Pricing and inventory decisions in the food supply chain with production disruption and controllable deterioration

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Abstract

Food deterioration is becoming a crucial problem for most countries in the world, which may cause both economical losses and environmental damages. In this paper, a Stackelberg gaming model for a three-level food supply chain (consists of one retailer, one vendor and one supplier) with production disruption is established, which aims to study the optimal pricing, inventory and preservation decisions that maximize the individual profit. In the decentralized supply chain, upstream firms act as leaders and downstream firms as followers. Due to the mathematical complexity, an illustrative algorithm is developed to solve the problem. Numerical tests show that retailer's preservation investment not only benefits itself, but also benefits the vendor and the supplier. Comparing the optimal decisions to that in the 'forward integration' and 'backward integration' model, supply chain members' vertical cooperation helps to enhance the total profit. Meanwhile, the carbon footprint of the food supply chain is also studied. It is found that, vertical cooperation contributes to the reduction of carbon emission. In most situations, 'forward integration' outperforms 'backward integration' strategy because it incents the retailer to invest more in preservation and reduce food deterioration. Other managerial implications are also shown in the paper.

Keywords: Deteriorating items, Game theory, Preservation investment, Cooperative strategies, Carbon emission

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1. Introduction

Food deterioration is becoming a great challenge for food industry in many countries. According to Ghare & Schrader (1963), deterioration is defined as decay, change or spoilage through which the quality and/or the quantity of the items are decreasing. There are many reasons for the high perishable rate for food products such as long distance transport, inappropriate preservation methods, poor sanitation standards or rapid change in demand and supply. Approximately, 15% of foods are deteriorated in the

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food retailing sector (Ferguson & Ketzenberg, 2005). About one third of food produced is perished or wasted during consumption globally, which accounts for 1.3 billion tons (FAO, 2011). Also, as reported by Martin (2015), in China, more than 25% of fruit and vegetables are deteriorated during transportation, at wholesale markets and in shops.

Food deterioration causes both economical and environmental damages. It is prevalent in industry that companies in food supply chains (including food producers, food distributors and food sellers) are suffering from high losses due to food deterioration. As reported, food spoilage in Australia costs about $ 10,000,000 annually in its food sectors (Pitt & Hocking, 2009). In addition to the economic damages, food deterioration also worsens greenhouse gas emissions and brings significant damages to natural resources, such as air, water and climate (Alex, 2013). The carbon emission of food produced and wasted is approximately 3.3 billion tons, which follows the total emission of the USA and China (FAO, 2013). Thus, reducing food deterioration is significantly important and meaningful for both the economy and the environment.

To reduce food deterioration, an applicable option is to *invest in preservation technologies* during manufacturing, storage, transportation, and in the supermarkets (See Blackburn & Scudder, 2009; Dye & Hsieh, 2012; Hsu et al., 2010; Kouki et al., 2013; Musa & Sani, 2012). Spoilage of foods mainly stems from several environmental factors, including temperature, relative humidity, air velocity, atmospheric composition and sanitation procedures (Qin et al., 2014). Thus, suitable preservation environment is required to reduce product deterioration, which can be achieved by utilizing various preservation technologies. For example, supermarkets use refrigerators to preserve meat, milk, eggs, fruits and vegetables; use drying machines to keep the breads or cakes dry; use humidifiers to keep fruits or flowers hydrated. However, to achieve a lower deterioration rate, more investments are required. For example, to maintain a lower temperature, more electricity will be consumed. In real practice, managers need to balance the cost of product deterioration and that of preservation to enhance total profit, which is challenging but meaningful for supply chain management.

Vertical cooperation can also reduce food deterioration by reducing production/transportation lead times or optimizing production and sales strategies. In the food industry in China, some food companies choose to integrate with downstream sellers. This type of integration is called 'forward integration'. An illustrative example is *Suguo Inc.* (a leading supermarket in eastern China) operates several large distribution centers by itself. After procuring varies food products (including fresh fruits, vegetables, meat, milk) from upstream suppliers, they store the products in their refrigerated warehouses, and deliver the food products to its own sales stores. Besides, some companies choose the 'backward integration' strategy, which means the collaboration between upstream producers or raw material suppliers and vendors. Taking two fresh meat providers, *Shuanghui Inc.* and *Yurun Inc.* in China as examples, both of them cooperate with the upstream farms and distribute fresh pork to downstream retailers through their own distribution systems. Either type of cooperation strategy has its own advantages and disadvantages. As Lin et al. (2014) demonstrates, 'forward integration' enables firms to better control the retail price, and to respond more effectively to the changes in market demand changes. However, 'backward integration' enables firms to better control the production process and quality of the products.

In food industry, *production disruption* happens frequently and has significant impacts. For example, the Typhoon Goni that raged in Northern Luzon caused significant agriculture losses in Philippines, which result in the rise of vegetable prices and shortage of supply (Pia, 2015). As one of the biggest citrus growers in the world, Chinese citrus industry is suffering from typhoons repeatedly and greening disease, which once took more than 10% of the total production away during two seasons in 2014 (Cherrie, 2015). Disruptions in production processes at farms and food processors not only cause breakdowns in production, but also delays in supply chains. The upstream supply chain disruption can have significant impacts on downstream operations, and can cause purchasing cost increase, the shortage of supply or the damage to firms reputations. It is therefore critical to study the interactive decision making in the supply chains when the partners face production disruption.

Previous research on food deterioration mainly concentrate on the analysis of economical impacts, while seldom consider its environmental impacts. To solve the real world problems and to fill the gap in literature, this paper also analyzes the carbon footprint of the food supply chain. Specifically, the main research targets are summarizes as follows

- To study the supplier's, the vendor's and the retailer's optimal prices, preservation investment and inventory decisions under the risk of upstream disruption and product deterioration.
- To investigate the impacts of critical parameters, such as producers reliability, inventory holding costs and production costs, to the optimal decisions, the maximum profits and carbon emissions.
- To investigate both the economical and environmental impacts of different cooperative strategies (i.e., forward integration and backward integration).

Focusing on the main research targets, a three level supply chain is modeled with a retailer, a vendor and a supplier. The main contributions of this paper are as follows. Firstly, a three level supply chain producing and selling deterioration products is studied, in which the supplier has production disruption risk and the retailer has controllable deterioration rate. The paper aims to fill the gap of supply chain management models for deteriorating items. Secondly, an illustrative algorithm is proposed to solve the complex multi-level gaming model. Thirdly, based on the numerical tests and sensitivity analysis, some important and interesting managerial insights for supply chain management of deteriorating items are identified, which can help to improve supply chain efficiency. Lastly, impacts of supply chain structure to the equilibrium results and carbon emission are studied.

1.1. Literature review

This research mainly involves three key elements: (1) preservation technology investment for deteriorating products (2) gaming models in multi-level supply chains with inventory decisions and (3) production disruption models with deteriorating products.

Table 1: Summary of the existing literature

	Preservation investment	Pricing	Supply Chain disruption	Supply chain type
Hsu et al. (2010)	√			One level
Dye & Hsieh (2012)	√			One level
Dye (2012)	√	√		One level
Dye (2013)	√			One level
Dye & Yang (2016)	√	√		One level
He & Huang (2013)	√	√		One level
Hsieh & Dye (2013)	√			One level
Yang et al. (2015)	√			One level
Zhang et al. (2016)	√	√		One level
Tayal et al. (2014)	√			Two level
Zhang et al. (2015)	√	√		Two level
Glock (2013)			√	One level
Abboud (1997)			√	One level
Abboud et al. (2000)			√	One level
Chung et al. (2011)			√	One level
Giri et al. (2005)			√	One level
Sana et al. (2007)		√	√	One level
Chakraborty et al. (2008)			√	One level
Jeang (2012)			√	One level
This paper	√	√	√	Three level

The first stream is about EOQ/EPQ models with product deterioration and preservation technology investment. In most literature, deterioration rate is assumed to be a constant parameter (see He & He, 2010; He & Wang, 2012; He et al., 2010; Liang & Zhou, 2011; Sana et al., 2004; Taleizadeh, 2014; Taleizadeh et al., 2013; Taleizadeh & Nematollahi, 2014; Taleizadeh et al., 2015; Thangam & Uthayakumar, 2009; Widyadana et al., 2011) or an exogenous time linked parameter (see Musa & Sani, 2012; Shah et al., 2013; Skouri et al., 2009; Tat et al., 2015). However, in real situations, deterioration rate can be reduced through various efforts such as procedural changes and specialized equipment installation. For products with high deterioration rates, such as fruits, vegetables or seafoods, firms usually adopt preservation technologies to reduce the deterioration rate. Some scholars found the links between investment and deterioration rate, and the reduced proportion of deterioration rate is a convex increasing function of the investment level (see Hsu et al., 2010; Dye & Hsieh, 2012). Blackburn & Scudder (2009) studied the optimal temperature control and delivery batch decision through the whole supply chain from picking stage, cooling stage to selling stage. Kouki et al. (2013) found that a continuous temperature control policy can be more efficient in warehouse management. Similar studies can be seen in Dye (2012), Dye (2013), Dye & Yang (2016), He & Huang (2013), Hsieh & Dye (2013), Yang et al. (2015) and Zhang et al. (2016), which all consider firms preservation investment decisions under different conditions. Some people studied the preservation investment problem in a two level supply chain, such as Tayal et al. (2014) and Zhang et al. (2015). In this research stream, previous studies seldom consider multi-level supply chain problems with preservation investment.

The second stream refers to the gaming models in multi-level supply chains on inventory and pricing decisions. Lee et al. (2016) studied a two level supply chain with VMI policy and limited storage capacity. They found that an inventory holding cost sharing policy can coordinate the supply chain efficiently when the vendor's reservation cost is equal to the minimum cost of integrated supply chain. Yu et al. (2012) studied an integrated supply chain with one manufacturer and multiple retailers. Numerical tests showed that VMI can achieve a lower cost comparing to the decentralized supply chain. Ghiami et al. (2013) studied an integrated supply chain inventory system with one supplier and one retailer, in which the retailer's warehouse has capacity constraint and also can rent a warehouse with higher holding cost. Cárdenas-Barrón & Sana (2014) also studied an integrated supply chain with one retailer and one supplier, in which the production rate is a decision variable and the production cost is linked to production rate. Lee & Moon (2006) Mitra (2012) studied a two level closed-loop supply chain with used products recovering and found that low recycling rate or high recovered product demand results in higher profit. Xu et al. (2012) studied a supplier's and a retailer's gaming problem with inventory inaccuracy. Some improvement strategies (e.g., information sharing, error estimation, RFID indicator application) are proposed to mitigate inaccuracy and to gain more profit. Lee & Moon (2006) studied a three echelon inventory system with a supplier, a manufacturer and a retailer considering product deterioration. They model the supply chain under different alliance settings, and studied the effect of the alliance style for each party's profit. A compensation policy is applied to achieve the perfect coordination of the whole supply chain. Wang et al. (2011) extended Lee & Moon (2006) by considering product deterioration and studied the joint impacts of product deterioration and alliance types to the optimal decisions. In this research stream, most papers consider integrated decisions, and seldom consider multi-level gaming problems for deterioration products.

