Information Aggregation through Stock Prices and the Cost of Capital

by

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This paper studies a firm's optimal capital structure in an environment where the firm's stock price serves as a public signal for its default risk. In equilibrium, the number of traders who find it profitable to trade the firm's stock increases as the firm issues more equity. In turn, the precision with which the stock price communicates the firm's fundamental to bond investors increases in the number of equity investors. Thus, through its capital structure, firms can internalize the informational externality that stock prices exert on bond yields. Strong firms therefore issue equity to reduce borrowing costs.

Keywords: information aggregation, capital cost, sequential markets, market depth

JEL classification code: C72, D53, G32

1 Introduction

Lehman's 2008 bankruptcy may have come as a surprise to those bondholders who believed in its A-ratings. It was less of a surprise to the bondholders who observed that Lehman's stock price had fallen from 62.19 at the beginning of 2008 to its 3.65 low on September 12, 2008, the day before Lehman filed for bankruptcy. Similarly, AIG's A-ratings were not more informative during the days before its bailout. The stock price, which had fallen by more than 90 percent in that year, provided a more

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informative signal. In both cases, the stock price served as a timely, costless, and arguably unbiased monitoring device for bondholders.

We study a firm's optimal capital structure choice in models with default and asymmetric information. Our model shows that the informational content of publicly observable prices is a crucial aspect that shapes a firm's financing decisions. In the present model, firms choose their capital structure to internalize the informational externality that stock prices have on equilibrium bond yields. Our main finding is that the informational spillover from stock price signals to bond yields makes it optimal for a firm that is financially strong to issue more equity and less debt than it would if it were financially weak. This finding relies on a positive relation between the informativeness of the firm's stock price and the amount of equity issued: as the firm issues more equity, it incentivizes more equity investors to research and trade the firm's stock. In turn, the firm's stock price becomes more informative and communicates the firm's strong financial situation more clearly to bond investors, who use the information contained in the stock price to calculate the firm's default risk and the corresponding equilibrium bond yield. In contrast with models where the firm's capital structure choice communicates insider knowledge to outside investors,[1] the present paper analyzes how different capital structures facilitate the information exchange *between* outside investors.

We use a stylized framework where a firm issues bonds B and sells a number of shares K to raise an exogenously given revenue I. The firm's objective is to minimize its cost of capital. The model is sequential, and at the beginning of time, the firm announces a capital structure (K, B). Subsequently, the stock (equity) market opens and the shares K are sold to risk-averse investors at a market-clearing price p. Stock investors observe private signals about the firm's fundamental θ, where θ determines the payoffs on the firm's equity and debt. The stock price then aggregates and publicizes the investor's private information and partially reveals the firm's strength θ. Subsequently, bond investors use the stock price p to calculate the firm's conditional default probability. In turn, bonds are traded at the corresponding market-clearing interest rate r. The firm's true strength θ is revealed in the final period where bond and equity returns materialize. Bonds pay a net return r if the fundamental is strong ($\theta \geq 0$). Otherwise, the firm declares bankruptcy and bond investors take a loss.

1.1 Related Literature

In terms of the survey on capital structures by Harris and Raviv (1991), the present analysis is closest to models of asymmetric information, where the firm's choice of a capital structure transmits insider information to outside investors. However, instead of a transfer of information from insiders to outsiders, the present analysis focuses on the transfer of information *between* outsiders, i.e., bond and stock investors.

[1] See Harris and Raviv (1991, pp. 306–315), for a survey of models where various aspects of asymmetric information shape optimal capital structures.

Closest to our model, Harris and Raviv (1990) develop a capital structure model where the firms' indebtedness influences information revelation.[2] Harris and Raviv (1990) assume that firms use a technology to produce output that is unknown to outside investors. However, outside investors do observe whether the firm can service its debt. That is, if a deeply indebted firm can meet its obligations, outside investors learn that it must be very productive. Accordingly, strong firms indebt themselves to communicate their strength. The opposite is true in our model, where a stock price, which aggregates dispersed private information, publicizes the firm's strength. That is, unlike Harris and Raviv (1990), we argue that the stock, rather than the bond, market is the main source of public information. This assumption is validated by a large empirical literature that shows that the stock market leads bond market returns: Kwan (1996), Forte and Peña (2009), Longstaff, Mithal, and Neis (2003), Norden and Weber (2009). In our two-period model, this is captured by the assumption that the stock market opens earlier than the bond market. Regarding this stock market, we employ the standard noise-trader models, Grossman and Stiglitz (1976, 1980), Green (1975), Hellwig (1980), and Vives (2008), which emphasize the aggregation of information.

Our paper is also related to the literature on security design. In particular, Boot and Thakor (1993) study a firm's incentive to divide its revenue stream into a risky "information sensitive equity claim" and a safe "information insensitive debt claim." The main similarity to the current model lies in the prediction that strong firms design their securities to amplify the informativeness of the price at which risky claims trade.[3] Firms in the model of Boot and Thakor (1993) cannot default. Put differently, the probability of a firm defaulting on its debt is always zero, and thus independent of the stock price. The effects of stock prices on default probabilities, which we study, are therefore excluded by construction in Boot and Thakor (1993). Finally, Che and Sethi (2013) study the cost of risky debt in models where agents use derivatives to trade default risks.

Our model generates several testable implications. We show that firms that believe that their fundamental is strong will issue equity to communicate their financial strength to investors. Namely, strong firms should operate with low leverage. This is consistent with the findings of Chadha and Sharma (2015), Pouraghajan et al. (2012), Olokoyo (2013), Twairesh (2014), Ahmed Sheikh and Wang (2013),

[2] See also Calomiris and Kahn (1991) for a related model. Admati et al. (2010, pp. 28–31) discuss the governance and informational role of a firm's debt. Albagli, Hellwig, and Tsyvinski (2011) develop a model where a firm interacts with stock prices that aggregate the investors' dispersed private information. In their model, however, firms issue no debt.

[3] Boot and Thakor (1993) discuss a noise-trader model with risk-neutral investors who can decide to purchase fully revealing private information on the firm's fundamental. This fundamental can take a high value $\bar{x} > 0$ or a low value $\underline{x} > 0$. The firm decides whether to sell one risky asset that yields either a high value $\bar{x} > 0$ or a low value $\underline{x} > 0$. Alternatively, it can sell a safe asset that always yields $\underline{x} > 0$, and a risky asset that yields either $\bar{x} - \underline{x} > 0$ or nothing. The safe asset always pays $\underline{x}$, and thus trades at price $\underline{x}$ regardless of the informational content of the risky asset's price.

and Mireku, Mensah, and Ogoe (2014), who identify a significant and negative correlation between leverage and firm performance in emerging markets. Similarly, our result – that stock prices of firms with a large stock market capitalization predict future earnings and dividends better than stock prices of firms with a small market capitalization – is in line with the empirical findings of Collins, Kothari, and Rayburn (1987).

The importance of the information channel described in the current paper should vary across firms. Namely, the stock price signal should be less important for firms whose financial strength is closely correlated with other observable market indicators. The car industry may be seen as an example where a firm's performance relies heavily on the overall business sentiment. Moreover, in the case of, e.g., Ford, GM, and Chrysler, the situation of, say, Ford can also be inferred from the stock prices of its two peers. Hence, the role of Ford's stock price is more subdued. Conversely, the effect described here should be of great importance to firms that produce specialized products. Indeed, Titman and Wessels (1988) find that firms with unique or specialized products have relatively low debt ratios.[4] Finally, Baker and Wurgler (2002) show that low-leverage firms are those that raised funds when their market valuations were high.[5] Conversely, highly leveraged firms raised funds when their market valuations were low. This observation is consistent with the negative relation between financial strength and leverage that our model implies.

