**Free vibration of wavy single-walled fluid-conveying** **carbon nanotubes under multi-physics fields**

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**Abstract**

Fluid-conveying carbon nanotubes (CNTs) have attracted intensive research and have been used in Nano-electromechanical systems (NEMS). In this paper, the free vibration of embedded single-walled fluid-conveying carbon nanotubes (SWCNTs) in magnetic and temperature fields is investigated. The CNTs are modelled as wavy Timoshenko beams. Based on the nonlocal beam theory, the governing equations of motion are derived using Hamilton’s principle. These equations are solved by the Galerkin approach, leading to a set of ordinary differential equations from the partial differential equations of motion. Numerical examples are analysed to assess the difference between the present model and some models reported in the published literature and the effects of the nonlocal parameter, the fluid velocity and flow density, the temperature and magnetic field flux change, and the surrounding elastic medium are discussed. The numerical results validate the analytical model proposed in the present study and lead to conclusions that are potentially useful for the application of fluid-conveying CNTs as NEMS devices.

Keywords: Wavy Single-walled carbon nanotubes; Nonlocal Timoshenko beam theory; Conveying fluid; Vibration; Multi-physical fields

## 1. Introduction

Nano-electromechanical systems (NEMS) have been widely used as sensors, switches, motors, robots, and transducers in physical, chemical, and biological applications, due to their advantages of high sensitivity and fast response [[1](#_ENREF_1)]. Carbon nanotubes (CNTs), owing to their extraordinary mechanical, electronic, thermal and other physical attributions, have been widely used in NEMS, for example in nano-biological devices [[2](#_ENREF_2)]. Because of their hollow cylindrical geometry, fluid-conveying CNTs can have many practical applications. They can be used in heat exchangers, as nano-pipes carrying fluids (drugs and chemicals) in chemical and power plants, and nano-containers for gas storage. Thus, it is very useful to study the mechanical behaviour of fluid-conveying CNTs. The fluid flow inside CNTs apparently will affect their vibration. In recent years, fluid-conveying CNTs have attracted many researchers [[3-11](#_ENREF_3)] to use theoretical and experimental methods to study their dynamic behaviour. There are still many interesting issues unexplored, such as theoretical modelling. Hence, it is very useful to improve the accuracy of theoretical models of fluid-conveying CNTs.

Except experimental work, there are generally two approaches to theoretical analysis of fluid-conveying CNTs’ mechanical behaviour. One is the Molecular Dynamic simulations (MD); the other is continuum mechanics. Though MD [[12](#_ENREF_12), [13](#_ENREF_13)] has been used to study the vibration response of fluid-conveying CNTs, the continuum mechanics approach has been more widely used for vibration analysis of CNTs, because it can provide a viable and efficient alternative method. Reddy and Lu [[14](#_ENREF_14)] developed Euler-Bernoulli beam theory to quantify natural frequency of fluid-conveying SWCNTs. On the basis of Euler-Bernoulli beam theory, Yoon et *al*. [[15](#_ENREF_15)] analysed the vibration and instability of CNTs as fluid-conveying nano-pipes, and their results showed that the internal moving fluid could substantially affect vibration frequencies and the critical flow velocity for structural instability in some cases could fall within the range of practical significance. Meanwhile, they suppressed the instability by embedding fluid-conveying CNTs in an elastic medium. Rasekh and Khadem [[16](#_ENREF_16)] employed Euler-Bernoulli beam theory to investigate the influence of internal moving fluid and compressive axial load on the nonlinear vibration and stability of CNTs. Compared with Euler-Bernoulli beam theory, it is more accurate to use Timoshenko beam theory to study the vibration of SWCNTs [[17](#_ENREF_17)], because the transverse shear deformation and rotary inertia are considered in a Timoshenko beam model. Khosravian and Raffi-Tabar [[18](#_ENREF_18)] developed Timoshenko beam theory to investigate the structural stability of multi-walled carbon nanotubes (MWCNTs) conveying a non-viscous fluid; and their results showed that Timoshenko beam theory is more appropriate for characterising the dynamic behaviour of nanotubes in some cases.

