

# **Essays on Social Security Insurance**

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# ABSTRACT

In the light of the rapid demographic changes of becoming the ageing society and the gradual increase in government expenditure on the social security system, this thesis conducts an in-depth evaluation in a wide range of aspects of the social insurance programme that covers five self-contained studies, each of which is mainly relevant to the old-age pension system, known as the notional (non-financial) defined contribution (NDC) scheme. The scheme is currently approaching the adult phase, and it is time to investigate lessons learned and to explore important issues that need to be addressed to enhance the existing scheme.

Given the financial information provided on balance sheets and income statements, the first study assesses the financial stability after the introduction of the Swedish NDC system and the negative effects of financial and economic crises in the recent years on pension benefits and notional accounts. Sweden also becomes the leader in providing extensive information to the NDC plan's members and the pensioners through the so-called *orange envelope*. As a result, the second study aims to examine the experience of public communication and information on the Swedish pension system.

The other part of the thesis focuses on the annuity divisors used to convert an accumulated notional capital into a life annuity. The use of unisex annuity divisors introduces an intra-generational redistribution of income from the short-lived toward long-lived individuals. It implies that persons with low mortality risk get back more money over their lifetime than they have contributed in the pension scheme. The third study intends to quantify the distributional effects of a generic NDC system using the demographic divisor and the economic divisor by the means of the money's worth ratio. Moreover, a complex actuarial model is developed in the computation of the annuity divisor for the case of the life annuity covering both old-age and survivors' benefits.

In addition, the thesis works on the multiple state model for disabled workers in order to explore the evolution of disability risk among the states of active, disabled, and dead. A generic method for the estimation of the one-year probabilities of transition; dying and surviving in a particular state, is proposed by using the cross-sectional data and making assumptions corresponding to the relative ratio between the probabilities of death for disabled and active persons.

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# CHAPTER 1

## Introduction

Social security programme – providing individuals with a degree of income security when faced with the contingencies of old age, survivorship, incapacity, disability, unemployment or sickness – continues to play a significant role in many countries. Over the last century, age distribution of population has dramatically shifted towards an ageing society as a consequence of unprecedented demographic changes with the increase in life expectancies and the decrease in fertility rates. One implication of the rapid population ageing and longevity growth is that more resources must be devoted to support the elderly and governments must frequently raise social security expenditure in particular on old-age pension benefits. Additionally, the ongoing process of population ageing and the increasing social security burden have recently required several adjustments of the social protection benefits and put enormous pressure on the sustainability of the public pay-as-you-go (PAYG) social security system (Attanasio *et al.*, 2007; Bloom and McKinnon, 2010; Ediev, 2014; Bouchet *et al.*, 2017; Vogel *et al.*, 2017).

The Organisation for Economic Co-operation and Development (OECD) member countries have confronted a high level of public social expenditure accounting for 21% of gross domestic product (GDP) on average where the pension is a major section of social security expenses (OECD, 2017a). Public expenditure on pensions is forecast to grow from around 9% of GDP in 2010-15 to 11.3% in 2060 (OECD, 2015). Besides, the financial and economic crises pose fundamental and far-reaching challenges to pension systems. The pension systems commonly respond to such challenges that have

resulted in long-term fiscal problems by means of parametric pension reforms aimed at maintaining the basic structure of the existing system. These parametric reforms include, for instance, changes in contribution rate, increases in pension eligibility age, changes in the number of years used in benefits calculation or decreases in indexation of pensions in payment so that the resources accumulated during working life are sufficient to ensure the payments promised to pensioners (Whiteford and Whitehouse, 2006; Martin and Whitehouse, 2008; Whitehouse, 2009; OECD, 2015).

Traditional defined benefit (DB) schemes with a PAYG financing – workers’ current contributions pay for pensioners’ current benefits – have also been criticised for creating strong incentives for early retirement and, thus, significant distortions in the labour market (Börsch-Supan and Schnabel, 1998; Gruber and Wise, 1998; Kapteyn and de Vos, 1998). As a consequence, the structural pension reform – partially or entirely – shifting from a DB PAYG to a funded pension scheme or a new innovative public pension programme known as ‘notional (non-financial) defined contribution’ (NDC) has been triggered in order to tackle the fall in labour force participation rates and the fiscal crisis.

Specifically, the basic idea of the NDC scheme is to reproduce the logic of a defined contribution pension plan within a pay-as-you-go (PAYG) framework. The notional account is a virtual one that records individual contributions, together with the fictitious return that they generate throughout each contributor’s working life. As opposed to the financial account scheme, the contributions, however, are not invested in financial assets. The returns on contributions are set by law reflecting the financial health of the pension system that typically coincides with some long-term average growth rates of the GDP, of the wage bill or of the income from total social security contributions. The account balance is notional because it does exist only for record keeping (i.e. the system does not invest funds as the scheme is based on PAYG financing). Once a worker reaches retirement age, the virtual accumulated notional capital divided by the so-called ‘annuity divisor’ is transformed into a lifelong annuity. The divisor is determined by the remaining life expectancy at retirement for a given cohort, potential future pension indexations and the technical interest rate used to discount the cash flows.

In the 1990s, the first group of countries, Italy (1995), Latvia (1996), Poland (1999) and Sweden (1999) has legislated the NDC scheme as the first pillar of their pension system. Other countries, such as the Kyrgyz Republic (1996)<sup>1</sup>, the Russia Republic (2002)<sup>2</sup>, Greece (2012)<sup>3</sup> and the Arab Republic of Egypt (2013)<sup>4</sup> have adopted some elements of the NDC, but informative and analytical outcomes have been scarcely found due to incomplete implementation of the system. Several countries have expressed interest in the NDC approach including Argentina, Uruguay, Portugal, Spain, Iran, and China (Holzmann, 2017).

The designs of the NDC schemes among countries exhibit both similarities and differences that can affect future levels of benefits as well as the overall performance of the scheme. The key components of the NDC pension schemes implemented in the four countries – Italy, Latvia, Poland and Sweden – are summarised in Table 1.1. It compares the main drivers of pension reform, the contribution rate, the notional rate of return applied to capitalise the contribution, the inheritance gains from the account holders who passed away during the accumulation phase, the indexation rate to adjust the amount of pension during the pay-out phase, the annuity divisor used to compute the initial pension as well as the retirement age<sup>5</sup>.

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<sup>1</sup> The Kyrgyz Republic's scheme, Palmer (2006a), is for new entrants and is incompletely designed, especially regarding the rate of return used. With the long transition period, the first group of persons will retire in 2039, given entry at age 21 in 1997 and retirement at age 63. Until around 2060, the entire population will be covered by NDC.

<sup>2</sup> Unlike in a typical NDC scheme, the pension benefits under the Russian system are inadequate design because of the low notional rate of return (below the average growth rate of wages) and are not adjusted in line with the changing (usually increasing) life expectancy at the time of retirement that means that newly granted annuities do not follow the development of life expectancy (Hauner, 2008).

<sup>3</sup> In March 2012, a reform of the secondary (auxiliary) pension as one type of the first pillar DB PAYG scheme was enacted in which many of the larger auxiliary pension funds of employees are merged into one and turned into a Balanced Notional Defined Contribution system, precluding any kind of fund transfer from the national budget (Symeonidis, 2016).

<sup>4</sup> The reformed system, combining non-financial (notional) defined contribution (NDC) and financial defined contribution (FDC) schemes, is mandatory for new entrants to the labour market and voluntary for current workers, starting from July 2013 (Maait and Demarco, 2012). However, implementation keeps being interrupted by the Arab Spring (Holzmann, 2017).

<sup>5</sup> Additional information on the NDC features in the different countries can be found in Daniele and Nicola (2006) and Gronchi and Nistico (2006) for Italy, Vanovska (2004) and Palmer *et al.* (2006) for Latvia, Chłoń-Domińczak and Góra (2006) and Chłoń-Domińczak *et al.* (2012) for Poland and Palmer (2000), Könberg *et al.* (2006) and Swedish Pensions Agency (2017) for Sweden.

**Table 1.1:** Main characteristics of four NDC countries

	<b>Italy</b>	<b>Latvia</b>	<b>Poland</b>	<b>Sweden</b>
Drivers to reform	<ul style="list-style-type: none"> <li>• Population ageing</li> <li>• Labour productivity ↓</li> </ul>	<ul style="list-style-type: none"> <li>• Population ageing</li> <li>• Adaptation of Soviet pension schemes to the market economy</li> <li>• Degree of formality ↑</li> </ul>	<ul style="list-style-type: none"> <li>• Population ageing</li> <li>• Adaptation of Soviet pension schemes to the market economy</li> <li>• Degree of formality ↑</li> </ul>	<ul style="list-style-type: none"> <li>• Population ageing</li> <li>• Labour productivity ↓</li> </ul>
Contribution rate	33% (employees) 20% (self-employed) 24% (atypical contracts)	14%	19.52%	16%
Notional rate of return	Five-year average of nominal GDP growth	Covered wage bill growth	Covered wage bill growth	Per capita wage growth + Automatic balancing mechanism (if triggered)
Inheritance gains	Increase in general reserve	Increase in general reserve	Increase in general reserve	Distributed on a birth cohort basis to the accounts of survivors in the cohort based on their relative share in the sum of all accounts in the cohort
Indexation of pensions	Price indexation	Price indexation plus 25% of wage sum growth	Price indexation	Price indexation ± difference between real per capita wage growth and 1.6% rate of return + balancing when solvency ratio falls below unity
Annuity divisor	Life expectancy adjusted with an imputed rate of return, at 1.5%	Life expectancy (unisex) at retirement age	Life expectancy (unisex) at retirement age	Life expectancy adjusted with an imputed rate of return, at 1.6%
Retirement age (men/women)	66.8/65.8 increased in line with life expectancy at 65	62.75/62.75	65/60	65/65 (minimum age of 61 for retirement)

Source: Own illustration based on Chłoń-Domińczak *et al.* (2012) and OECD (2017b)

The emergence of an NDC scheme primarily aims to encourage incentives to work, to delay retirement and, more especially, to stay in the formal labour market since benefits would depend on work record. A generic NDC system, in principle, is also intended to achieve financial stability in part through the dependence of pension annuities on life expectancy improvements together with the adjustable return on contributions reflecting the economic (or productivity) growth of the system. The scheme is designed to maintain long-run equivalence between assets and liabilities with a fixed contribution rate for all generations. In this sense, it makes the results of political intervention more transparent than the conventional DB PAYG does (Holzmann and Palmer, 2006a; Boeri and Galasso, 2013).

Notional (Non-financial) Defined Contribution (NDC) schemes are currently approaching the adult phase, and it is time to examine lessons learned and to take stock of important issues that need to be addressed to ensure a successful adulthood. The thesis, thus, covers self-contained studies mainly conducting an in-depth evaluation in a wide range of aspects of the NDC pension scheme, especially focusing on Sweden as a pioneer handful country where the ideas of the NDC public pension model have been introduced.

As part of the Swedish NDC reform, an actuarial balance sheet has been compiled since 2001 to measure the solvency of the system while it has been absent in the other NDC countries. The balance sheet is an instrument that provides a true and fair view of the pension system's assets and liabilities, addresses an extraordinary level of transparency in the management of the public finances, immunises the system against political risk and gains credibility among contributors and pensioners in the sense of harmonising their expectation with the economic realities of the pension plan (Boado-Penas *et al.*, 2008; Boado-Penas and Vidal-Meliá, 2013). In addition to this, Sweden has legislated an automatic balance mechanism (ABM) for financial imbalances in its pension system. This mechanism is based on the solvency indicator that emerges from the actuarial balance sheet, known as the solvency ratio (or balance ratio) – the ratio of assets to liabilities. When assets are shown to be less than liabilities, this deficit is eliminated by reducing the indexation of benefits and the notional pension accounts (Settergren, 2001; Vidal-Meliá *et al.*, 2009).

Given the financial information provided on the balance sheets and income statements, the first study presented in Chapter 2 aims to assess the financial stability over last decade between 2007–2015 as well as to investigate the negative effects of the financial and economic crises of 2008–2009 on the pension benefits and notional accounts. The policy responses to such crises are also discussed.

Furthermore, one useful way of reaching the better designed and managed public pensions is to provide a well-planned public communication and an improved information about the pension plans. Participants also need information on pensions, e.g. the contributions made, the probable amount of their future pensions, the replacement rate, the retirement options, and the accrued and consolidated rights in order to make decisions at which age to retire and how much to save. Sweden becomes a leader in providing extensive information to individuals via an annual financial statement, or the so-called '*orange envelope*', that contains information regarding contributions paid, an account statement, a fund report for the funded part and a forecast of the future pension. As a result, the purpose of Chapter 3 is to examine the issues of information that emerge when attempting to communicate with contributors and beneficiaries to explain pension schemes and pension reforms. The experience in Sweden after the introduction of an NDC plan and funded individual accounts is, thus, highlighted in the chapter.

The other part of the thesis, composed by Chapter 4 and Chapter 5, studies the annuity divisor used to convert an accumulated notional capital into a life annuity. In relation to annuitisation rules, all NDC countries require the use of unisex life tables that assume a common (average) survival probability for men and women. Due to the existence of mortality differential across gender and another socioeconomic status, the use of unisex annuity divisor introduces an intra-generational redistribution of income from short-lived towards long-lived individuals. This treatment implies that persons with low mortality get back more money over their lifetime than they have paid for.

Chapter 4 intends to quantify the distributional effects of a generic NDC system using two types of divisors – the demographic divisor related to the remaining life expectancy and the economic divisor found in the Swedish NDC system for a valuation

of pension liability associated with the size of pension payments and the pay-out period – to compute the amount of pension. The construction of the subgroup-specific cohort life tables across socioeconomic status that is assessed through educational attainment and used for the calculation of the annuity divisors and the money's worth of the system is also demonstrated.

Although all NDC countries share the same thing of using the unisex annuity divisor, Italy is the only country taking into account the survivors' pensions apart from the old-age pensions in the computation of the annuity divisor. The Italian NDC system provides an additional stream of payments to the surviving spouse and children after the death of the pensioner, but the formula of the annuity divisor created by law relies on the probabilities of surviving of the pensioner and his/her spouse. To fill this gap, the objectives of Chapter 5 are to develop a complex actuarial model for valuing the pension plan that covers the old-age and all possible survivors' benefits – surviving spouse, surviving children, and all surviving family members – and to explore the differences of the annuity divisors obtained by the Italian rules and the proposed model.

In addition to old-age pensions, the thesis also focuses on the incapacity benefits for disabled workers in Chapter 6. A thorough understanding of the transition of an individual into and out of a disabled state and the accurate estimation of the probability of becoming and remaining disabled are the essential information for the government to design a provision of disability benefits, to determine the demand for such benefits, and to project public expenditure on incapacity benefits.

A multiple state model is typically used to describe the transition of the disability risk among the states of active, disabled, and dead. Ideally, the estimations of transition probabilities and transition intensities rely on longitudinal data; however, the data required are often missing. This chapter intends to propose a generic method of estimating the one-year transition probabilities when the recovery from disability is allowed by using the cross-sectional data in order to overcome the limitation of the data. Under some plausible assumptions concerning the probabilities of death among

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disabled and non-disabled persons, the estimated prevalence rates of disability are used to decompose the survival probabilities in each state.

Finally, the main findings of the thesis are concluded in Chapter 7.

# CHAPTER 2

## Last Lessons Learned from the Swedish Public Pension System<sup>1</sup>

Among the countries with NDCs, Sweden is the only one where an automatic balancing mechanism goes hand in hand with the prior calculation of a financial solvency indicator that emerges from an actuarial balance sheet. This chapter describes the Swedish pension experience over the 2007–2015 period through its accounting method, together with the problems faced by the system and the policy responses.

During the financial crisis 2008, the imbalance of the Swedish NDC system has been chiefly caused by the change in indexation of the pension liability. Also, the system has done some policy responses with the aim to eliminate or at least reduce the volatility caused by the indexation itself. In particular, the use of smoothing balance ratio has been introduced from 2015, called as a damped balance ratio. In balancing the indexation should yearly be reduced by only one-third of the adjustment to achieve 100 percent solvency.

**Keywords:** accounting, balancing mechanism, public pensions, retirement, solvency, Sweden.

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<sup>1</sup> A version of the paper (joint work with María del Carmen Boado-Penas and Ole Settergren) will be published as a chapter of the book 'Public Pension systems: The greatest economic challenge of the 21st century' edited by Springer.

## 2.1 Introduction

Social security systems across the world are undergoing significant reforms to adapt their schemes to economic and demographic uncertainties.

In Europe, the common trend in the responses to the pension crisis is a wave of parametric adjustments that usually include changes in the contribution ceilings, increases in the retirement age or changes in the indexation of pensions<sup>2</sup>. Among the major changes carried out in pension reforms are the so-called notional defined contribution pension schemes (NDCs).

The notional model has some positive features, such as facing the ageing population more or less automatically or improving the relationship between contributions and pensions paid. However, these schemes do not guarantee sustainability due to the PAYG nature<sup>3</sup>. Valdés-Prieto (2000) shows that NDCs cannot generally provide financial equilibrium over the short term unless they are in the realistic steady state and have a notional rate equal to the covered salary bill. Hence, NDCs also require other financial modification mechanisms, such as government guarantees and repeated recourse to legislation – to be imposed in the same way as traditional defined benefit (DB) systems – or special measures, such as automatic balancing mechanisms (ABMs).

Sweden has gone beyond the NDC system that it implemented in 2001, in the sense that an income statement and a balance sheet<sup>4</sup> are annually published with the aim of analysing the system's solvency. Even so, an automatic balancing system that reduces the growth of the liabilities is triggered if the plan is not fully solvent. Sweden is claimed to be the only NDC country whose pension is financially sustainable over the long term in the sense that it is not necessary to make changes to the contribution rate<sup>5</sup>.

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<sup>2</sup> See Whitehouse (2009); Whitehouse *et al.* (2009) and OECD (2012).

<sup>3</sup> See Valdés-Prieto (2000) and Palmer (2013).

<sup>4</sup> Also called an actuarial balance sheet.

<sup>5</sup> According to Diamond (2004), a well-structured NDC system, with a decent-sized buffer stock of assets, will have little probability of needing legislative intervention as long as the economic growth is large enough.

This chapter first presents and discusses the Swedish accounting method (income statements and balance sheets over the 2007–2015 period), with special attention to changes in total assets and liabilities. Second, it describes and explains Sweden’s policy responses to the negative effects of the financial and economic crises on pension benefits and notional accounts, in combination with the effect of the new rules of automatic balancing.

The chapter is structured as follows. Section 2.2 describes the Swedish public pension system. Section 2.3 summarises the published balance sheets and income statements and analyses how they have evolved over the 2007–2015 period. Section 2.4 concludes with a discussion on the new responses to the economic crisis, which have been undertaken by the Swedish pension system since 2007.

## 2.2 Swedish public pension system<sup>6</sup>

The Swedish public pension system consists of two different earnings-related benefit schemes: an NDC pension (called the *inkomstpension*), on which this chapter focuses, and a fully funded financial defined contribution pension (the *premium pension*). A tax-financed guaranteed pension, annually adjusted according to the consumer price index, provides supplementary support for retirees with low NDC pensions.

Under the NDC scheme, both accounts and benefits are indexed by the change in the average income. When the initial pension is calculated – that is, when the notional account value is converted into an annuity – the pension is increased or front-loaded on the basis of an assumed annual real growth rate of 1.6 percent for the income index. This rate of advanced interest is then deducted every year from the increase in the income index. Thus, the NDC pension is indexed annually by the change in the income index reduced by 1.6 percent. In certain situations, exceptions to the regular income indexation of accounts and benefits apply. These exceptions are governed by the ratio

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<sup>6</sup> For a more detailed description of the Swedish pension system, see the Swedish Social Insurance Agency (2007–2015) and Barr (2013). The latter paper also evaluates the pension system in Sweden against the goals established at the time of the reforms in the late 1990s.

of assets to liabilities (balance ratio<sup>7</sup>) as provided in the legislation on the balancing mechanism. The balance ratio is an indicator that emerges from the actuarial balance sheet of the NDC scheme (presented in Section 2.3), expressed as:

$$\text{Balance ratio} = \frac{\text{Assets (e.g. fund assets + contribution asset)}}{\text{Pension liabilities}}$$

The balance ratio used in Sweden has a dual purpose – to measure whether the system can fulfil its obligations to its contributors and to decide whether an ABM should be applied.

### ***Balancing mechanism***

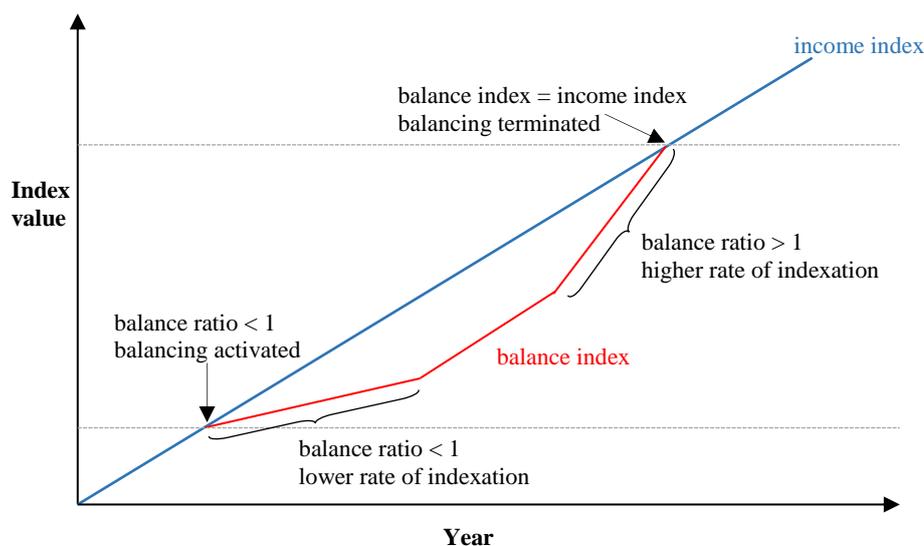
Following Settergren's (2001) work, if for some reason, the balance ratio is less than 1, the ABM is triggered as shown in Figure 2.1. This process basically consists of reducing the growth in pension liability (i.e. the pensions in payment and the pension account balances of the economically active population). Thus, the balance index (Figure 2.1), rather than the change in the average income (expressed by an income or a salary index), is used to revalue the pensions in payment and the notional account of each contributor.

This ABM can be considered asymmetric as it is only triggered when the solvency ratio is lower than 1. However, the Swedish ABM allows for recovery. After a period of low returns as a consequence of the mechanism, a period of higher-than-normal returns follows (Figure 2.1)<sup>8</sup>.

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<sup>7</sup> To indicate that the solvency ratio of a PAYG scheme is different from that of the *premium pension*, the *inkomstpension* system calls this ratio the balance ratio.

<sup>8</sup> For more detailed explanations, see Barr and Diamond (2011).

**Figure 2.1:** Illustration of the balancing mechanism

Source: Swedish Pensions Agency (2017)

### 2.3 Development of solvency: Balance sheets for 2007–2015

Sweden developed the new accounting rules with an almost identical discipline of the double-entry bookkeeping. The country also produces an actuarial balance sheet and an income statement every year. Its annual report has presented an overall picture of the financial health of the Swedish NDC pension system since 2001.

The balance sheet for the Swedish NDC pension system is a financial statement listing the pension system's obligation to contributors and pensioners on a particular date (also referred to as liabilities), together with the amounts of the various assets (financial assets and contribution amounts) that back up these commitments. The actuarial balance sheet mainly aims to provide a true and fair view of the pension system's assets and liabilities in the beginning and at the end of each fiscal year; by comparing these figures, it intends to determine the change in the net worth. The balance sheet also contributes to the management and disclosure of financial information because it is useful not only for the authority governing the system but also for contributors and pensioners in general and for the body that guarantees

payment (i.e. the state and the contributors it represents) (Boado-Penas *et al.*, 2008; Boado-Penas and Vidal-Meliá, 2013).

The system's assets comprise the value of future pension contributions, referred to as the contribution asset and the buffer fund. The first entry on the asset side is called the contribution asset, which is the turnover duration (TD) multiplied by the value of the contributions made in a specific period. The TD is the expected average length of time between the payment of a monetary unit of contribution into the system and the disbursement of the corresponding credit in the form of a pension. It thus reflects the difference between the weighted average age of pensioners and the weighted average age of contributors, assuming that economic, demographic and legal conditions are constant. The TD<sup>9</sup> is also the sum of the pay-in duration and the pay-out duration.

Additionally, the contribution asset is equivalent to the present value of the perpetual future flow of contributions discounted with the inverse of the TD, called the implicit discount rate. If the TD increases (decreases), the implicit discount rate decreases (increases), and the value of the contribution flow increases (decreases). The implicit discount rate varied between 3.13 and 3.29 (see Table 2.1). The implicit rate can also be regarded as the system's discount rate for contributions because it is defined by the contribution flows' capacity to amortise the pension debt.

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<sup>9</sup> After 2015, the disclosure about the TD has been calculated in terms of the difference between the weighted average ages of pensioners and contributors. See the Annual Report of the Swedish Pension System (2015), Appendix B, Formula B.3.1.

**Table 2.1:** Balance sheet of the Swedish NDC pension system on December 31, 2007–2015

Item	2007	2008	2009	2010	2011	2012	2013	2014	2015
<b>Assets (% of GDP)</b>									
Fund assets	27.2	20.9	25.1	25.4	23.9	26.0	28.1	30.1	29.4
Contribution asset	185.5	191.2	193.5	186.8	186.7	187.7	188.9	187.5	178.4
Total assets	212.7	212.1	218.6	212.2	210.6	213.7	217.0	217.6	207.8
<b>Liabilities and results brought forward (% of GDP)</b>									
Opening results brought forward	3.0	0.5	-7.4	-9.2	2.8	4.3	-2.1	3.2	10.1
Net income or loss for the year	-2.5	-7.7	-2.4	12.1	1.5	-6.4	5.5	7.5	-6.0
Closing results brought forward	0.5	-7.2	-9.8	2.9	4.3	-2.2	3.4	10.7	4.1
Pension liability	212.2	219.3	228.4	209.3	206.3	215.8	213.6	206.8	203.7
Total liabilities and results brought forward	212.7	212.1	218.6	212.2	210.6	213.7	217.0	217.6	207.8
<b>Financial Indicators</b>									
Balancing year	2009	2010	2011	2012	2013	2014	2015	2016	2017
Balance ratio, original definition <sup>a</sup>	1.0026	0.9672	0.9570	1.0138	1.0208	0.9901	1.0158	1.0521	1.0201
Balance ratio, modified legislation <sup>b</sup>	n.a.	0.9826	0.9549	1.0024	1.0198	0.9837	1.0040	1.0375	1.0067 <sup>c</sup>
Pay-in duration (years)	21.07	20.88	20.83	20.62	20.55	20.56	20.41	n.a.	n.a.
Pay-out duration (years)	10.69	10.79	10.83	10.88	10.89	10.92	10.99	n.a.	n.a.
Turnover duration (years)	31.76	31.67	31.66	31.51	31.44	31.48	31.40	30.37	n.a.
Smoothed turnover duration (years)	31.93	31.76	31.76	31.67	31.66	31.51	31.48	31.44	30.38
Implicit discount rate (1/TD) (%)	3.13	3.15	3.15	3.16	3.16	3.17	3.18	3.18	3.29
Income (or Balance) index (%)	4.5	6.2	-1.4	-2.7	5.2	5.8	-1.1	2.5	5.9
GDP (SKr billions)	3,297	3,388	3,289	3,520	3,657	3,685	3,770	3,937	4,181

Source: Authors' compilation based on the data from Swedish Pensions Agency (2008–2016)

Note: GDP = gross domestic product; n.a. = not applicable

<sup>a</sup> The balance ratio calculated according to the previous definition (2007). It is calculated solely on the basis of the buffer fund's market value as of December 31 of the corresponding year, formerly called the financial position.

