**Op1susMp1**)+3^(**Mp2susMp1**)+ 3^(**Mp1susMp1**)

However, for each section the number encoding the duration vector is assigned a different value.

For Ai:

Crotchet = 2 ; dotted crotchet = 3 ; dotted crotchet tied to a quaver = 5 ; dotted crotchet tied to a crotchet = 7

Ex.27 *Mutation of DM* Ai b.5. Encoding shown above clarinet.



For Aii: semiquaver = 2 ; quaver = 3 ; dotted quaver = 5 ; crotchet = 7

Ex.28 *Mutation of DM* Aii b.28. Encoding shown above new note. Piano only.

Thus, the symbolic coding for the start of Ai and Aii is similar but it can be seen in Examples 27 and 28 that the rhythm and therefore, the *HRS* changes in Aii due to the change in value of the duration vectors.

It is worth noting that *Transcendental Musical Platonism* states that the *ITS* of a composition is in a direct relationship to the *Form*. When a composition is *mutated* the original *ITS* informs an alternative *copy* of the original *Form*. However, it can clearly be seen that this procedure is not adhered to in *Mutation of Dowland's Midnight*. The *ITS* of *Mr Dowland's Midnight* generates all the musical material used in the mutation, this music material is re-arranged into a different *ITS* in the *mutation*. Each formal section begins with an extract of Dowland's original music meaning that the *ITS* of the *mutation* is different to the original music.

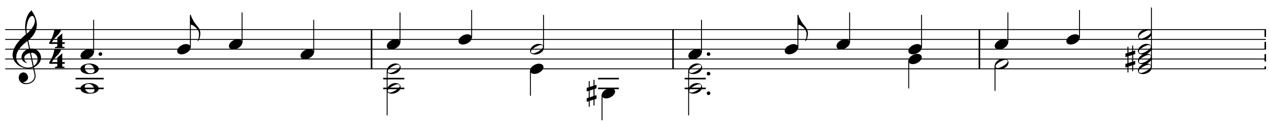
The reason for this is that *TMP* is an abstract, theoretical model. In the composing of a piece of music, musical considerations outweigh theoretical concerns. Therefore, the model of *TMP* is reconsidered to produce a well-formed composition. Regarding *Mutation of Dowland's Midnight,* extracts of Dowland's original music are heard for dramatic effect and to allow a comparison between Dowland's music and the *mutation*.

Chapter 23. Musical DNA

23i. *Musical Integers* and Musical DNA

Musical integers (MIs) are utilised to *remake* the musical DNA[40] of a pre-existing composition. The DNA is *mutated* to produce a new piece of music in the style of the new composer, informed by the DNA of a recognised pre-existing masterpiece. In *Mutation of Dowland's Midnight* and subsequent mutations, the DNA of pre-existing music controls the *mutation* of *harmo-rhythmic structures (HRS).* This allows for an exact *mutation* of pre-existing *HRS* using the techniques outlined in *Mutation of Dowland's Midnight*. The original *HRS* are altered by a change in *OTS* and *TS,* this is illustrated in Examples 27 and 29. The first bar in the Example 29 has a passing note **B** rising up to a consonant note **C** , encoded as: 2^(**Op1susMp5**). In the *mutated* version in Example 27 this is realised as a passing note **A** falling to a consonant **A**♭. Thus, any passing notes and suspensions in the original musical DNA have a symbolic equivalent in the *mutation*.

Ex.29 *Mr Dowland's Midnight* b.1



23ii. Genetic Code

MIs may be viewed as a genetic code showing the musical genes that shape the style, genre and ethnicity of the music. At various stages in the history of music only certain musical genes would be found if a *MI* was produced from a composition of that era.

This idea is demonstrated for Palestrina's motets *Stabat Mater (SM)* and *Super Flumina Babylonis (SFB).* Using the rules stated in 5iv. page 31, it will be possible to predict which musical genes will occur in the motets. This predicted musical gene will be shown to exist in the motets and illustrated with a short extract of music.

The *MIs* encode the choices made by Palestrina in following the rules of counterpoint. The rules governing the rhythmic treatment of dissonant intervals become an *outside time* construct which shape the music *in-time*. The use of a 4-3 suspension varies even within the same composition once the rules of counterpoint are realised *in-time*. Choices are made regarding voice leading, rhythmic configurations, tessitura etc. *MIs* reveal these choices by manipulating the number into a symbolic encoding.

[40] This concept is used by the author to equate the symbolic encoding in *MIs* with genetic sequences in strands of DNA

SM uses the Dorian mode on **D** with the additional pitches of **B**♭, **C**♯, **F**♯ and **G**♯. This limits the numbers encoding for pitch to:

5, 9, 12, 14, 16, 18, 22, 24, 26, 1, 3.

SFB uses the Phrygian mode on **E** with the additional pitches **C**♯, **F**♯ and **G**♯. The numbers in this case are:

9, 12, 14, 16, 18, 22, 26, 1, 3, 5.

A contrary motion chord progression, predicated from the rules of counterpoint, would have a coding of 512225 to 229221 and 916269 to 222219. This encodes contrary motion between parts or similar motion in thirds, sixths.
The extracts in Figures 36 and 37 show these encodings occurring in the motets.
The smallest rhythmic unit in both the motets is a semi-quaver. The *MI* for *SM* bar 49 beat 3 to 4 is:

$$2^{512225}3^{4}5^{229221}7^{8}$$

The extract from *SFB* bar 19 is more detailed:

$$2^{1169}3^{4}5^{9260169}7^{2}11^{2209169}13^{2}17^{2609169}19^{4}23^{221229}29^{4}$$

Following the procedure outlined previously these two *MIs* may be written as (see 21i. pg.131):

SM: **N**=51222; **M**=12221

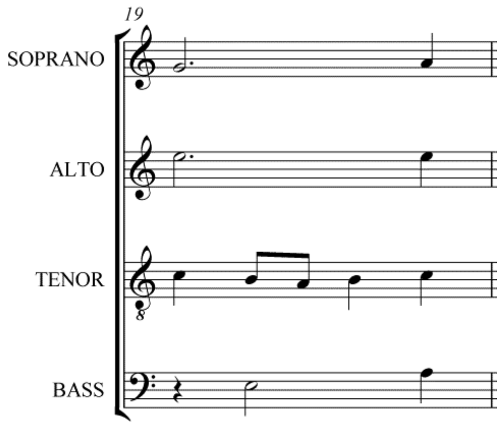
((**N**+**Np1**)**+**(**Np5+7+Np4+7**))+(4+8)

SFB: **N**=91626; **M**=12221

((**Mp5**+7+**Mp2+7**)+(**Np5susMp3+Mp2**)+(**Mp2susNp1+Mp3+Mp2**)+ **Np3sus Np1+Mp3+Mp2**)+(**Np4+5**+**Np5+5**))+(4+2+2+4+4)

Fig.36 *Stabat Mater* b.49

Fig.37 *Super Flumina Babylonis* b.19



The rest of this demonstration of the musical DNA of *SM* and *SFB* is in Appendix II, pg. 215.

Chapter 24. *Mutation of Missa Papae Marcelli*

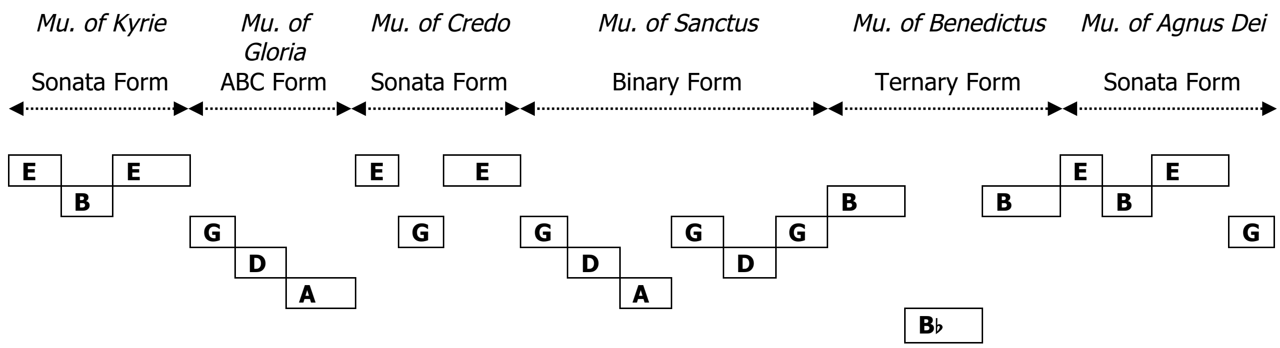
24i. *Mutation of Missa Papae Marcelli*

This composition has a grand design consisting of six movements in total that *mutate* extracts from Palestrina's *Missa Papae Marcelli (MPM)*. The composition exists in two different guises, the first is a six-part open score without specified instrumentation (the fifth movement is in four parts); the second is a realisation of the open score for any suitable ensemble. The reason it is done in this way is to produce an abstract piece of counterpoint which may be realised in any way that is conceivable. A precedent would be Bach's *Art of Fugue*. The analysis in this commentary indicates if the score discussed is the open score or the realisation.

24ii. Structure of *Mutation of Missa Papae Marcelli*

Table 18 shows the overall structure of *Mutation of MPM.* Movements 1,3 and 6 are in sonata form with dissonant pitch-organisation, movements 2,4 and 5 are in ABC, binary and ternary form respectively and have less dissonant pitch-organisation. Table 18 shows that an *artificial tonality* derived from a mode on the note **E** is the most common in terms of number of bars, a mode on the note **G** is the second most used and acts as an artificial relative major tonality. The piece ends with a tierce de Picardie[41] in a mode on the note **G**.

Table 18 Overall Structure of *Mutation of MPM*



N.B. Diagram not to scale

24iii. *Mutation of Kyrie*

Mutation of Kyrie extends the idea of using pre-existing music to produce an exact *mutation* of a *harmo-rhythmic structure (HRS)* using a *musical integer (MI)*. An extract from Palestrina's *Kyrie* bars 52 to 76 was encoded in a similar fashion to *Mr Dowland's Midnight*. Table 19 shows the symbolic encoding for first four bars of this extract, the complete table is in Appendix 1, pg.211.

[41] Tierce de Picardie in this sense means ending in a major tonality rather than a minor chord having a major third in the final cadence.

Table19 Symbolic Encoding of *Kyrie* b.52-76, (b.52-55 only)

Encoding for Each Bar	Bar No.
2^11^(**Np1**)+11^(**Mp1+7**sus**Np1**)	52
2^11^(**Mp2**sus**Mp1+7**)+11^(**Np4+7**)	53
2^7^(**Np4+Np1**)+3^(**Mp1**sus**Np4**)+3^(**Mp1+7+Mp4+7**sus**Np1**)+ 3^(**Np1+7**sus**Mp5+7+Mp1+7**)+3^(**Mp2+7**sus**Mp5+7+Mp1+7**)	54
2^5^(**Np4+2**sus(**Mp1+7+Mp1+7**)+5^(**Mp2+7**sus**Mp1+7**+**Np4+2**)+ 11^(**N+7+Np1+7**)	55

In the original music, the tonality is C major, the chords are shown in Table 20:

Table20 Symbolic Encoding of Chords of C Major Scale

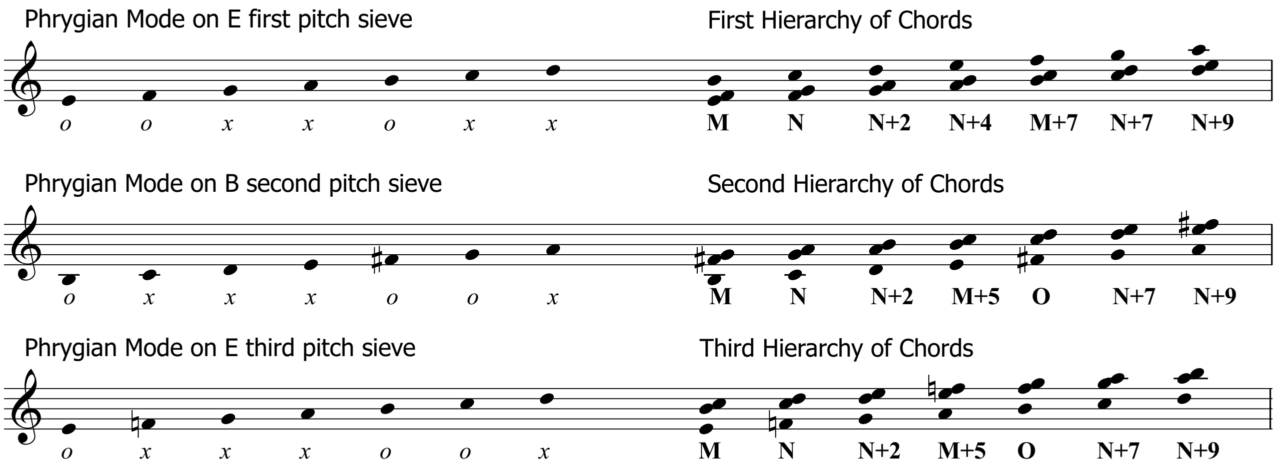
C E G	-	**M**
D F A	-	**N**
E G B	-	**N**+2
F A C	-	**M**+5
G B D	-	**M**+7
A C E	-	**N**+7
B D F	-	**O**

For the encoding the following duration vectors were used for Palestrina's music: Quaver = 2 ; crotchet = 3 ; minim = 5 ; dotted minim = 7 ; semibreve = 11

The symbolic encoding for bars 52 to 76 shown in Table 19 was used as the source material to generate a sonata form movement. Thus, the extract from Palestrina encoded in a *MI* produces a composition similar in conception to *Flumina* however, the *mutation* of *HRS* in this case uses an exact *mutation* of Palestrina's *HRS*.

The *mutation* used a Phrygian mode as an *OTS*, three different pitch sieves of the mode were used, Figure 38 shows the mode and chords for *Mutation of Kyrie*:

Fig.38 Mode, Chords: *Mutation of Kyrie*

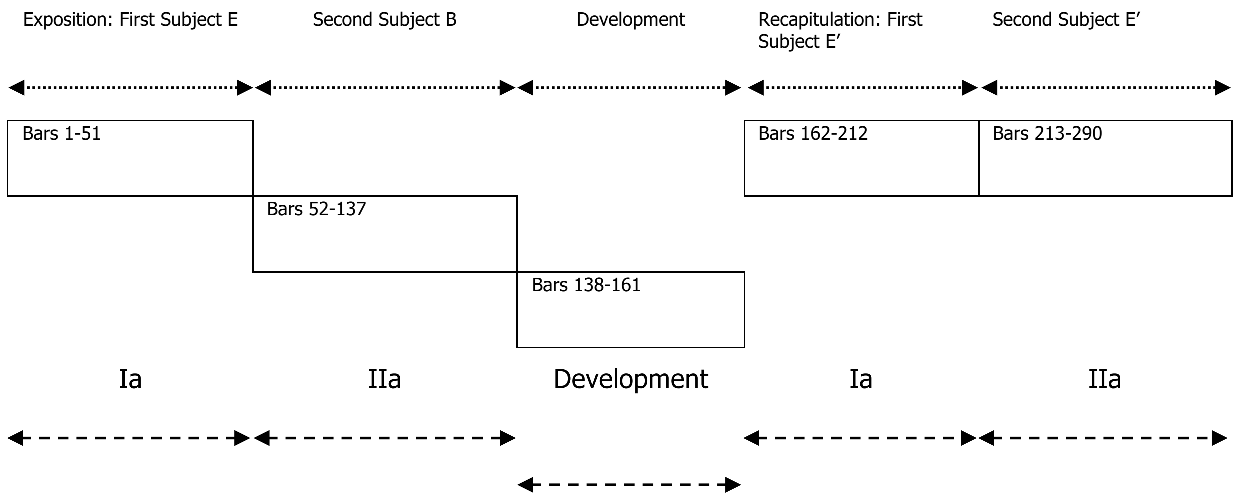


The Phrygian Mode on **E** and **B** generate an *artificial tonality* using the chords shown in Figure 38. The advance upon the *artificial tonality* found in *Mutation of Dowland's Midnight* is the use of a different pitch sieve of the Phrygian Mode on **E** in the recapitulation of the second subject. The change in chords enhances the contrast of *artificial tonality*.

24iv. Form of Mutation of Kyrie

Form chart 12 shows the form of *Mutation of Kyrie.*

Form Chart 12 *Mutation of Kyrie*



This composition uses a *MI* to precisely encode the *HRS* found in Palestrina's *Kyrie. Mutation of Kyrie*, in a similar fashion to *Mutation of Dowland's Midnight*, preserves the *HRS* of Palestrina. The idealised sonata form shown on pg.138, was used as a template for the form.

Ia (shown in Form Chart 12), uses the first pitch sieve of the Phrygian Mode shown in Figure 38. The chords for this section are shown in Table 21:

Table21 Symbolic Encoding of Chords of Phrygian Mode

E F B	**-**	**M**
F G C	**-**	**N**
G A D	**-**	**N**+2
A B♭ E*	**-**	**M**+5
B C F♯*	**-**	**M**+7
C D G	**-**	**N**+7
D E A	**-**	**N**+9

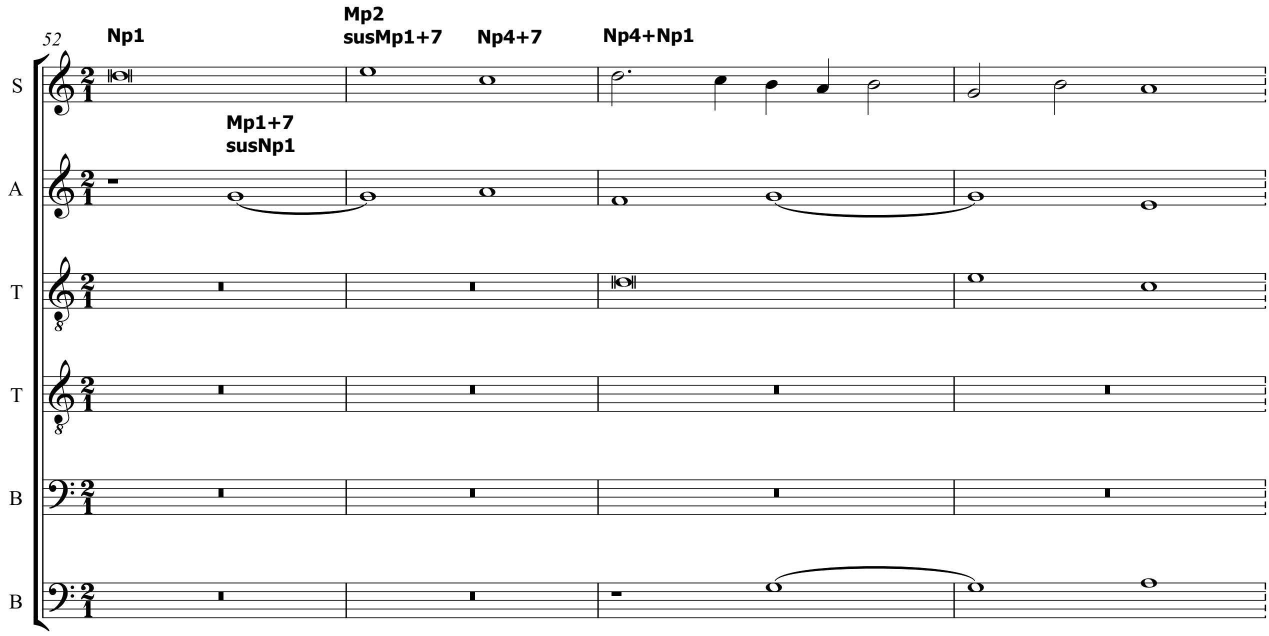
Chords marked with an asterisk are chromatically altered from the chords shown in Figure 38 to generate missing chords.