However, the dynamic behaviour of CNTs exhibits size effects [[19](#_ENREF_19)], for example, lattice spacing between individual atoms becomes important. So nonlocal theory of elasticity initiated by Eringen [[20](#_ENREF_20), [21](#_ENREF_21)], which admits size effects by introducing a small scale parameter, has been adopted to investigate the static and dynamic behaviour of CNTs. Soltani et *al*. [[22](#_ENREF_22)] used a nonlocal elastic beam theory to study the transverse vibration of SWCNTs conveying a viscous fluid; their results revealed that the natural frequencies could be affected by the damping of the medium. Following that work, they [[23](#_ENREF_23)] investigated the nonlinear vibration of fluid-conveying SWCNTs using an energy balance method. Chang [[10](#_ENREF_10), [24](#_ENREF_24)] introduced a nonlocal theory of elasticity to study the thermal-mechanical vibration and instability of fluid-conveying SWCNTs. Kiani [[25](#_ENREF_25)] studied the dynamic response of SWCNTs subjected to a viscous flow based on nonlocal Rayleigh beam theory. Ghavanloo and Fazelzadeh [[26](#_ENREF_26)] applied nonlocal elastic Timoshenko beam theory to analyse the flexural vibration of embedded CNTs conveying viscous fluid, and they discussed the effect of temperature change on vibrating CNTs.

Although plenty of studies have been made to investigate the dynamic behaviour (including wave propagation) of fluid-conveying CNTs, it is clear that further theoretical development is required to meet the need of NEMS, and there have been many trial and error attempts during design of NEMS devices. There are two main reasons. One is that the design process of NEMS devices is a multidisciplinary task. NEMS devices can work in complex physical environments, such as in magnetic field, temperature field and so on. However, there are few studies of fluid-conveying CNTs in multi-physical fields [[27](#_ENREF_27)]. The other reason is the fact that CNTs are not an entirely straight and perfect beam structure, due to their fabrication process using chemical vapour deposition, which can be seen from the images of CNTs taken by transmission electron microscopes.

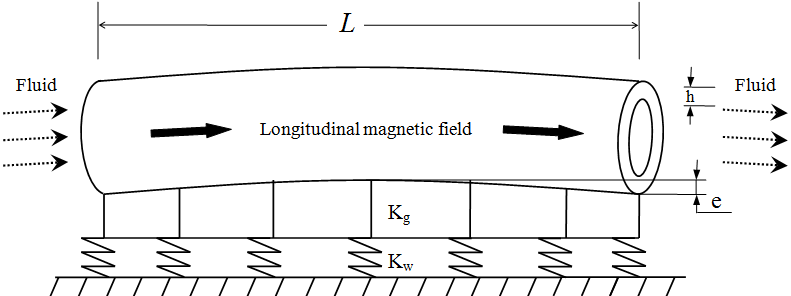
Some previous studies [[5](#_ENREF_5), [17](#_ENREF_17), [28](#_ENREF_28), [29](#_ENREF_29)] have shown that the initial geometrical imperfection (described as wavy or curved, in the same meaning) had influences on CNTs’ mechanical properties and dynamical behaviour. Xia and Wang [[5](#_ENREF_5)] explored the vibration characteristics of fluid-conveying CNTs with curved longitudinal shape. Fisher et *al*. [[28](#_ENREF_28)] discussed the influence of initial geometrical imperfection on the elastic modulus of CNTs; their results indicated that the theoretical modulus was in excellent agreement with experimental data, taking into account the initial geometrical imperfection. Ouakad and Younis [[29](#_ENREF_29)] applied Euler-Bernoulli beam theory to initially curved CNTs under electric excitation. Soltani et *al*. [[17](#_ENREF_17)] developed nonlocal elastic Euler-Bernoulli and Timoshenko beam theories for the transverse vibration of wavy SWCNTs; they found that the waviness had a significant effect on the vibration behaviour of SWCNTs, and Timoshenko beam model was highly recommended for stiffer boundary conditions and stocky SWCNTs. It is noted that Farshidianfar and Soltani [[30](#_ENREF_30)] investigated flow-induced vibration of wavy SWCNTs based on nonlocal Euler-Bernoulli beam theory by the perturbation method of multi scales and they discovered that the initial geometrical imperfection could cause the nonlinear frequency ratio to decrease at increasing fluid velocity. Ghavanloo et *al*. [[31](#_ENREF_31)] studied the influence of the internal moving fluid on in-plane vibration of curved CNTs using the finite element method, and their results revealed that the curved CNTs were unconditionally stable even at sufficiently high fluid velocity.