<sup>b</sup> The balance ratio calculated according to the new definition (December 31, 2008 onwards). It is calculated on the basis of a three-year average of the buffer fund's market value.

<sup>c</sup> The damped balance ratio has been used instead of the balance ratio from December 31, 2015 onwards. It is equal to 1 plus one-third of the difference between the balance ratio fixed for that year and the number 1.

The buffer fund<sup>10</sup>, also called the fund asset, aims to stabilise pension disbursements and/or pension contributions in relation to economic and demographic changes. The buffer fund of the Swedish NDC system consists of five separate national pension funds (also known as *Allmänna pensionsfonder* [AP] Funds): the First, Second, Third, Fourth and Sixth AP Funds. Pension contributions are deposited equally to the First to the Fourth AP Funds, which also equally finance the pension disbursements. In contrast, the Sixth<sup>11</sup> AP Fund is an evergreen one, which means that there are no contributions paid to and pension disbursements from the fund. The buffer fund's size is large, amounting to 20.9–30.1 percent of the GDP (see Table 2.1) and 9.5–14.5 percent as a share of the pension liability in 2007–2015.

In the balance sheet, the pension liability includes a liability to contributors and a liability to pensioners. The liability to contributors is the notional accumulated capital in the contributors' accounts. The liability to pensioners is the present value of the expected total of all pensions paid to current pensioners during their lifetimes, taking into account the current life expectancy and the real technical interest rate applied (1.6 percent) when the amount of the initial pension was calculated. The pension liability varies from 203.7 to 228.4 percent of the GDP (see Table 2.1).

Since the system was implemented in 2001, the contribution asset has always been less than the pension liability over time. For example, in 2015, the contribution asset accounted for SEK 7,457 billion, in contrast to SEK 8,517 billion in pension liability (see Table A.1 in Appendix A.1). The system would have been partially insolvent. However, by including the buffer fund that absorbs the differences between the inflow

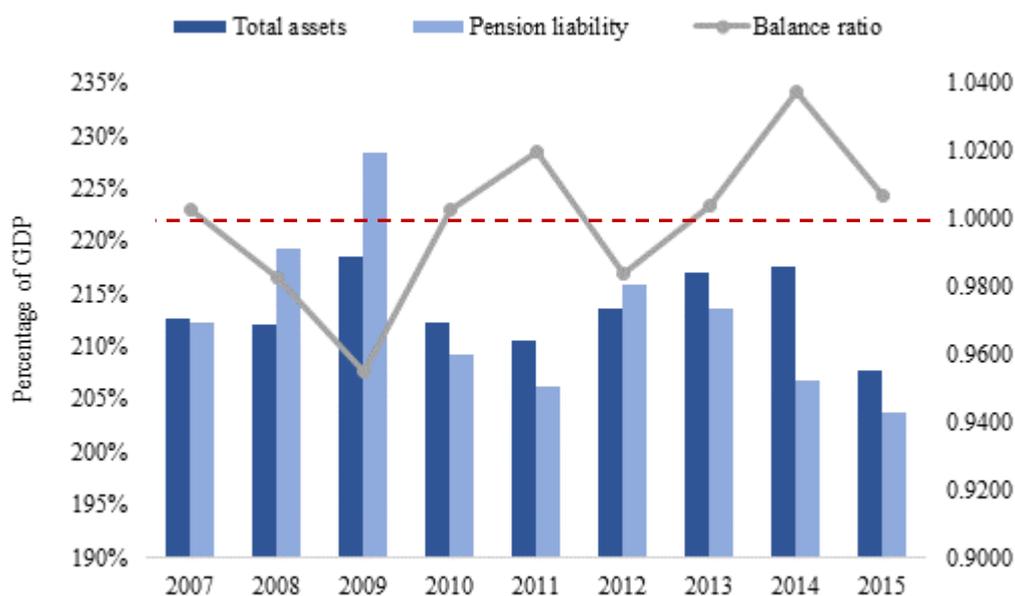
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<sup>10</sup> Sweden is the only one among NDC countries where has created the buffer fund. Several countries with the PAYG state pension system (e.g. Canada, France, Ireland, Japan, Norway, Spain, etc.) have established the public reserve funds (Yermo, 2008).

<sup>11</sup> The Sixth National Pension Fund, known as the Sixth AP Fund, operates as a long-term investor in unlisted companies. Profits can be reinvested, and any losses must be covered by the fund capital. The owner (i.e. the Swedish Parliament) may, by changing the law, decide to allow funds to be paid into or withdrawn from the Sixth AP Fund. The Sixth AP Fund invests in the private equity market, while the First to the Fourth AP Funds had identical investment rules and were allocated the same amount of capital in connection with the reorganisation of the AP Funds in 2001.

of contributions and the outflow of pensions, the total assets will likely exceed the pension liability.

**Figure 2.2:** Evolution of total assets, pension liability and balance ratio of the Swedish NDC pension system



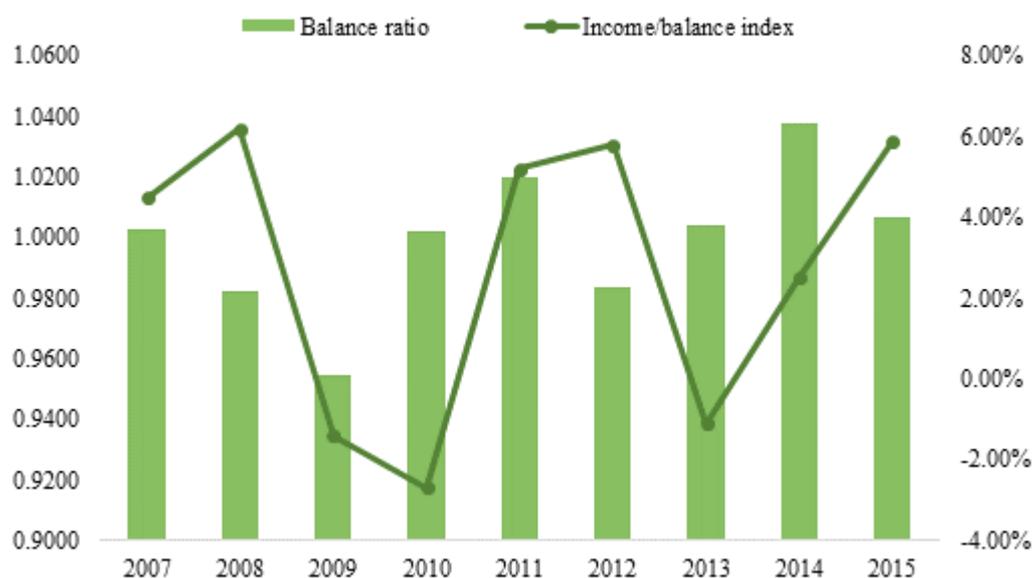
*Source:* Authors' compilation based on the data from Swedish Pensions Agency (2008–2016)

Figure 2.2 shows the evolution of the total assets, the pension liability and the balance ratio for the 2007–2015 period. Before 2008, the system's balance ratio was greater than 1, and the total assets and the pension liability had risen continuously, with a rather higher growth in liabilities than in total assets. In 2008, the financial position of the pension system substantially deteriorated. The balance ratio dropped below 1 for the first time, amounting to 0.9672 due to a huge net loss of SEK 261 billion, equivalent to 7.7 percent of the GDP (see Table 2.1). According to the original legislation, balancing was activated with a 3.28 percent reduction of the indexation of notional accounts and pensions in 2009/2010. However, in 2009, the parliament modified the new rule on the basis of the three-year average of the buffer fund for calculating the balance ratio. As a result, the modified balance ratio increased to 0.9826, and the balancing effect was reduced to 1.74 percent.

In 2009, the system still faced financial deficit, but the loss (2.4 percent of the GDP) was not as large as that of the previous year. The total assets were less than 4.3 percent of the pension liability, for the balance ratio of 0.9549. The pension liability was 228.4 percent of the GDP, the highest value during the period. The negative indexation of notional accounts and benefits in 2009 and 2010 forced a significant drop in the value of the pension liability, and then, assets exceeded liabilities at the end of 2010. This surplus was equal to 0.0024 percent, for the balance ratio of 1.0024.

The pension system had been restored for a couple of years, and at the end of 2012, the pension liability exceeded the total assets again, for the balance ratio of 0.9837. Balancing was activated, and the indexation of pension balances and pension disbursements was decreased in 2013/2014. The pension liability has been revalued at a slower rate, and the pension system has been strengthened financially since 2013. The pension liability reached a value of 213.6 percent of the GDP in 2013 and 206.8 percent of the GDP in 2014, while the balance ratio increased to 1.004 in 2013 and 1.0375 in 2014. With the aim of reducing the volatility caused by the indexation, the new form of smoothing the balance ratio (referred to as the damped balance ratio) was introduced and amounted to 1.0067, as of December 31, 2015. The damped balance ratio restricts balancing to one-third, resulting in less volatility in pension benefits when balancing is activated.

Figure 2.3 illustrates the relationship between the balance ratio and the income/balance index. When the balance ratio was less than 1, balancing was activated, and the income index was reduced in the next fiscal year. For example, in 2008 and 2009, the balance ratio was less than 1, and then, the income index was dropped in 2009 and 2010. After balancing was deactivated, the balance ratio was greater than 1, and the income index was increased. For instance, the balance ratio was higher than one in 2010 and 2011, and the income index rose in 2011 and 2012.

**Figure 2.3:** Balance ratio and income (or balance) index of the Swedish NDC pension system

Source: Authors' compilation based on the data from Swedish Pensions Agency (2008–2016)

### *Income statements of the NDC scheme*

A full explanation of the reasons for the changes in the NDC system's solvency is specified and quantified in the income statement. It is divided into three sections: change in fund assets, change in contribution asset and change in pension liability, each presented as a percentage of the GDP over the 2007–2015 period (Table 2.2).

- **Change in fund assets (buffer fund)**

The total annual pension contributions and disbursements have amounted to approximately 6 percent of the GDP since 2007. In the first eight years after the scheme was implemented, the net cash flow of the AP Funds (the difference between contributions and disbursements, commonly referred to as the buffer fund entry) was positive. However, since 2009, the pension disbursements have become greater than the pension contributions. One reason for the growth in pension benefits is the huge number of cohorts born in the 1940s, who have recently retired (Swedish Pensions Agency, 2010). The relatively substantial increase in the income index is another reason for the higher value of pension disbursements. For instance, in the beginning of

2009, the income index rose by 6.2 percent compared to 4.5 percent in the beginning of 2008, and then, the pension benefits grew by 9 percent in 2009, from SEK 199 billion in 2008 to SEK 217 billion in 2009.

However, the deficit of the net cash flow between pension contributions and benefits was offset by the returns on assets and capital in the buffer fund. From 2009 onwards, the positive return on capital has resulted in a relatively good overall change in fund assets; for example, the return on funded capital accounted for 3.76 and 1.60 percent of the GDP in 2014 and 2015, respectively. The variations in the returns on the buffer fund reflect the volatility in the assets' prices allocated in each AP Fund. The investment strategies for the funds are broadly proportioned to 60 percent of equities and 40 percent of bonds.

The administrative costs are deducted annually from the pension funds, only until a retiree begins to draw a pension. At the current cost level, the deduction for costs reduces the value of the funds by approximately 0.05 percent of the GDP. The administrative costs reported in the income statement contain the costs of insurance administration and the National Pension Funds' operating expenses. Asset management fees, performance-based fees and transaction costs, such as brokerage commissions, are not reported as direct costs by the funds but have a negative effect on the section of the return on funded capital.

**Table 2.2:** Income statement of the Swedish NDC pension system, 2007–2015

Item	2007	2008	2009	2010	2011	2012	2013	2014	2015
<b>Change in fund assets (% of GDP)</b>									
Pension contributions	5.76	5.99	6.17	5.82	5.91	6.02	6.02	5.99	5.88
Pension disbursements	-5.64	-5.87	-6.60	-6.25	-6.02	-6.40	-6.74	-6.48	-6.34
Return on funded capital	1.15	-5.73	4.14	2.41	-0.46	2.74	3.40	3.76	1.60
Administrative costs	-0.06	-0.03	-0.06	-0.06	-0.05	-0.05	-0.05	-0.05	-0.05
Total	1.24	-5.64	3.65	1.93	-0.60	2.31	2.65	3.23	1.10
<b>Change in contribution assets (% of GDP)</b>									
Value of change in contribution revenue	5.85	11.66	-3.50	6.59	6.97	3.26	5.70	6.76	7.96
Value of change in turnover duration	-0.67	-0.97	-0.03	-0.54	-0.05	-0.90	-0.16	-0.20	-6.12
Total	5.19	10.66	-3.50	6.05	6.92	2.36	5.52	6.53	1.84
<b>Change in pension liability<sup>a</sup> (% of GDP)</b>									
New pension credits and ATP points	-5.88	-6.44	-6.51	-6.11	-5.66	-6.19	-6.42	-5.84	-5.88
Pension disbursements	5.64	5.87	6.60	6.25	6.02	6.40	6.74	6.48	6.34
Indexation or change in value	-8.13	-11.36	-1.95	4.69	-4.79	-10.94	-2.55	-2.34	-9.04
Value of change in life expectancy	-0.52	-0.80	-0.70	-0.71	-0.38	-0.35	-0.42	-0.51	-0.36
Inheritance gains arising	0.30	0.32	0.33	0.31	0.30	0.30	0.32	0.30	0.29
Inheritance gains distributed	-0.33	-0.35	-0.40	-0.37	-0.33	-0.35	-0.37	-0.36	-0.33
Deduction for administrative costs	0.06	0.03	0.03	0.03	0.03	0.03	0.03	0.05	0.02
Total	-8.89	-12.72	-2.55	4.12	-4.84	-11.10	-2.68	-2.24	-8.97
Net income or loss for the year (% of GDP)	-2.49	-7.70	-2.40	12.07	1.48	-6.43	5.49	7.52	-6.03

Source: Authors' compilation, based on the data from Swedish Pensions Agency (2008–2016)

Note: GDP = gross domestic product; ATP = *Allmän tilläggspension*

<sup>a</sup> A negative item (-) increases the pension liability, and a positive item decreases it (by the shown amount).

- **Change in contribution asset**

In the income statement, the change in the contribution asset is divided into the value of the change in the contribution revenue and the value of the change in the TD.

The value of the change in the contribution revenue, representing how much more (or less) the liability can be financed by a higher (or lower) level of contributions relative to the previous year, is the monetary value as expressed by the growth in the value of the contributions per capita multiplied by the growth of the labour force (Settergren, 2013). Due to a gradual increase in pension contributions since the pension scheme was implemented, the value of the change in the contribution revenue has been positive and in 2008, reached the maximum value of 11.66 percent of the GDP. In 2009, the contribution asset fell to SEK 6,362 billion from SEK 6,477 billion in 2008, resulting in a negative value of the change in the contribution revenue (-3.50 percent of the GDP). A reduction in contributions was one reason for the loss incurred in the pension system and worsened the balance ratio. During the 2010–2015 period, the contribution revenue had slightly grown again at 3.26–7.96 percent of the GDP.

The value of the change in the TD, which is the two-year average of the smoothed contribution revenue<sup>12</sup> multiplied by the change in the TD, had dropped since 2005 and amounted to -6.12 percent of the GDP in 2015. The smoothed TD was 31.44 years in 2014 and 30.38 in 2015 (see Table 2.1). Therefore, the change in the TD dropped by 1.06 years, resulting in a sizable negative value of the change in the TD in 2015.

Taking into account the inverse relationship between the TD and the implicit discount rate, the marginally lower TD represents a slight increase in the implicit discount rate for the contribution flow, from 3.13 percent in 2007 to 3.29 percent in 2015 (see Table 2.1). The decreased TD and the increased implicit discount rate have reduced the contribution flow's capacity to finance the pension liability.

The pay-in duration, reflecting the difference in the number of years between the expected average age of earning a pension credit and the expected average age of

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<sup>12</sup> As of the financial year 2015, a smoothed contribution revenue is no longer used.

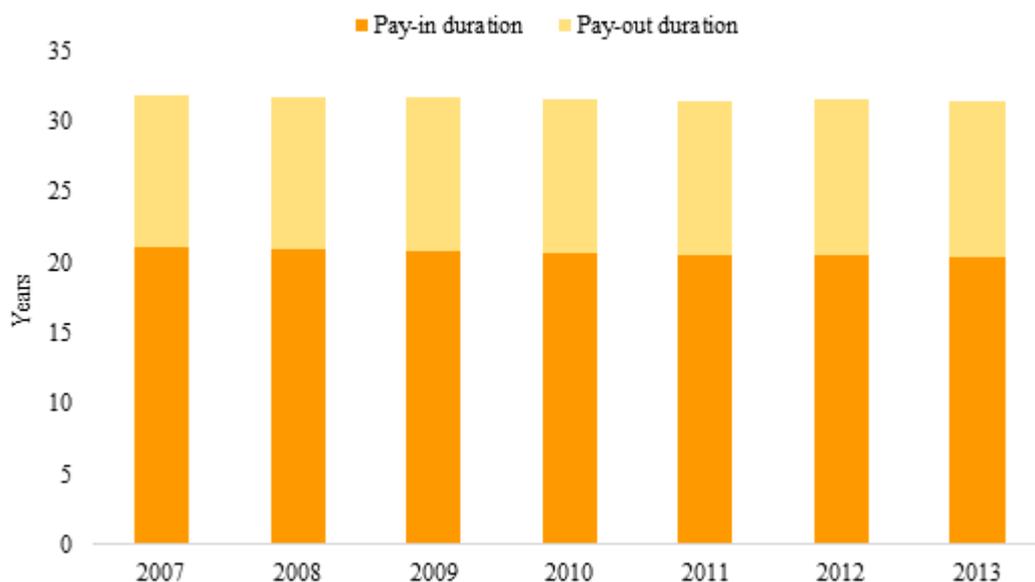
retirement, had decreased from 21.07 years to 20.41 years over the 2007–2013 period (see Table 2.1). This change represents an increase in the expected average age of contributors. However, the reasons for the negative trend have not been fully investigated<sup>13</sup> (Settergren, 2013).

On the other hand, the pay-out duration, which is the difference in the number of years between the expected average age of retirement and the expected average age of pension recipients, had risen from 10.69 years in 2007 to 10.99 years in 2013 (see Table 2.1). This means an increase in the expected money-weighted average age of retirees because of longevity. The net effect of the change in the expected ages leads to a moderate drop in the TD. Figure 2.4 also illustrates the evolution of the TD, which is the sum of the pay-in and the pay-out duration.

The improvement in life expectancy has a direct effect on the increased pension liability and also changes the pension liability's structure – the time profile of payments – in a way that does not need to be fully financed in a PAYG pension system. The increased TD implies that the same flow of contributions can finance a larger pension liability (Settergren, 2013). Besides, the net effect of the increase in life expectancy in a PAYG pension system, which is the difference of the increased value between the pension liability and the contribution flow, is associated with the higher TD. Settergren and Mikula (2005) and Vidal-Meliá and Boado-Penas (2013) describe how cash flows, the stock of assets and liabilities of the PAYG pension system are affected by a shift in the mortality pattern.

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<sup>13</sup> Settergren (2013) points out that the ages of entry to the labour force market have increased slightly, and the relative wages of younger employees seem to have decreased. This situation could be explained, at least partially, by the increase in immigration to Sweden. The adult immigrants might have pushed up the average age of entry to the labour market; thus, there has been a decrease in the TD.

**Figure 2.4:** Evolution of pay-in and pay-out duration of the NDC Swedish pension system

Source: Authors' compilation based on the data from Swedish Pensions Agency (2008–2016)

- **Change in pension liability**

The changes in the size of the pension liability are caused by seven main items: new pension credits and *Allmän tilläggspension* (ATP) points, pension disbursements, indexation or change in value, value of change in life expectancy, inheritance gains arising, inheritance gains distributed and deduction of administrative costs as presented in the income statement (Table 2.2).

The new pension credits and the ATP points had been in a range of 5.66–6.51 percent of the GDP over the 2007–2015 period. The pension credits are accumulated in the notional account, whereas the ATP points corresponding to the estimated value of the new pension credits earned in the DB system are being phased out<sup>14</sup>. When the ATP points disappear completely, the value of this item will be identical with the pension contributions in the section of the change in fund assets. The equality between the

<sup>14</sup> Currently, the value of new ATP points (that corresponding to the old pension system) is very small and will no longer be earned after 2017.

contributions paid and the new pension credits earned is one common characteristic of an NDC or a defined contribution scheme (Settergren, 2013).

As pension benefits are paid out by the AP Funds and also constitute an amortisation of the pension liability, the amount of pension disbursements then reduces both the values of the fund assets and the pension liability. The value of the pension disbursements included in the change in the fund assets (Table A.2 in Appendix A.1) has the same value as the change in the pension liability.

The pension liability changes primarily with the annual indexation of pensions and pension account balances. The higher value of the indexation increases the pension liability. Indexation is determined by the change in the income index, in combination with the balance ratio in the years when balancing is activated, referred to as the balance index. In 2008, the indexation produced an increase in the pension liability, amounting to 11.36 percent of the GDP. Since balancing was activated in 2009 and 2010, the indexation was reduced. As a result, the pension liability decreased to approximately 4.69 percent of the GDP in 2010. In 2012, the pension liability increased to 10.94 percent of the GDP because of the high indexation, 5.2 percent of the income index, and the financial position of the system deteriorated with a negative balance ratio. The balancing mechanism was then activated, and the indexation was dropped to restore the system, which increased the pension liability by 2.55 and 2.34 percent of the GDP in 2013 and 2014, respectively.

Additionally, the pension liability is positively related to the change in the life expectancy. Life expectancy, as used in the income statement, refers to the assumed length of time for which an average pension amount is disbursed (the so-called economic life expectancy). For the NDC system, a higher life expectancy will increase only the pension liability to retirees. According to the projected negative impact on pensions of younger cohorts as presented in the 2010 Orange Report (Swedish Pensions Agency, 2010), the monthly pension will be lower for younger cohorts if they retire at a fixed age when life expectancy is improving. Therefore, the pension liability to active workers is unaffected by mortality changes. The value of the change in life expectancy (Table A.2 in Appendix A.1) is the difference between the pension liability

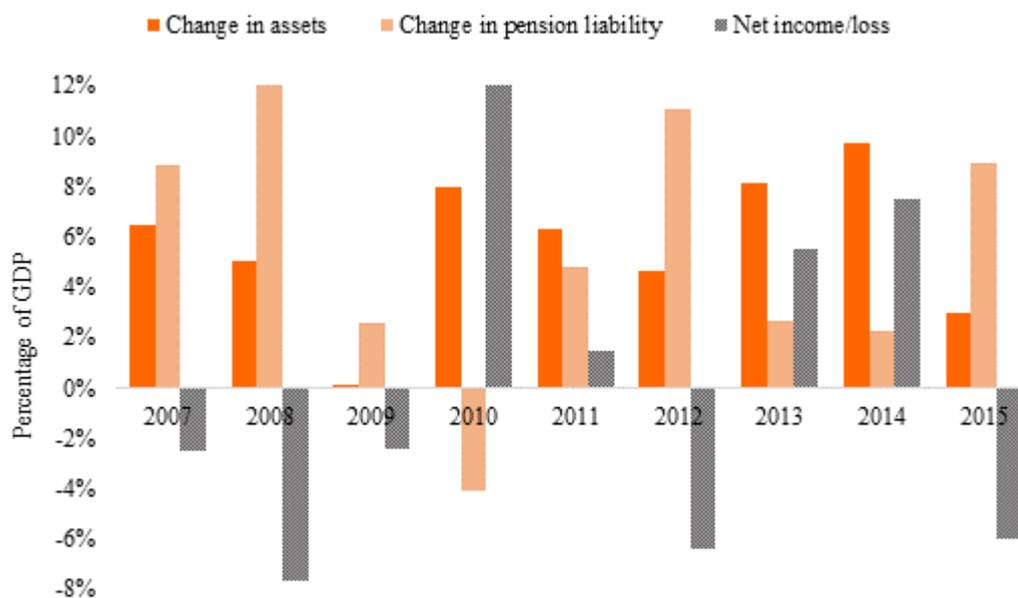
calculated with the economic life expectancy used in the financial year and that used in the previous year. Over the 2007–2015 period, the value of the change in life expectancy had varied between -0.35 and -0.80 percent of the GDP.

The remaining accounting item in Table 2.2, inheritance gains, represents the notional account balances of participants who do not survive to retirement age, distributed to the survivors belonging to the same birth cohort. The items vary between 0.3 and 0.4 percent of the GDP. The inheritance gains distributed are not those actually arising but those expected to arise. Hence, both items are not offset perfectly.

The administrative costs are deducted from the notional accounts to finance the administrative and the fund management costs. On average, the deduction amount ranged between 0.02 and 0.06 percent of the GDP in 2007–2015 and did not exactly match the cash withdrawals from the fund assets in the administrative cost item.

Eventually, the net income/loss for the year is the sum of the totals of three sections: change in fund assets, change in contribution asset and change in pension liability. However, regarding the double-entry accounting system, the net income/loss is equivalent to the difference between the change in assets (fund assets and contribution asset) and the change in liabilities. Figure 2.5 shows the evolution of the change in assets, the change in pension liability and the net income/loss of the Swedish NDC pension system. The value had been volatile, range from the loss of 7.70 percent of the GDP in 2008 and the gain of 12.07 percent of the GDP in 2010. The imbalance of the system's financial position is chiefly caused by the change in the indexation of the pension liability.

**Figure 2.5:** Evolution of change in assets, change in pension liability and net income/loss of the Swedish NDC pension system



Source: Authors' compilation based on the data from Swedish Pensions Agency (2008–2016)

## 2.4 Conclusions and new responses to economic crisis

This chapter has described the Swedish experience in its NDC pension system over the 2007–2015 period, with special attention to the balance ratio under economic and financial uncertainties. It has shown how in the post-2008 recession, the solvency of the Swedish NDC scheme dropped, and the balance ratio fell significantly below the unity in 2008, 2009 and 2012. The automatic balancing mechanism was triggered with the reduction in indexation of pensions and notional accounts in 2010, 2011 and 2014.

Three different policy responses had been carried out in this respect, as follows:

The first was to smooth the value of the buffer fund used in the balance ratio. Thus, instead of using the value of the buffer fund as of 31 December in a specific year, the average of the value of the buffer fund as of 31 December in the last three years was used. This change was legislated despite (the predecessor to) the Swedish Pensions Agency's objection about the measure's inefficiency. The agency proposed that rather

than smoothing the value of the buffer fund, the solvency deficit's effect on the pensions should be smoothed. According to the agency, this solution would result in a more efficient smoothing, partly due to considering all sources of solvency volatility, not only the volatility resulting from the value of the buffer fund.

The second policy response to the reduced pensions under the notional model was to cut taxes on pensions so as to counteract the net effect of the negative indexation. The retirees were more or less completely compensated for their losses in *inkomstpension* by the reduced taxes. This policy response implies that one of the main objectives of the NDC design – to insulate government finances from the development of the public pension plan – was not achieved.

Third, the volatility of the solvency measure – the balance ratio – triggered the Swedish Pension Agency's analysis of the reasons for the unanticipated level of volatility. This work revealed that the three-year smoothing of the income index caused a delay relative to the development of the contributions to the plan. This situation caused a significant real volatility in the ratio of the contribution asset to the pension liabilities. For example, a shift downwards in the nominal earnings growth would immediately result in a lower growth in the contributions and the contribution asset. The income index would fully react to this outcome with a four-year delay, causing a *ceteris paribus* permanent negative gap between assets and liabilities. The result would also be observed in traditional actuarial projections of the NDC plan.