The duration vectors used for Ia are:

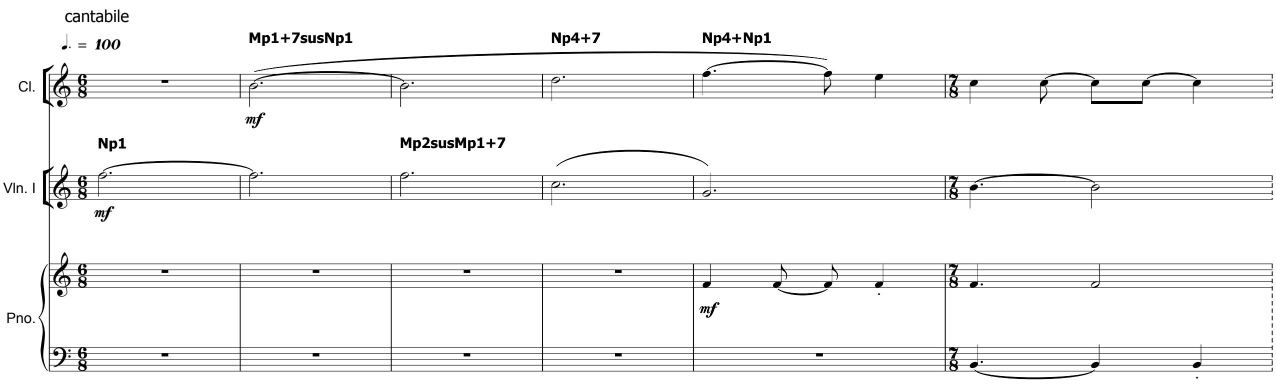
dotted quaver = 2 ; crotchet = 3 ; dotted crotchet = 5 ; dotted crotchet tied to quaver = 7 ; dotted crotchet tied to dotted crotchet = 11

Examples 30 and 31 show the first four bars of the extract from Palestrina's *Kyrie* bar 52-53 and the first six bars of the *mutation* respectively.

Ex.30 Palestrina *Kyrie* b.52-55. Encoding shown above new note.



Ex.31 *Mutation of Kyrie* b1-6. Encoding shown above new note.



IIa (shown in Form Chart 12) begins at bar 52, reusing the encoding of the extract from Palestrina's *Kyrie*. However, for IIa the Phrygian Mode modulates to a starting note of **B** and the chords used are the second pitch sieve shown in Figure 38:

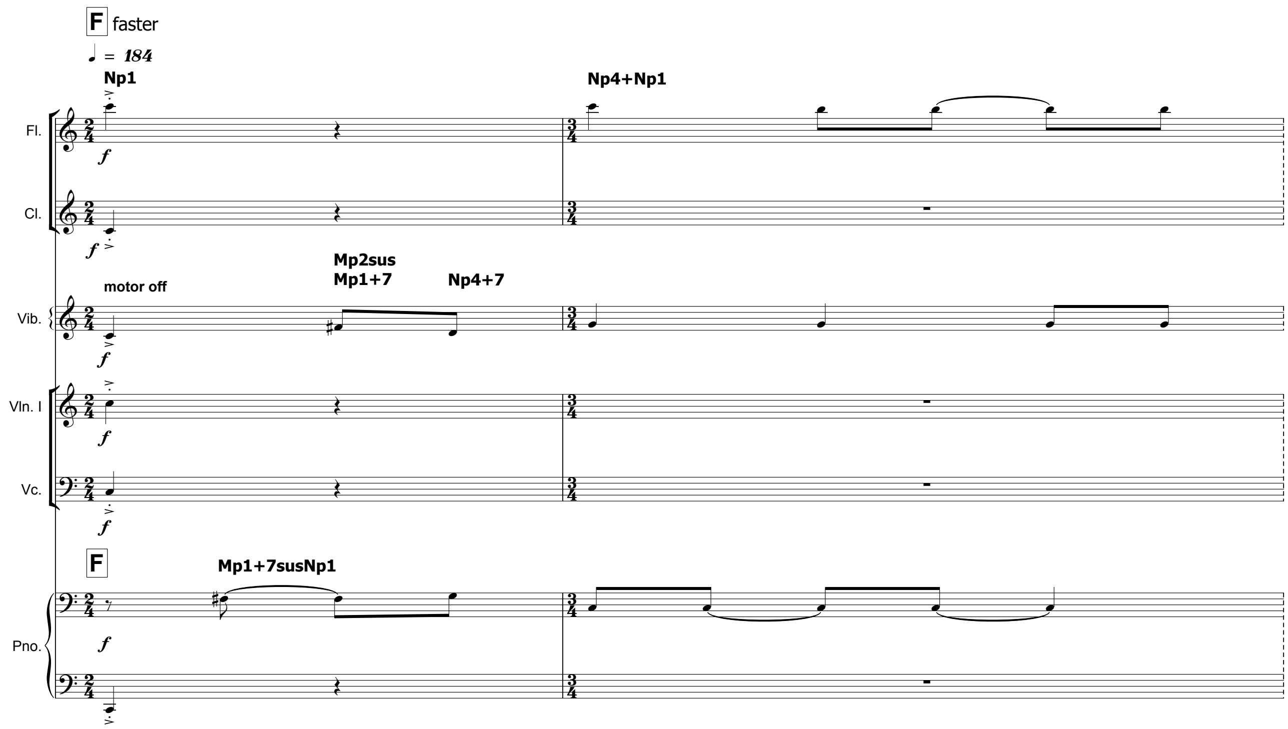
Table22 Symbolic Encoding of Chords of Phrygian Mode

B F♯G	-	**M**
C G A	-	**N**
D A B	-	**N**+2
E B C	-	**M**+5
F♯ C♯ D*	-	**M**+7
G D E	-	**N**+7
A E F♯	-	**N**+9

The chord with the asterisk is chromatically altered to generate a missing chord.

The duration vectors are inverted for IIa with the longest values in Palestrina's music becoming the shortest values in the mutation:
quaver = 11 ; crotchet = 7 ; dotted crotchet = 5 ; minim = 3 ; minim tied to a crotchet = 2
Example 32 shows the first two bars of IIa at bar 52.

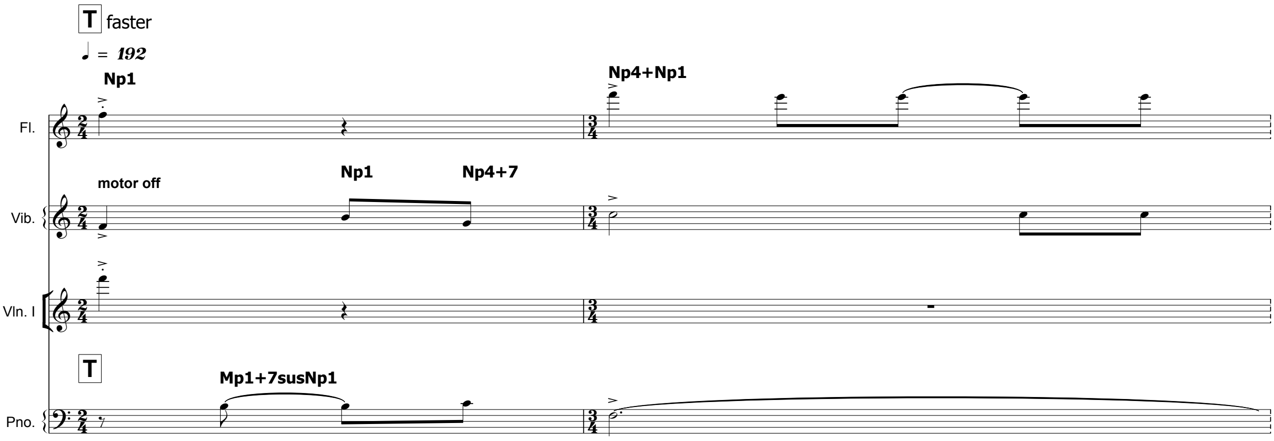
Ex.32 *Mutation of Kyrie* IIa b.52 Encoding shown above new note.



The change in duration vectors produces a different *HRS* to that found in Ia. This procedure mirrors that of previous sonata forms where the metre subdivided in a proportion to the number 2 in a new formal section. However, the *mutation* of pre-existing *HRS* means the change of rhythmic material is of a more complicated nature than simply dividing rhythm types. The change in duration vectors in IIa of *Mutation of Kyrie* produces a quicker harmonic rhythm than in Ia and a change in metric structure.
The development section (shown in Form Chart 12) at bar 138 reworks material from the second subject to bar 154 and from the first subject from bar 155 to 161. The chords are transposed by an augmented fourth from the original at bar 138 and up a third at bar 147. At bar 155 the material returns to the *artificial tonality* of the exposition Ia.
The recapitulation begins at bar 162 with a slightly altered Ia (shown in Form Chart 12) with a starting note of **E** for the mode and chords shown in Figure 38. At bar 213 IIa (shown in Form Chart 12) returns with the chords transposed down a fifth to give a starting note of **E** for the Phrygian Mode in a similar fashion to traditional sonata form. The difference here is the use of *alternative chords* in the IIa

recapitulation. The use of a Phrygian mode with a starting note of **E** in the recapitulation is constant however, the use of the third hierarchy *alternative chords* shown in Figure 38, in the IIa recapitulation helps to articulate the formal sections. Example 33 shows the beginning of the IIa recapitulation. This illustrates the change in *artificial tonality* with a new hierarchy of *alternative chords* in comparison to those used in the exposition.

Ex.33 *Mutation of Kyrie* IIa b.213 Recapitulation. Encoding shown above new note.



24v. Mutation of Gloria

This movement *remakes* bars 24 beat 4 to bar 35 beat 1 of the *Gloria*. There are several differences in the *Mutation of Gloria* in contrast with the *Kyrie*:

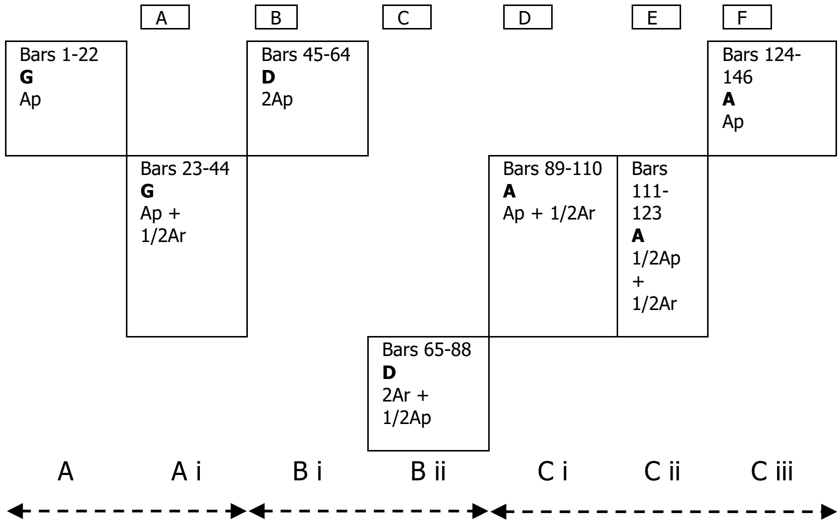
1. The *mutation* uses a short extract and reworks Palestrina's music into a three-section form ABC in which there is no return to the *artificial tonality* of the A section in the final section, the same form used in *Concertino*.
2. The duration vectors remain constant in the three sections, however, the rhythms of the theme in A are subdivided with the number 2 as a denominator.
3. The *mutation* makes use of retrograde, diminution and augmentation of the A theme rhythms.

The *mutation* in the A section of the extract from Palestrina has a change in *artificial tonality* in the B and C sections and is modified further using retrograde, augmentation and diminution of rhythm types. In practise the procedure was slightly more complicated. For each section the prime version of the theme in A is heard against its retrograde. Either the prime version or the retrograde is the preeminent version, if a conflict of pitch arises, the pitch reverts to this version. This is hard to explain in principle however, in practical terms the two versions were placed together and the chords of the preeminent version attracted all the pitches to its chords. This resulted in extra rhythms on the same pitch or an arpeggiation of the chord (see pg.160-161).

24vi. Form of *Mutation of Gloria*

Form Chart 13 shows the form of *Mutation of Gloria.*

Form Chart 13 *Mutation of Gloria*



Ap is the prime form of A

Ar is the retrograde of A

1/2Ap is the diminution of the prime form of A

1/2Ar is the diminution of the retrograde of A

2Ap is the augmentation of the prime form of A

2Ar is the augmentation of the retrograde of A

Ap + 1/2Ar is the prime form of A heard against the diminution of the retrograde of A etc.

The formal scheme works since 1/2Ar is half the length of Ap and thus would be stated twice against one statement of Ap etc. The scheme allows for peaks and relaxations of musical intensity to be built in to an overall ABC form.

In Palestrina's music, the two choirs exchange similar musical material up to bar 33 followed by a quick climax to a tutti at bar 34. Consequentially, in the A section the use of A and Ap + 1/2Ar means that the music builds to a climax at the end of A. The use of Ar means that this intensity level is sustained throughout the restatement of A. A similar procedure is followed for the B section with 2Ap followed by 2Ar + 1/2Ap. Another consequence of this formal structure is that certain motifs such as the ascending crotchet passage in bar 33 of Palestrina's *Gloria* reoccur with various transformations.

Example 34 shows the beginning of the A theme. The music mirrors the original in that the musical material swaps from the muted brass to the woodwind.

Ex.34 *Mutation of Gloria* A

The triads for Palestrina's original music are shown in Table 22. The duration vectors in Palestrina's *Gloria* are:

Quaver = 2 ; crotchet = 3 ; minim = 5 ; dotted minim = 7 ; semibreve = 11

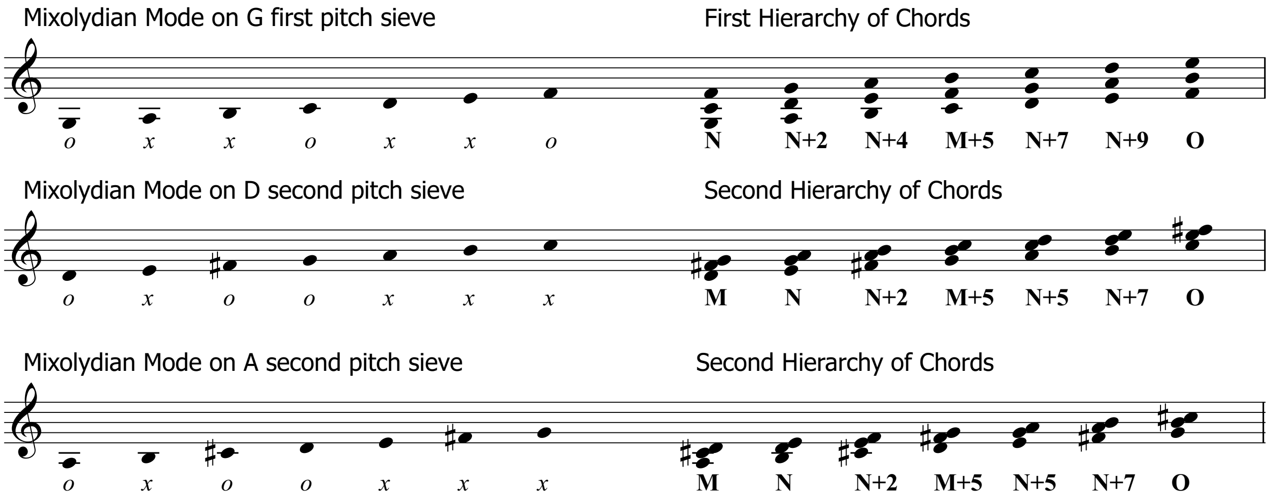
Following a similar procedure to *Mutation of Dowland's Midnight* and *Mutation of Kyrie* Table 23 gives the first four bars of the symbolic encoding for bars 24 beat 4 to bar 35 beat 1 of Palestrina's *Gloria,* the complete table is in Appendix 1, pg.213.

Table23 Symbolic Encoding of *Gloria* b.24-35, (b.24-27 only)

Symbolic Coding for Each Bar	Bar No.
2^7^(**Mp1+M**)	24
2^3^(sus**Mp1+M**)+5^(**M+7+Mp1+7**)+11^(**Mp1**+**M**)	25
2^5^(**M+5+Mp2+5**)+5^(**Np1**+**N**)+3^(**Np1+7**sus**N**)+ 3^(**Np6+7**sus**Np1+7+Np3**)+3^(**Ns1+7**sus**Np1+7**)**+** 2^(**Mp6+7**sus**Np1+7+Np1+7**)+2^(**Np4+7**sus**Np1+7**+**Np1+7**)	26
2^5^(**Mp6+7**+**Np1+7**)**+**5^(**M**)+5^(**Np4+7**sus**Mp3**)+ 5^(**Mp2+2**sus**Np4+**7)	27

For the *mutation,* the *OTS* of Palestrina's music was replaced two different pitch sieves of a Mixolydian Mode starting from the notes **G**, **D** and **A.** Figure 39 shows the mode and the chords:

Fig.39 Mode, chords: *Mutation of Gloria*



The chords used for the theme in the A section are shown in Table 24:

Table24 Symbolic Encoding of Chords of Mixolydian Mode

G C F♯*	-	**M**
A D G	-	**N**
B E A	-	**N**+2
C F B	-	**M**+5
D G C♯*	-	**M**+7
E A D	-	**N**+7
F B E	-	**O**

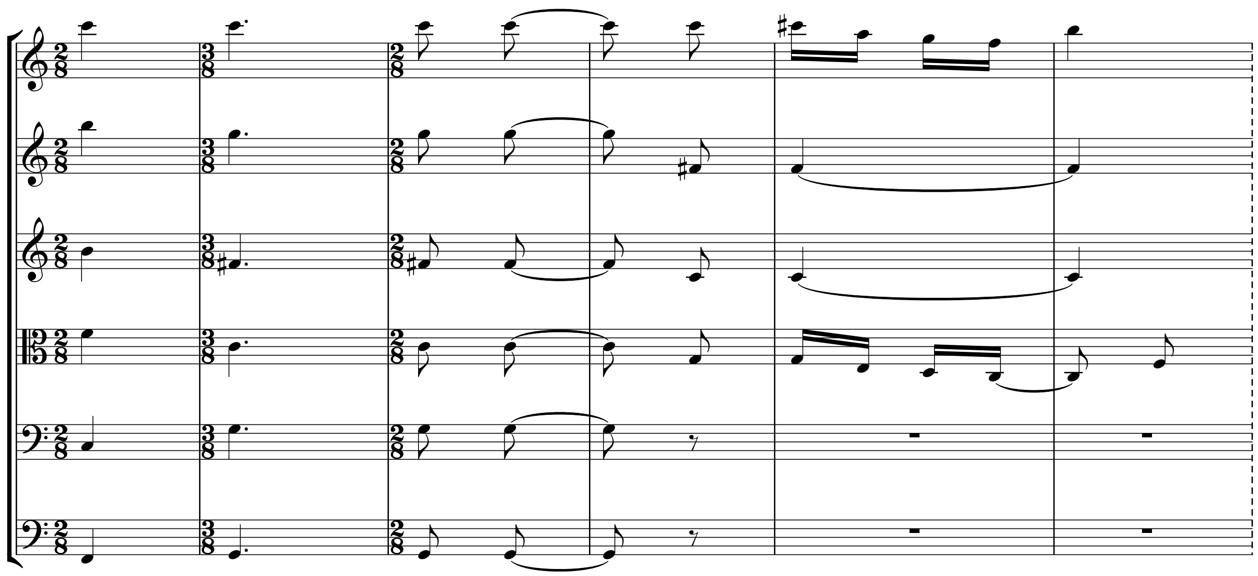
Chords marked with as asterisk have been chromatically altered from those shown in Figure 39 to generate missing chords.