Based on the aforementioned review, it can be concluded that there is not so much work reported on the vibration of wavy fluid-convening CNTs subjected to multi-physical fields. The purpose of this paper is to study free vibration of embedded fluid-conveying CNTs with a geometrical imperfection under thermal and magnetic fields. The paper is organized as follows: in Section 2 the theoretical formulations, which are based on nonlocal Timoshenko beam theory, are presented. In Section 3, in order to highlight the effects of initial geometrical imperfection, thermal and magnetic fields on the frequencies of embedded fluid-convening CNTs, numerical analysis is carried out and the obtained numerical results are discussed. Concluding remarks are given in Section 4.

## 2. Theoretical formulation

***2.1 Timoshenko beam model***

In this section, a simply supported Timoshenko beam model is constructed to characterise the vibration behavior of fluid-conveying SWCNTs subjected to temperature and magnetic fields. Figure 1 shows the geometry of SWCNTs with length embedded in an elastic medium modelled as a Pasternak foundation with elastic stiffness  and shear stiffness. SWCNTs have an initial geometrical imperfection described by a small rise function [[17](#_ENREF_17), [30](#_ENREF_30)] in the form of, where  is the amplitude of the imperfection. The initial imperfection function  is considered in the same way as in reference [[30](#_ENREF_30)]. In [[30](#_ENREF_30)], rise is defined as the maximum height of the midline of the model from the horizontal  axis and is assumed to be small with respect to the length of the nanotube.This means that the imperfection does not affect the bending moment and the fluid flow, which is the same assumption made in reference [[32](#_ENREF_32)]. The imperfection function only needs to satisfy the boundary conditions of the CNTs. Neglecting the gravity effect of the fluid, the fluid inside the nanotubes is assumed to be an ideal incompressible fluid. It is also assumed that the magnetic permeability of SWCNTs is equal to the magnetic permeability of the surrounding medium.



**Figure 1. Geometry of wavy fluid-conveying SWCNTs.**

Like references [[30](#_ENREF_30), [32](#_ENREF_32)], the imperfection does not affect the bending moment and the fluid flow. Thus, the effect of the initial imperfection on the exerted lateral forces on the nanotube due to fluid flow is not considered in this paper. According to Timoshenko beam theory [[33](#_ENREF_33)], the strains are given by

 (1)

where and are the axial strain (along the -axis) and shear strain, respectively.  is the transverse displacement in terms of the spatial coordinate  and time  at the neutral axis of the beam, and  is the rotation of the cross-section.

The energy principle and the variational approach are used to establish the governing equations of the beam. Hamilton’s principle is

 (2)

where  ,  and  denote the kinetic energy of the nanotube plus the fluid inside, the potential energy of SWCNTs and the work done by external loads, respectively.

The kinetic energy can be written as

 (3)

where is the mass of the fluid per unit length and is the flow velocity.  is the mass of SWCNTs per unit length,  and  represent mass moment of inertia for SWCNTs and fluid, respectively.

The strain energy of SWCNTs is given by

 (4)

where  is the cross sectional area. The normal resultant force, bending moment and transverse shear force are defined as flows

 (5)

The work done by external loads can be written as

 (6)

where  is the induced axial thermal force and  is the magnetic force per unit length.

According to the thermal theory of elasticity,  is expressed for a beam constrained at two ends as [[34](#_ENREF_34)]

 (7)

where is the Poisson’s ratio, is the coefficient of the thermal expansion in the  direction and represents the temperature change over the length of the beam.

For the present dynamic vibration problem, only a steady axial magnetic field is considered. Thus, the magnetic force per unit length can be written as [[35](#_ENREF_35), [36](#_ENREF_36)]

 (8)

where  is the magnetic field permeability and  is the magnetic flux vector component in the  direction. The detailed derivation of Eq.(8) is given in Appendix A.

The equations of motion corresponding to the above kinetic and potential energy are

 (9)

 (10)

The velocity of the fluid is constant in this paper, and the last term in Eq.(9) is zero.

