A HIGH PRECISION LAMINATED ANISOTROPIC THIN SHELL FINITE STRIP

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1. INTRODUCTION

The demand for light weight high performance structures has grown rapidly. Laminated and sandwich type constructions are ideally suited for the use as light weight prefabricated roof structures owing to their high strength-to-weight and stiffness-to-weight ratios. Such structures are typical to those used in ferrocement, corrugated sheets and fiber reinforced roof structures.

One of the main characteristics of such composite structures is its heterogeneous and anisotropic nature which require analysis techniques incorporating the coupling between bending and membrane actions which usually exist in such systems [1].

The number of analytical methods which are capable of accurately predicting the response of such anisotropic laminated systems are limited and for a certain class of laminated systems, rigorous techniques do not existed. Finite strip method appears to have a great potential in predicting the static response of the anisotropic structures under consideration since the computer storage requirements for the finite strip method is relatively small. In addition, the incorporation of the anisotropic and coupling effects in the finite strip formulation are rather straight forward.

The finite strip method was first introduced by Cheung [2], but was limited to the analysis of isotropic structures. The method was extended by Ibrahim and Monforton [3], to the analysis of certain types of anisotropic laminated roof structures.

The objective of this paper is to extend the application of the finite strip method to predict the actual behaviour of some types of laminated structural systems considered herein. Numerical results are presented to illustrate the potential of the method for predicting the structural response of laminated shell roof structures.

2. GENERAL FORMULATION

In the classical lamination theory [1,4], the following assumptions (Kirchoff hypothesis) are made:

(1) Normals to the middle surface before deformation remain straight and normal after deformation. This is equivalent to ignore the shearing strains in planes perpendicular to the middle surface, that is, $\gamma_{xz} = \gamma_{yz} = 0$ where z is the normal direction, and γ_{xz} and γ_{yz} are shearing strains in planes xz and yz respectively.

(2) The normals are presumed to have constant length so that the strain perpendicular to the middle surface is ignored, that is $\varepsilon_z = 0$. Therefore, the normal deflection w at any point in the laminate is equal to the normal deflection of

the corresponding point on the mid plane.

The formulation presented herein follows the approach of Ref.[3] and in the following, the formulation is summarized for the application of the finite strip method to laminated anisotropic thin shells.

2.1 STRAIN-DISPLACEMENT AND FORCE-DEFORMATION RELATIONS.

Based on the previously mentioned assumptions, the strain-displacement relations for a point at a distance z from the middle surface of the Laminate are given by :

$$\{\varepsilon\} = \{\varepsilon^{o}\} + z\,\{K\} \qquad \ldots\ldots\ldots\ldots(1)$$

Where $\{\varepsilon^{o}\}$ contains the normal and shearing strains of the reference surface and $\{K\}$ contains the changes in curvature and angle of twist of the reference surface during deformation. The components of $\{\varepsilon^{o}\}$ and $\{K\}$ are given by:

$$\varepsilon_x^{o} = u_x \;;\quad \varepsilon_y^{o} = v_y + \frac{1}{R} w \;;\quad \gamma_{xy}^{o} = v_x + u_y \;;$$
$$K_x = -w_{xx} \;;\quad K_y = -w_{yy} + \frac{1}{R} v_y \quad \text{and} \quad K_{xy} = -2(w_{xy} - \frac{1}{R} v_x)$$

In which, ($u_x = \partial u / \partial x$, $v_y = \partial v / \partial y$, ...) are the derivatives of the tangential midplane displacements, u and v and R is the radius of curvature of the laminate. Inthe case of plane surface $R = \infty$.

For a laminate composed of m orthotropic laminae with arbitrary orientations and thicknesses (Fig. 1a and 1b), the force and moment resultants of the cross section are given by,

$$\begin{bmatrix} N \\ M \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline B & D \end{array}\right] \begin{bmatrix} \varepsilon^{o} \\ K \end{bmatrix} \qquad \ldots\ldots\ldots(2a)$$

Or;

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \left[\begin{array}{ccc|ccc} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} \\ & A_{22} & A_{23} & B_{12} & B_{22} & B_{23} \\ & & A_{33} & B_{13} & B_{23} & B_{33} \\ \hline & & & D_{11} & D_{12} & D_{13} \\ & \text{sym.} & & & D_{22} & D_{23} \\ & & & & & D_{33} \end{array}\right] \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ K_x \\ K_y \\ K_{xy} \end{bmatrix} \quad \text{.........(2b)}$$

where, $\{N\}^T$ are the membrane forces, $[N_x, N_y, N_{xy}]$; $\{M\}^T$ are the bending stress resultants, $[M_x, M_y, M_{xy}]$; and $\{\varepsilon^o\}$ and $\{K\}$ are as defined earlier. [A], [D] and [B] are the effective membrane, bending and bending-membrane coupling stiffness matrices of angle-ply unbalanced laminated composite section respectively [1]. In the case of a cross-ply laminated composite section composed of two layers (plies), in its simplest shape as shown in Fig. 1c ; some terms of the matrices [A], [B], and [D] are setting to zero:

$$A_{13} = A_{23} = B_{12} = B_{13} = B_{23} = B_{33} = D_{13} = D_{23} = 0.0 \quad \text{.....(3)}$$

3. FORMULATION OF STRIP STIFFNESS MATRIX

3.1 BASIC ASSUMPTIONS:

1- Thin laminated cylindrical shell finite strip is the main unit in the analysis. The flat plate strip is a special case of the shell strip and is used in plate and folded plate analysis (Fig. 2a and 2b).

2- In the finite strip analysis of laminated systems, the whole structure is divided into a number of longitudinal strips by lines which are called "nodal lines", (Fig. 2c).

3- The cylindrical shell roof structures considered in the present work are assumed to be simply supported at the ends on transverse diaphragms that are infinitely rigid in their own plane but perfectly flexible normal to their plane. For such end supports the following boundary conditions can be applied at X = 0 and L ;

$$v = 0 \quad ; \quad w = 0 \qquad \text{.............(4a)}$$

$$N_x = M_x = 0 \qquad \text{.............(4b)}$$

where v, w, N_x and M_x are as defined before.