The duration vectors used in the *mutation* are:

Semiquaver = 2 ; quaver = 3 ; crotchet = 5 ; dotted crotchet = 7 ; dotted minim = 11

At bar 23 the A theme in its prime form is heard against the diminution and retrograde of the A theme. Example 35 shows the open score version of the start of 1/2Ar and Example 36 shows the realisation of A + 1/2Ar (shown in Form Chart 13) at bar 23.

Ex.35 *Mutation of Gloria*1/2Ar



By comparing 1/2Ar and the A theme shown in Example 34, the chords of the A theme are preserved and 1/2Ar either introduces a new rhythm or an arpeggiation of a chord. At bar 25 the ascending crotchet passage in bar 33 of Palestrina's *Gloria* reoccurs, *mutated* to a quicker, descending figure.

Ex.36 *Mutation of Gloria* Ai A+1/2Ar. en in bar 23 indicates the extra notes in comparison to the A theme bar 1 in Example 34.

At bar 45 the B section (shown in Form Chart 13) begins with the A theme in augmentation. For the B section the *artificial tonality* not only changes to a starting note of **D** for the Mixolydian Mode; the second pitch sieve shown in Figure 39 gives a set of *alternative chords* which helps to articulate the formal sections of the music. Table 25 shows the chords used in the B section:

Table25 Symbolic Encoding of Chords of Mixolydian Mode

D F♯G	-	**M**
E G A	-	**N**
F♯ A B	-	**N**+2
G B C	-	**M**+5
A C♯ D*	-	**M**+7
B D E	-	**N**+7
C E F♯	-	**O**

The chord marked with an asterisk is chromatically altered to generate a missing chord.

At bar 67 an augmented retrograde form of the theme in A is heard against 1/2Ap (shown in Form Chart 13). The chords used are the same as those used in Bi.

At bar 89 the *artificial tonality* changes to a Mixolydian Mode starting on **A**. The chords used are shown in Table 26.

Table26 Symbolic Encoding of Chords of Mixolydian Mode

A C♯ D	-	**M**
B D E	-	**N**
C♯ E D	-	**N**+2
D F♯ G	-	**M**+5
E G♯ A*	-	**M**+7
F♯ A B	-	**N**+7
G B C♯	-	**O**

The chord marked with an asterisk is chromatically altered to generate a missing chord.

These are the same triads as for the B section transposed up a fifth. The second movement of *Mutation of MPM* was intended to be calmer and less intense in comparison to the *mutated Kyrie* and *Credo* either side of it. The use of the same duration vectors throughout and the re-use of the same triads in the C section help to give a more static feel to the music.

Ci (shown in Form Chart 13) is an altered statement of Ap + 1/2Ar from Ai (shown in form Chart 13). The use of *alternative chords* and a different starting note for the Mixolydian Mode differentiate this statement of Ap + 1/2Ar. Example 39 shows the start of Ci at bar 89. Comparison with Ai in Example 34 shows that it is the chords used, that differentiates Ai from Ci.

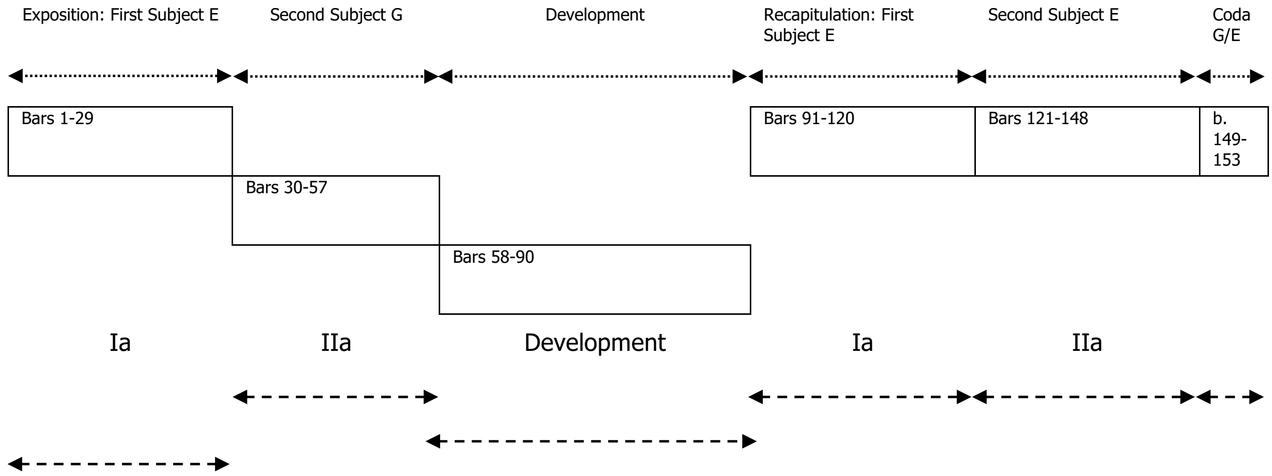
Ex.37 *Mutation of Gloria* Ci A+1/2Ar. Chords used are 2nd pitch sieve on A shown in Fig.39.

At bar 111 the intensity of the music increases with 1/2Ap + 1/2Ar meaning that a diminution of the A theme is heard against a diminution of the retrograde form of the A theme. At bar 124 the A theme returns with the original chords; however, the original music is transposed up a fifth to a starting note of **A** for the Mixolydian Mode. This transposition preserves the *tonality* of the C section and gives a contrast of musical material with the A section.

24vii. *Mutation of Credo*

Form Chart 14 shows the form of *Realisation* of *Mutation of Credo.*

Form Chart 14 *Mutation of Credo*



This movement is similar in design to *Mutation of Kyrie,* it is written in sonata form and has dissonant pitch-organisation. The *Credo* is the centre point of Palestrina's Mass with a lengthy libretto. To reflect this, *Mutation of Credo* has a more complex design than the other movements. Ia, (shown in Form Chart 14), *mutates* bars 116-131 of Palestrina's *Credo* using an **E** Dorian Mode **E F♯ G A B C♯ D E** to give the *artificial tonality:*

Table27 Symbolic Encoding of Chords of Dorian Mode

E F♯ G	-	**M**
F♯ G A	-	**O**
G A B	-	**N**
A B C*	-	**M**+5
A B C♯	-	**N**+2
B C♯ D	-	**M**+7
C♯ D E	-	**O**+7
C D E*	-	**N**+5
D E F♯	-	**N**+7

The chords marked with an asterisk are chromatically altered to generate missing chords. The Dorian Mode was chosen to contrast with the **E** Phrygian mode used in *Mutation of Kyrie.* The duration vectors used for Ia of the *Mutation of Credo* are those described on pg. 153 in Ia of the *Mutation of Kyrie.*

IIa, (shown in Form Chart 14), mutates bars 153-165 of Palestrina's *Credo.* This contrasts with *Mutation of Kyrie* where IIa *mutates* the same extract as Ia. IIa in *Mutation of Credo* uses a **G** Lydian mode: **G A B C♯ D E F♯** to give the *artificial tonality:*

Table28 Symbolic Encoding of Chords of Lydian Mode

G B C♯	-	**M**
A C♯ D	-	**N**
B D E	-	**O**
C♯ E F♯	-	**M**+6
D F♯ G	-	**N**+5
D F♯ G♯*	-	**M**+7
E G A	-	**O**+7
E G♯ A*	-	**N**+7
F♯ A B	-	**M**+e

The chords marked with an asterisk have been chromatically altered. **G** Lydian Mode was chosen in order to give IIa the equivalent of a relative major modulation i.e. from **E** minor to **G** major.

IIa of *Mutation of Credo* uses the same duration vectors as IIa of *Mutation of Kyrie* shown on pg.155.

Bar 66 in the development section, (shown in Form Chart 14), *mutates* bars 178-184 of Palestrina's *Credo.* This passage uses an *artificial tonality* generated from a **B** Phrygian Mode: **B C D E F# G A**

Table29 Symbolic Encoding of Chords of Phrygian Mode

B C D	-	**M**
C D E	-	**N**
D E F♯	-	**N**+2
E F G*	-	**M**+5
E F♯ G	-	**O**
F G A*	-	**N**+5
F♯ G A	-	**M**+7
G A B	-	**N**+7
A B C	-	**O**+5

Chords marked with an asterisk have been chromatically altered.

The duration vectors for the passage from bars 66-90 are the same for Palestrina's original music shown on pg.152.

Table 30 shows how the music from bars 66-90 uses a similar procedure to that described on pg.157 in *Mutation of Gloria.*

Table30 Form of *Mutation of Credo* b.66-90

P	P+Pr	Pr+1/2P	1/2P+1/2Pr
b. 66-72	b. 73-79	b. 80-86	b. 87-90

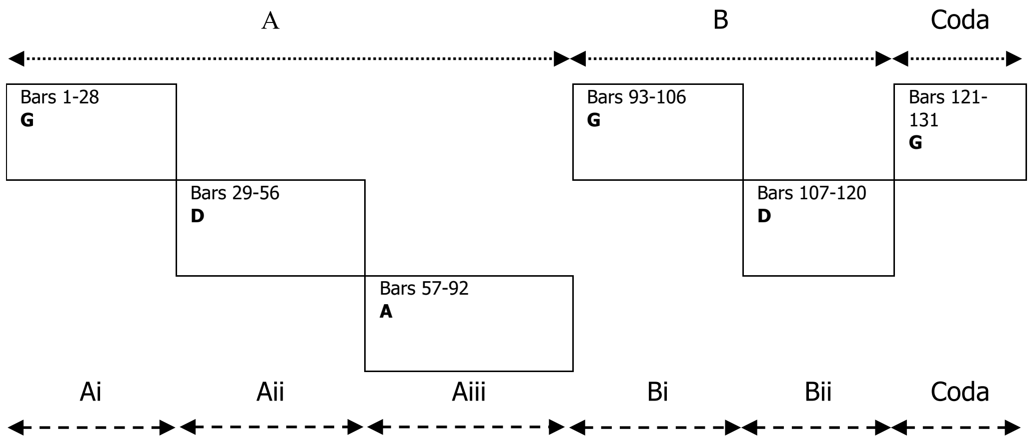
Where P is prime, Pr is retrograde of P, 1/2P is rhythmic diminution of P and 1/2Pr is rhythmic diminution of retrograde of P. Bars 66-90 of *Mutation of Credo* are a retransition to the return of Ia. Table 30 shows how the music is an accumulative crescendo to the recapitulation.

At bar 121 IIa, (shown in Form Chart 14), returns in the *artificial tonality* shown in Table 27 thus remaining in the same tonality as Ia. In the Coda at bar 149, (shown in Form Chart 14), there is a restatement of bars 87-90.

24viii. *Mutation of Sanctus*

Form Chart 15 shows the form of *Mutation of Sanctus Open Score.*

Form Chart 15 *Mutation of Sanctus*



Mutation of Sanctus mutates bars 1-14 of Palestrina's *Sanctus*, each of the sections shown in Form Chart 15 *mutate* this same extract. *Mutation of Sanctus* was regarded as an oasis of calm after the complex, dissonant *Mutation of Credo.* Hence, *Mutation of Sanctus* is in binary form and uses relatively consonant pitch-organisation. Ai (shown in Form chart 15), uses an *artificial tonality* derived from a **G** Mixolydian Mode: **G A B C D E F**

Table31 Symbolic Encoding of Chords of Mixolydian Mode

G A E	-	**M**
A B F	-	**O**
B C G	-	**N**
C D A	-	**M**+5
D E B	-	**M**+7
E F C	-	**N**+5
F G D	-	**O**+8
F♯ G E*	-	**N**+7

The chord marked with an asterisk has been chromatically altered.

Ai (shown in Form chart 15), uses the following duration vectors: quaver = 2; dotted quaver = 3, minim = 5 and dotted minim = 7.

Ai (shown in Form chart 15), uses an *artificial tonality* transposed from **G** to **D** with a different pitch sieve.

Table32 Symbolic Encoding of Chords of Mixolydian Mode

D F♯ B	-	**M**
E G C	-	**N**
F♯ A D	-	**N**+2
G B E	-	**M**+5
A C F♯	-	**O**
A C♯ F♯*	-	**M**+7
B D G	-	**N**+7
C E A	-	**M**+e

The duration vectors for Aii, (shown in Form Chart 15), are the same as A.
Aiii (shown in Form chart 15), uses an *artificial tonality* transposed to **A** with another pitch sieve.

Table33 Symbolic Encoding of Chords of Mixolydian Mode

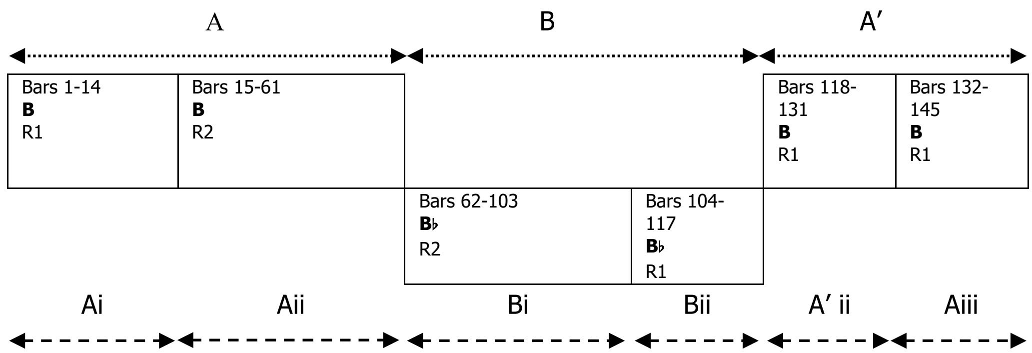
A E F♯	-	**M**
B F♯ G	-	**N**
C♯ G A	-	**N**+2
D A B	-	**M**+5
E B C♯	-	**O**
F♯ C♯ D	-	**M**+7
C E A	-	**M**+e

Aiii, (shown in Form Chart 15), uses the following duration vectors: minim = 2, crochet = 3, dotted crochet = 5, quaver = 7.
To enhance a mood of tranquillity in *Mutation of Sanctus* Bi, (shown in Form Chart 15), returns to an *artificial tonality* derived from a **G** Mixolydian Mode by transposing the chords shown in Table 33 down by a wholetone. The duration vectors for Bi are: quaver = 2, crochet = 3, dotted crochet = 5, minim = 7.
Bii, (shown in Form Chart 15), uses the *artificial tonality* shown in Table 32 and uses the same duration vectors as Bi.
The Coda, shown in form Chart 15, is a restatement of Ai bars 1-11.

24ix. *Mutation of Benedictus*

Form Chart 16 shows the form of *Mutation of Benedictus Open Score:*

Form Chart 16 *Mutation of Benedictus*



The *Benedictus* from Palestrina's *Missa Papae Marcelli* is shorter in length than the other movements and is in four-part writing rather than six-part. Mutation *of Benedictus* mutates the whole of Palestrina's *Benedictus* to give a ternary form ABA′. The A and A′ sections (shown in Form Chart 16), uses an *artificial tonality* generated from a **B** Lochrian Mode:

Table34 Symbolic Encoding of Chords of Lochrian Mode

B C D	-	**M**
C D E	-	**N**
D E F	-	**O**
E F G	-	**M**+5
F G A	-	**N**+5
F♯ G A*	-	**M**+7
G A B	-	**N**+7
A B C	-	**O**+7

The B section, (shown in Form Chart 16), uses a **B♭** in place of a **B** the consequence is that the Lochrian Mode used in section A becomes a **B♭** Phrygian Mode: **B♭ C D E F G A** and gives an *artificial tonality*:

Table35 Symbolic Encoding of Chords of Phrygian Mode

B♭ C D	-	**M**
C D E	-	**M**+2
D E F	-	**N**
E♭ F G*	-	**M**+5
E F G	-	**O**
F G A	-	**M**+7
G A B♭	-	**N**+5
A B♭ C	-	**O**+5
A B C*	-	**N**+7

The **B** to **B♭** semitone fall mirrors the **F** to **E** found in the **E** Phrygian Mode used in important structural points in *Mutation of MPM. Mutation of Benedictus* is the penultimate movement and the use of a mode based on the note **B** was chosen to sound like a dominant to the note **E** meaning the *artificial tonality* of the final movement would sound like a tonic resolution.

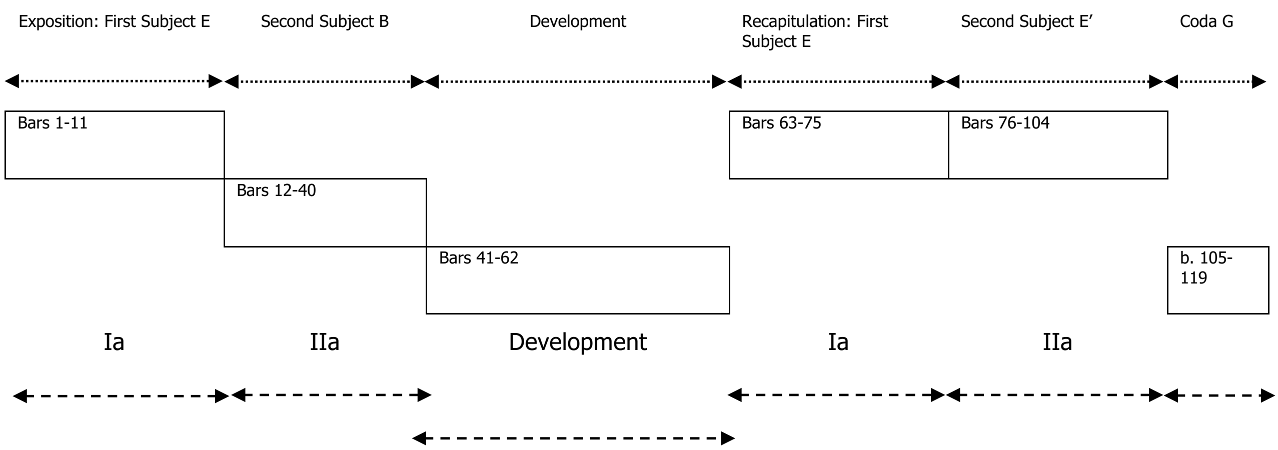
Form Chart 16 shows how the 39 bars of Palestrina's *Benedictus* were divided into three 13 bar segments. The first segment was *mutated* to give Ai, (shown in Form Chart 16), using the *artificial tonality* shown in Table 34 and durations vectors labelled R1 in Form Chart 16: semibreve = 2, dotted minim = 3, dotted crochet = 5, crochet = 7, quaver = 11, semiquaver = 13. R2 has the duration vectors: quaver = 2, crochet = 3, dotted crochet = 5, minim = 7, minim tied to a quaver = 11, dotted minim = 13. R2 was used in the *mutation* of the second and third 13 bar segments Aii and Bi respectively. Aii has the same *artificial tonality* as Ai, Bi uses that shown in Table 35. At bar 104 the *mutation* of the three 13 bar segments was repeated using the *artificial tonality* and duration vectors indicated in Form Chart 16 to give Bii, A'ii and Aiii.

