Adequacy, Fairness and Sustainability of Pay-As-You-Go-Pension-Systems: Defined Benefit versus Defined Contribution

Jennifer Alonso-García*
Maria del Carmen Boado-Penas†
Pierre Devolder‡

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Abstract

There are three main challenges facing pay-as-you-go public pension systems. First, pension systems need to provide an adequate income for pensioners in the retirement phase. Second, participants wish a fair level of benefits in relation to the contributions paid. Last but not least, the pension system needs to be financially sustainable in the long run. In this paper, we jointly analyse the adequacy, fairness and sustainability of defined benefit and defined contribution schemes. Also, risk sharing mechanisms, that involve changes in the key variables of the system, are designed to restore the financial sustainability while we study their consequences on the adequacy and fairness of the system.

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Keywords: Adequacy, Fairness, Pay-as-you-go, Public pensions, Risks, Sustainability.

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*(Corresponding author). ARC Centre of Excellence in Population Ageing Research (CEPAR), UNSW Sydney, Level 3, East Wing, 223 Anzac Parade, Kensington NSW 2033, Australia; School of Risk and Actuarial Studies, UNSW Sydney, Australia. Email: j.alonsogarcia@unsw.edu.au

†Institute for Financial and Actuarial Mathematics (IFAM), University of Liverpool, Mathematical Sciences Building, Liverpool, L69 7ZL, United Kingdom. Email: Carmen.Boado@liverpool.ac.uk

‡Institute of Statistics, Biostatistics and Actuarial Sciences (ISBA), Université Catholique de Louvain, Voie du Roman Pays, 20 bte L1.04.01 B-1348-Louvain-la-Neuve, Belgium. Email: pierre.devolder@uclouvain.be.

1 Introduction

Public pension systems are usually financed on a pay-as-you-go (PAYG) basis where pensions for retirees are paid by the contributions of the working-age population. About half of OECD countries have taken measures to improve the financial sustainability of their pension systems over the past few years (OECD 2015). The main objective of recent reforms is to delay retirement by gradually increasing the statutory retirement age from 64 in 2014 to 65.5 by 2060 on average in the OECD based on current legislation. Some other European countries have made some structural reforms by changing the formula to calculate the initial pension from a Defined Benefit (DB) to a Defined Contribution (DC) with the aim of controlling the expenditure on pensions (Whitehouse 2012).

DC unfunded pension systems (also called Notional Defined Contribution accounts or Nonfinancial Defined Contribution-NDC schemes) have some positive features, such as facing the population ageing more or less automatically or improving the relationship between contributions and pensions paid (Palmer 2006). However these schemes do not guarantee sustainability due to the PAYG nature of the system (Valdés-Prieto 2000; Palmer 2013; Alonso-García et al. 2017). In this line, Auerbach and Lee (2006), Auerbach and Lee (2011) and Auerbach et al. (2013) study numerically the fiscal sustainability of NDCs, their performance in regards of risk-spreading among generations and how economic and demographic shocks are spread among different generations.

Several papers study the sustainability of PAYG pension systems and how they react to changes in fertility and ageing. For instance, Bovenberg (2008), Cigno (2007), Sinn (2007) state that the combined effect of fewer births and longer lives is putting public pension finances under strain. Consequently, either the contribution rate is increased or the pensions relative to wages are reduced. However, Fanti and Gori (2012), through a two-period general equilibrium overlapping generations closed economy, show that a fertility drop does not necessarily cause financial problems in the pension system. In fact, the fewer young workers would also need less income to support their children, thus favouring the rise in PAYG pensions. Similarly, Cipriani (2014) find that population ageing may be a problem as soon as one introduces longevity in the baseline model since an increase in longevity has a negative effect on fertility and pension benefits.

The World bank report (Holzmann et al. 2008) suggests evaluating potential modalities for pension systems by applying a multi-pillar approach to potential reform designs . The authors propose to evaluate the different pension schemes against a set of primary and secondary evaluation criteria. The primary criteria are the ability of the scheme to maintain adequacy, affordability, sustainability, equitability, predictability and robustness, whereas the secondary criteria evaluate the system's contribution to output and growth. However, the definitions related to both criteria are quite broad.

A possible approach to control the public pension expenditure is to introduce a contingency fund in order to absorb unexpected events that might affect the liquidity of the PAYG pension system (Haberman and Zimbidis 2002; Pantelous and Zimbidis 2008; Gannon et al. 2016; Godínez-Olivares et al. 2016a,b). The aim of this non-zero buffer fund is to fluctuate in the short run and absorb partially or completely the uncertainty in mortality, fertility rate and other events. Similarly, Gannon et al. (2016) and Godínez-Olivares et al. (2016b) define this fund as the inter-temporal budget balance that equates expected future expenditures to expected future income from contributions.

For policymakers, a desirable pension system should be financially sustainable but at the same time should also provide an adequate income for pensioners in the retirement phase (adequacy), a fair level of benefits in relation to the contributions paid (actuarial fairness)[1]. There is an obvious trade-off between improving financial sustainability and pension adequacy at the same time, as increases in pension benefits (adequacy) deteriorate pension finances (sustainability).

With the aim of restoring the sustainability of the pension system some countries have incorporated automatic balance mechanisms (ABMs). These mechanisms can be defined as a set of pre-determined measures established by law to be applied immediately according to an indicator of the financial health of the system (Vidal-Meliá et al. 2009, 2010). The adjustments can be made on benefit levels, revaluation of contribution bases or indexation of pensions in payment (D'Addio and Whitehouse 2012; Godínez-Olivares et al. 2016a).

In practice, countries like Sweden, Canada, Germany and Japan, among others, have a combination of risk-sharing mechanisms (RSMs) that affect to both contributors and pensioners of state pension systems. In particular, in Sweden and Japan, an asymmetric[2] mechanism is applied to both the contribution bases and indexation of pensions while Canada and Germany adjust both contribution rate and indexation of pensions (Börsch-Supan et al. 2004; Vidal-Meliá et al. 2009). However, to the best of the authors' knowledge, these RSMs in place do not have any theoretical basis.

This paper designs, from a theoretical point of view, flexible and tractable RSMs that involve changes in the contribution rate and/or indexation of pensions to restore the sustainability of the DB and DC pension schemes. At the same time, we aim to shed some light on the consequences of such mechanisms on the adequacy and fairness of both schemes. This research will certainly contribute to the debate on pension finance in the sense that, for the very first time according to the authors' knowledge, sustainability and its effect on adequacy and fairness are studied under a dynamic set-up.

The remainder of the article is structured as follows. Section 2 describes the dynamic overlapping generation model and shows how sustainability, adequacy and fairness are measured. Section 3 develops a flexible RSM which restores the sustainability into the pension scheme. Section 4 provides an illustration of the impact of various RSMs on adequacy, actuarial fairness and sustainability. Section 5 and two appendices conclude.

2 A dynamic overlapping generation model

This section describes the demographic-economic structure of the system, the calculation of the sustainability indicator and finally the expressions to compute adequacy and fairness in DB and DC schemes. We also show the sustainability, fairness and adequacy for the steady state case and show that, under a set of pre-specified assumptions, the DB and DC schemes have the same equilibrium structure.

2.1 Population and salary dynamics

The demographic-economic structure at any time t is represented as follows:

Age:

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[1] See Queisser and Whitehouse (2006) for more details on actuarial fairness, also denoted as 'benefit to cost ratio'.

[2] The asymmetric mechanisms are designed to face adverse demographic and economic changes. On the contrary, the symmetric mechanism, Alho et al. (2013), is adjusted for both positive and negative deviation of the financial health of the system.

$$x = \overbrace{x_0, x_0+\Delta, ...x_r(t)-2\Delta, x_r(t)-\Delta,}^{\text{Contributors' ages}} \underbrace{x_r(t), x_r(t)+\Delta, ..., \omega-2\Delta, \omega-\Delta}_{\text{Pensioners' ages}},$$

where

x_0 is the fixed entry age,

Δ is the period considered (e.g. $\Delta = 1$ and $\Delta = 0.25$ represent a yearly or quarterly periodicity respectively),

$x_r(t)$ is the time-dependent retirement age,

and ω is the maximum lifespan.

Population at time t:

$$N_t^x = N_{t-x+x_0}^{x_0} \cdot {}_{x-x_0}p_{x_0}(t-x+x_0), \tag{2.1}$$

where

N_t^x denotes the population aged x for $x \in [x_0+\Delta, \omega-\Delta]$ who are alive at time $t > 0$ and joined the labour market at time $t-x+x_0$. We assume that individuals only enter the system at the age of x_0. In particular, the entries at age x_0 vary at the rate n_t between $t-\Delta$ and t as follows:

$$N_t^{x_0} = N_{t-\Delta}^{x_0}\left(1+n_t\right). \tag{2.2}$$

Finally, ${}_{x-x_0}p_{x_0}(t-x+x_0)$ is the probability of an individual to survive to age x by time t conditional on being alive at age x_0 at time $t-x+x_0$.

Formula 2.1 indicates that the population at time t only relies upon the survival of individuals, implying that the population is closed to migration[3].

Individual salaries at time t

$$W_t^x = W_{t-\Delta}^x\left(1+g_t\right), \tag{2.3}$$

where

W_t^x denotes average individual salaries for x =x_0,...,$x_r(t)-\Delta$ at time $t > 0$ which are earned by the active population and are assumed to be paid at the beginning of the calendar year. Salaries are age and time dependent in line with empirical evidence[4],

and g_t is the rate of salary variation from the period $t-\Delta$ to period t.

The retirement age is represented by $x_r(t)$ and is assumed to be linked to life expectancy (Knell 2012; Chłón-Domińczak et al. 2012; OECD 2015; Tyrowicz et al. 2016). The retirement age increases on a Δ period basis.

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[3] Following Settergren and Mikula (2005) and OECD (2015) we do not consider migration in our analysis. However, in practice migration plays an important role in the population dynamics of most European countries (Eurostat 2011, 2012).

[4] Some countries exhibit an inverted U-shaped wage path that peaks in middle age and declines smoothly thereafter (Blanchflower and Oswald 1990; Groot et al. 1992; Sessions 1993).

