**Vibration control of beams subjected to a moving mass using a successively combined control method**

Yangjun Pi a, Huajiang Ouyang b

a State Key Laboratory of Mechanical Transmission, Chongqing University, Chongqing 400044, China (Email:cqpp@cqu.edu.cn)

b School of Engineering, University of Liverpool, The Quadrangle, Liverpool L69 3GH, UK

(Corresponding author: Email:h.ouyang@liverpool.ac.uk)

**Abstract**

This study addresses the vibration control of beams subjected to a moving mass, which represent real applications such as vehicle-bridge interaction. Positive position feedback control (PPF) which has been successfully used in vibration control of flexible structures is found not suitable for the current control problem when the moving mass is travelling on the beam as it makes the structure more flexible but is found capable of reducing free vibration after the moving mass leaves the beam. Sliding mode control (SMC) is known to be a robust method to deal with parameter uncertainties and disturbances in vibration, however it is found to be likely to introduce some higher-frequency vibration which is detrimental to the beam. A method combining SMC and PPF is proposed to supress the vibration of the beam when the moving mass is on and off the beam, which overcomes the above problems and possesses the benefits of both SMC control and PPF control. Simulated numerical examples demonstrate the effectiveness of the proposed method.

**Keywords**

Vibration, beam, moving mass, sliding mode control, positive position feedback control

**1 Introduction**

Dynamic problems of structures subjected to moving loads have received great attention during the last decades [[1-7](#_ENREF_1)]. Applications include vehicle-bridge interaction, wood saws, computer discs, and vehicle disc and drum brakes [[8](#_ENREF_8)]. Vehicle-bridge interaction dynamics as a particular moving load problem has been intensively researched in civil engineering [[9](#_ENREF_9), [10](#_ENREF_10)]. Marchesiello et al. [[11](#_ENREF_11)] studied dynamics of multi-span bridges subjected to excitation of a multi-degrees of freedom moving vehicle. Fryba [[12](#_ENREF_12)] researched many simple moving-load problems and provided their analytic solutions in his monograph.

Besides the dynamic modelling and analysis of civil and mechanical structures, vibration control of these structures is another important issue. Some methods such as passive, semi-active, active and hybrid vibration control have been developed [[13-16](#_ENREF_13)]. There has been an increasing interest in active control of structures since it is proved to be a more efficient technique than passive control [[17](#_ENREF_17), [18](#_ENREF_18)]. There are a great deal of control algorithms based on control theory for active structural control, such as PID control [[13](#_ENREF_13)], optimal control [[19](#_ENREF_19)], adaptive control [[20](#_ENREF_20), [21](#_ENREF_21)], and intelligent control [[22](#_ENREF_22)]. A comprehensive review about active structural control was presented by Korkmaz [[18](#_ENREF_18)].

Recently, vibration control of beams subjected to a moving mass has gained much attention in both physics and engineering due to the increasing speed of moving mass and structural flexibility [[23-26](#_ENREF_23)]. Independent modal space control is an important method in vibration control of flexible structures as the analytic solution to the model of a flexible structure is commonly expressed in modal space. One of the major problems of the modal space control of flexible structures is the well-known phenomenon of spillover [[27](#_ENREF_27), [28](#_ENREF_28)]. Positive position feedback control introduced by Goh and Caughey is a simple and successful control algorithm in real applications, since the main advantage of PPF control is its insensitivity to control spillover, that is, it can control the target mode individually without disturbing other uncontrolled modes [[29](#_ENREF_29), [30](#_ENREF_30)]. Chuang et al. [[23](#_ENREF_23)] studied the active suppression problem of a beam under a moving mass by using a PPF controller. The PPF controller can increase the damping at a desired frequency, and dissipate the energy at this frequency. It is suitable to supress free vibration which contains harmonic components. However, the PPF controller has a disadvantage that it makes a structure more flexible, which can lead to larger steady state errors [[29](#_ENREF_29)]. It means that the PPF control is not suitable to deal with the control problem when the moving mass is on the beam, as the response of the beam may not have harmonic components within this duration, which can be seen from the numerical results given in Section 4.