***2.2 Nonlocal Timoshenko beam model***

For an elastic material in one-dimension, the nonlocal constitutive relations reduce to [[37](#_ENREF_37)]

 (11)

where  is a material constant which can be obtained by experiment or through correlation of theoretical results from the nonlocal model with those of an atomic model, and  is an internal characteristic length (e.g., lattice parameter, C-C bond length, granular distance, etc.).

Multiplying Eq. (11) by  and , and then integrating them over the area , the nonlocal Timoshenko constitutive relations are expressed as

 (12-1)

 (12-2)

 (12-3)

where  is the shear correction factor used to compensate for the error due to constant shear stress assumption. The value of  depends on the shape of the cross-section of the tubes.

From Eqs. (9-10) and Eq. (12), the equations of transverse vibration of nonlocal wavy fluid-convening SWCNTs are described by the following equations

(13-1)

 (13-2)

As the velocity of the fluid is constant in this paper, the terms **** and **** in Eq.(13-1) become zeroes, and thus these two terms are neglected in this paper.

The following dimensionless parameters are introduced

 (14)

Substituting the dimensionless parameters in Eq. (14) into Eq. (13), the linear dimensionless equations of wavy fluid-convening SWCNTs are obtained as

 (15-1)

 (15-2)

***2.3 Vibration analysis of wavy SWCNTs***

The Galerkin method is a powerful solution technique for solving differential equations. The Galerkin method of decomposition is used to obtain a set of algebraic equations from the partial differential equations of motion. For small vibration, the transverse deflection is assumed to be [[18](#_ENREF_18)]

 (16)

where  is the amplitude of deflections of the nanotubes and  is the amplitude of the rotational motion due to bending deformation of SWCNTs.  is dimensionless eigenvalue of order .

Substituting Eq. (16) into Eq. (15), then multiplying each side of the above two equations with  and , respectively, and finally integrating the resultant equations over the domain . In the present work, the response of the nanotubes at resonance is considered and the initial geometrical imperfection is described by one mode sinusoidal function. So it suffices to consider the case of , as in reference [[38](#_ENREF_38)].

 (17-1)

 (17-2)

Eq. (17) could be rewritten in a matrix form below:

 (18)

By setting the determinant of the coefficient matrix in Eq. (18) to zero, the characteristic equation is obtained as follows:

 (19)

As reported in [[39](#_ENREF_39)], there would be at most two wave modes for the vibration of single-Timoshenko-beam model and the higher frequency is the shear frequency while lower frequency is the flexural frequency. The dimensionless frequencies of vibration can be expressed as

 (20)

where ,

Letting, the critical velocity of the fluid can be easily obtained



## 3. Numerical Results and Discussions

In this section, the dynamic response of SWCNTs is studied on the basis of the formulations obtained above with the nonlocal Timoshenko beam model. Some numerical examples are presented. The validity of the suggested model is partly validated by comparing the obtained results with those given in the open literature. Moreover, the effects of the nonlocal parameter, flow velocity and density, temperature changes, axial magnetic field and surrounding elastic medium on the frequencies of SWCNTs are investigated.

The material and geometric properties of SWCNTs are given in Table 1 and Table 2 [[35](#_ENREF_35), [39-41](#_ENREF_39)] and the length of nanotubes is 50nm. The shear modulus is calculated as.As stated in [[35](#_ENREF_35)], a conservative estimate of the nonlocal parameter was  for SWCNTs in a magnetic field. It is thought that the nonlocal parameter range of is still valid. In this manuscript, we also study dynamic behaviour of fluid-conveying SWCNTs and hence the value of the nonlocal parameter is taken to be from 0 to 2.0 nm.

**Table 1. Material and geometry properties Ⅰ**

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**Table 2. Material and geometry properties Ⅱ**

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***3.1 Numerical verifications***

In order to validate the present formulations and the computing programs developed by the authors, a comparison is made with the results of Yoon et *al*. [[15](#_ENREF_15)]. Fig.2 presents this comparison of fundamental flexural frequency of wavy SWCNTs, with real part representing vibration frequency and imaginary part denoting decaying rate.