2.2 Liquidity (Sustainability) indicator

We measure the sustainability of the system in terms of the liquidity indicator that compares the income from contributions, together with financial assets, and pension expenditures in one particular year[5] Our scheme can be considered sustainable in the long run since liquidity is ensured on an annual basis. Formally, the ratio at time t, LR_t is represented as follows:

$$LR_t = \frac{C_t + F_t^-}{P_t}, \tag{2.4}$$

where

C_t represents the income from contributions at time t,

P_t represents the total pension expenditures at time t,

and F_t^- represents the value of the (buffer) fund at time t, also called reserve fund, before new contributions and benefits payments are considered. The value of the fund at time t after contributions and payments is given by

$$F_t^+ = F_t^- + C_t - P_t. \tag{2.5}$$

The buffer fund at time t, F_t^-, for an initial buffer fund F_0^-, can be rewritten using Equation (2.5) as follows:

$$\begin{aligned} F_t^- &= F_{t-\Delta}^+(1+i_t) = \left(F_{t-\Delta}^- + C_{t-\Delta} - P_{t-\Delta}\right)(1+i_t) \\ &= F_0^- \prod_{j=\Delta}^{t}(1+i_j) + \sum_{j=0}^{t-\Delta}(C_j - P_j) \prod_{k=j+\Delta}^{t}(1+i_k) \end{aligned} \tag{2.6}$$

where

i_t represents the financial rate of return of the fund from period $t - \Delta$ to t.

2.2.1 Income from contributions

The income from contributions received by the pension system at time t, C_t, is represented as follows:

$$C_t = \pi_t \sum_{x=x_0}^{x_r(t)-\Delta} W_t^x N_t^x \tag{2.7}$$

$$= \begin{cases} \pi_t \sum_{x=x_0}^{x_r(t)-\Delta} W_t^x N_t^x & \text{if } x_r(t) = x_r(t-\Delta), \\ \pi_t \sum_{x=x_0}^{x_r(t-\Delta)-\Delta} W_t^x N_t^x + \pi_t W_t^{x_r(t-\Delta)} N_t^{x_r(t-\Delta)} & \text{if } x_r(t) \neq x_r(t-\Delta), \end{cases} \tag{2.8}$$

where

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[5] The concept of one-period liquidity has been widely used in the literature in Haberman and Zimbidis (2002); Godínez-Olivares et al. (2016a); Alonso-García and Devolder (2016).

π_t is the contribution rate at time t. The expression (2.8) represents the income from contributions when the retirement age changes between two periods. If the retirement age increases at time t the cohort who at time $t-\Delta$ was expected to retire at age $x_r(t-\Delta)$ will contribute during one additional period instead of starting to receive pension payments. This evolution is highlighted in Remark 1.

Remark 1. The income from contributions (2.7) can be rewritten to highlight the effect of mortality and variations in the contribution rate as follows:

$$
\begin{aligned}
C_t =& \pi_t W_t^{x_0} N_t^{x_0} + \pi_t \sum_{x=x_0+\Delta}^{x_r(t-\Delta)-\Delta} W_t^x N_t^x + \left(\pi_t W_t^{x_r(t-\Delta)} N_t^{x_r(t-\Delta)}\right) \cdot 1_{x_r(t)\neq x_r(t-\Delta)} \\
=& \pi_t W_t^{x_0} N_t^{x_0} + \pi_t \sum_{x=x_0+\Delta}^{x_r(t-\Delta)-\Delta} W_{t-\Delta}^{x-\Delta} \left(1+g_t\right) N_{t-\Delta}^{x-\Delta} \cdot \underbrace{{}_{\Delta}p_{x-\Delta}(t-\Delta)}_{1-{}_{\Delta}q_{x-\Delta}(t-\Delta)} \\
&+ \left(\pi_t W_t^{x_r(t-\Delta)} N_t^{x_r(t-\Delta)}\right) \cdot 1_{x_r(t)\neq x_r(t-\Delta)} \\
=& \pi_t W_t^{x_0} N_t^{x_0} + \left(\pi_t W_t^{x_r(t-\Delta)} N_t^{x_r(t-\Delta)}\right) \cdot 1_{x_r(t)\neq x_r(t-\Delta)} + \frac{\pi_t}{\pi_{t-\Delta}} C_{t-\Delta}\left(1+g_t\right) \\
&- \pi_t \sum_{x=x_0+\Delta}^{x_r(t-\Delta)-\Delta} W_t^x N_{t-\Delta}^{x-\Delta} \cdot {}_{\Delta}q_{x-\Delta}(t-\Delta) - \pi_t W_t^{x_r(t-\Delta)-\Delta} N_{t-\Delta}^{x_r(t-\Delta)-\Delta}
\end{aligned}
$$

where

${}_{\Delta}q_{x-\Delta}(t-\Delta)$ is the mortality rate of individuals aged $x-\Delta$ at time $t-\Delta$ before attaining age x by time t.

The income from contributions at time t varies with the age-independent salaries' rate g_t and with the contributions paid by the new entrants aged x_0. If the retirement age increases in the year of study, the income from contributions will benefit from an additional contribution. It decreases with the contributions ceased to be paid by individuals who just retired and with the contributions ceased to be paid by individuals who deceased between $t-\Delta$ and t.

2.2.2 Pension expenditures

The pension expenditures paid by the pension system at time t, P_t, corresponds to the sum of the pensions paid to all retirees at t and is represented as follows:

$$
\begin{aligned}
P_t &= \sum_{x=x_r(t)}^{\omega-\Delta} P_t^x N_t^x && (2.9) \\
&= \begin{cases} \sum_{x=x_r(t)+\Delta}^{\omega-\Delta} P_t^x N_t^x + P_t^{x_r(t)} N_t^{x_r(t)} & \text{if } x_r(t) = x_r(t-\Delta), \\ \sum_{x=x_r(t)+\Delta}^{\omega-\Delta} P_t^x N_t^x + P_{t-\Delta}^{x_r(t)-\Delta} N_{t-\Delta}^{x_r(t)-\Delta}\left(1+\lambda_t\right) & \text{if } x_r(t) \neq x_r(t-\Delta), \end{cases} && (2.10)
\end{aligned}
$$

where

P_t^x represents the individual pension paid to retirees aged x at time t. The pension depends on the initial pension paid and the indexation during retirement as follows

$$
P_t^x = P_{t-\Delta}^{x-\Delta}\left(1+\lambda_t\right), x \in \left[x_r(t)+\Delta, \omega-\Delta\right], \tag{2.11}
$$

where

λ_t is the pension's indexation rate from period $t-\Delta$ to period t. The initial pension paid to individuals who have just retired is denoted by $P_t^{x_r(t)}$, for $t>0$ and is calculated according to the pension scheme design.

Equation (2.10) shows the effect of an increasing retirement age on pension expenditures. If the retirement age increases at time t then the cohort which was supposed to retire at t according to the previous rules will contribute during one additional period as indicated in the Equation (2.8). Therefore, the pension expenditures at time t in this context correspond to those paid one period earlier which are indexed. The following Remark 2 highlights the evolution of the pension expenditures.

Remark 2. The pension expenditures (2.9) can be rewritten to highlight the effect of mortality and variations in the indexation rate as follows:

$$\begin{aligned}
P_t &= \sum_{x=x_r(t)+\Delta}^{\omega-\Delta} P_t^x N_t^x + P_t^{x_r(t)} N_t^{x_r(t)} \cdot 1_{x_r(t)=x_r(t-\Delta)} + P_{t-\Delta}^{x_r(t)-\Delta} N_{t-\Delta}^{x_r(t)-\Delta} \left(1+\lambda_t\right) \cdot 1_{x_r(t)\neq x_r(t-\Delta)} \\
&= P_t^{x_r(t)} N_t^{x_r(t)} \cdot 1_{x_r(t)=x_r(t-\Delta)} + P_{t-\Delta}^{x_r(t)-\Delta} N_{t-\Delta}^{x_r(t)-\Delta} \left(1+\lambda_t\right) \cdot 1_{x_r(t)\neq x_r(t-\Delta)} \\
&+ \sum_{x=x_r(t)+\Delta}^{\omega-\Delta} P_{t-\Delta}^{x-\Delta} N_{t-\Delta}^{x-\Delta} \left(1+\lambda_t\right) - \sum_{x=x_r(t)+\Delta}^{\omega-\Delta} P_t^x N_{t-\Delta}^{x-\Delta} \cdot {}_{\Delta}q_{x-\Delta}(t-\Delta)
\end{aligned}$$

The pension expenditures at time t increase with the age-independent indexation of pensions λ_t and with the pensions paid to the new retirees aged $x_r(t)$ if the retirement age remains constant. It decreases with the pensions ceased to be paid to retirees who died at age ω and with the pensions ceased to be paid by retirees who deceased between $t-\Delta$ and t.

Obviously, the expression of the initial pension $P_t^{x_r(t)}$ depends on whether the pension's design is DC or DB. The remainder of this section develops the first pension for three different schemes: one DB and two DC (with and without the survivor dividend).

Defined Benefit

DB pension systems are usually based on a percentage K_t, commonly known as replacement rate, of a wage-dependent amount $PS_t^{x_r(t)}$, which we name pensionable salary. Mathematically, the initial pension for a retiree at time t is expressed as follows:

$$P_t^{x_r(t)} = K_t \cdot PS_t^{x_r(t)} \tag{2.12}$$

The most common expressions for the pensionable salary are:

$$PS_t^{x_r(t)} = \begin{cases} \frac{\sum_{x=x_0}^{x_r(t)-\Delta} W_{t-x_r(t)+x}^x \prod_{j=t-x_r(t)+x+\Delta}^t (1+g_j)}{x_r(t)-x_0} & \text{for mean wage revalorized,} \\ W_{t-\Delta}^{x_r(t)-\Delta}\left(1+g_t\right) = W_t^{x_r(t)-\Delta} & \text{for last wage revalorized} \end{cases} \tag{2.13}$$

Defined Contribution

In the case of pay-as-you-go DC, also known as notional or non-financial DC, the pension at retirement depends on the notional capital saved throughout the working career $NC_t^{x_r(t)}$ and the annuity factor $a_{x_r(t)}$. The pension capital in DC can be calculated in two ways: with or without the survivor dividend (SD), also called inheritance gains.