The rest of the paper is organized as follows. In section 2, we outline the model. In section 3, we derive the main results. In a separate section 4, we comment on our assumptions and replace some of them to demonstrate the robustness of our findings. Finally, we present a more general specification for the bond market. Section 5 offers concluding remarks.

2 *Model*

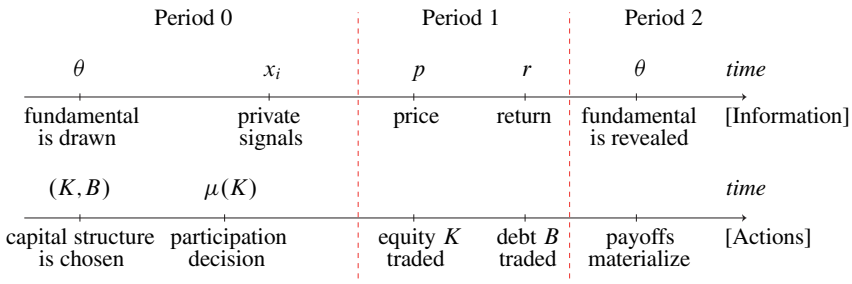
Our model consists of the firm, a unit measure of potential stock investors, and a unit measure of bond investors. The returns earned by equity and bond investors depend on the unknown financial strength of the firm, θ. In our baseline specification, this fundamental θ is exogenous and independent of the capital structure. Positive values of θ correspond to the case when the firm is solvent; the firm's bond investors then receive a net return r, determined endogenously as a market-clearing rate. Negative θ corresponds to the state of firm's default, where investors incur a loss. Regarding equity returns, we study a standard CARA-normal model, where $(\theta - p)k_i$ is an agent's return who bought a (possibly negative) number k_i of shares at price p.

The current payoff structure emphasizes that a higher expected θ makes default less likely, which is what we require for our results. Regarding equity payoff, how-

[4] Uniqueness is measured by the firms' expenditures on research and development, their selling expenses, and the rate at which employees voluntarily leave their jobs.

[5] As measured by the market-to-book ratio.

Figure
Time Line



ever, we note that we use the payoff structure of Grossman and Stiglitz (1980) and Hellwig (1980), which violates limited liability. Regarding bonds, the current specification implies that bondholders get paid in full even if the amount of resource is below $B + rB$, where B is the total debt outstanding; this is not consistent with a traditional budget constraint. In section 4.3, we show that the incorporation of a resource constraint regarding the payoffs to bonds strengthens the effects that we derive for the simplified setting.

The model is characterized by the following time line:

Period 0. The firm decides on the capital structure (K,B) that minimizes the expected cost of capital, $\mathbb{E}^f[C] = \mathbb{E}^f[K(\theta - p) + Br]$, subject to the revenue constraint $I = \mathbb{E}^f[p]K + B$, where $\mathbb{E}^f$ denotes the expectation operator associated with the firm's beliefs over θ. We assume that the firm believes that $\theta > 0$ with probability one.[6] After the firm announces a particular plan, (K,B), equity investors decide whether to participate or to abstain from the equity market. In equilibrium, the mass of participating agents, μ, depends on the size K of the equity market. Finally, equity investors receive private signals x_i on the firm's strength and submit demand schedules.

Period 1. The equity market opens, and an equilibrium stock price p is observed. The debt market opens after the equity market, and bonds are traded at the equilibrium yield r, which depends on p.

Period 2. The firm's unknown fundamental θ is revealed, and all payoffs are realized.

[6] A large literature argues in favor of the assumption that the firm is overly optimistic: Heaton (2002); Malmendier and Tate (2005, 2008); Malmendier, Tate, and Yan (2011); Goel and Thakor (2008); Graham, Harvey, and Puri (2013).

Due to the information spillover between stock market and bond market, we solve the model recursively. Proceeding in line with backward induction, we begin with the bond market; subsequently, we introduce the equity market. Finally, we analyze the ex ante decision (K, B) of the firm.

2.1 Debt Market

When the bond market opens, bond investors use the stock price p to compute the probability that the firm is solvent and form their demand. The demand for bonds, $B^D(\pi_p, r)$, is given by a continuously differentiable function of two arguments: the probability $\pi_p \equiv \Pr(\theta \geq 0|p)$ with which the firm is solvent, and the net interest rate r. A greater probability that the firm is solvent (i.e., does not default on its debt) implies a higher demand for bonds. Bond demand is also increasing in the net interest rate r that bond investors receive if the firm is solvent:

$$\frac{\partial B^D}{\partial \pi_p} > 0, \quad \frac{\partial B^D}{\partial r} > 0.$$

The demand for bonds, B^D, would result from payoff maximization by the mass of investors in the bond market. We do not specify the underlying optimization problem in this paper, but Gorelkina and Kuhle (2013) show that the demand function can come from the maximization of CARA utilities by bond investors who face a piecewise-constant payoff to debt.

Moving on to the other properties of the demand function, we assume that the demand is negative when the interest rate is zero, since there is a risk of default on repaying the principal ($\pi_p < 1$ for any real p):

$$B^D(\pi_p, 0) < 0.$$

We define the limit values of the bond demand function to ensure that the bond market clears for all price signals:

$$\lim_{r \to \infty} B^D(\pi_p, r) = \infty, \quad \lim_{r \to -\infty} B^D(\pi_p, r) = -\infty.$$

For a given bond supply B, market clearing requires

$$B^D(\pi_p, r) = B. \tag{1}$$

Differentiation of (1) yields the bond market's comparative statics:

LEMMA 1 *The equilibrium rate of return r decreases in the firm's survival probability: $\partial r / \partial \pi_p < 0$. Returns increase in the debt supply: $\partial r / \partial B > 0$.*

2.2 Equity Market

Equity is traded in the standard linear CARA-normal noise-trader market. Equity investors can engage in short selling, and there are no borrowing constraints. They

hold an uninformative prior and receive noisy private signals $x_i = \theta + \sigma_x \xi_i$, which reveal the true θ with precision $\alpha_x = 1/\sigma_x^2$. Private noise $\xi_i \sim \mathcal{N}(0,1)$ is idiosyncratic.[7] Finally, noise-trader demand for the firm's equity is given by $\sigma_\varepsilon \varepsilon$, where $\varepsilon \sim \mathcal{N}(0,1)$.