24x. *Mutation of Agnus Dei*

Missa Papae Marcelli by Palestrina has two setting of *Agnus Dei.* The *Mutation of Agnus Dei* uses extracts from both settings. Ia and IIa (shown in Form Chart 17), *mutate* bars 1-15 of Palestrina's *Agnus Dei I.* The coda, (shown in Form Chart 17), *mutates* bars 46-53 of *Agnus Dei II* with the consequence that the final bars of *Mutation of MPM* are a *mutation* of the final bars of Palestrina's music.

Form Chart 17 shows the form of *Realisation of Mutation of Agnus Dei*

Form Chart 17 *Mutation of Agnus Dei*



Ia (shown in Form Chart 17), uses an *artificial tonality* derived from an **E** Phrygian Mode

Table36 Symbolic Encoding of Chords of Phrygian Mode

E A B	-	**M**
F B C	-	**N**
G C D	-	**M**+3
A D E	-	**M**+5
B E F	-	**O**
B E F♯	-	**M**+7
C F G	-	**M**+8
C F♯ G	-	**N**+7
D G A	-	**M**+t

For duration vectors for Ia are: semibreve = 2, minim = 3, dotted crochet = 5, semiquaver = 7.

IIa (shown in Form Chart 17), uses the same *artificial tonality* derived from a **B** Phrygian Mode as IIa from *Mutation of Kyrie,* see pg.154. The duration vectors for IIa are: semiquaver = 2, dotted quaver = 3, crochet = 5, dotted crochet = 7.

At bar 52 in the development section of *Mutation of Agnus Dei* there is a false recapitulation using the duration vectors of Ia but the chords shown in Table 36 are replaced with the chords shown in Table 37 derived from a **G** Mixolydian Mode

Table37 Symbolic Encoding of Chords of Mixolydian Mode

G B C	-	**M**
A C D	-	**N**
B D E	-	**N**+2
C E F	-	**M**+5
D F G	-	**N**
D F♯ G	-	**M**+7
E G A	-	**N**+5
F A B	-	**N**+7

At bar 76, (shown in Form Chart 17), IIa returns with the chords shown in Table 22 replaced with an artificial tonality derived from an **E** Phrygian Mode:

Table38 Symbolic Encoding of Chords of Phrygian Mode

E B D	-	**M**
F C E	-	**N**
G D F	-	**M**+3
A E G	-	**M**+5
B F A	-	**O**
B F♯ A	-	**M**+7
C G A	-	**N**+7
D A B	-	**M**+t

The durations vectors remain the same as for IIa in the exposition.
The coda, (shown in Form Chart 17), at bar 132 uses a modified version of the chords in Table 38 derived from a **G** Mixolydian Mode:

Table39 Symbolic Encoding of Chords of Mixolydian Mode

G D F	-	**M**
A E G	-	**M**+2
B F A	-	**O**
C G B♭	-	**M**+5
C G B	-	**N**
D A C	-	**M**+7
E B D	-	**M**+9
F C E	-	**N**+5
G D F♯	-	**N**+7

The durations vectors in the coda (shown in Form Chart 17), are the same as Palestrina's original music, see pg.152.
The final section of *Mutation of Agnus Dei* bars 112-119 uses the chords from the false recapitulation at bar 52 shown in Table 37.

24xi. *This Day Dawns*

Another composition to utilise *artificial tonality* is a piece for SATB setting an anonymous text *This Day Day Dawns,* this composition is submitted as an appendix. Figure 24 shows the Phrygian Mode on **E** and **B** and the *alternative chords* used in *This Day Dawns*. The refrain of *This Day Dawns* uses the first pitch sieve shown Figure 24 which generates an *artificial tonality*

Table40 Symbolic Encoding of Chords of Phrygian Mode

E G A	-	**M**
F A B	-	**O**
G B C	-	**N**
A C D	-	**M**+5
B D E	-	**M**+7
C E F	-	**N**+5
D F G	-	**M**+t

In the verse section of *This Day Dawns* the music modulates to an *artificial tonality* from the Phrygian Mode on **B**. For this section, the first chord is transposed to the note **G** to give the *alternative chords*

Table41 Symbolic Encoding of Chords of Phrygian Mode

G B C	-	**M**
A C D	-	**N**
B D E	-	**N**+2
C E F	-	**M**+5
D F♯G	-	**M**+7
E G A	-	**N**+7
F A B	-	**O**

This Day Dawns applies techniques acquired in the *mutation* of past music using *MIs.* A chord sequence built from the first pitch sieve in Figure 24 is reused in the verse section, however, the chords from the first pitch sieve are replaced with the chords from the second pitch sieve in Figure 24. The technique of symbolically encoding the triads in the refrain to reuse the same pitch-organisation in the verse, replicates the procedure used in the *remaking* of past music found in *Mutation of Dowland's Midnight* and *Mutation of MPM.* This is an example where a theoretical model leads to a methodology that may be freely applied in a usage that differs from its original conception.

24xii. Reflections on the Use of *MIs* and *Artificial Tonality*

MIs and *artificial tonality* are the practical application of the theories of *Transcendental Musical Platonism* and *MIs*. The methodology offers the possibility of *mutating* an entire composition with alternative *OTS* and *TS*, however, a more creative approach is to treat the *MI* as a source of musical DNA which may be *mutated* into a composition in the style of the modern composer.

The advantage of using the compositional methodology outlined in this chapter is the encoding and decoding of the *MI* is symbolic rather than literal. It is like the translation of the English sentence: "This Sentence ends with the French word *chien"* into French: "Cette phrase se termine par la mot Français chien." This is a literal translation however, it fails to convey the feel and meaning of the original sentence. A symbolic translation would be: "Cette phrase se termine par la mot Anglais dog." In the symbolic translation "Anglais dog" replaces "Français chien" in order to preserve the mixture of languages in the original sentence and thus convey a similar ambience in the new language.

The symbolic nature of *MIs* means the *mutation* of past music is like the second, symbolic translation of the sentence above. A chord in the new *OTS* has a symbolic counterpart in the original music. A chord sequence encoded and decoded from an *MI* is likely to be more successful in conveying the feel of the original music since a symbolic relationship exists between the chord of the original music and the *mutation.* The use of *MIs* simplifies the translation of the original version to the *mutated* version. Using *MIs* gives a method in which both semiotic and semantic content is preserved in the *mutation.*

24xiii. *MIs* and *Artificial Tonality* in Context

The compositional methodology outlined in this chapter is a summation of all the previous achievements described in earlier chapters. These are listed as follows:

1. *Harmo-rhythmic structures*: *MIs remake* pre-existing *HRS.* This allows for the rhythmic control of dissonance informed by historical precedence.
2. *Tonality.* The use of *artificial tonality* articulates formal sections through the contrast of chord. The use of *triadic hierarchy modes* gives the chords used a built-in hierarchy.
3. *Remaking the Past*. *MIs* allow for a precise *mutation* of pre-existing music by the symbolic encoding and decoding of music.
4. *Symbolic Music*. *MIs* extend *Symbolic Music* by allowing a calibrated expression for duration vectors.
5. *Musical Platonism*. A relativistic model regards *MIs* as a model of the *ITS* of a piece of music, the *ITS* is in a direct relationship to the *Form*. The realisation of this *Form* to produce a composition is a defective *copy* of the *Form*.

The *mutation* process outlined in this commentary is in terms of pitch and duration only. The methodology could be expanded to allow the encoding of dynamics and instrumentation, it is a trade-off between the amount of encoding undertaken and the complexity of the *MI*. Similarly, the *MIs* described in this commentary are reduced to a single octave in the encoding process, the encoding could be made more complex to indicate the exact pitch of the original music. The view was taken that the compositional methodology should allow a certain amount of freedom to the composer in the decoding process. The ordering of the pitches from the bass upwards is preserved however, the composer has a limited choice in terms of the register of the pitches in the decoding dependent upon the instruments used.
The instrumentation, dynamics, articulation etc. used in the decoding are choices made by the composer rather than emerging from the system employed, the composer is allowed creative freedom in the decoding process.

Chapter 25. *Evolution of the Cosmos*

25i. Introduction

Palestrina's *Missa Papae Marcelli* sets the words of the Ordinary of the Latin Mass *Kyrie, Gloria, Credo, Sanctus, Benedictus and Angus Dei* (Roche, Elizabeth and Alex Lingas. "Mass." *The Oxford Companion to Music*. *Oxford Music Online*. Ed. Alison Latham (Accessed 25 March 2017). The Latin Mass is sung before and during the Eucharist which is a re-enactment of the Last Supper when Jesus gave bread and wine to the disciples saying:

> "This is my body given for you; do this in remembrance of me." In the same way, after the supper he took the cup, saying, "This cup is the new covenant in my blood, which is poured out for you. "
> (Luke 22:7-23)

The libretto for *Evolution of the Cosmos* (*Eotec*) was compiled by the author from a variety of sources, these are listed in the bibliography, see pg.221. The words of the Latin Mass influenced the form and structure of the libretto. The six sections of the mass *Kyrie, Gloria, Credo, Sanctus, Benedictus and Angus Dei* are mirrored with six sections in *Evolution of the Cosmos* (*Eotec*) *The Universe Began, The First Stars, The Laws of Science, The Solar System, Planet Earth* and *The Evolution of Life.*

Gloria, Sanctus and Benedictus were regarded as lighter, celebratory sections with the *Kyrie, Credo and Angus Dei* viewed as more serious by contrast. This is reflected in the libretto with *The First Stars, The Solar System* and *Planet Earth* having lighter, less involved content with *The Universe Began, The Laws of Science* and *The Evolution of Life* having greater detail and theoretical complexity in comparison.

In certain sections of *Eotec* the words of the Latin mass are sung, see pg.178.

The Liturgy set by Palestrina states key doctrines of the Catholic faith. *Gloria* has the words:

> Domine Deus, Agnus Dei,
> Filius Patris.
> Qui tollis peccata mundi,
> miserere nobis.
> Qui tollis peccata mundi,
> suscipe deprecationem nostram.
> - Lord God, Lamb of God,
> Son of the Father.
> Thou that takest away the sins of the world, have mercy upon us.
> Thou that takest away the sins of the world, receive our prayer.

These words echo the description of the Last Supper celebrating the feast of the Passover. Jesus offers himself as a sacrificial lamb to atone for the sins of the world. The *Credo* sets out the key areas of belief for Catholics:

> Credo in unum Deum;
> Patrem omnipotentem,
> factorem coeli et terrae,
> visibilium omnium et invisibilium.
> - I believe in one God;
> the Father almighty,
> maker of heaven and earth,
> and of all things visible and invisible.
>
> Crucifixus etiam pro nobis
> sub Pontio Pilato,
> passus et sepultus est.
> Et resurrexit tertia die
> secundum Scripturas.
> - He was crucified also for us,
> suffered under Pontius Pilate,
> and was buried.
> And on the third day He rose again
> according to the Scriptures:
>
> Credo in unam sanctam
> catholicam et apostolicam Ecclesiam.
> Confiteor unum baptisma,
> in remissionem peccatorum.
> Et expecto resurrectionem mortuorum
> et vitam venturi sæculi.
> - And I believe in one holy
> catholic and apostolic Church.
> I acknowledge one baptism
> for the remission of sins.
> And I await the resurrection of the dead
> and the life of the world to come.

The libretto of *Eotec* sets out a scientific explanation of the evolution of the cosmos according to theoretical physics and studies of the fundamental forces of nature. A scientific *credo* is outlined in *The Laws of Science.* It is a matter of faith whether one believes these laws to be the workings of "the Father almighty maker of heaven and earth, and of all things visible and invisible."

25ii. Inspiration

The derivation of the *Mutation of MPM* from Palestrina's mass is outlined in Chapter 24, *Mutation of MPM,* see pg.151-171. A number of devices used in *Eotec* were inspired by a study of the *War Requiem*[42] by Britten[43]. The *War Requiem* contrast the sacred text of the Latin Liturgy with the secular poetry of Wilfred Owen[44]. Britten uses lines from Owen's poetry to comment and reflect upon the Latin text (Evans P, Britten's War Requiem*, Tempo No. 61/62,* 1962). *Eotec* uses a contrast of sacred and secular. The sacred content is the instrumental sections *mutated* from Palestrina's mass with some passages quoting the sacred text from the movements used for the *mutation*. The settings of the secular libretto are freely composed using pitch-organisation and durations derived from Palestrina's music.
The *War Requiem* uses the interval of a Lydian augmented fourth as a structural device in the large-scale form (Ibid.). *Eotec* uses a similar strategy in the use of the note **F** to begin *The Universe Began, The Laws of Science* and *The Evolution of Life.* The repetition of this pitch gives a unity between these three movements all of which use sonata form. The start of the exposition and recapitulation sound a prominent **F** in these structurally significant places.
Britten uses instrumentation and timbre to unify the large-scale form of the *Requiem.* The boys choir, mixed chorus and solo soprano sing all of the Latin text, the only exception is the solo tenor singing *Dona nobis pacem* (Britten B, *War Requiem,* op. 66, Score Preface, n.p. B&H, 1961). The solo baritone and tenor sing the lines from Owen's poetry. Further differentiation is achieved by the use of an off-stage organ to accompany the boys choir, the orchestra to accompany the mixed choir and solo soprano and a chamber orchestra to accompany the solo baritone and tenor. Britten uses the timbre of bells, often playing the interval of a Lydian augmented fourth at structurally significant points in the music (Evans P, Britten's War Requiem, *Tempo No. 61/62*, 1962).
In *Eotec* the beginning of the sonata form of *The Universe Began, The Laws of Science* and *The Evolution of Life* uses a crescendo on the tam-tam as an introduction. The second subject of these three movements uses a bass clarinet and vibraphone, timbre and instrumentation signify this section within the overall sonata form structure. The freely composed sections of *The Universe Began, The Laws of Science* and *The Evolution of Life* employ a drum kit as a cultural signifier*,* the derivative rock, jazz and Latin beats are seen as a striking contrast to the vocal polyphony of Palestrina. The climax of the final movement *The Evolution of Life,* is the only time that un-tuned percussion and a *mutation* of Palestrina's music are

[42] Britten B, *War Requiem,* op. 66, 1961, B&H
[43] British composer Benjamin Britten 1913-1976
[44] British war poet 1893-1918

heard together. A contrast of vocal texture was used to differentiate the separate movements of *Eotec. The Universe Began, The Laws of Science* and *The Evolution of Life* use complex, polyphony vocal textures at certain points while *The First Stars, The Solar System* and *Planet Earth* employ monody or homophonic textures. Britten evokes the sound of a Gamelan orchestra in his setting of *Sanctus* on page 140 (Britten B, *War Requiem,* op. 66, n.p. B&H, 1961). This inspired the decision to use gongs and tuned percussion in the sections of *Eotec mutated* from Palestrina, only the freely composed sections use un-tuned drums and cymbals.

25iii. Setting the Libretto of *Evolution of the Cosmos*

The re-working of *Mutation of Missa Papae Marcelli* to produce *Evolution of the Cosmos (Eotec)* produced a fundamental problem, the libretto of *Eotec* is in technical language and only certain rhythms and phrase lengths may be employed in the word setting. There is no guarantee that these rhythms and phrases will join effectively with the *mutated* music of Palestrina. Three strategies were used to address this difficulty:

1. The pitch-organisation of Palestrina was modified rhythmically to produce a sequence of chords that informed the vocal writing.
2. The pitch-organisation and durations of Palestrina produced a sequence of chords that informed the vocal writing.
3. Palestrina's *mutated* music was combined with freely composed vocal writing.

This approach leads to five distinct types of music in *Eotec* these are listed in Table 42.

Table42 Five Types of music used in *Evolution of the Cosmos*

Music Type	**Where it Occurs in *Eotec***
1. *Mutation* of Palestrina's music without SATB choir	*The Universe Began* b. 35-52, b. 91-97 *The First Stars* b. 1-8, b. 93-115 *The Laws of Science* b. 105-111, b. 126-129, b. 155-167 *The Solar System* b. 23-33, b. 58-73, b. 81-101 *Planet Earth* b. 19-39, b. 47-79, b. 95-103, 125-145 *The Evolution of Life* b. 62-73, b. 79-84

2. *Mutation* of Palestrina's music with sung Latin text	*The Universe Began* b. 53-72, b. 144-172 *The Laws of Science* b. 26-39, b. 75-104, b. 112-125, 196-208 *The Solar System* b. 102-109 *Planet Earth* b. 40-46, b. 80-94 *The Evolution of Life* b. 74-78, b. 85-95
3. *Mutation* of Palestrina's music combined with freely composed vocal writing	*The First Stars* b. 9-92 *The Solar System* b. 1-22, b. 34-57, b. 74-81, b. 110-119 *Planet Earth* b. 1-18, b. 104-124 *The Evolution of Life* b.109-171
4. Freely composed vocal writing informed by the durations and pitch-organisation of Palestrina's music	*The Laws of Science* b. 1-25, b. 40-74, b. 130-154, b. 168-195, b. 209-229 *The Evolution of Life* b. 1-61, b. 96-108
5. Freely composed vocal writing informed by the pitch-organisation of Palestrina's music	*The Universe Began* b.1-35, b. 73-90, b. 98-143

Pg.180-200 explain how these five music types were utilised in the composing of *Eotec.*

Missa Papae Marcelli was viewed as a work of absolute music with an emphasis upon compositional technique rather than an expressive setting of the sacred text of the Latin mass. This absolute approach was applied to the setting of the libretto of *Eotec.* The question of research undertaken was to explore the possibility of using *harmo-rhythmic structures* in the writing of polyphonic music. The freely composed sections in *Eotec* sought to employ this compositional technique in which pitch-organisation was controlled rhythmically. This leads to similarity of approach between the word setting in *Missa Papae Marcelli* and *Eotec.*

25iv. The Form of *Evolution of the Cosmos (Eotec)*

The consequence of using *Mutation of Missa Papae Marcelli* as the framework for *Eotec* is the formal design of the two works is similar. The overall form and form charts shown on pg.151, 153, 157, 164, 166, 168, 169 applies to *Eotec* in addition to *Mutation of MPM.* Sections 25v to 25x describe how the form of *Eotec* was modified from that of *Mutation of MPM* in order to achieve a coherent setting of the libretto.