The sum of the contributions of individuals who do not survive until retirement represent the survivor dividend (SD). The government can choose to redistribute the balances within the same cohort, increasing the notional return (Vidal-Meliá et al. 2015). In fact, this is the approach considered in Sweden, the only country which redistributes explicitly the balance of the deceased (Chłón-Domińczak et al. 2012)[6]. In this paper, we study as well the effect of distributing the SD within the cohort in terms of adequacy and actuarial fairness[7].

Mathematically the expression for the initial pension for the *individual* approach is expressed as follows:

$$P_t^{x_r(t)} = \frac{NC_t^{x_r(t)}}{a_{x_r(t)}} \tag{2.14}$$

where the notional capital for the *individual* $NC_t^{x_r(t)}$ is expressed as follows:

$$NC_t^{x_r(t)} = \sum_{x=x_0}^{x_r(t)-\Delta} \pi_{t-x_r(t)+x} \cdot W_{t-x_r(t)+x}^{x} \prod_{j=t-x_r(t)+x+\Delta}^{t} (1+nr_j) \tag{2.15}$$

where

$\pi_{t-x_r(t)+x}$ is the contribution rate[8] at time $t-x_r(t)+x$ and nr_i is the notional (virtual) rate of return on the pay-as-you-go contributions for the period $j-\Delta$ to j. This notional rate is usually set by law and is equal to an indicator of the financial health of the system, such as, growth rate of GDP, average wages or total income from contributions.

Mathematically the expression for the initial pension for the *cohort* approach which includes the SD is expressed as follows:

$$P_t^{x_r(t)} = \frac{NC_t^{x_r(t)}}{a_{x_r(t)} N_t^{x_r(t)}} \tag{2.16}$$

where the notional capital for the *cohort* $NC_t^{x_r(t)}$ is expressed as follows:

$$NC_t^{x_r(t)} = \sum_{x=x_0}^{x_r(t)-\Delta} \pi_{t-x_r(t)+x} \cdot W_{t-x_r(t)+x}^{x} \cdot N_{t-x_r(t)+x}^{x} \prod_{j=t-x_r(t)+x+\Delta}^{t} (1+nr_j) \tag{2.17}$$

The SD is included because we account for the contributions for all members of the cohort, even those that do not survive to retirement, and because the initial pension is dependent on the number of individuals retiring the same year as highlighted in formula (2.17).

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[6] Arnold et al. (2015) state that the SD could be used to finance unexpected longevity increases instead.

[7] In fact, we show that the DC with SD is actuarially fair on a cohort basis whenever the population is closed, as already shown in Boado-Penas and Vidal-Meliá (2014).

[8] Classical notional DC consider that the contribution rate is constant over time, shifting most of the financial burden on the retirees, see Palmer (2013).

The annuity factor depends on the indexation of pensions and discount rate as well as the life tables chosen. Note that the annuity factor does not depend on whether we decide to distribute the SD or not. Mathematically the annuity for the retiring cohort aged $x_r(t)$ at time t is represented as follows:

$$a_{x_r(t)} = \sum_{x=x_r(t)}^{\omega-\Delta} {}_{x-x_r(t)}p_{x_r(t)}(t-x+x_r(t)) \prod_{j=x_r(t)}^{x} \frac{1+\lambda_j}{1+nr_j} \tag{2.18}$$

As seen in Equation (2.18) when the indexation equals the discounting rate the annuity $a_{x_r(t)}$ is then reduced to the expression of the life expectancy at retirement $e_{x_r(t),t}$[9].

2.3 Adequacy and actuarial fairness

According to Chomik and Piggott (2016), adequacy refers to poverty alleviation or income replacement. The level deemed sufficient to maintain a reasonable standard of living can be measured as the proportion of the pension to average or minimum wages, or alternatively by creating a budget standard for pensioners which covers an adequate basket of goods and services (Stiglitz et al. 2010)[10].

Income replacement rates in pension fund management are sometimes represented as the amount of the initial pension over the last salary. In this paper, we define the replacement rate as the ratio of the pension to the average income for the same year. It is thus related to the 'Benefit ratio' found in Aggregate Accounting methods (Roseveare et al. 1996; Boldrin et al. 1999; Dang et al. 2001; Jimeno et al. 2008). Mathematically, the replacement rate RR_t^x for an individual aged x at time t is represented as follows:

$$RR_t^x = \frac{P_t^x}{\sum_{x=x_0}^{x_r(t)-\Delta} W_t^x}\,(x_r(t)-x_0) \tag{2.19}$$

The replacement rate RR_t^x does not provide a longitudinal measure of the pension system. A way to solve this problem is to calculate the individual actuarial fairness by means of the benefit to cost ratio (Queisser and Whitehouse 2006). This ratio studies the relationship between the present value of the benefits paid during retirement and the contributions paid during their working career. A value of 1 indicates that the system is actuarially fair for the specific individual, that is, she receives pension benefits which correspond to her contributions. A value greater (lower) than 1 indicates that the individual receives more (less) than she contributed.

Mathematically, the actuarial fairness for an individual retiring at age $x_r(t)$ at time t, denoted as AF_t, can be expressed as follows:

$$AF_t^{x_r(t)} = \frac{\sum_{x=x_r(t)}^{\omega-\Delta} {}_{x-x_r(t)}p_{x_r(t)}(t-x+x_r(t)) P^x_{t-x_r(t)+x} \prod_{i=t}^{t-x_r(t)+x+\Delta} \frac{1}{1+nr_i}}{\sum_{x=x_0}^{x_r(t)-\Delta} {}_{x-x_r(t)}p_{x_r(t)}(t-x+x_r(t)) \pi_{t-x_r(t)+x} W^x_{t-x_r(t)+x} \prod_{i=t-x_r(t)+x+\Delta}^{t} (1+nr_i)} \tag{2.20}$$

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[9] This is the case in countries such as Poland and Latvia (Chłón-Domińczak et al. 2012).

[10] The basket of goods and services may differ per individual, since some may find some goods and services more important than others. In this vein, OECD (2017) allow individuals to create their own "better life index" based on their preferences.

2.4 Particular case: Steady State

Public pension systems are often studied when the economy and population are in the steady state with the aim of deriving elegant conclusions on their design and dynamics. In our context we say that our system is in 'steady state' when the wage, population growth and contribution rates are constant and when the survival probability and retirement age is time-independent. In this subsection we first present the formulae of the income from contributions and pension expenditures in steady state and discuss the role of the buffer fund on the sustainability in Remark 3. Finally, Proposition 1 shows that defined benefit and contribution schemes in steady state and how they yield similar results under some circumstances. Mathematically, the population (2.1) and wages (2.3) in steady state are expressed as follows:

$$\begin{aligned} N_t^x &= N_{t-x+x_0}^{x_0} \cdot {}_{x-x_0}p_{x_0}, && (2.21)\\ N_{t-x+x_0}^{x_0} &= N_{t-x+x_0-\Delta}^{x_0}\left(1+n\right), && (2.22)\\ W_t^x &= W_{t-\Delta}^x\left(1+g\right). && (2.23) \end{aligned}$$

The evolution of the income from contributions C_t (2.7) simplifies to $C_t = C_{t-\Delta}\left(1+g\right)\left(1+n\right)$. Similarly, the pensionable salary in the DB case evolves with wages increase as follows $PS_t^{x_r} = PS_{t-\Delta}^{x_r}\left(1+g\right)$. Finally, the notional capital for DC schemes (2.24), and the initial pension for both DB and DC schemes (2.25) can be represented as follows:

$$NC_t^{x_r} = \begin{cases} \overbrace{\sum_{x=x_0}^{x_r-\Delta} \pi W_{t-x_r+x}^x N_{t-x_r+x}^x \left(1+nr\right)^{x_r-x}}^{(1+g)(1+n)NC_{t-\Delta}^{x_r}} & \text{if DC with SD,}\\ \underbrace{\sum_{x=x_0}^{x_r-\Delta} \pi W_{t-x_r+x}^x \left(1+nr\right)^{x_r-x}}_{(1+g)NC_{t-\Delta}^{x_r}} & \text{if DC without SD.} \end{cases} \tag{2.24}$$

$$P_t^{x_r} = \begin{cases} P_{t-\Delta}^{x_r}\left(1+g\right) & \text{if DB,}\\ \frac{NC_{t-\Delta}^{x_r}(1+g)(1+n)}{a_{x_r} N_{t-\Delta}^{x_r}(1+n)} = P_{t-\Delta}^{x_r}\left(1+g\right) & \text{if DC with SD,}\\ \frac{NC_{t-\Delta}^{x_r}(1+g)}{a_{x_r}} = P_{t-\Delta}^{x_r}\left(1+g\right) & \text{if DC without SD.} \end{cases} \tag{2.25}$$

Pension expenditures at time t, P_t (2.9), for a general pension system are then expressed as follows:

$$P_t = \sum_{x=x_r}^{\omega-\Delta} P_{t-\Delta}^x N_{t-\Delta}^x \left(1+g\right)\left(1+n\right) = P_{t-\Delta}\left(1+g\right)\left(1+n\right). \tag{2.26}$$

The following Remark 3 highlights the fund's role on the sustainability of the system in a steady state context.

Remark 3. It is straightforward to note that the fund F_t^+ (2.5), assuming that the fund at inception is zero, i.e. $F_0^- = 0$, is simplified as follows:

$$
\begin{aligned}
F_t^+ = \sum_{j=0}^{t} \left(C_j - P_j\right)(1+i) &= \left(C_0 - P_0\right)(1+i)^t \sum_{j=0}^{t} \left(\frac{(1+g)(1+n)}{1+i}\right)^j \\
&= \left(C_0 - P_0\right)(1+i)^t \, \frac{\left(\frac{(1+g)(1+n)}{1+i}\right)^{t+\Delta} - 1}{\frac{(1+g)(1+n)}{1+i} - 1} \qquad (2.27)
\end{aligned}
$$

The above mentioned expression shows that if the initial contribution rate is chosen such that there is a systematic surplus, i.e. $C_0 > P_0$, then the fund will be systematically accumulating funds. Alternatively, if the opposite holds, the fund will be systematically in debt. In particular, when the contribution rate is chosen such that $C_0 = P_0$, the fund will be equal to zero at all times. The expression (2.27) shows that it may be worth using RSMs when the initial equilibrium is not guaranteed even when the system is in steady state. In this non-dynamic environment, the RSM only works in one way, either reducing the benefits and increasing the contribution rate for a systematic deficit or increasing the benefits and reducing the contribution rate for a systematic surplus.

