Structural Modifications for Torsional Vibration of Shafting Systems Based on Torsional Receptances

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**Abstracts**：

Torsional vibration of shafts is a very important problem in engineering, in particular in ship engines and aero engines. Due to their high levels of integration and complexity, it is hard to get their accurate structural data or accurate modal data. This lack of data is unhelpful to vibration control in the form of structural modifications. Besides, many parts in shaft systems are not allowed to be modified, such as rotary inertia of a pump or an engine, which is designed for achieving certain functions.

This paper presents a strategy for suppressing torsional vibration of shaft systems in the form of structural modifications based on receptances, which does not need analytical or modal models of the systems under investigation. It only needs the torsional receptances of the system, which can be obtained by testing a simple auxiliary structure attached to relevant locations of the shaft system, and using the finite element model (FEM) of the simple structure. An optimisation problem is constructed to determine the required structural modifications, based on the actual requirements of modal frequencies and mode shapes. A numerical experiment is set up and the influence of several system parameters are analysed. Several scenarios of constraints in practice are considered. The numerical simulation results demonstrate the effectiveness of this method and its feasibility in solving torsional vibration problems in practice.

Key words: Inverse problem, Torsional vibration, Shaft, Measured torsional receptance, Structure modification.

1. **Introduction**

Dynamic performance of structures plays an important role in engineering; however, there are always some circumstances in which structural dynamic performance does not meet the design requirements or actual situations in practice. Therefore, it is common that some existing structures need to be modified in order to acquire desired dynamic performance [[1](#_ENREF_1)]. Many researchers have put forward many methods for the eigenstructure assignment problems [[2-5](#_ENREF_2)]. One major way of doing that is to assign a structure suitable natural frequencies and modes through structural modifications as a typical vibration control strategy, which usually requires knowledge of accurate structural parameters (e.g., mass, stiffness and damping matrices [[6](#_ENREF_6), [7](#_ENREF_7)]) or modal data [[8-10](#_ENREF_8)]. However, in most engineering problems, it is very difficult to gain such knowledge. Usually modal tests must be conducted [[11](#_ENREF_11)] and model updating carried out [[12](#_ENREF_12)], which is expensive and tedious for complicated structures. Moreover, the application of modal data in practice has a number of difficulties, which was discussed in [[13](#_ENREF_13)]. On the other hand, some structural modification methods directly based on the measured system receptances or Frequency Response Functions (FRFs) [[14-17](#_ENREF_14)] overcome those difficulties and provide effective solutions to this kind of problems, which belong to inverse structural dynamics. Structural modifications based on the measured receptance or FRFs were studied in forward analysis for prediction of receptances of the modified structure [[18](#_ENREF_18)], and in inverse analysis for assigning natural frequencies and vibration nodes [[19](#_ENREF_19)], and eigenstructures [[13](#_ENREF_13), [20](#_ENREF_20)].

For rotating machines, one of the most important problems is torsional vibration of shafts. The adverse impact caused by torsional vibration includes: vibration of the whole machine, damage in the transmission system, excessive wear of bearings and gears, and even shaft fracture [[21](#_ENREF_21)]. For shaft structures, numerical models are usually needed to evaluate torsional vibration characteristics in the engineering design stage, but some structural parameters (such as the rotary inertia of the motor, the actual torsional stiffness of gears) cannot be accurately obtained easily. Therefore, there are inevitably considerable discrepancies between the designs and the actual structures based on such imperfect theoretical models. So suppression of torsional vibration is a big challenge. If there is a method not required an accurate theoretical system model in solving the torsional vibration problems and can also achieve structural modifications for the system based on measured data, this method will bring lots of advantageous in practice.

However, it should be noted that, perhaps partly resulting from difficulties in accurately measuring torsional FRFs, inverse structural dynamics problems based on measured rotational receptance nearly have never been applied to rotating machineries before [[22](#_ENREF_22)].

Although many researchers have put forward a number of methods for measuring transfer functions for rotational degrees of freedom (DoFs), for example, Mottershead et al. [[23](#_ENREF_23)] proposed one indirect method based on an T-block for obtaining rotational receptances, the progress in measuring torsional transfer functions in shaft systems is still very limited [[23-27](#_ENREF_23)], and nearly none of these papers are about torsional vibration measurement of shaft structures. Recently Lv et al. [[22](#_ENREF_22)] put forward an indirect method to measure the torsional receptance. The method was implemented by using a T-shaped simple auxiliary structure attached to one end of the shaft system, and the torsional system receptances could be obtained accurately through combining the auxiliary structure’s finite element model (FEM) and test data of the whole structure.