3.2 DISPLACEMENT FUNCTIONS

Following Ref. [3], the displacement functions for the strip can be written as follows:

$$u = \sum_{n=1,2...}^{N} \sum_{j=1}^{2} [H_{0j}^{(1)}(y)u_j^{(n)} + H_{1j}^{(1)}(y)u_{j,y}^{(n)}] \cos\frac{n\Pi X}{L} \qquad \text{.......(5a)}$$

$$v = \sum_{n=1,2...}^{N} \sum_{j=1}^{2} [H_{0j}^{(1)}(y)v_j^{(n)} + H_{1j}^{(1)}(y)v_{j,y}^{(n)}] \sin\frac{n\Pi X}{L} \qquad \text{.......(5b)}$$

$$w = \sum_{n=1,2...}^{N} \sum_{j=1}^{2} [H_{0j}^{(2)}(y)w_j^{(n)} + H_{1j}^{(2)}(y)w_{j,y}^{(n)} + H_{2j}^{(2)}(y)w_{j,yy}^{(n)}] \sin\frac{n\Pi X}{L} \qquad \text{.......(5c)}$$

Where $H_{fj}^{(1)}$ and $H_{fj}^{(2)}$ are one-dimensional third- and fifth- order interpolation polynomials respectively defined in Appendix A. The undetermined coefficients ;

$(u_{j,y}^{(n)} = \partial u_j^{(n)} / \partial y,\ v_{j,y}^{(n)} = \partial v_j^{(n)} / \partial y,\ w_{j,y}^{(n)} = \partial w^{(n)} / \partial y$, and

$w_{j,yy}^{(n)} = \partial^2 w^{(n)} / \partial y^2$)

are derivatives of the displacement parameters ($u_j^{(n)}$, $v_j^{(n)}$, $w_j^{(n)}$) associated with the nth term of the basic functions at each of the two edges (j=1,2) of the strip. A total of 14 independent displacement coefficients (7 on each edge) are involved in the strip for each value of n.

For the boundary conditions prescribed (eq. 4), the displacement functions are orthogonal; therefore, the terms of the basic functions are uncoupled in the potential energy expression and solutions for each cycle can be obtained independently. In addition to satisfying the boundary conditions of eq. 4, the displacement functions proposed in eq.5, provide the capability of imposing membrane strain continuity as well as continuity of curvature at common edges of adjacent strips.

3.3 <u>STRIP POTENTIAL ENERGY</u>

The strain energy of a thin laminated shell strip i is given by the following integral over the strip ;

$$U_i = \frac{1}{2} \iint_s \{\{N\}^T\{\varepsilon^o\} + \{M\}^T\{K\}\} \, ds. \qquad \text{.........(6)}$$

Substituting from eqn 2 into eqn 6;

$$U_i = \frac{1}{2} \iint_s \{\{\varepsilon^o\}^T[A]\{\varepsilon^o\} + \{\varepsilon^o\}^T[B]\{K\} + \{K\}^T[B]\{\varepsilon^o\} + \{K\}^T[D]\{K\}\} \, ds. \qquad \text{.........(7)}$$

in which s = the middle surface area of the laminated surface.

The potential of the loads applied to the face of the strip can be expressed as :

$$W_i = \iint_s [Xu + Yv + Zw] \, ds. \qquad \text{.........(8)}$$

in which X, Y, and Z are the components of the loads applied to the strip and are generated using a work equivalent load approach and u,v,and w are the corresponding displacement functions.

The resulting total potential energy of the laminated system is : $\Pi_{pi} = (U_i - W_i)$ ....(9)

Due to the orthogonal properties of the displacement functions, Π_{pi} may be represented as:

$$\Pi_{pi} = \sum_{n=1,2...}^{N} \Pi_{pi}^{(n)} \qquad \text{........(10a)}$$

in which $\Pi_{pi}^{(n)} = U_i^{(n)} - W_i^{(n)}$ ....(10b)

Eq. 10b represents the potential energy associated with the nth term of the displacement functions for strip i, $U_i^{(n)}$ and $W_i^{(n)}$ refer to strain energy and potential energy of the external loads for strip i, respectively.

By integrating eqn. 7 over the surface of the strip i, it takes the form:

$$U_i = \sum_{n=1,2...}^{N} U_i^{(n)} \qquad \text{........(11a)}$$

in which $U_i^{(n)} = \frac{1}{2} \underset{(1X14)}{\{d_i^{(n)}\}^T} \underset{(14X14)}{[K_i^{(n)}]} \underset{(14X1)}{\{d_i^{(n)}\}}$ ....(11b)

where $[K_i^{(n)}]$ refers to the stiffness matrix for one strip associated with the nth term of the displacement functions and $\{d_i^{(n)}\}$ is the corresponding vector of undetermined displacement coefficients for the strip associated with the nth term of the displacement functions ; thus:

$$\{d_i^{(n)}\} = \{u_j^{(n)}, u_{j,y}^{(n)}, v_j^{(n)}, v_{j,y}^{(n)}, w_j^{(n)}, w_{j,y}^{(n)}, w_{j,yy}^{(n)}\}^T, \; j=1,2 \qquad \text{....(12)}$$

Similarly by integrating eqn. 8 over the surface of the strip i, it takes the form:

$$W_i = \sum_{n=1,2...}^{N} W_i^{(n)} \qquad \text{.......... (13a)}$$

in which $W_i^{(n)} = [P_i^{(n)}]^T \{d_i^{(n)}\}$ .... (13b)

$\qquad\qquad\qquad\qquad (1X14) \quad (14X1)$

where $[P_i^{(n)}]$ refers to the work equivalent load vector matrix and $\{d_i^{(n)}\}$ is as defined previously.