The third stream of literature is the EOQ/EPQ models considering production disruption. A common assumption in the studies has been that, when a disruption happens, the production rate drops to zero (Glock, 2013). Abboud (1997) established an EMQ model by considering machine failure during production under Poisson distribution. Then, Abboud et al. (2000) developed an economic lot sizing model with the consideration of random machine unavailability time. Later, Chung et al. (2011) extended the model by considering product deterioration with stochastic machine unavailability time and shortage. Wang (2004) developed an EPQ model where production shifts from an in-control state to an out-of-control state with an exponential shift distribution. Giri et al. (2005) developed EPQ model with machine failure and general time. Sana et al. (2007) developed an EPQ model with unreliable production process and assumed that some of the imperfect quality items can be sold at a lower price. Chakraborty et al. (2008) studied an EPQ model considering production system with process deterioration and machine breakdown. Jeang (2012) assumed that the quality of the products drops with time and considered about the determination of production lot size and process parameters under process breakdown and process deterioration simultaneously. In this research stream, seldom papers consider multi-level inventory models with production disruption.

A summary of the existing literature is shown in Table 1. This paper aims to fill the gaps in the above streams by considering preservation investment and gaming in a three level supply chain. The remainder of the paper is structured as follows. Section 2 is the assumptions and notations of the model. Section 3 is the model formulation. Section 4 is the numerical tests for decentralized case. In section 5, models for the forward integrated supply chain and the backward integrated supply chain are studied, along with the comparison of the three cases. Section 6 presents the carbon footprint analysis of the supply chain. Section 7 is the conclusion for this paper.

2. Notation

The notation is presented in Table 2.

3. Model formulation

A three level inventory system (see Figure 1) is studied, which is comprised of a supplier, a vendor and a retailer. The supplier produces and sells a kind of deteriorating food products to an intermediate vendor, who re-sells the products to a downstream retailer. Then, the retailer sells the products to end customers. All of the firms adopt the 'lot-to-lot' policy (Lee & Moon, 2006; Lee et al., 2016) and the decision time

Table 2: Notation

Decision Variables	
u	Retailer's preservation investment.
p	Retailer's selling price.
T_r	Retailer's ordering cycle length.
q	Retailer's ordering quantity.
w_v	Vendor's wholesale price.
T_v	Vendor's production time length.
n	Vendor's shipment times to the retailer in one cycle.
Q	Vendor's ordering quantity.
w_s	Supplier's wholesale price.
T_s	Supplier's production time length.
System Parameters	
g:	The unit cost of preservation investment.
γ:	The preservation investment efficiency.
θ_r, θ_v, θ_s:	Retailer, vendor and supplier's deterioration rate respectively.
h_r, h_v, h_s:	Retailer, vendor and supplier's unit inventory holding respectively.
A_r, A_v, A_s:	Retailer, vendor and supplier's lump sum ordering (or starting) cost respectively.
c:	Supplier's unit production cost.
P:	Supplier's production rate.
M:	Supplier's production line recovery cost.
μ:	Supplier's replenishment cost from an secondary market, $\mu > c$.
s:	Supplier's production line's 'in-control' time length, a random variable following an exponential distribution with parameter λ. (Chakraborty et al., 2008; Jeang, 2012) The probability density and distribution function is $g(s) = \frac{1}{\lambda} e^{\frac{-s}{\lambda}}$ and $G(s) = \int_0^s g(s)ds$ respectively.
$\widetilde{D}$	Maximum truck load capacity.
$e_p, e_{t1}, e_{t2}, e_s, e_d$	Carbon emission parameters during production, transportation, storage, and deterioration.
Dependent Variables	
$D(p)$:	Customers' demand rate, a linear function of selling price $D(p) = a - bp$.
$f(u)$:	The reduced deterioration rate ($0 < f(u) = 1 - e^{-\gamma u} < 1$), a concave increasing function w.r.t. preservation investment u. $f^{'} > 0$,$f^{''} < 0$.
$I_r(t), I_v(t)$:	Retailer and vendor's inventory level w.r.t. time t.
$I_s^1(t), I_s^2(t)$:	Supplier's inventory level w.r.t. time t in the 'in-control' and 'out-of-control' states respectively.
TP_r, TP_v:	Retailer and vendor's profit.
TP_s^i:	Supplier's profit under under scenario i, where $i = ND$ denotes the scenario without disruption and $i = D$ denotes the scenario with disruption.
TP_s:	Supplier's expected profit.
TP_{sc}:	Supply chain's total expected profit.
E_p, E_t, E_s, E_d	Total carbon emission during production, transportation, storage, and deterioration respectively.
TE	Total average carbon emission amount.

horizon is infinite. At every time instance, customers come to the retailer's store and purchase the food products with price p, which yields a demand rate of $D(p) = b - ap$.

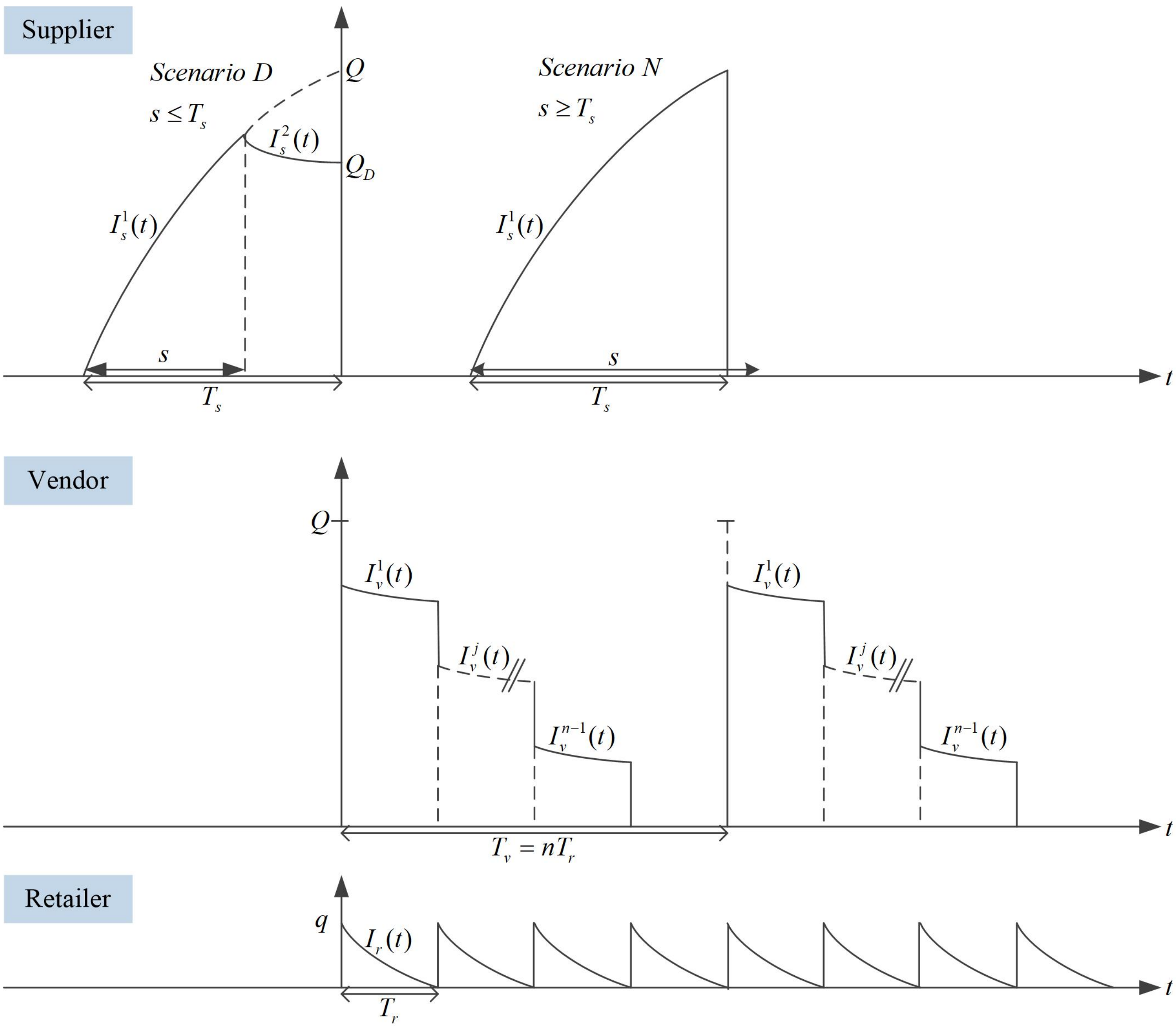


Figure 1: Supplier, vendor and retailer's inventory systems.

To maximize the unit time profit, the retailer needs to optimally set sales price (p) together with ordering cycle length (T_r) and ordering quantity (q). In addition, to reduce the deterioration rate and cut down the deterioration cost, the retailer can also invest in preservation. The retailer can reduce the initial deterioration rate θ_r to $\theta_r f(u)$, in which $f(u)$ is constrained by $0 < f(u) < 1$ and determined by the investment level u. Following previous studies on preservation investment (e.g. Zhang et al., 2015), the function of $f(u)$ satisfies

$$f(u) = 1 - e^{-\gamma u}, u \in [0, +\infty), \tag{1}$$

which is exponentially linked to the investment level u. Parameter γ denotes the efficiency of preservation investment. For investment level u, the unit time investment cost is gu. In this study, only retailer's preservation investment strategies is considered. The supplier's and vendor's deterioration rates are assumed to be much lower than that of the retailer, thus the deterioration rates are treated as constants.

Following the studies of Ben-Daya et al. (2008) and Huang et al. (2017), the supplier's production line is unreliable, which may switch from the 'in-control' state to the 'out-of-control' state during the production run. In the 'in-control' state, the production line functions well; however, in the 'out-of-control' state, the production line stops working. The 'in-control' time length s is a random variable, with probability density function $g(s)$ and cumulative distribution function $G(s)$. Assuming s follows a exponential distribution with parameter λ (Huang et al., 2017), which means

$$g(s) = \frac{1}{\lambda} e^{-\frac{s}{\lambda}}, \qquad G(s) = 1 - e^{-\frac{s}{\lambda}}. \tag{2}$$

The expectation of s is $E[s] = \lambda$. When the expected 'in-control' time length is longer, the supplier's production system is more reliable; otherwise, the production system will be less reliable.