The most commonly used method in vibration control of a beam subjected to a moving mass is optimal control in modal space [[19](#_ENREF_19), [31](#_ENREF_31), [32](#_ENREF_32)]. It assumes that the whole controlled states can be completely known from measured or estimated data. However, it may not be possible to install all sensors to obtain the full states [[33](#_ENREF_33)]. For practical implementations, there are ineluctable modelling errors and measurement errors. Then a robust control algorithm is needed to deal with modelling errors caused by modal truncation and measurement errors. Sliding mode control is an effective robust control technique which can deal with modelling uncertainty and external disturbances [[34](#_ENREF_34), [35](#_ENREF_35)]. A sliding mode control method for seismically excited nonlinear and hysteretic civil engineering structures was proposed by Yang et al. [[33](#_ENREF_33)]. Qiu et al. [[36](#_ENREF_36)] introduced a kind of discrete-time sliding mode control algorithm to supress vibration of a flexible plate. However, the main disadvantage of sliding mode control is that it may introduce higher-frequency excitation which may be detrimental to the structure. It seems that sliding mode control has not been applied in vibration control of moving load dynamic problems.

The moving mass problem can be partitioned into two sub-problems. One is the time varying linear problem when the mass is travelling on the beam, and the other is the time invariant problem after the mass leaves the beam [[32](#_ENREF_32)]. The first problem is a self-excited vibration problem, while the second one is a free vibration problem. Then, it is natural to use different control laws to deal with the two different problems. In this paper, a method combining both sliding mode control and positive position feedback control is proposed to deal with the moving load problem. The sliding mode control is used to decrease the deflection of the beam when the mass is on the beam. After the moving mass leaves the beam, PPF controller is introduced to supress the vibration caused by the moving mass. The main contributions of this paper are: 1) This paper firstly introduces the sliding mode control to the vibration control of beams subjected to a moving mass, whose study has gained much attention in both physics and engineering due to the increasing speed of vehicles in modern transportation and structural flexibility in real engineering problems. 2) The proposed combined method exploits the benefits of both sliding mode control and positive position control to suppress vibration excited by a moving mass, which can be partitioned into two sub-problems that are self-excited vibration problem and free vibration problem. Results of the simulated numerical examples show that the combined control method (CCM) can significantly suppress the beam vibration.

**2 Theoretical formulations**

Some practical problems such as a vehicle travelling on a bridge can be approximated as a simple beam subjected to a moving mass illustrated in Fig.1. Assuming the moving mass is small compared with the mass of the beam and the beam damping is proportional to the velocity of vibration, vibration of the moving mass problem under control can be described by the equation below [[8](#_ENREF_8)]

 (1)

where *w* is the transverse motion, is the density of the beam, *A* is the cross-sectional area, *E* is the Young’s modulus, *I* is the second moment of area of the beam’s cross section, *c* is the damping of the beam, *N* is a constant force, *u* is the travelling speed which is considered constant in this paper, is the Dirac function.  is the force of the *i*th actuator at the horizontal position . *k* is the number of actuators.

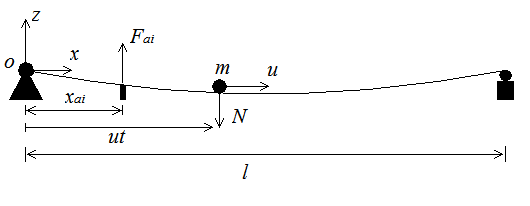


Figure 1. A model for a beam subject to a moving mass.

Assuming that the mass does not separate from the beam, the instantaneous vertical motion of the moving mass *v* can be associated with the transverse motion at a moving coordinate as

 (2)

then

 (3)

Substituting Eq. (3) into (1), now the equation of motion becomes

 (4)

The analytic solution of Eq. (4) can be expressed as

 (5)

where is the beam’s *j*th orthogonal modal shape, and is the *j*th modal coordinate.