To characterize the market price signal, we guess and verify that there exists a linear price function, $p = \eta_1 \theta + \eta_2 \varepsilon + c$, where η_1, η_2, c are the coefficients that need to be determined. Regarding θ, the price contains the same information as a signal $Z \equiv (p-c)/\eta_1 = \theta + (\eta_2/\eta_1)\varepsilon$. Given the price function, we can characterize individual demands based on the information x_i, Z. Investors choose their individual stock holdings k_i to maximize expected CARA utility $U(y) = -e^{\gamma y}$, where γ is the coefficient of absolute risk aversion:[8]

$$\begin{aligned} k_i &= \operatorname*{argmax}_{k_i} \left\{ \mathbb{E}\left[-e^{-\gamma(\theta-p)k_i} \,\middle|\, x_i, Z \right] \right\} \\ &= \operatorname*{argmax}_{k_i} \left\{ e^{\gamma \mathbb{E}[(\theta-p)k_i \,|\, x_i, Z] - (\gamma^2/2)\operatorname{var}[(\theta-p)k_i \,|\, x_i, Z]} \right\} \\ &= \operatorname*{argmax}_{k_i} \left\{ \gamma \left(\frac{\alpha_x}{\alpha} x_i + \frac{\alpha_z}{\alpha} Z - p \right) k_i - \frac{\gamma^2}{2\alpha} k_i^2 \right\}, \end{aligned}$$

where α_x and $\alpha_z = \eta_1^2/\eta_2^2$ are the precisions of the signals x_i and Z, respectively, and $\alpha = \alpha_x + \alpha_z$. Consequently, the individual demand function becomes

$$k_i^d = \frac{\alpha}{\gamma} \left(\frac{\alpha_x}{\alpha} x_i + \frac{\alpha_z}{\alpha} Z - p \right). \tag{2}$$

In the appendix section A.1, we use (2) to (i) compute aggregate demand for a given mass μ of investors and (ii) determine the price function's coefficients as $\eta_1 = 1$ and $\eta_2 = \gamma \sigma_\varepsilon / \alpha_x \mu$. Concerning the stock price's informational content, this implies that

$$p = \theta + \alpha_z^{-1/2} \varepsilon - \frac{K\gamma}{\alpha\mu}, \quad Z = \theta + \alpha_z^{-1/2} \varepsilon.$$

Importantly, the precision $\alpha_z = \alpha_x^2 \mu^2 / \gamma^2 \sigma_\varepsilon^2$, with which Z reveals the true fundamental θ, increases in the mass μ of equity investors.

2.2.1 Equilibrium Participation.

Equity investors have to decide between (i) receiving a signal and buying shares, on the one hand, and (ii) receiving utility U_0, which represents an outside option, on the other. Agents are thus just indifferent between the two options if

$$\mathbb{E}[\mathbb{E}[U \,|\, Z, x_i]] = U_0. \tag{3}$$

[7] Hammond and Sun (2008) characterize the validity of a framework with a random macro state and idiosyncratic micro shocks via Monte Carlo simulation as well as event-wise measurable conditional probabilities.

[8] See Raiffa and Schlaifer (2000, p. 250) for the standard results on prior and posterior distributions of normally distributed variables that we use throughout the paper.

In the appendix section A.2, we show that the agent's ex ante expected utility in (3) can be rewritten as a function of market size $|K|$ and participation μ:

$$\mathbb{E}[\mathbb{E}[U|Z,x_i]] = F(|K|,\mu), \quad \frac{\partial F}{\partial |K|} > 0, \quad \frac{\partial F}{\partial \mu} < 0. \tag{4}$$

For a given mass μ of traders, trade becomes more profitable as the firm issues more shares. Trade becomes less profitable as the mass μ of competing traders increases. Hence, we have the following:

PROPOSITION 1 *The mass μ of equity investors and the price signal's precision $\alpha_z = \alpha_x^2\mu^2/\gamma^2\sigma_\varepsilon^2$ increase with the equity market's depth $|K|$.*

PROOF It follows from (3) and (4) that the break-even level of μ must satisfy $U_0 \equiv F(|K|,\mu)$. By the implicit-function theorem, we have $\partial\mu/\partial|K| = -(\partial F/\partial|K|)/(\partial F/\partial\mu) > 0$. *Q.E.D.*

That is, as the size of the stock market increases, there are larger rents, which attract additional equity investors to the market. Hence, the announcement of different capital structures (K,B) influences the price signal's precision and thus the strength of the informational spillover.

2.3 Stock Price Signal and Bond Returns

Once we recall that the stock investors' mass μ increases the informativeness $\alpha_z = \mu^2\alpha_x^2/\gamma^2\sigma_\varepsilon^2$ of the price signal $Z = \theta + \varepsilon/\sqrt{\alpha_z}$, bond investors can compute the firm's default probability π_p as a function of Z. This yields the following

PROPOSITION 2 *The cost of debt, r, decreases (increases) in the mass of equity investors, μ, if the fundamental θ is positive (negative).*

Proposition 2 (for the proof, see the appendix section A.4) shows that a deeper equity market, in which more investors collect information about the firm's unknown fundamental, reduces the noise trader's influence and thus causes the stock price to reveal the firm's fundamental with greater precision to bond investors.

3 Optimal Capital Structure

As the firm chooses its capital structure, it anticipates the interaction between the stock and bond markets. In particular, the firm knows that the mass of equity investors and thus the informativeness of the price signal changes with the amount of equity issued, as in section 2.2.1. That is, every capital structure choice implies a distinct informational environment. In this section, we show that a firm that believes that it is strong issues more equity and less debt. Put differently, strong firms amplify the precision with which their fundamental strength is communicated to bond investors. This reduces expected borrowing costs.

The firm is assumed to be risk-neutral. Moreover, it requires external financing $I > 0$, which it can attract by issuing shares K and debt B. The firm announces its ex ante optimal capital structure K, B at $t = 0$ before markets open. That is, the firm chooses its capital structure subject to the anticipated informational interaction between the two markets described above. Finally, the firm believes that it will not default, i.e., it believes $\theta > 0$. The expectation operator associated with the firm's beliefs is denoted $\mathbb{E}^f[\,]$, where the superscript f refers to the firm.[9]

Taking into account the equilibria in the bond and equity markets, the firm's optimization problem reads

$$\min_{K,B} C = \min_{K,B} \mathbb{E}^f[(\theta - p)K + rB] \tag{5}$$

subject to

$$\begin{aligned}
&\mathbb{E}^f[p]K + B = I && \text{(revenue constraint)},\\
&\mu = \mu(K) && \text{(equilibrium participation)},\\
&p = \theta + \frac{1}{\sqrt{\alpha_z}}\varepsilon - \frac{\gamma K}{\alpha\mu},\ \alpha = \alpha_x + \alpha_z,\ \alpha_z = \frac{\alpha_x^2\mu^2}{\gamma^2\sigma_\varepsilon^2} && \text{(equity market equilibrium)},\\
&r = r(B,\mu) && \text{(debt market equilibrium)},
\end{aligned} \tag{6}$$

where $p = p(K,\mu)$ is the solution for the equity market equilibrium (A3), and $r = r(B,\mu)$ is the bond market equilibrium (1). The problem (5)–(6) yields a first-order condition for the firm's optimal capital structure:

$$\frac{dC}{dK} = \underbrace{\frac{\partial C}{\partial K}}_{\text{direct effect}} + \underbrace{\frac{\partial C}{\partial \mu}\frac{\partial \mu}{\partial K}}_{\text{info externality}} = 0. \tag{7}$$

The condition (7) singles out two effects. First, there is a *direct effect*, which describes how issuing equity impacts the cost of capital for a given information structure. Second, there is the *informational externality*, which shows that the stock price's informational content varies with the firm's capital structure. Regarding the direct effect, we note:

LEMMA 2 *For any positive revenue I, there exists a finite pair $K^* > 0, B^* \gtreqless 0$ that solves $\partial C/\partial K = 0$. This capital structure minimizes the firm's capital costs for every exogenously given mass μ of equity investors. If I is sufficiently high, we have $B^* > 0$ and $K^* > 0$.*

[9] In equilibrium, stock and bond investors can use the optimal plan (K^*, B^*) to make inferences about the firm's belief f over θ. As we point out in the introduction, we assume that the firm's belief is uninformative (to investors), to separate the effects of information spillovers from stock prices from the signaling effects.