To focus on the main research targets, additional assumptions should be made. Firstly, shortage is not allowed for all the three firms. When disruption happens, the supplier has to replenish from a secondary market with a high cost to fill the shortages caused by production disruption. Secondly, when the production line is disrupted, the supplier needs to pay a lump sum cost to recover it at the end of each production cycle. Thirdly, the production rate is higher than the demand rate, which makes sure the supplier has enough time to produce the vendor's orders in a production cycle. Lastly, upper stream firms act as Stackelberg leaders, while lower stream firms act as followers. Thus, the gaming sequence is

- Firstly, the supplier announces its wholesale price w_s.
- Secondly, by knowing the supplier's wholesale price, the vendor sets its wholesale price w_v and shipment multiplier n.
- Thirdly, by knowing the vendor's wholesale price w_v, the retailer determines the selling price p, the preservation investment u and the ordering cycle T_r simultaneously.
- Lastly, customers arrives and purchase with price p.

3.1. Retailer's problem

Figure 1 shows that the retailer's inventory depletes due to two reasons: the demand and product deterioration. At time $t = 0$, the retailer receives q products from the vendor. During the time interval $t \in [0, T_r]$, its inventory level depletes and finally reaches zero at time $t = T_r$. For notational convenience, $D(p)$ is denoted by D and $f(u)$ is denoted f. According to previous research on deteriorating inventory models (e.g., Ghare & Schrader, 1963), the retailer's inventory level satisfies the following differential equation

$$\dot{I}_r(t) = -D - \theta_r(1-f)I_r(t), t \in [0, T_r], \tag{3}$$

with boundary conditions $I_r(T_r) = 0$ and $I_r(0) = q$. Solving the differential equation yields the inventory level *w.r.t.* time t as

$$I_r(t) = \frac{D}{\theta_r(1-f)}\left(e^{\theta_r(1-f)(T_r-t)} - 1\right), t \in [0, T_r]. \tag{4}$$

Hence, the retailer's ordering quantity q can be given by

$$q = I_r(0) = \frac{D}{\theta_r(1-f)}\left(e^{\theta_r(1-f)T_r} - 1\right) \tag{5}$$

During each ordering cycle, the retailer's unit time average profit can be expressed as

$$\begin{aligned} TP_r(p, u, Tr) &= \frac{1}{T_r}\left(pDT_r - w_v q - A_r - h_r\int_0^{T_r} I_r(t)\mathrm{d}t - guT_r\right) \\ &= pD - w_v\frac{D(e^{\theta_r(1-f)T_r} - 1)}{\theta_r(1-f)T_r} - \frac{A_r}{T_r} - h_r\frac{D(e^{\theta_r(1-f)T_r} - \theta_r(1-f)T_r - 1)}{\theta_r^2(1-f)^2T_r} - gu \end{aligned} \tag{6}$$

In the right side of (6), the first part denotes the average sales revenue. The second part denotes the average purchasing cost. The third part is the average ordering cost. The fourth part represents the average inventory holding cost. The last part is the unit time preservation investment cost. The retailer's problem is to find the optimal decisions of p, u and T_r that maximize the profit function by knowing the vendor's wholesale price w_v, which is summarized in (7).

$$\begin{aligned} \textbf{P.1} \qquad & \max_{p,u,T_r}\{TP_r(p, u, T_r; w_v)\}, \\ & s.t.\ \ u \geq 0, \quad w_v \leq p \leq \frac{b}{a}, \quad T_r > 0. \end{aligned} \tag{7}$$

By calculation, properties of the optimal decisions can be obtained, which are presened in the following propositions.

Proposition 1. *For constant p, T_r and w_v, the retailer's optimal preservation investment u^d can be derived from $\frac{\partial TP_r}{\partial u}=0$, namely*

$$\frac{w_v D}{\theta_r T_r}(\frac{\theta_r T_r f' e^{\theta_r(1-f)T_r}}{1-f}-\frac{f'(e^{\theta_r(1-f)T_r}-1)}{(1-f)^2})+\frac{h_1 D}{\theta_r^2 T_r}(\frac{\theta_r T_r f'(e^{\theta_r(1-f)T_r}-1)}{(1-f)^2} -\frac{2f'(e^{\theta_r(1-f)T_r}-\theta_r(1-f)T_r-1)}{(1-f)^3})-g=0 \tag{8}$$

if $u^d \geq 0$;otherwise, the optimal preservation investment is $u^d=0$.

Proof. See Appendix A.1. □

Proposition 2. *For constant u, T_r and w_v, the retailer's optimal price p^d can be derived from $\frac{\partial TP_r}{\partial p}=0$, namely*

$$p^d=\frac{1}{2a}(b+aw_v\frac{e^{\theta_r(1-f)T_r}-1}{\theta_r(1-f)T_r}+ah_r\frac{e^{\theta_r(1-f)T_r}-\theta_r(1-f)T_r-1}{\theta_r^2(1-f)^2T_r}) \tag{9}$$

if $w_v < p^d \leq \frac{b}{a}$; otherwise, the optimal price is $p^d=\frac{b}{a}$.

Proof. See Appendix A.2. □

Proposition 3. *For constant u, p and w_v, the retailer's optimal ordering cycle T_r^d can be derived from $\frac{\partial TP_r}{\partial T_r}=0$, namely*

$$(\frac{w_v D}{\theta_v(1-f)}+\frac{h_r D}{\theta_r^2(1-f)^2})(\theta_r(1-f)T_r^d e^{\theta_r(1-f)T_r^d}-e^{\theta_r(1-f)T_r^d}+1)-A_r=0. \tag{10}$$

Proof. See Appendix A.3. □

In Proposition 1-3, the existence of optimal decisions p, u and T_r that maximize the retailer's profit function is proved. Due to the mathematical complexity, analytical results for the retailer's optimal decisions cannot be obtained. The uniqueness of the optimal decisions is proved numerically in the numerical analysis. Solving the functions of (8)-(10), the response functions $\hat{p}^d(w_v)$, $\hat{u}^d(w_v)$, $\hat{T}_r^d(w_v)$ and $\hat{q}_r^d(w_v)$ for wholesale price w_v can be obtained.

3.2. Vendor's problem

By anticipating the retailer's responsive decisions, the vendor needs to optimally determine wholesale price (w_s), ordering quantity (Q) and inventory parameter (n). The inventory pattern of the vendor is clearly depicted in Figure 1. The ordering cycle for the vendor is nT_r. The inventory depletes due to product deterioration and retailer's batch orders. Suppose the inventory level of the vendor during one ordering cycle in the jth phase is $I_v^j(t)$, in which $j = 1, 2, ..., n$. Then, vendor's inventory level in the jth $(j = 1, 2, ..., n)$ phase satisfies the differential equation

$$\dot{I}_v^j(t) = -\theta_v I_v^j(t), \qquad t \in [(j-1)T_r, jT_r]. \tag{11}$$

with boundary conditions

$$\begin{aligned} I_v^1(0) &= Q - q, \\ I_v^{j-1}((j-1)T_r) - q &= I_v^j(jT_r), \quad j = 1, 2, .., n, \\ I_v^n((n-1)T_r) &= 0. \end{aligned} \tag{12}$$

By calculation, the inventory level can be expressed as

$$I_v^j(t) = \begin{cases} qe^{-\theta_v t} \sum_{i=j}^{n-1} e^{i\theta_v T_r}, & t \in [(j-1)T_r, jT_r], \quad j = 1, 2, ..., n-1. \\ 0, & t \in [(j-1)T_r, jT_r], \quad j = n. \end{cases} \tag{13}$$

The ordering quantity Q w.r.t. n and T_r can be expressed as

$$Q = I_v^1(0) + q = q\frac{e^{n\theta_v T_r} - 1}{e^{\theta_v T_r} - 1}. \tag{14}$$

The total inventory holding quantity can be expressed as

$$\sum_{j=1}^{n} \left(\int_{(j-1)T_r}^{jT_r} I_v^j(t)dt \right) = \frac{q}{\theta_v} \left(\frac{e^{\theta_v n T_r} - e^{\theta_v T_r}}{e^{\theta_v T_r} - 1} - n + 1 \right). \tag{15}$$

The vendor's profit function consists of four parts: the sales revenue, the purchasing cost, the fixed ordering cost and the inventory holding cost. The overall average profit function can be expressed as

$$TP_v(w_v, n) = \frac{1}{nT_r} \left(w_v q n - w_s q \frac{e^{\theta_v n T_r} - 1}{e^{\theta_v T_r} - 1} - A_v - h_v \frac{q}{\theta_v} \left(\frac{e^{n\theta_v T_r} - e^{\theta_v T_r}}{e^{\theta_v T_r} - 1} - n + 1 \right) \right). \tag{16}$$

By substituting $\hat{p}^d(w_v)$ and $\hat{q}^d(w_v)$ into vendor's profit function,the supplier's profit function TP_v

w.r.t. decision variable w_v and n can be derived. The optimization problem is

$$\begin{aligned} \mathbf{P.2} \quad & \max_{w_v,n} \left\{TP_v\left(w_v, n, \hat{q}^d(w_v), \hat{p}^d(w_v), \hat{u}^d(w_v), \hat{T}_r^d(w_v); w_s\right)\right\}, \\ & s.t.\ \ w_s < w_v < \hat{p}^d(w_v), \quad n \in N^+. \end{aligned} \tag{17}$$

The wholesale price is constrained by $w_s < w_v < \hat{p}^d(w_v)$ to insure positive sales margin. Due to the complexity, analytical results cannot be obtained. Using computation software, the responsive functions $\hat{w}_v^d(w_s)$ and $\hat{n}^d(w_s)$ *w.r.t.* supplier's selling price w_s can also be derived numerically. The optimal ordering quantity $\hat{Q}_v^d(w_s)$ can also be obtained with (14).

3.3. Supplier's problem

After receiving the vendor's orders $\hat{Q}_v^d(w_s)$ (denoted by Q for expositional simplicity in this subsection), the supplier starts its production at time point $t = T_v - T_s$. As depicted in Figure 1, in every production cycle T_v, due to the uncertainty of 'in-control' time period, the production line may be disrupted, which yields two scenarios: No disruption (N) scenario and Disruption (D) scenario.