Substituting eqns. 11b and 13b into eqn. 10b, it takes the form:

$$\pi_{pi}^{(n)} = \frac{1}{2} \underset{(1X14)}{\{d_i^{(n)}\}^T} \underset{(14X14)}{[K_i^{(n)}]} \underset{(14X1)}{\{d_i^{(n)}\}} - \underset{(1X14)}{\{P_i^{(n)}\}^T} \underset{(14X1)}{\{d_i^{(n)}\}} \quad \ldots(14)$$

The total potential energy, Π_p, of an assemblage of I laminated shell roof strips can be obtained by invoking the appropriate continuity conditions of displacement, slope, strain, and curvature between adjacent strips and satisfying the boundary conditions along the longitudinal edges of the structure. It can be expressed as;

$$\Pi_p = \sum_{n=1,2\ldots}^{N} \Pi_p^{(n)} \quad \ldots\ldots\ldots\ldots(15a)$$

in which

$$\Pi_p^{(n)} = \sum_{i=1,2\ldots}^{I} \pi_{pi}^{(n)} = \frac{1}{2} \underset{(1xm)}{\{D^{(n)}\}^T} \underset{(mxm)}{[K^{(n)}]} \underset{(mx1)}{\{D^{(n)}\}} - \underset{(1xm)}{\{P^{(n)}\}^T} \underset{(mx1)}{\{D^{(n)}\}} \quad \ldots\ldots(15b)$$

Eq. 15b represents the total potential energy of the laminated cylindrical shell structure associated with the nth term of the displacement functions ; $[K^{(n)}]$ and $\{P^{(n)}\}$ are the master stiffness matrix and load vector, assembled from the strip stiffness matrix and work equivalent load vector respectively ; and $\{D^{(n)}\}$ represents the vector containing the m independent degrees of freedom associated with the assembled structure.

3.4 MASTER STIFFNESS MATRIX

Applying the minimum total potential energy theorem leads to: $[K^{(n)}] \{D^{(n)}\} = \{P^{(n)}\}$ ....(16)
Equation 16 is the overall matrix equation which represents a set of linear simultaneous equations relating all the unknown parameters to the applied loads.

As mentioned previously, the master stiffness matrix for any one term of the series (basic functions) $[K^{(n)}]$ can be assembled quite easily by the Variable Correlation Table scheme, which is a procedure used for numbering Degrees of Freedom, NDF. With this procedure, the boundary conditions for nodal lines of strips can be set automatically during the assemblage of master stiffness matrix. This matrix has narrow half-band width, so that it can be stored in the memory as a rectangular array of N X HB (N is the total number of unknowns for one term and HB is the half band width) and solved directly by band matrix solution techniques. Solution of eqn. 16 for $\{D^{(n)}\}$ and substitution of the results into eqn.5 give the required solutions for displacements. Differentiation of the results and substitution into eqn. 2 give the force and moment values for one term of the series. The results for each of the various terms of the series are summed to give the final displacements and internal forces.

4. NUMERICAL EXAMPLES

4.1. SIMPLY SUPPORTED SQUARE PLYWOOD PLATE

In order to verify that the above formulations can be applied to analyze laminated balanced plate structures as a special class of shell structures, a simply supported plywood plate which was analyzed previously by Timoshinko [5] was reanalyzed again using the proposed method. The plate is subjected to a transverse distributed load of 100 Ib/in^2. Loading, plate dimensions, and finite strip simulation can be seen in Fig. 3a. The type of plywood material used in this example is Maple, 5-ply and the elastic properties of this material are: E_{11}=1.87 X 10^6 psi; E_{22}= 6 X 10^5psi; and G = 1.59 X 10^5 psi. Where $E_{11}$is the material modulus of elasticity in X - direction, E_{22} is the modulus of elasticity in Y- direction and G is the material shear modulus. The resulting stiffnesses for the plywood material constructed of 5-plies (Fig. 3b) are as follows:

1. Membrane stiffnesses :

A_{11}= 1.87 X 10^5 Ib/in. ; A_{12}= 7.3 X 10^3 Ib/in. ;
A_{22}= 6. X 10^4 Ib/in. ; A_{33}= 1.59 X 10^4 Ib/in.

2. Bending stiffnesses :

D_{11}= 1.56 X 10^2 in.-Ibs ; D_{12}= 6.08 in.-Ibs ;
D_{22}= 50.0 in.-Ibs ; D_{33}= 13.25 in.-Ibs

One should observe that there is no coupling between membrane and bending actions in laminated balanced sections thus, the elements of the coupling stiffness matrix [B] will be equal to zero.

In order to assess the coupling effect in laminated unbalanced sections, the simply supported plywood plate was reanalyzed for the same dimensions and loading but with 4-plies only, $0^o/+90^o/0^o/+90^o$. The resulting stiffnesses for this case are as follows:

1. Membrane stiffnesses :

A_{11}= 1.55 X 10^5 Ib/in. ; A_{12}= .6 X 10^5 Ib/in. ;

$A_{22} = 1.55 \times 10^5$ Ib/in. ; $A_{33} = 1.59 \times 10^4$ Ib/in.

2. Bending stiffnesses :

$D_{11} = 1.29 \times 10^2$ in.-Ibs ; $D_{12} = 50.34$ in.-Ibs ;

$D_{22} = 1.29 \times 10^2$ in.-Ibs ; $D_{33} = 13.25$ in.-Ibs

3. Coupling stiffnesses :

$B_{11} = -998.86$ Ibs ; $B_{22} = -B_{11}$

For analysis, 4 finite strips were used as illustrated in Fig. 3a. The vertical deflection, w, longitudinal moment, M_x, and transverse bending moment, M_y, at the center of the span are given in Table 1 together with the results of Ref. [5]. In this table, L_o refers to the results obtained from the laminated balanced plate, L_1 refers to the results obtained from the laminated unbalanced plate when the coupling stiffnesses are considered and L_2 represents those results when the coupling stiffnesses are neglected. The variation in deflection quantities across the width of the plate at its midspan together with the results of Ref. [5] are shown in Fig. 3c.

A comparison of these results indicates that, in general, an excellent agreement exists between the predictions of the proposed method and that of the other method of Ref. [5] for laminated balanced case. It is observed that the effect of coupling in laminated unbalanced plate is to reduce the overall deflection, w, longitudinal moment, M_x, and to increase the transverse moment, M_y (M_x and M_y will be nearly the same). The proposed model utilizes 4 strips with only 29 degrees-of-freedom and 7 harmonic cycles. The execution time on the PC 88 IBM 640 KB is 7 minutes.