This paper presents a theoretical strategy of structural modifications for suppression of torsional vibration of shaft systems, using ‘measured’ torsional receptances and a structural optimization method. One main advantage of this method proposed in this paper is that it does not need any knowledge of mass and stiffness parameters, or even analytical or modal models of the system under investigation; instead ‘measured’ torsional receptance data are used, which can be obtained from measured translational vibration data obtained through an additional structure. In this paper, structural modifications for suppressing torsional vibration of a simplified model of a ‘real’ rotating machine are studied. Several scenarios of practical constraints are considered. Theoretical results show the effectiveness of this method.

**2. Receptance-based Method**

A shaft system under a harmonic excitation, treated as a general linear discrete conservative dynamic system without damping, can be described by

 (1)

where **J** is the mass (moment of inertia) matrix,  is the acceleration vector, **K** is the stiffness matrix , **x** is the displacement vector, and **f** is the force amplitude vector, e is the , i is the , *ω* is the, *t* is the .

Denote the change in mass matrix and change in stiffness matrix due to structural modifications asandrespectively. Then Eq.(1) becomes

 (2)

It can be assumed that the response is harmonic in the form of , which u is the . Substituting it into Eq. (2) yields

 (3)

The original system FRF matrix is defined as . Eq. (3) can then be re-written as

 (4)

For the eigenvalue problem, it is assumed that the desired natural frequency and mode are respectively and, then the following equation is derived:

 (5)

In order to obtain the FRFs of the unknown shaft system, a simple auxiliary structure needs to be attached to the shaft at one end, as shown in Fig.1.

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Fig.1. The schematic of the auxiliary structure, and the shaft structure under study (not in scale)

Next, the T-shaped simple auxiliary structure is divided into two parts to be considered. One part is the very short OD structure whose torsional vibration about the z-axis (not including beam AOB) in only considered, the other part is beam AOB whose bending vibration is only considered. A good FE model of auxiliary structure must be established, which is fairly easy, given its simple geometric shape. The whole derivation details are given in Lv et al.’s paper [[22](#_ENREF_22)].

Firstly, the force analysis for torsional vibration has been done on the OD structure, and the shaft system receptances at point D had been given in reference [22] by:

 (6)

Next step is to consider only the bending vibration of the AOB beam. By taking *n* times average of the test results of two different loading conditions (from points A and B), a estimation formula of  could be described by

 (7)

where,

 ,  ,

and matrix **G** are auto-spectrum or cross spectrum of the excitation and response. Taking the sub-matrix  as an example, it could be obtained by the following formula:

  (8)

Where x is

By substituting  into Eq.(6), then the needed torsional receptances is found. Therefore, Eq.(5) can be rewritten as

 (9)

For the sake of convenience in presenting the method and without losing generality, it is assumed that the structural modifications are to be made to the last few DoFs. Then **M** and **K** have only non-zero elements in those rows and columns corresponding to the modified DoFs. As a result,

 (10)

where *x* denotes non-zero terms, which correspond to the modified DoFs and involve the products of the modifications and modal displacements at only those modified DoFs.

Consequently, the *i*th row of Eq. (9) can be written as

 (11)

where *j* denotes the modified (last few) DoFs.

Equation (11) is worth a close examination. If certain *i* modal displacements of a mode are to be assigned certain values, equation (1) reveals that only the **H** elements in those *i* rows and *j* columns are required, which means only *i*×*j* number of **H** elements, which can be a very small number even for a very complicated structure. *j* should be those locations where structural modifications are allowed and easy to do, while *i* should be either some interesting locations of the structure where the modal displacements need to take certain specified values, or any convenient DoFs if there are not particular interesting locations. For different modes, assigned modal displacements could even be at different locations. It should be pointed out that for certain assigned frequencies and associated modes, there is no guarantee that there is a solution or a unique solution.

The eigenstructure assignment problem can finally be cast as

 (12)

where  is the weighting coefficient (a positive scalar).