Scenario 1: No production disruption

In this scenario, $s \geqslant T_s$, which means production disruption does not happen and the production line is smooth during the production cycle. The inventory level of the supplier rises due to production rate and deterioration rate. Thus, the inventory evolves according to the differential equation

$$\dot{I}_s^1(t) = P - \theta_s(1-f)I_s^1(t), t \in [T_v - T_s, T_v], \tag{18}$$

with boundary conditions $I_s^1(t)(t = T_v - T_s) = 0$ and $I_s^1(T_v) = Q$.

The production time length *w.r.t.* Q can be obtained as

$$T_s(Q) = \frac{1}{\theta_s} ln\left(\frac{\theta_s Q}{P} - 1\right). \tag{19}$$

The inventory level for the supplier can be expressed as

$$I_s^1(t) = \frac{P}{\theta_s}\left(1 - e^{-\theta_s(t - T_v + T_s(Q))}\right), t \in [T_v - T_s(Q), T_r]. \tag{20}$$

The total average profit for the supplier under scenario 1 can be expressed as

$$
\begin{aligned}
TP_s^N(w_s) &= \frac{1}{T_v}\left(w_sQ - cPT_s(Q) - A_s - h_s\int_{T_r-T_s(Q)}^{T_r} I_s^1(t)\mathrm{d}t\right) \\
&= \frac{1}{T_v}\left(w_sQ - cPT_s(Q) - A_s - \frac{h_sP(e^{-\theta_sT_s(Q)} + \theta_sT_s(Q) - 1)}{\theta_s^2}\right).
\end{aligned} \tag{21}
$$

The first part is the wholesale revenue, the second part is the production cost, the third part is the lump sum starting cost, the last part is the inventory holding cost.

Scenario 2: With production disruption

In this scenario, disruption happens ($s < T_s(Q)$) during the production run, and the final production quantity cannot satisfy the vendor's ordering quantity. Before the breakdown time point $t = s$, the inventory level $I_s^1(t)$ increases due to production and product deterioration, which satisfies the differential equation

$$\dot{I}_s^1(t) = P - \theta_s(1-f)I_s^1(t), t \in [T_v - T_s(Q), T_v - T_s(Q) + s], \tag{22}$$

with boundary condition $I_s^1(T_v - T_s(Q)) = 0$. Solving the equation yields

$$I_s^1(t) = \frac{P}{\theta_s}\left(1 - e^{-\theta_s(t-T_v+T_s(Q))}\right), t \in [T_v - T_s(Q), T_v - T_s(Q) + s]. \tag{23}$$

After production breakdown, inventory level $I_s^2(t)$ depletes due to deterioration and satisfies the differential equation

$$\dot{I}_s^2(t) = -\theta_s(1-f)I_s^2(t), t \in [T_v - T_s(Q) + s, T_v], \tag{24}$$

with boundary condition $I_s^1(t = T_v - T_s(Q) + s) = I_s^2(t = T_v - T_s(Q) + s)$. Solving the equation yields

$$I_s^2(t) = \frac{P}{\theta_s}(e^{\theta_s s} - 1)e^{-\theta_s(t-T_v+T_s(Q))}, t \in [T_v - T_s(Q) + s, T_v]. \tag{25}$$

The final production quantity when facing production disruption is

$$Q_D = I_s^2(T_v) = \frac{P}{\theta_s}\left(e^{\theta_s s} - 1\right)e^{\theta_sT_s(Q)}. \tag{26}$$

It is straight forward that $Q_D < Q$, which means the supplier has to replenish from the secondary market to fill the vendor's orders when disruption happens.

The average profit is

$$TP_s^D(w_s) = \frac{1}{T_v}\left(w_sQ - cPs - \mu(Q-Q_D) - M - A_s - h_s\int_{T_v-T_s(Q)}^{T_v-T_s(Q)+s} I_s^1(t)\mathrm{d}t - h_s\int_{T_v-T_s(Q)+s}^{T_v} I_s^2(t)\mathrm{d}t\right)$$
$$= \frac{1}{T_v}\left(w_sQ - cPs - \mu(Q-Q_D) - M - A_s - h_s\frac{P(e^{\theta_s s}+\theta_s s-1)}{\theta_s^2} - h_s\frac{P(1-e^{-\theta_s s})(1-e^{-\theta_s T_s(Q)+\theta_s s})}{\theta_s^2}\right). \tag{27}$$

The first part is the wholesale revenue. The second part is the production cost. The third part is the replenishment cost from the secondary market to fill the vendor's orders. The fourth part is the recovering cost when production disruption happens. The fifth part is the lump sum starting cost, and the last part is the inventory holding cost.

Based on the analysis of two scenarios (i.e., scenario N and scenario D) *w.r.t.* different values of s, the expected average profit for the supplier can be finally expressed as

$$TP_s(w_s) = \int_{T_s(Q)}^{\infty} TP_s^N(w_s)\mathrm{d}G(s) + \int_0^{T_s(Q)} TP_s^D(w_s)\mathrm{d}G(s)$$
$$= \frac{1}{T_v}\int_{T_s(Q)}^{\infty}\left(w_sQ - cPT_s(Q) - A_s - \frac{h_sP(e^{-\theta_s T_s(Q)}+\theta_s T_s(Q)-1)}{\theta_s^2}\right)\mathrm{d}G(s)$$
$$+ \frac{1}{T_v}\int_0^{T_s(Q)}\left(w_sQ - cPs - \mu(Q-Q_D) - M - A_s - h_s\left(\frac{P(e^{\theta_s s}+\theta_s s-1)}{\theta_s^2} + \frac{P(1-e^{-\theta_s s})(1-e^{-\theta_s T_s(Q)+\theta_s s})}{\theta_s^2}\right)\right)\mathrm{d}G(s). \tag{28}$$

The supplier's optimization problem can be expressed as

$$\textbf{P.3}\quad \max_{w_s}\left\{TP_s\left(w_s, \hat{Q}^d(w_s), \hat{w}_v^d(w_s), \hat{n}^d(w_s)\right)\right\},$$
$$s.t.\quad c < w_s < \hat{w}_v(w_s). \tag{29}$$

3.4. Solution procedure for the decentralized model

Backward induction is used to solve the game model. The solution procedure begins from the downstream retailer, to the intermediate vendor, and finally to the upstream supplier. Thus, an illustrative algorithm with two level iteration is proposed to solve the model, which is shown in Table 3.

4. Numerical examples and sensitivity analysis for decentralized model

Numerical examples are presented to illustrate the above propositions and the algorithm.

4.1. Numerical example

Parameter settings are as follows.

Retailer $\theta_r = 0.005$, $h_r = 0.01$, $A_r = 10$, $b = 10$, $a = 1$, $g = 0.05$, $\gamma = 0.5$.

Table 3: Algorithm 1

Algorithm 1	
Step 1	Initialize $[TP_s]_{w_s} = 0$;
Step 2	Start with $j = 1$ and initialize $w_{s,j} = 0$.Given a small positive number δ_1 as step size;
Step 3.1	Initialize $[TP_v]_{w_v,n_0} = 0$;
Step 3.2	Start with $i = 1$ and initialize $w_{v,i} = 0$ and $n_0 = 1$. Given a small positive number δ_2 as step size;
Step 3.2.1	Calculate the optimal price p^d, the ordering cycle length T_r^d and preservation investment u^d from proposition 1-3, then the profit of the vendor $[TP_v]_{w_{v,i},n_0}$ for a given set of $w_{v,i}, n_0$;
Step 3.2.2	If $[TP_v]_{w_{v,i},n_0} - [TP_v]_{w_{v,i-1},n_0} > 10^{-6}$, then set $w_{v,i+1} = w_{v,i} + \delta_2$, i=i+1, and go to ***Step 3.2.1***;
Step 3.2.3	Otherwise, denote $w_{v,n_0} = w_{v,i}$, $[TP_v]_{w_{v,n_0},n_0}$, $n_0 = n_0 + 1$, and go to ***Step 3.2.1***;
Step 3.3	If $[TP_v]_{w_{v,n_0},n_0} > [TP_v]_{w_{v,n_0-1},n_0-1}$ and $[TP_v]_{w_{v,n_0},n_0} > [TP_v]_{w_{v,n_0+1},n_0+1}$, then denote $w_v^d = w_{v,n_0}$ and $n^d = n_0$.
Step 4	If $[TP_s]_{w_{s,j}} - [TP_s]_{w_{s,j-1}} > 10^{-6}$, set $w_{s,j+1} = w_{s,j} + \delta_1$, $j = j + 1$ and go to ***Step 3.1***.
Step 5	Otherwise, denote $w_s^d = w_{s,j}$;
Step 6	Output the equilibrium decisions $(p^d, u^d, T_r^d, w_v^d, w_s^d, n^d)$.

Vendor $\theta_v = 0.002$, $h_v = 0.005$, $A_v = 100$.

Supplier $\theta_s = 0.002$, $h_s = 0.005$, $A_s = 200$, $c = 1$, $M = 40$, $P = 6$, $\mu = 4$, $\lambda = 40$.

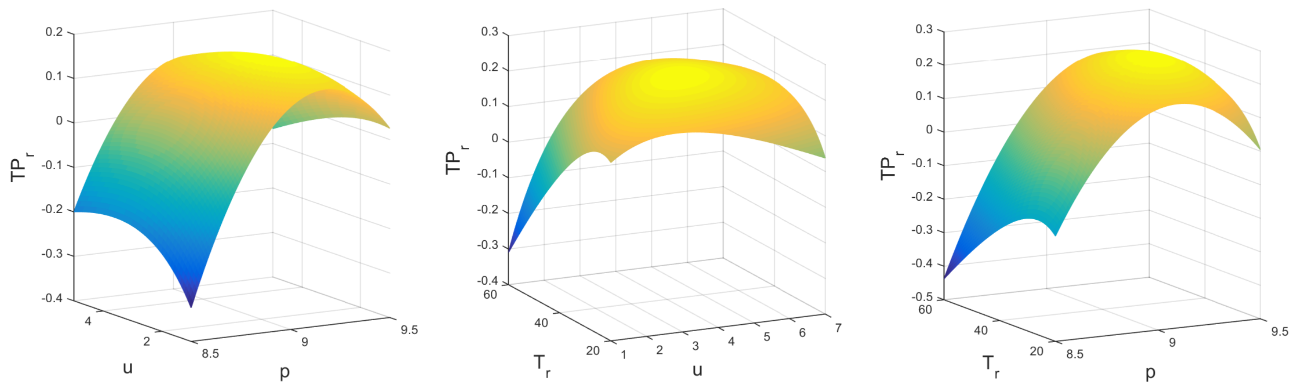
With the above parameters, algorithm 1 is adopted to search for the optimal solutions. As illustrated in Figure 2(a)-(c), when the vendor's wholesale price is fixed ($w_s = 8.00$), the retailer's profit function is concave in parameters p, u and T_r. So there is an optimal set p, u and T_r that maximize the retailer's profit function.