For the sake of comparison, the effects of initial geometrical imperfection, surrounding elastic medium, temperature change and magnetic flux on the flexural frequency are temporarily neglected. It can be seen that the present model has a very similar tendency to that of Yoon et al.’s model. In the present study, the small scale effect on the model has been considered, so our results are smaller than those by Yoon et *al*.’s model, as the small scale is known to soften nanotubes [[42](#_ENREF_42)]. Though the effect of nonlocal parameter on flexural frequency is not so significant, it should be considered, because the small scale can affect the critical flow velocity, which can be seen in Fig.3 and explained by Eq. (21).

From Fig.2, it can also be observed that the flexural frequency of the present model excluding the small scale effect is greater than the Yoon et *al.*’s result and the decaying rate is showing the opposite trend to that of the vibration frequency. Furthermore, when the flow velocity increases up to 1890 m/s (the critical velocity in Yoon et *al*.’s model) and 2100 m/s (the critical velocity in the present model) excluding the waviness and the small scale effect in Fig.3, both the real part and imaginary part of the flexural frequency are equal to zero for the first mode, which corresponds to a point of bounded neutral stability. Once the velocity of fluid is greater than the critical velocity, the vibration frequency is zero and the decaying rate is positive, which means SWCNTs are in divergence instability. In addition, from Fig.3 it is clear that the small scale effect is significant on short CNTs [[33](#_ENREF_33)] . Meanwhile, the critical velocity is dependent on the nonlocal parameter and waviness, which can be seen in Eq. (21).

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**Figure 2. Comparison of the real and imaginary parts of flexural frequency between the Yoon et .*al.*’s model and the present model.**

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**Figure 3. Effect of nonlocal parameter on the critical flow velocity of wavy fluid-conveying SWCNTs for different model and different values of aspect ratio.**

***3.2 The effect of flow velocity and density***

Firstly, SWCNTs with a nonlocal parameter ****nm are considered. Except the small scale effect, there is no other effect on SWCNTs in this example, as represented by****.

The dimensionless shear and flexural frequencies of wavy fluid-conveying SWCNTs are shown as functions of the amplitude of waviness in Fig.4, with different flow velocity values of 0, 500, 1000 and 2000 m/s. It can be observed that with an increase of the fluid velocity, both frequencies decrease, which reveals that the fluid can make the nanotube more flexible [[24](#_ENREF_24)]. Furthermore, with the increase of the amplitude of waviness, the dimensionless shear and flexural frequencies increase. From Fig. 4.b, the dimensionless flexural frequency is zero at fluid velocity 2000 m/s, which is smaller than the critical velocity 2100 m/s in Fig.2 when the small scale effect is excluded.

Fig.5 shows the variations of the dimensionless shear and flexural frequencies with the fluid velocity, at different waviness ratios. They exhibit a similar tendency to the results in Fig. 4 that the fluid can ‘soften’ the nano-pipe. However, the waviness has the opposite effect. In addition, the waviness has a significant influence on dimensionless frequency at lower fluid velocity, which can be found in the inset of Fig.4.

Fig.6 shows the effect of fluid density  on the frequency characteristics.**** is taken. It is very interesting to see that the fluid density has a strong effect on the dimensionless shear frequency. As the fluid density increases, the shear and flexural frequencies decrease. Meanwhile, the critical flow speed is dependent on the fluid density, which can be inferred from Eq. (21).

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**Figure 4. Variation of shear and flexural frequencies of wavy SWCNTs as a function of amplitude of waviness for different fluid velocity: (a) Dimensionless shear frequency; (b) Dimensionless flexural frequency**

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**Figure 5. Fundamental frequencies of wavy fluid-conveying SWCNTs as a function of flow velocity for different waviness ratios: (a) Dimensionless shear frequency; (b) Dimensionless flexural frequency**

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**Figure 6.** **Variation of shear and flexural frequencies of wavy SWCNTs as a function of amplitude of waviness for fluid density: (a) Dimensionless shear frequency; (b)Dimensionless flexural frequency**

***3.3 The thermal effect on the vibration of fluid-conveying SWCNTs***

Fig.7 shows the effect of temperature change on vibration response of wavy SWCNTs conveying fluid at 1000 m/s. In this section, the coefficient of thermal expansion is chosen as  to simulate the room temperature case. **** , ****and ****nm are taken. It can be observed that with an increase of temperature, the dimensionless frequencies increase, which have a similar tendency to those reported in [[43](#_ENREF_43)]. Excluding the results without temperature change, the results of dimensionless shear and flexural frequencies are greater than those in Fig.4 and Fig.5. The critical flow velocity increases as temperature increases.