Steady state pension systems can be sustainable in the long run whenever the contribution rate is chosen such that the initial contributions, C_0, are sufficient to pay for initial pension expenditures, P_0 as shown in Equation (2.27). This result holds for the three pension systems separately. In terms of adequacy and actuarial fairness, the following Proposition 1 shows that the DB and the DC with SD provide the same amount of pension under certain pre-specified assumptions[11].

Proposition 1. *The amount of the initial pension for the DB and DC with SD schemes are equal whenever the DB pensionable salary evolves with wages, the notional DC scheme pays the canonical notional rate*[12] *and the DB contribution rate is chosen to ensure financial equilibrium, that is,* $C_0 = P_0$.

Proof. See Appendix A. ☐

3 Risk-sharing mechanisms

This section develops tractable RSMs (RSM) for unfunded pension schemes. Note that some countries[13] do not solely rely on pay-as-you-go to finance their pension commitments but also on funded capital because it enhances the welfare for individuals and allows for diversification benefits between demographic and market risks (De Menil et al. 2006; Knell 2010; Alonso-García and Devolder 2016; Lever et al. 2017). However, since the funded part of the pension commitments will be inherently liquid as capitals paid at retirement are based on underlying funds[14], we focus on the pay-as-you-go part of these systems.

---

[11] Note that Queisser and Whitehouse (2006) show a similar result when comparing DB, points systems and notional-accounts, whereas Vidal-Meliá et al. (2010) prove an equivalent result when comparing DB and notional DC under similar assumptions.

[12] This rate, known as the 'natural rate' of the NDC scheme (Valdés-Prieto 2000; Börsch-Supan 2006) or the 'biological rate' of the economy (Samuelson 1958), corresponds to the growth of the total contribution base.

[13] Some examples of mixed funding are Sweden (Könberg et al. 2006), Latvia (Dundure and Pukis 2015) and Poland (Chłoń-Domińczak and Strzelecki 2013).

[14] If pensions from the funded part are paid as a guaranteed lifetime annuity, the funded pension scheme may also encounter difficulties to honour their commitments in the presence of (unexpected) longevity improvements.

The RSM presented here changes the level of the contribution rate and the indexation of pensions when salaries and population dynamics are exogenous. Please note that we do not control for the retirement age as done in Godínez-Olivares et al. (2016a,b) and Gannon et al. (2016) but rather choose to link the effective retirement age to the life expectancy as done in practice (OECD 2015).

Let $\beta_t \in [0,1]$ be the time-dependent risk-sharing coefficient between the contributors and the pensioners and $F_t^- = 0$ the fund when the RSMs are put in place. The one-period deficit or surplus D_t^* (2.5) before any adjustments are made is then denoted by:

$$D_t^* = C_t^* - P_t^*. \tag{3.1}$$

The government shares the burden between the contributors and pensioners as follows:

- $\beta_t D_t^*$ is the share of the surplus/deficit borne by the contributors, and
- $(1-\beta_t)\, D_t^*$ is the share of the surplus/deficit borne by the pensioners.

Before showing in Proposition 2 the adjustments needed in the contribution and indexation rates to achieve liquidity, we highlight the evolution of the income from contributions and pension expenditures before (C_t^* and P_t^*) and after the application of the RSMs (C_t and P_t). C_t^* and P_t^* are the income from contributions and pension expenditures respectively before the application of the RSMs which ensure one-period liquidity. The income from contributions at time t is calculated with the contribution rate corresponding to the previous period $\pi_{t-\Delta}$ while pension expenditures are calculated based on the observed indexation rate λ_t^* in absence of RSMs. The expression are given as follows:

$$C_t^* = \pi_{t-\Delta} \sum_{x=x_0}^{x_r(t)-\Delta} W_t^x N_t^x, \tag{3.2}$$

$$P_t^* = \begin{cases} \sum_{x=x_r(t)+\Delta}^{\omega-\Delta} P_{t-\Delta}^{x-\Delta}\left(1+\lambda_t^*\right) N_t^x + P_t^{x_r(t)} N_t^{x_r(t)} & \text{if } x_r(t) = x_r(t-\Delta), \\ \sum_{x=x_r(t)+\Delta}^{\omega-\Delta} P_{t-\Delta}^{x-\Delta}\left(1+\lambda_t^*\right) N_t^x + P_{t-\Delta}^{x_r(t)-\Delta} N_{t-\Delta}^{x_r(t)-\Delta}\left(1+\lambda_t^*\right) & \text{if } x_r(t) \neq x_r(t-\Delta). \end{cases} \tag{3.3}$$

C_t and P_t are given by (2.7) and (2.9) and represent respectively the income from contributions and pension expenditures after the application of the RSM. The evolution of the contribution and indexation rates are represented as follows:

$$\pi_t = \pi_{t-\Delta}\left(1+\alpha_t^\pi\right), \tag{3.4}$$

$$\lambda_t = \left(1+\lambda_t^*\right)\left(1+\alpha_t^\lambda\right) - 1, \tag{3.5}$$

where

α_t^π (resp. α_t^λ) is the rate of increase of the contribution rate (resp. indexation rate) after risk-sharing.

Proposition 2 (Risk-sharing)**.** *The RSMs at time t related to the contribution rate,* α_t^π*, and related to the indexation rate,* α_t^λ*, are expressed as follows:*

$$\alpha_t^\pi = \beta_t \left(\frac{1 - LR_t^*}{LR_t^*} \right), \tag{3.6}$$

$$\alpha_t^\lambda = \begin{cases} \beta_t + (1-\beta_t)\, \frac{C_t^* - P_t^{xr(t)} N_t^{xr(t)}}{P_t^* - P_t^{xr(t)} N_t^{xr(t)}} - 1 & \textit{if } x_r(t) = x_r(t-\Delta); \\ \beta_t + (1-\beta_t)\, \frac{C_t^*}{P_t^*} - 1 & \textit{if } x_r(t) \neq x_r(t-\Delta), \end{cases} \tag{3.7}$$

where LR_t^* *corresponds to the liquidity ratio in absence of a buffer fund prior to the application of the RSM.*

Proof. See Appendix B. □

Proposition 2 shows the rate of variation of the contribution and indexation rate needed to restore the liquidity when β_t% of the surplus or deficit is borne by contributors and the remainder by pensioners. For instance, when the income from contributions before RSM is greater than the pension expenditures, the liquidity ratio LR_t^* is higher than 1, which indicates that there is a surplus. The parameter α_t^π (3.6) is then negative and the contribution rate is then reduced by $1 + \beta_t \left(\frac{1-LR_t^*}{LR_t^*} \right)$.

In the same vein, the parameter affecting the indexation rate, α_t^λ (3.7), is positive and the indexation rate is increased by the liquidity ratio without buffer fund corrected by the first pension paid, that is, $1 + \beta_t + (1-\beta_t)\, \frac{C_t^* - P_t^{xr(t)} N_t^{xr(t)}}{P_t^* - P_t^{xr(t)} N_t^{xr(t)}}$ if the retirement age remains constant between two subsequent periods. The liquidity ratio under the RSM is corrected by the initial pension because the first pension is not affected by the indexation rate. However, this is not longer the case when $x_r(t) \neq x_r(t-\Delta)$. This follows from the evolution of pension expenditures when the retirement age increases: the first cohort of retirees after a change corresponds to those retired in the previous period and are therefore affected by the indexation rate as well. Corollary 1 indicates how the RSMs presented in Proposition 2 simplify to the well known 'pure-DB' and 'pure-DC' cases when the risk-sharing coefficient β_t is carefully chosen.

Corollary 1 (Particular cases: $\beta_t = 0$ and $\beta_t = 1$)**.** *The risk-sharing coefficients* α_t^π (3.6) *and* α_t^λ (3.7) *simplify when the risk-sharing coefficient* β_t *is equal to* 0 *or* 1 *as follows:*

- *when the risk-sharing coefficient* β_t *is equal to* 0*, that is, when the surplus or deficit is solely borne by pensioners, the expressions of* (3.6) *and* (3.7) *become:*

$$\alpha_t^\pi = 0, \tag{3.8}$$

$$\alpha_t^\lambda = \begin{cases} \frac{C_t^* - P_t^{xr(t)} N_t^{xr(t)}}{P_t^* - P_t^{xr(t)} N_t^{xr(t)}} - 1 & \textit{if } x_r(t) = x_r(t-\Delta), \\ \frac{C_t^*}{P_t^*} - 1 & \textit{if } x_r(t) \neq x_r(t-\Delta). \end{cases} \tag{3.9}$$

- *Alternatively, when the risk-sharing coefficient* β_t *is equal to* 1*, that is, when the surplus or deficit is solely borne by contributors, the expressions of* (3.6) *and* (3.7) *become:*

$$\alpha_t^\pi = \frac{1 - LR_t^*}{LR_t^*}, \tag{3.10}$$

$$\alpha_t^\lambda = 0. \tag{3.11}$$

Corollary 1 shows two extreme cases of our general RSM framework. In the first case, the contribution rate does not change over time and the whole deficit or surplus of the system is borne or benefited by the pensioners through an adjusted indexation. This case relates to the classical notional DC as the contribution rate is fixed and constant by definition (Palmer 2013; Chłón-Domińczak et al. 2012; Alonso-García and Devolder 2017). However, it can also be used in DB schemes whenever the contribution rate remains constant. The second case presents the opposite situation where the deficit or surplus is borne or benefited by the contributors. In particular, this RSM adjusts the contribution rate while maintaining the pension benefits promised by the system. This relates to a classical DB scheme in absence of structural of parametric reforms.