TABLE 1. EXAMPLE 1- Deflection, Longitudinal and Transverse Moments for Plywood Plate

Method of analysis	Deflection w (inches) x=a/2, y=b/2	Longitudinal moments Mx (Ib.in/in) x=a/2, y=b/2	Transverse moments My (Ib.in/in) x=a/2, y=b/2
L_o	-60.013	-918.0	-295.82
Ref. [5]	-60.0	-912.14	-293.95
L_1	-41.63	-645.8	-646.63
L_2	-39.51	-651.71	-652.54

4.2. LAMINATED UNBALANCED CYLINDRICAL SHELL ROOF

In order to show the capability of the proposed finite strip model to analyze cylindrical shells, two examples indicating the effect of the material strength on the coupling between membrane and bending actions in cross-ply laminated composites were solved. The two investigated examples have the same cross-section and dimension as shown in Fig. 4a.

The structure shown in Fig. 4 is 60 in. long and is constructed of cross-ply laminated composite section. The material of composites used in the first example (Ex. 2) consists of two graphite-epoxy plies, each of 0.125 in. thick. The elastic properties of which are E_{11}= 30 X 10^6 psi; E_{11}/E_{22}= 40; G_{12}/E_{22}= 1; and ν_{12}=0.25. The total thickness of the cross section is 0.25 in. and the resulting stiffnesses for the two lay-ups considered in Fig. 1c are as follows;

1. Membrane stiffnesses :

A_{11}= 3.85 X 10^6 Ib/in. ; A_{12}= 4.69 X 10^4 Ib/in. ;
A_{22}= 3.85 X 10^6 Ib/in. ; A_{33}= 1.88 X 10^5 Ib/in.

2. Bending stiffnesses :

D_{11}= 2.01 X 10^4 in.-Ibs ; D_{12}= 2.45 X 10^2 in.-Ibs ;

D_{22}= 2.01 X 10^4 in.-Ibs ; D_{33}= 9.77 X 10^2 in.-Ibs

3. <u>Coupling stiffnesses</u> :

$B_{11} = \pm$ 3.3 X 10^5 Ibs

in which the $\pm$ sign of B_{11} is shown in Fig. 1c, and $B_{22} = -B_{11}$

The elastic properties of the material of the second example (Ex. 3) which consists of two glass-epoxy plies are E_{11}= 7.5 X 10^6 ; E_{11}/E_{22}= 3 ; G_{12}/E_{22}= 0.4 ; and ν_{12}= 0.25. The resulting stiffnesses for this case are as follows :

1. <u>Membrane stiffnesses</u> :

A_{11}= 1.28 X 10^6 Ib/in. ; A_{12}= 1.6 X 10^5 Ib/in. ;

A_{22}= 1.28 X 10^6 Ib/in. ; A_{33}= 2.5 X 10^5 Ib/in.

2. <u>Bending stiffnesses</u> :

D_{11}= 6.65 X 10^3 in.-Ibs ; D_{12}= 8.34 X 10^2 in.-Ibs ;

D_{22}= 6.65 X 10^3 in.-Ibs ; D_{33}= 1.3 X 10^3 in.-Ibs

3. <u>Coupling stiffnesses</u> :

$B_{11} = \pm$ 3.99 X 10^4 Ibs ; $B_{22} = -B_{11}$

The shell roof is subjected to a load of 1 Ib/in^2 of projected area as shown in Fig. 4a. Taking symmetry into consideration, one-half of the shell was modeled using 5 strips (Fig. 4b). The longitudinal edge (ϕ = 0) was considered free while the symmetry conditions were imposed at the crown (ϕ = 30):

$$v = -\frac{\partial w}{\partial y} = 0 \qquad \text{.......................} \quad (17)$$

where v and $\partial w/\partial y$ are the in-plane displacement in the direction y and the rotation around x axis respectively.

Results of the longitudinal force, N_x, circumferential force resultants, N_ϕ, and transverse moment, M_ϕ, at midspan and the shear force resultant, $N_{x\phi}$, at the supports are plotted against ϕ in Fig. 5 for Ex. 2 (Graphite-Epoxy laminated shell roof). It has to be mentioned that in Fig. 5, $N_x^{(L_o)}$, $N_\phi^{(L_o)}$, $N_{x\phi}^{(L_o)}$, and $M_\phi^{(L_o)}$ refer to the results obtained when the coupling stiffnesses are neglected and $N_x^{(L_1)}$, $N_\phi^{(L_1)}$, $N_{x\phi}^{(L_1)}$, $M_\phi^{(L_1)}$, $N_x^{(L_2)}$, $N_\phi^{(L_2)}$, $N_{x\phi}^{(L_2)}$, and $M_\phi^{(L_2)}$ correspond to the lay-ups shown in Fig. 1c. Table 2 and 3 comprise comparisons between the results of the midspan longitudinal force, N_x and transverse moment, M_ϕ of examples 2 and 3.

Inspection of Tables 2 and 3 indicates that the effects of the coupling stiffnesses are generally small for glass-epoxy material (week material) while being significant for graphite-epoxy material (strong material). The biggest influence in this last case lies in the longitudinal force resultant, N_x, for lay-up L_1. The results plotted in Fig. 5, indicate that the coupling effect is significant for the transverse moments, especially for lay-up L_1, where the orthotropic solution (L_o) underestimates this moment resultant at the crown by approximatly 12 % .

TABLE 2. - Mid-span Longitudinal Force Resultant (Nx)

ϕ, deg.	EXAMPLE 2 - Graphite-epoxy			EXAMPLE 3 - Glass-epoxy		
(from edge)	$N_x^{(L_o)}$, in pounds/inch	$N_x^{(L_1)}/N_x^{(L_o)}$	$N_x^{(L_2)}/N_x^{(L_o)}$	$N_x^{(L_o)}$, in pounds/inch	$N_x^{(L_1)}/N_x^{(L_o)}$	$N_x^{(L_2)}/N_x^{(L_o)}$
30	-551.63	0.96	0.97	-604.89	0.99	1.0
24	-502.90	0.98	0.99	-537.66	0.99	1.0
18	-341.98	1.04	1.04	-330.99	1.0	1.0
12	-23.62	2.27	2.09	29.18	0.75	1.07
6	528.38	0.98	0.99	563.49	0.99	1.0
0	1413.3	1.05	1.05	1297.4	1.01	1.0

lb/in = 1.7513 X 10^2 N/m

TABLE 3. - Mid-span Transverse Moment (M_ϕ)