This pre-crisis (unidentified but obvious from the experience) source of solvency volatility triggered the legislative attempt to eliminate or at least reduce the volatility caused by the indexation itself. The Swedish Pension Agency proposed measures that would eliminate the volatility caused by the indexation. Its main component was to eliminate the smoothing in the indexation and to make it slightly forward looking. Rather than using the change in income in the current year relative to the previous one, the (projected) change in income in the coming year relative to the current one should be used. The difference between the projected and the actual income growth should affect the indexation the following year. The agency claimed that combined with some compensating adjustments, this solution would completely eliminate the avoidable

solvency volatility caused by the delay in indexation relative to the contribution development. To further reduce volatility, the agency proposed that balancing should only come into effect if the balance ratio was below the unity for three years in a row, and that in balancing, the indexation should yearly be reduced by only one-third of the adjustment necessary to achieve 100 percent solvency.

The Pensionsgroup (set up by the government to negotiate any changes to the political agreement on the pension reform in 1994) decided to propose to the parliament a simplified version of the technical adjustments recommended by the Swedish Pensions Agency. The smoothing of the income index was abolished, but the forward-looking design proposed by the agency was not enacted. The government/Pensionsgroup judged the forward-looking proposal (the increased use of projections in the indexation) as a risk and also disliked the technical changes forced by its design. Neither did the government/Pensionsgroup adopt the agency's proposed rule of only using the balance mechanism if the solvency was less than 100 percent for three consecutive years. However, the new form of smoothing (reducing the indexation by only one-third of the necessary reduction to achieve 100 percent solvency) was implemented. The new indexation and the new balance ratio (called as a damped balance ratio) were introduced in 2015.<sup>15</sup>

The volatility of the indexation of pensions caused by the balance mechanism has been criticised by various groups in Sweden. Perhaps the most vocal critique has come from groups representing retirees, but other groups have also expressed disapproval. The political parties supporting the pension agreement have not criticised the balance mechanism's goal to secure an automatic, financially stable pension scheme. As evident from their legislative actions, they have disliked the volatility and tried to reduce it while adhering to the principle of automatic financial stability.

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<sup>15</sup> Another change in legislation addressed the inefficiency in the indexation/balance mechanism that implied that pension credits earned during a balancing period, and thus not subjected to a downward adjustment of the indexation, would still benefit from balancing the mechanisms upward. This inefficiency was already acknowledged by legislators when the balance mechanism was legislated in 2000, but at that time, no satisfactory solution to the problem was identified – partly due to time constraints. Barr and Diamond (2011) criticise this inefficiency.

Somewhat surprisingly, there has been almost no professional or popular debate on the relevance of the solvency measure used. Considering the positive projections of the pension plan (the actual and the projected sizes of the buffer fund) – indicating that the present solvency deficit will be overcome without reducing pensions – the lack of debate is even more astonishing. This situation can possibly be attributed to the human tendency to accept a number as a fact, more or less regardless of the underlying ambiguities of the calculation. Another or complementary interpretation of this absence of a more profound and philosophical critique is that the use of only factual transactions and events in estimating solvency, without any projections, has worked.

# CHAPTER 3

## Sweden's Fifteen Years of Communication Challenges: What are the Challenges Looking Forward?<sup>1</sup>

It is clear that pension reforms require the solid backing of the population. This is only possible if the participants of the system are well informed and understand pension rules from an economic and financial point of view. In this sense, the Swedish social security administration publishes an annual actuarial balance of the solvency of the whole public system and at the same time sends out to each participant the *orange envelope* with information on the individual accumulated notional balance and funded accounts, movements during the year and also estimates of the individual pension amount under different scenarios. This chapter describes the Swedish pension experience on communication to its participants over the last fifteen years and discusses its effectiveness in terms of credibility.

**Keywords:** accounting, individual information, public pensions, retirement, solvency, Sweden

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<sup>1</sup> A version of the paper (joint work with Ole Settergren, María del Carmen Boado-Penas and Erland Ekheden) was presented at the NDC III Conference, Rome, Italy, October 5-6, 2017 and will be published as a chapter in a book on NDCs edited by the World Bank.

### 3.1 Introduction

Pensions are sufficiently complex to be very hard to understand. In particular, Barr and Diamond (2008), public pension systems are likely to need to be adjusted due to changes in demographic and economic conditions and may also change with political circumstances, what adds more complexity. New (1999) states that the problem may not be lack of information but an information processing problem. With an information processing problem, the problem is too complex for many agents to make rational choices even when they have the necessary information. Specifically, for pension products, Larsson *et al.* (2009), the long-time horizon between the payment of contributions and benefits produces inherent difficulties in understanding the product.

According to Fornero (2014), political parties however do not rely on ideological message to communicate the reform and if the participants of the system do not understand its basic principles, it risks having little effects or even being repealed. Information is thus important not only for individual wellbeing but also for society. For an individual planning, knowledge of the system is essential to avoid mistakes as for the difference between the expected and actual pension benefits. Information on the financial sustainability of the pension system is also fundamental in the sense that if participants misinterpret the system and the need for reform they will try to reverse it. Also, Lusardi and Mitchell (2007a; 2007b) and Biggs (2010) state that access to financial information and appropriate planning may have a positive impact on decision-making concerning retirement. Moreover, information about the pension benefits influences the age at which individuals retire (Sundén, 2013). Similarly, Boeri and Tabellini (2012) points out that reforms can obtain the popular support if they are well described, explained and understood. However, empirical evidence (e.g. Mitchell, 1988; Lusardi and Mitchell, 2007a; Lusardi and Mitchell, 2011) amongst others indicates that most individuals have very limited information about the core elements of social insurance systems and on the key variables that define the amount of their pensions.

In the last years, governments in several countries try to facilitate decision making to the contributors by regularly sending statements about their individual pension position and estimates of the expected pension benefits. For example, the *social security statement* in the USA, the *orange envelope* in Sweden or the *yellow envelope* in Germany.<sup>2</sup>

Information on the financial sustainability of the system can be described, Boado-Penas and Vidal-Melía (2013), by means of the actuarial balance sheet<sup>3</sup> – mainly explained as an aggregate accounting projection model. Allowing for particular differences between countries, actuarial balances are compiled, on a regular basis, in countries such as US, Japan or Canada, amongst others to reveal the financial position of the pension system.<sup>4</sup>

It is clear that there is social demand of transparency regarding the management of the finances of pension systems and also a need of credibility in the sense that promises of payment are rationally respected. With all of this in mind, the aim of this chapter is to describe the Swedish pension experience on both individual information and information on the financial sustainability in terms of its effectiveness.

The chapter is structured as follows; Section 3.2 describes the Swedish public pension system. The main channels of communications: the actuarial balance together with its main financial indicators over the 2007–2015 period (global information) and the orange report (individual information) are explained in the next section. The effectiveness of the Swedish information by means of survey results is illustrated in Section 3.4. and Section 3.5 concludes the paper.

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<sup>2</sup> For more details, see Kritzer and Smith (2016).

<sup>3</sup> In Sweden, an actuarial balance sheet, in the accounting sense of the term, is used in the Swedish notional pension system.

<sup>4</sup> The actuarial balance of the US social security programs, OASDI (2015), is aimed at measuring the system's financial solvency over a 75-year time horizon. In Japan, the Actuarial Affairs Division of the Ministry of Health, Labour and Welfare (Actuarial Affairs Division, 2014) compiles an actuarial balance at least every five years with a 95-year time that includes an automatic balancing mechanism to make the system sustainable in such horizon of time. In Canada, actuarial valuation reports on the Canada Pension Plan (CPP) are prepared by the Office of the Chief Actuary (Office of the Chief Actuary, 2015) every three years. These reports determine a minimum contribution rate and show projections of the Plan's contributions, expenditures and assets for the next 75 years.

## 3.2 Swedish public pension system<sup>5</sup>

The Swedish public pension system consists of two different earnings-related benefit schemes: a Notional Defined Contribution (NDC) scheme on pay-as-you-go financing (called the *inkomstpension*) and a fully funded financial defined contribution pension (called the *premium pension*). The contribution rates for two schemes are 18.5 percent of the pension base, with a split of 16 percent for the NDC pension and 2.5 percent for the FDC scheme. A tax-financed guaranteed pension, annually adjusted according to the consumer price index, also provides supplementary support for retirees with low NDC pensions.

- *Notional defined contribution (NDC) scheme*

Under the Swedish NDC scheme, both accounts and benefits are indexed by the change in the average income. When the initial pension at retirement age<sup>6</sup> is calculated – that is, when the notional account value is converted<sup>5</sup> into an annuity – the pension is increased or front-loaded on the basis of an assumed annual real growth rate of 1.6 percent for the income index. This rate of advanced interest is then deducted every year from the increase in the income index. Thus, the NDC pension is indexed annually by the change in the income index reduced by 1.6 percent.

### *The Automatic Balancing Mechanism (ABM) applied to the NDC scheme*

In certain situations, exceptions to the regular income indexation of accounts and benefits apply. These exceptions are governed by the ratio of assets to liabilities (known as balance ratio<sup>7</sup>) as provided in the legislation on the balancing mechanism. The balance ratio is an indicator that emerges from the actuarial balance sheet of the NDC scheme (see more info in Section 2.2) and is used in Sweden in order to measure

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<sup>5</sup> For a more detailed description of the Swedish pension system, see the Swedish Social Insurance Agency (2007–2015) and Barr (2013). The latter paper also evaluates the pension system in Sweden against the goals established at the time of the reforms in the late 1990s.

<sup>6</sup> In Sweden, retirement is flexible and pension benefits can be withdrawn from age 61. When converting benefits into the whole life annuities, the life expectancy of the cohort is taken into account.

<sup>7</sup> To indicate that the solvency ratio of a PAYG scheme is different from that of the *premium pension*, the *inkomstpension* system calls this ratio the balance ratio.

whether the system can fulfil its obligations to its contributors and to decide whether an ABM should be applied. If the balance ratio is less than 1, the ABM is activated (see how the 'Balancing mechanism' works in Section 2.2).

- ***Funded financial defined contribution (FDC) scheme***

In the FDC scheme, participants have the individual financial account and their pension contributions are invested in a large number of funds chosen by the members themselves. The rate of return on the individual accounts is determined by the return on the funds that the individual has chosen. The FDC pension can be drawn in either of traditional insurance with profit annuity or fund insurance. In contrast to the NDC scheme, the annuity divisor used to convert the value of the pension account into an annuity is based on forecasts of future life expectancy rather than the current remaining life expectancy of the person's birth cohort. The initial pension of both forms of insurance is credited with an interest rate of 1.75 percent and a deduction for costs of 0.1 percent (Swedish Pensions Agency, 2017).

If the FDC pension is drawn in the form of traditional insurance with profit annuity, the pension is calculated as a guaranteed lifelong annuity. The shares in the individual's accumulations are traded and the Swedish Pensions Agency assumes responsibility for the investment as well as the, very limited, financial and longevity risk. Thus, the person's annuity is based on the performance of the fund as a whole. While, the fund insurance (or known as unit-linked insurance) means that a person's accumulation remains in the pension funds chosen by the participant and a sufficient fund shares are sold to finance the payment of the premium pension. The pension increases with stock market gains, or vice versa. If the value of the fund shares increases, fewer shares are sold and if it decreases, more shares are sold. Changes in prices of fund shares have an effect on the value of the premium pension in the following year.

### 3.3 Channels of communication

In order to make decisions about at which age to retire and how much to save, participants in the Swedish pension system need information about how the level of benefits are affected by the number of years of contributions and the retirement age. Also, a challenge for the communication strategy is to convey that the automatic balancing mechanism is a regular component of the indexation of earned pension rights. The *annual report* (that includes the actuarial balance sheet) and the *orange envelope* provide communication with participants regarding their individual pension and the sustainability of the whole pension system.

#### 3.3.1 Accounting information: The actuarial balance sheet

Since 2001, the Swedish pension system has produced an actuarial balance sheet and income statement every year based on the principle of double-entry bookkeeping. Under the NDC scheme, the balance sheet can be defined as a financial statement listing the pension system's obligation to contributors and pensioners (liabilities to contributors and pensioners) on a particular date together with the amounts of the various assets (financial assets and contribution amounts) that back up these commitments. The details about the accounting information of the NDC scheme is based on Section 2.3 of the development of solvency: balance sheets for 2007–2015 (see Section 2.3 p.14-19).

Under the fully-funded pension scheme, the insurance assets, which consist of fund insurance, conventional insurance, and temporary management, are reported at their true value, or accrued acquisition cost. As of the balance sheet date, the true value of the assets is evaluated at their price on the last trading day of the year, while the assets reported at accrued acquisition cost are the difference between acquisition cost and redemption price periodised as interest revenue for the remaining time to maturity. Fund insurance assets refer to the investments in funds by pension savers and are reported at redemption price for fund assets. Temporary management is made up of pension contributions paid in periodically during the year in which pension credit is

earned and are reported at their accrued acquisition value. Conventional insurance assets are invested in equity and interest funds and are reported at their true value.

The balance sheet for the Swedish funded pension as the percentage of GDP is illustrated in Table 3.1. The insurance assets have continuously increased since 2007 and in 2015 the value of the insurance assets was SEK 896 billion, accounting for 21.44 percent of GDP. The main component of the insurance assets of the fully-funded system is fund insurance amounted to almost 94 percent of total assets, which is invested 90 percent in stocks and shares and 10 percent in bond and other interest-bearing securities. The change in insurance assets chiefly refers to newly-earned pension credit, positive changes in value, allocated management fees and pension disbursements.

The pension liability to pension savers and retirees is related primarily to fund shares and is affected by the development of the market value of the funds chosen. With traditional insurance, the pension liability is determined for each insurance policy as the capital value of the remaining guaranteed disbursements in which their value is computed on assumptions about the future returns, life expectancy and operating expenses. The return is dependent on the market rates of interest on government bonds of varying maturities. Pension liability in 2015 increased SEK 84 billion, about 10 percent.

In the funded pension scheme, all payments in and out of the system and all changes in value have in principle the same effect on the assets and liabilities of the system. Changes in the value of fund shares affects the assets of participants and pensioners in the system. The positive result of the scheme belongs to the pension savers and the pensioners, and is invested in the consolidation fund as owner equity. The amount in the consolidation fund for traditional insurance with profit annuity are disbursed as a bonus rate in connection with pension disbursements. Money in the consolidation fund for fund insurance is deducted from the following year's contributions to cover operational costs.

**Table 3.1:** Balance sheet of the Swedish FDC pension system on December 31, 2007–2015

Item	2007	2008	2009	2010	2011	2012	2013	2014	2015
<b>Assets (% of GDP)</b>									
<b>Insurance assets</b>	<b>10.27</b>	<b>7.72</b>	<b>11.29</b>	<b>12.59</b>	<b>11.86</b>	<b>13.97</b>	<b>17.20</b>	<b>20.63</b>	<b>21.44</b>
Fund insurance	9.38	6.84	10.38	11.64	10.79	12.82	16.01	19.33	20.12
Traditional insurance	0.04	0.05	0.07	0.14	0.24	0.29	0.34	0.46	0.50
Temporary management	0.84	0.83	0.84	0.81	0.83	0.85	0.85	0.84	0.82
<b>Other assets</b>	<b>0.06</b>	<b>0.04</b>	<b>0.06</b>	<b>0.07</b>	<b>0.07</b>	<b>0.08</b>	<b>0.10</b>	<b>0.11</b>	<b>0.10</b>
<b>Total assets</b>	<b>10.33</b>	<b>7.76</b>	<b>11.35</b>	<b>12.67</b>	<b>11.93</b>	<b>14.05</b>	<b>17.30</b>	<b>20.74</b>	<b>21.54</b>
<b>Liabilities and results brought forward (% of GDP)</b>									
<b>Closing results brought forward</b>	<b>-0.04</b>	<b>-0.04</b>	<b>-0.02</b>	<b>0.01</b>	<b>0.04</b>	<b>0.06</b>	<b>0.10</b>	<b>0.15</b>	<b>0.15</b>
Opening results brought forward	-0.04	-0.03	-0.04	-0.02	0.01	0.03	0.05	0.09	0.13
Net income/loss for the year	0.01	0.00	0.02	0.04	0.03	0.03	0.04	0.06	0.02
<b>Liabilities</b>	<b>10.36</b>	<b>7.80</b>	<b>11.37</b>	<b>12.65</b>	<b>11.89</b>	<b>13.99</b>	<b>17.20</b>	<b>20.59</b>	<b>21.39</b>
Pension liability	9.41	6.88	10.43	11.73	11.79	13.88	17.08	20.46	21.27
Other liabilities	0.95	0.91	0.95	0.92	0.10	0.11	0.12	0.13	0.12
<b>Total Liabilities and results brought forward</b>	<b>10.33</b>	<b>7.76</b>	<b>11.35</b>	<b>12.67</b>	<b>11.93</b>	<b>14.05</b>	<b>17.30</b>	<b>20.74</b>	<b>21.54</b>

Source: Authors' compilation based on the data from Swedish Pensions Agency (2008–2016)

### 3.3.2 Individual information to participants: The orange envelope

In 1999, as part of the reform of the Swedish pension system, a so-called *orange envelope* (see Appendix B.1 for the sample of *Orange envelope*) was introduced to provide information to the participants as well as projection of benefits. All insured persons who have not started to receive a full pension receive an annual statement with four pages containing account information on the changes and current status of their notional and fully-funded accounts as well as a projection of pension benefits. In addition to providing information on the expected benefits, the orange envelope summarises how the new pension system works and highlights to participants that benefits are determined by lifetime earnings. The pension account is presented as a bank account so that the participants see the public pension system as savings for retirement.

The first page displays the monthly national public pension forecast that the member is expected to receive before tax with the different age when the participant decides to retire, e.g. age 61, 65, 68 and 3 months or 70. This page also illustrates the hierarchy of the pension sources that the participant would earn. The first order indicates the national public pension, both NDC and FDC, while the occupational pension is in the middle of the hierarchy and followed by the private pension. Moreover, the participant can see her entire pension through the website<sup>8</sup> by using the personal code attached in this page to log in.

In the second page, the evolution of the pension values of each account – income pension and premium pension account – during the year is presented (based on information from two years before). The statement consists of the account value of the previous year, the contributions assigned, the amount received for the survivor dividend – the pension balance of contributors who have died before reaching the age of retirement, which is distributed among the surviving members of their birth cohorts – and together with the administrative and fund fees charged. Furthermore, this page illustrates the values of the premium pension account with the breakdown of the portfolio, the allocation of each fund that the accountholder chooses and the actual distribution. The contributor will know the development of the premium pension funds in more details, in particular where the money is invested and how much she pays in fees. The changes in the value are also shown in percent which can be compared with the data for the average participant.

The third page provides forecasts of the individual monthly pension amount with different retirement ages, i.e. 61, 65, 68 and 3 months and 70 under the assumption made on that there is the same pensionable income per year – 0 percent wage growth.<sup>9</sup>

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<sup>8</sup> The Swedish Social Insurance Agency and the Premium Pension Authority together with the insurance companies for the occupational plans launched the website <https://secure.pensionsmyndigheten.se/B3>. This website presents individual projections of both the public pension and occupational pension benefits and the total projected pensions.

<sup>9</sup> Before 2011, the statement presented estimates under two scenarios: (1) 0 percent wage growth and (2) 2 percent wage growth. However, the higher assumption was removed in 2011 because surveys indicated that it was too confusing.

The last page gives the total pension credits, which basically means the money paid in during the year, and decomposes the contributions made for each account. The pensionable income from different sources is also contained.

### 3.3.3 Information on pension projections

The pension projection is perhaps the most important line of communication between the pension provider and pension savers. A projection is also the only thing that gives the insured persons a rational basis for decision making on matters affecting their work and savings. The projection must be easy to understand and to communicate and it must be calculated as accurately as possible. Under a notional scheme, projections are needed to give information on expected benefits to the system's contributors.

The forecasts in the *orange envelope* since 2011 has been based on only one scenario, 0 percent average wage growth. The rate of return on the funded individual account is assumed to be 3.5 percentage points higher than salary growth. All calculations are made in fixed prices, i.e. inflation is set to zero.

Consequently, this wage growth influences the size of contributions, or pension rights paid to notional or funded pensions. Within the national pension scheme, the growth is also employed as interest rate in the notional pension that means the pension balance (accumulated pension rights) is recalculated upwards each year by an interest rate equal to the assumed growth.<sup>10</sup>

If the real calculation is done without growth, this is, the projection uses zero growth, the forecasted pension amount is expressed in terms of the same price and wages levels as at time of the projection. In this scenario, there is no income or standard increase during the time leading up to retirement. Zero growth over a longer period is highly improbable; however, the reason of using this scenario is simply to provide a

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<sup>10</sup> The growth is measured in the national pension scheme by the income index, which is the development of average pensionable income in Sweden, including income above the earnings ceiling.

projection amount expressed in current price and wage terms which can be directly associated with the current income.

When the automatic balancing mechanism is activated within the public pension scheme, it is unclear how long and how fast the financial balance is recovering and when income indexation should apply again. Only known values of balance ratio and balance index are used in the projection for the national pension.

Every year since 1999, the pension projection in the orange envelope has used the retirement ages of 61, 65 and 70 with the following reasons: The age of 61 is the earliest possible age at which old-age pension may be received. The age of 65 is chosen because it was the 'normal' retirement age, being the retirement age under the old system. The age of 65 is also the age at which certain social insurance benefits, such as sickness and activity compensation and unemployment insurance come to an end and others such as guarantee pension and housing supplement for pensioners start. The age of 70 was chosen to provide a retirement age after 65 corresponding to the 61-year alternative. Having several different retirement ages is to show how the retirement age impacts the size of monthly pension payments. This reminds the fact that the longer the contributor works the higher the amount of pension will be and also that remaining life expectancy at retirement greatly influences the size of pension.

Life-expectancy assumptions and projection interest rates are used to ensure a desired payment profile during retirement. The Swedish Pensions Agency's projection assumptions concerning life expectancy are based on Statistics Sweden's projections of future mortality, which means projections for younger generations are based on the longer lifespans these are expected to have. The forecasted divisors set by the Swedish Pensions Agency are different for the notional pension and the funded pension that rely on actual divisors for the public pension but taking into account Statistics Sweden's mortality projections. The new projection divisors are determined each year in November.

### 3.3.4 The role of the Swedish Pensions Agency

In 2010, the Swedish Pensions Agency was established, taking over administration of the national retirement pension, which was previously Försäkringskassan's responsibility, and the premium pension, which had previously been handled by the Premium Pensions Agency. The information challenge was one of the main reasons to establish the new Agency. Therefore, an important task for the Swedish Pensions Agency is to work towards accessible and simple information on the total pension (including public pension, occupational pensions and private pensions).

To meet the needs on information the operational customer service operations of the Agency provide face-to-face meetings, telephone customer services, e-services and printed reports such as the orange envelope, annual report, statistics, amongst others.

The Swedish Pensions Agency has also made the webpage [www.minpension.se](http://www.minpension.se) (containing information on both public and occupational schemes), available as an embedded service from their own site [www.pensionmyndigheten.se](http://www.pensionmyndigheten.se).

## 3.4 Survey results: Does the information work?

Since 1999 the Swedish Social Insurance Agency<sup>11</sup> conducts an annual survey about the orange envelope to evaluate to what extent the participants open the envelope, read and understand the contents. The sample consists of 2,200 individuals interviewed by telephone and includes three different target groups such as old-age pensioners (27 percent of the sample), general pension savers (46 percent of the sample) and new pension savers (27 percent of the sample).

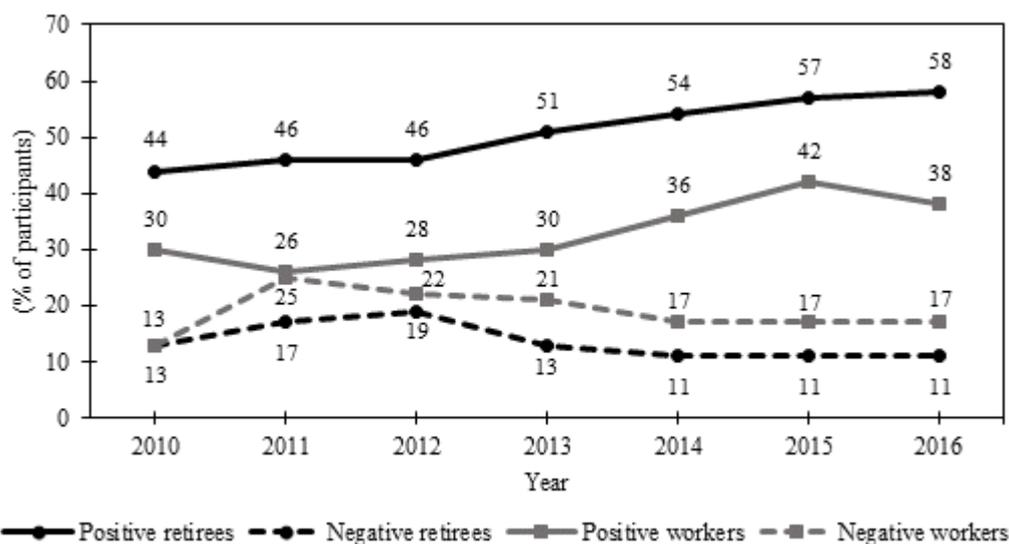
Currently, three-fourths of participants confirm that they have opened the orange envelope and half of them read some of the content. Figure 3.1 shows that the reported confidence in the Pensions Agency has been slowly but steadily increasing over time for both retirees and workers. In particular, in 2016, more than half of retirees and a

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<sup>11</sup> The name was changed to 'Swedish Pensions Agency' in 2010.

third of workers had some or great confidence in the pension system administration (including information and service) while the share of those with no confidence has slightly decreased.

**Figure 3.1:** Level of confidence in the Swedish Pensions Agency, 2010–2016

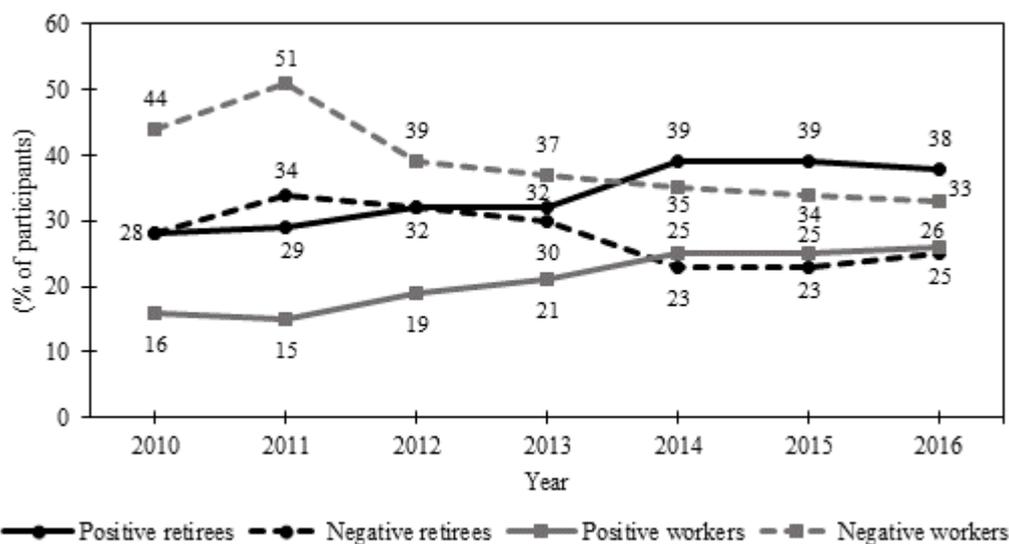


Source: Annual Orange Envelope Survey

Note: The grades to answer this question are 1-5. Grade 1 and 2 are grouped as negative while 4 and 5 as positive. Grade 3 is rated as neutral as is disregarded from the graphics.

However, the share of participants with confidence in the pension system, Figure 3.2, only reaches the value of 38 percent for pensioners and 26 percent for contributors in 2016. At the same time, the share of participants with no confidence has slightly decreased over the period 2010–2016.