Then, when the supplier's selling price is fixed ($w_v = 6.00$), the vendor's profit function is concave in w_v and n (See Figure 3(a)), which means there is an optimal set of w_v and n that maximize the vendor's profit. Lastly, after the iteration of parameter w_s, it shows in Figure 3(b) that the profit function is concave in w_s and the optimal point is $w_s^d = 5.75$. The optimal solutions of the numerical example is shown as follows.

Retailer $p^d = 9.17$, $u^d = 3.78$, $T_r^d = 38.36$, $q^d = 32.30$, $TP_r^d = 0.2391$.

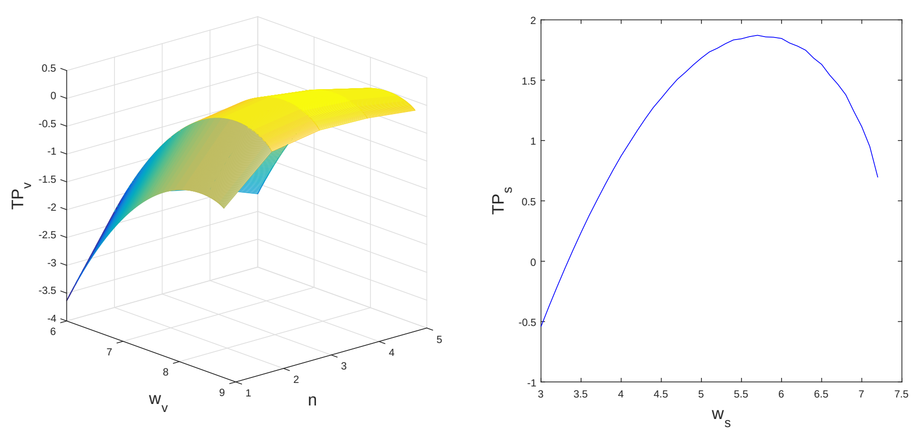
Vendor $w_v^d = 8.03$, $n^d = 3$, $Q^d = 104.83$, $TP_v^d = 0.4814$.

Supplier $w_s^d = 5.75$, $TP_s^d = 1.8854$.



(a) $T_r = 20$ (b) $p = 9.00$ (c) $u = 3.00$

Figure 2: The retailer's profit change w.r.t. (a) p and u (b) u and T_r (c) p and T_r when vendor's wholesale price is $w_v = 8.00$



(a) (b)

Figure 3: (a) The vendor's profit change $w.r.t.$ w_v and n when supplier's selling price is $w_s = 6.00$ (b) The supplier's profit change $w.r.t.$ w_s

4.2. Sensitivity analysis of the equilibrium results

Sensitivity analysis on the equilibrium strategies *w.r.t.* key system parameters g, γ, λ, M, μ, θ_r, θ_v and θ_s is carried out by varying only one parameter once and keep others fixed. The results are obtained by applying Algorithm 1, which are presented in Table 4. Some important managerial insights are also provided.

Table 4: Sensitivity analysis results for the decentralized model

Decentralized model													
Parameters	Change	p^d	u^d	T_r^d	q^d	n^d	Q^d	w_v^d	w_s^d	TP_r^d	TP_v^d	TP_s^d	TP_{sc}^d
Default		9.17	3.78	38.36	32.30	3	104.83	8.03	5.75	0.2391	0.4814	1.8854	2.6059
$g = 0.05$	-50%	9.19	5.39	43.69	35.86	3	117.71	8.09	5.79	0.3002	0.4854	2.0237	2.8093
	50%	9.15	2.76	33.81	29.36	3	94.38	7.96	5.70	0.2196	0.4641	1.7329	2.4166
$\gamma = 0.5$	-50%	9.15	3.93	30.22	26.43	3	84.32	7.92	5.73	0.1949	0.3526	1.5745	2.1220
	50%	9.19	3.15	41.97	34.43	3	112.60	8.08	5.80	0.2639	0.4659	1.9781	2.7079
$\lambda = 40$	-50%	9.26	3.68	39.96	30.21	3	98.38	8.18	6.03	0.1192	0.2391	1.4382	1.7965
	50%	9.11	3.84	37.32	33.75	5	197.02	7.92	5.34	0.3361	0.6084	2.5557	3.5002
$M = 40$	-50%	9.17	3.78	38.41	32.23	4	145.21	8.03	5.66	0.2349	0.4768	2.2468	2.9585
	50%	9.19	3.76	38.69	31.85	2	66.27	8.06	5.82	0.2116	0.2780	0.9950	1.4846
$\mu = 4$	-50%	9.10	3.84	37.23	33.88	3	109.70	7.91	5.52	0.3451	0.6991	2.2620	3.3062
	50%	9.26	3.68	39.96	30.21	3	98.38	8.18	6.03	0.1192	0.2391	1.5521	1.9104
$\theta_r = 0.005$	-50%	9.16	2.40	38.23	32.47	2	67.53	8.02	5.73	0.3194	0.3562	1.0550	1.7306
	50%	9.22	4.54	39.18	31.21	4	140.93	8.11	5.79	0.1330	0.3569	2.1776	2.6675
$\theta_v = 0.002$	-50%	9.16	3.79	38.12	32.62	4	138.28	8.01	5.81	0.2595	0.6691	2.2007	3.1293
	50%	9.19	3.76	38.69	31.85	2	67.63	8.06	5.75	0.2116	0.2354	1.0648	1.5118
$\theta_s = 0.002$	-50%	9.16	3.79	38.17	32.55	3	105.61	8.01	5.71	0.2552	0.5181	1.8954	2.6687
	50%	9.18	3.77	38.48	32.14	3	104.33	8.04	5.77	0.2290	0.4633	1.8688	2.5611

(1) *sensitivity analysis of g.*

It is shown in Table 4 that under the decentralized scenario, as the investment cost g increases, p^d, u^d, T_r^d, q^d, Q^d, w_v^d, w_s^d, TP_r^d, TP_v^d, TP_s^d, TP_{sc}^d decrease. This implies that when facing a higher investment cost, retailer is not willing to invest more in preservation, and this leads to a higher deterioration rate and deterioration cost. To reduce the deterioration cost, the retailer will set a lower price to obtain a higher market demand, also sets a lower ordering quantity to keep a lower inventory level. For the vendor and supplier, to stimulate the retailer to order more products, they set lower wholesale prices. As a result, the profits of all the three supply chain members decrease due to the increase of preservation investment cost.

(2) *sensitivity analysis of γ.*

A higher value of parameter γ means more efficiency of one unit investment. Under the decentralized scenario, Table 4 shows that when γ increases, p^d, T_r^d, q^d, Q^d, w_v^d, w_s^d, TP_r^d, TP_v^d, TP_s^d, TP_{sc}^d increases, however u^d decrease. This implies that when the efficiency of investment increases, the retailer can invest less on preservation to achieve a lower deterioration rate, thus the deterioration cost and the preservation cost decrease. Then the retailer can set a higher price, longer ordering cycle and a higher inventory level under the lower deterioration cost. This gives the vendor and supplier chances to increases the wholesale price. As a result, all the three members can benefit from the increase of parameter γ.

(3) *sensitivity analysis of* λ.

The parameter λ is the expected reliable time before disruption, and a higher value of λ means a more reliable production system. As Table 4 shows, when the suppliers production line is more reliable, p^d, T_r^d, w_v^d, w_s^d decease, while u^d, q^d, Q^d, TP_r^d, TP_v^d, TP_s^d, TP_{sc}^d increase. Additionally, with the more reliable supplier's production line, the recovering cost and the replenishment cost decrease, so it can set a lower wholesale price to the downstream members to stimulate more product orders from the retailer and the vendor. However, when setting a higher ordering quantity and under a higher inventory level, the retailer should invest more on preservation technology to reduce the deterioration cost. As a result, all the three members can benefit from a more reliable production system.

(4) *sensitivity analysis of* M *and* μ .

For a higher recovering cost M and replenishment cost μ, it shows in Table 4 that, p^d, T_r^d, w_v^d, w_s^d increase, while u^d, q^d, Q^d, TP_r^d, TP_v^d, TP_s^d, TP_{sc}^d decrease. When suppliers cost increase, to get more profit, the supplier need to set a higher wholesale price, which results higher selling prices and lower ordering quantity of the retailer and the vendor. For the retailer, with a lower ordering quantity, it will set a lower preservation investment level. In addition, all the three members profit decreases with higher recovering cost and extra purchasing cost.

(5) *sensitivity analysis of* θ_r, θ_v *and* θ_s.

In Table 4, it is presented that, when the retailers deterioration rate θ_r increases, q^d and TP_r^d decrease, while the other parameters increase. This is because, with higher deterioration rate, the retailer will set a higher price, a lower inventory level and a higher investment level to reduce its deterioration cost. However, the increase of the selling price gives chance for the vendor and supplier to set higher wholesale prices. It is interesting that higher deterioration rate in the retail side can benefit the upper stream firms. On the contrary, the increase of vendors and suppliers deterioration rates (θ_v and θ_s) can be harmful for all the supply chain members.

5. Cooperative strategies: forward v.s. backward integration

5.1. Model FI: The forward integration

In the forward integration model, the vendor cooperates with the retailer and they make decisions together. Such case is prevalent in industry, especially for some leading grocery companies. For example, the supermarket *Suguo Inc.* is a large grocery company in China, which is operating several distribution centers in eastern China and distributes its products to its own retail stores. Based on equations (6) and (16), the profit functions for the vendor-retailer alliance is $TP_{rv}(p,u,T_r,n;w_v) = TP_r(p,u,T_r;w_v) + TP_v(w_v,n)$. The gaming sequence is: firstly, the supplier announces its selling price w_s; then the retailer-vendor alliance set the optimal price p, preservation investment level u, ordering cycle length T_r and multiplier parameter n. The optimization problem is summarized as follows

$$w_s^{FI} = arg \begin{cases} \max_{w_s} \left\{ TP_s(w_s, \hat{p}^{FI}(w_s), \hat{u}^{FI}(w_s), \hat{T}_r^{FI}(w_s), \hat{n}^{FI}(w_s)) \right\} \\ s.t. \begin{cases} c \le w_s \le \hat{p}^{FI}(w_s), \\ \left(\hat{p}^{FI}(w_s), \hat{u}^{FI}(w_s), \hat{T}_r^{FI}(w_s), \hat{n}^{FI}(w_s)\right) = \\ \quad arg \begin{cases} \max_{p,u,T_r,n} \{TP_{rv}(p,u,T_r,n;w_s)\} \\ s.t. \quad w_s \le p \le \frac{b}{a}, u \ge 0, T_r > 0, n \in N^+. \end{cases} \end{cases} \end{cases} \tag{30}$$

5.2. Model BI: The backward integration

In the backward integration model, the vendor cooperates with the supplier and they make decisions together. Backward integration is also prevalent in industry. A representative example is two leading companies, *Yurun Inc.* and *Shuanghui Inc.*, in the Chinese pork industry. Both of the firms cooperate with upstream farms and sell fresh pork to downstream retailers through their own distribution centers. Based on equations (16) and (28), the profit functions for the alliance of the vendor and the supplier is $TP_{vs}(w_v,n) = TP_v(w_v,n;w_s) + TP_s(w_s)$. The gaming sequence is: firstly, the vendor-supplier alliance announces the selling price w_s and determines the multiplier parameter n; then the retailer sets the optimal price p, preservation investment level u and ordering cycle T_r.