ϕ, deg. (from edge)	EXAMPLE 2 - Graphite-epoxy			EXAMPLE 3 - Glass-epoxy		
	$M_\phi^{(L_o)}$, in lb.in/in.	$M_\phi^{(L_1)}/M_\phi^{(L_o)}$	$M_\phi^{(L_2)}/M_\phi^{(L_o)}$	$M_\phi^{(L_o)}$, in lb.in/in.	$M_\phi^{(L_1)}/M_\phi^{(L_o)}$	$M_\phi^{(L_2)}/M_\phi^{(L_o)}$
30	19.46	1.12	0.88	19.5	1.04	0.93
24	17.62	1.13	0.87	17.64	1.04	0.93
18	12.85	1.13	0.86	12.74	1.05	0.92
12	6.8	1.13	0.83	6.65	1.05	0.91
6	1.83	1.12	0.75	1.71	1.06	0.86
0	-0.1	1.02	0.0	-0.1	0.0	1.25

lb. in/in = 4.448 N. m/m

5. Conclusions

The main advantage of the proposed model lies in its simplicity, its high accuracy, and to a certain extent, its versatility. The time and effort required to obtain a solution on a personal computer of relatively limited size is minimal in comparison to the other methods.

The results of the treated examples indicate that the effect of the coupling stiffnesses in laminated unbalanced sections may be significant and that the influence of this coupling decreases rapidly with the increase in the number of layers (plies) while it increases in sections of layers made of strong materials. Using the proposed model, the convergence of the results for stresses and displacements can be obtained with a reasonable limited number of strips and terms of the basic displacement functions.

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APPENDIX A-INTERPOLATION POLYNOMIALS

The definition of the interpolation polynomials is as follows:

A.1- Third-order polynomials:

$$H_{01}^{(1)}(y) = \frac{b^3 - 3by^2 + 2y^3}{b^3} \quad \text{.............. (A.1a)}$$

$$H_{02}^{(1)}(y) = \frac{3by^2 - 2y^3}{b^3} \quad \text{.............. (A.1b)}$$

$$H_{11}^{(1)}(y) = \frac{b^2y - 2by^2 + y^3}{b^2} \quad \text{..............(A.1c)}$$

$$H_{12}^{(1)}(y) = \frac{-by^2 + y^3}{b^2} \quad \text{..............(A.1d)}$$

A.2- Fifth-order polynomials:

$$H_{01}^{(2)}(y) = \frac{b^5 - 10b^2y^3 + 15by^4 - 6y^5}{b^5} \quad \text{..............(A.2a)}$$

$$H_{02}^{(2)}(y) = \frac{10b^2y^3 - 15by^4 + 6y^5}{b^5} \quad \text{..............(A.2b)}$$

$$H_{11}^{(2)}(y) = \frac{b^4y - 6b^2y^3 + 8by^4 - 3y^5}{b^4} \quad \text{..............(A.2c)}$$

$$H_{12}^{(2)}(y) = \frac{-4b^2y^3 + 7by^4 - 3y^5}{b^4} \quad \text{..............(A.2d)}$$

$$H_{21}^{(2)}(y) = \frac{\frac{1}{2}\,(b^3y^2 - 3b^2\,y^3 + 3by^4 - y^5)}{b^3} \quad \text{..............(A.2e)}$$

$$H_{22}^{(2)}(y) = \frac{-\frac{1}{2}(b^2y^3 - 2by^4 + y^5)}{b^3} \quad \text{..............(A.2f)}$$

where for the cylindrical shell strip

$$y = R\,\phi \quad \text{and} \quad b = R\,(\Delta\,\phi) \quad \text{...............(A.3)}$$

APPENDIX B- NOTATION

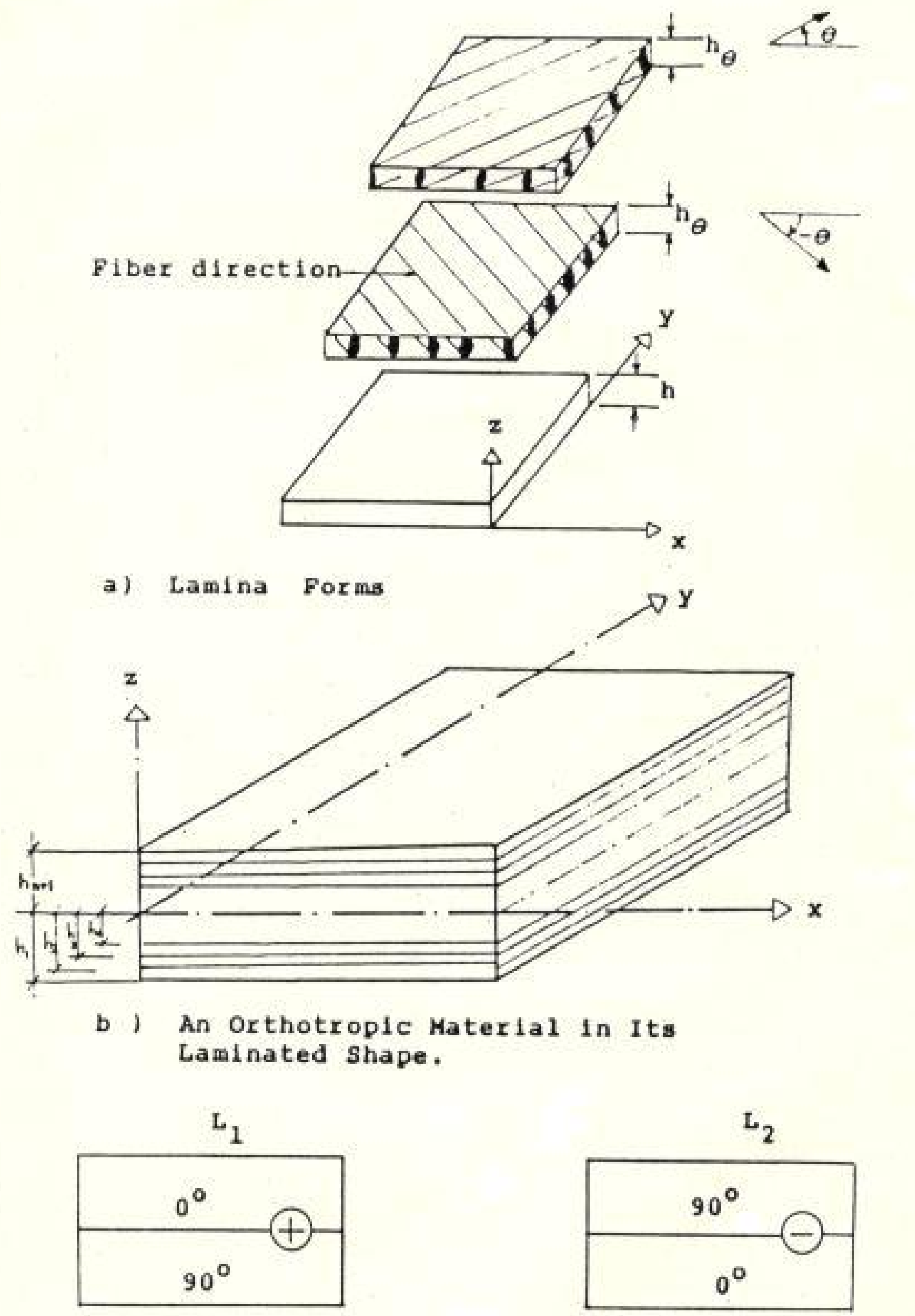
The following symbols are used in this paper:

A_{IJ}	Membrane stiffnesses of laminated cross-section.
B_{IJ}	Coupling stiffnesses of anisotropic laminates.
D_{IJ}	Bending stiffnesses of laminated cross-section.
$E_{11}, E_{22}, G_{12}, \upsilon_{12}$	Elastic constants of each layer of the laminates.
$H_{0j}^{(1)}, H_{1j}^{(1)}$	Third-order interpolation polynomials
$H_{0j}^{(2)}, H_{1j}^{(2)}, H_{2j}^{(2)}$	Fifth-order interpolation polynomials
M_x, M_y, M_{xy}	Moment resultants of laminated plate
N_x, N_y, N_{xy}	Force resultants of laminated plate
b	Width of finite strip
h	Total thickness of the cross-section
h_k	Distance between the kth layer and the middle surface of the cross-section.
L	Length of finite strip
m	Number of layers
n	Denotes nth term of basic functions
R	Radius of curvature of cylindrical strip.
u, v, w	Displacement components of a plate in local strip coordinate system.
$\bar{u}, \bar{v}, \bar{w}$	Displacement components of a plate in reference coordinate system.
x, y, z	Local strip coordinate system
x, η, ζ	Reference coordinate system

Subscripts and Superscripts:

i	Subscript or superscript denoting ith strip.
j	Subscript denoting two edges of strip (j =1, 2).
k	Subscript denoting kth layer

Additional symbols used are explained in the text.

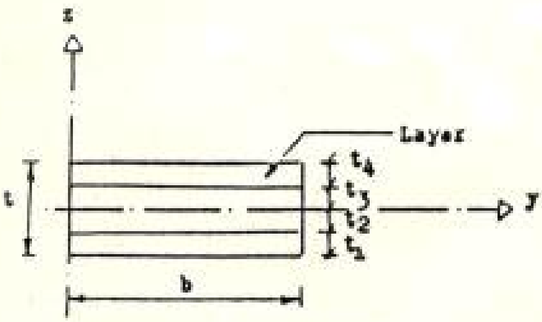


a) Lamina Forms

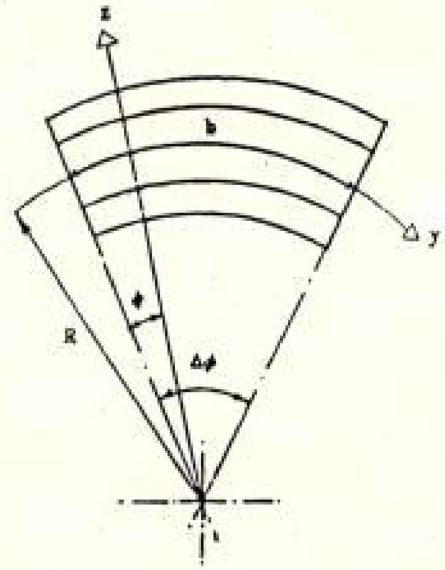
b) An Orthotropic Material in Its Laminated Shape.

c) Cross-ply Laminated System in its Simplest Shape (⊕ and ⊖ indicate the sign of B_{11}

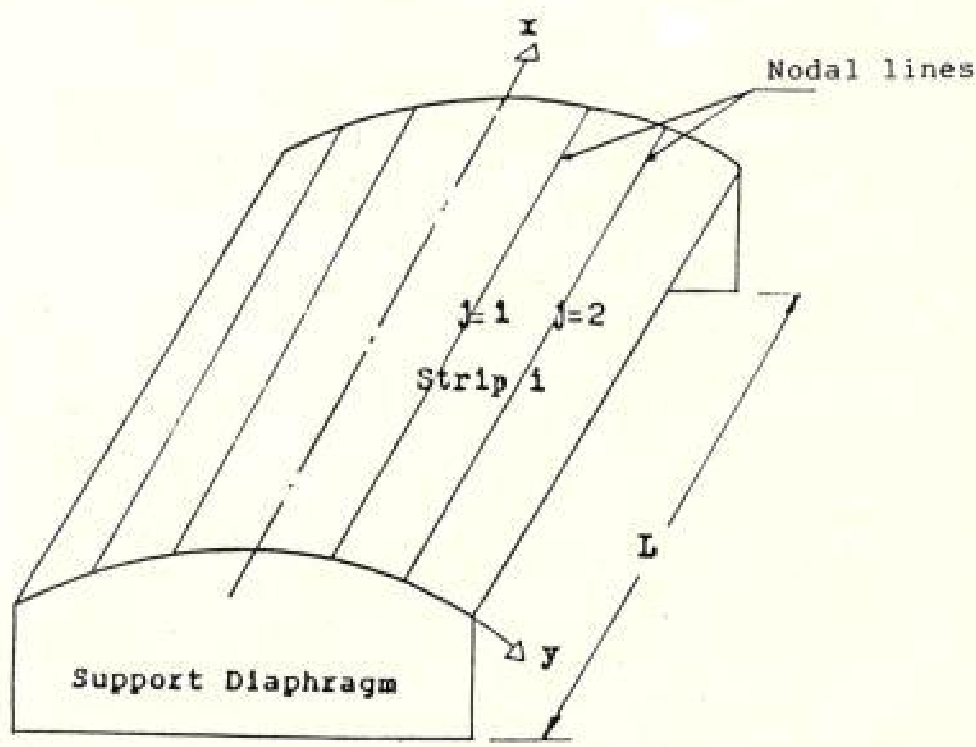
Fig. 1 LAMINATED COMPOSITE CROSS SECTION.