We can observe, Figure 3.1 and Figure 3.2, that the confidence level on both the pension system and the pension system administration got worse in 2011 because of the negative income indexation that year (and the year before) as a result of the automatic balancing mechanism being triggered.

**Figure 3.2:** Level of confidence in the Swedish Pension System, 2010–2016

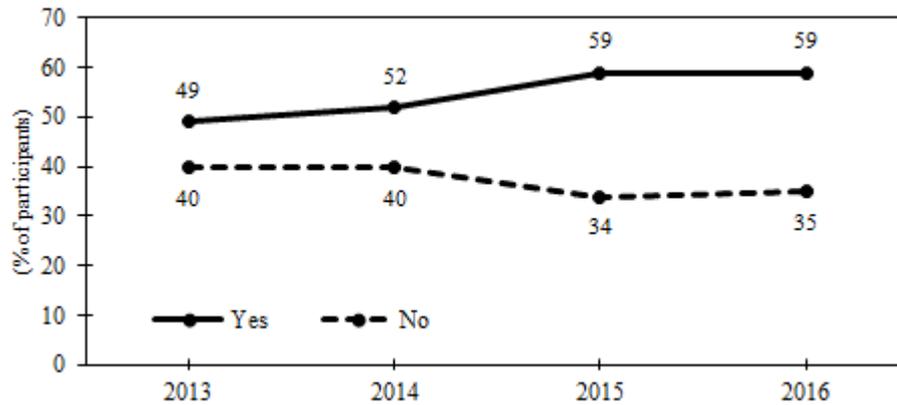
Source: Annual Orange Envelope Survey

Note: The grades to answer this question are 1-5. Grade 1 and 2 are grouped as negative while 4 and 5 as positive. Grade 3 is rated as neutral as is disregarded from the graphics.

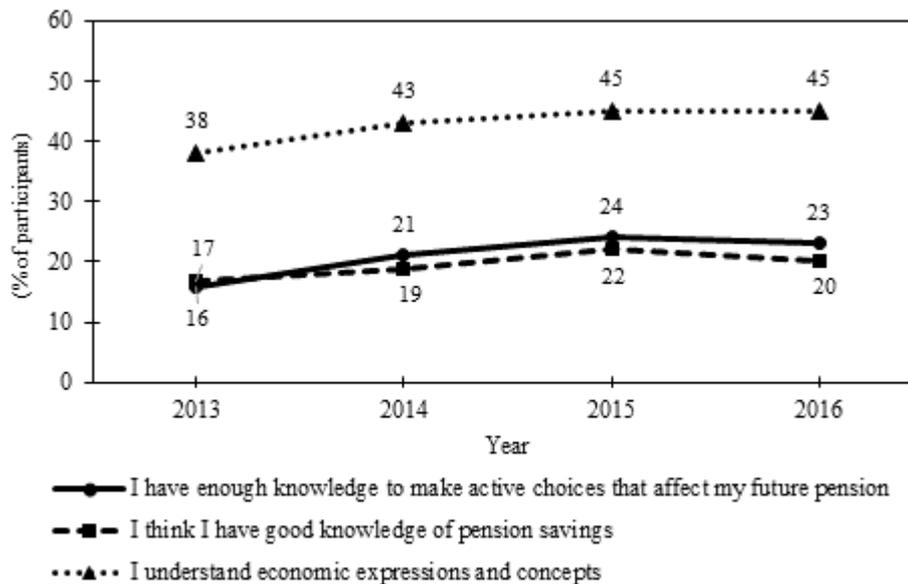
The survey also includes a set of questions to gauge the support provided by the Swedish Pensions Agency and understanding and use of the information received by the participants. The share of participants who value the information and support provided has been continuously increasing. In 2016, 59 percent of respondents reported that the Swedish Pensions Agency has provided information and support needed to make decisions on retirement as shown in Figure 3.3-a. The survey contains a question to categorise workers into three different target groups based on their understanding and interest in financial and pension issues. Studies show that these questions predict ability and probability of gathering and understanding information regarding pensions. As shown in Figure 3.3-b, knowledge on issues relating to pension savings is increasing over time and the share of workers with some knowledge on pension issues in 2016 is 88 percent. According to this 88 percent, the percentage of respondents with enough or good knowledge to make active choice is 43 while 45 percent of workers report to have some grasp on economic and financial concepts.

**Figure 3.3:** Questions to assess the knowledge about the pension system, 2013–2016

a. Do you, today, have the information and support need to make decision on retirement?



b. Self-reporting understanding in financial and pension issues 2013–2016



Source: Annual Orange Envelope Survey

### 3.5 Conclusions

The notional and financial defined contribution scheme puts more risk – or at least does so more explicitly than a defined benefit scheme. It places more (explicitly) responsibility on participants to plan for retirement. Therefore, it is possible to argue that in DC schemes it is more important that participants of the system are well informed and understand pension rules from an economic and financial point of view,

than it is in a DB scheme. We realise that this judgment is not free from criticism, same or similar information might be argued is equally relevant in DB schemes and only partly for different reasons.

This chapter shows the main channels of communication used by the Swedish administration, i.e. the annual report on the solvency of the public system and the orange envelope with information on the individual accumulated capital and forecasts of expected benefits.

The main challenge for the Swedish Pensions Agency is to know how the participants use the information provided and how well it communicates information about the pension system. At the same time, the new notional system puts additional demands on participants about how the level of benefits varies according to the number of years of contributions – or more exactly their life earnings, the number of years is not a direct parameter, it is only relevant in the impact it has on life earnings – and the retirement age. Projections of future pension benefits and how the amount of benefits changes with retirement ages is necessary information for contributors in order to make good decisions about number of years at work and savings.

The chapter shows that self-reporting use and understanding of the information received has been slowly increasing. However, the confidence on both the pension system and the pension system administration gets worse when the automatic balancing mechanism is triggered. It seems that there is still a lack of understanding on how the system works and why the activation of the mechanism is needed when the solvency is compromised. It is not surprising that to most people the abstract issue of the financial situation of the public pension plan is not understood or accepted as a viable argument for reducing what for most people amounts to an important benefit.

# CHAPTER 4

## Fairness and Annuitisation Divisors for Notional Defined Contribution Pension Schemes<sup>1</sup>

The use of a gender-neutral annuity divisor in the conversion of the accumulated contributions into the initial pension introduces an intra-generational redistribution from short-lived toward long-lived individuals; that entails a transfer of wealth from males (who on average live shorter) to females (who on average live longer) and from low-socioeconomic groups (who live shorter) to high classes. The aim of this study is to quantify the lifetime redistribution of a generic NDC system using two types of divisors – the demographic and the economic – to compute the amount of the initial pension. With this in mind, we first build differential mortality tables, by both level of education and gender, to compute the annuity divisors. Second, by calculating the ratio between the present value of expected pension benefits and contributions we assess the redistribution between the different subgroups of the system under the different divisors.

**Keywords:** annuity divisor, notional defined contribution, mortality forecasts, pay-as-you-go, public pension, redistribution, socioeconomic groups

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<sup>1</sup> A version of the paper (joint work with María del Carmen Boado-Penas and Steven Haberman) was presented at the 3rd European Actuarial Journal (EAJ) Conference, Lyon, France, September 5–8, 2016 and the 21st International Congress on Insurance: Mathematics and Economics (IME 2017), Vienna, Austria, July 3–5, 2017.

## 4.1 Introduction

On the basis of gender equality, all countries with NDC schemes in place use unisex (average) life tables to compute the annuity divisor, instead of using gender-specific life tables. Consequently, the use of unisex life expectancy introduces an explicit intra-generational redistribution from short-lived towards long-lived individuals that entails a transfer of wealth from males (who on average live shorter) to females (who on average live longer) (Palmer, 2006b; Alho *et al.*, 2013; James, 2013).

Another heterogeneity in life expectancy, in addition to age and gender, arises from socioeconomic classes – normally assessed through income, education or occupation. A close link between level of education and mortality has been well documented (Borrell *et al.*, 1999; Huisman *et al.*, 2004; Lleras-Muney, 2005; Miech *et al.*, 2011; Kaplan *et al.*, 2014) and we focus on education attainment in this study<sup>2</sup>. The evidence suggests that the mortality rates are inversely correlated to social-economic status, i.e. individuals with higher socioeconomic status live longer than those belonging to lower socioeconomic groups and the size of the mortality differential across subgroups has increased over time<sup>3</sup>. The net effect of considering both gender and socioeconomic group differential mortality rates is not clear for subgroups such as females with low socioeconomic status or men from a high socioeconomic class.

Although the evaluation of redistributive impacts on the pension system has attracted wide attention among scholars<sup>4</sup>, there has been a little recent literature on the lifetime income redistribution in the NDC scheme. Several studies have compared the

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<sup>2</sup> Education can capture both direct and indirect effects on mortality. Typically, individuals with an advanced education degree, having better knowledge on how to behave and enjoying a healthier lifestyle, are likely to live longer than those with lower education (Friedman *et al.*, 1995; Kern and Friedman, 2008; Cutler and Lleras-Muney, 2010). In addition, education can be used as a proxy for an individual's lifetime resources since more highly educated people earn, on average, higher incomes (Brown, 2003).

<sup>3</sup> Whitehouse and Zaidi (2008) and Ayuso *et al.* (2016) provide comprehensive reviews of mortality differentials across socioeconomic groups and the implications of heterogeneity for pension reform and design.

<sup>4</sup> See for example of the works on measuring distributional effects in the pension system Cubeddu (2000), Börsch-Supan and Reil-Held (2001), Gustman and Steinmeier (2001), Barnay (2007), Cremer *et al.* (2010), Forteza and Ourens (2012), Le Garrec (2012), Gustman *et al.* (2013), Platanakis and Sutcliffe (2016).

distributional impacts between the old defined benefit and the new NDC scheme<sup>5</sup>. Also, the other related researches have investigated the income redistribution that occurs from differential mortality across the population when annuities are priced uniformly in the social insurance programmes with mandatory retirement saving accounts (Brown, 2002; 2003; Gong and Webb, 2008; Finkelstein *et al.*, 2009) and the collective pension schemes (Hári *et al.*, 2006; Bonenkamp, 2009; Hoevenaars and Ponds, 2008).

Due to lack of exploring the in-depth analysis with respect to the magnitude and the direction of the intra-generational transfers under a generic NDC pension scheme in the earlier studies, we then intend to complete this gap. Also, the idea of using another annuity divisor, called as an economic annuity divisor, reflecting the expected number of years that the pension benefits will be disbursed rather than the number of years that a person will survive stimulates our attention. We, therefore, aim to quantify redistribution arising from the use of gender-neutral annuity divisors for various socioeconomic groups. In this study, each subpopulation is distinguished by gender and level of education to measure cross-gender redistribution – between males and females – together with intereducational redistribution – between individuals with different levels of education. The full life cycle is incorporated in the analysis, rather than the annual effects, by using an indicator namely the present value ratio that enables us to assess the expected money's worth of participation to the pension system.

In addition to focusing on the effect of the demographic annuity divisor related to the remaining life expectancy, we examine the distributional impact of an alternative divisor – an economic annuity divisor – that is currently applied for the valuation of liabilities in the Swedish NDC system and associated with the size of pensions and the expected number of years that pension will be disbursed. We also construct the gender- and education-specific cohort life tables to be used for the computations of the annuitisation divisors and the expected lifetime income cash flows.

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<sup>5</sup> See for example Borella (2004), Borella and Coda Moscarola (2006), Belloni and Maccheroni (2013) for the Italian case.

The chapter is organised as follows; we describe the framework of a generic NDC scheme on how to calculate notional capital, demographic annuity divisor, and economic annuity divisor in Section 4.2. The next section discusses the measurement of income redistribution in the pension scheme with the gender-neutral annuity pricing when the heterogeneity in mortality exists in the population. In the subsequent section, we provide a numerical illustration of the money's worth of using the average (unisex) annuity divisors for each subpopulation under different scenarios. Finally, the chapter ends with conclusions.

## 4.2 Methodology to calculate initial pension under an NDC framework: Demographic versus economic divisors

The amount of pension benefits under an NDC pension scheme is based on the concept of an individual lifetime account. Contributions are periodically noted on the personal account over the whole life career and the account balance is credited with a (notional) rate of return. At retirement, the amount of the account balance is converted into a lifetime pension according to actuarial practice. The initial pension at retirement can be expressed in the general form as:

$$\text{Initial pension at retirement} = \frac{\text{Accumulated notional capital}}{\text{Annuity divisor}} \quad (4.1)$$

This section describes the mathematical formulae used to calculate the notional account balance and the demographic annuity divisor. As an alternative to the demographic divisor, we explain the concept of economic annuity divisor, currently used by the Swedish NDC system to calculate retirees' pension liability.

### 4.2.1 Notional capital

Under an NDC scheme, the individual notional account grows with new contributions plus the (notional) returns on the account. Additionally, among countries implementing this system, Sweden is the only country which distributes the notional

capital of persons who die during the accumulation phase to the account values of those survivors belonging to the same birth cohort. This capital, known as ‘inheritance gains’ or ‘survivor dividend’<sup>6</sup>, constitutes a part of notional capital used to calculate the initial pension when people retire. Following Boado-Penas and Vidal-Meliá (2014), we describe the calculation of the accumulated capital at retirement age with both the inclusion and non-inclusion of the survivor dividend. Contributions and pension benefits are assumed to be payable yearly in advance.

- **Notional capital, excluding the survivor dividend**

If the accumulated survivor dividend is not included, the accumulated notional capital at retirement age  $x_e + n$  in year  $t + n$  for an individual with gender  $i$  and level of education  $j$ , denoted as  $K_{(x_e+n,t+n)}^{i,j}$ , can be calculated as

$$K_{(x_e+n,t+n)}^{i,j} = \sum_{k=0}^{n-1} \pi \cdot s_{(x_e+k,t+k)}^{i,j} \cdot (1 + r_{t+k})^{n-k} \quad (4.2)$$

where

$i$  denotes the gender indicator with man ( $m$ ) and woman ( $w$ );

$j$  denotes the indicator with respect to educational attainment: low ( $L$ ), medium ( $M$ ) and high ( $H$ ) education;

$n$  represents the number of year that contributions are paid from the age of entry into the system  $x_e$  in year  $t$  until reaching retirement age  $x_e + n$  in year  $t + n$ ;

$\pi \cdot s_{(x_e+k,t+k)}^{i,j}$  is the contribution paid in year  $t + k$  by an individual with gender  $i$  and education  $j$  at age  $x_e + k$ . The contribution is as a fixed fraction  $\pi$  of the annual earnings (or salary)  $s_{(x_e+k,t+k)}^{i,j}$ ;

$r_{t+k}$  is the (notional) annual rate of return in year  $t + k$ .

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<sup>6</sup> This entitlement, Chłoń-Domińczak *et al.* (2012), is different from the inheritance options in a prefunded scheme, where the assets of deceased participants are distributed to inheritors. For more details of survivor dividend in the Swedish NDC system, see Boado-Penas and Vidal-Meliá (2014), Arnold *et al.* (2016), Vidal-Meliá *et al.* (2016).

- **Notional capital, including the survivor dividend**

If the survivor dividend is included, then the individual amount of notional capital at retirement increases in line with the accumulated survivor dividend at retirement age. Thus, the accumulated notional capital including the survivor dividend at retirement age  $x_e + n$  in year  $t + n$  for an individual with gender  $i$  and level of education  $j$ , denoted  $K_{(x_e+n,t+n)}^{*i,j}$  can be calculated as follows:

$$K_{(x_e+n,t+n)}^{*i,j} = K_{(x_e+n,t+n)}^{i,j} + D_{(x_e+n,t+n)}^{ac} \quad (4.3)$$

with

$$D_{(x_e+n,t+n)}^{ac} = \sum_{k=0}^{n-1} \frac{\pi \cdot s_{(x_e+k,t+k)} \cdot (1+r_{t+k})^{n-k}}{n-k P_{(x_e+k,t+k)}} - \sum_{k=0}^{n-1} \pi \cdot s_{(x_e+k,t+k)} \cdot (1+r_{t+k})^{n-k} \quad (4.4)$$

where

$D_{(x_e+n,t+n)}^{ac}$  is the accumulated survivor dividend at retirement age for an individual who has contributed for  $n$  years since entering the system;

$s_{(x_e+k,t+k)}$  is the average earnings (salary) for an individual aged  $x_e + k$  in year  $t + k$ ;

$n-k P_{(x_e+k,t+k)}$  is the probability that an individual aged  $x_e + k$  in year  $t + k$  will be alive at age  $x_e + n$  in year  $t + n$ .

### 4.2.2 Annuity divisor

Two different types of divisors are studied. First, the demographic divisor that takes into account the remaining future lifetime at retirement is described. Second, we show the mathematical formulae of the economic divisor associated to the expected number of years of pension disbursements.

- **Demographic annuity divisor**

The demographic annuity divisor is equivalent to the present value of a whole life annuity-due at retirement age  $x_e + n$  in year  $t + n$  and is calculated as

$$DD_{(x_e+n,t+n)} = \sum_{k=0}^{m-1} k P_{(x_e+n,t+n)} \cdot \left( \frac{1 + \lambda_{t+n+k}}{1 + r'_{t+n+k}} \right)^k = \sum_{k=0}^{m-1} \frac{l_{(x_e+n+k,t+n+k)}}{l_{(x_e+n,t+n)}} \cdot \left( \frac{1 + \lambda_{t+n+k}}{1 + r'_{t+n+k}} \right)^k \quad (4.5)$$

where the pension benefits are disbursed for  $m$  years from the retirement age until the maximum age that an individual can survive. The pensions settled after the first payment are revalued by the indexation rate,  $\lambda_{t+n+k}$  and discounted with the technical interest rate  $r'_{t+n+k}$ . Also,  $l_{(x_e+n+k,t+n+k)}$  is the number of individuals surviving at age  $x_e + n + k$  in year  $t + n + k$ .

In practice<sup>7</sup>, the annuity divisors are equal for both sexes, being based on unisex (average) life tables for that particular cohort.

- **Economic annuity divisor<sup>8</sup>**

The Swedish NDC system is the only country that applies the economic divisor to calculate the pension liability toward a retiree. For the calculation of the economic annuity divisor it both the size of pensions and the pay-out period are considered.

According to Swedish Pensions Agency (2017), the formula for the demographic annuity divisor is modified to get the formula for the economic divisor. The formula to calculate the economic divisor at retirement age  $x_e + n$  in year  $t + n$  on a birth cohort basis, denoted by  $ED_{(x_e+n,t+n)}$ , is calculated as follows:

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<sup>7</sup> The formulae of the annuity divisor are differentiated among countries; for example, in Latvia and Poland (called a G-value), this factor corresponds to the life expectancy of the retirees in the individual's birth cohort at retirement age. In Italy and Sweden, the divisor includes the assumed rate of real GDP growth (or wage growth) with 1.5% in Italy and 1.6% in Sweden, respectively (Chłoń-Domińczak *et al.*, 2012).

<sup>8</sup> See formula B.7.5 in Appendix B, Swedish Pensions Agency (2017).

$$ED_{x_e+n,t+n} = \sum_{k=0}^{m-1} \frac{l_{x_e+n+k,t+n+k}^*}{l_{x_e+n,t+n}^*} \cdot \left( \frac{1}{1+r'_{t+n+k}} \right)^k \quad (4.6)$$

where  $l_{(x_e+n+k,t+n+k)}^* = l_{(x_e+n+k,t+n+k)} \cdot \bar{P}_{(x_e+n,t+n)} \cdot (1+\lambda_{t+n+k})^{k-1}$  (4.7)

$l_{x_e+n+k,t+n+k}^*$  represents the total pension disbursements at time  $t+n+k$  for all retirees at age  $x_e+n+k$ ;

$\bar{P}_{(x_e+n,t+n)}$  is the average initial pension at retirement age  $x_e+n$ . The amount of pension benefits is indexed by  $\lambda_{x_e+n+k}$ .

### 4.3 Measurement of income redistribution

NDCs narrow the ratio between benefits and contributions, achieving greater actuarial fairness. Participants who have paid the same amount of contributions and retire at the same age should be entitled to the same pension benefits, while the individuals with longer earnings histories, or higher earnings who have been contributing more will receive higher benefits.

A pension scheme is viewed as actuarially fair if for all individuals the (expected) present value of lifetime contributions is equal to the (expected) present value of pension benefits. Any difference between these two present values is defined as an income redistribution towards or away from the individual.

Boado-Penas and Vidal-Meliá (2014) and Alonso-García *et al.* (2017) have demonstrated that the NDC scheme considering the survivor dividend is actuarially fair – pension rights equivalent to contributions actually paid – on a cohort basis when the population is closed – neither immigration nor emigration occurs. Due to the heterogeneity in mortality across the population, the use of average (unisex) life expectancy as one factor of the actuarial divisor would introduce the explicit income redistribution from high-mortality risk to low-mortality risk groups within the same

generation (also known as intra-generational redistribution). People who expect to live longer would get a better deal out of the pension system than those who are expected to die earlier, given that they receive the same amount of initial pension at the same retirement age.

The measurement of the lifetime redistribution from an individual's perspective can be investigated through the difference between an individual's lifetime contributions and lifetime retirement benefits – in this way, we would capture the money's worth of participating to the pension system. This concept is expressed in absolute term and known as the Social Security Wealth (SSW) (Börsch-Supan and Reil-Held, 2001; Bonenkamp, 2009; Belloni and Maccheroni, 2013). However, we use its variant based on the principle of relative equivalence, called as the benefit to cost ratio (or present value ratio) defined as the ratio between the present value of benefits paid during the retirement and the contributions paid during their working career (Brown, 2002; Liebman, 2002; Brown, 2003; Borella and Coda Moscarola, 2006; Barnay, 2007; Mazzaferro *et al.*, 2012; Belloni and Maccheroni, 2013). Thus, this ratio shows how much the system returns to the participant for each euro paid.

A value of 1 indicates that the system is actuarially fair for an individual, i.e. she received pension benefits which correspond to her contributions – by definition, there is no redistribution towards or away from any person (Queisser and Whitehouse, 2006). A value greater (lower) than 1 indicates that an individual receives more (less) than she has been contributing – an individual faces an expected gain (loss) from the pension system.

The present value ratio, PVR, for a specific individual with gender  $i$  and level of education  $j$  who pays the contributions in the scheme for  $n$  years throughout the working life and receives the pension benefits at any age  $x_e + n + k$  in year  $t + n + k$  denoted as  $P_{(x_e+n+k, t+n+k)}^{i,j}$  for  $m$  years from retirement age until the maximum age of surviving, is evaluated at the age of entry into the system  $x_e$  in year  $t$  and is given by

$$PVR_{(x_e,t)}^{i,j} = \frac{\sum_{k=0}^{m-1} {}_{n+k}P_{(x_e,t)}^{i,j} \cdot (1+r_{t+n+k}^*)^{-(n+k)} \cdot P_{(x_e+n+k,t+n+k)}^{i,j}}{\sum_{k=0}^{n-1} {}_kP_{(x_e,t)}^{i,j} \cdot (1+r_{t+k}^*)^{-k} \cdot \pi \cdot S_{(x_e+k,t+k)}^{i,j}} \quad (4.8)$$

where  ${}_kP_{(x_e,t)}^{i,j}$  is the probability that an individual entering into the scheme at age  $x_e$  in year  $t$  will be surviving at age  $x_e+k$  in year  $t+k$  and  $1+r_{t+k}^*$  is the real discount rate in year  $t+k$ .

## 4.4 Numerical illustration

This section presents a numerical example to quantify the redistributive effects between subgroups within the same cohort of using the unisex (average) demographic and economic divisor in the calculation of the initial pension. We discuss the data, based on the Swedish data, in the first sub-section. Then, the main results of income redistribution by using the demographic and the economic divisors are presented. This section also provides a sensitivity analysis of the underlying assumptions.

### 4.4.1 Data

The participants in this study are classified by gender and educational attainment with three groupings: low, medium and high level of education<sup>9</sup>. These persons have differences in terms of income profiles and survival probabilities which are the main factors to determine the direction and magnitude of the intra-generational transfers.

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<sup>9</sup> According to the International Standard Classification of Education (ISCED 2011) classification, the Eurostat provides data by education into three categories: 1) less than primary, primary and lower secondary education (levels 0-2); 2) Upper secondary and post-secondary non-tertiary education (levels 3-4); and 3) Tertiary education (levels 5-8). We then refer those three groups as low, medium and high level of education, respectively.

The main assumptions and the data sources made for this numerical example are:

- A participant joins the labour market at the age of  $x_e = 25$  with a particular level of education that does not change over her working life.
- The retirement age  $x_r$  is equal to 65. Therefore, a participant pays contributions for  $n = 40$  years as no unemployment is considered.
- The maximum age that an individual can survive is  $95^{10}$  so the pension benefits are disbursed for  $m = 31$  years.
- The contribution rate  $\pi$  is fixed at 16%<sup>11</sup> of the individual's earnings.
- The notional rate of return  $r$  for the accumulated notional capital, the discount rate  $r'$  credited in the annuity divisor, the discount rate  $r^*$  for the present value ratio (PVR) and the indexation rate  $\lambda$  to revalue the pension benefits are set equal to 1.6% and constant over time.
- The cross-sectional average annual income for the Swedish population by age group, gender and level of education in 2013<sup>12</sup> provided in the Eurostat Database has been used<sup>13</sup>.
- The projected group-specific cohort life tables for an individual who was born in 1988 and started working at age 25 in 2013 explained in the next sub section are used to calculate the annuity divisors and the present value ratio (PVR).

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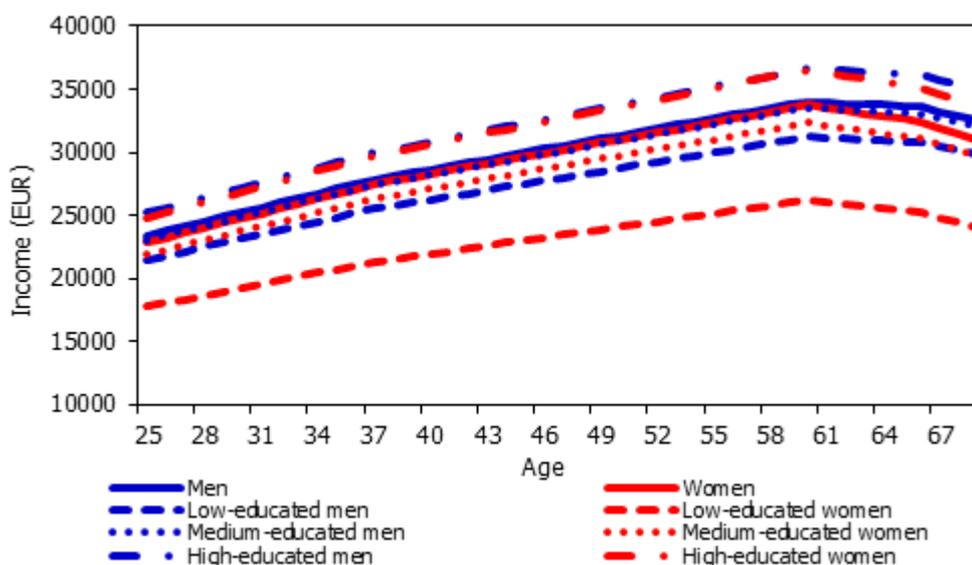
<sup>10</sup> This is the maximum age at which mortality data by level of education is available in the Eurostat Database.

<sup>11</sup> This value is coherent to the contribution rate of the Swedish NDC system.

<sup>12</sup> We consider the year of 2013 as the base year because the latest data on mortality by socioeconomic status (level of education) provided by the Eurostat has been updated until this year.