The optimization problem is summarized as follows:

$$\left(w_s^{BI}, n^{BI}\right) = arg \begin{cases} \max_{w_v,n} \{TP_{vs}(w_v, n, \hat{p}^{BI}(w_v), \hat{u}^{BI}(w_v), \hat{T}_r^{BI}(w_v), \hat{q}^{BI}(w_v)\} \\ s.t. \begin{cases} c \le w_v \le \hat{p}^{BI}(w_v), n \in N^+, \\ \left(\hat{p}^{BI}(w_v), \hat{u}^{BI}(w_v), \hat{T}_r^{BI}(w_v), \hat{q}^{BI}(w_v)\right) = \\ \quad arg \begin{cases} \max_{p,u,T_r,n} \{TP_r(p,u,T_r;w_v)\} \\ s.t. \quad w_v \le p \le \frac{b}{a}, u \ge 0, T_r > 0 \end{cases} \end{cases} \end{cases} \tag{31}$$

5.3. Numerical studies

To investigate the impacts of cooperative strategies, numerical studies are conducted. The same parameters in the decentralized model are also used in model FI and model BI. The optimal decisions for model FI and model BI can be obtained, which is shown in Table 5. The following observations can be obtained.

Table 5: Optimal decisions for model FI and model BI

Optimal decisions of model FI		
Retailer-vendor	$p^{FI} = 8.34$, $u^{FI} = 6.77$, $T_r^{FI} = 109.01$, $n^{FI} = 1$, $q^{FI} = 182.37$, $Q^{FI} = 182.37$.	$TP_{rv}^{FI} = 1.3999$.
Supplier	$w_s^{FI} = 6.08$.	$TP_s^{FI} = 4.7653$.
Optimal decisions of model BI		
Retailer	$p^{BI} = 8.44$, $u^{BI} = 4.16$, $T_r^{BI} = 29.86$, $q^d = 47.13$.	$TP_r^{BI} = 1.9021$.
Vendor-supplier	$w_v^{BI} = 6.67$, $n^{BI} = 4$, $Q^{BI} = 206.62$.	$TP_{vs}^{BI} = 3.7526$.

Observation 1. *Vertical integration generates more profit to the whole supply chain.*

It is shown that the system profit for the decentralized model is 2.6059. While, after supply chain integration, the system profit increases to 6.1652 in model FI and to 5.6547 in model BI. In other words, when the vendor cooperates with the retailer, the profit of the system is increased by 136.59%; when the vendor cooperates with the supplier, the profit of the system is increased by 116.99%. There are two reasons for the enhancement of profit. Firstly, supply chain integration helps to mitigate double marginalization effects, thus increases the total efficiency of the total system. It is sown that, comparing the results in model FI and model BI to that in the decentralized model, the selling price drops and the total sales rises. Secondly, supply chain integration incents the retailer to investment more in food preservation, thus reduces the total deterioration cost. It can be observed that the investment level is 3.78, which then rises to 6.77 in model FI and 4.16 in model BI. When the deterioration rate drops, the retailer can store more products and also reduces the ordering cost.

Observation 2. *Forward integration benefits the supplier, and backward integration benefits the retailer, which is counter intuitive.*

It is shown in Table 6 that in model FI, supplier's profit increases from 1.8854 (in decentralized model) to 4.7653; and in model BI, the retailer's profit increases from 0.2391 (in decentralized model) to 1.9021. It is because supply chain integration always mitigates the double marginalization effects. Thus, in model FI, the retail price drops and more products wil be sold, which benefits the supplier as well; in model BI, the retailer will be beneficial from the dropped wholesale price offered by the vendor.

Observation 3. *Forward integration outperforms backward integration for the whole supply chain.*

Comparing the results in model FI and model BI, the total profit for model FI is 6.1652 while that for model BI is 5.6547. This is because forward integration will incents the retailer to invest more in preservation technology, thus reduces the total wastes of food deterioration.

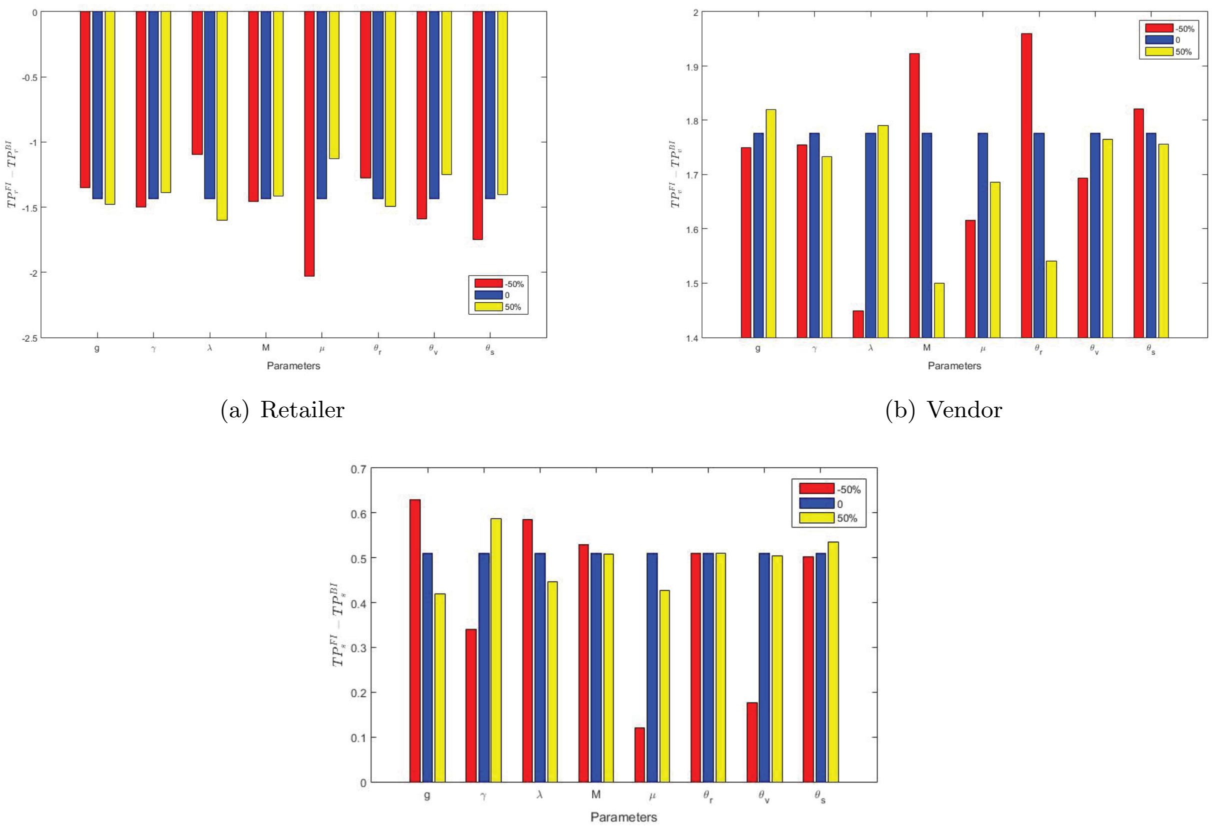
Table 6: Sensitivity analysis results for the forward integration model

Forward integration model											
Parameters	Change	p^{FI}	u^{FI}	T_r^{FI}	q^{FI}	n^{FI}	Q^{FI}	w_s^{FI}	TP_{rv}^{FI}	TP_s^{FI}	TP_{sc}^{FI}
	Default	8.34	6.77	109.01	182.37	1	182.37	6.08	1.3999	4.7653	6.1652
$g = 0.05$	-50%	8.35	8.20	112.16	186.44	1	186.44	6.10	1.5517	4.7791	6.3308
	50%	8.34	5.90	105.98	178.41	1	178.41	6.06	1.2757	4.7493	6.0250
$\gamma = 0.5$	-50%	8.33	10.56	103.05	174.56	1	174.56	6.04	1.1700	4.7312	5.9012
	50%	8.34	5.07	111.04	185.18	1	185.18	6.09	1.5025	4.7748	6.2773
$\lambda = 40$	-50%	8.50	6.75	114.42	172.83	1	172.83	6.37	0.9382	3.8046	4.7428
	50%	8.25	6.77	106.38	187.38	1	187.38	5.92	1.6746	5.2525	6.9271
$M = 40$	-50%	8.33	6.77	108.50	183.32	1	183.32	6.05	1.4503	4.8653	6.3156
	50%	8.35	6.77	109.36	181.73	1	181.73	6.10	1.3665	4.6658	6.0323
$\mu = 4$	-50%	8.11	6.76	102.51	195.18	1	195.18	5.66	2.1512	5.9029	8.0541
	50%	8.55	6.75	116.09	170.08	1	170.08	6.45	0.8192	3.8197	4.6389
$\theta_r = 0.005$	-50%	8.34	5.38	109.01	182.37	1	182.37	6.08	1.4692	4.7653	6.2345
	50%	8.34	7.58	109.01	182.37	1	182.37	6.08	1.3593	4.7653	6.1246
$\theta_v = 0.002$	-50%	8.34	6.77	109.01	182.37	1	182.37	6.08	1.3999	4.7652	6.1651
	50%	8.32	4.95	42.96	72.80	2	155.62	5.85	1.1766	4.6149	5.7915
$\theta_s = 0.002$	-50%	8.33	6.77	108.50	183.32	1	183.32	6.05	1.4503	4.8164	6.2667
	50%	8.35	6.77	109.36	181.73	1	181.73	6.10	1.3665	4.7128	6.0793