a) Laminated Plate Strip

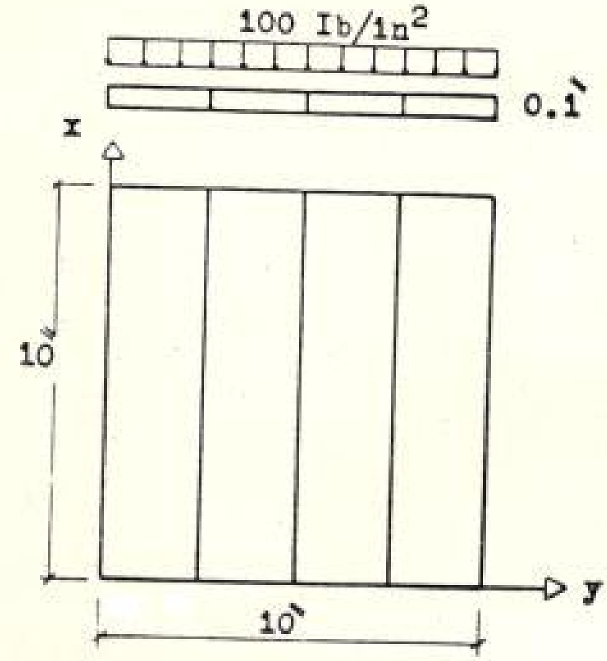


b) Laminated Cylindrical Strip

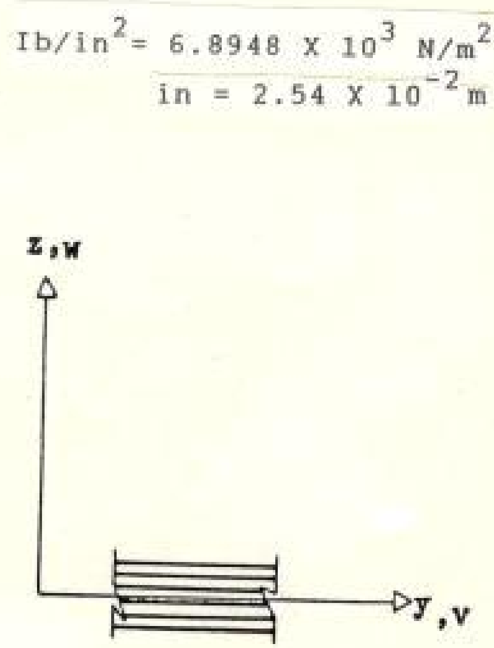


c) Cylindrical Shell Roof

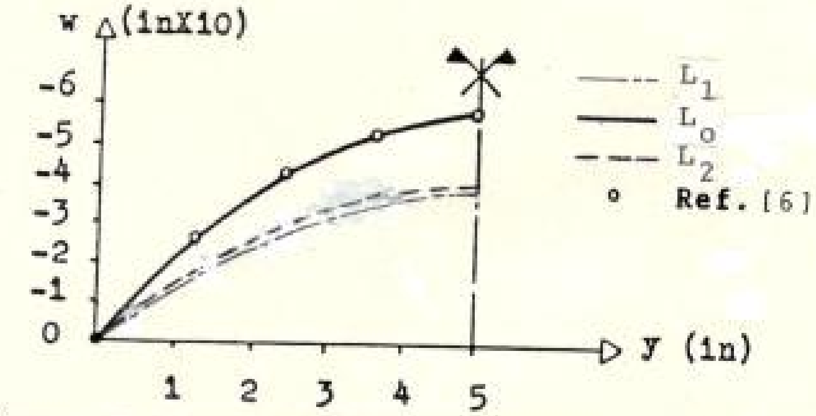
Fig. 2 LAMINATED ROOF SYSTEMS.



a) Simply Supported Plate and its Finite Strip Simulations (4 strips)