4 Numerical illustration

This section presents a numerical example using Belgian data under the generic DB and DC pension systems developed in Section 2. First, the main data and assumptions for the base case are presented. Then we discuss the results under different RSMs. The base case scenario assumes that the retirement age increases in line with the current life expectancy and that the annuities are based on projected life tables. Then we present in Section 4.4.1 the effects of using current tables to analyse the effect of unexpected longevity improvements. Finally, Section 4.4.2 briefly presents the consequences of a constant retirement age.

4.1 Data and assumptions

- The demographic structure of the Belgian population from 1935-2016 is obtained from the Human Mortality Database (2016b). Note that the population before 2016 is open as it represents the total observed population for each age. The forecasted population relies upon data from Eurostat (2013). New entries are assumed to join the system at age 20 and population after 2016 is closed, that is, exits are only due to death.

- The belgian salary structure[15] as of 2010 is taken from Eurostat (2010). Historical salaries growth is based on Statbel (2016) while future salary increases are based on the forecast for labour productivity per hour from the European Commission (2014).

- Historical mortality tables are taken from the Human Mortality Database (2016a) and Statbel (2014) while projected values are obtained from the Belgian Federal Planning Bureau (2016).

- For the DB pension system, the initial pension is set at 60% of average revalorized salary in line with the current Belgium DB formula (Federal Pension Service 2016).

- For both DB and DC pension systems, the initial contribution rate in 2016 is the rate that makes the DB system balanced in this particular year, i.e, 19.02%. Note that this contribution rate is higher than the currently used in Belgium, i.e. 16.86%.

- The indexation of pensions is equal to the rate of increase of the income from contributions. Furthermore, the discount rate in the annuity calculation for the DC schemes is set equal to the indexation rate. The value of the annuity is therefore equal to the life expectancy at retirement based on the projected life table.

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[15] The wage structure is based on the mean annual earnings by sex, age and economic activity including industry, construction and services and excluding public administration, defense and compulsory social security.

- The retirement age increases in line with the current life expectancy as follows:

$$x_r(s) = 65 + e_{65,s} - e_{65,2016}.$$

  where $e_{65,s}$ is the life expectancy at time s using the current life table from Statbel (2014), that is, retirement age increases based on verifiable current life expectancy and does not incorporate expected or unexpected longevity improvements. Changes in retirement age are made on a trimester basis, therefore $\Delta = 0.25$.

- The interest rate of the buffer fund is assumed to be equal to 0%[16].
- No minimum and maximum pension are considered in our analysis[17].
- The replacement rate is calculated at retirement age and age 85 according to the formula (2.19). The replacement rate at retirement age assesses the initial generosity of the scheme whereas the replacement rate at 85 investigates the impact of subsequent applications of the RSM. We may find that DC schemes, despite having a lower initial replacement rate, perform better in the long run through a lower volatile benefit payment.
- The actuarial fairness is calculated as the relative difference between the value of the benefits received and the contributions made at the retirement age $x_r(t)$ as shown in Equation (2.20).
- The RSM β_t is equal to the ratio of the working population to the retirees, that is, the inverse of the dependency ratio. In other words, the higher the share of contributors to the economy, the higher the cost they have to bear compared to the retirees.

4.2 Base scenario

This section presents the results for the base case scenario under the assumptions presented in Subsection 4.1. Historical data is used for the population, wages increase and mortality experience while forecasted values from various statistical sources are used for the analysis exercise from 2016 to 2060. In this section we do not consider RSMs, retirement age is linked to the current life expectancy and the annuity considered is based on forecasted tables, capturing the future longevity improvements.

Figure 1 shows the contribution and indexation rate, the replacement rate at age of retirement and 85 as well as the actuarial fairness for the three different pension schemes: DB with a replacement rate of 60%, a DC with SD and a DC without SD. The first row shows that, in absence of RSM, the contribution rate is kept constant and equal to 19.02% during our study. However, the indexation rate presents spikes that correspond to the years when the retirement age increases. The working population is then increased by one cohort which was not present in the previous year, leading to a one-off substantial increase in the total contribution base.

The effect of these spikes is clear in Figure 1c. We see that, after each increase in retirement age, the additional contributions increase the ratio of contributions to pension expenditures

---

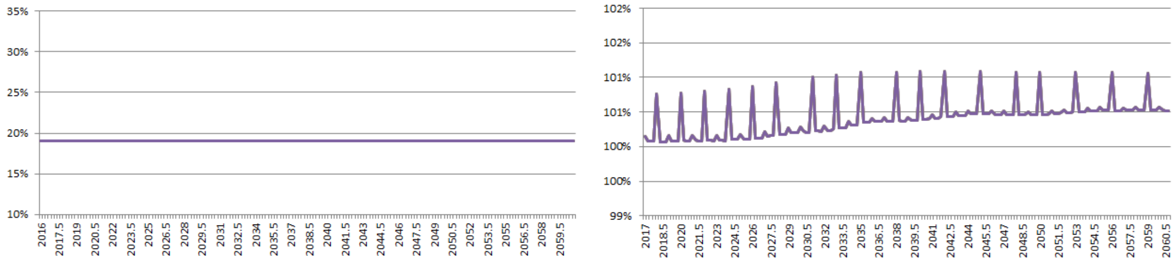
[16] The interest rate affects the buffer fund only for the base case scenario in absence of RSM. Once the RSMs are in place the system will always be liquid and the fund has a value of 0 and will not be affected by the interest rate.

[17] The RSM affecting the indexation rate may reduce the pensions, especially after subsequent periods of negative indexation. However, in our case, the inclusion of a minimum pension in our analysis has no impact because the wages considered for pension calculation almost double the minimum wages. If different income categories were considered, minimum pension might have an impact.

Figure 1: No RSM: The figure depicts the contribution and indexation rate, the liquidity ratio, the replacement rate at age of retirement and 85 as well as the actuarial fairness for three different pension schemes: DB (green), DC with SD (red) and DC without SD (blue). The contribution and indexation rate in absence of RSM is depicted in purple.

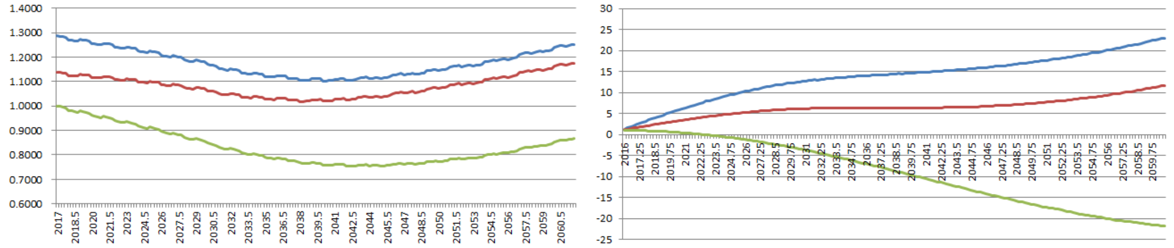
a) Contribution rate

b) Indexation rate

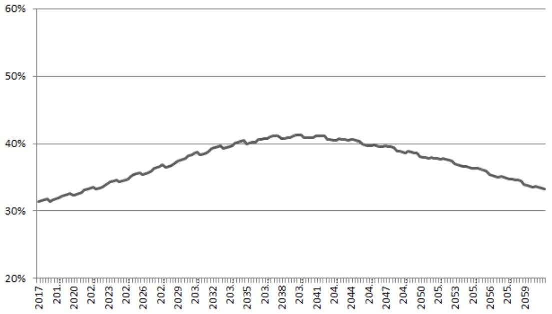


c) Liquidity ratio without fund

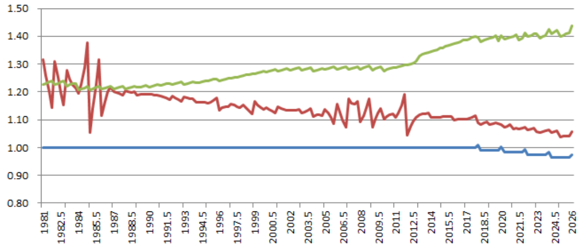
d) Liquidity ratio with fund



e) Dependency ratio

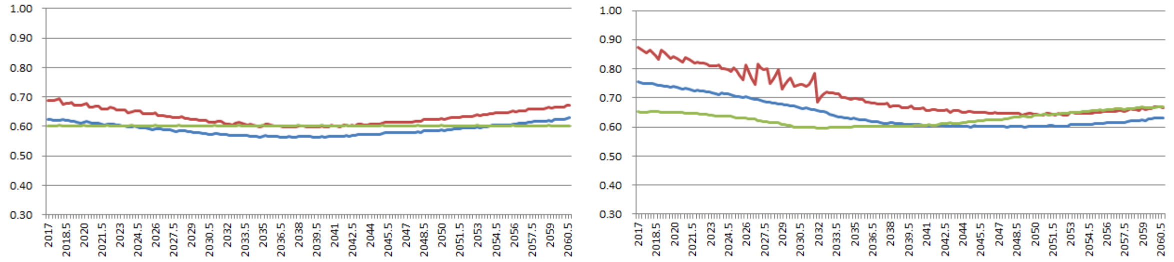


f) Actuarial fairness (benefit to cost ratio)



g) Replacement rate at retirement age

f) Replacement rate at 85



Notes: The spikes in the indexation rate are caused by the time-dependent retirement age. When the retirement age increases it increases with an additional contribution from the cohort which was expected to retire one year earlier. This produces a spike in the working population increase which translates to the indexation rate which is a combination of the wages and working population increase. These spikes affect the dependency ratio and liquitidy rate without fund as well. The replacement rate at 85 corresponds to the pension paid to a 85 year retiree in the indicated year relative to the average of the wages in the economy.

and positively affect the liquidity of the system. However, these parametric reforms are not sufficient to attain liquidity (Figure 1c and 1d). Despite the initial liquidity, the DB scheme enters debt very quickly. This is mainly caused by the benefit structure, independent of the contributions made, as well as the dependency ratio (Figure 1e). Indeed, the decrease of the liquidity ratio without fund goes hand in hand with an increase in the dependency ratio from 30% in 2015 to 41% in 2040. It is noteworthy that the increasing retirement age does not make the system sustainable or lower the dependency ratio. The dependency ratio also draws the liquidity down for DC schemes. However, these DC schemes have been accumulating a large surplus to finance the effect of the baby boom combined with the fertility bust. Recall that the contribution rate is the same in the three schemes presented and is equal to the rate that makes the DB initially liquid as indicated in Subsection 4.1. Figure 1d further highlights the effect of the accumulation of the debt in the system. By 2040 the system would have a debt equivalent to 10 times the income from contributions for the same year.