Lemma 2 (for the proof, see the appendix section A.3) describes the capital structure choice of a firm that does not internalize the informational externality that stock prices exert on bond yields. A firm that internalizes this externality chooses a capital structure that solves (7):

PROPOSITION 3 *To internalize the informational externality, the firm chooses an optimal capital structure* $(K^{**}, B^{**}) \in \mathbb{R}^2_+$*, such that* $K^* < K^{**}$*,* $B^* > B^{**}$*, and* $\mu(K^*) < \mu(K^{**})$*.*

Proposition 3 (for the proof, see the appendix section A.4) shows that the firm has an incentive to issue equity to internalize the informational externality. That is, as the firm issues more equity, the stock price conveys its strong financial position more clearly to bond investors. This reduces expected borrowing costs. To understand Proposition 3, we discuss the informational externality in greater detail:

$$\frac{\partial C}{\partial \mu}\frac{\partial \mu}{\partial K} = \underbrace{-\left(1+\mathbb{E}^f\left[r+\frac{\partial r}{\partial B}B^*\right]\right)\frac{\gamma K^{*2}}{\alpha^2\mu^2}\left(\alpha_x+\frac{3\alpha_x^2\mu^2}{\gamma^2\sigma_\varepsilon^2}\right)}_{+}\underbrace{\frac{\partial \mu}{\partial K}}_{+} + \underbrace{\mathbb{E}^f\left[\frac{\partial r}{\partial \mu}\frac{\partial \mu}{\partial K}\right]B^*}_{-} < 0. \tag{8}$$

The first term in (8) shows that increases in stock supply raise participation in the stock market. This increases the stock price's informational content, and thus stock investors who also condition their demand on the stock price's informational content take larger positions. That is, there are more and better-informed buyers, so that a given stock supply K^* can be sold at a higher price

$$\frac{\gamma K^{*2}}{\alpha^2\mu^2}\left(\alpha_x+\frac{3\alpha_x^2\mu^2}{\gamma^2\sigma_\varepsilon^2}\right)\frac{\partial \mu}{\partial K}.$$

Moreover, $\mathbb{E}^f[r+(\partial r/\partial B)B^*]$ reflects that this increase in stock prices allows one to reduce debt, which in turn reduces borrowing costs on debt, B^*. The second term in (8) shows that the information externality, which goes from the stock price to the bond yield, strengthens as the firm issues additional stocks: The stock price communicates the firm's strong financial position more clearly to bond investors, which reduces borrowing costs.

In summary, we find that the firm issues equity to increase the number of investors who find it profitable to research and trade the firm's stock. This makes the equity market deeper and increases the precision with which the stock price communicates the firm's fundamental. In turn, bond investors who learn the firm's strength from the stock price are willing to lend at lower rates.

4 Discussion

A number of our previous assumptions were made to simplify the exposition. First, we discuss the assumption that the stock market rather than the bond market is the main source of public information. Second, we show that the firm's dividend θ, and thus the survival probability π_p, can be derived from a consistent budget constraint. Finally, we present a more general specification for the bond market.

4.1 Direction of the Spillover

Currently, the stock price is the main source of public information. In principle, it is possible to construct an alternative model where bond investors research the firm's financial strength and stock investors use the equilibrium yield to infer the firm's fundamental. Alternatively, we could also study the intermediate case where both p and r carry information. While such analysis is possible, we believe that the current specification, where the stock price signal influences bond yields, is likely the more relevant one. Empirical evidence suggests that, even though many firms operate with a capital structure where the value of the debt far exceeds the value of the equity, it is the stock price and not the bond yield (of some reference maturity) that is published most prominently in the media. The financial data provided by Google Finance, Reuters, or Bloomberg make it straightforward to observe the latest stock prices; at the same time, it is difficult to inquire about bond yields. Moreover, a firm's bond market is fragmented into bonds with various maturities. The market for a particular issue is accordingly very small compared to the more homogeneous equity that is traded in the stock market. According to Gebhardt, Hvidkjaer, and Swaminathan (2005), research on bonds is mostly limited to the firms covered by rating agencies, and those do not revise a firm's bond ratings as often as an equity analyst might change her forecasts and recommendations. Empirically, Gebhardt, Hvidkjaer, and Swaminathan (2005) show that stock prices adjust to the firm-specific information more quickly than do bond prices. Furthermore, Kwan (1996) finds that lagged stock returns have explanatory power for current bond yield changes, while current stock returns are unrelated to lagged bond yield changes. Forte and Peña (2009) confirm this finding on a sample of North American and European companies. Longstaff, Mithal, and Neis (2003) and Norden and Weber (2009) use different regression techniques and reach the same conclusion.

Finally, from a theoretical perspective, Dang, Gorton, and Holmström (2009) and Gorelkina and Kuhle (2013, pp. 30–34) show that the coarse (the linear) payout profile of debt (equity) discourages bond investors from researching a firm's financial position. On the contrary, equity investors have a strong incentive to acquire the detailed information that is aggregated and publicized through the stock price.

4.2 Segmentation of Investors

We assume that bond and stock investors belong to two different groups. Alternatively, we could assume that there is a group of bond investors and another group of investors who decide to participate *either* in the stock market *or* in the bond market.

For our results to hold, it suffices that the mass μ of investors who trade the stock is increasing in equity issued. That is, an increase in the amount of equity issued would have to increase investors' utility in the stock market and decrease utility from participating in the bond market. In Proposition 1, we have already established that increases in the amount of equity issued increase the rents in the stock market. When the survival probability of the firm is fixed, the same is true of the bond market by Lemma 1, i.e., if the number of bonds sold decreases, the interest rates and thus the buyers' utility decrease. By the law of iterated expectations, ex ante utility in the bond market also decreases in the supply of bonds; thus traders would migrate from bonds to equity until the expected utility from participating in the two markets is equalized. We obtain that increases in equity issued would reduce the number of bonds and that investors would therefore migrate from debt to equity, and our results would carry over.

4.3 The Firm's Resource Constraint and General Bond Demand

To simplify the exposition in the main text, we treated the firm's dividend θ as independent of the capital structure. In this section, we show that the results are indeed strengthened once we add a consistent budget constraint. To do so, we assume that the firm's aggregate resources are given by Y. In turn, these resources are used to service the debt B and to pay a dividend θ per share:

$$Y = K\theta + (1+r)B \quad \Leftrightarrow \quad \theta = \frac{Y}{K} - \frac{(1+r)B}{K}. \tag{9}$$

The comparative statics of the equity market obtain as before, once agents receive noisy private signals over the new θ.[10] In the appendix section A.5, we show that the informational externality, which induces firms to issue more equity and less debt, is strengthened once dividends θ, and thus the firm's survival probability π_p, depend on the interest rate as in (9). In particular, we find that the reduction of expected interest rates, which is associated with more precise stock prices, now also reduces the default probability $1-\pi_p$.

To complete the discussion of the resource constraint (9), it remains to discuss the implications for bond demand. For our results to hold, it is sufficient that de-

[10] The key observation in this context is that the return $r(p)$ is a deterministic function of the stock price. Hence, equity investors who condition their demand on the informational content of the stock price p *always* correctly anticipate the interest rate $r(p)$, so that there is no additional nonlinearity in the equity investors' utility maximization problem.

mand is given by a continuously differentiable function $B^D(\pi_p, r)$:

$$\frac{\partial B^D}{\partial \pi_p} > 0, \quad \frac{dB^D}{dr} = \frac{\partial B^D}{\partial \pi_p}\frac{\partial \pi_p}{\partial \theta}\frac{\partial \theta}{\partial r} + \frac{\partial B^D}{\partial r} > 0, \tag{10}$$

where θ is defined as in (9). The assumptions regarding the demand function's derivatives indicate that, for every given interest rate, demand increases in the firm's survival probability. Moreover, increases in the rate of return increase bond demand. In equilibrium, where the supply of bonds is B, we have

$$B^D(\pi_p, r) = B \;\Leftrightarrow\; r = r(B, \pi_p) > 0, \quad \frac{\partial r}{\partial B} > 0, \quad \frac{\partial r}{\partial \pi_p} < 0,$$

where the signs of $\partial r/\partial B$, and $\partial r/\partial \pi_p$, which are required for our main Proposition 3, follow from (10).