<sup>13</sup> As shown in Figure 4.1, the pattern of age-earnings profile is an inverted U-shape where the annual income increases in the early stages of a worker's career, peaks around the middle-age and finally declines as a worker approaches retirement. Women earn less than men at all educational levels and the gender gap is wider for persons with high school education than those with advanced degrees. Also, see more details of the estimation of age-earnings profile in Appendix C.4.

**Figure 4.1:** Age-earnings profile estimates during working life, age 25-69 by gender and level of education (in EUR)



#### 4.4.2 Results

- **Modelling and forecasting mortality heterogeneity across socioeconomic groups**

When projecting the heterogeneity in mortality among socioeconomic groups (e.g. income, occupation, or level of education), we require the methods that enable capturing both mortality level differentials and mortality improvement differentials between subpopulations. Some studies applied the survival models under the different mortality laws, e.g. Gompertz, Makeham, Perks and Makeham-Perks etc. (Richards, 2008) and the generalised linear models (Madrigal *et al.*, 2011) to assess the level of mortality differentials but ignoring the mortality improvement by socioeconomic profiles and the projection of their future evolution. However, several papers use multiple population extensions of the Lee-Carter Model (Lee and Carter, 1992) allowing to model and forecast mortality for two or more related populations

simultaneously<sup>14</sup>. Additionally, Villegas and Haberman (2014) introduced a relative modelling approach where the mortality of socioeconomic subgroups is modelled relative to the mortality of a larger reference population (generally referred to the national population). Their study concludes that mortality level differentials have a stronger impact on expectations of life and annuity values than mortality improvement.

In this study, we employ the extension of the Lee-Carter model, known as the ‘common factor’ model proposed by Li and Lee (2005) to estimate and forecast age- and gender-specific mortality rates across three levels of education. This model assumes all subpopulations have the same mortality improvements at all times, described by the common factor  $\beta_x$  and  $K_t$ . In practice, it is unlikely that all subgroups would have the identical tendency, however, because of the restriction of the available mortality statistics across socioeconomic status (only for five years, 2009-2013) it is necessary to make the assumption that the related populations share a common time trend in the long run<sup>15</sup>.

The mortality data of this analysis consist of the national mortality experience for calendar years 1960–2013 covering the age range 25–95 obtained from the Human Mortality Database (HMD), and the subpopulation dataset by educational attainment from the Eurostat having a shorter observation period 2009–2013 with the age range 25-95. The set of calendar years of subpopulation data is a subset of the corresponding calendar years in the national population and the ages available within each subgroup is a subset of those available in the national population as well. Then, we can apply the common factor model in the form of the relative mortality approach between the reference (national) population and the subpopulation introduced by Haberman *et al.* (2014). The procedures for the estimation and projections of the group-specific

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<sup>14</sup> For instance, the quantification of mortality differentials across countries (Li and Lee, 2005; Li and Hardy, 2011; Enchev *et al.*, 2017), regions within a country (Debón *et al.*, 2011; Danesi *et al.*, 2015) and socioeconomic subgroups (Villegas and Haberman, 2014; van Baal *et al.*, 2016).

<sup>15</sup> We have also modelled the mortality rates by using the other extensions of Lee and Carter (1997), which is known as the Joint- $\kappa$  model assuming the differential mortality improvements among groups, described in Appendix C.2. However, the forecasted rates are unreasonable in the long run; for example, the forecasted mortality rates of the low-educated group will be less than those of medium-educated. With regard to the papers of Haberman *et al.* (2014) Villegas *et al.* (2017) that comment on the difficulties with small data sets and less than about 8-10 years of data, our results might be caused by the very short time period of available data on education-specific mortality.

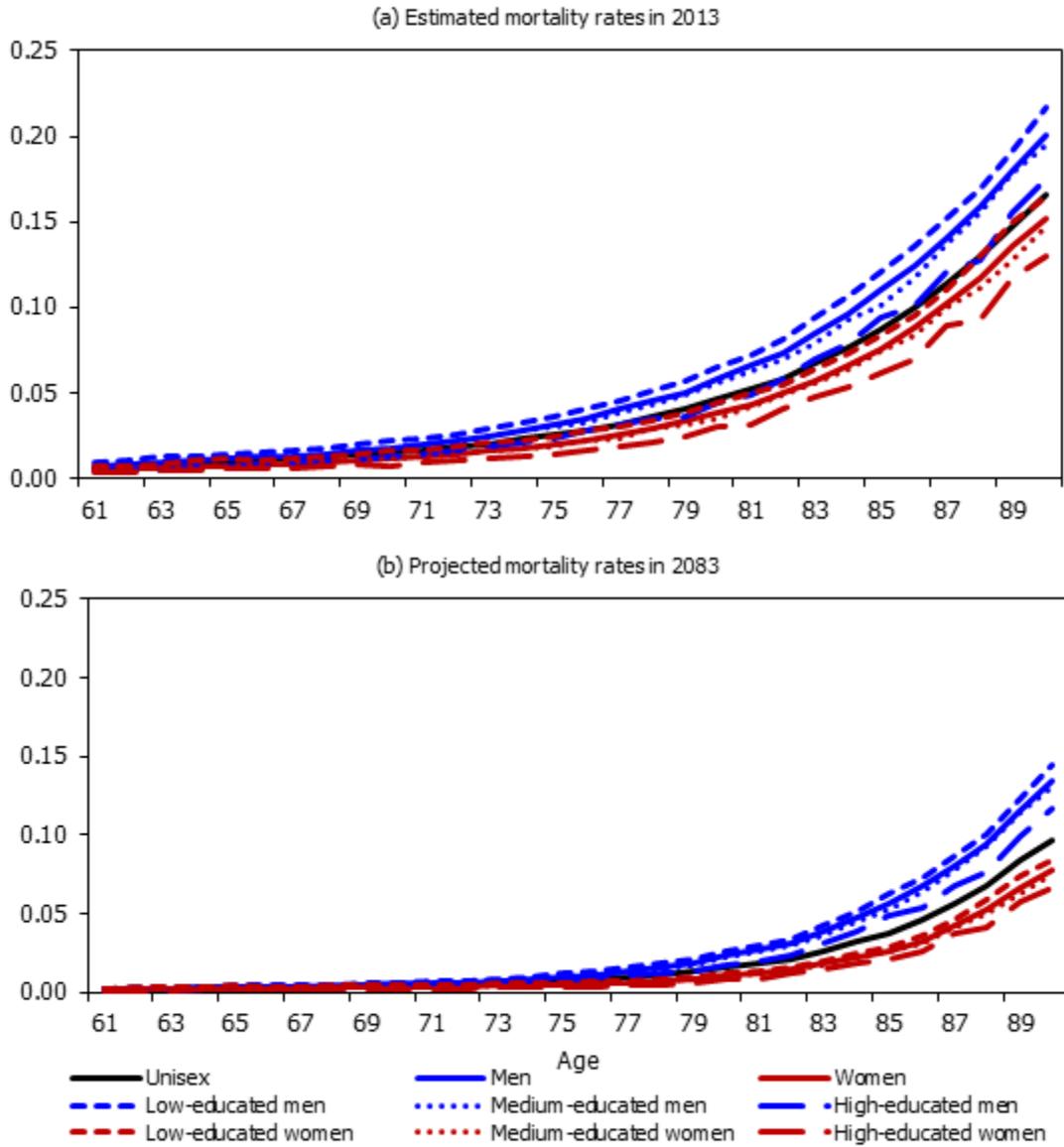
mortality rates and the construction of the cohort life tables are described in Appendix C.1.

The Statistics Sweden reports that life expectancy at birth in 2015 is 84.0 years for women and 80.3 years for men, while at age 65 these values are 21.3 and 18.7, respectively. During the last five years, the life expectancy for men has grown up by 1 year, while a rise in women has been by 0.6 years. There are clear and growing differences in remaining life expectancy between groups with different educational levels. Specifically, over the period 2012-2014, remaining life expectancy at age 65 for males with compulsory education (primary school) was 17.9 years and those with a degree were expected to live 20.2 years. Low educated women at age 65 are expected to survive 20.4 years and the life expectancy will rise to 22.9 years for those women graduated from the university. Apparently, the difference between the groups with the highest and the lowest educational level was about 5.5 years throughout the country (The Statistics Sweden, 2016).

As shown in Figure 4.2, the mortality across subgroups of males (females) is above (below) the average mortality. Furthermore, Figure 4.2 shows that the average mortality rates for men and women are quite close to those with medium education and the mortality in high-educated men and low-educated women are similar to the rates for the overall population.

In 2083, the age-specific mortality rates for all subgroups are projected to continuously decline, while maintaining the existing patterns of differential mortality. Also, because of the coherent approach to modelling the mortality trends for subpopulations, independently for each gender, the projected future subgroup-specific mortality rates do not diverge in the long run and the education- and age-specific mortality rates for each gender deviate constantly from the general (reference) mortality rates over time.

**Figure 4.2:** Evolution of mortality rates across gender and three levels of education

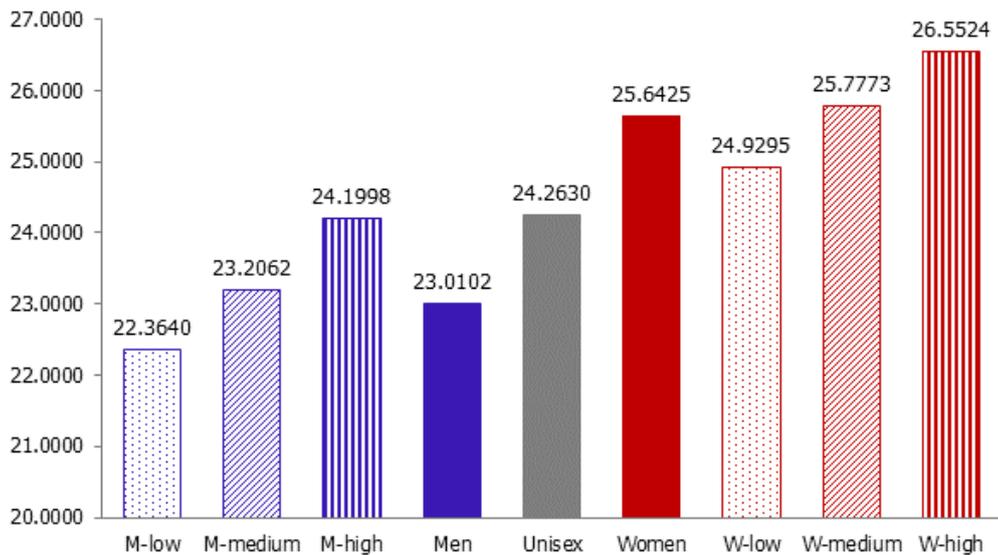


- **Application of the demographic annuity divisor**

Under the conditions defined above, the unisex demographic annuity divisor for an individual aged 65 is equal to 24.2630. Since we define the indexation rate,  $\lambda$ , to be identical to the discount rate,  $r'$ , for the divisor, then the annuity divisor is equivalent to the remaining life expectancy at age 65. The values of this annuity divisor across subpopulations compared to the unisex (average) annuity divisor are illustrated in

Figure 4.3. It is noticeable that the value of the demographic divisors for all subgroups of women (men) are greater (lower) than the unisex divisor. The values of the demographic annuity divisors across the subgroups have a pattern that is consistent with mortality. The value of the divisor for the medium-educated women group, i.e. 25.7773, is close to the value of overall women, i.e. 25.6425. As expected, the value of the divisor for women with high-education is the greatest with the value of 26.5524 as they have the lowest mortality among the whole population. In a similar way, the annuity divisor for men in the group of medium education with the value of 23.2062 is quite close to the value for all men, i.e. 23.0102 while the lowest value corresponds to the low-educated group with the value of 22.3640 as those individuals have the highest mortality compared to all subpopulations. We can expect that the groups of participants, particularly all females and probably the high-educated males, who have the divisor greater than the unisex (average) divisor would have a PVR higher than 1 and receive a greater income transfer than the others.

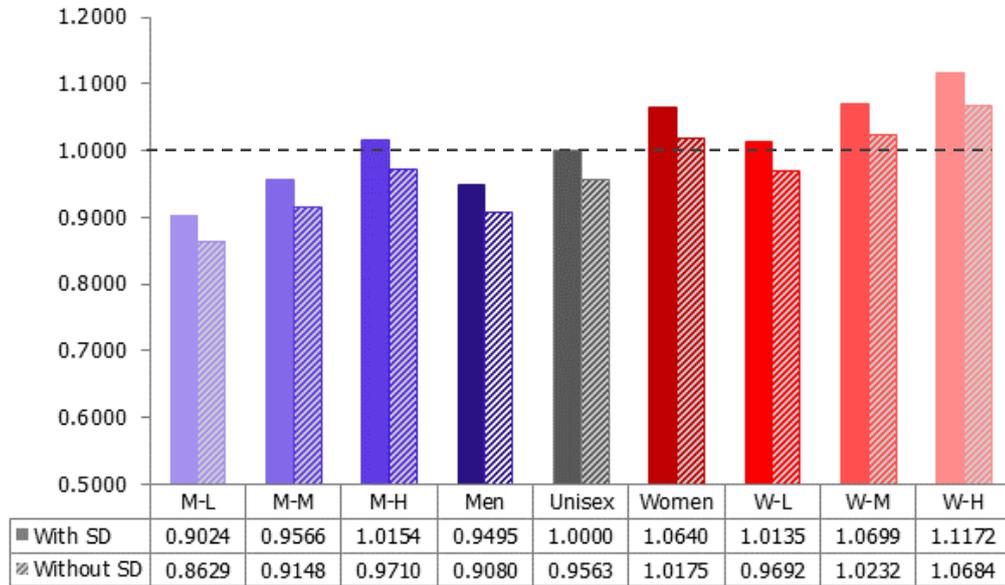
**Figure 4.3:** Annuity divisors under the baseline assumptions by gender and education



The money's worth of the pension benefits for an individual who retires at age 65, given the unisex (average) annuity divisor at 24.2630, is measured through the PVR as exhibited in Figure 4.4. When the survivor dividend (SD) is included in the

calculation of the initial pension and if there is no heterogeneity in mortality across populations, the PVR is equal to 1 indicating that the individuals will receive pension benefits identical to what they have been contributing over their working lives.

**Figure 4.4:** PVR across subpopulations in the cases of notional capital including and excluding survivor dividend



Considering differences in levels of mortality rates per groups, the PVR is higher for females than for males and it increases with the level of education, ranging from 0.9024 for a low-educated man to 1.1127 for a high-educated female worker. Moreover, it is worth noting that for most of the subgroups of females (males) participants, the PVRs are higher (less) than 1 as shown in Figure 4.4, what implies that they receive (pay for) a pension transfer.

A low-educated man worker is the largest loser, with the PVR of 0.9024 which is about 4.96% less than the PVR for the average man and 9.76% less than the average population, while a high-educated man receives a transfer of income from the system of 1.54%. In contrast, all women obtain welfare gains when the unisex (average) demographic annuity divisor is used, i.e. 1.35% of transfer for a low-educated woman, 6.99% for medium-educated and 11.72% for high-educated.

If the unisex (average) demographic divisor is used, it is remarkable that the gender has a larger impact on the income redistribution as even for women with low education the PVR is higher than the case for the average population.

Figure 4.4, when the survivor dividend is not included in the calculation of the pension benefits, the PVR for each subgroup decreases by 4.4% compared to the case of including the survivor dividend. However, even without the inclusion of the survivor dividend, women – specifically with medium and high education – are still benefitted by the system with a PVR higher than 1.

- **Application of the economic annuity divisor**

The value of the economic divisor for a particular cohort corresponding to the equation (4.6) depends on the population composition. Therefore, some plausible assumptions on the size of the different subgroups are needed. As stated in Section 4.2, the value of the economic divisor increases when the pension disbursement is higher and the payout period is longer.

Table 4.1 shows the results when the number of females increases (decreases) by 20% with respect to the number of males, i.e.  $l_{x_e+n}^{female} = 1.2l_{x_e+n}^{male}$  (or  $l_{x_e+n}^{female} = 0.8l_{x_e+n}^{male}$ ), keeping the equal proportion of each level of education. If we calculate the economic divisor only considering gender differences in mortality, we see that the value of the economic annuity, 24.4383, is higher than the demographic divisor when the number of women is larger since the number of years that pensions will be disbursed increases – women are generally living longer than men. In contrast, when the number of women is smaller, the value of the economic divisor is lower than the demographic divisor.

Consequently, as shown in Table 4.1, when the value of the economic divisor is higher (lower), both the amount of the pension and the PVR decrease (increase) compared to the case of using the unisex demographic annuity divisor.

**Table 4.1:** PVR for different subgroups when only gender differences in mortality taking into account in the calculation of the divisors

Sup-group	Demographic divisor	Unisex economic divisor					
		Females < Males		Females = Males		Females > Males	
<b>Divisor</b>	<b>24.2630</b>	<b>24.1725</b>		<b>24.3186</b>		<b>24.4383</b>	
<b>PVR and % relative change</b>							
Men	<b>0.9495</b>	<b>0.9530</b>	0.37%	<b>0.9473</b>	-0.23%	<b>0.9427</b>	-0.72%
M-L	<b>0.9024</b>	<b>0.9058</b>	0.37%	<b>0.9003</b>	-0.23%	<b>0.8959</b>	-0.72%
M-M	<b>0.9566</b>	<b>0.9602</b>	0.37%	<b>0.9544</b>	-0.23%	<b>0.9497</b>	-0.72%
M-H	<b>1.0154</b>	<b>1.0192</b>	0.37%	<b>1.0131</b>	-0.23%	<b>1.0081</b>	-0.72%
Women	<b>1.0640</b>	<b>1.0680</b>	0.37%	<b>1.0616</b>	-0.23%	<b>1.0564</b>	-0.72%
W-L	<b>1.0135</b>	<b>1.0173</b>	0.37%	<b>1.0112</b>	-0.23%	<b>1.0062</b>	-0.72%
W-M	<b>1.0699</b>	<b>1.0739</b>	0.37%	<b>1.0675</b>	-0.23%	<b>1.0623</b>	-0.72%
W-H	<b>1.1172</b>	<b>1.1214</b>	0.37%	<b>1.1146</b>	-0.23%	<b>1.1092</b>	-0.72%

The economic annuity divisor is also associated with the different sizes of the socioeconomic groups in the total population. As expected, if the economic divisor considers that all individuals are low-educated, its value would be lower, i.e. 23.5324, than the unisex demographic divisor, as illustrated in Table 4.2. The use of the economic divisor, in this particular case, provides a higher initial pension and consequently all subgroups would be benefitting from the application of the economic divisor. Conversely, the value of the economic divisor increases when its calculation considers that all individuals are high-educated people and thus the PVRs for all subpopulations decrease 4.38% compared to the case of using demographic annuity divisor.

**Table 4.2:** PVR for different subgroups when all individuals are considered to have the same education level (low, medium or high education) for the calculation of the economic divisor

Sup-group	Demographic divisor	Unisex economic divisor					
		Low education		Medium education		High education	
<b>Divisor</b>	<b>24.2630</b>	<b>23.5324</b>		<b>24.4664</b>		<b>25.3731</b>	
<b>PVR and % relative change</b>							
Men	<b>0.9495</b>	<b>0.9789</b>	3.10%	<b>0.9416</b>	-0.83%	<b>0.9079</b>	-4.38%
M-L	<b>0.9024</b>	<b>0.9304</b>	3.10%	<b>0.8949</b>	-0.83%	<b>0.8629</b>	-4.38%
M-M	<b>0.9566</b>	<b>0.9863</b>	3.10%	<b>0.9486</b>	-0.83%	<b>0.9147</b>	-4.38%
M-H	<b>1.0154</b>	<b>1.0469</b>	3.10%	<b>1.0069</b>	-0.83%	<b>0.9710</b>	-4.38%
Women	<b>1.0640</b>	<b>1.0971</b>	3.10%	<b>1.0552</b>	-0.83%	<b>1.0175</b>	-4.38%
W-L	<b>1.0135</b>	<b>1.0450</b>	3.10%	<b>1.0051</b>	-0.83%	<b>0.9692</b>	-4.38%
W-M	<b>1.0699</b>	<b>1.1032</b>	3.10%	<b>1.0610</b>	-0.83%	<b>1.0231</b>	-4.38%
W-H	<b>1.1172</b>	<b>1.1519</b>	3.10%	<b>1.1079</b>	-0.83%	<b>1.0683</b>	-4.38%

Table 4.3 extends Table 4.2 and assumes six different compositions of the population when calculating the economic divisor<sup>16</sup>. As shown in Table 4.3, the values of the economic divisor increase with a high level of education in the population. Specifically, the lowest value of the economic divisor, i.e. 23.9604, results for a combination of medium-educated males together with low-educated females. The values of economic divisors increase with level of education and we can categorise the results into three groups: the combination of population structure of low- and medium-educated (23.9604 and 24.0893), the low and high education (24.5034 and 24.6203) and the medium and high education (24.9382 and 24.9482), respectively.

<sup>16</sup> Table 4.3 involves the women with different education level from men, while Table 4.2 assumes that both males and females are having the same level of education.

**Table 4.3:** PVR for different subgroups when six different combinations of population are considered for the calculation of the economic divisor<sup>17</sup>

Sub-group	Demographic divisor	Unisex economic divisor					
		ML&WM	ML&WH	MM&WL	MM&WH	MH&WL	MH&WM
<b>Divisor</b>	<b>24.2630</b>	<b>24.0983</b>	<b>24.6203</b>	<b>23.9604</b>	<b>24.9489</b>	<b>24.5034</b>	<b>24.9382</b>
<b>PVR</b>							
Men	<b>0.9495</b>	0.9560	0.9357	0.9615	0.9234	0.9401	0.9238
M-L	<b>0.9024</b>	0.9085	0.8893	0.9138	0.8776	0.8935	0.8779
M-M	<b>0.9566</b>	0.9631	0.9427	0.9687	0.9303	0.9472	0.9307
M-H	<b>1.0154</b>	1.0223	1.0006	1.0282	0.9875	1.0054	0.9879
Women	<b>1.0640</b>	1.0713	1.0486	1.0775	1.0348	1.0536	1.0352
W-L	<b>1.0135</b>	1.0204	0.9988	1.0263	0.9857	1.0036	0.9861
W-M	<b>1.0699</b>	1.0772	1.0544	1.0834	1.0405	1.0594	1.0410
W-H	<b>1.1172</b>	1.1248	1.1010	1.1313	1.0865	1.1062	1.0869

Furthermore, we show that there are slight differences in the economic divisor values when the population consists of the same combination of level of education no matter the gender. For example, the value of economic divisor for the case of a population consisting of high-educated males and low-educated females in the system, 24.5034, is quite similar than the case of low-educated males and high-educated females, 24.6203. With the same combination of education, the value of the economic divisor is lower for the case where women have lower level of education than men due to their longer life expectancy.

As expected, the lower the value of the economic divisor is, the higher the amount of the initial pension and it leads to the higher PVR for all subgroups and vice versa. Although, the values of economic divisors vary depending on the population structure,

<sup>17</sup> Each case represents that (1) ML&WM – all men are low-educated and all women are medium-educated; (2) ML&WH – all men are low-educated and all women are high-educated; (3) MM&WL – all men are medium-educated and all women are low-educated; (4) MM&WH – all men are medium-educated and all women are high-educated; (5) MH&WL – all men are high-educated and all women are low-educated; (6) MH&WM – all men are high-educated and all women are medium-educated.

females with medium and high education benefit from the system no matter which assumptions are made to calculate the divisor.

### 4.4.3 Sensitivity analysis

This section presents a sensitivity analysis of some variables that might affect the redistribution of the NDC scheme such as the retirement age, notional rate, discount rate and indexation of pensions in payment. The results are illustrated only for the case of the unisex demographic divisor.<sup>18</sup>

- **Changes in retirement age**

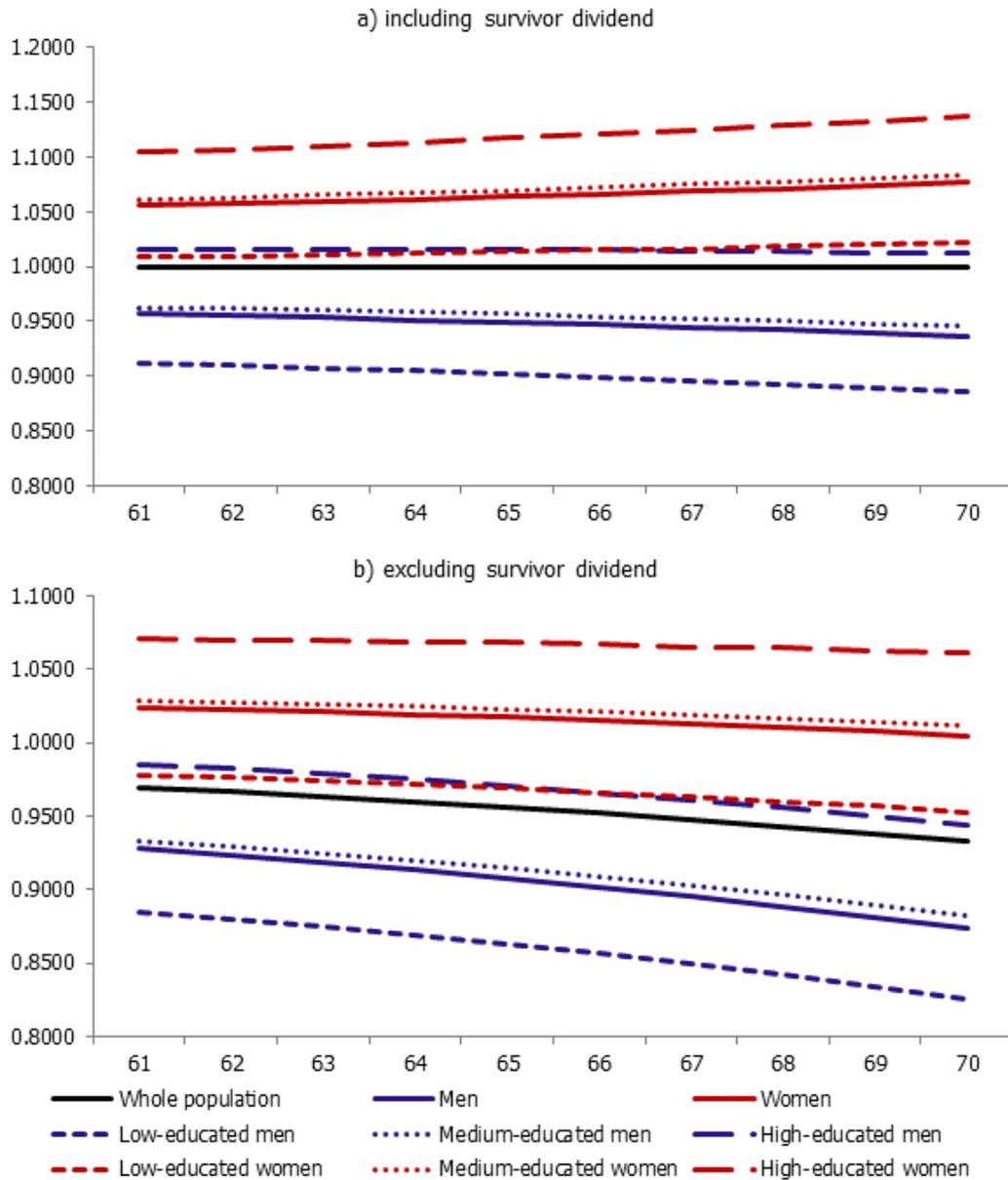
As seen in the equation (4.2), (4.3) and (4.5), when the retirement age increases the accumulated notional capital increases and the demographic annuity divisor decreases due to a shorter remaining life expectancy. As a result, the amount of the initial pension increases but the effect on the PVR is not clear. When the survivor dividend is not distributed, the PVR would decrease if the retirement age is delayed (see Figure 4.5b).