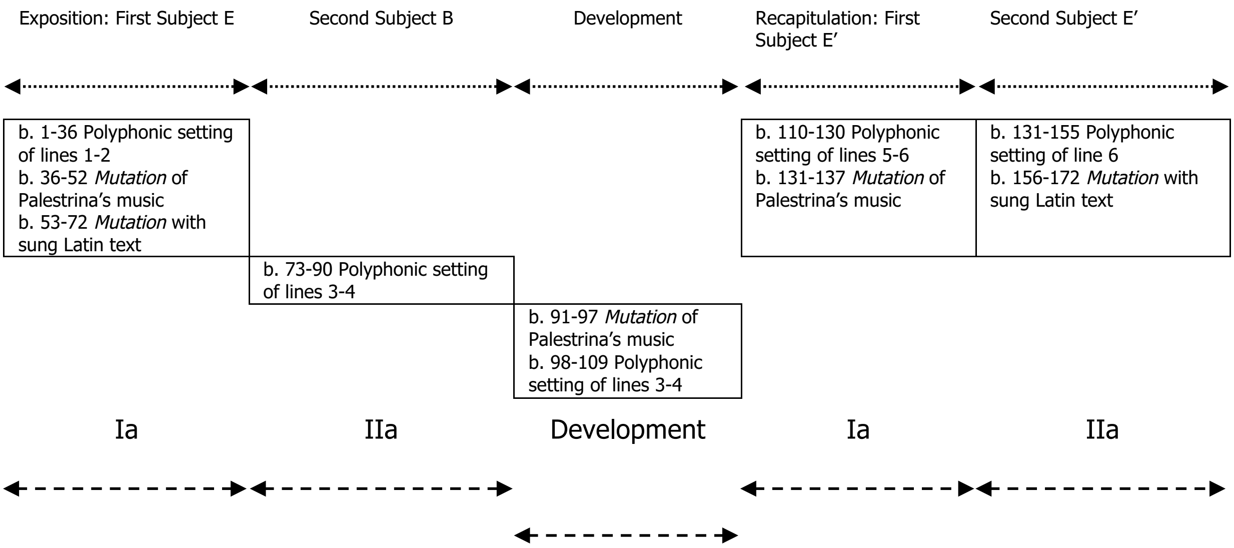
25v. Form of *The Universe Began*

Sets lines 1-6 of the libretto

1 The Universe began as a singularity
2 All of space and time in a single point
3 A sea of neutrons, protons, electrons, positrons, photons and neutrinos
4 In a high-energy state began cooling and expanding
5 To a form a cold World of matter
6 Stars, planets and Galaxies.

Form Chart 18 shows the sonata form of *The Universe Began.*

Form Chart 18 *The Universe Began*



In order to facilitate the setting of the words, chord sequences were derived from *Mutation of Kyrie.* Example 38 shows the chord sequence generated from Ia of *Mutation of Kyrie.* The chords change on every semibreve up to bar 24 and every minim thereafter. At bar 25 the pitch-organisation changes to the chords used in IIa of *Mutation of Kyrie.* The change of harmonic rhythm at bar 25 follows the procedure of earlier compositions in subdividing rhythms in proportion to the number 2. This change of harmonic rhythm and pitch-organisation reflects the nature of the libretto since the music is in a state of flux and continual changing with the passage of time.

Ex.38 *The Universe Began* Ia

The chord sequence shown in Example 38 informed the setting of lines 1-2 of the libretto. The four-part writing followed the rules outlined on pg.46 regarding the rhythmic placing of consonant and dissonant intervals. The chord sequence shown in Example 38 represented a relative consonant sound which was altered by weak beat passing notes and strong beat prepared and resolved suspensions.

At bar 37 of *The Universe Began* the remainder of Ia of *Mutation of Kyrie* not used to generate the chord sequence shown in Musical Example 38 is played by instruments only initially and with sung Latin text at bar 53. This procedure was adopted throughout the rest of *Eotec.* A chord sequence was generated from a formal section of *Mutation of Missa Papae Marcelli,* the remainder of the formal section not used for the chord sequence was sounded after the setting of the libretto, with or without sung Latin text.

The chord sequence used for IIa of *The Universe Began* is shown in Example 39. The harmonic rhythm changes to a chord change every crochet at bar 73 and a chord change every minim at bar 76. At bar 76 the pitch organisation changes to that used in Ia.

Ex.39 *The Universe Began* IIa

Since the chord sequence shown in Example 39 was insufficient to complete the settings of lines 3-4 of the libretto the chord sequence repeats at bar 82. This procedure was used for the rest of *Eotec,* if the libretto was incompletely set by the end of the chord sequence it was repeated partially or completely until the setting of the libretto was finished. If a partial repeat was used the remainder of the formal section played by instruments or instruments with sung Latin text began from the point of partial completion rather than that of the completed chord sequence. However, in *The Universe Began* IIa proceeds straight to the development section without an instrumental completion of IIa from *Mutation of Kyrie.*

The development section begins at bar 91 with instruments playing the beginning of the development of *Mutation of Kyrie.* At bar 98 there is a polyphonic setting of lines 3-4 of the libretto with chords derived from those shown in Example 39. The recapitulation begins at bar 110 with a polyphonic setting of lines 5-6. Ia in the recapitulation uses the chord sequence shown in Example 38. IIa at bar 131 uses the chord sequence shown in Example 40 for a polyphonic setting of line 6. Like in *Mutation of Kyrie* the pitch-organisation for IIa remains from a starting note of **E** and doesn't modulate. Both Ia and IIa in the recapitulation use new material in the vocal parts in comparison with Ia and IIa of the exposition.

Ex.40 *The Universe Began* IIa Recapitulation

At bar 156 there is a completion of the remainder of IIa played by instruments with sung Latin text.

25vi. Form of *The First Stars*

Sets lines 7-13 of the libretto

7 Gravity slowed the expansion of swirls of gas

8 Causing them to collapse inwards and grow hotter

9 Igniting the first stars in the Universe

10 A small number of first generation stars

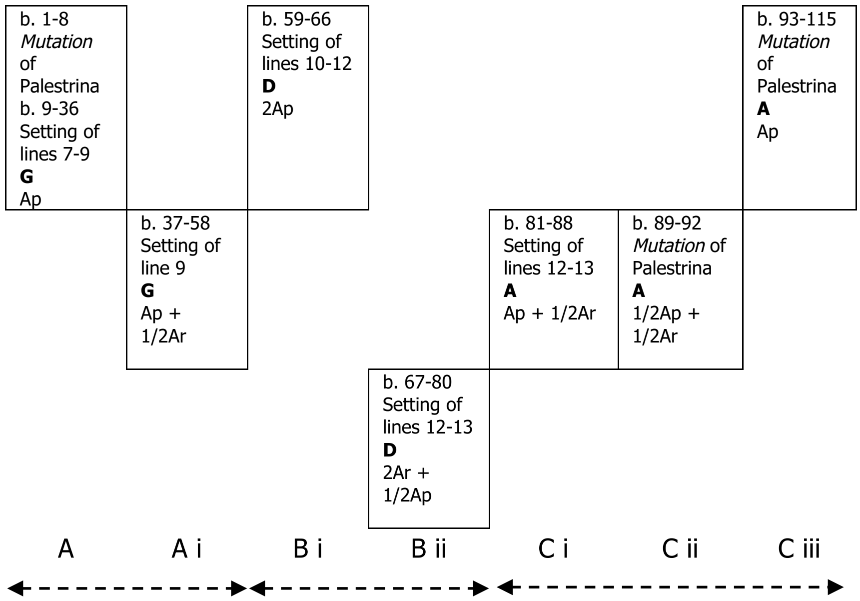
11 Evolved into supernova that fused together

12 Lighter elements to form the elements heavier than iron

13 Found in later stars and planets

Form Chart 19 below shows the form of *The First Stars.*

Form Chart 19 *The First Stars*



Similarly, to *The Universe Began* a chord sequence was derived from *Mutation of Gloria* to facilitate a setting of the words that would fit together musically with the *mutated* music. Example 41 shows the chord sequence used the set lines 7-9 of the libretto. In contrast with the chord sequences shown in Examples 38, 39 and 40 all subsequent chords sequences used to set the libretto preserved both the pitch-organisation and durations mutated from Palestrina's music.

Ex.41 *The First Stars* A

The use of both the durations and pitch-organisation *mutated* from Palestrina's *Gloria* allows the freely composed vocal lines to be sounded simultaneously with the *mutation* of Palestrina's music at bar 9 and in all subsequent vocal sections in *The First Stars*. Since the setting of the libretto was incomplete after the chord sequence shown in Example 41 the sequence repeats at bar 23.

Lines 10-12 of the libretto were set using the chord sequence shown in Example 42.

Ex.42 *The First Stars* Bi

At bar 81 the chord sequence shown in Example 42 is used for a setting of lines 12-13. The chords are transposed to a starting note of **A** following the formal structure of *Mutation of Gloria,* see pg.162. *The First Stars* ends with instruments playing a *mutation* of Palestrina's music at bars 89 to 115.

25vii. Form of *The Laws of Science*

Sets lines 14 to 23ii

> 14 Darwin: "If it could be demonstrated that any complex organ existed, which could not possibly
>
> 14i have been formed by numerous, successive, slight modifications, the theory of evolution would
>
> 14ii break down. But we can find no such case."
>
> 15 Life on Earth has evolved by the natural selection of inherited characteristics.
>
> 16 Heisenberg: "There is an inherent limit of uncertainty if we try to measure both the position and
>
> 16i momentum of a particle."
>
> 17 The building blocks of matter may be modelled as both waves and particles. Waves and particles

18 obey quantum mechanics and evolve in accordance with the probability of a wave function.

19 Einstein: "The speed of light is a constant for all observers. Matter warps space-time."

20 Gravity was infinite in the singularity at the beginning of the Universe and in singularities found

20i inside black holes.

21 Hubble: "Hubble's law shows that the universe is expanding. Objects recede at a faster velocity as

21i their relative distance increases."

22 Space time is expanding.

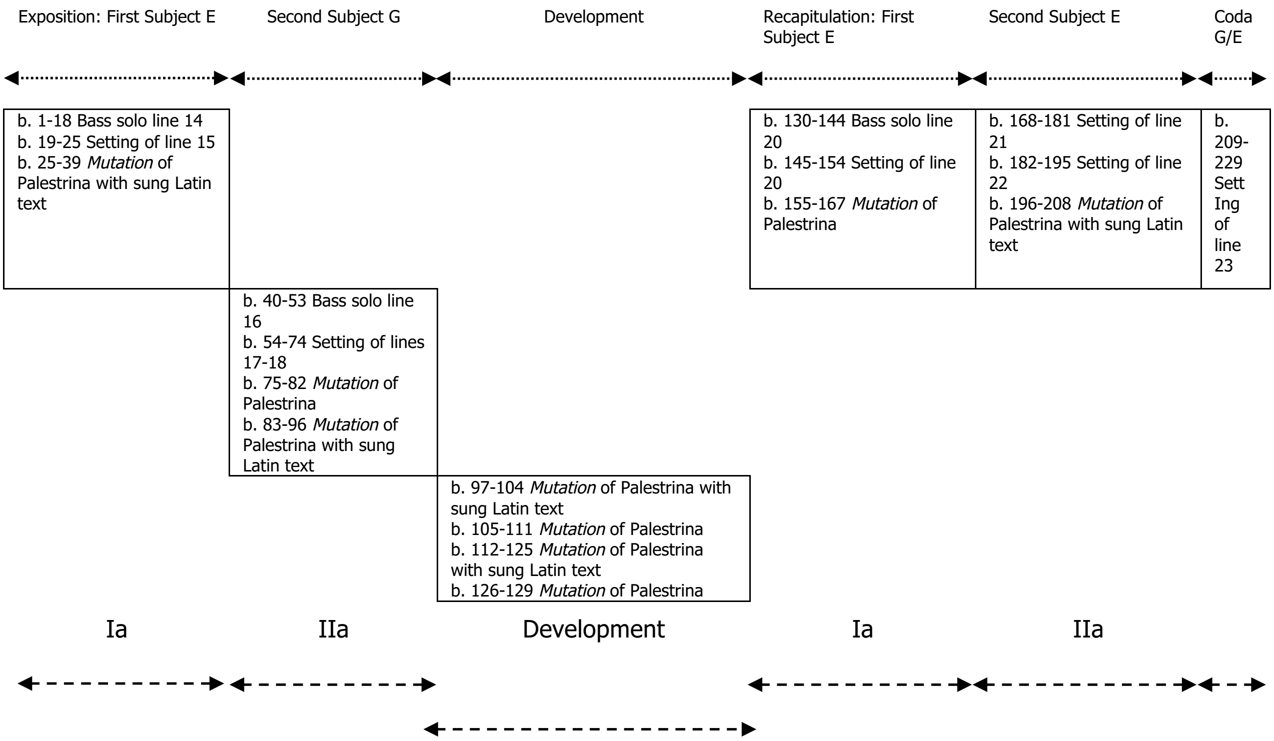
23 Mathematical models predict the behaviour of matter and energy; space and time. The

23i mathematics holds true throughout the universe except in singularities. The laws of science will

23ii change when new theories come to light.

Form Chart 20 shows the form of *The Laws of Science.*

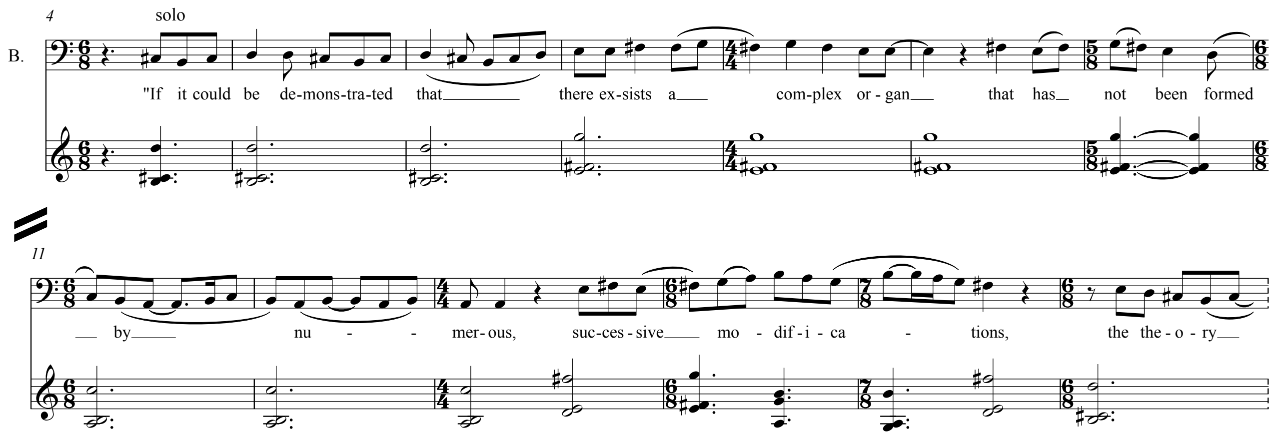
Form Chart 20 *The Laws of Science*



The Laws of Science uses solo voice for the first time in *Eotec,* a solo bass sings quotes from Darwin, Newton and Heisenberg in recitative. Solo voice is used at significant points in the setting of the libretto of *Eotec* to provide a contrast of dynamic, timbre and register. It is the words and nature of the libretto rather than a preordained scheme that determines when a solo voice is used in *Eotec*.

Ia (shown in Form Chart 20), uses the chord sequence shown in Example 43. This sequence was derived from the pitch-organisation and durations of Palestrina's *mutated* music.

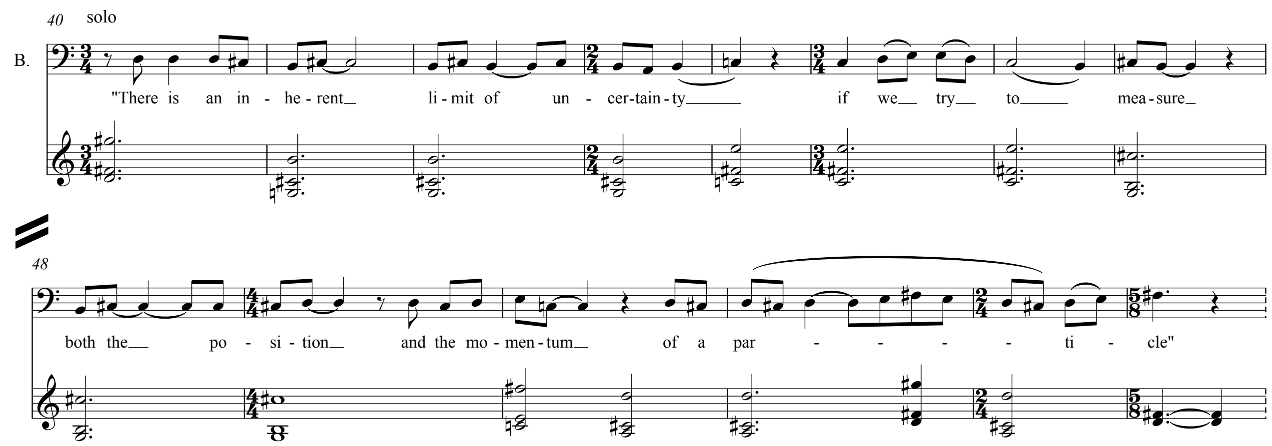
Ex.43 *The Laws of Science* Ia



The chord sequence shown in Example 43 repeats at bar 14.

Similarly, to the *Mutation of Credo, The Laws of Science* modulates to an *artificial tonality* based on the note **G**. The chord sequence used in IIa (shown in Form Chart 20), is derived from the pitch-organisation and durations *mutated* from Palestrina's *Credo.* IIa begins at bar 40 with a solo recitative. The sequence shown in Example 44 repeats at bars 54 and 68.

Ex.44 *The Laws of Science* IIa



The development section of *The Laws of Science* closely follows that of *Mutation of Credo* with addition of sung Latin text. The recapitulation at bar 130 uses three repeats of the sequence shown in Example 43. The chord sequence is the same as Ia in the exposition however, in the recapitulation the freely composed lines for the bass solo and SATB choir are different.

IIa shown in (Form Chart 20), begins at bar 168 and similarly to *Mutation of Credo,* the music does not modulate and remains in an artificial tonality based on the note

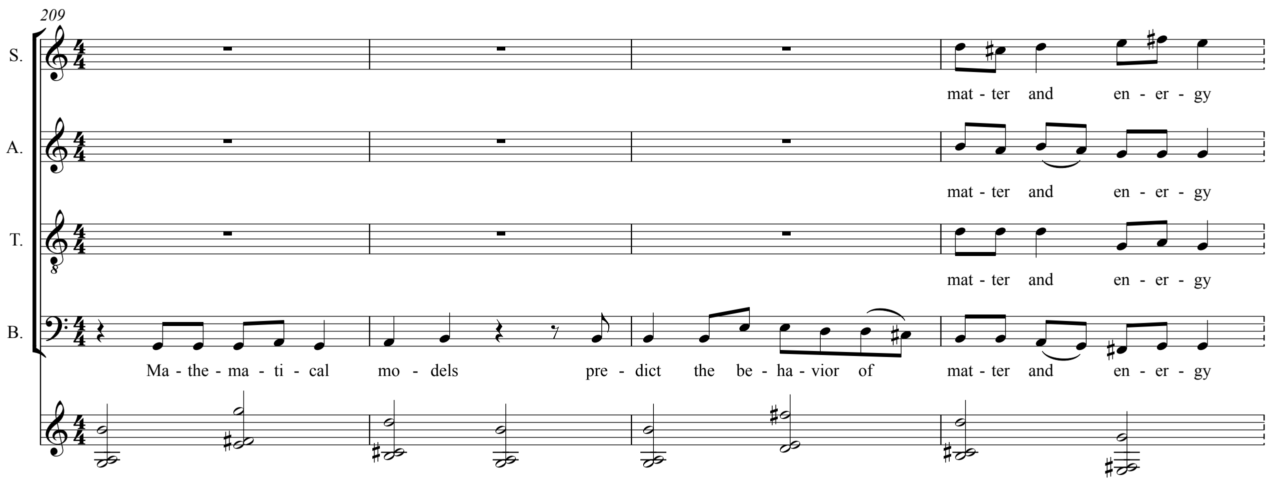
E. IIa uses the chord sequence shown in Example 45 derived from the pitch-organisation and durations of Palestrina's *mutated* music.

Ex.45 *The Laws of Science* IIa Recapitulation

The chord sequence shown in Example 45 repeats at bar 182. Similarly, to Ia, in the recapitulation IIa uses different lines in the vocal writing in comparison to IIa of the exposition.