5 *Conclusion*

The present paper provides a simple equilibrium model in which the firm's stock price serves as a rating device for investors who buy its debt. The main motivation for our analysis lies in the observation that stock investors, as opposed to rating agencies (which rely on fees from the firms they rate), have no incentive to misreport their information on the firm's financial position. That is, they do not buy stocks at inflated prices to mislead bond investors.

To study the spillover from stock price signals to bond yields, we have assumed the perspective of a firm that minimizes its capital cost subject to the informational connection between the bond and stock markets. In a first step, we have shown that a firm that is financially strong benefits from an informative stock price that communicates its financial strength clearly to bond investors, who rely on price signals to infer the firm's default probability. In a second step, we endogenize the strength of the informational spillover. In an economy where equity investors can choose whether to take a position in the firm's stock, increases in the amount of equity issued incentivize more stock investors to trade the firm's stock, which increases the stock price signal's precision. Hence, the strength of the informational spillover varies with different capital structures. Consequently, firms have an incentive to choose their capital structure to internalize the informational externality that stock prices exert on bond yields. Firms with a strong fundamental will therefore issue equity to generate stock price signals that communicate their strong financial position on average more clearly to bond investors.

The contribution of this paper is therefore twofold. First, it shows how the informational content of a firm's stock price influences its borrowing cost. Second, we find that strong firms have an incentive to issue equity, which increases their stock price's informational content and reduces their borrowing costs.

Appendix

A.1 Stock Market Price

Using (2), we write the aggregate demand K^D as

$$K^D = \int_{[0,\mu]} k_i \, di + \sigma_\varepsilon \varepsilon = \frac{\mu\alpha}{\gamma}\left(\frac{\alpha_x}{\alpha}\theta + \frac{\alpha_z}{\alpha}Z - p\right) + \sigma_\varepsilon \varepsilon.$$

Market clearing requires that

$$\text{(A1)} \qquad K^D = K^S \quad \Leftrightarrow \quad \frac{\mu\alpha}{\gamma}\left(\frac{\alpha_x}{\alpha}\theta + \frac{\alpha_z}{\alpha}Z - p\right) = K - \sigma_\varepsilon \varepsilon.$$

To close the argument, we now resubstitute $Z = (p-c)/\eta_1$ and calculate η_1 and η_2. First, we solve (A1) for p to obtain

$$\text{(A2)} \qquad p = \frac{\alpha_x}{\alpha - \alpha_z/\eta_1}\theta + \frac{\gamma\sigma_\varepsilon}{\mu(\alpha - \alpha_z/\eta_1)}\varepsilon + \frac{1}{1-\alpha\eta_1/\alpha_z}\left(c + \frac{K\gamma\eta_1}{\mu\alpha_z}\right).$$

Comparison of (A2) with our initial guess, $p = \eta_1\theta + \eta_2\varepsilon + c$ indicates that η_1, η_2 must satisfy

$$\eta_1 = \frac{\alpha_x}{\alpha - \alpha_z/\eta_1}, \quad \eta_2 = \frac{\gamma\sigma_\varepsilon}{\mu(\alpha - \alpha_z/\eta_1)}, \quad c = \frac{1}{1-\alpha\eta_1/\alpha_z}\left(c + \frac{K\gamma\eta_1}{\mu\alpha_z}\right); \quad \alpha = \alpha_x + \alpha_z.$$

The solution to the first equality is $\eta_1 = 1$; thus $\eta_2 = \gamma\sigma_\varepsilon/\alpha_x\mu$ and $c = -K\gamma/\mu\alpha$. Accordingly, p and $Z = (p-c)/\eta_1 = \theta + (\eta_2/\eta_1)\varepsilon$ are given by

$$\text{(A3)} \qquad p = \theta + \alpha_z^{-1/2}\varepsilon - \frac{K\gamma}{\alpha\mu}, \quad Z = \theta + \alpha_z^{-1/2}\varepsilon, \quad \alpha_z = \frac{\alpha_x^2\mu^2}{\gamma^2\sigma_\varepsilon^2}.$$

Q.E.D.

A.2 Stock Market Participation

The agent's ex ante expected utility is

$$U_i^e(\mu, K) = \mathbb{E}U((\theta - p)k_i).$$

Using the properties of CARA utility functions and the law of iterated expectations, we have

$$U_i^e(\mu, K) = -\mathbb{E}[\mathbb{E}[\exp(-\gamma(\theta - p)k_i)|x_i, Z]].$$

Since k_i, p are constant conditional on (x_i, Z) and θ is conditionally normally distributed, we can write

$$U_i^e(\mu, K) = -\mathbb{E}\left[\exp\left(-\gamma(\mathbb{E}[\theta|x_i, Z] - p)k_i + \frac{1}{2}\gamma^2 k_i^2 \operatorname{var}(\theta|x_i, Z)\right)\right].$$

Denoting $\mathbb{E}[\theta|x_i,Z]=\theta_i$, $\mathrm{var}(\theta|x_i,Z)=\alpha^{-1}$, so that $k_i=k_i(x_i,Z)=(\alpha/\gamma)(\theta_i-p)$, we have

$$\begin{aligned}U_i^e(\mu,K)&=-\mathbb{E}\left[\exp\left(-\gamma(\theta_i-p)\frac{\alpha}{\gamma}(\theta_i-p)+\frac{1}{2}\gamma^2\frac{\alpha^2}{\gamma^2}(\theta_i-p)^2\alpha^{-1}\right)\right]\\&=-\mathbb{E}\left[\exp\left(-\frac{\alpha}{2}(\theta_i-p)^2\right)\right].\end{aligned}$$

Substituting in the demand function $\theta_i=(\alpha_x/\alpha)x_i+(\alpha_z/\alpha)Z$ and $p=\theta+\alpha_z^{-1/2}\varepsilon-K\gamma/\mu\alpha$, we obtain

$$U_i^e(\mu,K)=-\mathbb{E}\exp\left(-\frac{\alpha}{2}\left(\frac{\alpha_x}{\alpha}x_i+\frac{\alpha_z}{\alpha}Z-\theta-\alpha_z^{-\frac{1}{2}}\varepsilon+\frac{K\gamma}{\mu\alpha}\right)^2\right).$$

Substitute $x_i=\theta+\alpha_x^{-1/2}\xi_i$, $Z=\theta+\alpha_z^{-1/2}\varepsilon$, and recall that $\alpha=\alpha_x+\alpha_z$ (thus θ cancels out):

$$U_i^e(\mu,K)=-\mathbb{E}\exp\left(-\frac{1}{2\alpha}\left(\alpha_x^{\frac{1}{2}}\xi_i+\alpha_z^{\frac{1}{2}}\varepsilon-\alpha\alpha_z^{-\frac{1}{2}}\varepsilon+\frac{K\gamma}{\mu}\right)^2\right).$$

Note that $\alpha_z^{1/2}\varepsilon-\alpha\alpha_z^{-1/2}\varepsilon=\alpha_z^{-1/2}(\alpha_z-\alpha)\varepsilon=-\alpha_z^{-1/2}\alpha_x\varepsilon$:

$$U_i^e(\mu,K)=-\mathbb{E}\exp\left(-\frac{1}{2\alpha}\left(\alpha_x^{\frac{1}{2}}\xi_i-\alpha_z^{-\frac{1}{2}}\alpha_x\varepsilon+\frac{K\gamma}{\mu}\right)^2\right). \tag{A4}$$