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**Figure 7.** **Variation of shear and flexural frequencies of wavy SWCNTs as a function of amplitude of waviness for different temperature changes in room temperature: (a) Dimensionless shear frequency; (b) Dimensionless flexural frequency**

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**Figure 8.** **Variation of shear and flexural frequencies of wavy SWCNTs as a function of amplitude of waviness for different temperature changes in high temperature: (a) Dimensionless shear frequency; (b) Dimensionless flexural frequency**

Fig.8 presents the effect of temperature change on vibration response of wavy SWCNTs conveying fluid at 1000 m/s at high temperature. In other words, thermal expansion  is taken. From Fig.8, it is noted that at the same fluid velocity, with an increase of temperature ****, both the shear frequency and the flexural frequency increase, which is opposite to the tendency of Fig.7. A similar phenomenon was reported in [[24](#_ENREF_24)]. Meanwhile, the critical flow velocity increases as temperature increases.

***3.4 Influence of the longitudinal magnetic field***

Another interesting study is carried out to investigate the influence of the longitudinal magnetic field on frequency characteristics of fluid-conveying SWCNTs with waviness ratio, temperature, nonlocal parameter, the coefficient of thermal expansion and fluid velocity****, **** K, ****nm,  and 1000 m/s. From Fig.9, it can be found that the natural frequency is significantly influenced by the longitudinal magnetic field. As the longitudinal magnetic field flux increases, the frequencies increase. Meanwhile, the critical flow velocity is much influenced by the longitudinal magnetic field too. Thus, adding a longitudinal magnetic flux is a simple method to keep the fluid-conveying wavy SWCNTs stable. In comparison, Yoon et al. [[15](#_ENREF_15)] discussed the effect of the surrounding elastic medium on the vibration of fluid-conveying CNTs and they found that the critical flow velocity increased when embedding CNTs in an elastic medium.

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**Figure 9.** **Variation of shear and flexural frequency of wavy SWCNTs as a function of amplitude of waviness for different values of magnetic field: (a) Dimensionless shear frequency; (b) Dimensionless flexural frequency**

***3.5 The effect of Pasternak-type foundation***

As mentioned in [[15](#_ENREF_15)], the surrounding elastic medium (such as polymer matrix) can significantly reduce the effect of internal moving fluid on vibration frequencies, and suppress or eliminate structural instability within the practical range of flow velocity. In this subsection, the influence of the Pasternak-type foundation on the dynamic behaviour is studied. The Pasternak-type foundation [[44](#_ENREF_44)] simulates the interaction between the surrounding medium and fluid-conveyingSWCNTs by two different parameters which describe resistance not only to the normal pressure  but also the transverse shear stress , respectively.

Fig.10 depicts the effect of elastic stiffness  on the dynamic behaviour. ****,**** nm, ,  and  A/m, and flow speed of 1000 m/s are taken. The shear stiffness  is temporarily neglected. It can be found that the dimensionless frequencies increase with the increase of elastic stiffness values, which is evidence that the elastic medium makes the nano-pipe stiffer [[45](#_ENREF_45)] .

Fig.11 examines the effect of the shear stiffness of the surrounding medium on the frequency characteristic. It can be clearly seen that the shear resistance of the elastic medium plays an important role on the dynamic behaviour [[45](#_ENREF_45)] .