The coda at bar 209, (shown in Form Chart 20), is based on the coda of *Mutation of Credo*. The chord sequence used is derived from the pitch-organisation and durations of Palestrina's *mutated* music*.* This sequence is shown in Example 46, repeats five times in total.

Ex.46 *The Laws of Science* Coda



The coda uses instrumental lines derived from the coda of *Mutation of Credo.* Comparison will show that the coda of *The Laws of Science* varies from the coda of *Mutation of Credo* meaning this section's music was regarded as music type four in Table 42

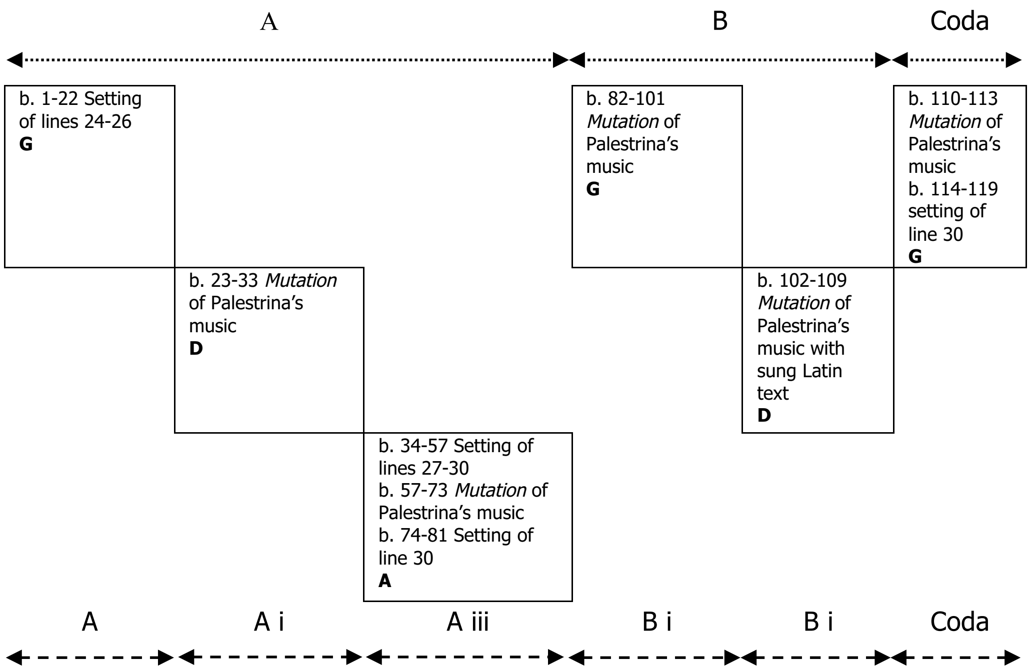
25viii. Form of *The Solar System*

Sets lines 24-30 of the libretto

24 The Solar System formed when a cool, swirling cloud of
25 Gas and dust collapsed inwards due to gravity.
26 The cloud span faster and faster as it collapsed
27 The dust and gas cloud heated up until
28 The cloud was hot enough for the spontaneous
29 Nuclear fusion of hydrogen into Helium
30 The Sun was born.

Form Chart 21 shows the form of *The Solar system*

Form Chart 21 *The Solar System*



This movements represents a change of emphasis in *Eotec* since the libretto describes the human experience of the solar system rather than the vast totality of the universe controlled by the fundamental laws of science. Additionally, it begins a process in the libretto that describes how the evolution of the cosmos gave rise to the evolution of life on Earth leading to the emergence of homo sapiens. The solo soprano at key points in the libretto represents a human voice in the vastness of the cosmos.

Similarly, to *The First Stars* the monodies and homophonic textures of *The Solar System* allow the freely composed vocal lines to be sounded simultaneously with the music *mutated* from Palestrina's *Sanctus.* Section Ai (shown in Form Chart 21), sets lines 24-26 of the libretto using the chord sequence shown in Example 47.

Ex.47 *The Solar System* Ai

Section Aiii uses an artificial tonality based on the note **A.** The chord sequence used to set lines 27-30 is shown in Example 48.

Ex.48 *The Solar System* Aiii

In the coda at bar 110, (shown in Form Chart 21), a setting of line 30 of the libretto is combined with the *mutated* music of Palestrina's *Sanctus* at bar 114*.* This is classed as music type three in Table 42 since the libretto is sounded with Palestrina's *mutated* music rather than Latin text.

25ix. Form of *Planet Earth*

Sets lines 31-41 of the libretto

31 The cloud of gas and dust orbiting the Sun

32 Began to accrete into planetesimals

33 Collisions and gravity attracted the planetesimals into

34 The eight planets of the solar system

36 The planets swept all material from their orbits

37 Solar winds blew away the gas from the inner

38 Solar system giving four rocky planets near

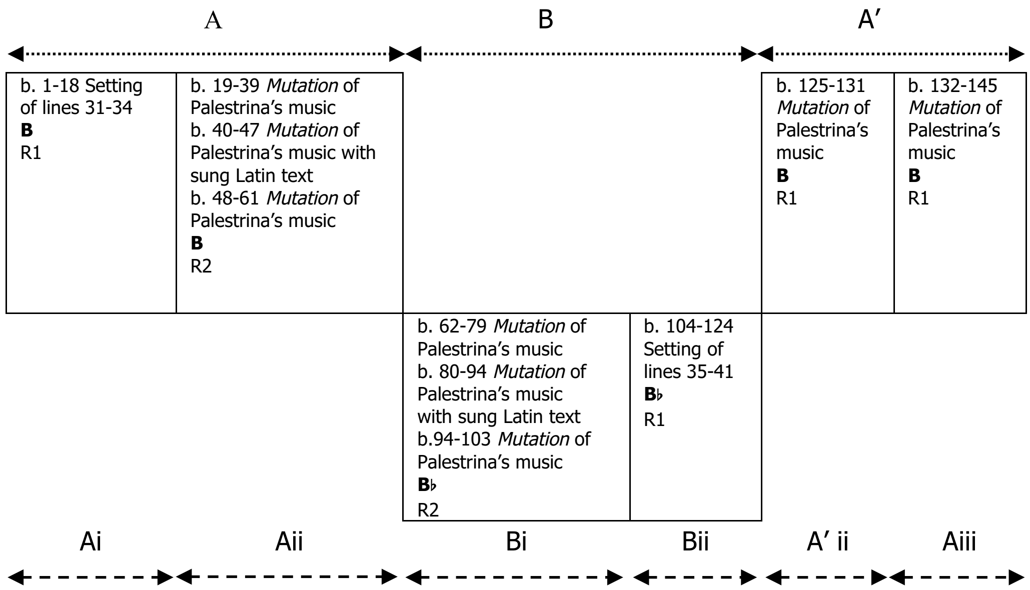
39 To the Sun and four gaseous planets in the

40 Outer solar system

41 Earth is the third rocky planet from the Sun

Form Chart 22 shows the form of *Planet Earth*

Form Chart 22 *Planet Earth*



Palestrina's *Benedictus* is set for four-part harmony rather the six-part harmony used in the rest of *Missa Papae Marcelli.* For this reason, *Planet Earth* uses string orchestra, tuned percussion and SAT choir without bass, some of the setting is for solo alto voice. The unique sound of this movement reflects the libretto in its description of the evolution of planet Earth, the only known place in the universe that contains life.

Similarly, to *The First Stars* and *The Solar System* the freely composed vocal lines in *Planet Earth* occur simultaneously with the *mutated* music of Palestrina's *Benedictus.* The chord sequence shown in Example 49 was used to set lines 31-34 of the libretto.

Ex.49 *Planet Earth* Ai

The chord sequence shown in Example 50 was used to set lines 35-41 of the libretto in Bii, (shown in Form Chart 22). Similarly, to *Mutation of Benedictus* this section uses an artificial tonality based on the note **Bb.**

Ex.50 *Planet Earth* Bii

25x. Form of *The Evolution of Life*

Sets lines 42-53 of the libretto

42 Three and a half Billion years ago Life was present on Earth

43 This age saw the split of the Universal common ancestor into bacteria and archaea

44 Three billion years ago photosynthesise began in cyanobacteria

45 Two billion years ago multicellular organisms emerged

46 Two hundred million years ago the first dinosaurs walk the Earth

47 Sixty-five million years ago the extinction of the dinosaurs occurred

48 The age of mammals began

49 Two million years ago, Man’s ancestors first appeared

50 Two hundred and fifty thousand years ago Homo Sapiens evolved

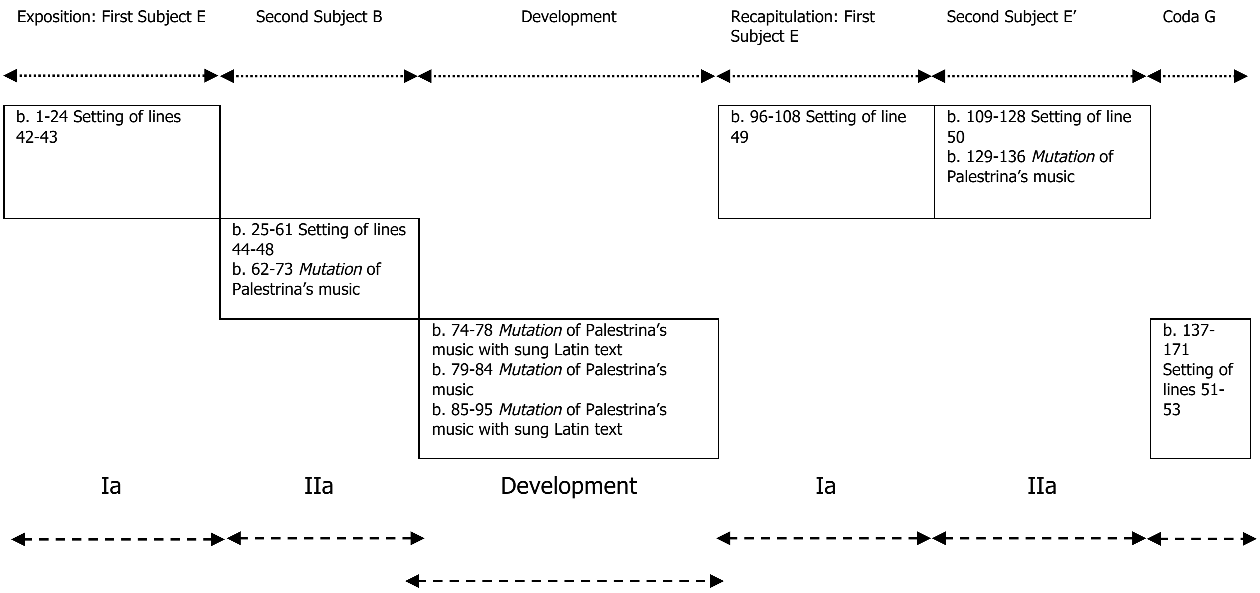
51 The evolution of the Cosmos gave rise to the evolution of life on Earth

52 The solar system, Sun, Planets and all known life are formed from

53 Primordial hydrogen and star dust.

Form Chart 23 shows the form of *The Evolution of Life*

Form Chart 23 *The Evolution of Life*



The repeat of the time measurements in the libretto of *The Evolution of Life* is a device to give a narrative structure to unfolding of the words. A solo tenor sings the time measurement in years at the start of lines 42 and 44-47. The tenor becomes a narrator for the unfolding story of the evolution of life. The recapitulation of *The Evolution of Life* is the climax of *Eotec* with the arrival of homo sapiens in the universe. The music is polyphonic throughout the recapitulation. The setting of the line 53 is sombre and downbeat since the evolution of the cosmos leads to the destruction of the solar system and the heat death of the universe. Star dust was formed in the explosion of a star, the solar system and any remaining lifeforms will become formless star dust when the Sun dies.

Ia (shown in Form Chart 23), uses the chord sequence shown in Example 51.

Ex.51 *The Evolution of Life* Ia

Similarly, to IIa of *Mutation of Agnus Dei,* IIa (shown in Form Chart 23), modulates to an artificial tonality based on the note **B**. The chord sequence used in this section is shown in Example 52.

Ex.52 *The Evolution of Life* IIa

Like IIa in *Mutation of Agnus Dei* IIa in the recapitulation of *The Evolution of Life* remains in an artificial tonality based on the note **E**. The chord sequence used in this section is shown in Example 53

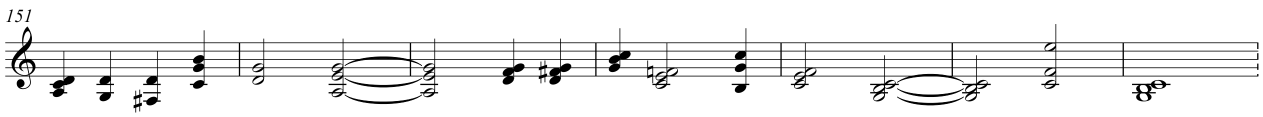
Ex.53 *The Evolution of Life* IIa Recapitulation

The freely composed sections in the recapitulation of Ia and IIa are played simultaneously with *mutated* music from Palestrina's *Agnus Dei.* Up to bar 108 the music is regarded as type 4 in Table 42. Bars 109-171 is classed as type 3 in Table 42, the difference of categorisation is one of compositional strategy since until bar 108 the intention was for freely composed music that might combine with the *mutated* music, the vocal setting after bar 108 was conceived as being sounded with Palestrina's mutated music.

The coda of *The Evolution of Life* like that of *Mutation of Agnus Dei, mutates* the final bars of *Missa Papae Marcelli.* The music modulates to an artificial tonality based on the note **G**, behaving like a tierce de Picardie, the equivalent of a relative major tonality in traditional music. The chord sequences used in the coda are shown in Examples 54 and 55.

Ex.54 *The Evolution of Life* Coda

Ex.55 *The Evolution of Life* Coda



25xi. Reflections on *Evolution of the Cosmos (Eotec)*

Eotec is an application of the use of *musical integers* to *re-make* past music. The musical DNA of Palestrina's *Missa Papae Marcelli* is encoded and has a symbolic equivalent in the *harmo-rhythmic structures* used in *Eotec.* The freely composed sections are informed by the *mutated* music of Palestrina and become a mutation of a mutation.

The use of a libretto and orchestral colour in the composition of *Eotec* places this piece within an established genre and the musical decision taken were informed by reference to Britten's *War Requiem.* This represents a change of emphasis in the research, a compositional methodology has been established, *Eotec* applies this methodology to achieve a setting of a libretto taking into account large scale formal considerations, use of instrumental and vocal resources, effective vocal textures that are appropriate to the text. *Eotec* anticipates post-PhD research, with considerations of how to apply the compositional methodology effectively.

Chapter 26. Conclusion

26i. Reflections on the Portfolio and Appendices.

The Intervallic Compositions demonstrated it was possible to produce coherent and consistent music using modified rules from Fux *Gradus* to rhythmically control dissonant intervals. However, *String Quartet I* revealed that a listener has expectations from historical precedent. The sounding of a major or minor triad sets up the anticipation of hearing functional harmony in the music; if this is not fulfilled the listener compares the music unfavourably in comparison to music based on traditional chord progressions.

The use of modes and pitch-organisation based on the contrast of chords helped to overcome the difficulty of what might be called historical listening, in which the influence of the past cannot be forgotten. *Two Explorations of a Chord Sequence* and *Mutation of Mozart's Jupiter Symphony* used four-note chords derived from modes cd and dd; *Flumina* and *Concertino* used three-note chords derived from mode cd and mode cc.

Musical Integers encapsulate the theories of musical Platonism and Xenakis' *Symbolic Music* allowing for a precise encoding and decoding of the *in-time structures* of pre-existing music. *The mutation* of pre-existing music into new compositions can be thought of as both a dialogue between music of past and present and as a new *copy* of an external, Platonic Form. *Mutation of Dowland's Midnight* and *Mutation of Missa Papae Marcelli* symbolically encoded and decoded pitch and duration vectors of pre-existing music to utilise historical *harmo-rhythmic structures* in their pitch-organisation.

26ii. Achievements

Schoenberg sought to liberate all twelve pitches of the chromatic scale in his serial music, if each note is heard as being equal, there will be no sense of tonality or key. However, Schoenberg did not consider the effect of the fracture between rhythmic placement and pitch-organisation. If all combinations of notes are dissonant there is no inherent contrast between metric strong and weak beats as each will contain similar pitch-organisation.

The music submitted in the portfolio and appendix links rhythmic placement and pitch-organisation in the use of *harmo-rhythmic structures.* This allows for extreme dissonance in the music without an overly unpleasant effect, while the pitch-organisation sounds almost traditional at times, analysis would show the chords might be clusters of semitones or dissonant chords.

The use of rhythms and metres that subdivide or augment in a proportion to the number 2, in the music submitted in the portfolio and appendix, leads to a style that is perhaps more accessible than music using irrational subdivisions of rhythms and metres. The listener can perceive changes of rhythm that use subdivisions in proportion to the number 2, more easily than irrational subdivisions (see 8i. pg.44). However, some of the music in the portfolio and appendix takes an intermediate position by subdividing prime number metres such as $\frac{5}{4}$ and $\frac{7}{8}$, with the number 2 as a denominator.

26iii. Closing Thoughts

The compositional style of Palestrina does not reside in a set of rules and cannot be apprehended or pinned down by a set of rules. This has an interesting parallel with Gödel's Theorem which states that no formal system can be both complete and consistent. If ever more complex rules strengthen a formal system it still cannot overcome Gödel's Theorem[45]. A set of rules about the polyphonic style of Palestrina, no matter how complex and thorough, cannot fully reproduce music that sounds like that of Palestrina. However, a symbolic encoding of Palestrina's music in an *MI* does contain the musical DNA encoding the choices made by Palestrina *in-side time*. Thus, if a *MI* is used to remake Palestrina's music, the new composition is controlled in a precise fashion by a Palestrina composition.

The compositions in the portfolio and appendix have a reproducible compositional methodology. This commentary outlines strategies to rhythmically control pitch-organisation that might be of interest and use to other composers. This methodology is an attempt to amalgamated the achievements of C20th music into a style with a more traditional musical aesthetic. Boulez writes

> "We have already stressed the cardinal fact that the series dilutes opposition between horizontal and vertical, just as it creates a universe where consonance and dissonance are abolished."
> (Boulez P, *On Music Today,* trans. Bennett RR, Bradshaw S, London, 1971, pg. 132)

The compositional methodology outlined in this commentary sets out strategies to re-establish considerations of horizontal and vertical, consonance and dissonance in post-tonal, post-serial music. The extension of the harmonic language by amongst others, Berg, Schoenberg and Webern is utilised in an *artificial tonality* which informs *harmo-rhythmic structures,* medium and large scale formal structures. The use of irregular rhythms and metre found in the music of Stravinsky and other C20th composers, informs the subdivision of metre and rhythm in proportion to the number 2. These subdivisions offer a method of varying *harmo-rhythmic structures* regarding the contrasting placement of metric strong beats.