Eq.(12) can be solved by optimisation algorithms, which have already been applied in various fields[[28](#_ENREF_28)]. It should be noted that the focus of this paper is to establish a novel strategy for reducing torsional vibration of shaft structures through structural modifications and show its effectiveness. Hence, the algorithms for solving this optimisation problem will not be studied in this paper.

The proposed receptance-based strategy does not require information of the mass or stiffness matrices of the original system, and it only needs a few receptances that can be measured relatively easily, for example, by using the method presented in [[22](#_ENREF_22)]. This method is particularly suitable in dealing with those shaft systems that suffer from torsional vibration problems in which mass (moment of inertia) and stiffness matrices are difficult to measure.

1. **Numerical experiment**

The shaft modification method proposed in this paper is based on ‘measured’ torsional receptances. During the numerical simulation stage, a shaft system is chosen as an example of the numerical experiments to verify the feasibility of method proposed. The require data, torsional system receptances of the shaft, are obtained directly from the system parameters in the simulation.

3.1 Model setup

The model reflects a ship propulsion shafting, including air compressors, pumps, cylinders, flywheels, couplings, propeller and other components in series, and the whole structure is shown in Figure.2.

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Fig.2. Schematic diagram of the simulated 12-dof system.

According to the structure and properties of materials, the model is simplified to get each structural inertia and torsional stiffness, as shown in Table 1. The natural frequencies of the original system are given in Table 2, and the first and the third modes are shown in Fig. 3.

For the numerical application of the proposed method, the system FRFs are acquired by solving matrixat frequencies of desired modes  and (desired modes frequencies are two safe-frequencies, 30 and 90 Hz respectively, and the reason of choosing the values will be explained in next subsection). The 2nd natural frequency  does not need to be considered, which does not have high risks of resonance when the shaft working in the rated rotation speed.

**Table 1.** System parameters

|  |  |  |  |
| --- | --- | --- | --- |
| Parameter | Value [kg·m2] | Parameter | Value [105 N·m] |
| *J*1\* | 5 | *k*12\* | 10 |
| *J*2\* | 2 | *k*23\* | 200 |
| *J*3\* | 1 | *k*34 | 150 |
| *J*4 | 3 | *k*45 | 100 |
| *J*5 | 3 | *k*56 | 100 |
| *J*6 | 3 | *k*67 | 100 |
| *J*7 | 3 | *k*78 | 100 |
| *J*8 | 3 | *k*89 | 200 |
| *J*9 | 40 | *k*9 10 | 7 |
| *J*10 | 5.5 | *k*10 11 | 4 |
| *J*11 | 3.5 | *k*11 12 | 50 |
| *J*12 | 8 |  |  |

Where the superscript \* means this parameter is an [equivalent](javascript:void(0);) value after considering the influence of gears ratio.

**Table 2. Original system modes**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Mode number | 1 | 2 | 3 | 4 | 5 |
| *f* [Hz] | 24.96 | 57.52 | 74.65 | 108.25 | 232.69 |

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Fig.3. Original system 1st and 3rd order modes shapes

3.2 Modification goals

It is assumed that the rated speed of the shaft system is 1500 r/min, so that 25Hz is the main shafting vibration excitation frequency. Moreover, excitations from subharmonics and superhamonics of the rated rotation speed (0.5, 1 and 2 times of 25 Hz), shown by red dotted lines in Fig.4, are also possible. In addition, for the 3-blade propeller, another main excitation frequency is three times of the 25Hz. It is important to ensure that the natural frequencies of the structure do not coincide with the excitation frequencies[[29](#_ENREF_29)]. So there are at least two must-avoid frequencies, 25Hz and 75Hz, shown with thicker red dotted lines in Fig.4.

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Fig.4. FRF with desired frequencies.

During its run-up, the shaft system needs to go through a speed-increasing process. If some natural frequencies are lower than the rotation frequency, then these frequencies will be excited, which may lead to the resonance of the shaft and bring about some serious damage.

Therefore, considering both points above, the first natural frequency of the shaft is preferably greater than 25Hz. Therefore, the modification goal is to shift the 1st and 3rd natural frequencies to 30Hz and 90Hz respectively, as shown by the green dashed lines in Fig.4, which aims to move the system away from these potentially damaging frequencies to two ‘safe’ frequencies. In addition, 2nd natural frequency (57.52 Hz) is not located around any [harmonic](javascript:void(0);) [frequencies](javascript:void(0);) of the basic frequency 25 Hz, so it is not necessary to be modified. In this example, because there are no special requirements on mode shapes, for simplicity, the mode shapes, as shown in Fig.3, remain unchanged.