Substituting Eq. (5) into (4), then multiplying the resultant equation with and integrating it over the beam length, one can obtain

 (6)

where the dot denotes derivative with respect to time and the prime to *x*; the time-invariant terms which depended on the properties of the beam can be expressed as

,, (7)

and the time dependent terms introduced by the moving mass are

,, (8)

By using the eigenfunction expansion, the equation in the physical coordinates is now transferred to that in the modal coordinates.

For simplification, Eq. (6) can be expressed as

 (9)

This is a nonstationary system, as the coefficients of the equation are functions of time. Theoretically, the continuous structure has an infinite dimension. In practice, only the first few modes need to be considered in the analysis and control by using modal truncation.

**3 Controller design**

In practical implementations, the modal coordinates cannot be directly measured. Although one can use enough sensors to estimate them, the observer spillover cannot be avoided [[27](#_ENREF_27)]. Furthermore, the actuator spillover is also introduced if the vibration controller is designed in the modal space.

3.1 Sliding Mode Control

Suppose that there are *k* actuators with *k* collocated sensors, which are usually used in vibration control of flexible structures. Considering the first *n* vibration modes，

 (10)

where is the *k*-dimensional vector of transverse displacements of the beam which can be measured by the sensors,  is the modal vector matrix which links modal space to physical space and can be obtained from Eq. (5), and is the vector of the first *n* modal coordinates.

Substituting Eq. (10) into (9),

 (11)

in which is the *k*-dimensional actuator vector,  is the modelling errors introduced by modal truncation and other disturbances, such as external disturbances. It should be noted that the number of modal coordinates can typically be chosen as the same as that of actuators for the convenience of controller implementation, as the modal truncation error can be treated as a modelling error with sliding mode control.

The design of sliding mode controller can be divided into two steps. One is to design a sliding surface, which has some desired properties; the other is to design a controller to drive the states onto the sliding surface and keep them there [[33](#_ENREF_33)].

The sliding surface is selected as

 (12)

where  is a positive constant. The transverse displacement vector  asymptotically converges to zero with the time constant  when this sliding surface is reached.

Differentiation of  with respect to *t* yields

 (13)

Defining the Lyapunov function as

 (14)

Differentiation of *V*

 (15)

 (16)

Control force is designed as

 (17)

where

 (18)

 (19)

 (20)

where *p* is the control gain, is the boundary layer of sliding mode, and sat(\*) is the saturation function.

Taking into account Eqs. (18) - (20), and substituting Eq. (17) into (16), the time derivative of Lyapunov function *V* is

= (21)

If the number of modes considered in analysis is big enough, then the modal truncation error is quiet small. If control gain *p* is chosen to be big enough, the condition  can be guaranteed. It means that the reaching condition  is satisfied, and the states can be driven onto the sliding surface and maintained there. The controller is proposed to guarantee asymptotic convergence to zero of the transverse motion [[37](#_ENREF_37)].

3.2 Positive Position Feedback Control

Positive position feedback control is a simple but successful method to supress vibration of flexible structures. It can add damping to the eigenvalue of a target mode, and does not disturb the other modes.

The PPF control can be described by the following equation [[29](#_ENREF_29)]

 (21)

and

 (22)

where is the feedback gain, *b* determines the level of force into the mode of interest,  is the position of the structure, and is the position of the compensator, and are the structure’s and compensator’s damping ratios, respectively, and  and  are the structure’s and compensator’s natural frequencies, respectively.

By using the PPF control, the displacement response of the structure is positively fed back to produce a suitable control force to the structure. The maximum active damping efficiency is obtained by adjusting the compensator’s natural frequency in relation to the structure’s natural frequency. The PPF controller is like a low pass filter, whose transfer function rolls off quickly at high frequencies. The PPF controller has two advantages: one is that it does not affect the high frequency dynamics, and the other is that it does not introduce a new eigenvalue to the system.