[45] This theorem is set out in Gödel K, *On Formally Undecidable Propositions of Principia Mathematica and Related Systems,* 1931; trans. Martin Hirzel, 2000

The use of extended instrumental techniques and timbre as a structural device found in amongst others Xenakis, Ligetti and Stockhausen, is utilised as a method to articulate *harmo-rhythmic structures* and in the decoding of existing *in-time structures.*

The author's intention is to employ the compositional methodology developed during the research undertaken, in future compositions; and to further develop and enhance the philosophies and ideas used in the compositional methodology. Elliott Carter (1908-2012) produced an exhaustive list of combinations of intervals that might be employed in serial music in his *Harmony Book* (Carter E, *Harmony Book,* ed. Hopkins N & Link JF, NY, 2002). The author's intention is to produce a list of all possible *triadic hierarchy modes* and all possible *artificial tonalities* that might be derived from these *triadic hierarchy modes.* This would give composers a resource if they wished to employ *artificial tonalities* in their music. The idea for this list was inspired by Carter's *Harmony Book.*

Glossary of Terms and Ideas

Artificial Tonality
A term used by Sallis to indicate a tonality built from the contrast of *chords* unrelated to the harmonic series.
See: Chords/Triads, Tonality, Triadic Harmony Modes

Compositional Methodology
This may be viewed as the strategies and theories used by Sallis to compose in a polyphonic style that employs *HRS.* Each composition will be constructed from an *OTS* and a *TS.* These structures inform the *ITS* of the music*. HRS* are used to control the pitch-organisation within the metre or rhythmic cycle.
On a larger scale an *artificial tonality* and the contrast of *HRS* articulate the form of the music. Usually this form will be *sonata form* or *ABC form.*
See: ABC Form, Chords/Triads, Harmo-rhythmic Structures, In-time Structures, Outside Time Structures, Sonata Form, Temporal Structures, Tonality,

Chords/Triads
Used in the traditional sense to mean the triads that form the basis of chords in functional harmony.
In the compositions of Sallis *chords* means a collection of *chords* which represent the relative *consonant* sound of the music. The *chords* are generated in a systematic way from the *OTS* of the music.
See: *Consonance, Dissonance*, *Outside time Structures, Tonality, Triadic Harmony Modes*

Consonance
This has a historic meaning applied to the music of Palestrina and a specific meaning in the compositions of Sallis. In the music of Palestrina, the intervals of a minor and major 3rd; 5th; minor and major 6th and the compound of these intervals are consonant intervals.
In the music of Sallis the historical consonance of Palestrina applies to some early compositions, in later music a set of chords represent a relative, consonant sound, this is the reference against which any alteration of the chords is considered dissonant.
See: Chords/Triads, Dissonance, Palestrina, Passing notes, Suspensions

Dissonance
This has a historic meaning applied to the music of Palestrina and a specific meaning in the compositions of Sallis. In the music of Palestrina, the intervals of a minor and major 2nd; 4th; augmented 4th; minor and major 7th and the compound of these intervals are dissonant intervals.
In the music of Sallis the historical dissonance of Palestrina applies to some early compositions. In later music, a set of chords represent a relative, consonant sound; any deviation from this set of chords by the use of passing notes or suspensions is considered dissonant.
See: Chords/Triads, Consonance, Palestrina, Passing notes, Suspensions

Duration Quanta
The smallest, indivisible unit of duration used to produce metres, rhythmic cycles etc. by adding together this quantum of unit of duration. All durations may be produced from the iambic rhythm of Ancient Greece in which the long and short rhythm are in the ratio of 2:1.
In Sallis' extension of *Symbolic Music*; Platonic *Forms* may be modelled as ordered pitch and duration sets based on pitch and duration quanta.
See: *Gödel Numbering, Music as an Integer, Musical Platonism*

Gödel Numbering
A technique invented by the logician and mathematician Gödel in which strings from number theory are encoded into integers by expressing them as powers of prime numbers. Each character in the string is expressed as a power of successive prime numbers. These prime numbers are multiplied together to give a product or Gödel Number.
Sallis used the technique of *Gödel Numbering* to extend the *Symbolic Music* of Xenakis. This produces an integer which encodes the *ITS* of the music.
See: In-time structures, Musical Integers, Outside Time Structures, Symbolic Music, Temporal Structures

Harmo-Rhythmic Structures (HRS)
A term used by Sallis to describe the rhythm control of dissonance in the music of Palestrina. This not only regulates the harmonic content of the music; across time a sense of pulse and metric strong beat is reinforced by the periodic sounding of consonant harmony or suspensions. The music of Palestrina could be viewed as a continuous concatenation of *HRS*.
The compositional methodology of Sallis uses various strategies to utilise *HRS* in modern music.
See: *Chords/Triads, Consonance, Dissonance*, *Gödel Numbering, In-time Structures, Music as an Integer, Outside Time Structures, Palestrina, Passing Notes, Suspensions, Symbolic Music, Temporal Structures*

In-time Structures (ITS)
One of the three logical levels identified by Xenakis in his book *Formalized Music. ITS* are the correspondence between the *TS* and the *OTS* to give an architecture *in-side time*.
In Sallis' extension of *Symbolic Music* using the technique of Gödel Numbering; *ITS* may be encoded as an integer using the technique of Gödel Numbering.
See: *Gödel Numbering, Musical Integers, Outside Time Structures, Symbolic Music, Temporal Structures*

Metre
Metre is defined as the periodic sounding of strong and weak beats. Consonances or suspensions occur on strong metric beats; dissonant passing notes may occur on metric weak beats.
See: *Consonance, Dissonance, Palestrina, Passing notes, Suspensions*

Musical Integers (MIs)
Sallis used the technique of *Gödel Numbering* to encode the *ITS* of a piece of music as an integer. This integer is the product of successive prime numbers raised to a power. Each entity in the *OTS* and *TS* are assigned a unique whole number, this number becomes the power of a new, successive prime number. In this way, a large integer encoding the entire piece of music may be produced.
See: Gödel Numbering, In-time structures, Outside Time Structures, Symbolic Music, Temporal Structures

Musical Platonism
A view of music in which a composition pre-exists its *discovery* by a composer. All music has an ideal, Platonism existence; a composition is a token or kind of this ideal music.
See: *Gödel Numbering, Musical Integers, Symbolic Music, Transcendental Musical Platonism*

Outside Time Structures (OTS)
One of the three logical levels identified by Xenakis in his book *Formalized Music.* For Xenakis *OTS* are unaltered by temporal considerations or their ordering in time. They are constructed by logical, theoretical or cultural means and consequentially; exist independently of musical time.
In Sallis' extension of *Symbolic Music* using the technique of Gödel Numbering; *OTS* are considered as variable and may be substituted without altering the essence of the music. The integer encoding the music does not alter if the *OTS* is replaced with another.
See: *Gödel Numbering, In-side Time Structure, Music as an Integer, Symbolic Music, Temporal Structures*

Passing Notes
Used in the traditional sense as a dissonant interval moving by step between two consonant intervals.
In the music of Sallis a passing note moves between the pitches of a relatively consonant chord.
See: Chords/Triads, Consonance, Dissonance

Pitch Quanta
The smallest, indivisible unit of pitch used to produce modes, scales etc. by adding together this quantum of pitch. To give an example Aristoxenos used the twelfth tone, a whole tone divided into twelve equal segments of pitch, as his pitch quanta.
In Sallis' extension of *Symbolic Music*; Platonic *Forms* may be modelled as ordered pitch and duration sets based on pitch and duration quanta.
See: *Gödel Numbering, Musical Integers, Musical Platonism*

Remaking the Past
The title of a book by the American music theorist Straus which outlines how composers such as Stravinsky and Schoenberg utilised triads, tonality and sonata form in their compositions in order to remake past music to their own designs.
In the music of Sallis there are three different strategies to remake the past: i. The use the music from the past to inform new compositions. ii. A more systematic approach to use pre-existing music as a model to create new music. iii. The use of *Gödel Numbering* to encode music as an integer. This integer encodes the *ITS* of the music. The *ITS* is mutated by altering the *OTS* and *TS* of the original music.
See: *Chords/Triads, Gödel Numbering, Harmo-rhythmic structures, Music as an Integer, Sonata Form, Tonality*

Sonata Form
Sallis uses various strategies to remake traditional sonata form: i. the contrast of *HRS* to articulate the second subject and development sections. ii. The contrast of *tonality.* iii. The use of *artificial tonality*.
See: *Harmo-rhythmic structures, Tonality, Triadic Hierarchy Modes*

Suspensions
Used in the traditional way to mean a note from a consonant interval sounded on a weak metric beat is suspended into a dissonant interval on a strong metric beat before resolving by step into a consonant interval on a weak metric beat.
In the music of Sallis a note not belonging to a chord which is sounded on a weak metric beat is suspended into the chord on the strong metric beat before resolving into a note of the chord on a weak metric beat.
See: *Chords/Triads, Consonance, Dissonance, Metre*

Symbolic Music
A theory found in Xenakis' book *Formalized Music.* There two related issues: i. to define three logical levels found in music: an *OTS; a TS* and an *ITS.* ii. To use these logical levels to find means to express music symbolically. This may be seen in the compositions *Herma, Nomos Alpha and Nomos Gamma* by Xenakis.
Sallis sought to extend the ideas of Xenakis by expressing *music as an integer* using the technique of *Gödel Numbering*. This extension of the theory allows for a calibrated expression for duration which in turn means the *ITS* of the music may be precisely encoded.
See: *Gödel Numbering, In-time structures, Musical Integers, Outside Time Structures, Temporal Structures.*

Temporal Structures (TS)
One of the three logical levels identified by Xenakis in his book *Formalized Music.* Xenakis describes how time signatures, rhythmic cycles; the rhythmic control of entities without regard to the entities themselves; are different to durations of these entities which are measured in seconds against metric time. Xenakis calls these structures controlling the rhythm of entities *Temporal Structures.*
In Sallis' extension of *Symbolic Music* using the technique of Gödel Numbering; *TS* are considered as variable and may be substituted without altering the essence of the music. The integer encoding the music does not alter if the *TS* is replaced with another.
See: *Gödel Numbering, In-side Time Structure, Musical Integers, Outside Time Structures, Symbolic Music*

Tonality
This term is used in the traditional sense and to describe the use of chords to articulate the form in the compositions of Sallis.
In the music of Sallis chords are used structurally to contrast formal sections in the music. The later music of Sallis uses *artificial tonality* in which the chords from a *THM* replace the triads used in traditional harmony.
See: *ABC Form, Chords/Triads, Consonance, Sonata Form, Triadic Hierarchy Modes*

Transcendental Musical Platonism (TMP)
A term used by Sallis to indicate a musical Platonism which employs Plato's epistemology and thus, indicates knowledge about music is to be found in a system outside of musical practice.
See: *In-time Structures, Musical Integers, Musical Platonism*

Triadic Hierarchy Modes (THM)
A term used by Sallis to describe a mode or scale which produces a hierarchy of chords/triads. The major scale is the best know example which yields a hierarchy of major and minor triads plus a seventh chord. Sallis claims; without proof; that there are a limited number of modes or pitch sets which produce a coherent hierarchy of chords/triads. Usually, what seems to be a new mode turns out to be a known mode beginning from a different starting pitch. A property of *THM* is there are several ways to pitch sieve the mode to give what Sallis calls *alternative chords.*
See: *Triads, Tonality*

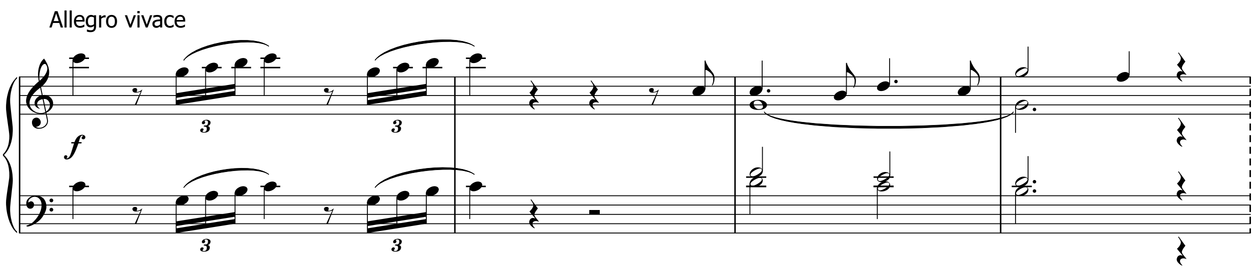
Appendix 1. Tables of Symbolic Encodings

Mozart's *Symphony 41* K.551 Encoding

The methodology outlined in 21ii, pg.135, will be used to encode Ia from Mozart's *Symphony 41,* K.551 first movement. Example 56 shows a piano reductions of the opening four bars of Ia.

Since the encoding is to express the overall form of the piece, notes that repeat in other parts will be ignored. Thus, in the first bar the opening crotchet will be encoded as one note of **C** only despite the fact it appears in each instrumental part. This gives a more compact encoding and will not affect the shape of the overall form.

Ex.56 *Symphony 41*, K.551 Ia



The numbers representing the pitch vectors are those used previously in Table 13 and for rhythm vectors:

2 = demi-semiquaver
3 = semiquaver triplet
5 = semiquaver
7 = quaver
11 = crotchet
13 = dotted crotchet
17 = minim
19 = semibreve

The unique sequence for each of the twenty-three bars of the first subject, first idea are generated in a similar manner. This enables an encoding for the complete section b.1 -23 shown in Table 43.

The sequences written out in Table 43 may be substituted for the letters a,b,c etc. and the prime numbers Pb_1 etc. put in an equation to generate a second level *MI*. This would give an expression for the exponent for Ps1, the first prime to encode for the overall form:

$$Ps_1^{\,2^a3^b5^c7^d11^e13^f17^g19^h23^i29^j31^k37^l41^j43^k47^m53^n59^o61^p67^q71^r73^s}$$

If this process is continued for all the sections listed in the analysis above an expression for the overall form may be written:

Exposition: $Ps_1^{\,Ia} Ps_2^{\,Ib} Ps_3^{\,Ic} Ps_4^{\,IIa*} Ps_5^{\,IIb*} Ps_6^{\,IIc*}$

Development: $Ps_1^{\,dev}$

Recapitulation: $Ps_1^{\,Ia} Ps_2^{\,Ib} Ps_3^{\,Ic} Ps_4^{\,IIa} Ps_5^{\,IIb} Ps_6^{\,IIc}$

Where Ia = exponent of Ps1 etc. dev = development section.
These encodings give an expression for Mozart's *Symphony 41,* K.551 first movement, similar to the idealised equation for sonata form:

$$A^{Q1} B^{Q2} C^{Q3}$$

where Q1 = 2^Ia + c_1Ib + c_2Ic+ c_3IIa* + c_4IIb* + c_5IIc*

Q2 = development section

Q3 = 2^Ia + c_1Ib + c_2Ic+ c_3IIa + c_4IIb + c_5IIc

c_1, c_2 etc. are constants

Table43 Encoding of Form of Mozart's *Symphony 41*, K.551

Pb$_1$ ^ $2^\wedge 11^{1}+c_1 7^{28}+c_2 3^{16}+c_3 3^{33}+c_4 3^{26}+c_5 11^{1}+c_6 7^{28}+c_7 3^{16}+c_8 3^{22}+c_9 3^{26}$

Pb$_2$ ^ $2^\wedge 11^{1}+c_1 11^{28}+c_2 11^{28}+c_3 7^{28}+c_4 7^{1}$

Pb$_3$ ^ $2^\wedge 13^{512161}+c_1 7^{26051216}+c_2 13^{195016}+c_3 7^{101916}$

Pb$_4$ ^ $2^\wedge 17^{26516016}+c_1 11^{12026516}+c_2 11^{28}$

Pb$_5$ ^ $2^\wedge 11^{16}+c_1 7^{28}+c_2 3^{5}+c_3 3^{9}+c_4 3^{14}+c_5 11^{16}+c_6 7^{28}+c_7 3^{5}+c_8 3^{9}+c_9 3^{14}$

Pb$_6$ ^ $2^\wedge 11^{16}+c_1 11^{28}+c_2 11^{28}+c_3 7^{28}+c_4 7^{5}$

Pb$_7$ ^ $2^\wedge 13^{9165}+c_1 7^{10916}+c_2 13^{52616016}+c_3 7^{12051626}$

Pb$_8$ ^ $2^\wedge 17^{11222}+c_1 11^{91601}+c_2 11^{28}$

Pb$_9$ ^ $2^\wedge 11^{1169}+c_1 7^{1916}+c_2 2^{16}+c_3 2^{1201916}+c_4 2^{15916}+c_5 2^{101916}+c_6 11^{1916}+c_7 11^{1916}$

Pb$_{10}$ ^ $2^\wedge 11^{11222}+c_1 7^{11651226}+c_2 2^{1011222}+c_3 2^{24011222}+c_4 2^{12212}+c_5 2^{16012212}+c_6 11^{11222}$
$+c_7 11^{11222}$

Pb$_{11}$ ^ $2^\wedge 11^{12616512}+c_1 7^{11651226}+c_2 2^{5011651226}+c_3 2^{1011651226}+c_4 2^{11651226}+c_5 2^{22011651226}$
$+c_6 11^{11651226}+c_7 11^{11651226}$

Pb$_{12}$ ^ $2^\wedge 11^{19161}+c_1 7^{11691}+c_2 2^{16011691}+c_3 2^{12011691}+c_4 2^{11691}+c_5 2^{5011691}+c_6 11^{11691}+c_7 11^{11691}$

Pb$_{13}$ ^ $2^\wedge 11^{11222}+c_1 7^{11222}+c_2 2^{1011222}+c_3 2^{24011222}+c_4 2^{11211}+c_5 2^{16011222}+c_6 11^{11222}+c_7 11^{11222}$

Pb$_{14}$ ^ $2^\wedge 11^{12616512}+c_1 7^{11651226}+c_2 2^{5011651226}+c_3 2^{1011651226}+c_4 2^{11651226}+c_5 2^{22011651226}$
$+c_6 11^{11651226}+c_7 11^{11651226}$

Pb$_{15}$ ^ $2^\wedge 11^{116926}+c_1 7^{911626}+c_2 2^{1209161}+c_3 2^{909161}+c_4 2^{509161}+c_5 2^{109161}+c_6 11^{16265}+c_7 7^{16526}$
$+c_8 2^{16016526}+c_9 2^{12016526}+c_{10} 2^{9016526}+c_{11} 2^{5016526}$

Pb$_{16}$ ^ $2^\wedge 11^{1169}+c_1 7^{9116}+c_2 2^{1209161}+c_3 2^{909161}+c_4 2^{509161}+c_5 2^{109161}+c_6 11^{16265}+c_7 7^{16526}$
$+c_8 2^{16016526}+c_9 2^{12016526}+c_{10} 2^{9016526}+c_{11} 2^{5016526}$

Pb_{17} ^ $2\hat{}11^{11691}+c_1 5^{16}+c_2 5^{16016}+c_3 5^{16}+c_4 5^{16016}+c_5 5^{1}+c_6 5^{101}+c_7 5^{1}+c_8 5^{101}+c_9 5^{16}+c_{10} 5^{16016}+c_{11} 5^{16}+c_{12} 5^{16016}$

Pb_{18} ^ $2\hat{}5^{1}+c_1 5^{101}+c_2 5^{1}+c_3 5^{101}+c_4 5^{16}+c_5 5^{16016}+c_6 5^{16}+c_7 5^{16016}+c_8 5^{9}+c_9 5^{909}+c_{10} 5^{9}+c_{11} 5^{909}+c_{12} 5^{1}+c_{13} 5^{101}+c_{14} 5^{1}+c_{15} 5^{101}$

Pb_{19} ^ $2\hat{}11^{16}+c_1 7^{26516}+c_2 11^{16526026516}+c_3 11^{11691}+c_4 7^{19161}+c_5 7^{191019161}$

Pb_{20} ^ $2\hat{}11^{16526}+c_1 7^{26516}+c_2 11^{16526026516}+c_3 11^{11691}+c_4 7^{19161}+c_5 7^{191019161}$