Surprisingly, when the survivor dividend is considered, Figure 4.5a, the PVRs for all subgroups of females increase with retirement age due to the higher value of the notional capital after the inclusion of the dividend. For males, we see that the higher values of the accumulated balances – resulting to the higher pensions – do not compensate the shorter lifespan of a late retirement and therefore the PVRs decrease when the retirement age increases.

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<sup>18</sup> Although the results of the sensitivity analysis using the economic divisor are different in absolute terms, the conclusions are the same as for the demographic divisor.

**Figure 4.5:** PVR when using unisex demographic divisors by different retirement ages in the case of including and excluding survivor dividend



- **Changes in notional rate**

Changes in the notional rate have a positive effect on the amount of the accumulated notional capital at retirement age and as a result there is a direct change in the amount of the initial pension. As expected, when the notional rate increases, the expected present value of pension benefits would also increase, while the expected present value

of contribution payments remains unchanged. Consequently, the PVRs increase with the (almost) same percentage for all subgroups as shown in Table 4.4 and there will be more favourable in welfare.

**Table 4.4:** PVR with different notional rates and % of relative change compared to the case of using demographic divisor under the main assumption

Sub-group	Demographic divisor	Notional rate					
		1%		2%		3%	
<b>Divisor</b>	<b>24.2630</b>	<b>24.2630</b>		<b>24.2630</b>		<b>24.2630</b>	
<b>PVR and % relative change</b>							
Men	<b>0.9495</b>	<b>0.8380</b>	<b>-11.74%</b>	<b>1.0340</b>	<b>8.91%</b>	<b>1.2891</b>	<b>35.78%</b>
M-L	<b>0.9024</b>	<b>0.7965</b>	<b>-11.74%</b>	<b>0.9828</b>	<b>8.91%</b>	<b>1.2252</b>	<b>35.78%</b>
M-M	<b>0.9566</b>	<b>0.8443</b>	<b>-11.74%</b>	<b>1.0418</b>	<b>8.91%</b>	<b>1.2988</b>	<b>35.78%</b>
M-H	<b>1.0154</b>	<b>0.8962</b>	<b>-11.74%</b>	<b>1.1058</b>	<b>8.91%</b>	<b>1.3786</b>	<b>35.78%</b>
Women	<b>1.0640</b>	<b>0.9393</b>	<b>-11.72%</b>	<b>1.1587</b>	<b>8.90%</b>	<b>1.4441</b>	<b>35.72%</b>
W-L	<b>1.0135</b>	<b>0.8947</b>	<b>-11.72%</b>	<b>1.1037</b>	<b>8.90%</b>	<b>1.3755</b>	<b>35.72%</b>
W-M	<b>1.0699</b>	<b>0.9445</b>	<b>-11.72%</b>	<b>1.1651</b>	<b>8.90%</b>	<b>1.4521</b>	<b>35.72%</b>
W-H	<b>1.1172</b>	<b>0.9862</b>	<b>-11.72%</b>	<b>1.2166</b>	<b>8.90%</b>	<b>1.5162</b>	<b>35.72%</b>

- **Changes in discount rate**

A higher discount rate will reduce the value of demographic annuity divisor and both the expected present value of contribution payments and pension benefits would decrease. This would lead to the reduction in the PVRs as exhibited in Table 4.5. The changes in the discount rate have the relatively larger effect on the pension rights of females than males and the same happens for individuals with high education relative to those who are poorly educated.

When the discount rate increases to 2%, the cross-gender redistribution declines by 7.97% for males and by 8.33% for females and considering within females, the PVR drops by 8.30% for the low-educated and 8.41% for those with high-educated. Therefore, an increase in the rate of discount brings about more welfare loss for all subgroups of population.

**Table 4.5:** PVR with different discount rates and % relative change compared to the case of using demographic divisor under the main assumption

Sub-group	Demographic divisor	Discount rate					
		1%		2%		3%	
<b>Divisor</b>	<b>24.2630</b>	<b>26.1943</b>		<b>23.0914</b>		<b>20.5127</b>	
<b>PVR and % relative change</b>							
Men	<b>0.9495</b>	<b>1.0719</b>	12.89%	<b>0.8738</b>	-7.97%	<b>0.7047</b>	-25.78%
M-L	<b>0.9024</b>	<b>1.0181</b>	12.83%	<b>0.8308</b>	-7.94%	<b>0.6706</b>	-25.69%
M-M	<b>0.9566</b>	<b>1.0805</b>	12.95%	<b>0.8801</b>	-8.00%	<b>0.7091</b>	-25.87%
M-H	<b>1.0154</b>	<b>1.1486</b>	13.12%	<b>0.9332</b>	-8.09%	<b>0.7502</b>	-26.11%
Women	<b>1.0640</b>	<b>1.2086</b>	13.59%	<b>0.9754</b>	-8.33%	<b>0.7793</b>	-26.76%
W-L	<b>1.0135</b>	<b>1.1506</b>	13.52%	<b>0.9294</b>	-8.30%	<b>0.7433</b>	-26.66%
W-M	<b>1.0699</b>	<b>1.2158</b>	13.63%	<b>0.9806</b>	-8.35%	<b>0.7830</b>	-26.81%
W-H	<b>1.1172</b>	<b>1.2708</b>	13.75%	<b>1.0232</b>	-8.41%	<b>0.8157</b>	-26.98%

- **Changes in indexation rate**

We finally present the distributional effects if we change the rate of indexation to adjust the amount of pension benefits in each year. As expected, the higher indexation rate gives the higher values of the annuity divisor and therefore the amount of the initial pension would be made to be lower. In particular, in the case that there is no indexation – the pension benefits remain constant over time – the unisex annuity divisor decreases from 24.2630 to 19.9924 as illustrated in Table 4.6.

However, we show that the PVRs do not change in the same direction for males and females, i.e. PVRs decrease for males and increase for women when indexation of pension increases. This is due to the fact that the EPV of pensions for women increases with high value of indexation since this group benefitted from having life expectancy longer than average.

**Table 4.6:** PVR with different indexation rates and % relative change compared to the case of using demographic divisor under the main assumption

Sub-group	Demographic divisor	Indexation rate								
		0%		1%		2%		3%		
<b>Divisor</b>	<b>24.2630</b>	<b>19.9924</b>		<b>22.5277</b>		<b>25.5208</b>		<b>29.0658</b>		
<b>PVR and % relative change</b>										
Men	<b>0.9495</b>	<b>0.9578</b>	0.88%	<b>0.9527</b>	0.34%	<b>0.9472</b>	-0.23%	<b>0.9415</b>	-0.84%	
M-L	<b>0.9024</b>	<b>0.9133</b>	1.21%	<b>0.9066</b>	0.47%	<b>0.8995</b>	-0.32%	<b>0.8921</b>	-1.14%	
M-M	<b>0.9566</b>	<b>0.9638</b>	0.75%	<b>0.9594</b>	0.29%	<b>0.9547</b>	-0.20%	<b>0.9497</b>	-0.72%	
M-H	<b>1.0154</b>	<b>1.0181</b>	0.27%	<b>1.0165</b>	0.11%	<b>1.0146</b>	-0.07%	<b>1.0126</b>	-0.27%	
Women	<b>1.0640</b>	<b>1.0563</b>	-0.73%	<b>1.0610</b>	-0.28%	<b>1.0661</b>	0.19%	<b>1.0713</b>	0.68%	
W-L	<b>1.0135</b>	<b>1.0091</b>	-0.44%	<b>1.0118</b>	-0.17%	<b>1.0147</b>	0.12%	<b>1.0177</b>	0.41%	
W-M	<b>1.0699</b>	<b>1.0612</b>	-0.82%	<b>1.0666</b>	-0.31%	<b>1.0722</b>	0.21%	<b>1.0782</b>	0.77%	
W-H	<b>1.1172</b>	<b>1.1043</b>	-1.15%	<b>1.1122</b>	-0.44%	<b>1.1206</b>	0.30%	<b>1.1293</b>	1.08%	

## 4.5 Conclusions

The use of gender-neutral annuity divisor in the conversion of the accumulated notional capital into a whole life annuity under the NDC pension scheme introduces an intra-generational income redistribution when heterogeneity in mortality exists in the population. This chapter examines the redistributive effects across subpopulations – differentiated by gender and educational attainment – of using the unisex demographic annuity divisor – related to the remaining life expectancy – and the alternative divisor, namely the economic annuity divisor – involving the size of pension disbursements and the pay-out period.

The value of the alternative divisor, used in Sweden to calculate the pension liabilities, depends on the composition of the population. In this respect, for a particular gender or level of education, the values of economic divisors tend to be higher if the pension system has a large size of population with low-mortality risk, i.e. females or high-educated people. Moreover, we demonstrate that when mortality differential by gender and level of education are taken into account, the economic divisors are mostly driven by the longevity effect corresponding to the gender.

The income redistribution is measured through the concept of the money's worth for participation in the pension system on a lifetime basis. As expected, the numerical illustration strongly confirms that the expected transfer of wealth away from high-mortality risk groups to low-mortality risk groups. Also, all subgroups of women and high-educated men are benefitted from the use of unisex annuity divisors – both demographic and economic divisors – as their lifespans are longer than the average life expectancy. This leads to present value ratios greater than the case for the average population.

The other consequence of using the economic divisor is that, with changes in the value of economic divisor, the amount of initial pension and the PVRs would change in the opposite direction with the same relative percentage of change for all subgroups.

In addition, the initial pension taking into account the survivor dividend would create an actuarially fair pension system – an individual will receive pension benefits identical to what she has been contributing over her working life – if there is no heterogeneity in mortality across subpopulations. Surprisingly, the survivor dividend also produces a trade-off between the PVRs for males and females at the different ages of retirement. Specifically, the PVRs for all subgroups of females increase if the retirement age is delayed due to the higher value of the notional capital distributed by more capital from the deceased participants. In contrast, the higher values of the accumulated balances for males do not compensate the shorter lifespan of a late retirement and thus the PVRs decrease when the retirement age increases. Furthermore, the distribution of survivor dividend would reduce the effect of differential mortality on the income transfer since the PVRs when including the survivor dividend for all subgroups are higher than those of without the survivor dividend.

This chapter evaluates income redistribution on a purely financial basis. One potential direction for future research would be to investigate the effect of mortality heterogeneity on the utility gains associated to the use of unisex annuity divisors. Different policy options could also be designed along the contribution-benefit lifecycle with the aim of reducing the distributional effects of the system.

# CHAPTER 5

## Non-financial Defined Contribution (NDC) Pension Schemes: Construction of Annuity Divisors Using the Direct Method<sup>1</sup>

Under the non-financial (notional) defined contribution (NDC) pension scheme, the accumulated contributions of an individual are converted into a life annuity by a so-called annuity divisor regularly taking into account life expectancy at retirement, indexation of pension and interest rate. The annuity divisor, however, is differently formulated among countries depending on the types of benefits. In particular, the Italian NDC scheme combines the old-age with the survivors' benefits in the calculation of the annuity divisor that makes the actuarial value of pension benefits related to multiple lives. This paper aims to construct the annuity divisors for the Italian NDC scheme using a simulation technique, a so-called the direct method, which works on the mean values of the variables. The direct method follows all possible trajectories and its probability is known, hence we can compute the actuarial value of annuity being paid to a pensioner and her dependents.

**Keywords:** annuity divisor, notional defined contribution, retirement, simulation, survivors' benefits, multiple state model

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<sup>1</sup> A version of the paper (joint work with María del Carmen Boado-Penas, Jacques Janssen and Raimondo Manca) was presented at the International Actuarial Association (IAA) Life Colloquium, Barcelona, Spain, October 22–23, 2017.

## 5.1 Introduction

The annuity divisor, according to the actuarial principles, is typically referred to the expected present value (also known as the actuarial value) of a life annuity payable periodically – usually monthly or annually – throughout the pensioner’s life. Although the four pioneer NDC countries share the same idea of using unisex (average) annuity divisors, these countries consider different elements in the divisor’s calculation based on the provision of pension benefits<sup>2</sup>. Specifically, Italy is the only country that combines the old-age with survivors’ benefits – involving life contingencies of more than two lives, e.g. pensioner, spouse, and children – in the computation of the annuity divisor.

After the reform in 1995, the Italian pension system has still retained the previous rules for survivors. On the death of the pensioner, his/her remaining notional balance is transferred to the family members in the form of life annuity. The surviving spouse receives 60 percent of the deceased’s entitlement. The pension amount is enlarged to 70 percent if the recipient has only one child. If there are more dependents, the pension will be allocated depending on the number of dependents, with the ceiling becoming 100 percent of the original pension. However, the method for computing the transformation coefficients created by the Italian government assumes that the insured person has only a spouse and the transformation coefficients, thus, rely on the probabilities of the survival of the pensioner and his/her spouse (Chłoń-Domińczak *et al.*, 2012).

It is commonly known that the actuarial value of the whole life annuity for a single life is always less than those actuarial values for two lives or more. Thus, if the annuity divisor considering a single life is used to determine the initial pension benefits related to multiple beneficiaries, the amount of pension payment (and therefore the liabilities of the system) will be higher than it should be. We, therefore, aim to fill in this gap

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<sup>2</sup> These NDC countries provide old-age, disability and survivors’ pensions but most of them do not include other than death of the pensioner in the calculation of the annuity divisor. In particular, Latvia, Poland and Sweden take into account the remaining future lifetime for a single life – a pensioner only, while Italy incorporates the survival probabilities of a pensioner and of his/her spouse after a pensioner’s death (Gronchi and Nistico, 2006; Chłoń-Domińczak *et al.*, 2012).

by developing a complex actuarial model for the annuity divisor of the NDC scheme that includes the old-age and all possible survivors' benefits. We also explore the differences of the annuity divisors obtained under the current Italian rule and the proposed model.

In the model, we incorporate the survivors' pensions for the widow and the orphan together with the old-age benefits in the multiple state framework, and the formulae of the expected present values of the annuity paid to each beneficiary are constructed in terms of the one-year transition probabilities. In the computation of the annuity divisors, we apply the simulation technique called the 'direct method', which works on the mean values of the variables rather than their distribution functions. The direct method follows all possible trajectories that a person would occupy during his/her lifetime, and its probability is known. The results achieved in the numerical example endorse the fact that the model does really work, and show the evolution of the longevity improvement affecting the annuity divisors.

The chapter is organised as follows; after this introduction, Section 5.2 presents the formula underlying the annuity divisors of the Italian rule. In the next section, we describe the model to value the unitary pension of the old-age together with the survivors' benefits based on the discrete time multiple state framework. Then, the details of how to construct the algorithm of the expected payments for each type of the pension recipient by using the direct method are given in Section 5.4. Afterwards, Section 5.5 shows the results of the annuity divisors obtained from the numerical example representative of the Italian rule and of the proposed model. The paper ends with the main conclusions and the possible directions for the future research.

## 5.2 The transformation coefficients of the Dini reform

The Italian state pension system was shifted from a DB PAYG to an NDC scheme in 1995 as a part of the so-called Dini reform (Law335/95) in which the pension at retirement age  $x_r$  can be calculated as the product of the accumulated notional capital

over the course of working life and the transformation coefficient<sup>3</sup>,  $TC_{x_r}$ , expressed as:

$$\text{Initial pension at age } x_r = \text{Accumulated notional capital} \times TC_{x_r} \quad (5.1)$$

This transformation coefficient, according to the actuarial rules, takes into account the life expectancy of both a pensioner and her spouse (after her death). Although the scheme provides the orphan benefits as well, the formula (5.3) is not very theoretical since it does not include the actuarial value of the annuity paid to the children.

The formulae underlying the transformation coefficient of the Dini reform for the retirement at age  $x_r$  is defined as following:

$$TC_{x_r} = \frac{1}{AD_{x_r}} \quad (5.2)$$

$$AD_{x_r} = \frac{\sum_{g=m,f} (a_{x_r,g}^{\text{old-age}} + a_{x_r,g}^{\text{survivor}})}{2} - k \quad (5.3)$$

$$a_{x_r,g}^{\text{old-age}} = \sum_{t=0}^{w-x_r} \frac{l_{x_r+t,g}}{l_{x_r,g}} \left( \frac{1+r'}{1+\lambda} \right)^{-t} \quad (5.4)$$

$$a_{x_r,g}^{\text{survivor}} = \eta \delta_g \sum_{t=0}^{w-x_r} \frac{l_{x_r+t,g}}{l_{x_r,g}} q_{x_r+t,g} \left( \frac{1+r'}{1+\lambda} \right)^{-t} \theta_{x_r+t,g} \cdot a_{x_r+t+1,g'}^{\text{spouse}} \quad (5.5)$$

$$a_{x_r+t+1,g'}^{\text{spouse}} = \sum_{\tau=1}^{w-x_r-t+\varepsilon_g} \frac{l_{x_r+t+\tau-\varepsilon_g,g'}}{l_{x_r+t+1-\varepsilon_g,g'}} \left( \frac{1+r'}{1+\lambda} \right)^{-\tau} \quad (5.6)$$

where:

$TC_{x_r}$  is the average (unisex) transformation coefficient at retirement age  $x_r$ ;

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<sup>3</sup> It is the inverse form of an annuity divisor. In this study, we focus mainly on the annuity divisor – the actuarial present value of a whole life annuity – rather than the transformation coefficient.

$AD_{x_r}$  is the average (unisex) annuity divisor at retirement age  $x_r$ , which represents the average expected present value of a lifelong annuity consisting of old-age and survivors' benefits for males and females;

$a_{x_r, g}^{\text{old-age}}$  is the expected present value of a life annuity of 1 monetary unit for a pensioner (old-age benefits) at retirement age  $x_r$  and gender  $g$  that will be paid as long as he/she is alive;

$a_{x_r, g}^{\text{survivor}}$  is the expected present value of a life annuity of 1 monetary unit for a surviving spouse (if there is any) after a pensioner's death, conditional on that a pensioner has been received the old-age benefits since retirement age  $x_r$ . The pension will be paid as long as a spouse is alive;

$k$  is the actuarial adjustment factor to take into account the frequencies of pension payments (e.g. annually, quarterly, monthly and semi-monthly) made to the pensioner each year and it is specified by law<sup>4</sup>;

$l_{x_r, g}$  is the number of people surviving at age  $x_r$  and gender  $g$ ;

$w$  is the maximum reachable age;

$\eta$  is the percentage of pension paid to the surviving spouse and in Italy it is defined at  $\eta = 0.6$ ;

$\delta_g$  is the earning-related reduction factor for the widow/widower having the survivor pension entitlement, a value of 0.9 for a male pensioner (paid to the widow) and 0.7 for a female pensioner (paid to the widower). Thus,  $\eta\delta_g$  represents the fraction of the pension paid to the widow/widower;

$\theta_{x_r, g}$  is the probability that the pensioner aged  $x_r$  and gender  $g$  got married;

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<sup>4</sup> In the Italian case, it was previously considered bimonthly with the value of  $k = 0.4231$  and since 2006 it has been payable monthly with the value of  $k = 0.4615$ .

$a_{x_r+t+1, g'}^{\text{spouse}}$  is the expected present value of a whole life annuity of 1 monetary unit paid to the surviving spouse when the age of a pensioner is  $x_r + t + 1$ . If a pensioner is gender  $g$ , then the gender of his/her spouse is  $g'$ ;

$\varepsilon_g$  is the age difference between a pensioner and his/her spouse with the value of 3 for a male pensioner and -3 for a female pensioner. This assumption is made by law;

$r'$  is the technical rate of return;

$\lambda$  is the inflation rate to adjust the amount of pension in payment;

$\frac{1+r'}{1+\lambda}$  is assumed constant over time with 1.015 and is equivalent to the long-run expected GDP growth rate (as a five-year moving average);

$g$  denotes the gender of the pensioner with  $g = \{male(m), female(f)\}$ ;

$g'$  denotes the gender of his/her spouse with  $g' = \{f \text{ if } g = m, m \text{ if } g = f\}$ ;

It is noticeable from the formula (5.3) that the annuity divisor of the unitary pension benefits is derived from two components: the expected present value of a lifetime annuity for a pensioner and the expected present value of a whole life annuity for a surviving spouse. Also, due to the gender equality, the expected present values of the unitary annuity for males and females are averaged to obtain the single value for both genders at the same age.

The transformation coefficients of the Dini reform were initially computed in 1995 (Law335/95), and should be updated every ten years, but the first revision in 2005 was not made. However, the new coefficients were established during the 2007 reform (Law247/07), and were in force between 2010–12. The coefficients are expected to be revised every three years, depending on the changes in life expectancy and the new GDP growth rates projections. According to the 2011 reform (Law214/11), the revised coefficients were in force between 2013–15, and the latest available coefficients are applicable from January 1, 2016 to December 31, 2018. Table 5.1 reports the previous and current transformation coefficients at retirement age 57–65.

**Table 5.1:** The official Italian transformation coefficients (in %)

Age	1995–2009	2010–2012	2013–2015	2016–2018
57	4.720	4.419	4.304	4.246
58	4.860	4.538	4.416	4.354
59	5.006	4.664	4.535	4.468
60	5.163	4.798	4.661	4.589
61	5.334	4.940	4.796	4.719
62	5.514	5.093	4.940	4.856
63	5.706	5.257	5.094	5.003
64	5.911	5.432	5.259	5.159
65	6.136	5.620	5.435	5.326

Source: Aben (2011), OECD (2015) and The Research Center of Itinerari Previdenziali (2017)

### 5.3 The theoretical model to value old-age and survivors' benefits

This section describes a theoretical model including both retirement and survivors' benefits. First, we describe the multiple state model related to a general pension plan. Then, the actuarial values – also called as the expected present values or the annuity divisors – of those pension benefits are then presented.

#### 5.3.1 The multiple state model for old-age combined with survivors' benefits

When a worker reaches retirement age, the pension is paid periodically throughout his/her life. In addition, upon the pensioner's death, the reversibility pension – a proportion of the original pension transferred to the survivors – will be paid to his/his family members either spouse or children. The pension transition diagram is shown in Figure 5.1. The state of being a retiree is denoted as ' $r$ ' and he/she can have survivors who are classified into three groups: i) spouse and children (complete family), ii) only spouse and iii) only children. If the death of a retiree occurs, the benefit entitlement can be changed into four different options:

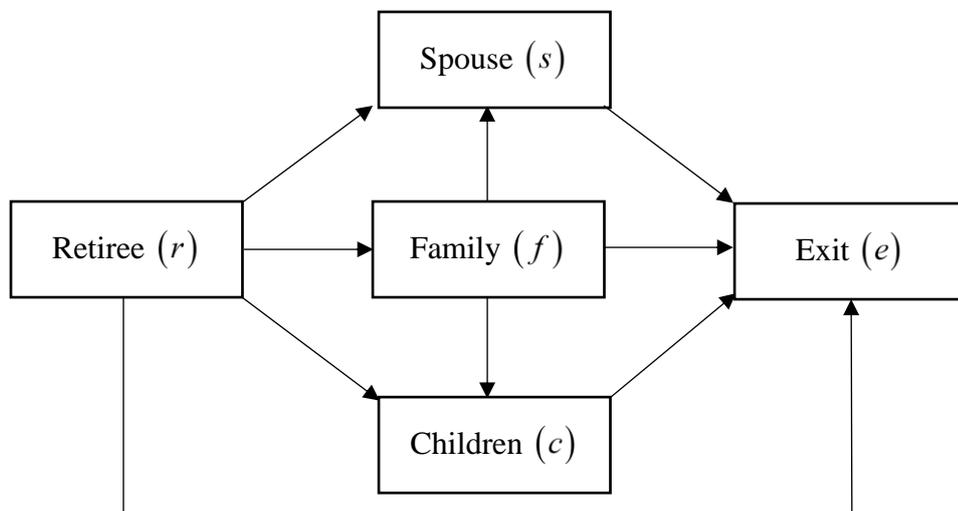
i) In case of a complete family, the reversibility pension will be paid to the surviving members of a family consisting of a spouse and the children denoted as the complete family state, ' $f$ '. The transition from the state  $r$  to  $f$  is activated if the pensioner has the surviving spouse and young children and the average age of the children does not exceed the maximum age of having the pension right.

ii) In case of only spouse, the reversibility pension will be paid to the only surviving spouse denoted as the spouse state, ' $s$ '. The transition from the state  $r$  to  $s$  is activated if the pensioner got married – the spouse is still being alive – and either he/she has no children or the average age of the children exceeds the maximum age of eligibility for the pension benefits.

iii) In case of only children, the reversibility pension will be paid to the only children denoted as the children state, ' $c$ '. The transition from the state  $r$  to  $c$  is activated if the pensioner has a family with only children and the age of children does not exceed the maximum age of having the right for the pension.

iv) No reversibility pension (the termination of the pension plan) is denoted as the exit state, ' $e$ '. The transition from the state  $r$  to  $e$  is activated if the pensioner has no family.

**Figure 5.1:** The pension transition for old-age combined with survivors' benefits



Additionally, the transition from the complete family,  $f$ , state to the spouse,  $s$ , state will occur when the children reach the maximum age of having the pension right. On the other side, the complete family,  $f$ , state will transfer to the children,  $c$ , state if the spouse dies. The survivors' benefit payments will be terminated when all children reach the maximum age of receiving the pension and the spouse dies. Then, their occupied states will be changed to the absorbing exit,  $e$ , state.

Mathematically, we consider a time-discrete stochastic process of the random state occupied by the pension recipient  $\{Y(t): t=0,1,2,\dots\}$ , with state space  $\Omega = \{r, f, s, c, e\}$ . For states  $i, j \in \Omega$  in a multiple state model and for the integer age  $x_i$  at time  $t \geq 0$ , we define

$${}_t p_{x_i}^{ij} = \Pr[Y(x_i + t) = j | Y(x_i) = i] \quad (5.7)$$

where  ${}_t p_{x_i}^{ij}$  is the probability that a person aged  $x_i$  in state  $i$  is in state  $j$  at age  $x_i + t$  and  $j$  can be equal to  $i$ .

Our model involves more than two lives including the pensioner, the spouse and the children and we assume that the future lifetime of each person is not affected in any way by the other life. For a one-year period, the relationships of the probabilities for each type of an annuitant, conditional on that he/she is in his/her own state  $i$  at age  $x_i$  are given by:

$$\text{Pensioner aged } x_r : p_{x_r}^{rr} + p_{x_r}^{rf} + p_{x_r}^{rs} + p_{x_r}^{rc} + p_{x_r}^{re} = 1 \quad (5.8)$$

$$\text{Spouse aged } x_s : p_{x_s}^{ss} + p_{x_s}^{se} = 1 \quad (5.9)$$

$$\text{Children aged } x_c : p_{x_c}^{cc} = 1 \quad (5.10)$$

We suppose the transition from state  $i$  into state  $j$  and  $i \neq j$  will occur in the middle of the year. Under the uniform distribution of the transition assumption, for an integer age  $x_i$  and for  $0 \leq a < 1$ , we can obtain

$${}_a P_{x_i}^{ij} = a \cdot p_{x_i}^{ij} \quad (5.11)$$

Thus, the mid-year transition probability for an annuitant aged  $x_i$  and  $i \neq j$  is defined as:

$${}_{1/2} P_{x_i}^{ij} = \frac{1}{2} p_{x_i}^{ij} \quad (5.12)$$

With regard to the one-year probabilities of remaining in the same state  $i$  for any age  $x_i + n$ ,  $p_{x_i+n}^{ii}$ , the following multi-year probabilities can be derived:

$${}_t P_{x_i}^{ii} = \prod_{n=0}^{t-1} p_{x_i+n}^{ii} \quad (5.13)$$

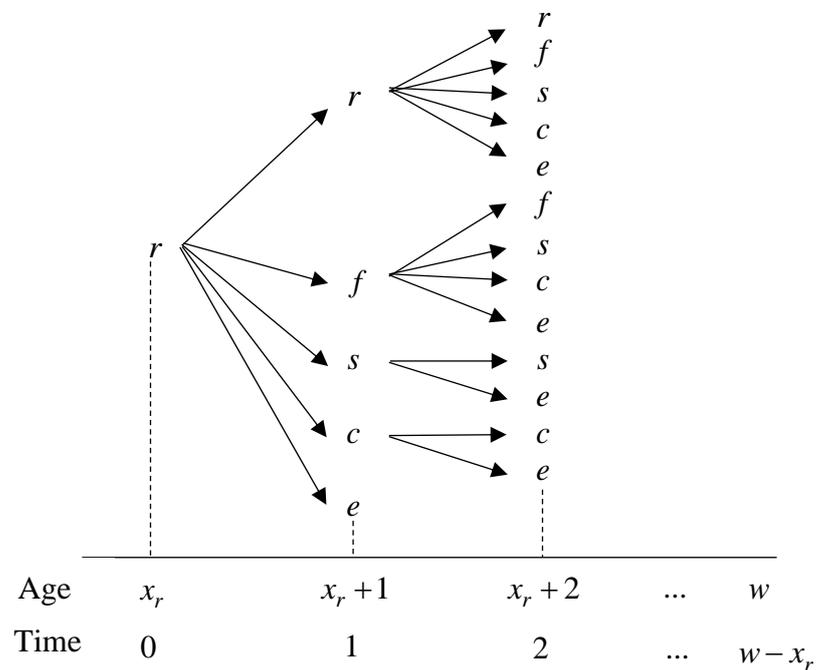
and 
$${}_0 P_{x_i}^{ii} = 1 \quad (5.14)$$

### 5.3.2 Actuarial values of unitary pension benefits

With the aim of calculating the actuarial present value of the unitary pension benefits, we initially compute the actuarial values of the life annuity for the spouse, for the children and for the complete family separately. Then, these three actuarial values are incorporated in the calculation of the annuity divisor for the retired person at a particular retirement age.

