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Applications of Signed Graphs to Portfolio Turnover Analysis

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Abstract

Portfolio turnover is an important area for portfolio managers and investors, since it significantly impacts returns through higher trading costs and taxes. Currently, methods for assessing the possibility of portfolio turnover are practically non-existent. Using the concept of signed graphs one can assess the stability of portfolios and thereby the likelihood of portfolio turnover. We demonstrate our method using empirical data from the Indian Stock Exchange and show that portfolios focusing on risk alone can result in higher portfolio turnover, causing misleading portfolio management.

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Keywords: Signed graph; portfolios; balance; risk management; portfolio turnover.

1. Introduction

Portfolio risk analysis involves multidisciplinary perspectives, combining statistical modelling and financial theory to name just a couple of areas (Connor, Goldberg & Korajczyk, 2010). Other researchers such as (Jorion. 2007) have examined risks further into distinctive components. In addition to risk analysis, an important aspect of portfolio performance is portfolio turnover: the frequency that assets within a fund are bought and sold by managers to adjust its composition. Ideally a manager would like zero turnover or adjustments to its portfolio because each adjustment incurs trading costs (taxes, brokerage fees etc.), thus reduces the overall fund's return to investors. This can represent

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a major source of loss in portfolios and can render many potential investment strategies highly unprofitable.

Portfolio turnover occurs whenever the asset's properties in the portfolio change (e.g. correlations, variance etc.) because such changes mean that the manager must adjust the portfolio to take account of the new properties. Prior to the portfolio's actual adjustment, current methods for assessing the future possibility of a portfolio to turnover are practically non-existent. Most methods on assessing portfolio turnover rely on examining the historical trades of the fund, after the turnover has actually occurred, hence do not provide a priori guidance to managers.

Using Signed Graph Theory it is possible to represent a portfolio as a Signed Graph as in (Harary, Lim & Wunsch, 2002). By (Structural) Balance Theory it is known that unbalanced signed graphs are unstable and so are subject to change. This change can take a variety of forms (e.g. correlation changes, etc.) but regardless of the type of the change, the change in the context of portfolios implies the assets will undergo changes. Consequently, this implies unbalanced signed graph portfolios will incur higher turnover than balanced portfolios because unbalanced portfolios will be subject to more change and so prompt the portfolio manager to adjust his portfolio.

In this paper, we apply signed graph theory to portfolios of arbitrary asset size and assess their stability (in terms of signed graph theory) to determine their vulnerability to portfolio turnover. This method also has implementation advantages. Firstly the computation time and estimation difficulty does not necessarily increase with portfolio asset size, hence our method does not suffer from the 'curse of dimensionality'. This is important in finance where portfolios can realistically consist of 100-1000 assets. Secondly, our method is not dependent on robust or accurate forecasting or estimation methods; hence it is not penalized by estimation or forecasting errors. Such a property is highly desirable given that estimation and forecasting is non-trivial in finance.

The rest of the paper is organized as follows: in the next section we explain our method of analyzing portfolios using signed graphs. In the third section we apply our method by conducting a numerical experiment and then analyse our results. We then end with a conclusion.

2. Signed Graph Portfolio Analysis

Graph Theory and signed graph representation have been used to investigate complex structures of inter-related entities. For example (Harary, 1977) analyses international relations between countries; further he (Harary, 1961) investigated the Middle East in 1956 and the shifts in alliance between entities involved in the conflict. A graph G consists of a finite nonempty set V, whose elements are called vertices, and a set E of unordered pairs of distinct elements of V, whose elements are called edges. For example a communication network can be modeled as a graph, where the vertex set V is the set of all communication stations and an edge joining two vertices indicates the existence of a direct communication link between the corresponding stations. A signed graph is a graph $G = (V,E)$ where each edge in E is labeled as positive or negative.