Another main reason using the original modes as the desired modes is that, after moving the natural frequencies to a safe region, vibration response becomes much reduced and mode shapes are no longer a concern. In the event that certain modal displacement should take some desirable values, the method presented in this paper is equally applicable. This will be demonstrated by an example.

In addition, the method proposed in this paper especially suits for assigning only a few modal displacements of a mode; on the other hand, a whole mode also could be assigned if receptances at all these DoFs are available, which means that more test data are required. The method proposed in this paper theoretically is fully capable of assigning any frequencies and modes for vibration reduction, which will be proved by another example.

**4. Results**

In comparison with other complicated structures (such as engines, air compressors, pumps and so on), couplings are easy to be replaced in shaft systems and there are many types. In another word, for a complicated equipment (for example, an engine), once the type has been decided upon in the design stage of a shaft system, it would be very difficult or unrealistic to modify any inertia or stiffness of it. Thus, in this paper, the chosen specific strategy is to modify the coupling, which contains two inertias and one torsional stiffness between them[[30](#_ENREF_30)].

In this paper, the optimization problem is solved by a genetic algorithm. Other effective optimisation algorithms can also be used.

4.1 Assignment of frequencies and part of modes

As the method proposed in section 2, aiming at only considering 10th and 11th DoFs, the modified parameters are *J*10,  *J*11,  *k*10 11. In other words, the subscripts *i* and *j* in Eq.(11) are equal to either 10 and 11, and the **H** elements only in those *i* rows and *j* columns are required. For convenience of mathematical treatment and computer coding, these two DoFs are moved to the last two elements of **v** (see Eq.(10)). But for the description of the process of using this method, there is no need to do so.

For this optimisation problem, the modification bounds and desired values of modal displacements are listed in Table 3.

**Table 3****.** Modification bounds

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Parameter | | Lower bound | | Upper bound | |
| Moment of inertia  [kg·m2] | *J*10 | | -0.5 | | 0 | |
| *J*11 | | -1 | | 0 | |
| Stiffness  [kN·m] | *k*10 11 | | 0 | | 500 | |
|  | | Desired mode number, *h* | | *uh*,10 | | *uh*,11 |
| desired modal displacements | | 1 | | -0.4765 | | -0.9432 |
| 3 | | 1 | | -0.1513 |

After solving the optimisation problem, a solution for structural modifications in the form of a group of two moments of inertia and one stiffness are obtained, as shown in Table 4, and the system FRF, *H*11 are shown in Fig. 5, respectively. The difference between the desired natural frequencies and obtained frequencies is shown as | *fh−fi* | in Table 4, and the difference between desired**u***h* and obtained**u***i* are also listed in the last two lines.

**Table 4.** Obtained frequencies and required structural modifications

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | | Parameter | Initial value | Obtained Value |
| Moment of inertia [kg·m2] |  | | *J*10 | 5.5 | 5.15 |
|  | | *J*11 | 3.5 | 2.65 |
| Stiffness [kN·m] |  | | *k*10 11 | 400 | 800.01 |
|  | Desired mode number, *h* | | Goal | Proposed method | Error |
| | *fh*−*fi*| [Hz] | 1 | | 30 | 30.01 | 0.01 |
|  | 3 | | 90 | 90.02 | 0.02 |
| **u***h*−**u***i* | 1 | *u*1,10 | -0.4765 | -0.4766 | 0.0001 |
| *u*1,11 | -0.9432 | -0.9431 | -0.0001 |
| 3 | *u*3,10 | 1 | 1 | 0 |
| *u*3,11 | -0.1513 | -0.1512 | -0.0001 |

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Fig.5. Frequency response comparison

It can clearly be seen from the results that the performance of the method proposed is excellent for achieving the desired modal behaviour for this shaft system: the 1st and 3rd natural frequencies of the original shaft system are now shifted to 30Hz and 90Hz and four modal displacements are assigned. What is impressive is that only 4 ‘experimental’ data of receptance are used in this example, which means several benefits for real engineering applications.