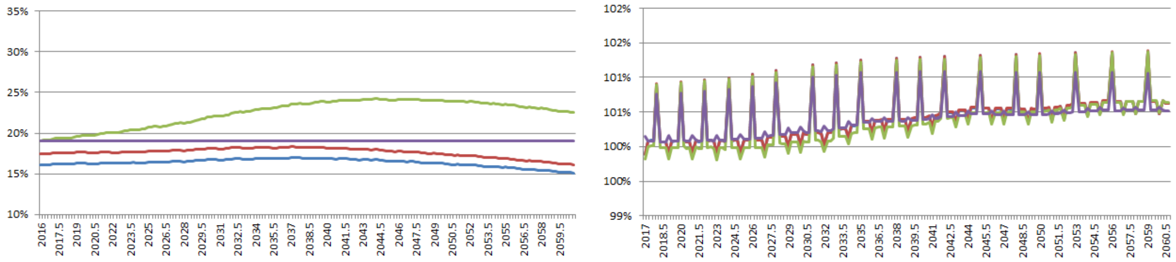
Figure 1f compares the value at retirement of the benefits paid relative to the value at retirement of the contributions made (actuarial fairness). A value of 1 indicates that the system is actuarially fair. A value higher (lower) than 1 indicates that the individual gets more (less) than they paid for. The DC scheme with SD and DB schemes pay more to individuals compared to what they have been contributing. As expected, the SD increases the return on contributions for individuals in a DC scheme with SD. The DB becomes increasingly more unfair for older cohorts since they receive pension payments for a longer period in line with the increase in life expectancy. On the other hand, the DC scheme without SD is practically actuarially fair. Indeed, the pensions received correspond exactly to the capital at retirement because the first pension payment depends on an annuity factor based on the forecasted mortality experience. However, this equality does not hold when the retirement age varies, because the retirement age varies according to current life table whereas the annuity is based on the forecasted life table. This mismatch between the tables renders a non-actuarially fair system for individuals under a DC scheme without SD.

Finally, the replacement rate at retirement also evolves in time. Figure 1g shows that the DC schemes are sensitive to the dependency ratio. The DC schemes earn a notional rate which is linked to the income from contributions. The slow growth of the working population leads to an increasingly lower return on contributions which provides a lower capital at retirement. This, combined with the increasing life expectancy, lowers the replacement rate. However, changes in the retirement age positively affect the replacement rate. In fact, we see how the replacement rate for the DC system recovers and even outperforms the initial DB pension. This is, however, at the expense of contributing an additional period and receiving the pension for a shorter period of time. At the age of 85, in absence of RSMs, the DC schemes outperform the DB because of the high notional rates paid during the baby boom period. For instance, someone aged 85 in 2017 corresponds to an individual who retired at the age of 65 in 1997 and contributed between 1952 and 1997, benefiting from post-war inflationary periods and population increase. Note that the DB scheme provides a higher replacement rate at 85 than it does at 65 under this scenario. This is caused by the indexation rate which accounts for both the (positive) increase in population and the wages increase. Therefore, the value of the pension relative to the wages increases during retirement.

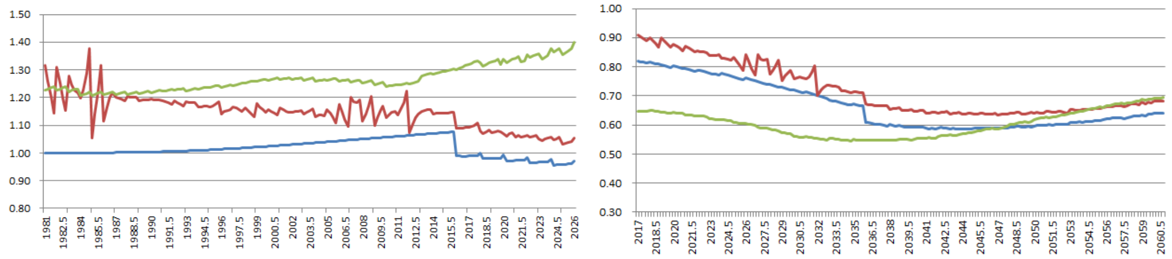
4.3 Risk-sharing mechanisms

After introducing a sustainability-ensuring RSM, the adequacy and actuarial fairness will behave differently. The RSM does not alter the population structure which affects the notional return paid to contributions and the dependency ratio. However, it does affect the contribution and indexation rate and subsequently the actuarial fairness and replacement rate at 85. Figure 2 shows that the contribution rate increases over time for the DB scheme which pays for very high pensions compared to the contributions paid. On the other hand, the DC schemes see their contribution rate lowered because of the surplus. However, all schemes are affected by the baby boom as shown in Figure 2a. Indeed, the contribution rate reaches its maximum

Figure 2: RSM with β_t equal to the inverse of the dependency ratio: The figure depicts the contribution and indexation rate, the actuarial fairness and replacement rate at 85 for three different pension schemes: DB (green), DC with SD (red) and DC without SD (blue). The contribution and indexation rate in absence of RSM is depicted in purple.


a) Contribution rate

b) Indexation rate


c) Actuarial fairness (benefit to cost ratio)

d) Replacement rate at 85

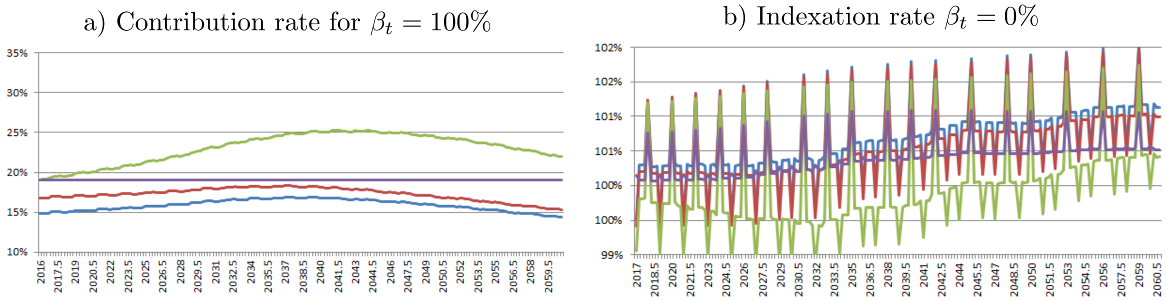
Notes: The spikes present in the indexation rate are caused by the time-dependent retirement age. When the retirement age increases it accounts for an additional contribution for the cohort which was expected to retire one year earlier. This produces a spike in the working population increase which translates to the indexation rate which is a combination of the wages and working population increase. Note that these spikes affect the dependency ratio and liquitidy rate without fund as well. The replacement rate at 85 corresponds to the pension paid to a 85 year retiree in the indicated year relative to the average of the wages in the economy.

level around year 2040 after which it starts to decrease due to the baby boom cohort leaving the system. The indexation rate, Figure 2b, is very erratic due to the combined effect of the increasing retirement age and RSMs.

The adequacy and fairness of the scheme are affected by the RSM put in place. We observe that the DC scheme without SD, which was practically actuarially fair in absence of an RSM, increases its unfairness. The continuing surplus increases the indexation rate such that the individuals from most cohorts receive more than they paid for. The cohorts which have a longer exposure to the RSMs are the most affected by this. A shortcoming of the actuarial fairness ratio is that we cannot fully assess the effect of parametric changes made today since we have a limited forecasting horizon. Therefore we only illustrate the actuarial fairness for the generations with a complete contribution and payment story going from 1981 to 2026. The adequacy of the system is affected as well as indicated by Figure 2. We observe that the DB scheme lowers its pension value after the subsequent negative indexation produced by the increasing dependency ratio. DC schemes are less affected since they also benefit from the periods with a higher than expected pension indexation (cf. discussion on actuarial fairness).

When we only adjust the contribution rate in order to attain sustainability ($\beta_t = 100\%$), Figure 3a, we observe that the contribution rate for the DB scheme increases substantially and that by construction the indexation rate remains unchanged. In fact, it would be needed an annual increase of the contribution rate until reaching a level of 25% by 2050. We observe that the contribution rate should also increase for the DC pension scheme. Due to the initial surplus depicted in Figure 1c, the contribution rate would immediately decrease to a level of

Figure 3: RSM with β_t equal to 100% (a) and 0% (b): The figure depicts the contribution and indexation rate for three different pension schemes: DB (green), DC with SD (red) and DC without SD (blue).

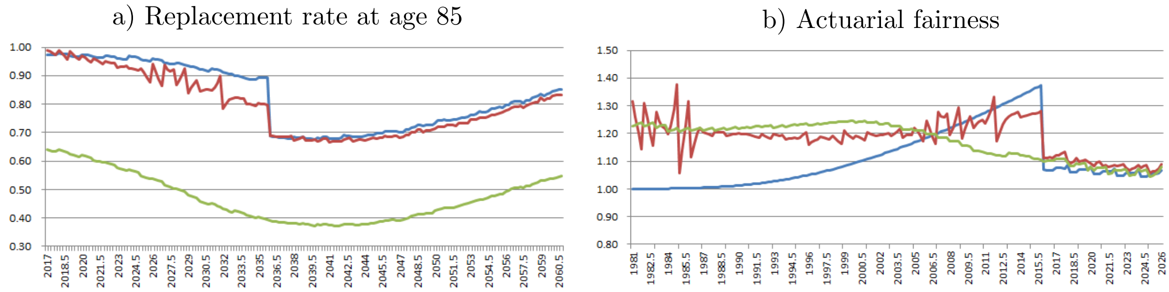

a) Contribution rate for $\beta_t = 100\%$

b) Indexation rate $\beta_t = 0\%$

Notes: A coefficient β_t equal to 100% indicates that the surplus or deficit is enjoyed or borne respectively by the working population only. This aligns with a DB system where the benefits remain untouched. Alternatively, a coefficient β_t equal to 0% corresponds to a RSM that puts all the weight to the retirees. This extreme case mimics the classical DC schemes more closely.