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**Figure 10.** **Effect of elastic stiffness on the fundamental frequency of wavy fluid-conveying SWCNTs: (a) Dimensionless shear frequency; (b) Dimensionless flexural frequency**

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**Figure 11.** **Effect of shear stiffness on the fundamental frequency of wavy fluid-conveying SWCNTs: (a) Dimensionless shear frequency; (b) Dimensionless flexural frequency**

## 4. Conclusions

In this paper, the influence of waviness on frequencies of fluid-conveying SWCNTs is investigated. The nonlocal Timoshenko beam model has been introduced to analyse the effect of multiple physical fields on the free vibration of wavy SWCNTs. The main aim is to improve the accuracy of theoretical models of fluid-conveyingSWCNTs and the main contributions are determination of the influence of the small scale effect, temperature change, axial magnetic field and the surrounding elastic medium on the frequency characteristics of fluid-conveying wavy SWCNTs. The effectiveness of the present model has been validated against results from the open literature. The effects of waviness, temperature change, magnetic flux and surrounding elastic medium are analysed on several numerical examples. The results can reveal that with the increase of the amplitude of waviness, the shear and flexural frequencies increase. Moreover, the fluid flowing inside the nanotubes can make the tubes more flexible. The frequencies and critical flow velocity are much influenced by temperature change, magnetic flux and Pasternak-type foundation, which can make fluid-conveying wavy SWCNTs stiffer.

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**Appendix A: the detailed derivation of the Lorentz force in z direction**

Like in references [35,36], for the present dynamic vibration problem, only a steady axial magnetic field is considered. For connivance, the Lorentz force in the z direction is written as [35,36],

 (8)

The detailed derivation of Eq.(8) is given below following [35,36].

Denoting  as current density matrix,  as the disturbing vector, and  as the strength vector of electric field, the Maxwell equations are given by [35,36].

**** (A.1)

**** (A.2)

**** (A.3)

**** (A.4)

**** (A.5)

where **** is the gradient operator. **** are the unit vectors.  is the magnetic field permeability. For simplifying the analysis, a longitudinal magnetic field vector **** is exerted on the carbon nanotubes. The displacement vector is defined as ****, then

**** (A.6)

**** (A.7)

The Lorentz force induced by the longitudinal magnetic field is given as

**** (A.8)

Therefore Lorentz forces along the x, y and z directions are

**** (A.9)

For the present dynamic vibration problem, only a steady axial magnetic field in the z direction is considered. Thus, the magnetic force per unit length can be written as

 (A.10)

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Figure captions

**Figure 1 Geometry of wavy fluid-conveying SWCNTs.**

**Figure 2 Comparison of the real and imaginary parts of flexural frequency between the Yoon et .al.’s model and the present model.**

**Figure 3 Effect of nonlocal parameter on the critical flow velocity of wavy fluid-conveying SWCNTs for different model and different values of aspect ratio.**

**Figure 4 Variation of shear and flexural frequencies of wavy SWCNTs as a function of amplitude of waviness for different fluid velocity: (a) Dimensionless shear frequency; (b) Dimensionless flexural frequency**

**Figure 5 Fundamental frequencies of wavy fluid-conveying SWCNTs as a function of flow velocity for different waviness ratios: (a) Dimensionless shear frequency; (b) Dimensionless flexural frequency**

**Figure 6 Variation of shear and flexural frequencies of wavy SWCNTs as a function of amplitude of waviness for fluid density: (a) Dimensionless shear frequency; (b) Dimensionless flexural frequency**

**Figure 7 Variation of shear and flexural frequencies of wavy SWCNTs as a function of amplitude of waviness for different temperature changes in low temperature: (a) Dimensionless shear frequency; (b) Dimensionless flexural frequency**

**Figure 8. Variation of shear and flexural frequencies of wavy SWCNTs as a function of amplitude of waviness for different temperature changes in high temperature: (a) Dimensionless shear frequency; (b) Dimensionless flexural frequency**

**Figure 9 Variation of shear and flexural frequency of wavy SWCNTs as a function of amplitude of waviness for different values of magnetic field: (a) Dimensionless shear frequency; (b) Dimensionless flexural frequency**

**Figure 10 Effect of elastic stiffness on the fundamental frequency of wavy fluid-conveying SWCNTs : (a) Dimensionless shear frequency; (b) Dimensionless flexural frequency**

**Figure 11 Effect of shear stiffness on the fundamental frequency of wavy fluid-conveying SWCNTs: (a) Dimensionless shear frequency; (b) Dimensionless flexural frequency**

Table captions

**Table 1 Material and geometry properties Ⅰ**

**Table 2 Material and geometry properties Ⅱ**

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