A path P in a graph G is a sequence of distinct vertices $(v_0, v_1,\ldots,v_k)$ such that $v_iv_{i+1}\in E$ for all i, $0 \leq i \leq k-1$. The integer k is called the length of the path. A cycle in G is given by $(v_0, v_1,\ldots,v_{k-1},v_0)$ where $v_0,v_1,\ldots, v_{k-1}$ are distinct vertices and v_iv_{i+1}, $1\leq i\leq k-1$ and $v_{k-1}v_0$ are edges in G. Motivated by Heider's theory, (Harary, 1953) introduced the following mathematical formulation of the notion of balanced signed graphs. *A signed graph G is balanced if every cycle in G has an even number of negative edges and is unbalanced if there exists at least one cycle with an odd number of negative edges*. It has been proved that (Harary, 1953) a signed graph is balanced if and only if the vertex set V can be partitioned into two subsets V_1 and V_2 such that any two individuals a and b with $a\in V_1$ and $b\in V_2$ have a negative tie and two individuals in the same group have a positive tie.

A portfolio of n assets is a vector $\mathbf{w} = (w_1, w_2,\ldots,w_n)$ where $w_i > 0$ and $\sum_{i=1}^{n} w_i = 1$.

The number w_i is the weight or fraction of the investors capital in the i^{th} asset. A portfolio of securities, represented as a signed graph G, has vertex set V as the set of securities of the portfolio and the edges are determined by the correlation coefficient between the securities' returns. A signed graph model for portfolio analysis in risk management was proposed in (Harary, Lim & Wunsch, 2002); they constructed a signed graph whose vertex set equals the portfolio stocks, and two vertices are joined by a positive edge if the corresponding correlation between the two assets is positive and larger than a pre-fixed threshold value and are joined by a negative edge if the correlation coefficient is negative and is less than another prefixed threshold value. If the correlation coefficient lies between the threshold values, then no edge is added. This process gives an elegant signed graph representation of a portfolio of assets.

The importance of unbalanced signed graphs is that by Structural Balance Theory (Harary, Lim & Wunsch, 2002) such graphs tend to be unstable, in that they undergo changes to push them towards becoming a balanced signed graph. In fact a theory in social psychology proposed in (Heider, 1946) states that there is a natural tendency towards balance in any society. In terms of portfolios that have unbalanced signed graphs, such assets in the portfolios will tend to change so that the graph *G* becomes balanced. This implies that portfolios with unbalanced signed graphs may exhibit changes in asset correlations, asset composition to name a couple of possibilities. Since changes in asset properties causes portfolio managers to adjust their portfolio composition, this implies that such portfolios are more susceptible to higher portfolio turnover. Hence signed graphs enable us to assess a priori the portfolio turnover likelihood of any given portfolio.

An n-asset portfolio is represented by an n vertex signed graph G with each vertex connected by signed edges to some other vertices in G. Consequently large asset portfolios possess graphs that are increasingly intractable as n increases. For example realistic portfolios contain 100 to 1000 assets. This does not make our method unusable because we do not need to examine the entire graph G for balance. Since a graph is unbalanced if there exists at least one cycle with an odd number of negative edges, if a 3 vertex size subgraph of G is unbalanced then any vertex size subgraph of G containing this unbalanced subgraph will also be unbalanced. Hence portfolio turnover (or balance) analysis is reduced to examining the 3 vertex size subgraphs that exist in any size portfolio.

The number of three vertex size signed subgraph structures is 10. These are given in Figure 1.