Since ξ_i and ε are independent and identically distributed as $N(0,1)$, the random component in (A4) is normally distributed around zero with variance $(\alpha_{xi}^{1/2})^2+(\alpha_z^{-1/2}\alpha_x)^2=\alpha_x+\alpha_x^2/\alpha_z=\alpha_x\alpha/\alpha_z$. We normalize the expression to obtain

$$U_i^e(\mu,K)=-\mathbb{E}\exp\left(-\frac{\alpha_x}{2\alpha_z}\left(\varsigma+\frac{K\gamma}{\mu}\sqrt{\frac{\alpha_z}{\alpha_x\alpha}}\right)^2\right),$$

where the new random variable $\varsigma\sim N(0,1)$. Denoting

$$\begin{aligned}\lambda&=\left(\frac{K\gamma}{\mu}\sqrt{\frac{\alpha_z}{\alpha_x\alpha}}\right)^2=\frac{K^2\gamma^2}{\mu^2}\Big/\left(\alpha_x\left(1+\frac{\alpha_x}{\alpha_z}\right)\right)\\&=\frac{K^2\gamma^2}{\mu^2}\Big/\left(\alpha_x\left(1+\frac{\gamma^2\sigma_\varepsilon^2}{\alpha_x\mu^2}\right)\right)=\frac{K^2\gamma^2}{\mu^2}\Big/\left(\alpha_x+\frac{\gamma^2\sigma_\varepsilon^2}{\mu^2}\right),\end{aligned}$$

where we substitute $\alpha_z=\alpha_x^2\mu^2/\gamma^2\sigma_\varepsilon^2$, we observe that ς^2 follows a noncentral χ^2 distribution with one degree of freedom and the noncentrality parameter λ. We can therefore use the moment-generating function $\mathbb{E}e^{bz^2}=(1-2b)^{-1/2}e^{b\lambda/(1-2b)}$, with $b=-\alpha_x/2\alpha_z=-(1/2)\gamma^2\sigma_\varepsilon^2/\alpha_x\mu^2$, for the noncentral χ^2 distribution. The moment-

generating function can be derived as follows:

$$\begin{aligned}\mathbb{E}e^{bz^2} &= \frac{1}{\sqrt{2\pi_p}}\int e^{bz^2}e^{-\frac{(z-\sqrt{\lambda})^2}{2}}dz = \frac{1}{\sqrt{2\pi_p}}\int e^{-\frac{(1-2b)z^2-2\sqrt{\lambda}z+\lambda}{2}}dz \\ &= \frac{1}{\sqrt{2\pi_p}}\int e^{-\frac{(\sqrt{1-2b}z-\sqrt{\lambda/(1-2b)})^2-\frac{\lambda}{1-2b}+\lambda}{2}}dz \\ &= \frac{1}{\sqrt{1-2b}\sqrt{2\pi_p}}\int e^{\frac{\lambda b}{1-2b}}e^{-\frac{(\sqrt{1-2b}z-\sqrt{\lambda/(1-2b)})^2}{2}}d(\sqrt{1-2b}z) = \frac{1}{\sqrt{1-2b}}e^{\frac{\lambda b}{1-2b}}.\end{aligned}$$

This moment-generating function can be used to rewrite the utility as

$$U_i^e(\mu,K) = -(1-2b)^{-\frac{1}{2}}e^{\frac{\lambda b}{1-2b}}.$$

The derivative w.r.t. μ (denote $b' = \partial b/\partial\mu$, $\lambda' = \partial\lambda/\partial\mu$) is

$$\begin{aligned}\frac{\partial U_i^e(\mu,K)}{\partial\mu} &= -(1-2b)^{-\frac{3}{2}}b'e^{\frac{\lambda b}{1-2b}} - (1-2b)^{-\frac{1}{2}}e^{\frac{\lambda b}{1-2b}}\left(\frac{(\lambda b)'(1-2b)+\lambda b\times 2b'}{(1-2b)^2}\right) \\ &= -(1-2b)^{-\frac{3}{2}}e^{\frac{\lambda b}{1-2b}}\left(b'\left(1+\frac{\lambda}{1-2b}\right)+\lambda' b\right) < 0,\end{aligned}$$

since $b' > 0$, $b < 0$, $\lambda > 0$, $\lambda' < 0$. To see $\lambda' < 0$, note that

$$\begin{aligned}\frac{\partial\lambda}{\partial\mu} &= -2\frac{K^2\gamma^2}{\mu^3}\left(\alpha_x+\frac{\gamma^2\sigma_\varepsilon^2}{\mu^2}\right)^{-1} + 2\frac{K^2\gamma^2}{\mu^2}\left(\alpha_x+\frac{\gamma^2\sigma_\varepsilon^2}{\mu^2}\right)^{-2}\frac{\gamma^2\sigma_\varepsilon^2}{\mu^3} \\ &= 2\frac{K^2\gamma^2}{\mu^3}\left(\alpha_x+\frac{\gamma^2\sigma_\varepsilon^2}{\mu^2}\right)^{-2}\left(-\alpha_x-\frac{\gamma^2\sigma_\varepsilon^2}{\mu^2}+\frac{\gamma^2\sigma_\varepsilon^2}{\mu^2}\right) \\ &= -2\frac{\alpha_x K^2\gamma^2}{\mu^3}\left(\alpha_x+\frac{\gamma^2\sigma_\varepsilon^2}{\mu^2}\right)^{-2} < 0.\end{aligned}$$

Thus, the ex ante utility decreases in μ.

Regarding the derivative w.r.t. $|K|$ (note $\partial b/\partial K = 0$, $\lambda_k = \partial\lambda/\partial K^2$), we have

$$\frac{\partial U_i^e(\mu,K)}{\partial K^2} = -(1-2b)^{-\frac{3}{2}}e^{\frac{\lambda b}{1-2b}}(\lambda_k b) > 0,$$

since $b < 0$ and $\lambda' > 0$. Thus, the ex ante utility increases in $|K|$. *Q.E.D.*

A.3 Proof of Lemma 2

The first-order condition for the cost-minimizing K^* in (A9) is

$$\begin{aligned}\frac{\partial C}{\partial K} &= \mathbb{E}^f[\theta]-\mathbb{E}^f[p] \\ &\quad + \left(\frac{\partial C}{\partial\mathbb{E}^f[p]} - \frac{\partial C}{\partial\mathbb{E}^f[r]}\frac{\partial\mathbb{E}^f[r]}{\partial B}K\right)\frac{\partial\mathbb{E}^f[p]}{\partial K} - \frac{\partial C}{\partial\mathbb{E}^f[r]}\frac{\partial\mathbb{E}^f[r]}{\partial B}\mathbb{E}^f[p] = 0,\end{aligned}$$

for we have

$$\begin{aligned}
\frac{\partial C}{\partial K} &= \mathbb{E}^f[\theta] - \mathbb{E}^f[p] - \mathbb{E}^f[r]\mathbb{E}^f[p] \\
&\quad + \left(-K - \mathbb{E}^f[r]K - B\frac{\partial \mathbb{E}^f[r]}{\partial B}K\right)\left(-\frac{\gamma}{\alpha\mu}\right) - B\frac{\partial \mathbb{E}^f[r]}{\partial B}\mathbb{E}^f[p] \\
&= \mathbb{E}^f[\theta] - \mathbb{E}^f[p](1+\mathbb{E}^f[r]) + \frac{\gamma}{\alpha\mu}\left(1+\mathbb{E}^f[r]+\frac{\partial \mathbb{E}^f[r]}{\partial B}B\right)K - B\frac{\partial \mathbb{E}^f[r]}{\partial B}\mathbb{E}^f[p] \\
&= \mathbb{E}^f[\theta] + \left(\frac{\gamma}{\alpha\mu}K - \mathbb{E}^f[p]\right)\left(1+\mathbb{E}^f[r]+\frac{\partial \mathbb{E}^f[r]}{\partial B}B\right) \\
\text{(A5)} &= \mathbb{E}^f[\theta] + \left(2\frac{\gamma}{\alpha\mu}K - \mathbb{E}^f[\theta]\right)\left(1+\mathbb{E}^f[r]+\frac{\partial \mathbb{E}^f[r]}{\partial B}B\right) = 0.
\end{aligned}$$