We assume that an annuity of 1 monetary unit is payable annually in the middle of the year throughout the lifetime of a retired person aged  $x_r$ . If he/she dies during the pay-out period, the reduced (reversibility) annuity, as a fraction of the basic amount of 1 monetary unit, will be paid to the surviving members of the family during their

lifetime. For the case of the surviving spouse pension, the initial payment of  $SB\%$  of the basic amount of 1 is paid in the mid-year after the pensioner's death and the life annuity of  $SB$  monetary unit per annum continues to be paid as long as the spouse aged  $x_s$  is still being alive. For the children benefits, the annuity of  $CB\%$  of the basic amount of 1 is payable annually until the children aged  $x_c$  have reached the maximum age denoted as  $M$  and we suppose that they do not die during the pay-out period. For the case of the complete family pension, the annuity of  $FB\%$  of the basic amount of 1 is paid to the family in which a spouse aged  $x_s$  is considered as a chief recipient for the benefit valuation and the payments will be terminated if either the children reach the maximum age or the spouse dies.



**Figure 5.2:** The pension benefits' trajectory tree

The time-line diagram for possible events of the benefit payments since an individual has retired at age  $x_r$  until she has reached the maximum age of surviving, denoted as  $w$ , is illustrated in Figure 5.2. The expected benefit payments are evaluated at time  $t=0$  and discounted with the fixed annual interest rate, set as  $r'$ . The expected

(actuarial) present value of the whole unitary pension benefits for a retired person aged  $x_r$ ,  $EPV_{x_r}$ , can be expressed as:

$$\begin{aligned}
 EPV_{x_r} = & \overbrace{\sum_{t=0}^{w-x_r} {}_tP_{x_r}^{rr} \left[ p_{x_r+t}^{rr} + \frac{1}{2} \left( p_{x_r+t}^{rf} + p_{x_r+t}^{rs} + p_{x_r+t}^{rc} + p_{x_r+t}^{re} \right) \right]}^{\text{The expected pension payments for a pensioner paid until the mid-year of death}} v^{(t+0.5)} \\
 & + \overbrace{\sum_{t=0}^{w-x_r} {}_tP_{x_r}^{rr} \cdot \frac{1}{2} \left( FB \cdot p_{x_r+t}^{rf} + SB \cdot p_{x_r+t}^{rs} + CB \cdot p_{x_r+t}^{rc} \right)}^{\text{The expected payments for dependents in the mid-year of a pensioner's death}} v^{(t+0.5)} \\
 & + \overbrace{\sum_{t=1}^{w-x_r} {}_{t-1}P_{x_r}^{rr} \left[ FB \cdot p_{x_r+t-1}^{rf} \cdot EPV_{x_s+t}^f \right]}^{\text{The expected pension for a family after a pensioner's death}} v^{(t+0.5)} \\
 & + \overbrace{\sum_{t=1}^{w-x_r} {}_{t-1}P_{x_r}^{rr} \left[ SB \cdot p_{x_r+t-1}^{rs} \cdot EPV_{x_s+t}^s \right]}^{\text{The expected pension for a spouse after a pensioner's death}} v^{(t+0.5)} \\
 & + \overbrace{\sum_{t=1}^{w-x_r} {}_{t-1}P_{x_r}^{rr} \left[ CB \cdot p_{x_r+t-1}^{rc} \cdot EPV_{x_c+t}^c \right]}^{\text{The expected annuities-certain for children after a pensioner's death}} v^{(t+0.5)} \tag{5.15}
 \end{aligned}$$

Note that while a pensioner is aged  $x_r$ , his/her spouse is aged  $x_s$  and the average age of children is  $x_c$ . The payments are discounted with  $v = \frac{1}{1+r'}$ . The  ${}_tP_{x_r}^{rr}$  is the probability that a pensioner aged  $x_r$  is being alive at time  $t$ . The  $p_{x_r+t}^{rf}$ ,  $p_{x_r+t}^{rs}$  and  $p_{x_r+t}^{rc}$  represents the probability that a pensioner aged  $x_r + t$  dies within one year and the benefit payments will be transferred to the family, to the spouse, and to the children, respectively. The  $p_{x_r+t}^{re}$  is the probability that a pensioner aged  $x_r + t$  dies

within one year and no payment is made to the survivors because a pensioner has no family.

After the pensioner's death, the amount of an annuity paid to his/her family, spouse and children is reduced to  $FB$ ,  $SB$  and  $CB$  as a percentage of the 1 monetary unit, respectively. In addition, the benefit cash flows will depend on the expected payments made to the pensioner's dependents, which are composed of three components: i) the expected present value of a lifelong annuity for the complete family, denoted as  $EPV_{x_s+t}^f$ , that we mainly evaluate at the spouse's age  $x_s + t$ ; ii) the expected present value of a lifelong annuity for the spouse aged  $x_s + t$ , denoted as  $EPV_{x_s+t}^s$ ; and iii) the expected present value of an annuity-certain paid to the children from age  $x_c + t$  until reaching the maximum age  $M$  of having the pension right, denoted as  $EPV_{x_c+t}^c$ . The formulae of the expected present values of the survivor pension for each type of a beneficiary plugged in the above formula (5.15) are defined as follows:

- *The case of spouse*

The expected present value of a lifetime annuity of 1 monetary unit payable annually in the middle of the year for a spouse aged  $x_s + t$  is given by

$$EPV_{x_s+t}^s = \sum_{h=0}^{w-(x_s+t)} {}_hP_{x_s+t}^{ss} \left( P_{x_s+t+h}^{ss} + \frac{1}{2} P_{x_s+t+h}^{se} \right) v^{(h+0.5)} \quad (5.16)$$

As we suppose that the transition occurs in the middle of the year, the probability of an annuity paid to a spouse aged  $x_s + t$  at time  $h = 0$  is  $P_{x_s+t}^{ss} + \frac{1}{2} P_{x_s+t}^{se}$ . At any time  $h > 0$ , the payment is made with the probability of  ${}_hP_{x_s+t}^{ss} \left( P_{x_s+t+h}^{ss} + \frac{1}{2} P_{x_s+t+h}^{se} \right)$ . The  ${}_hP_{x_s+t}^{ss}$  represents the probability that a spouse aged  $x_s + t$  survives to at least age  $x_s + t + h$  and the  $P_{x_s+t+h}^{se}$  is the probability that a spouse aged  $x_s + t + h$  dies within one year.

- *The case of children*

The annuity is paid to the children up to a given maximum age,  $M$  and during the pay-out period we assume the children are being alive – the probability of survival is set as equal to 1. Thus, the expected present value of an annuity-certain of 1 monetary unit payable annually in the mid-year for  $M - (x_c + t)$  years paid to the children aged  $x_c + t$  can be determined as

$$\begin{aligned} EPV_{x_c+t}^c &= \left( v^{0.5} + v^{1.5} + v^{2.5} + \dots + v^{M-(x_c+t)+0.5} \right) \cdot no_{x_c+t} \\ &= \dot{a}_{\overline{M-(x_c+t)}|r'} \cdot no_{x_c+t} \end{aligned} \quad (5.17)$$

We also take into account the average number of children at a given age without distinguishing the sex of the child, denoted as  $no_{x_c+t}$ , in the formula of the expected present value for the children benefits. The age of children  $x_c + t$  used in this study represents the average age of children of the pensioner aged  $x_r + t$ . The expected present value of an annuity-certain of 1 payable annually in the middle of the year for  $n$  year discounted with the rate of interest  $r'$  can be also defined in the term of the actuarial notation as  $\dot{a}_{\overline{n}|r'}$ .

- *The case of complete family*

In this case, the annuity valuation is considered until the year when the children have reached the maximum age  $M$ . The actuarial values can be divided into two parts: the first is the expected present value of the annuity before the children have reached the maximum age  $M$ , denoted as  ${}^1EPV_{x_s+t}^f$ , and the second is the expected present value of the annuity during the year when the children have reached the maximum age  $M$ , denoted as  ${}^2EPV_{x_s+t}^f$ . Since the probabilities of payments to the family depend mainly on the spouse's life contingency, we use the age of a spouse  $x_s + t$  representing the age of the family in this valuation. The actuarial value of a life annuity of 1 monetary

unit payable annually in the middle of the year for the complete family formed by the spouse and the children can be expressed as:

$$EPV_{x_s+t}^f = {}^1EPV_{x_s+t}^f + {}^2EPV_{x_s+t}^f \quad (5.18)$$

Before the children have reached the maximum age, the life annuity is paid to the family as long as the spouse survives and the transition can move to the children state,  $f \rightarrow c$ , when the spouse dies. For  $h = 0, 1, 2, \dots, (M-1) - (x_c + t)$ , the expected present value of the pension payments made to the family is given by

$$\begin{aligned} {}^1EPV_{x_s+t}^f &= \overbrace{\sum_{h=0}^{(M-1)-(x_c+t)} h P_{x_s+t}^{ff} \left[ p_{x_s+t+h}^{ff} + \frac{1}{2} p_{x_s+t+h}^{fc} \right] v^{(h+0.5)}}^{\text{The expected pension payments made to a family until a spouse's death}} \\ &+ \frac{1}{2} CB \overbrace{\sum_{h=0}^{(M-1)-(x_c+t)} h P_{x_s+t}^{ff} \cdot p_{x_s+t+h}^{fc} \cdot v^{(h+0.5)}}^{\text{The expected payments made to children at time of a spouse's death}} \\ &+ \sum_{h=1}^{(M-1)-(x_c+t)} \overbrace{CB \cdot v^{(h+0.5)} \sum_{k=0}^{h-1} k P_{x_s+t}^{ff} \cdot p_{x_s+t+k}^{fc}}^{\text{The expected pension payments made to children after a spouse's death}} \quad (5.19) \end{aligned}$$

When the children have reached the maximum age, for  $h = M - (x_c + t)$ , the expected present value of the benefit payments made to the family can be defined as

$$\begin{aligned} {}^2EPV_{x_s+t}^f &= h P_{x_s+t}^{ff} \overbrace{\left[ \frac{1}{2} p_{x_s+t+h}^{ff} + \frac{1}{4} p_{x_s+t+h}^{fc} + \frac{1}{4} CB \cdot p_{x_s+t+h}^{fc} \right.}^{\text{The expected payments made to a family in the year reaching the maximum age of the children}} \\ &\quad \left. + \frac{1}{2} SB \cdot p_{x_s+t+h}^{fs} + \frac{1}{4} SB \cdot p_{x_s+t+h}^{fe} \right] v^{(h+0.5)}} \\ &+ \frac{1}{2} CB \overbrace{\sum_{k=0}^{h-1} k P_{x_s+t}^{ff} \cdot p_{x_s+t+k}^{fc} \cdot v^{(h+0.5)}}^{\text{The expected payments made to children after a spouse's death at the first half year of reaching the maximum age of children}} \end{aligned}$$

$$+ \overbrace{SB \cdot {}_h P_{x_s+t}^{ff} \cdot p_{x_s+t+h}^{fs} \cdot EPV_{x_s+t+h}^s \cdot v^h}^{\text{The expected pension payments made to the spouse once children exit}} \quad (5.20)$$

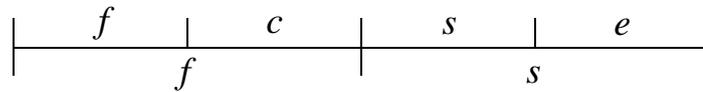
In the year  $h = M - (x_c + t)$  when the children have reached the maximum age, the payment is made to the family in the middle of the year with the probability of  ${}_h P_{x_s+t}^{ff} \left( \frac{1}{2} P_{x_s+t+h}^{ff} + \frac{1}{4} P_{x_s+t+h}^{fc} \right)$  meaning that the spouse is still alive until time  $h + \frac{1}{2}$  or dies within the second quarter after time  $h$ . After the spouse's death, the payment is transferred to the children with the probability of  $\frac{1}{4} CB \cdot {}_h P_{x_s+t}^{ff} \cdot p_{x_s+t+h}^{fc}$ . During the second half of the year, the recipient is changed from the complete family to the spouse and the payment is made with the probabilities that the spouse is alive for the half-year or die within a quarter of the year, set as  ${}_h P_{x_s+t}^{ff} \left( \frac{1}{2} SB \cdot p_{x_s+t+h}^{fs} + \frac{1}{4} SB \cdot p_{x_s+t+h}^{fe} \right)$ .

Once the children have reached the maximum age (the termination of their pension right), the subsequent disbursements are multiplied by the expected present value of the pension paid to the spouse aged  $x_s + t + h$ ,  $EPV_{x_s+t+h}^s$ , obtained in the equation (5.16) with the probability that the spouse survives from the beginning of the period up to the time when the children exit the system.

We can outline that  $p_{x_s+t+h}^{ff} = p_{x_s+t+h}^{fs}$  represents the probability that a spouse aged  $x_s + t + h$  is alive for one year. Also,  $p_{x_s+t+h}^{fc} = p_{x_s+t+h}^{fe}$  refers to the probability of transition to the children state or the termination of the payments that occurs when the spouse dies. It is equivalent to the probability of dying within one year of the spouse.

The behaviour of the last year of a complete family – the year when children have reached the maximum age  $M$  – can be displayed as Figure 5.3. For the first half of the year, the system considers that the annuity is paid to the family in the  $f$  state, and is paid to the spouse in the  $s$  state for the other half of the year. Hence, the probability of changing the state in this year can be divided into four parts: the first quarter belongs

to the  $f$  state, the second quarter moves to the  $c$  state, the third quarter transfers to the  $s$  state, and the last quarter goes to the  $e$  state.



**Figure 5.3:** Behaviour of the last year of a complete family

## 5.4 Simulation method

The expected present value of pension benefits paid to a retired person, the so-called annuity divisor (the inverse of the transformation coefficient), can be viewed as the pension liabilities – the future pension payments will be made to the retiree and his/her dependents (if provided). In the valuation of the pension liabilities, the model can be run in the different simulation methods depending on the assumptions used to generate the liabilities that can be either deterministic or stochastic. We can apply the simulation technique used in the construction of algorithms for the valuation of pension liabilities to compute the actuarial values of the unitary pension described in the previous section.

The actuarial model of pension benefits in our study is related to many random variables, especially mortality, interest rate, age of retirement and family composition. A stochastic process environment proposed by Balcer and Sahin (1983), Balcer and Sahin (1986), De Dominicis *et al.* (1991), Janssen and Manca (1997), and Janssen and Manca (1998) can simulate all aspects of the pension scheme but these models are very complicated and quite theoretical. As a result, they have not been yet totally applied in the practical way. For these reasons, the professionals usually employ the two fundamental approaches: the Monte Carlo method and the other known as the semi-deterministic model, also named the '*direct method*', to analyse the future benefit payments of the pension system.

The Monte Carlo method is commonly used to simulate the evolution of assets and liabilities of a pension scheme (see, e.g. Racinello, 1988; Chang and Cheng, 2002; Joshi and Pitt, 2010; Kryger, 2010; Müller and Wagner, 2017). This technique builds the model by substituting a range of random values from the distribution functions and deals with thousands or tens of thousands of recalculations to obtain the possible results; however, it requires a more time-consuming algorithm for a more accurate outcome.

On the contrary, the direct method works on the mean values of the variables rather than their distribution functions. In the case of the computation of the actuarial value of the unitary pension annuity, this method follows all possible trajectories that a person would occupy during his/her lifetime and its probability is known. The simulation model begins with the computation of the expected first year cost which is the product of the related pension payment and the corresponding survival probability. This cost is discounted at the beginning of the period of study,  $t = 0$ . We then compute the expected cost of the second year and discount it at time 0. In this way, these two results will be summed to obtain the mean cost of the first two years. Iterating the algorithm up to  $w$  (maximum reachable age) the total mean present value of the pension benefits will be obtained. The Monte Carlo, conversely, considers only one possible trajectory for the whole period to compute the present value of the total cost in each repetition and the calculation of the random variables given by each payment level and the related probability is repeated in a loop of a great number of times. We thus employ the direct method to compute the actuarial present values of the multiple life contingent annuities in this study since this technique is relatively simple and less time-consuming. More details of the direct method for the computation of the pension funds can be found in Volpe di Prignano and Manca (1988) and Devolder *et al.* (2012).

As the formulae of the expected present values of the pension benefits were explained in the previous section, we illustrate how to construct the algorithm path of the expected annual pension payment of 1 monetary unit paid to the retiree aged  $x_r$  in the middle of the year  $t$ , denoted as  $Y^r(x_r, t)$ . Each row of Table 5.2 represents the age of retiree and contains all pensions that the scheme will pay to the retiree of a given

age during his/her lifetime. All expected payment elements in each cell, corresponding to the same row of a specified age  $x_r$ , are discounted at time  $t = 0$  and these results are summed to obtain the actuarial value of the annuities for the pensioner who retires at age  $x_r$ .

**Table 5.2:** The algorithm path of the annual unitary pension for a pensioner

Age	Time ( $t$ )					
	0	1	2	...	$w - x_r - 1$	$w - x_r$
$\vdots$	$\dots$					
$x_r$	$Y^r(x_r, 0)$	$Y^r(x_r, 1)$	$Y^r(x_r, 2)$	$\dots$	$Y^r(x_r, w - x_r - 1)$	$Y^r(x_r, w - x_r)$
$x_r + 1$	$Y^r(x_r + 1, 0)$	$Y^r(x_r + 1, 1)$	$Y^r(x_r + 1, 2)$	$\dots$	$Y^r(x_r + 1, w - x_r - 1)$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$		
$w - 3$	$Y^r(w - 3, 0)$	$Y^r(w - 3, 1)$	$Y^r(w - 3, 2)$	$Y^r(w - 3, 3)$		
$w - 2$	$Y^r(w - 2, 0)$	$Y^r(w - 2, 1)$	$Y^r(w - 2, 2)$			
$w - 1$	$Y^r(w - 1, 0)$	$Y^r(w - 1, 1)$				
$w$	$Y^r(w, 0)$					

In the computation of the expected annual pension in each cell, we use the backward recursion by starting from the expected payment for the maximum reachable age,  $w$ . Then, the expected payment for a pensioner aged  $w - 1$  at time  $t = 1$  is related to  $Y^r(w, 0)$  with the probability that a pensioner aged  $w - 1$  has been alive for one year and is given by

$$Y^r(w - 1, 1) = p_{w-1}^{rr} \cdot Y^r(w, 0) + FB \cdot p_{w-1}^{rf} \cdot EPV_{x_s+1}^f + SB \cdot p_{w-1}^{rs} \cdot EPV_{x_s+1}^s + CB \cdot p_{w-1}^{rc} \cdot EPV_{x_c+1}^c$$

We also show some relations of the last three rows of the Table 5.2 as follows:

$$Y^r(w - 2, 1) = p_{w-2}^{rr} \cdot Y^r(w - 1, 0) + FB \cdot p_{w-2}^{rf} \cdot EPV_{x_s+1}^f + SB \cdot p_{w-2}^{rs} \cdot EPV_{x_s+1}^s + CB \cdot p_{w-2}^{rc} \cdot EPV_{x_c+1}^c$$

$$Y^r(w-2, 2) = p_{w-2}^{rr} \cdot Y^r(w-1, 1)$$

$$Y^r(w-3, 2) = p_{w-3}^{rr} \cdot Y^r(w-2, 1)$$

$$Y^r(w-3, 3) = p_{w-3}^{rr} \cdot p_{w-2}^{rr} \cdot p_{w-1}^{rr} \cdot Y^r(w, 0) = p_{w-3}^{rr} \cdot Y^r(w-2, 2)$$

Recalling the formula (5.15), the expected present value of the entire pension covering the old-age and survivor benefits for the pensioner aged  $x_r$  can be expressed in the term of the expected annual payment at time  $t$ ,  $Y^r(x, t)$  as:

$$EPV_{x_r} = \sum_{t=0}^{w-x_r} Y^r(x_r, t) \cdot v^{t+0.5} \quad (5.21)$$

For the case of the surviving spouse and the complete family, we can construct the related algorithm of the annuity by the similar way as presented in the previous case of the pensioner. Thus, the expected present values of the pension benefits paid to the spouse and the complete family defined in the equation (5.16) and (5.18) can be rewritten as:

$$EPV_{x_s+t}^s = \sum_{h=0}^{w-(x_s+t)} Y^s(x_s+t, h) \cdot v^{h+0.5} \quad (5.22)$$

and

$$EPV_{x_s+t}^f = \sum_{h=0}^{M-(x_s+t)} Y^f(x_s+t, h) \cdot v^{h+0.5} \quad (5.23)$$

where  $Y_{x_s+t}^s(x_s+t, h)$  is the expected annual survivor pension of 1 monetary unit for the spouse aged  $x_s+t$  at time  $h$  and  $Y_{x_s+t}^f(x_s+t, h)$  is the expected annual payment for the complete family at time  $h$ , conditional on that the main recipient is the spouse aged  $x_s+t$ ,  $t = 0, 1, 2, \dots, w-x_r$  and  $h = 0, 1, 2, \dots, w-(x_s+t)$ .

The expected annual pension for the pensioner, the spouse and the complete family expressed in the form of the related survival probabilities and the other triangular

matrices of all future payments to the surviving spouse and the complete family are demonstrated in Appendix D.1. We do not construct the algorithm path of the annual payment for the children because the calculation of the annuity-certain is simpler and is not associated with the survival probabilities.

Finally, the average (unisex) expected present value of the unitary pension benefits for the pensioner aged  $x_r$  is the mean value between males and females, according to the current Italian rule, defined as:

$$\overline{EPV}_{x_r} = \frac{EPV_{x_r, male} + EPV_{x_r, female}}{2} \quad (5.24)$$

## 5.5 Numerical illustration

This section presents a numerical example using Italian data to calculate the values of the annuity divisors – the actuarial (expected) present values or the inverse of the transformation coefficients – of the old-age together with survivors' pensions according to our proposed model by using the direct method described in Section 5.4. Additionally, we compare the results of the actuarial values between our proposed formulae and the Italian rules explained in Section 5.2. An additional objective of this section is to assess the changes in the value of divisors if some of the key features of the plan vary. Thus, the results are divided into two different parts: the divisors under the baseline assumptions and the changes in the features of the pension benefits.

### 5.5.1 Baseline case

The main assumptions according to the Italian rule and the data sources made for this numerical example are follows:

- The annuity divisors are calculated over the period 1995–2015.
- The retirement age,  $x_r$ , is a range from 57 to 65.

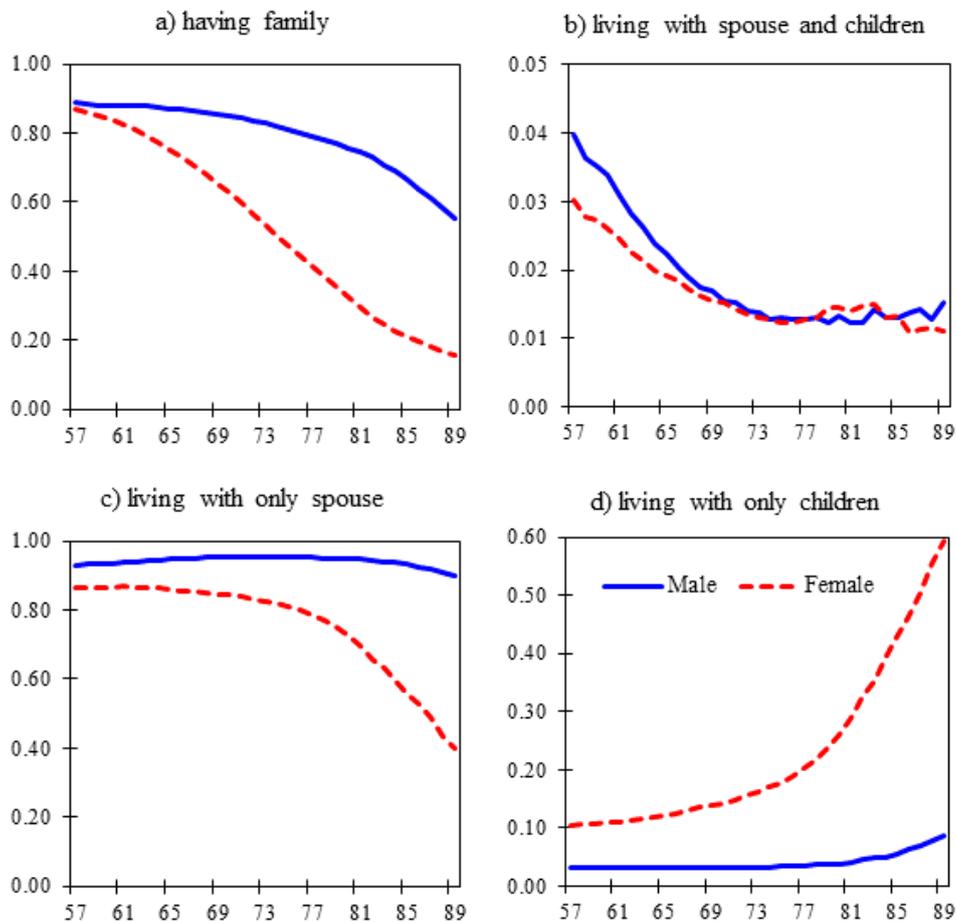
- The maximum age of surviving,  $w$ , is 110.
- The maximum age of children receiving the benefits,  $M$ , is 23.
- Average number of children is set as 1.
- The percentage of pension paid to the surviving spouse,  $SB$ , is 60% of the basic pension.
- The percentage of pension paid to the children,  $CB$ , is 70% of the basic pension.
- The percentage of pension paid to the complete family,  $FB$  is 60% of the basic pension.
- The reduction factor of the survival probability of the spouse (also representing the probability of having the right to receive the survivor benefits) is set at 0.7 for male and 0.9 for female.
- The age difference between the pensioner and his/her spouse is 3 for male and -3 for female.
- The technical interest rate,  $r'$ , is fixed at 1.5% over time.
- We use the official period life tables provided by the Italian National Institute of Statistics (ISTAT) and the transition probabilities are estimated from the rates of having a family, the rates of living with spouse only, the rates of living with children only and the rates of living with spouse and children carried out by the latest available data from the Italian 2001 Census Population and Household reported in the Eurostat Database. We summarise how to obtain the related transition probabilities as follows:
  - $p_{x_r}^{rf}$  is calculated by the probability that a pensioner aged  $x_r$  dies within one year,  $q_{x_r}$ , multiplied by the rate that a pensioner aged  $x_r$  has a family and the rate that a pensioner aged  $x_r$  lives with his/her spouse and children, conditional on having a family.