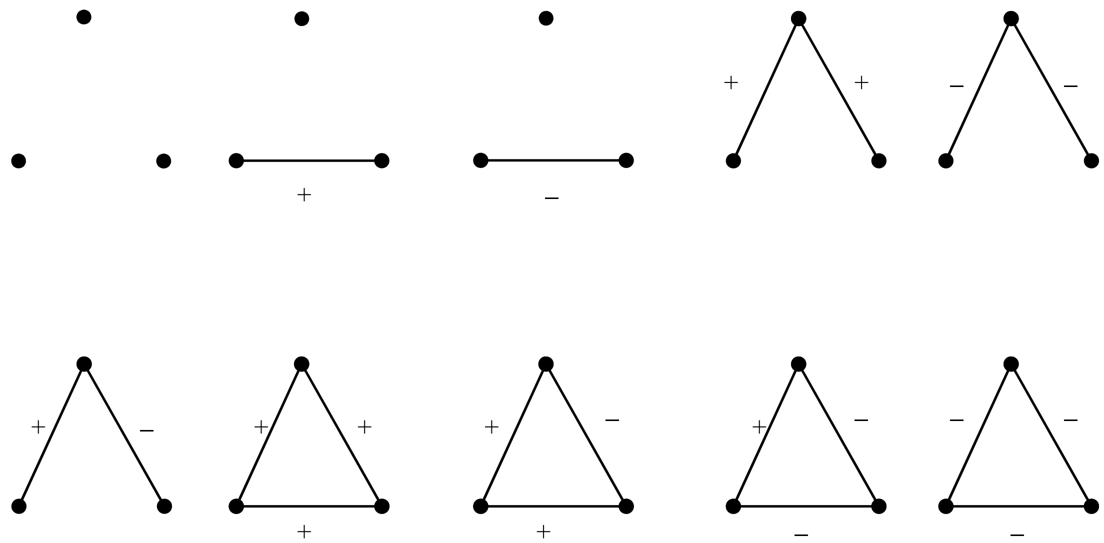

Figure 1: Signed graphs on three vertices

The ten signed graph structures are represented by the triples (0,0,0), (1,0,0), (−1, 0, 0), (1,1,0), (−1, −1, 0), (1,–1, 0), (1,1,1), (1,1,–1), (1,–1, −1) and (−1, −1, −1), where 0 represents no edge, 1 represents a positive edge and −1 represents a negative edge. From the definition of unbalance, graphs (−1,–1, −1) and(1,1,–1) are unbalanced. Hence any portfolio containing graphs (−1, −1, −1) and (1,1, −1) will be unbalanced; we note that (1,–1, 0) has no cycles and therefore is balanced. Hence we can determine the portfolio turnover likelihood using these unbalanced graphs for *any* portfolio of *any* asset size.

An important insight from signed graph analysis of portfolios is that portfolio managers that focus on minimizing risk can mislead portfolio management. This is because not all minimum risk portfolios will have balanced graphs; those portfolios with unbalanced graphs will be unstable and lead to higher turnover, which will reduce returns. Consequently, focusing on risk alone without due analysis of portfolio balance does not necessarily optimize a portfolio's return.

The signed graph method provides computational advantages. As explained before, the portfolio's balance is determined by the 10 graphs in figure 1 regardless of the number of assets *n* in the portfolio; hence our method does not suffer from the 'curse of dimensionality'. Such a property is particularly useful for portfolios when realistic portfolio asset sizes can be *n*=100 to *n*=1000; the number of calculations for correlations alone would be

$$\frac{(n^2-n)}{2},$$

that is 4950 (*n*=100) and 499500(*n*=1000). This clearly represents a non-trivial computational and estimation task.

In addition to computation advantages, the signed graph method also has implementation advantages. To obtain the signed graphs one does not require a large number of variables to estimate: only the correlation between assets. Additionally, the correlations between assets do not need to be accurately estimated; An implementation of signed graphs portfolio with a simple threshold function for correlations, whereby the correlation only needs to be categorized into 1 of 3 groups (positive, negative or 0) has been proposed in (Harary, Lim & Wunsch, 2002). Such parsimonious estimation and non-dependence on forecasting is particularly appealing to finance applications, whereby robust estimation and forecasting of any variable can be highly challenging.

3. Numerical Experiment

In this section we now demonstrate the signed graph method of determining portfolio stability or balance by applying it to a real world portfolio. We also demonstrate the importance of understanding the portfolio's (un)balance graph in portfolio risk management, in that by focusing on risk alone can lead to misleading portfolio management.