4.2 Assignment of whole modes

If a whole **u***h*needs to be assigned, the receptance-based structural modification method is also applicable. In this case, a whole column of matrix **H** in Eq.(11) is required, that is, in the same size as vector **u***h*. This means that more receptance data would be needed, which would bring a large amount of measurement work in practice. However, in this paper, this is not an issue in this section, as the purpose is to demonstrate its power in assigning whole modes.

The whole modal shapes are calculated by using obtained system parameters in section 4.1, taken as the desired modes in this section to ensure that the optimisation problem has at least one group of solution. The modification bounds and desired modes are listed in Table 5.

**Table 5.** Modification bounds

|  |  |  |  |
| --- | --- | --- | --- |
|  | Parameter | Lower bound | Upper bound |
| Moment of inertia  [kg·m2] | *J*10 | -1 | 0 |
| *J*11 | -1 | 0 |
| Stiffness  [kN·m] | *k*10 11 | 0 | 500 |

The solution for structural modifications obtained from the optimization method, as shown in Table 6, and receptance H11, and the mode shapes of the modified system are shown in Fig. 6(a) and Fig. 6(b), respectively. The difference between the desired natural frequencies and obtained frequencies is shown as | *fh−fi* | in Table 6. It can clearly be seen from the results that the performance of the method proposed is excellent for achieving the desired modal behaviour for this shaft system: the 1st and 3rd natural frequencies of the original shaft system are now shifted to 30Hz and 90Hz; and the obtained mode shapes are nearly the same as the desired ones. It proves that the proposed method is also capable of assigning whole mode shapes through modifications at only a few DoFs.

**Table 6.** Obtained frequencies and required structural modifications

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | Parameter | Initial value | Obtained Value |
| Moment of inertia [kg·m2] |  | *J*10 | 5.5 | 5.16 |
|  | *J*11 | 3.5 | 2.60 |
| Stiffness [kN·m] |  | *k*10 11 | 400 | 800.00 |
|  | Desired mode number, *h* | Goal | Proposed method | Error |
| | *fh*−*fi*| [Hz] | 1 | 30 | 30.05 | 0.05 |
|  | 3 | 90 | 89.95 | 0.05 |

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Fig.6. (a) Frequency response comparison, (b) Mode shape comparison

It should be pointed out that, for the proposed method, there is a possibility that an optimal solution under certain constraints (such as the bounds of certain system parameters) may not exist. On the other hand, it is also likely to get multiple optimal solutions in some cases [[13](#_ENREF_13), [20](#_ENREF_20)]. The latter means that, if choose different desired shape values in the needed DoFs in mode vector **u**h, the proposed strategy may provide a variety of good modification schemes, which may bring about significant advantages in practical applications.

**5. Conclusions**

For rotating machines, torsional vibration of shafts is one of the most important problems. One major barrier in vibration reduction is the difficulty in accurately obtaining structural parameters of components (moment of inertia and torsional stiffness) or system FRFs data for a shaft system in practice. A powerful method for vibration reduction is structural modifications based on measured receptances. One recently proposed method provides an indirect way of measuring torsional receptances of shaft systems and provides the required data for receptance-based structural modifications.

This paper presents a strategy of structural modifications for suppressing torsional vibration of shaft systems through assigning desired frequencies and modes, based on a recently proposed method of measuring torsional receptances and structural optimization. It needs only a few receptances that can be measured relatively easily, whose number is equal to the number of assigned modal displacements. In this paper, the assignment is formulated as a structural optimisation problem and a numerical experiment is conducted. Since some parameters of some complicated components of shaft systems (e.g. the engine, compressor, pumps, etc.) commonly are not allowed to be modified in practice, the chosen strategy is to modify the coupling (which contains two inertia and one torsional stiffness in between), which is easy to be replaced in shaft systems and there are many types. Under certain reasonable bounds, the numerical simulation leads to two sets of good results: the 1st and 3rd natural frequencies (25Hz and 75Hz) of the original system are both accurately shifted to two ‘safe’ frequencies (30Hz and 90Hz) respectively; two modal displacements of the corresponding modes and the wholes of the corresponding modes are also accurately assigned It is thus proved that the proposed method is excellent for achieving the desired modal behaviour and provides a valid solution to suppress torsional vibration for shaft systems in engineering.

**Competing Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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