Combining equations (21) and (22), the closed-loop system can be expressed as

 (23)

In order for the system governed by equation (23) to be asymptotically stable, its eigenvalue must not have a positive real part. Based on Routh-Hurwitz criterion, the stability of the closed loop system can be guaranteed when the feedback gain is chosen within 0<g<1.

The design of the PPF controller is to find the target natural frequency (usually for the first mode) of the system, and then choose a proper damping ratio of a compensator. The control performance is sensitive to the compensator’s natural frequency that should be chosen as near the target natural frequency of the system, which can be easily and accurately measured in practice. Also one restriction of the PPF controller is that its filter frequency should be tuned to the natural frequency of the target mode. The amount of damping needed depends on how accurately the filter frequency is tuned to the natural frequency of the target mode. The natural frequency can be found experimentally, or an adaptive positive position controller can be used to solve this problem. More details about the PPF control can be found in [[29](#_ENREF_29), [30](#_ENREF_30)].

3.3 Combined control method

As can be seen, for the vibration control of beams subjected to a moving mass, the dynamic model when the moving mass is on the beam is different from that after the moving mass leaves the beam. In this study, a combined control method is used. It means that when the moving mass is on the beam, the sliding mode control is adopted; otherwise the positive position feedback control is used.

The combined control method uses two successive controllers to suppress the vibration of beams subjected to a moving mass. When the moving mass leaves the beam, the controller switches from SMC to PPF, and there may be a discontinuous change in the control output. However, this change is quiet small and can be treated as an impulse to the system, and then the subsequent free vibration can be suppressed by the positive position control.

**4 Numerical simulations**

In this study, only the first two modes of the beam are used, since the other modes do not have a significant influence on the accuracy of the beam deflection. A uniform simple supported Euler-Bernoulli beam with length *l* =1m, mass per length unit =0.22 , and flexural rigidity *EI* =3.66 is considered.

The placement of the sensors and actuators is an important issue for structural control. In this study, the actuators and sensors are collocated at two fixed positions on the beam:  =0.157m and =0.710m (see Fig. 2).

It must be pointed out that although there are two collocated sensors and actuators, only one collocated sensor and actuator at = 0.157m is used in the PPF control as one actuator is enough to suppress the free vibrations excited by the moving mass. The target mode to be supressed is also only the first mode, as its contribution to the dynamic response of the beam is much larger than those from the other modes in this study, which can be seen in the following section.