In this paper we study creating a portfolio from 35 different assets. We recall from the previous section that any graph containing a 3 vertex size subgraph that is unbalanced is also unbalanced. Hence to determine a portfolio's balance (of any arbitrary size) requires determining the portfolios containing any of the 3 vertex subgraphs in Figure 1. As we have 35 assets this means the total number of graphs that can be produced (from all the different assets) is

$$\binom{35}{3} = 6545.$$

Since the graph's balance is determined by the 3 vertex graphs, this corresponds to 3 assets in the portfolio. Rather than analysing portfolios of larger asset sizes than 3, we analyse the risk of the portfolios required to produce the 3 asset portfolios (hence the total number of possible portfolios analysed is 6545). This is because the aim of the analysis is to demonstrate that lower risk portfolios do not necessarily provide balanced or stable portfolios, hence will incur higher turnover. By increasing the number of assets in the portfolio does not affect the graph's balance (as it is independent of vertex size), hence adding additional assets does not invalidate our analysis. Secondly, creating larger portfolios greatly increases the number of computations e.g. 10 asset size portfolio gives

$$\binom{35}{10} = 1.8\text{x}10^8$$

portfolios. Therefore, rather than examining10 asset size portfolios, we restrict our analysis to 3 asset size portfolios.

To calculate the risk of portfolios requires specifying a risk measure. We use the most widely and commonly accepted risk measure: portfolio variance. We note that any risk measure could have been chosen as it would not invalidate the signed graph approach to turnover analysis. The variance of the portfolio consisting of investments in three assets is computed using the following formula. If *A*, *B* and *C* are assets, w_a, w_b and w_c are the portfolio weights on each asset then the variance of the portfolio is

$$\sigma^2(r_p) = w_a^2\sigma^2(r_a)+ w_b^2\sigma^2(r_b)+ w_c^2\sigma^2(r_c)+ 2w_aw_b \operatorname{cov}(r_a, r_b)+ 2w_aw_c \operatorname{cov}(r_a, r_c) + 2w_bw_c \operatorname{cov}(r_b, r_c)$$

For the benefit of convenience we assume all 3 assets are equally weighted, that is $w_a=w_b=w_c=1/3$.We have also computed the covariance matrix of the weekly returns for any two assets.The correlation coefficients between the weekly returns for each pair of assets have been computed using the threshold values of −0.1 and 0.1.

The assets chosen for the portfolio were stocks listed on the Indian Stock Exchange. The stocks chosen were highly liquid and so are priced efficiently, hence the stock prices are not distorted by low volume trading. The return data and calculation for each asset follows the method in (Harary, Lim & Wunsch, 2002): it is collected on a weekly basis over a period of one year. We note that the time period, frequency and asset choices (stocks) are not important factors in the experiment, in that they could easily be varied without invalidating the method.

4. Results and Analysis

In our empirical data we observed that there are no signed graph structures corresponding to (0,0,0), (0,0,–1), (0, –1, –1) and (–1, –1, –1) since the correlations between assets did not match these graphs. Hence in our study there are only 6 classes remaining in total and they fundamentally determine the portfolio’s balance. The number of portfolios in each of the remaining six classes, the average portfolio variance for each of the classes and the maximum and minimum value of portfolio variance for each of the classes are given in Table 1.

Table 1: Portfolio variance data in six classes

	1,0,0	**1,1,0**	**1,1,1**	**1,1,-1**	**1,-1,-1**	**1,-1,0**
	C_1	C_2	C_3	C_4	C_5	C_6
AVG	17.50664	16.51561	18.7511	17.38628	18.05309	18.20043
MIN	11.7517	9.641176	9.206417	10.27562	10.03876	11.72408
MAX	23.49003	26.03316	31.48543	27.04934	26.00619	24.44621
count	130	318	5092	311	480	214

We observe that for the graphs representing the classes C_1, C_2 and C_3 in Table 1, all the edges are positive and hence these graphs are trivially balanced. Also the signed graph representing the class C_6 has no cycles and hence is balanced. This signed graph has one negative edge and one positive edge. The signed graph representing the class C_5 is also balanced. However, the signed graph representing the class C_4 is unbalanced and is the only unbalanced graph in Table 1.