Table 7: Sensitivity analysis results for the backward integration model

Backward integration model											
Parameters	Change	p^{BI}	u^{BI}	T_r^{BI}	q^{BI}	n^{BI}	Q^{BI}	w_v^{BI}	TP_r^{BI}	TP_{vs}^{BI}	TP_{sc}^{BI}
	Default	8.44	4.16	29.86	47.13	4	206.62	6.67	1.9021	3.7526	5.6547
$g = 0.05$	-50%	8.45	5.72	32.82	51.01	4	225.77	6.72	1.9451	3.7556	5.7007
	50%	8.42	3.18	27.24	43.63	4	189.75	6.62	1.8906	3.7143	5.6049
$\gamma = 0.5$	-50%	8.40	4.88	24.89	40.60	4	175.24	6.56	1.9177	3.6428	5.5605
	50%	8.45	3.39	31.78	49.70	4	219.25	6.70	1.9314	3.7586	5.6901
$\lambda = 40$	-50%	8.61	4.12	31.32	44.00	4	193.83	7.00	1.4098	2.7469	4.1567
	50%	8.34	4.17	29.15	48.75	4	213.28	6.49	2.1946	4.2857	6.4802
$M = 40$	-50%	8.43	4.16	29.78	47.31	4	207.38	6.65	1.9337	3.8517	5.7854
	50%	8.45	4.15	29.94	46.94	4	205.88	6.69	1.8706	3.6541	5.5247
$\mu = 4$	-50%	8.18	4.18	28.06	51.44	5	288.63	6.18	2.7380	5.1961	7.9341
	50%	8.58	4.12	31.08	44.49	3	142.22	6.95	1.4807	2.7305	4.2111
$\theta_r = 0.005$	-50%	8.44	2.77	29.86	47.13	4	206.63	6.67	1.9714	3.7526	5.7240
	50%	8.44	4.97	29.86	47.13	4	206.63	6.67	1.8615	3.7526	5.6141
$\theta_v = 0.002$	-50%	8.41	4.16	29.66	47.58	4	199.10	6.62	1.9816	4.0063	5.9879
	50%	8.47	4.15	30.11	46.57	4	214.41	6.73	1.8083	3.4794	5.2877
$\theta_s = 0.002$	-50%	8.42	4.16	29.74	47.40	4	207.76	6.64	1.9496	3.8141	5.7638
	50%	8.45	4.15	29.98	46.85	4	205.50	6.70	1.8549	3.6895	5.5445

The sensitivity results for model FI and model BI are respectively given in Table 6 and Table 7. The impacts of parameter change in model FI and model BI are similar to that in the decentralized model. Also, the observations in section 5.3 still hold for most of cases with different values of parameters, which means the results are robust. Figure 4 shows the impacts of forward integration and backward integration to the retailer, the vendor and the supplier's profit change. It is straight forward that the vendor prefers the forward integration strategy. However, a counter intuitive result is obtained that the supplier prefers forward integration strategy, but the retailer prefers the backward integration strategy.



(a) Retailer (b) Vendor

(c) Supplier

Figure 4: The profit gap between FI and BI for retailer, vendor and supplier.

6. Carbon footprint analysis of the food supply chain

In industry, during production, transportation, and storage, greenhouse gases will be emitted, which will hurt the environment. In addition, food deterioration also worsens greenhouse gas (mainly CO_2) emissions and badly hurts natural resources, such as air, water and climate. The carbon emission of the whole supply chain in one cycle can be calculated as follows.

(1) Carbon emission of production

According to the studies of Bazan et al. (2015) and Jaber et al. (2013), the CO_2 generating rate is quadratically linked to the production rate, which is given as $e_p = aP^2 + bP + c$. Thus, the total carbon emission amount during production can be formulated as

$$E_p = e_p \underbrace{\left\{ \int_0^{T_s(Q)} PsdG(s) + \int_{T_s(Q)}^{\infty} PT_s(Q)dG(s) \right\}}_{Vendor's\ total\ production\ quantity} \tag{32}$$

(2) Carbon emission of transportation

During transportation, the consumption of fuels also leads to the emissions of CO_2. Following the study of Bazan et al. (2017) and Tang et al. (2015), the total emission is comprised of two parts: the per unit item's transportation emission and the fixed cost of per truck load. Assuming $\widetilde{D}$ as the maximum capacity of a truck, then the total truck load is $(Q + nq)/\widetilde{D}$. The total carbon emission during transportation can be expressed as

$$E_t = e_{t1} \underbrace{(Q + nq)}_{Total\ transported\ quantity} + e_{t2} \underbrace{\frac{Q + nq}{\widetilde{D}}}_{Total\ numbers\ of\ truck\ load} \tag{33}$$

(3) Carbon emission of storage

Store the food products will also generate carbon emissions due to the consumption of fuel (Tang et al., 2015). The total carbon emission during storage of the supply chain can be expressed as follows.

$$\begin{aligned} E_s =& e_s \underbrace{\left\{ \frac{D(e^{\theta_r(1-f)T_r} - \theta_r(1-f)T_r - 1)}{\theta_r^2(1-f)^2} n \right\}}_{Retailer's\ inventory\ holding\ quantity} + e_s \underbrace{\left\{ \frac{q}{\theta_v} \left(\frac{e^{\theta_v n T_r} - e^{\theta_v T_r}}{e^{\theta_v T_r} - 1} - n + 1 \right) \right\}}_{Vendor's\ inventory\ holding\ quantity} \\ &+ e_s \underbrace{\left\{ \begin{array}{l} \int_{T(Q)}^{\infty} \left(\frac{P(e^{-\theta_s T_s(Q)} + \theta_s T_s(Q) - 1)}{\theta_s^2} \right) dG(s) + \\ \int_0^{T(Q)} \left(\frac{P(e^{\theta_s s} + \theta_s s - 1)}{\theta_s^2} + \frac{P(1 - e^{-\theta_s s})(1 - e^{-\theta_s T_s(Q) + \theta_s s})}{\theta_s^2} \right) dG(s) \end{array} \right\}}_{Supplier's\ inventory\ holding\ quantity} \end{aligned} \tag{34}$$

(4) Carbon emission of product deterioration

As discussed in the introduction, product deterioration also generates carbon emissions. Assuming the emission of unit deteriorated product is e_d, then the total carbon emission of deterioration can be expressed as

$$E_d = e_d \underbrace{\left\{ \left\{ \int_0^{T_s(Q)} (Ps + (Q - Q_D))dG(s) + \int_{T_s(Q)}^{\infty} PT_s(Q)dG(s) \right\} - DnT_r \right\}}_{Total\ deteriorated\ quantity} \tag{35}$$

The unit final product carbon footprint for final products consumed customers can be expressed as

$$TE = \frac{E_p + E_t + E_s + E_d}{DnT_r}, \tag{36}$$

in which, DnT_r represents the total sales of the products.

The parameters are set as follows: $a = 0.01$, $b = 0.2$, $c = 1.4$, $e_{t1} = 0.5$, $e_{t2} = 151.4$, $\widetilde{D} = 100$, $e_s = 0.01048$, $e_d = 1.00$. Other parameters are the same to the base model.

Table 8: Average Carbon Emission in the decentralized, forward integrated and backward integrated supply chain

	Production	Transportation	Storage	Deterioration	Total Amount
Decentralized Model	3.2105	4.2535	0.7272	0.5161	8.7073
Forward Integration Model	2.5657	4.0594	0.7055	0.6006	7.9312
Backward integration Model	2.7062	4.2710	0.8261	0.8217	8.6250

Observation 4. *Forward integration is the greenest strategy.*

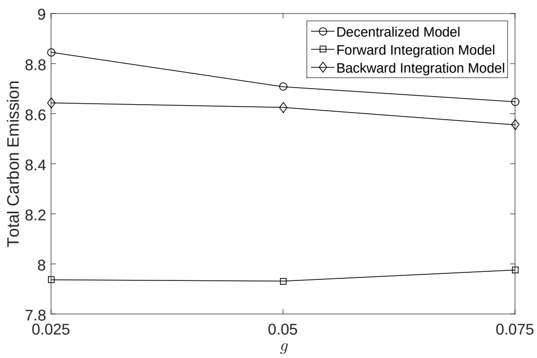
In Table 8, comparing the three models, forward integration is efficient to reduce carbon emission (7.9312, comparing to 8.7073 in the decentralized model). However, backward integration is less efficient (with total emission 8.6250). Also shown in Figure 5 that the forward integration is always the greenest among the three models. Figure 5 also tells the sensitivity of critical parameters on the total emissions. It can observed that higher preservation investment cost or lower investment efficiency leads to more emissions in the forward integration model, while leads to lower emission in decentralized model and backward integration model (see Figure 5(a)-(b)).

Observation 5. *Higher deterioration rate results in higher carbon emissions.*

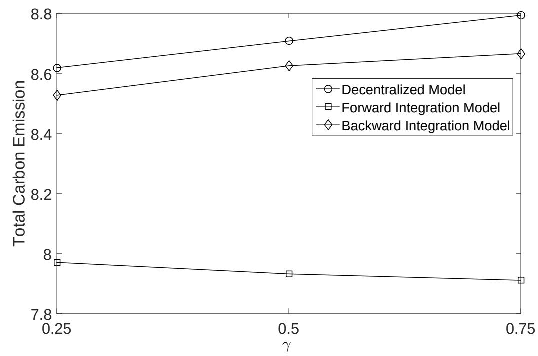
It also shows that, higher deterioration rate in the retailer side (θ_r) leads to higher emissions in all the three models. In addition, integration may be less green comparing to the decentralized case. In Figure 5(c)-(d) that when $\theta_r = 0.005$ *or* 0.0075, or $\theta_v = 0.003$, the total emission backward integration results in the highest emission. The reason is: when deterioration cost is high, backward integration leads to more storage emission and deterioration emission, which leads to the rise of the total emission (see Table B.1).