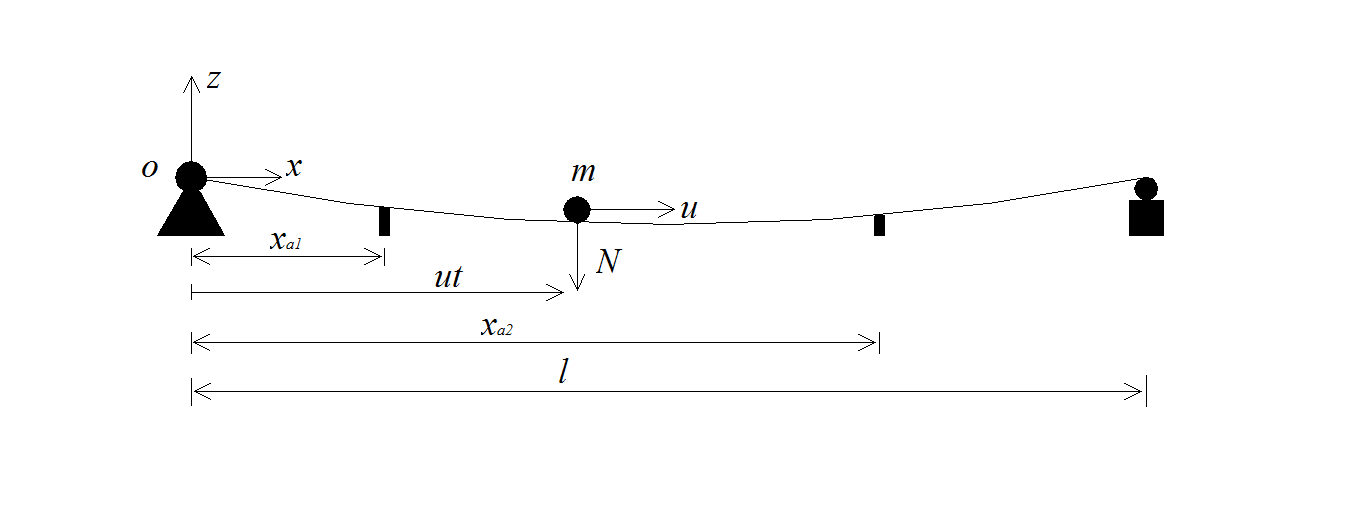


Figure 2. Actuator positions for a single span beam subjected to a moving mass.

As mentioned above, if the SMC controller drives the states onto the sliding surface, the deflection tends exponentially to zero with time constant. However, control parameter  which is called control bandwidth, is typically limited by structural resonant modes, time delays, and sampling rate of the controller in real applications [[37](#_ENREF_37)]. The boundary layer of sliding mode  is introduced to eliminate chattering, which should be chosen to achieve a trade-off between accuracy and robustness. The control gain of SMC is determined by the estimation of model uncertainty.

In this section, the control performance of the SMC controller with different control parameters is compared. One group of parameters (PARAM1) is set as , and, and the other group of parameters (PARAM2) are set as , , . The mass is 0.6kg and the velocity of the moving mass is set as 2.5m/s. Fig.3 shows that the deflection response at the positions where the actuators are placed. It is obvious that the deflections with PARAM2 are smaller than those with PARAM1. However, the deflections at the mid-span are similar for the two different controllers (see Fig.4). It means that there is no need to decrease the deflections at the actuator positions too much by using too much control force (thus incurs much cost) for the moving load problem. Although there are only two groups of control parameters shown in the paper, it indeed needs a trial-and-error process (which is common practice) to get a suitable group of control parameters. For example, the choice of the width of the boundary layer in the controller design is to achieve a trade-off between control precision and robustness to unmodelled dynamics.



Figure 3. Comparison of the response of the beam at the actuators’ positions under different control parameters: a- deflection at actuator 1 with PARAM1; b- deflection at actuator 1 with PARAM2; c- deflection at actuator 2 with PARAM1; d- deflection at actuator 2 with PARAM2.



Figure 4. Comparison of the response of the beam at the mid-span under different control parameters.

In the following, three controllers which implement respectively sliding mode control, positive position feedback control, and the combined control methods are designed. The detailed controller parameters are given in Table 1. It should be noticed that the combined control method adopt the same control parameters of SMC and PPF control given in Table 1.

Table 1 The controller parameters.

|  |  |
| --- | --- |
| Controller | Control parameters |
| Sliding mode control | ; ; |
| Positive position feedback control |  |

4.1 Example 1

In this case, to test the performance of PPF controller and SMC controller，an impulse excitation is exerted on the midpoint of the beam. Fig.5 shows the time domain vibration responses of the beam under different controllers. It is clear that both PPF controller and SMC controller can suppress the vibration significantly, and the SMC controller is more efficient in this case. Fig.6 shows the performance of both PPF controller and SMC controller in the frequency domain. It is clear that the frequency of the first mode of the beam is around 6.33Hz. From Fig.6, the first mode is significantly supressed by the PPF controller while higher modes are not affected. Although the SMC controller is more efficient to supress the vibration of beams, it should be pointed that the control force in the SMC controller is also significantly higher than that in the PPF controller (see Fig.7). Moreover, the PPF controller only uses one actuator in this study, while the SMC controller uses two actuators. It should be noted that a new peak around 34.77Hz (see Fig.6) is introduced by the SMC controller, which may be detrimental to the structure under control in some applications However, the PPF controller does not have this disadvantage.



Figure 5. Comparison of the mid-point deflection of the beam.



Figure 6. Vibration control results in frequency domain.



Figure 7. Control force of the controllers.