Out of the total number of 6545 portfolios, the majority of portfolios (95%) are categorized within balanced graphs, hence they are not unstable and are less likely to be exposed to higher portfolio turnover. However, there are 311 portfolios in C_4 which is the unbalanced graph; these represent 5% of the total number of portfolios and so does not represent an insignificant percentage of possible portfolios with instability. As can be seen from Table 1, even though C_4 is the only unbalanced graph portfolio it does not have the lowest or highest average, minimum or maximum variance. Consequently, risk management through risk (variance) can be misleading because it may lead to constructing portfolios with higher turnovers and lower returns.

To investigate whether the correlation mapping function may potentially increase the possibility of unbalanced graphs occurring, we constructed signed graphs based on the correlation coefficients, without using any threshold value, so that an edge between two assets is either positive or negative depending on the sign of the correlation coefficient. In this situation the total number of possible classes is now reduced to three: (1,1,1), (1,1,-1) and (1,-1,-1).The corresponding results are given in Table 2

Table 2: Portfolio variance data in three classes

	1,1,1	**1,1,-1**	**1,-1,-1**
	C_3	C_4	C_5
AVG	18.66665	17.26973	18.157
MIN	9.206417	10.27562	10.03876
MAX	31.48543	27.04934	26.00619
Count	5303	636	606

As before we have 6545 portfolios in total and C_4 is still the only unbalanced graph. From Table 2 we see that the majority of portfolios are in balanced graphs but now the percentage has significantly reduced to 90% of all portfolios. Now 10% of portfolios are in C_4 or unbalanced graphs, hence the possibility of constructing portfolios with instability is far higher in Table 2. Hence graph balance is affected by the correlation threshold function.

With regard to the impact of risk (variance) for Table 2 the results are somewhat inconclusive. The class C_4 has the lowest average variance but not the lowest minimum or maximum variance. However, even with focusing on risk management one can still produce portfolios with unbalanced graphs, hence this can be misleading unless one examines the signed graph structure.

5. Conclusion

In this paper we propose a new method of determining portfolio turnover a priori to the portfolio adjustment. We demonstrate this method by applying it to empirical data from assets on the Indian stock exchange. Our results demonstrate that picking portfolios based on risk (variance) alone can result in portfolios with unbalanced or unstable graphs. Consequently such portfolios can result in sub-optimal portfolio management due to incurring higher trading costs from brokers, taxes etc..

We show that our method has significant computation and implementation advantages. Firstly, for any asset sized portfolio, the balance of the portfolio can be determined by examining its signed graph and Figure 1. Hence our method is not affected by the 'curse of dimensionality'. Secondly, our method does not rely on accurate estimation and forecasting accuracy and so our method is robust to forecasting and estimation errors. We have also shown that graph balance is also affected by the correlation mapping function, hence this should be chosen with consideration. Future areas of research would be to apply other areas of Graph Theory to finance and to explore other topics in risk management.

References

Connor, G., Goldberg, L. and Korajczyk, R. (2010). *Portfolio Risk Analysis*, Princeton University Press .

Harary, F. (1953). On the notion of balance of a signed graph, *Michigan Math. J.,* 2, 143-146.

Harary, F. (1961). A structural analysis of the situation in the Middle East in 1956, *The Journal of Conflict Resolution,* 5(2), 167-178.

Harary, F. (1977). Graphing conflict in international relations, *The Papers of the Peace Science Society*, 27, 1-9.

Harary, F., Lim, M. and Wunsch, D.C. (2002). Signed graphs for portfolio analysis in risk management, *IMA Journal of Management Mathematics,*13, 201-210.

Heider, F. (1946). Attitude and Cognitive Organization, *Psychology*, 21, 107-112.

Jorion, P. (2007) *Vale at Risk, the New Benchmark for Managing Financial Risk*, 3rd Edition, New York, McGraw Hill.