To prove that K^* is positive if $I > 0$, we show that (A5) is strictly negative for $K \leq 0$. That is, issuing more equity reduces capital costs and we have $K^* > 0$. For the following steps, we only require that $\mu \in (0,1)$. Hence, we can neglect that μ changes with $|K|$ in the following arguments.

We rearrange the terms of (A5) as follows:

$$\text{(A6)} \qquad \frac{\partial C}{\partial K} = 2\frac{\gamma}{\alpha\mu}K\mathbb{E}^f\left[1+r+\frac{\partial r}{\partial B}B\right] - \mathbb{E}^f[\theta]\mathbb{E}^f\left[r+\frac{\partial r}{\partial B}B\right] = 0.$$

The first term, $2(\gamma/\alpha\mu)K\mathbb{E}^f[1+r+(\partial r/\partial B)B]$, is nonpositive for $K \leq 0$, since we know (i) from Lemma 1 (debt market equilibrium) that $r > 0$ and $\partial r/\partial B > 0$, and (ii) that the revenue constraint implies that $B \geq I > 0$ for $K \leq 0$. The second term of (A6), $\mathbb{E}^f[\theta]\mathbb{E}^f[r+(\partial r/\partial B)B]$, is positive as long as $K \leq 0$, since $\mathbb{E}^f[\theta] > 0$. Hence, dC/dK is negative for $K \leq 0$, implying that the cost-minimizing K^* is positive.

It remains to show that $B^* > 0$ if the financing requirement I is sufficiently high: It follows from the revenue constraint $I = \mathbb{E}^f[p]K + B$, the price function $\mathbb{E}^f[p] = \mathbb{E}^f[\theta] - (\gamma/\alpha\mu)K$, and the assumption $\mathbb{E}^f[\theta] > 0$ that if $B \leq 0$, then $\mathbb{E}^f[p] > 0$ and $K > 0$.[11] It therefore follows from (A6) that we have

$$\left.\frac{dC}{dK}\right|_{B=0,K=I/\mathbb{E}^f[p]} = 2\frac{\gamma}{\alpha\mu}\frac{I}{\mathbb{E}^f[p]}\mathbb{E}^f[1+r] - \mathbb{E}^f[\theta]\mathbb{E}^f[r].$$

One can now show that, for every given expectation $\mathbb{E}^f[\theta] > 0$, a sufficiently large financing requirement I ensures $dC/dK|_{B=0,K=I/Ep} > 0$. That is, a reduction in K, which implies an increase in B, reduces capital costs so that $B^* > 0$.

[11] By contradiction, we find that an allocation where $\mathbb{E}^f[p] < 0$ and $K < 0$, so that $I = \mathbb{E}^f[p]K > 0$, is impossible, since we have assumed that $\mathbb{E}^f[\theta] > 0$, so that $\mathbb{E}^f[p] = \mathbb{E}^f[\theta] - \gamma K/\alpha > 0$ for $K < 0$, which contradicts the initial assumption that $\mathbb{E}^f[p] > 0$.

Finally, we show that there are no corner solutions, i.e., that the pair K^*, B^* is finite. We use the revenue constraint to eliminate B from (A5):

$$= \mathbb{E}^f[\theta] + \left(2\frac{\gamma}{\alpha\mu}K - \mathbb{E}^f[\theta]\right)\left(1 + \mathbb{E}^f\left[r + \frac{\partial r}{\partial B}(I - K\mathbb{E}^f[p])\right]\right)$$

$$= \mathbb{E}^f[\theta] + \left(2\frac{\gamma}{\alpha\mu}K - \mathbb{E}^f[\theta]\right)\left(1 + \mathbb{E}^f\left[r + \frac{\partial r}{\partial B}\left(I - K\left(\mathbb{E}^f[\theta] - \frac{\gamma}{\alpha\mu}K\right)\right)\right]\right). \tag{A7}$$

In equation (A7), K^3 has the largest exponent, it has a nonnegative coefficient, and (A7) is therefore positive in the limit as $K \to +\infty$.[12] Moreover, we know from Lemma 2 that (A7) is negative for $K \leq 0$. The continuity of (A7) then ensures that it must equal 0 at (at least) one positive finite K^*, which is a solution to the first-order condition. *Q.E.D.*

A.4 Proof of Propositions 2 and 3

Proof of Proposition 2 The firm's survival probability is given by

$$\pi_p = \Pr(\theta \geq 0|p) = \Pr(\theta \geq 0|Z) = \Pr(Z - \alpha_z^{-1/2}\varepsilon \geq 0) = \Phi(\alpha_z^{1/2}Z) = \Phi\left(\frac{\mu\alpha_x}{\gamma\sigma_\varepsilon}Z\right),$$

where $\Phi()$ denotes the standard normal cumulative distribution function. From Lemma 1 we know $dr/d\pi_p < 0$ and therefore

$$\frac{dr}{d\pi_p}\frac{d\pi_p}{d\mu} = \frac{dr}{d\pi_p}\phi\left(\frac{\mu\alpha_x}{\gamma\sigma_\varepsilon}\theta + \varepsilon\right)\frac{\alpha_x}{\gamma\sigma_\varepsilon}\theta \lesseqgtr 0 \quad \text{if} \quad \theta \gtreqless 0.$$

Q.E.D.

Proof of Proposition 3 The first-order derivative,

$$\frac{dC}{dK} = \frac{\partial C}{\partial K} + \frac{\partial C}{\partial \mu}\frac{\partial \mu}{\partial K},$$

shows that the capital cost minimization problem consists of a direct effect $\partial C/\partial K$ and an indirect effect $(\partial C/\partial \mu)(\partial \mu/\partial K)$, which represents how the informational content of prices changes with the capital structure. Next, we recall Lemma 2, i.e., there exist values $K^* > 0$ and $B^* > 0$ for which $\partial C/\partial K = 0$. We also recall that $K^* > 0$ and $B^* > 0$ represent a global cost minimum.