4.2 Example 2

### In this example，the vibration control of the beam subjected to a moving mass is addressed. The simulation parameters are the same as those in [[38](#_ENREF_38)] for comparison, and the mass of the moving mass is 0.6 kg and its velocity is 2.5 m/s. The control performance of the PPF controller is shown in Fig.8 (the dashed line denotes of time instant when the moving mass exits from the beam). One can see that the PPF controller can supress the vibration after the moving mass leaves the beam, which is after 0.4 second in this case. However, the PPF controller is not effective when the moving mass is on the beam, as the PPF controller makes the beam more flexible.

Fig.9 shows that both the SMC controller and CCM controller can significantly supress the vibration whether the moving mass is on the beam or not. The maximum mid-span deflection of the beam decreases from 4.75cm to 0.23cm, which is much smaller than that in [[38](#_ENREF_38)] for the same vibration problem. The main difference between the SMC controller and CCM controller is that the control force of CCM controller after the moving mass leaves the beam is significantly small than that for SMC controller alone (see Figs.10-13).

Another indicator of efficient control is the actuation energy required by controllers. The control energy requirement is defined as [[28](#_ENREF_28)]

 (24)

where *T* is a time duration that should cover a few periods of the vibration. The energy requirements of different controllers are compared in Table2. One can see that the energy requirement of CCM controller after the moving mass leaves the beam is significantly small than that in SMC controller, which is very important in some applications such as railway bridges.

Table 2 The energy requirement with *m*=0.6kg, *u*=2.5m/s.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Actuator 1 | | Actuator 2 | |
| 0-0.4s | 0.4-1s | 0-0.4s | 0.4-1s |
| CCM | 7.53 | 1.46 | 5.62939 | 0 |
| SMC | 7.53 | 8.68 | 5.62939 | 1.69 |



Figure 8. The response of the beam at the mid-span under PPF controller.



Figure 9. The response of the beam at the mid-span under SMC controller and CCM controller.



Figure 10. Comparison of the actuator force under different controllers (actuator 1).



Figure 11. Comparison of the actuator force after the mass leaves the beam (actuator 1).



Figure 12. Comparison of the actuator force under different controllers (actuator 2).



Figure 13. Comparison of the actuator force after the mass leaves the beam (actuator 2).

To extensively evaluate the performance of the CCM controller, a different set of simulation parameters are used. The new mass is set as 1 kg, and the velocity of moving mass is 4 m/s. The mid-point deflections of the beam with different controllers are given in Fig.14 and Fig.15. The results show that both SMC controller and CCM controller can suppress the beam vibration significantly, as in the previous example. The maximum mid-span deflection of the beam decreases from 6.52cm to 0.44cm. However, the control force of SMC is significantly bigger than that of CCM after the moving load leaves the beam, (see Fig.16 and Fig.17). The energy requirement of CCM controller after the moving mass leaves the beam is quiet small compared with the SMC controller, which can be found in Table 3.

Table 3 The energy requirement with *m*=1kg, *u*=4m/s.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Actuator 1 | | Actuator 2 | |
| 0-0.25s | 0.25-1s | 0-0.25s | 0.25-1s |
| CCM | 13.99 | 6.66 | 14.20 | 0 |
| SMC | 13.99 | 0.13 | 14.20 | 0.05 |



Figure 14. The response of the beam at the mid-span under PPF controller (*m*=1kg, *u*=4m/s).



Figure 15. The response of the beam at the mid-span under SMC controller and CCM controller (*m*=1kg, *u*=4m/s).



Figure 16. Comparison of the actuator force under different controllers (actuator 1, *m*=1kg, *u*=4m/s).



Figure 17. Comparison of the actuator force under different controllers (actuator 2, *m*=1kg, *u*=4m/s).