14% after the inception of the RSM. However, this low contribution rate does not suffice to cover the pension expenditures when the dependency ratio increases (Figure 1e). Therefore the contribution rate increases every year until 2040 (Figure 3e). In terms of adequacy and fairness there are no significant changes in the trend compared to the cases without RSMs as the indexation of pensions is not affected[18]. However, the contribution rate will increase the fairness, especially in the DB case, as individuals will contribute more to the system and get the same pension in exchange. For DC schemes, the actuarial fairness properties remain unchanged because an increase in contributions corresponds to an increase in pensions at retirement. However, we are not able to see these effects because of the time horizon studied: changes to the contribution rate affect cohorts which retire after 2025, for which we do not have complete career trajectories given that our study finishes in 2060.

Figure 4: RSM with $\beta_t = 0$: The figure depicts the actuarial fairness and replacement rate at 85 for three different pension schemes: DB (green), DC with SD (red) and DC without SD (blue).


a) Replacement rate at age 85

b) Actuarial fairness

Notes: A coefficient β_t equal to 100% indicates that the surplus or deficit is benefited or borne respectively by the working population only. This aligns with a DB system where the benefits remain untouched. Alternatively, a coefficient β_t equal to 0% corresponds to a RSM that puts all the weight to the retirees. This extreme case mimics the classical DC schemes more closely. The replacement rate at 85 corresponds to the pension paid to a 85 year retiree in the indicated year relative to the average of the wages in the economy.

Figure 3b shows the path of the indexation of pensions if this was the only variable adjusted to restore the liquidity of the system ($\beta = 0\%$). As shown in Figure 3b, the DB scheme needs to keep a much lower negative indexation, compared to the DC schemes, over the whole

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[18] These figures are available upon request.

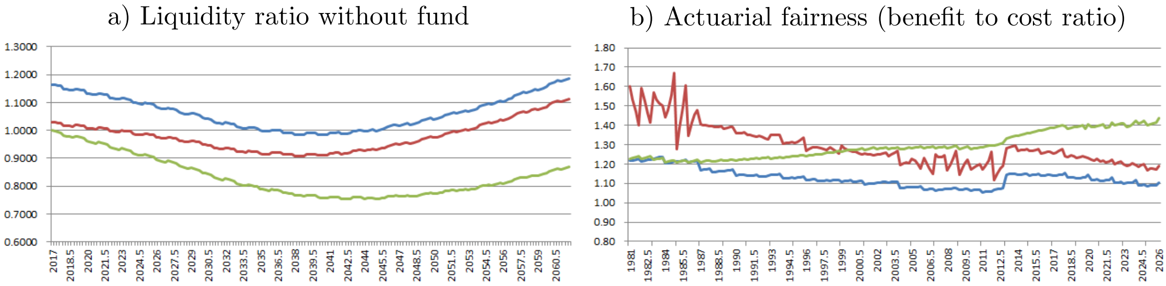
analysed period to guarantee the financial sustainability of the system. On the contrary, the indexation of pension would slightly increase for the DC scheme after 2038 as a result of the decreasing dependency ratio. The subsequent negative indexation will negatively affect the replacement rate at age 85, which reaches a value of 40% by 2040, increasing afterwards (Figure 4a). The major drop in the replacement rate is a consequence of the implementation of the RSM. This clarifies the big drop from the earlier generations who were less affected by the indexation rate. In the same vein, the actuarial fairness, Figure 4b, significantly improves after the inception of the RSMs in 2016 for the DB scheme mainly as a result of a reduction in the indexation of pensions. We observe that once liquidity is ensured via the indexation rate the actuarial fairness also changes accordingly. In fact, most schemes will have a ratio closer to 1 indicating a fair relationship between benefits received and contributions paid.

4.4 Sensitivity analysis

This subsection presents a sensitivity analysis of our main assumptions. Subsubsection 4.4.1 presents the effect of using current instead of forecasted life tables in the annuity calculation and subsubsection 4.4.2 illustrates the effect of a constant retirement age on the sustainability of the system.

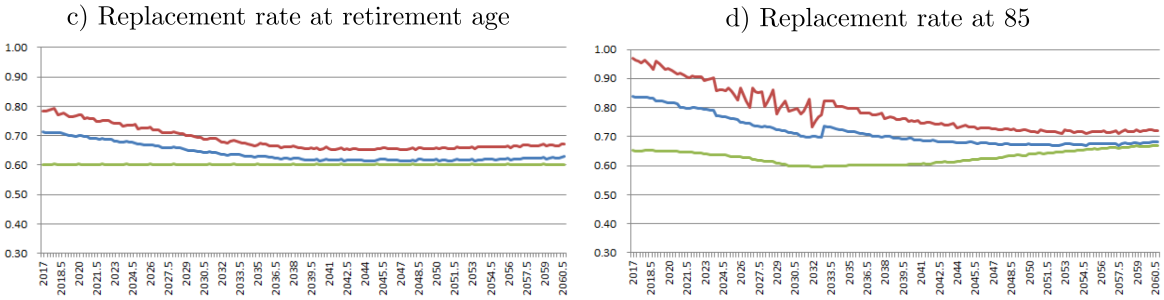
4.4.1 RSM in the presence of ageing

Figure 5: No RSM and ageing: The figure depicts the liquidity ratio, the replacement rate at age of retirement and 85 as well as the actuarial fairness in the presence of ageing for three different pension schemes: DB (green), DC with SD (red) and DC without SD (blue).



a) Liquidity ratio without fund

b) Actuarial fairness (benefit to cost ratio)



c) Replacement rate at retirement age

d) Replacement rate at 85

Notes: We denote as 'ageing' the fact that the policymakers do not consider future mortality improvements into account and pay pensions which may be too generous.

The previous subsection analyses the effect of RSMs when the retirement age is linked to the current life table and the annuity factor incorporates future longevity improvements from the Belgian Federal Planning Bureau (2016), which in our case coincides with the actual mortality experience. In contrast, this section considers that the annuity factor uses the current life table, that is, expected mortality improvements will not be accounted for in the annuity. This leads to higher pension payments compared to those which are actuarially fair. This assumption aligns with what is done in practice in Sweden and Italy (Chłón-Domińczak et al. 2012)[19].

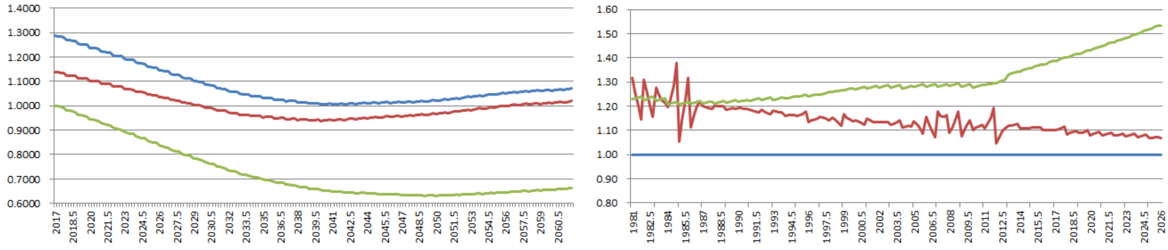
Figure 5 shows the ratio of contributions to expenditures, actuarial fairness and replacement rate for two ages: retirement age and 85. The contribution and indexation coincides with the other base case scenario since these are not affected by the annuity assumptions. A comparison between Figure 5a with Figure 1c shows that the liquidity of the two DC schemes is much lower than in the scenario without ageing improvements. This is caused by the higher pensions paid during a period which is longer than expected. In particular, the DC scheme with SD will be in debt only after 5 years from the inception and will restore sustainability when the baby boom cohorts retire. The increasing generosity in this context is further illustrated in Figure 5b, 5c and 5d. The lower annuity factor will increase the initial pension, especially for the younger cohorts who will have benefited from the higher post-war notional rates. The replacement rate for the DC schemes reaches a replacement rate similar to the DB scheme after 2040, due to the increasing retirement age. It follows from this description that the contribution and indexation rate will be more affected the risk-sharing mechanism in an ageing context.

4.4.2 RSM with a constant retirement age

Figure 6: No RSM and constant retirement age: The figure depicts the liquidity ratio, the replacement rate at age of retirement and the dependency ratio for a constant retirement age for three different pension schemes: DB (green), DC with SD (red) and DC without SD (blue).

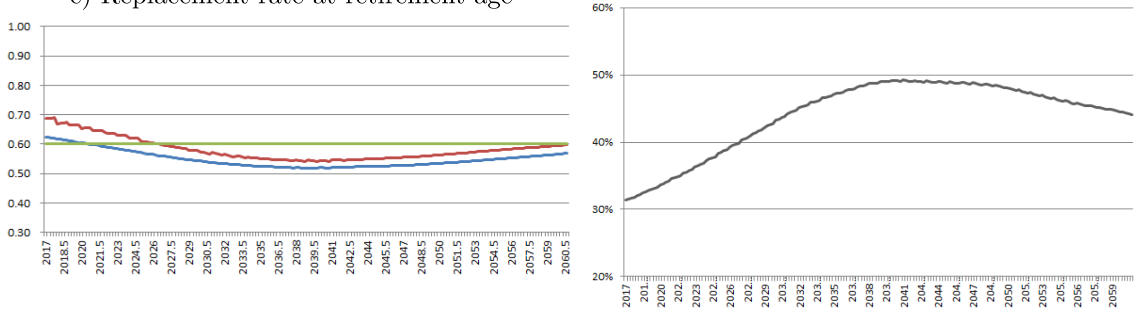
a) Liquidity ratio without fund

b) Actuarial fairness (benefit to cost ratio)



c) Replacement rate at retirement age

d) Dependency ratio



Notes: We illustrate the effect of a constant retirement age despite increases in life expectancy.