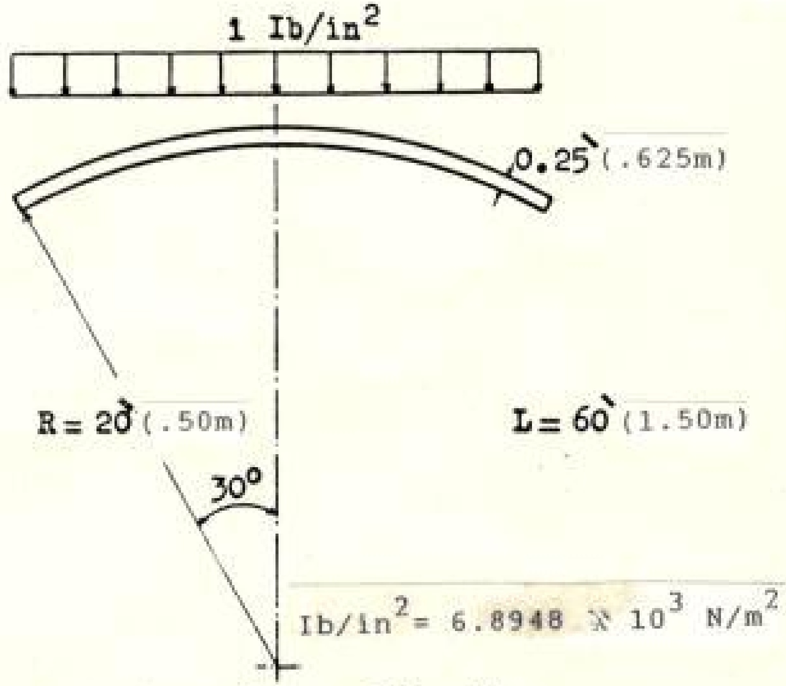


b) Cross-section of Maple, 5-ply

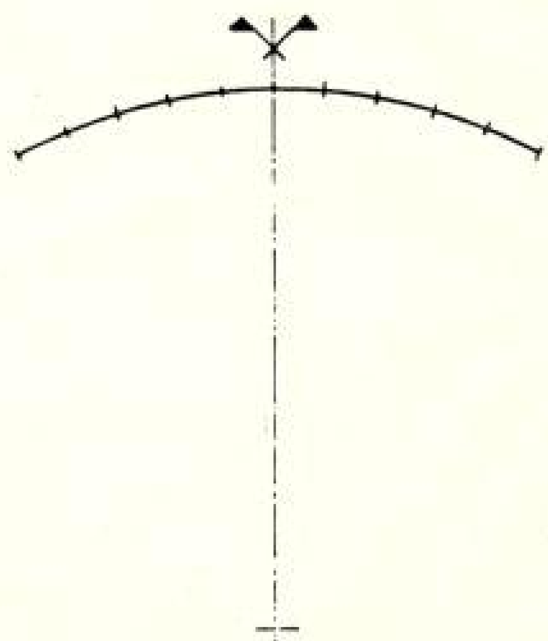


c) Midspan Deflection Distribution at $(x = \frac{a}{2})$

Fig 3 EXAMPLE 1 - SIMPLY SUPPORTED PLYWOOD PLATE.

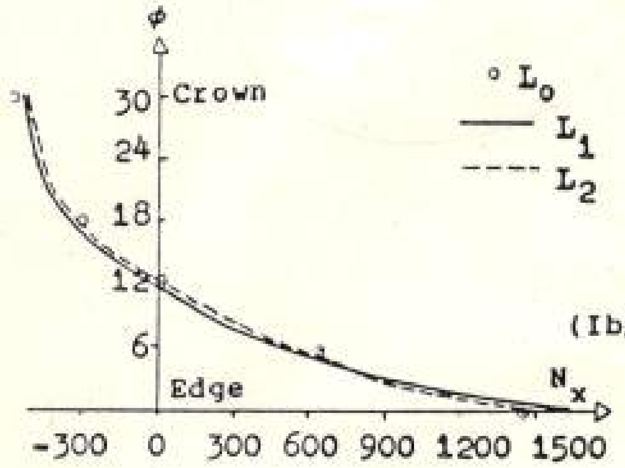


a) Dimensions and Loading

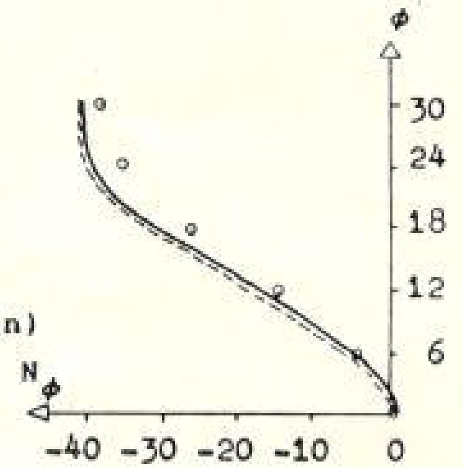


b) Finite Strip Simulations (5 strips in half of the shell)

Fig. 4 EXAMPLES 2 AND 3 - LAMINATED UNBALANCED SHELL ROOF STRUCTURE.



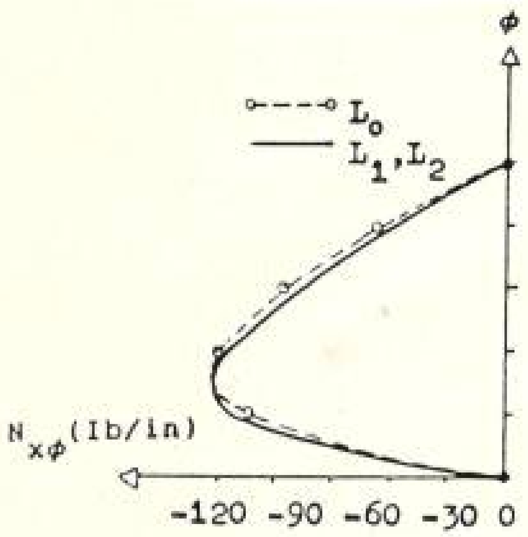
a) Longitudinal Force Resultants at Midspan.



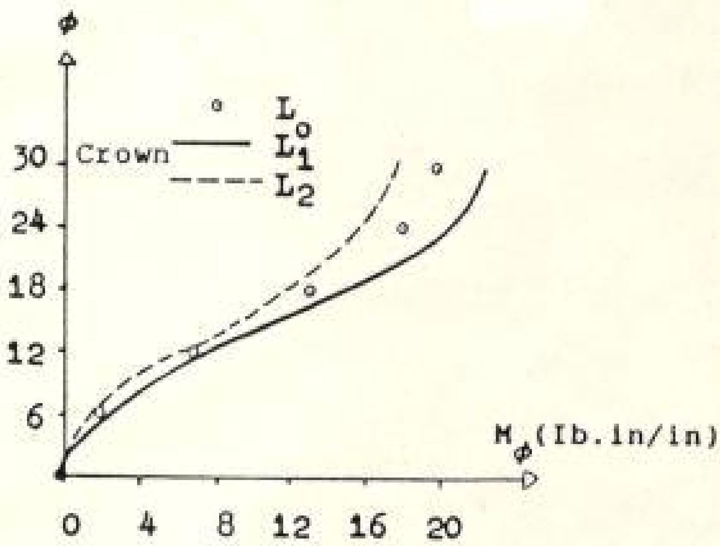
b) Circumferential Force Resultants at Midspan.

Ib/in = 1.7513 X 10^2 N/m

Ib.in/in = 4.448 N.m/m



c) Shear Stress Resultants at Supports.



d) Transverse Bending Moment at Midspan.

Fig. 5 EXAMPLE 2 - STRESS AND MOMENT RESULTANTS FOR GRAPHITE-EPOXY LAMINATED SHELL ROOF.