**5 Conclusions**

This paper studies the vibration control of beams subjected to a moving mass. It is a particular problem of vibration control of flexible structures. Two control algorithms are combined to solve this problem. The advantages and disadvantages of the two controllers are extensively studied in this paper. It is found that the PPF controller is efficient to supress the free vibration after the moving mass leaves the beam, however, it is not suitable to reduce the deflection when the moving mass is on the beam. The SMC controller can significantly reduce the deflection when the moving mass is on the beam, however, the energy requirement is significant higher than that with the PPF controller after the moving mass leaves the beam. Moreover, the SMC controller may introduce some higher frequency vibration which can be detrimental to the structure. The combined control method which incorporates the benefits of both SMC control and PPF control is introduced. The effectiveness of the proposed controller is demonstrated by the simulation examples.

**Acknowledgments**

The first author gratefully acknowledges the ﬁnancial support from the National Basic Research Program of China (No. 2014CB049404), the National Natural Science Foundation of China (No. 51105389) and the Special Research Fund for the Doctoral Program of Higher Education (No. 20130191110004). This work is carried out at the University of Liverpool during the visit by the first author sponsored by Chinese Scholarship Council.

**References**

[1] D. Stăncioiu, H. Ouyang, J.E. Mottershead, Vibration of a beam excited by a moving oscillator considering separation and reattachment, J. Sound Vib. 310 (2008) 1128-1140.

[2] L. Sun, Dynamic displacement response of beam-type structures to moving line loads, Int. J. Solids. Struct. 38 (2001) 8869-8878.

[3] S.-Y. Lee, S.-S. Yhim, Dynamic analysis of composite plates subjected to multi-moving loads based on a third order theory, Int. J. Solids. Struct. 41 (2004) 4457-4472.

[4] A.V. Pesterev, L.A. Bergman, A Contribution to the Moving Mass Problem, J. Vib. Acoust. 120 (1998) 824-826.

[5] N. Azizi, M.M. Saadatpour, M. Mahzoon, Using spectral element method for analyzing continuous beams and bridges subjected to a moving load, Appl. Math. Model. 36 (2012) 3580-3592.

[6] M. Dehestani, M. Mofid, A. Vafai, Investigation of critical influential speed for moving mass problems on beams, Appl. Math. Model. 33 (2009) 3885-3895.

[7] A. Nikkhoo, F.R. Rofooei, M.R. Shadnam, Dynamic behavior and modal control of beams under moving mass, J. Sound Vib. 306 (2007) 712-724.

[8] H. Ouyang, Moving-load dynamic problems: A tutorial (with a brief overview), Mech. Syst. Signal. Pr. 25 (2011) 2039-2060.

[9] F.T.K. Au, Y.S. Cheng, Y.K. Cheung, Vibration analysis of bridges under moving vehicles and trains: an overview, Prog. Struct. Eng. Mat. 3 (2001) 299-304.

[10] Y. Yang, J. Yau, Y. Wu, Vehicle-bridge interaction dynamics, World Scientific, 2004.

[11] S. Marchesiello, A. Fasana, L. Garibaldi, B.A.D. Piombo, Dynamics of multi-span continuous straight bridges subject to multi-degrees of freedom moving vehicle excitation, J. Sound Vib. 224 (1999) 541-561.

[12] L. Fryba, Vibration of solids and structures under moving loads, Thomas Telford, 1999.

[13] Z.-c. Qiu, X.-m. Zhang, H.-x. Wu, H.-h. Zhang, Optimal placement and active vibration control for piezoelectric smart flexible cantilever plate, J. Sound Vib. 301 (2007) 521-543.

[14] J.F. Wang, C.C. Lin, B.L. Chen, Vibration suppression for high-speed railway bridges using tuned mass dampers, Int. J. Solids. Struct. 40 (2003) 465-491.

[15] C.M. Casado, I.M. Díaz, J. Sebastián, A.V. Poncela, A. Lorenzana, Implementation of passive and active vibration control on an in‐service footbridge, Struct. Control Hlth. 20 (2013) 70-87.

[16] D. Younesian, E. Esmailzadeh, R. Sedaghati, Passive vibration control of beams subjected to random excitations with peaked PSD, J. Vib. Control 12 (2006) 941-953.