Section 4.3 considers that the retirement age is linked to the increases in life expectancy (OECD 2015). This assumption tackles the intrinsic non-sustainability of most systems by letting individuals contribute for a longer period of time in exchange of a higher pension, which is paid during a lower period of time in average. This subsection illustrates the effect

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[19] Note that the other two unfunded DC countries, Latvia and Poland, use forecasted life tables (Chłón-Domińczak et al. 2012).

of keeping the retirement age constant. We assume that the annuity factor uses forecasted life tables as in the base case.

Figure 6 shows the ratio of contributions to expenditures, actuarial fairness and replacement rate at retirement age, which in this case is constant and equal to 65. The contribution and indexation rates coincide with those presented in Figure 1. As expected, keeping the retirement age constant affects more negatively the DB scheme. This, combined with the increasing dependency ratio (Figure 6d), leads to a liquidity ratio of around 60% in 2040.

Maintaining the retirement age fixed does not substantially affect the actuarial fairness of DC schemes due to the payments being linked to the contributions made. However, the DB scheme will become increasingly unfair in absence of RSM reaching the value of 1.5 in 2025, that is, individuals retiring in 2026 will receive in average 50% more than they have contributed into the system (Figure 6b). Note that the individuals retiring in 2026 will live, on average, 11 years longer than the pensioners who left the labour force in 1981. The increase in life expectancy decreases the replacement rate at 65 for the DC schemes (Figure 6c) linked to the also increasing annuity factor which transforms their notional capital into a retirement income stream.

5 Conclusion

In this paper we study the sustainability of unfunded public pension systems, that is, systems where the pension expenditures are financed by the contributions of the working age population making it prone to demographic, wage and longevity risk. Around half of the OECD countries have taken measures to improve the financial sustainability of their systems by increasing the retirement age while others, such as Poland, Latvia, Sweden, Norway or Italy, have taken structural reforms by changing the benefit formula from defined benefit to contribution based (DC) systems (Chłón-Domińczak et al. 2012).

One of the reasons to switch from DB to DC is to improve the sustainability of the system while increasing the fairness of the pensions paid by the system. Indeed, by creating a link between the contributions and the benefits paid at retirement, policymakers make the system more transparent and less prone to political risk and discretionary occupational-specific schemes (Barr and Diamond 2006).

This paper develops a general dynamic and tractable framework to analyse the sustainability, defined as the equilibrium between the revenue and the pension expenditures, adequacy and actuarial fairness of three different schemes: one defined benefit paying benefits based on average careers, one defined contribution which accounts for survivor dividends, and one individualized defined contribution without survivor dividends. Survivor dividends increase the return on contributions and therefore the capital at retirement.

We show that in the steady state, under certain circumstances, the amount of the pension for DB and DC schemes might be equivalent. However, under a dynamic environment, DB and DC schemes reach significantly different values for the initial pension and consequently for sustainability, adequacy and fairness. This paper designs, from a theoretical point of view, flexible RSMs, involving variables such as the contribution rate and/or indexation of pensions, which restore the financial sustainability of a pension system in the short and long run (Valdés-Prieto 2000). This is done on top of a parametric reform linking the retirement age to the life expectancy (Knell 2012; OECD 2015).

We show that, considering economic and demographic projections for the case of Belgium, the sustainability of a generic DB pension scheme is seriously compromised unless some mechanisms are implemented immediately. These mechanisms affect negatively the benefits paid due to an increasing life expectancy and dependency ratio. In most cases, these mechanisms lower the pension level and improve the actuarial fairness of the system. As expected, DC schemes are more sustainable and fair compared to the DB schemes before considering any risk-sharing mechanisms and provide a more regular stream of payments when introducing mechanisms to restore sustainability.

We also show the effect of an ageing population in the pension scheme by assuming that the annuity factor that converts the capital into a pension stream does not account for expected mortality improvements. We show that despite being initially more liquid, DC schemes are in debt only after 5 years from inception and remain in debt until the baby boom cohorts leave the system. This indicates that, despite their link between pensions and contributions, they are very sensitive to the pricing of the annuities at retirement.

If a constant retirement age is considered, DB schemes are more negatively affected since their benefit formula is not linked to life expectancy in contrast to the DC schemes. In absence of risk-sharing mechanism a DB scheme would need twice their revenue to finance their pension expenditures by 2040 and individuals would receive around 50% more than they have contributed.

As expected, DC schemes perform better in terms of sustainability and fairness. Their good properties are less affected by the policy around retirement age while they are very sensitive to the assumptions made to calculate the annuity factor. In fact, when they do not account for longevity improvements they reach similar levels of (un)sustainability as the DB systems. Finally, it is noteworthy that despite controlling for liquidity, the risk-sharing mechanism linked to the indexation rate seems to achieve two goals at the same time: ensuring liquidity while improving the actuarial fairness substantially for the three systems.

The framework presented, despite being general and dynamic, is unable to address in its current form other policy issues such as inequality at retirement. We acknowledge that there are groups who are more at risk of inadequate retirement income such as women and individuals from low socio-economic status (James 2013). Furthermore, we assume that individuals have an homogeneous mortality experience, while in practice this is not the case and it may be the case that individuals earning higher incomes have a longer average lifespan (Kaplan et al. 1996; Madrigal et al. 2011). These issues can be considered as important directions for future research.

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A Proof Proposition 1

Let the initial pension for the DB and the DC with SD be represented by P_t^{DB,x_r} and P_t^{DC,x_r} respectively:

$$P_t^{DB,x_r} = K \cdot PS_t^{x_r} = K \cdot PS_{t-\Delta}^{x_r} (1+g) \tag{A.1}$$

$$P_t^{DC,x_r} = \frac{NC_t^{x_r}}{a_{x_r} N_t^{x_r}} = \frac{\pi \sum_{x=x_0}^{x_r-\Delta} \overbrace{W_{t-x_r+x}^{x} (1+g)^{x-xr}}^{W_t^x} \overbrace{N_{t-x_r+x}^{x} (1+n)^{x-xr}}^{N_t^x}}{a_{x_r} N_t^{x_r}} \tag{A.2}$$

Let the contribution rate π equal to the one that initially ensures the financial equilibrium under the DB scheme:

$$\pi = \frac{\sum_{x=x_r}^{\omega-\Delta} P_t^{DB,x} N_t^x}{\sum_{x=x_0}^{x_r-\Delta} W_t^x N_t^x}$$

In order to obtain the desired result, we need to rewrite the DB initial pension as follows:

$$P_t^{DB,x} = P_{t-x+x_r}^{DB,x_r} (1+\lambda)^{x-x_r} = P_t^{DB,x_r} \left(\frac{1+\lambda}{1+g}\right)^{x-x_r}$$

Then the contribution rate can be rewritten as follows:

$$\pi = \frac{P_t^{DB,x_r} \sum_{x=x_r}^{\omega-\Delta} \overbrace{N_t^x (1+n)^{x-x_r}}^{N_{t-x_r+x}^x} \left(\frac{1+\lambda}{(1+n)(1+g)}\right)^{x-x_r}}{\sum_{x=x_0}^{x_r-\Delta} W_t^x N_t^x}$$

Finally, if we replace the contribution rate in (A.2) we observe that the amount of the pension under the DC with SD is equal to the DB pension scheme:

$$\begin{aligned} P_t^{DC,x_r} &= P_t^{DB,x_r} \frac{\sum_{x=x_r}^{\omega-\Delta} N_{t-x_r+x}^x \left(\frac{1+\lambda}{(1+n)(1+g)}\right)^{x-x_r}}{\sum_{x=x_0}^{x_r-\Delta} W_t^x N_t^x} \frac{\sum_{x=x_0}^{x_r-\Delta} W_t^x N_t^x}{a_{x_r} N_t^{x_r}} \\ &= P_t^{DB,x_r} \underbrace{\frac{1}{a_{x_r}} \sum_{x=x_r}^{\omega-\Delta} \frac{N_{t-x_r+x}^x}{N_t^{x_r}} \left(\frac{1+\lambda}{(1+n)(1+g)}\right)^{x-x_r}}_{1} \end{aligned}$$

B Proof of Proposition 2

The expression for α_t^π is obtained by forcing the income from contributions C_t after RSM to be equal to the income from contributions before the application of the RSM reduced by the amount of the one-period buffer fund. Mathematically, this is expressed as follows:

$$\begin{aligned} C_t &= C_t^* - \beta_t D_t^*, \\ (1+\alpha_t^\pi) &= \frac{C_t^* - \beta_t D_t^*}{C_t^*} = 1 - \beta_t \frac{C_t^* - P_t^*}{C_t^*}, \\ \alpha_t^\pi &= \beta_t \left(\frac{1-LR_t^*}{LR_t^*}\right). \end{aligned}$$

The expression for α_t^λ is obtained in a similar manner by forcing the pension expenditures P_t after RSM to be equal to the pension expenditures before the application of the RSM reduced by the amount of the one-period buffer fund. Mathematically, this is expressed as follows:

$$P_t = P_t^* - (1-\beta_t)\, D_t^*,$$

$$P_t^{x_r(t)} N_t^{x_r(t)} + \sum_{x=x_r(t)+\Delta}^{\omega-1} P_{t-\Delta}^{x-1}\,(1+\lambda_t)\, N_t^x = P_t^* - (1-\beta_t)\, D_t^*,$$

Since the expression of the pension paid to the first generation retirees depends on whether the retirement age changes in the studied year, we will have two expressions for the RSM related to the indexation rate:

$$\alpha_t^\lambda = \begin{cases} \beta_t + (1-\beta_t)\, \frac{C_t^* - P_t^{x_r(t)} N_t^{x_r(t)}}{P_t^* - P_t^{x_r(t)} N_t^{x_r(t)}} - 1 & \text{if } x_r(t) = x_r(t-\Delta); \\ \beta_t + (1-\beta_t)\, \frac{C_t^*}{P_t^*} - 1 & \text{if } x_r(t) \neq x_r(t-\Delta). \end{cases}$$