Pb_{21} ^ $2\hat{}7^{16526}+c_1 7^{11691}+c_2 7^{16526}+c_3 7^{11691}+c_4 7^{16526}+c_5 7^{11691}+c_6 7^{16526}+c_7 7^{11691}$

Pb_{22} ^ $2\hat{}11^{16526}+c_1 11^{1652616}+c_2 11^{16}+c_3 11^{16}$

Pb_{18} ^ $2\hat{}19^{16}$

Table44 Symbolic Encoding of *Kyrie* b.52-76

Encoding for Each Bar	Bar No.
2^11^(**Np1**)+11^(**Mp1+7**sus**Np1**)	52
2^11^(**Mp2**sus**Mp1+7**)+11^(**Np4+7**)	53
2^7^(**Np4+Np1**)+3^(**Mp1**sus**Np4**)+3^(**Mp1+7+Mp4+7**sus**Np1**) + 3^(**Np1+7**sus**Mp5+7 +Mp1+7**)+3^(**Mp2+7**sus**Mp5+7+Mp1+7**)	54
2^5^(**Np4+2**sus**Mp1+7+Mp1+7**)+5^(**Mp2+7**sus**Mp1+7+Np4+2**)+11^(**N+7+Np1+7**)	55
2^5^(**Np2+Np4**sus**Np1+7**) +3^(**Mp1susNp2**)+(**Np2+Np1+7**)+3^(**Mp2**sus**Np2+Mp1+ Np1+7**)+5^(**Mp5+7+Mp5+7**sus**Mp1**)+ 3^(**Mp2+7**sus**Mp5+7+Mp5+7)**+ 3^(**Np1**sus**Mp4+7+Mp5+7**)	56
2^7^(**Mp1+Mp1+Mp1+Mp4**sus**Mp1+7**)+3^(**Mp2+7**sus**Mp1+Mp5+Mp4**)+ 11^(**Mp5+5 +Mp2+5+Mp1**sus**Mp1**)	57
2^5^(**Np1+Np4+Np2**)+3^(**Np1**sus**Np1+Np4+Np2**))+3^(**Mp2**sus**Np1+Np4+Np1**)+ 5^(**Mp4+7+Mp1+7+Mp5+7**sus**Np1**) +3^(**Np1+Mp3**sus**Mp4+7+Mp1+7+Mp1+7**)+ 3^(**Mp1+7+Mp3**sus**Mp6+7+Mp1+7+Mp1+7**)	58
2^3^(**Mp1+Mp4+Mp6**sus**Mp1+7**)+3^(**Np4**sus**Mp1+M**)+3(**Mp4+Mp2**sus**Mp1+Mp5**)+ 3^(**Mp6+7**sus**Mp1+Mp4+Mp3**)+ 3(**Mp6+5+M+5**)+3^(**Mp4+7**sus**Mp3+5+Mp4+5**) + 3^(**Mp4+5**sus**Mp3+5+Mp4+5**)+2^(**Mp2**sus**Mp5+5+Mp4+5+Mp2+5**)+2^(**Np2**sus **Mp5+5+Mp4+5+Mp2+5**)	59
2^3^(**Ms1+5**sus**Mp3+5**)+3^(**Mp2+7**sus**Ms1+5**)+5^(**Mp1**sus**Ms1+5**)+5^(**Mp5+7+ Mp1+7+Mp1+7**sus**Mp1**)+3^(**Mp2+7**sus**Mp5+7+Mp1+7+Mp1+7**)+3^(**Np4**sus**Mp1+7 +Mp4+7**)	60
2^3^(**Mp1+Mp1+Mp4+Mp1**sus**Mp1+7**)+3^(**Np1**sus**Mp1+Mp1+Mp5+Mp1**)+ 3(**Mp1**sus**Mp1+Mp1+Mp5+Mp1**)+3^(**Mp2+7**sus**Mp1+Mp1+Mp5+Mp1**)+5^(**Mp4+5+ Mp2+5**sus**Mp1+Mp1**)+5^(**Mp2+5**sus**Mp1+Mp4+5+Mp2+5+Mp1**)	61
2^5^(**Np1+N**sus**Mp2+5**)+5^(**Np1**sus**Np1+N**)+3^(**M+7+Mp5+7**sus**Np1**)+ 3^(**Mp1+7**sus**M+7+Mp5**)+ 3^(**Mp1+7**sus**M+7+Mp5**)+3^(**Mp1+7+Mp1+5**sus**Mp2+7+ Mp5+5+Mp3+7**)	62
2^3^(**Mp1+M+Mp2**sus**Mp1+7**)+3^(**Mp1+5+Np1**sus**Mp1+Mp5+Mp2**) 3^(**Mp2+**	63

Mp2sus**Mp1+Mp5+Mp2**)+3^(**Mp2+7+Np1**sus**M+Mp2**)+5^(**N+7+Np1+7**sus**Mp2**
+Mp2)+3^(**Np1+7**sus**Np4+7+Mp2+Np5+7**)+3^(**Np3+7**sus**Np4+7+Np1+7+Np5+7**)
2^7^(**Np1+Np1+N+Np2**)+3^(**Mp2**sus**Np1+Np1+N**)+5^(**M+7+M5+7**)+3^(**Mp2+7+** 64
Mp1+7sus**Mp5+7+Mp5+7**)+3^(**Mp1+7+Mp1+7**sus**Mp2+7+Mp5+7+Mp3+7**)
2^5^(**Mp1+M**sus**Mp2**)+3^(**Mp2+Mp1**sus**Mp1+M**)+3^(**Mp2+7**sus**M+Mp2+Mp1**)+ 65
5^(**Ms1+5**sus**Mp1**)+5^(**Mp2+7**sus**Ms1+5**)
2^5^(**Mp1+7+Mp3+7+Mp4+7**)+3^(**Mp1+7**sus**Mp1+7+Mp3+7+Mp4+7**) +3^(**Mp1+7**
+ Mp1+5+Mp3+7sus**Mp4+7**)+3^(**Mp1+Mp4+Mp1**sus**Mp1+7**)+3^(**Mp3+7**sus**Mp1** 66
+Mp5+Mp1)+5^(**Mp1+Mp4**sus**Mp1+Mp1+Mp1**)
2^5^(**Ms1+5**sus**Mp1**)+3^(**Mp2+7+Mp1+7+Mp1+7+Mp4+7**)+3^(**Np1+7**sus**Mp2+7+** 67
Mp1+7+Mp1+7+Mp1+7)+5^(**M**sus**Mp1+7**)+5^(**Mp1+Mp4**sus**Mp5**)
2^3^(**Mp4+5+Mp4+5**sus**Mp1**)+3^(**Np1**sus**Mp4+5+Mp4+5**)+3^(**Ns1+2+Np4+2)**+
3^(**Np1**sus**Ns1+2+Np2+2**)+5^(**Np4+7 Np2+7**sus**Np4+2**)+5^(**Np4+7+** 68
Mp1+4sus**Np4+7**)
2^5^(**M+7+Mp1+7**)+5^(**Mp2+Mp1+M**sus**Mp1+7**)+5^(**Ns1+Np2**)+3^(**M+7+** 69
Mp1+7sus**Np1**)+ 3^(**Np1**sus**M+7+Mp1+7**)
2^5^(**Mp1+Mp1+Mp4**sus**Mp1+7**)+5^(**Mp4+Mp1**sus**Mp1+Mp1+Mp1**)+5^(**Np2+N**)+
3^(**Mp1+7+Mp1+7+Np1+2+Mp2+7**sus**Np1**)+3^(**Mp3+7**sus**Mp1+7+Mp1+7+Np1+** 70
Mp2+7)
2^5^(**N+7+Np2+7**)+5^(**Ms2+5**sus**Np4+7**) +5^(**M+7+Mp4+7**)+5^(**Mp2+Mp1+** 71
Msus**Mp1+7**)
2^5^(**Np2+Np3+N**)+3^(**M+7+Mp1+7+Mp1+7**sus**Np1**)+3^(**Np4**sus**M+7+Mp1+7**)+ 72
5^(**Mp1+Mp1+Mp4+Mp2**sus**Mp1+7**)+5^(**Mp5+Mp2**sus**Mp1+Mp4**)
2^5^(**Mp4+5+M+5**)+5^(**M+7+Mp1+7+Mp4+7**)+5^(**Np4+7+Ns1+7**sus**Mp1+7**)+ 73
5^(**Mp1+5+Mp5+5**sus**Np4+7+Np2+7**)
2^5^(**Mp1+7+Np1+2+Mp1+7**sus**Mp3+5**)+3^(**Mp5+7+Mp2+7**sus**Mp1+7+Mp1+7**) +
3^(**Np2+7+Np1+7**sus**Mp1+Mp1+Mp1**)+5^(**Mp1+7+Np1+7+Mp3+7+** 74
Mp4+7sus**Mp1+7**)+3^(**Mp1+7+Mp1+7**sus**Mp1+7+Mp5+7 Mp2+7**)+5^(**Mp5+7+**
Mp2+7sus**Mp1+7+Mp1+7+Mp1+7**)
2^5^(**Np2+7+N+7+Np2**sus**Mp1+7**)+5^(**Mp1+5 Mp1+5**sus**Np2+7+Np4+7 Np2+7**)+
5^(**Mp1+7 + Mp1+7**sus**Np2+7 + Np2+7 + Mp1+5 + Np2+7**) + 5^(**Np2+7 +** 75
Mp2sus**Np2+7+Np2+7+Mp1+7+Np2+7**)
2^3^(**Mp1+5+Mp4+5**sus**Np2+7+Np2+7+Np2+7**)+3^(**Mp2+7**sus**Mp1+5+Np2+7+**
Mp4+5+Np2+7)+3^(**Np1+7**sus **Mp1+5+Np2+7+Mp4+5+Np2+7**)+3^(**Mp1+7+**
Mp2sus**Mp1+5+Np2+7+Mp2+5+Np2+7**)+ 5^(**Ms1+5**sus**Mp1+5+Np2+7+Np2+7**)+
5^(**Mp4+5+Np1**sus**Np2+7+Mp1+5+Np2+7**) 76

Table45 Symbolic Encoding of *Gloria* b.24-35

Encoding for Each Bar	Bar No.
2^7^(**Mp1+M**)	24
2^3^(sus**Mp1+M**)+5^(**M+7+Mp1+7)**+11^(**Mp1+M**)	25
2^5^(**M+5+Mp2+5**)+5^(**Np1+N**)+3^(**Np1+7**sus**N**)+3^(**Np6+7**sus**Np1+7+Np3**)+ 3^(**Ns1+7**sus**Np1+7**)+2**^**(**Mp6+7**sus**Np1+7+Np1+7**)+2^(**Np4+7**sus**Np1+7+ Np1+7)**	26
2^5^(**Mp6+7+Np1+7)**+5^(**M**)+5^(**Np4+7**sus**Mp3**)+5^(**Mp2+2**sus**Np4+**7)	27
2^5^(**Mp4+7+Mp1+7**)+5^(**Mp5+7+Mp2+7**sus**Mp4+7+Mp1+7**)+ 5^(sus**Mp5+7+Mp2+7**)+(**Mp2+2+Mp5+2**)	28
2^11^(**Mp5+7+Mp2+7**)+5^(**Np1+2+Np4+2**)+5^(**Np4+7**sus**Np1+2**)	29
2^3^(**Np1+2**sus**Np4+7**)+3^(**Mp4+7**sus**Np1+2**)+5^(**Np4+2**sus**Np1+2**)+5^(**Np2+ Np5**)+3^(**Mp4+7**sus**Np1**)+3^(**Np1**sus**Mp4+7**)	30
2^5^(**Mp1+Mp1+Mp1**)+11^(**M+Mp1**sus**Mp1**)+5^(**M+Mp1**sus**Mp1**)	31
2^11^(**M+Mp1**sus**Mp1**)+5^(**Mp2+5+M+5**)+5^(**Mp1+5**sus**M+5)**	32
2^3^(**Mp2+5**sus**Mp1+5+Mp4+5**)+3^(**Mp4+7**sus**Mp4+5**) +3^(**Np4+7**sus**Mp4+5)** +3^(**Mp6+7**sus**Mp4+5**)+5^(**M+Mp2**+5^(**Mp1+M+Mp4**)	33
2^5^(sus**Mp1+M+Mp4**)+5^(**Mp1+M+Mp4**)+11^(**Mp1+M+Mp4)**	34
2^5^(**Mp2+5+M+5+Mp3+5+Mp1+5)**	35

Appendix 2. Musical DNA in *Stabat Mater* and *Super Flumina Babylonis*

Dissonant notes may only be sounded as passing notes on weak beats or as a suspension on strong beats. The occurrence of a dissonant note may be predicted as occurring as part of a **sus** encoding with a prepared suspension and resolution; whilst a passing note dissonance will be an isolated note on a weak beat with other pitches sustained against it. The extracts in Figures 40-43 shows where this occurs in the motets.

Fig.40 *SM* b.15, 7-6 suspension in alto

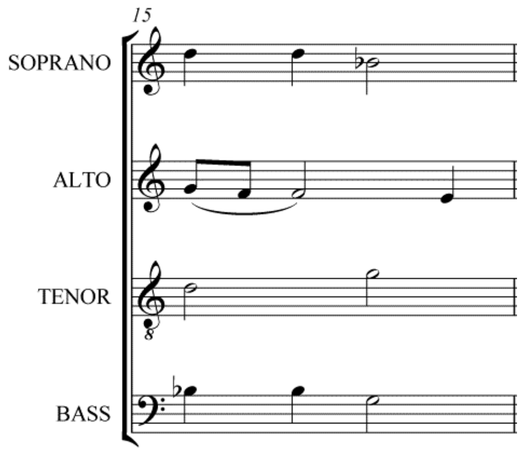


Fig.41 *SFB* b.10, 7-6 suspension in soprano

Fig.42 *SM* b.3, 4-3 suspension in tenor

Fig.43 *SFB* b.67, 4-3 suspension in soprano

Using the same procedure for the contrary motion chords these suspensions may be written as:

SM b.15, 7-6:	((**Ns1+5**+**Np3+5**)+(**Np2susNp6+5**+**Np3+5)**+(**Mp5+5+Mp2+5 susNp3+5**)+(**Np1+5**+**Np4+5susNp3+5**)+(**Mp2+7susNp1+5**+ **Np4+5**))+(2+2+4+4+4)
SFB bar 10, 7-6:	((**Mp2**+**Mp2**+**Mp5**)+(**Mp2+7+Mp1+7susMp2**+**Mp2)**+(**Np2**+ **susMp2+Mp2+7+Mp1+7**)+(**Mp4susMp2+7+Mp1+7**)+(**Mp3+2 +Mp2+2susMp4**)+(**Mp2susMp4+Mp3+2**)+(4+2+2+4+2+2)
SM bar 3, 4-3:	(((**Np5**+5)+(**Np4**+5))+((**Mp1**+**Mp5**)+7)**sus**5+(**Mp2**+ 7)**sus**22922)+(8+4+4)

Fig.44 *SM* b.20

Fig.45 *SFB* b.47

The encoding for the passing notes shown in figures 44 and 45 are:

SM bar 20:	(((**Mp5**+2)+(**Mp1**+2)+(**Mp1**+2))+**Mp2sus**16516+((**Mp4**+ 2)+(**Mp1**+2)+(**Mp1**+2)**sus**161616)+(**6**+2+**4**+4)
SFB bar 47 – 48 Beats 4 to 1 only	((**Np5**+**Np4**)+**Mp1sus**9269+(**N**+5)**sus**9)+(2+2+**4**)

Thus, the *MIs* encoding the fragments of Palestrina's music are sequences of musical DNA characteristic of C16th polyphonic music. The sequences are the genetic code of musical genes unique to Palestrina.

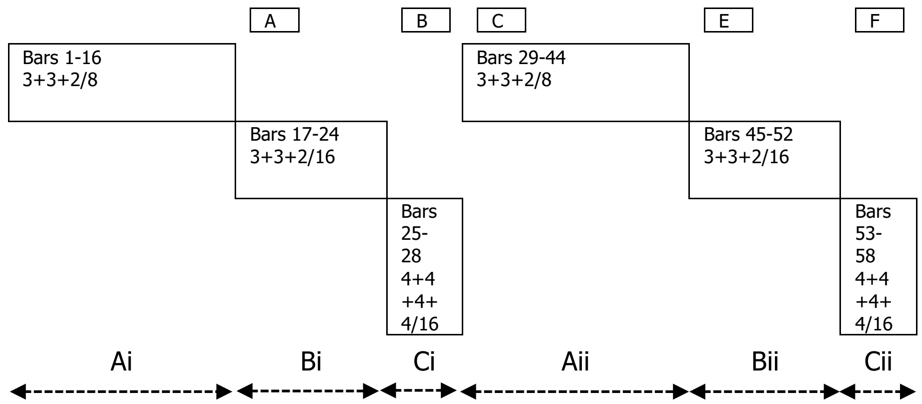
Appendix 3. *Two Explorations of a Chord Sequence*

13ii. Pg.73 explains the conception and pitch-organisation used.

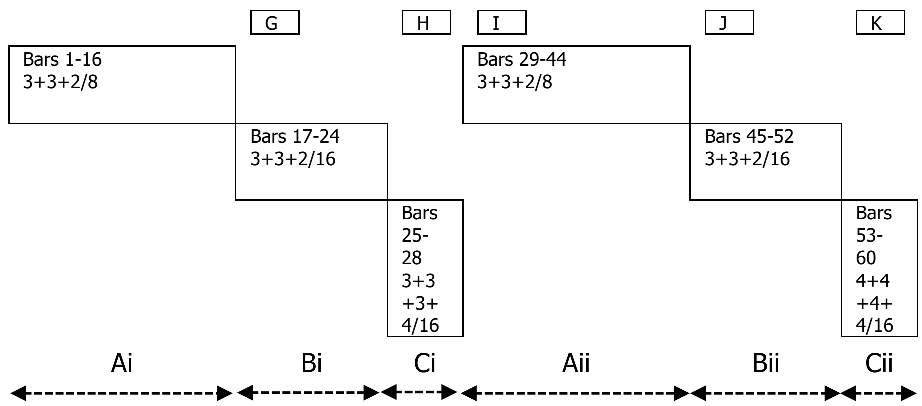
Form of *Two Explorations of a Chord Sequence*

Form Charts 1 and 2 shows the form of the two movements. At structural points in the form, the rate of harmonic change augments or decreases in proportions to the number 2, the chord changes shown in Figure 12 occur on every semibreve then every minim then every crotchet. This piece utilises a 3 + 3 + 2 quaver or semiquaver pattern. This rhythm was chosen since it is a pervasive rhythm in Latin and African music.

Form Chart 1 *1. Two Explorations of a Chord Sequence*



Form Chart 2 *2. Two Explorations of a Chord Sequence*



Appendix 4. *This Day Dawns*

24xi. pg.171 explains the conception and pitch-organisation used.

This Day Dawns uses a text by an anonymous author. The music is based on an extract from an antiphon *O Virtus Sapiente* by Hildegard von Bingen (Furore Ed. 1997) this is stated in the soprano part in bars 1 -6. The music uses non-retrogradable rhythms and is mostly polyphonic in texture with occasional homophonic passages.

Libretto

[refrain:] This day day *daws, [*dawns
This gentle day daws,
And I must home gone.
[verse 1:]
In a glorious garden green
Saw I sitting a comely queen
Among the *flow'r-es that fresh been. [*flowers

She gathered a flower and set between.
The lily-white rose me-thought I saw,
The lily-white rose me-thought I saw,
And ever she sang:
[refrain: 'This day day daws', etc.]

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