[17] R. Alkhatib, M. Golnaraghi, Active structural vibration control: A review, The Shock and Vibration Digest 35 (2003) 367-383.

[18] S. Korkmaz, A review of active structural control: challenges for engineering informatics, Comput. Struct. 89 (2011) 2113-2132.

[19] A. Nikkhoo, Investigating the behavior of smart thin beams with piezoelectric actuators under dynamic loads, Mech. Syst. Signal. Pr. 45 (2014) 513-530.

[20] I.D. Landau, A. Constantinescu, D. Rey, Adaptive narrow band disturbance rejection applied to an active suspension—an internal model principle approach, Automatica 41 (2005) 563-574.

[21] J. Liu, W.L. Qu, Y.L. Pi, Active/Robust Control of Longitudinal Vibration Response of Floating-type Cable-stayed Bridge Induced by Train Braking and Vertical Moving Loads, J. Vib. Control 16 (2010) 801-825.

[22] B. Xu, Z. Wu, K. Yokoyama, Neural Networks for Decentralized Control of Cable-Stayed Bridge, J. Bridge Eng. 8 (2003) 229-236.

[23] K.C. Chuang, C.C. Ma, R.H. Wu, Active suppression of a beam under a moving mass using a pointwise fiber Bragg grating displacement sensing system, IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control 59 (2012) 2137-2148.

[24] M.H. Ghayesh, H.A. Kafiabad, T. Reid, Sub- and super-critical nonlinear dynamics of a harmonically excited axially moving beam, Int. J. Solids. Struct. 49 (2012) 227-243.

[25] A.K. Mallik, S. Chandra, A.B. Singh, Steady-state response of an elastically supported infinite beam to a moving load, J. Sound Vib. 291 (2006) 1148-1169.

[26] S. Zheng, J. Lian, H. Wang, Genetic algorithm based wireless vibration control of multiple modal for a beam by using photostrictive actuators, Appl. Math. Model. 38 (2014) 437-450.

[27] D.J. Inman, Vibration with Control, John Wiley & Sons, Ltd, 2006.

[28] T. Soong, Active structural control: theory and practice, Longman Scientific & Technical, Harlow, 1990.

[29] M.I. Friswell, D.J. Inman, The relationship between positive position feedback and output feedback controllers, Smart. Mater. Struct. 8 (1999) 285-291.

[30] J.L. Fanson, T.K. Caughey, Positive position feedback control for large space structures, AIAA J. 28 (1990) 717-724.

[31] Y.G. Sung, Modelling and Control with Piezoactuators for a Simply Supported Beam under a Moving Mass, J. Sound Vib. 250 (2002) 617-626.

[32] D. Stancioiu, H. Ouyang, Optimal vibration control of beams subjected to a moving mass, in: Noise and Vibration: Emerging Methods(NOVEM 2012), Sorrento, Italy, 2012, pp. 070-071:076.

[33] J. Yang, J. Wu, A. Agrawal, Sliding mode control for nonlinear and hysteretic structures, Journal of Engineering Mechanics 121 (1995) 1330-1339.

[34] K.D. Young, V.I. Utkin, U. Ozguner, A control engineer's guide to sliding mode control, IEEE T. Contr. Syst. T. 7 (1999) 328-342.

[35] Y. Pi, X. Wang, Trajectory tracking control of a 6-DOF hydraulic parallel robot manipulator with uncertain load disturbances, Control Eng. Pract. 19 (2011) 185-193.

[36] Z.-c. Qiu, H.-x. Wu, D. Zhang, Experimental researches on sliding mode active vibration control of flexible piezoelectric cantilever plate integrated gyroscope, Thin. Wall. Struct. 47 (2009) 836-846.

[37] J.-J.E. Slotine, W. Li, Applied nonlinear control, Prentice-Hall Englewood Cliffs, NJ, 1991.

[38] D. Stăncioiu, H. Ouyang, Application of a state-dependent riccati equation based technique for control of bridge vibrations due to moving loads, in: Asia Pacific Vibration Conference, Jeju, Korea, 2013.