Consider the firm minimizing expected capital costs:

$$\min_{K,B} C(K, \mathbb{E}^f[p], \mathbb{E}^f[r], \mu) = \min_{K,B}(\mathbb{E}^f[\theta] - \mathbb{E}^f[p])K + \mathbb{E}^f[r]B \tag{A8}$$

[12] If $2(\gamma/\alpha)^2(\partial r/\partial B) = 0$, then K has the largest exponent.

subject to the market system

$$\begin{aligned}
&\mathbb{E}^f[p]K + B = I && \text{(revenue constraint)},\\
&\mu = \mu(K) && \text{(equilibrium participation)},\\
&p = Z - \frac{\gamma K}{\alpha\mu} && \text{(equity market equilibrium)},\\
&r = r(B,\mu) && \text{(debt market equilibrium)}.
\end{aligned}$$

On substituting for the level of debt $B = I - \mathbb{E}^f[p]K$, the problem (A8) becomes a problem in K alone:

$$\text{(A9)} \quad \min_K C(K,\mathbb{E}^f[p],\mathbb{E}^f[r],\mu) = \min_{K,B}(\mathbb{E}^f[\theta] - \mathbb{E}^f[p])K + \mathbb{E}^f[r](I - \mathbb{E}^f[p]K),$$

where $p = p(K,\mu)$ is the solution for the equity market equilibrium (A3), and $r = r(I - \mathbb{E}^f[p]K,\mu)$ is the bond market equilibrium (1).

To prove that firms that internalize the informational externality issue more equity and less debt, so that $K^{**} > K^*$, it remains to show that $(\partial C/\partial\mu)(\partial\mu/\partial K) < 0$ at K^*. Put differently, if $dC/dK|_{K^*} = 0 + (\partial C/\partial\mu)(\partial\mu/\partial K) < 0$, the firm issues more equity and less debt to internalize the informational externality. Regarding this externality, we note that

$$\frac{\partial C}{\partial\mu}\frac{\partial\mu}{\partial K} = \left(\frac{\partial C}{\partial\mathbb{E}^f[p]} - \frac{\partial C}{\partial\mathbb{E}^f[r]}\frac{\partial\mathbb{E}^f[r]}{\partial B}K\right)\frac{\partial\mathbb{E}^f[p]}{\partial\mu}\frac{\partial\mu}{\partial K} + \frac{\partial C}{\partial\mu}\frac{\partial\mu}{\partial K}.$$

Differentiation of the constraints in (6) yields a more explicit expression:

$$\text{(A10)} \quad \frac{\partial C}{\partial\mu}\frac{\partial\mu}{\partial K} = -\mathbb{E}^f\left[1 + r + \frac{\partial r}{\partial B}B\right]\frac{\gamma K^2}{\alpha^2\mu^2}\left(\alpha_x + \frac{3\alpha_x^2\mu^2}{\gamma^2\sigma_\varepsilon^2}\right)\frac{\partial\mu}{\partial K} + B\mathbb{E}^f\left[\frac{\partial r}{\partial\mu}\frac{\partial\mu}{\partial K}\right].$$

To sign (A10), we first recall that it follows from Lemma 2 that $K^* > 0$, and hence we know from Proposition 2 that participation is increasing in K, i.e., $\partial\mu/\partial K_{|K^*>0} > 0$. Moreover, we recall Lemma 1, which implies that the expected equilibrium rate of return on debt decreases in the price signal's precision: $\partial r/\partial\mu < 0$ if the fundamental θ is positive. Thus the sign of (A10), at the exogenous information optimum K^* where $\partial C/\partial K = 0$, is

$$\underbrace{-\left(1 + \mathbb{E}^f\left[r + \frac{\partial r}{\partial B}B^*\right]\right)\frac{\gamma K^{*2}}{\alpha^2\mu^2}\left(\alpha_x + \frac{3\alpha_x^2\mu^2}{\gamma^2\sigma_\varepsilon^2}\right)}_{+}\underbrace{\frac{\partial\mu}{\partial K}}_{+} + \underbrace{\mathbb{E}^f\left[\frac{\partial r}{\partial\mu}\frac{\partial\mu}{\partial K}\right]B^*}_{-} < 0,$$

where the negative sign in (8) is ensured if the financing requirement I is sufficiently high that, by Lemma 2, B^* is positive. Hence, at the exogenous information optimum, we have $(dC^{**}/dK)(K^*) < 0$, and an increase in K from K^* towards K^{**} decreases capital costs. *Q.E.D.*

A.5 Informational Externality with Budget Constraint

In this section, we derive the informational externality for the modified specifications given in section 4.3. The capital cost problem

$$\min_{K,B} C = \min_{K,B} \mathbb{E}^f[(\theta - p)K + rB] \quad \text{s.t.} \quad I = pK + B$$

is equivalent to

$$\min_{K,B} C = \min_{K,B} \mathbb{E}^f\left[\left(\theta - \theta - \alpha_z^{-\frac{1}{2}}\varepsilon + \frac{\gamma}{\alpha\mu}K\right)K + rB\right] \quad \text{s.t.} \quad I = pK + B,$$

$$\min_{K,B} C = \min_{K,B} \mathbb{E}^f\left[\frac{\gamma}{\alpha\mu}K^2 + rB\right] \quad \text{s.t.} \quad I = pK + B. \tag{A11}$$

From (A11), we calculate

$$\frac{dC}{dK} = \mathbb{E}^f\left[\frac{\partial C}{\partial K} + \frac{\partial C}{\partial \mu}\frac{\partial \mu}{\partial K}\right],$$

where the externality $(\partial C/\partial\mu)(\partial\mu/\partial K)$ induces firms to issue more equity and less debt, since[13]

$$\frac{\partial C}{\partial \mu}\frac{\partial \mu}{\partial K} = \mathbb{E}^f\left[\left[-\frac{\gamma K^2}{\alpha^2\mu^2}\left(\alpha_x + \frac{3\alpha_x^2\mu^2}{\gamma^2\sigma_\varepsilon^2}\right)\left(1 + r + \frac{\partial r}{\partial B}B\right) - \frac{\partial r}{\partial \mu}B\right]\frac{\partial \mu}{\partial K} - \left(r + \frac{\partial r}{\partial B}B\right)\frac{\partial \theta}{\partial \mu}\frac{\partial \mu}{\partial K}K\right] < 0.$$

The first term,

$$\mathbb{E}^f\left[-\frac{\gamma K^2}{\alpha^2\mu^2}\left(\alpha_x + \frac{3\alpha_x^2\mu^2}{\gamma^2\sigma_\varepsilon^2}\right)\left(1 + r + \frac{\partial r}{\partial B}B\right) - \frac{\partial r}{\partial \mu}B\right]\frac{\partial \mu}{\partial K} < 0,$$

is identical to the effect (8) from the baseline model. The second term,

$$\mathbb{E}^f\left[-\left(r + \frac{\partial r}{\partial B}B\right)\frac{\partial \theta}{\partial \mu}\frac{\partial \mu}{\partial K}K\right] < 0,$$

originates from the budget constraint (9). It is negative because we have shown earlier that $r > 0$, $\partial r/\partial B > 0$, $\partial\mu/\partial K > 0$, and $\mathbb{E}[\partial\theta/\partial\mu] = \mathbb{E}[-[(\partial r/\partial\mu)B]/K] > 0$, since $\mathbb{E}^f[\theta] > 0$. Put differently, the reduction in borrowing costs allows us to increase dividends, which increases the price at which shares sell, $(\partial\theta/\partial\mu)(\partial\mu/\partial K)K > 0$. In turn, the firm can sell fewer bonds, which reduces interest expenses $-r$. Finally, the reduction in borrowing reduces interest costs on the outstanding debt, $-(\partial r/\partial B)B$.

Adding a budget constraint (9) therefore amplifies the conclusion that the firm issues more equity to internalize the informational externality that stock prices have on bond yields.

[13] Note that $\mathbb{E}^f[r] = r(\pi_p, B) = r(\pi_p(\mu(K)), I - \theta K + cK^2)$. Participation μ is once again increasing in K. The proof that information participation increases in K is parallel to that in section A.2 (once we note that $r = r(p)$ is known once stocks are traded at price p).

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