7. Conclusions

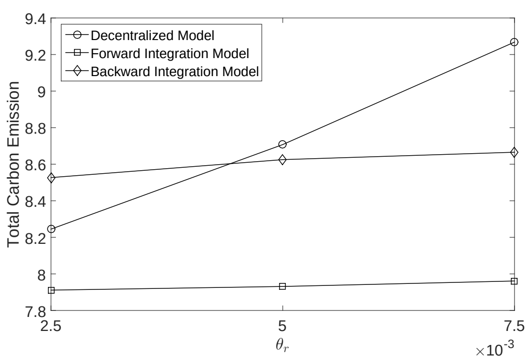
In this paper, a three level food supply chain with controllable deterioration rate and production disruption is modeled. Three types supply chain (i.e., decentralized model, forward integration model and backward integration model) are studied. The main conclusions are obtained through numerical



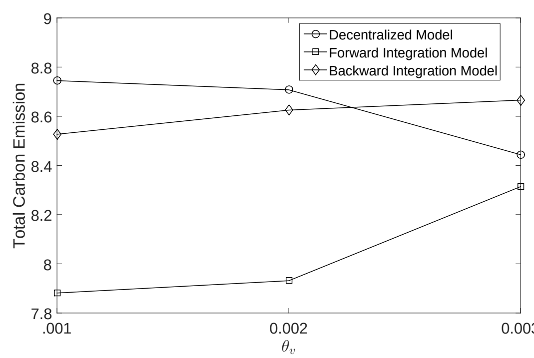
(a) Carbon emission change with g



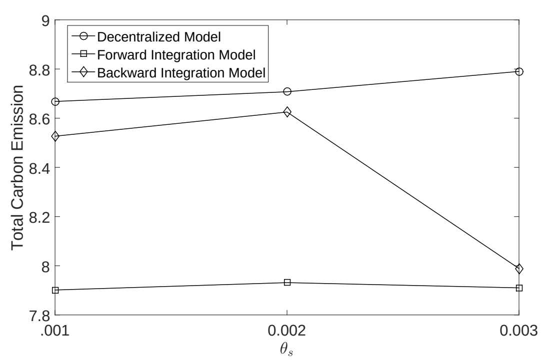
(b) Carbon emission change with γ



(c) Carbon emission change with θ_r



(d) Carbon emission change with θ_v



(e) Carbon emission change with θ_s

Figure 5: The carbon emission change with parameters g, γ, θ_r, θ_v and θ_s in the three models.

tests. Firstly, comparing to the decentralized model, supply chain integration helps to increases supply chain's total profit and individual firm's profit. This is because integration can not only mitigate double marginalization effects, but also incents the retailer to invest more in preservation, thus reduces the total wastes. Secondly, compare the forward and backward integration models, it shows that forward integration not only brings higher profit for the total supply chain, but also has the lowest carbon emission. Thirdly, it is interesting that, sometimes, the retailer can benefit more from backward integration, while the supplier can benefit more from forward integration.

In the future, this paper can be extended in several directions. Firstly, demand uncertainty for the retailer can be considered. Secondly, in real industry, multi retailers, multi vendors or multi suppliers in the food supply chain can be considered. At the same time, competitions between retailers, vendors or supplier can also be considered. Thirdly, supplier's production reliability control strategies can be studied. Lastly, the dynamic pricing policy can be considered which may linked to the food quality and inventory levels.

Acknowledgments

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Appendix A.

Appendix A.1. ***Proof of Proposition 1***

Proof. The exponential term in the equation (3) can be approximated by using the Taylor series expansion, i.e.

$$e^{\theta_r(1-f)T_r} = 1 + \theta_r(1-f)T_r + \frac{(\theta_r(1-f)T_r)^2}{2} + \frac{(\theta_r(1-f)T_r)^3}{6} + O(\theta_r(1-f)T_r) \tag{A.1}$$

. By ignoring the fourth and higher order terms of the Taylor expansion equation of $e^{\theta_r(1-f)T_r}$, the equation (4) can be simplified as

$$\begin{aligned} TP_r(p,u) =& pD - w\frac{D(1+\theta_r(1-f)T_r + \frac{(\theta_r(1-f)T_r)^2}{2} + \frac{(\theta_r(1-f)T_r)^3}{6} - 1)}{\theta_r(1-f)T_r} \\ & - \frac{A_r}{T_r} - h_1\frac{D(1+\theta_r(1-f)T_r + \frac{(\theta_r(1-f)T_r)^2}{2} + \frac{(\theta_r(1-f)T_r)^3}{6} - \theta_r(1-f)T_r - 1)}{\theta_r^2(1-f)^2T_r} - gu \end{aligned} \tag{A.2}$$

The first order derivative can be expressed as

$$\frac{\partial TP_r}{\partial u} = Dw(\frac{1}{2} + \frac{\theta_r(1-f)T_r}{3})\theta_r f^{'} T_r - \frac{Dh_1T_r}{6}\theta_r f^{'} T_r - g \tag{A.3}$$

The second order derivative is

$$\frac{\partial^2 TP_r}{\partial u^2} = Dw(\frac{\theta_r T_r}{2} + \frac{\theta_r^2(1-f)T_r^2}{3})f^{''} - Dw(\frac{\theta_r^2 T_r^2}{3})f^{'2} + \frac{Dh_1T_r^2}{6}\theta_r f^{''} \tag{A.4}$$

As $f^{'} > 0$ and $f^{''} < 0$, $\frac{\partial^2 TP_r}{\partial u^2} < 0$ holds.

This ends the proof of Proposition 1. □

Appendix A.2. ***Proof of Proposition 2***

Proof. The first and second order derivative can be expressed as

$$\frac{\partial TP_r}{\partial p} = b - 2ap + aw\frac{e^{\theta_r(1-f)T_r} - 1}{\theta_r(1-f)T_r} + ah_1\frac{e^{\theta_r(1-f)T_r} - \theta_r(1-f)T_r - 1}{\theta_r^2(1-f)^2T_r} \tag{A.5}$$

and

$$\frac{\partial^2 TP_r}{\partial u^2} = -2a < 0 \tag{A.6}$$

So the retailer's profit function is concave in p.

This ends the proof of Proposition 2. □

Appendix A.3. ***Proof of Proposition 3***

Proof. The first and second order derivative can be expressed as

$$\frac{\partial TP_r}{\partial T_r} = -(\frac{w_v D}{\theta_v(1-f)} + \frac{h_r D}{\theta_r^2(1-f)^2})\frac{\theta_r(1-f)T_r e^{\theta_r(1-f)T_r} - e^{\theta_r(1-f)T_r} + 1}{T_r^2} + \frac{A_r}{T_r^2} \tag{A.7}$$

and

$$\frac{\partial^2 TP_r}{\partial T_r^2} = -(\frac{w_v D}{\theta_v(1-f)} + \frac{h_r D}{\theta_r^2(1-f)^2})\frac{\theta_r^2(1-f)^2 T_r^2 e^{\theta_r(1-f)T_r} - 2\theta_r(1-f)T_r e^{\theta_r(1-f)T_r} + 2e^{\theta_r(1-f)T_r} - 2}{T_r^2} - \frac{2A_r}{T_r^3} \tag{A.8}$$

To prove the concavity of the profit function, we need to prove that the second order derivative is negative. Assuming $\Gamma[\theta_r(1-f)T_r] = \theta_r^2(1-f)^2T_r^2e^{\theta_r(1-f)T_r} - 2\theta_r(1-f)T_re^{\theta_r(1-f)T_r} + 2e^{\theta_r(1-f)T_r} - 2$ and $x = \theta_r(1-f)T_r$. The first order derivative of function $\Gamma(k)$ can be expressed as $\Gamma' = x^2e^x > 0$, which means Γ is increasing in the interval $x \in (0, +\infty)$. Also, when $x \to 0$, we have $\Gamma \to 0$. So $\Gamma > 0$ always holds. As $\frac{w_vD}{\theta_v(1-f)} + \frac{h_rD}{\theta_r^2(1-f)^2} > 0$ and $\frac{2A_r}{T_r^3} > 0$, the second order derivative is negative and the function is concave in T_r.

This ends the proof of Proposition 3. □

Appendix B. Numerical results of carbon emissions

Symbols 'PE', 'TE', 'SE', 'DE' and 'TE' denote carbon emission of production, transportation, storage, deterioration and the total emission respectively.

Table B.1: Numerical results of carbon emissions in the three models

		Decentralized Model					Forward Integration Model					Backward Integration Model				
		PE	TE	SE	DE	TE	PE	TE	SE	DE	TE	PE	TE	SE	DE	TE
	-50%	3.1681	4.2737	0.8293	0.5739	8.8450	2.5465	4.0579	0.7226	0.6095	7.9365	2.6198	4.2540	0.9020	0.8673	8.6431
g	0	3.2105	4.2535	0.7272	0.5161	8.7073	2.5657	4.0594	0.7055	0.6006	7.9312	2.7062	4.2710	0.8261	0.8217	8.6250
	50%	3.2642	4.2622	0.6439	0.4770	8.6473	2.5997	4.0848	0.6898	0.6011	7.9754	2.7699	4.2614	0.7547	0.7693	8.5553
	-50%	3.3237	4.2759	0.5774	0.4412	8.6182	2.6180	4.0857	0.6737	0.5922	7.9696	2.8359	4.2688	0.6933	0.7286	8.5266
γ	0	3.2105	4.2535	0.7272	0.5161	8.7073	2.5657	4.0594	0.7055	0.6006	7.9312	2.7062	4.2710	0.8261	0.8217	8.6250
	50%	3.1843	4.2633	0.7951	0.5508	8.7935	2.5450	4.0466	0.7162	0.6021	7.9099	2.6569	4.2730	0.8777	0.8578	8.6654
	-50%	3.2948	4.1539	0.4701	0.3259	8.2447	2.5457	4.0594	0.7055	0.6006	7.9112	2.6862	4.2711	0.8261	0.8218	8.6052
θ_r	0	3.2105	4.2535	0.7272	0.5161	8.7073	2.5657	4.0594	0.7055	0.6006	7.9312	2.7062	4.2710	0.8261	0.8217	8.6250
	50%	3.1591	4.3787	1.0189	0.7112	9.2679	2.5957	4.0594	0.7055	0.6006	7.9612	2.7362	4.2711	0.8261	0.8218	8.6552
	-50%	2.9724	4.2260	0.9516	0.5948	8.7448	2.5457	4.0594	0.7055	0.5706	7.8812	2.6096	4.1576	0.7904	0.7165	8.2741
θ_v	0	3.2105	4.2535	0.7272	0.5161	8.7073	2.5657	4.0594	0.7055	0.6006	7.9312	2.7062	4.2710	0.8261	0.8217	8.6250
	50%	3.3806	4.2199	0.4821	0.3610	8.4436	2.8776	4.2028	0.5917	0.6417	8.3138	2.8015	4.3793	0.8634	0.9308	8.9750
	-50%	3.1844	4.2558	0.7239	0.5038	8.6679	2.5445	4.0752	0.7017	0.5798	7.9012	2.6621	4.2578	0.8190	0.7809	8.5198
θ_s	0	3.2105	4.2535	0.7272	0.5161	8.7073	2.5657	4.0594	0.7055	0.6006	7.9312	2.7062	4.2710	0.8261	0.8217	8.6250
	50%	3.2503	4.2711	0.7320	0.5366	8.7900	2.5942	4.0567	0.7092	0.6285	7.9886	2.7343	4.2571	0.8290	0.8531	8.6735