- $p_{x_r}^{rs}$  is calculated by the probability that a pensioner aged  $x_r$  dies within one year,  $q_{x_r}$ , multiplied by the rate that a pensioner aged  $x_r$  has a family and the rate that a pensioner aged  $x_r$  lives with his/her spouse only, conditional on having a family.
- $p_{x_r}^{rc}$  is calculated by the probability that a pensioner aged  $x_r$  dies within one year,  $q_{x_r}$ , multiplied by the rate that a pensioner aged  $x_r$  has a family and the rate that a pensioner aged  $x_r$  lives with his/her children only, conditional on having a family.
- $p_{x_r}^{re}$  is calculated by the probability that a pensioner aged  $x_r$  dies within one year,  $q_{x_r}$ , multiplied by the rate that a pensioner aged  $x_r$  has no a family, equivalent to one minus the rate of having a family.
- $p_{x_r}^{rr}$  ( $p_{x_s}^{ss}$ ) is the probability that a pensioner (a spouse) aged  $x_r$  ( $x_s$ ) is alive at age  $x_r + 1$  ( $x_s + 1$ ), equivalent to  $p_x$  in the life table, and the  $p_{x_s}^{ff} = p_{x_s}^{ss}$ .
- $p_{x_s}^{se}$  is the probability that a spouse aged  $x_s$  dies within one year, equivalent to  $q_x$  in the life table, and the  $p_{x_s}^{fc} = p_{x_s}^{fe} = p_{x_s}^{se}$ .

The information on the family structure that we use to estimate the transition probabilities is shown in Figure 5.4. A family is defined as a couple without children, or a couple with one or more children, or a lone parent with one or more children. Obviously, the rates of having a family are higher for men than women and gradually decrease with age, from the rate of 0.89 to 0.55 for men and 0.87 dropping to 0.15 for women (see in Figure 5.4-a). It can imply that most of elderly women are living alone due to the widowhood circumstance. Women are more likely to be widowed than men because women outlive men.

Moreover, those people who live in a family can be classified by their family status, according to the 2001 Census: Population and Household, into three groups: i) children living with their parents – a person living with his/her spouse and children, ii) husband/wife in a couple married – a person living with only spouse, and iii) children living with lone father/mother – a person living with only children. The rates of people living in the complete family with their spouse and children are relatively low, and there is a very small difference between males and females as shown in Figure 5.4-b. The rates also slightly drop with age from 0.04 to 0.015 for males and 0.03 to 0.011 for females. There are a lot of individuals living with either only spouse or only children.

**Figure 5.4:** Rates related to family structure by age and gender



Source: Own calculations based on the Italian 2001 Census: Population and Household

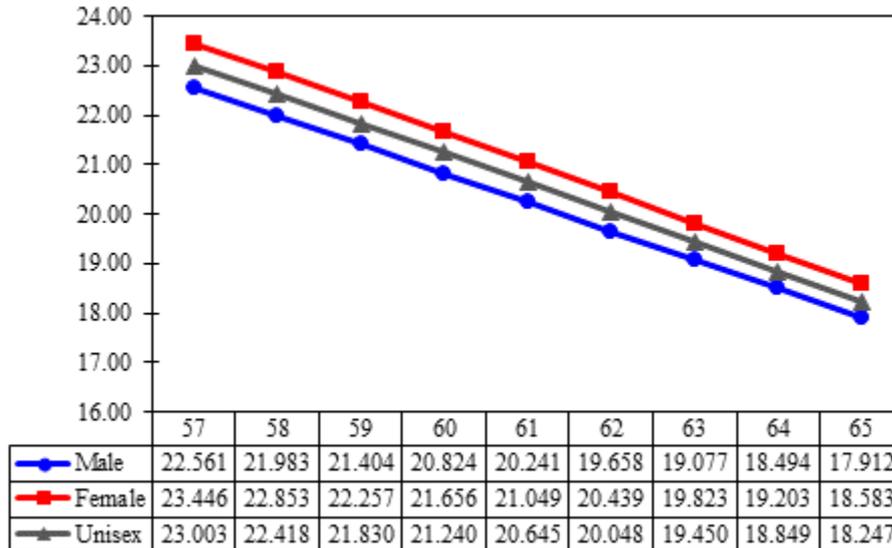
Figure 5.4-c displays the rates that the married couple stays together but without children. The husbands usually predecease their wives; hence the rates of men living with their wives are higher than the rates of women living with their husbands. There is a small decrease in the rates for men from 0.95 to 0.85, and the rates for women significantly drop with age from 0.87 to 0.28. In contrast to the rates of living with only children, females are more likely to live with her children than males. It is the consequence of the husband's death before his wife. After the father's death, the parenting will be taken by the single mother. Thus, the rates that females are living with her children are higher than those for males, and increase with age from 0.03 to 0.09 for males and 0.10 to 0.59 for females as displayed in Figure 5.4-d.

Next, we show the average (unisex) annuity divisors of the proposed model using the direct method in the calculation as exhibited in Table 5.3. The unisex annuity divisor for the pensioners in Italy is computed by averaging two divisors for males and females, see in the formula (5.24). We plot the gender- and age-specific annuity divisors comparing with the unisex value in a selected year of 2015 to illustrate the differences of the divisors among genders and ages shown in Figure 5.5. Apparently, the annuity divisors of the pension benefits for females are higher than the divisors for males – since females outlive males – and the unisex divisors are in between both genders. The annuity divisors are also highly age-linked representing that the actuarial values decrease with age because of the reduction in remaining life expectancy. At age 57, the average (unisex) annuity divisor equals to 23.003 and drops to 18.247 at age 65.

We, furthermore, illustrate the average annuity divisors from the Italian rule in Table 5.4 to compare with the results obtained from our proposed model – the direct method. Remarkably, the actuarial values computed from the Italian rule (Dini reform) are always greater than those values from our proposed formulae because the model supposes that the annuity and the transition of the payment from the pensioner to the dependents occur in the middle of the year whilst the Italian case is assumed payable annually in advance. As a consequence, the amount of the initial pension from the direct method is higher than the pension payments under the Italian rule. In addition, when we consider the different ages of retirement, the annuity divisors of both rules

decrease with age due to the decline in the survival probabilities, and in this way the workers who decide to extend their working life will receive a great amount of pension.

**Figure 5.5:** Annuity divisors obtained by the direct method, age 57-65 in 2015



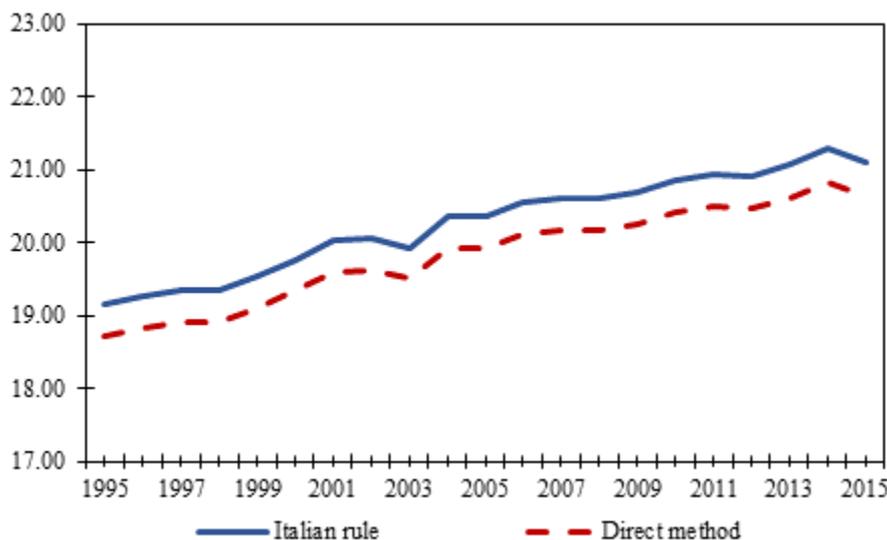
**Table 5.3:** Average annuity divisors obtained from the direct method

Year	Retirement age								
	57	58	59	60	61	62	63	64	65
1995	21.077	20.490	19.902	19.314	18.727	18.141	17.556	16.971	16.386
1996	21.182	20.595	20.007	19.419	18.834	18.248	17.660	17.074	16.491
1997	21.275	20.689	20.098	19.507	18.915	18.326	17.738	17.151	16.565
1998	21.279	20.693	20.104	19.511	18.917	18.323	17.731	17.141	16.551
1999	21.454	20.871	20.283	19.693	19.100	18.503	17.907	17.311	16.718
2000	21.683	21.103	20.518	19.928	19.335	18.739	18.142	17.546	16.951
2001	21.923	21.340	20.757	20.172	19.583	18.991	18.393	17.795	17.196
2002	21.971	21.387	20.800	20.213	19.622	19.027	18.430	17.829	17.227
2003	21.859	21.270	20.680	20.088	19.498	18.904	18.307	17.707	17.102
2004	22.283	21.696	21.107	20.519	19.928	19.340	18.749	18.154	17.555
2005	22.277	21.691	21.100	20.508	19.916	19.321	18.726	18.127	17.526
2006	22.474	21.892	21.304	20.713	20.119	19.524	18.927	18.331	17.733
2007	22.513	21.927	21.341	20.749	20.153	19.555	18.956	18.355	17.756
2008	22.531	21.948	21.360	20.771	20.177	19.577	18.976	18.374	17.770
2009	22.592	22.007	21.419	20.828	20.235	19.638	19.038	18.437	17.836
2010	22.769	22.184	21.596	21.005	20.412	19.817	19.216	18.611	18.005
2011	22.832	22.248	21.662	21.073	20.482	19.889	19.293	18.691	18.086
2012	22.833	22.248	21.658	21.066	20.472	19.878	19.281	18.683	18.078
2013	22.971	22.388	21.801	21.209	20.616	20.021	19.426	18.829	18.231
2014	23.182	22.600	22.014	21.425	20.832	20.238	19.642	19.045	18.446
2015	23.003	22.418	21.830	21.240	20.645	20.048	19.450	18.849	18.247

**Table 5.4:** Average annuity divisors obtained from the Italian rule

Year	Retirement age								
	57	58	59	60	61	62	63	64	65
1995	21.507	20.920	20.332	19.743	19.156	18.569	17.983	17.397	16.812
1996	21.613	21.026	20.438	19.850	19.265	18.678	18.089	17.502	16.918
1997	21.706	21.120	20.529	19.937	19.345	18.755	18.166	17.578	16.991
1998	21.709	21.124	20.534	19.941	19.345	18.751	18.158	17.568	16.977
1999	21.887	21.303	20.715	20.124	19.530	18.933	18.336	17.740	17.145
2000	22.118	21.538	20.952	20.361	19.768	19.171	18.573	17.977	17.381
2001	22.362	21.778	21.195	20.609	20.020	19.427	18.829	18.230	17.631
2002	22.409	21.825	21.238	20.650	20.059	19.463	18.865	18.263	17.661
2003	22.292	21.702	21.111	20.519	19.928	19.334	18.736	18.135	17.529
2004	22.724	22.137	21.548	20.959	20.368	19.780	19.188	18.592	17.993
2005	22.715	22.127	21.537	20.943	20.351	19.755	19.159	18.560	17.958
2006	22.915	22.333	21.745	21.153	20.558	19.963	19.364	18.768	18.169
2007	22.952	22.366	21.779	21.187	20.590	19.992	19.392	18.790	18.191
2008	22.969	22.386	21.798	21.208	20.613	20.013	19.410	18.808	18.203
2009	23.032	22.446	21.858	21.266	20.673	20.074	19.474	18.872	18.271
2010	23.211	22.625	22.036	21.445	20.851	20.256	19.654	19.048	18.441
2011	23.275	22.691	22.105	21.515	20.924	20.330	19.734	19.131	18.526
2012	23.275	22.689	22.100	21.507	20.913	20.318	19.720	19.121	18.515
2013	23.416	22.833	22.245	21.653	21.059	20.464	19.869	19.271	18.672
2014	23.631	23.048	22.462	21.872	21.280	20.685	20.089	19.491	18.891
2015	23.446	22.861	22.272	21.681	21.086	20.489	19.890	19.288	18.686

It is widely known that nowadays people are living longer and the probability of survival tends to expand and, therefore, leads to a continuous increase in the values of annuity divisor. As shown in Figure 5.6, we depict the annuity divisors obtained from the Italian rule versus the direct method for a selected age of 61 to present the evolution of the longevity risk during the year 1995-2015. There have been the upward tendencies in the annuity divisors for both formulae. In the case of the Italian rule, the actuarial values have increased from 19.156 in 1995 to 21.086 in 2015, accounting for 10.07 percent of surge, while the values of the annuity divisor from the direct method have been 18.55 in 1995 and 20.485 in 2015, growing up by 10.24 percent. We can also see that the annuity divisors based on the Italian rule are a bit higher than those values of the direct method by around 0.4 units. It presents that the amount of the initial pension computing from the Italian rule is less than the pension payment under our proposed plan which offers more survivors' benefits by about 2 percent when assuming that other things being equal.

**Figure 5.6:** Average annuity divisors of a selected age of 61, in 1995–2015

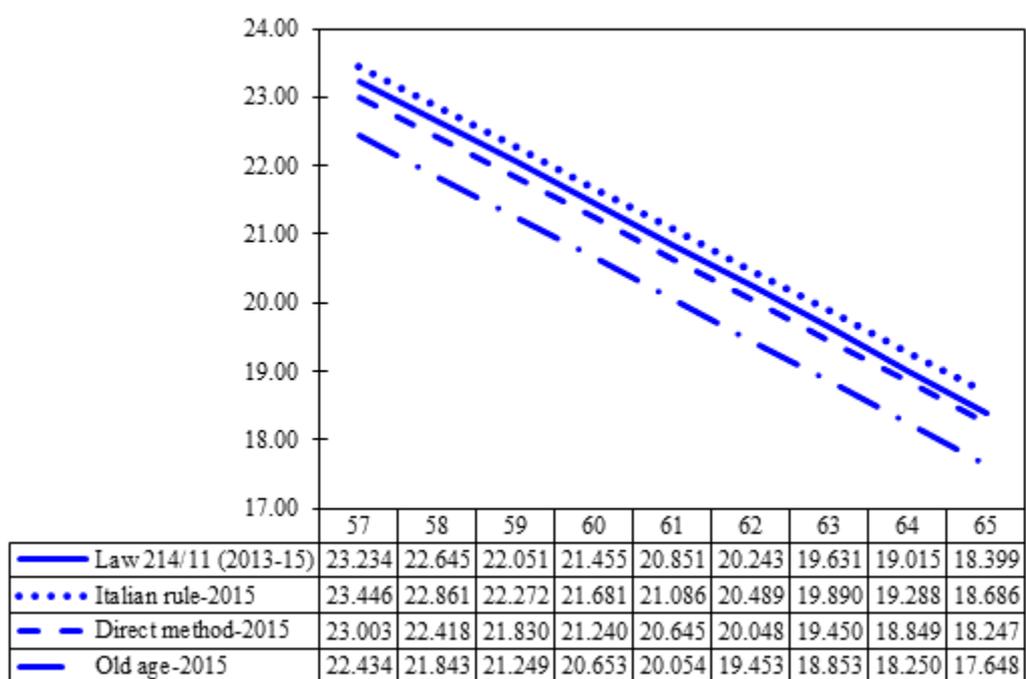
According to the transformation coefficients under the 2011 reform (Law214/11) presented in Table 5.1, we convert those coefficients into the annuity divisors, and compare them with our own calculations corresponding to the Italian rule in a selected year of 2015 as displayed in Figure 5.7. The annuity divisors followed by Law 214/11 were used to transform the accumulated contributions paid throughout working life into the initial pension for the pensioners who retired between 2013 and 2015. Clearly, the annuity divisors published by the government are lower than our own results<sup>5</sup>. Hence, the retirees have received more amount of pension than they should get. As a result, the incurred pensions by law would be higher than the pension amount with respect to our own calculation based on the Italian rule by around 1 percent in 2015.

In principle, the actuarial values of the annuity for a single life are always less than those actuarial values for multiple lives. From Figure 5.7, it is obvious that the average annuity divisors of the pension plan offering the old-age together with survivors' benefits for both cases of the Italian rule and the direct method are significantly greater

<sup>5</sup> As the transformation coefficients for the people who retired in 2013–15 reported by the law were calculated by using the life table in the earlier period – probably before 2013 – than the life table that our calculation was used – in 2015, and thus the survival probabilities of the prior year are lower than the following year – due to the effect of mortality improvement.

than the divisors of the only old-age pension<sup>6</sup> since the former case must be escalated by the weight of the continuation of the pension payment to the survivors. The Italian rule and the direct method<sup>7</sup>, for instance, give higher values of annuity divisors than the case of old-age pension in 2015, with a range of 4-6 percent (about 1 unit) and 2-3 percent (about 0.6 units), respectively.

**Figure 5.7:** Average annuity divisors from the Italian rule, the direct method and the old-age benefits in a selected year of 2015 versus the Law 214/11



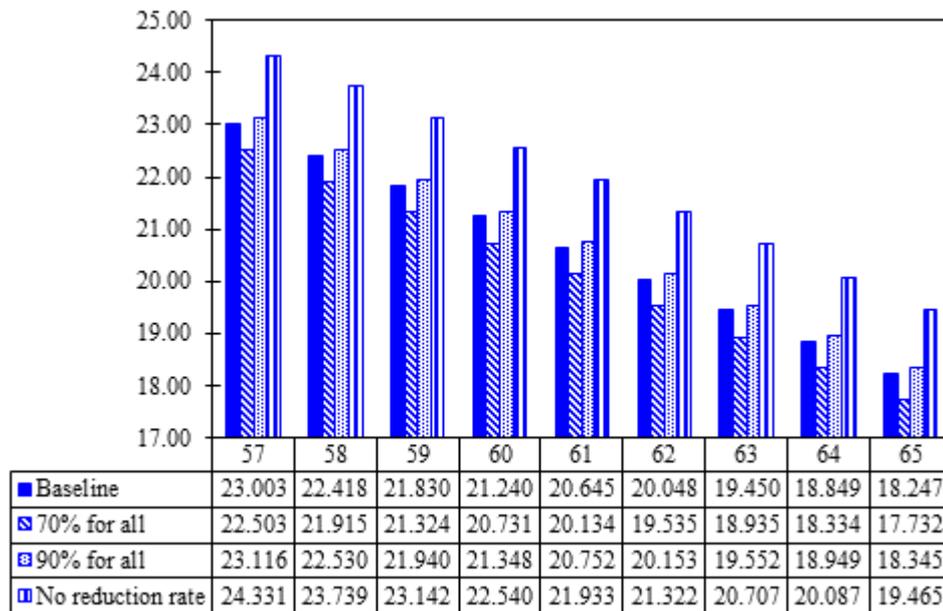
<sup>6</sup> We aim to illustrate the actuarial value of the whole life annuity-due paid to a pensioner as long as she survives, representing the divisor of the old-age pension, in order to compare with the actuarial values of the life annuity paid to the multiple lives – the old-age combined with the survivors' benefits.

<sup>7</sup> We also consider the same time point of the pension payable annually in the middle of the year for all three cases of the pension plan. The average annuity divisors of our proposed model (the direct method) are higher than those divisors of the old-age pension around 1.09 unit and just only 0.14 unit for the Italian rule at retirement age 61 in 2015, see more details in Appendix D.2.

### 5.5.2 Annuity divisors when benefit structure changes

We also examine how the size and direction of the average annuity divisors of our proposed model change with respect to the assumptions on the different features of the benefit levels. Firstly, the reduction rates of the widow benefits for each gender vary as shown in Figure 5.8. Under the baseline case, the reduction rates used to multiply the age-specific probabilities of survival of the spouse, in a similar way representing the probabilities of having the right to receive the survivor benefits, are equal to 70 percent for the widower (the spouse of the female pensioner) and 90 percent for the widow (the spouse of the male pensioner).

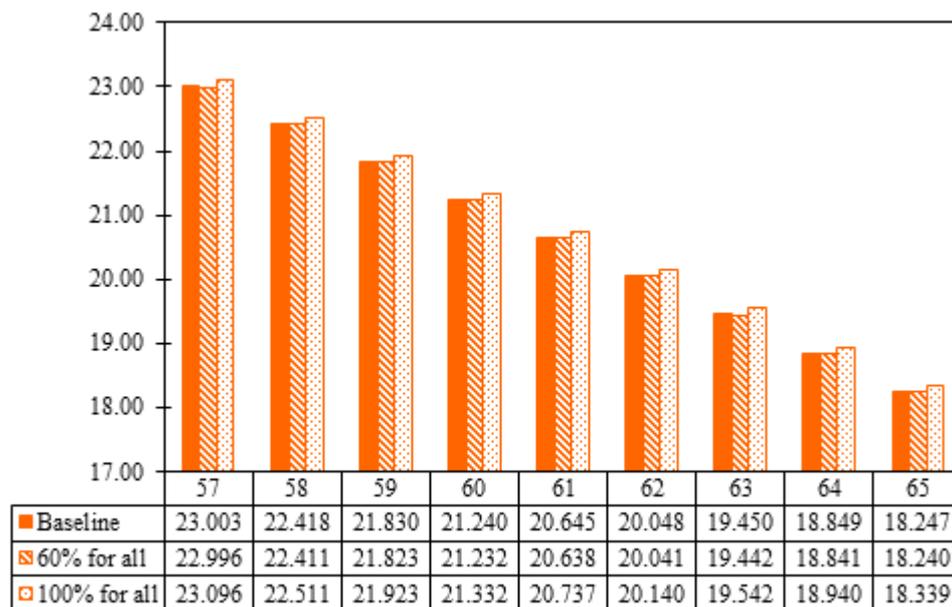
**Figure 5.8:** Average annuity divisors with different reduction rates in a selected year of 2015



If the reduction rates of receiving the widow benefits is set at 70 percent for males and females, the average divisors are slightly dropping by 0.5 units, around 2-3 percent for the retirement ages interval of 57-65, comparing to the baseline values (see Figure 5.8). With regard to the annuity divisors by gender, we found that the divisors for females remain unchanged while the values for males are significantly decreasing by approximately 1 unit, or almost 6 percent. In the case that the reduction rate is set at

90 percent for all spouses, the average annuity divisors are slightly higher than the divisors of the baseline case by approximately 0.1 units, or 0.5 percent. The annuity divisors are increasing for females only while the values for males keep constant. We also suppose that there is no reduction rate, which means that all spouses are eligible for the widow benefits once the pensioner dies. The annuity divisors are definitely increasing because the pension is expected to be paid more. The divisors are higher than the baseline case by almost 1.3 units, increasing by 6 percent. It is clear that the reduction rate crucially affects the gender-specific annuity divisors, particularly for the male pensioners.

**Figure 5.9:** Average annuity divisors with different percentages of survivor benefits in a selected year of 2015



Also, we investigate the changes in the levels of survivor benefits for the spouse, the children and the complete family as illustrated in Figure 5.9. Comparing to the baseline case where the survivors' benefits are 60 percent of the basic pension for the spouse, 70 percent for the children and 60 percent for the complete family respectively, the annuity divisors are smaller than the baseline case by around 0.007 units if all percentages of survivors' benefits are set at 60 percent. Moreover, there will be a very

small increase in the annuity divisors, by approximately 0.09 units, if the levels of the survivors' benefits for all three types of recipients are equal to 100 percent of the original pension. We can conclude that the changes in the percentages of the survivor benefits hardly influence the values of the annuity divisor.

## 5.6 Conclusions

Under the NDC pension scheme, the annuity divisor, which is mainly determined by the average life expectancy of the retiree and an assumed rate of return (an expected long-term real growth rate of the economy set by the policymaker), is used to convert the individual notional capital at retirement into the whole life annuity. However, the annuitisation divisor rules to calculate the annuity pension are constructed differently among countries. Specifically, Italy is taking into account both old-age and survivors' benefits in the calculation of the initial pension. The Italian NDC system provides the survivors' pensions for the surviving spouse and children once the retiree dies, but the official formula of the transformation coefficients considers the probabilities of paying benefits to the spouse only. Thus, this chapter develops a complex actuarial model for valuing the pension benefits covering for the retiree and all members of his/her family.

The computation of the annuity divisors in this study involves the probabilities that the pension will be paid to the retiree, the surviving spouse, the surviving children and the complete family if both spouse and children are still alive. The multiple life contingent model is complex and requires transition probabilities that we estimate from the observed data related to having family consisting of the rates of having family, the rates of living with spouse, the rates of living with children, and the rates of living together with spouse and children. We use the direct method to obtain the actuarial values since the probabilities of all possible trajectories that a person would occupy during his/her lifetime are known. This simulation technique is more flexible and less time-consuming.

In our proposed model, the unisex annuity divisors, which are derived from the mean of divisors between males and females in the Italian NDC system, decrease with age

because of the reduction in the remaining future lifetime of the retiree and his/her spouse. As a consequence, if the workers decide to extend their working life, they will receive greater amount of pension. Furthermore, the human mortality improvement over the past few decades has influenced the continuous increase in the annuity divisors, with more than 10 percent of the growth in the divisors from 1995 until 2015.

In addition, the comparative results of the annuity divisors between the Italian rule and the Law 214/11 indicate that our own calculations under the Italian rule are higher than those values published by the government. It results in the current retirees receiving 5 percent more of initial pension than they should actually receive. If the annuity divisors are underestimated, the pension liabilities of the system will increase and could be a disaster for the Italian public budget. Therefore, the government needs to revise the divisors more often to reflect the rapid changes in life expectancy and to maintain the financial sustainability.

Another main result that can be highlighted is that the annuity divisors of the pension benefits involving multiple lives are greater than divisors considering merely a single life. However, as the time of payment of the two methods – Italian rule versus direct method – are different, the divisors of our proposed model are lower than the divisors based on the Italian rule. In order to make the direct method comparable to the Italian rule, we adjust the time of payment to the middle of the year for both methods. After this adjustment, the divisors under the direct method are higher than the Italian rule by around 0.14 units. This means that the current retirees are benefitting from the Italian rule gaining approximately 1 percent more of initial pension than the corresponding value under the direct method.

The further valuable lesson to be drawn from the numerical example is transition probabilities that require to be estimated accurately before developing the model into the practice. It is really difficult to find the accurate and up-to-date data on relevant aspects of the family structure, for instance, the rates of having a family, the rates of living with either a spouse or children, the average age of children living in a family, and the average number of children per family.

In practice, the results achieved in the numerical illustration make sense and provide us with some useful guidance regarding the effect of changes in the benefits structure on the actuarial values. It confirms that our model works reasonably well and is close to the reality of the social insurance. Combining the survivors' benefits with the old-age pension in the determination of the initial pension, thus, would enhance the political attractiveness of the pension design. However, the policymaker must realise the social acceptability of the idea that the government would introduce the new higher annuity divisors along with the drop in the pension payments.

Finally, based on the model presented in this paper, we suggest a few important directions for the future research. First, the effect of longevity risk on the annuity divisors would be studied by applying the stochastic mortality model to capture the evolution of the life expectancy. The model would also be extended to consider the probability of remarriage of the widow/widower and the reduction in the spouse's pension if the survivors' benefits are his/her additional income. The last direction would be incorporating the permanent disability with old-age and survivors' benefits in the model.

## CHAPTER 6

# A Multiple State Model for Working-Age Disabled Population Using Cross-Sectional Data

A multiple state model describes the transitions of the disability risk among the states of active, disabled and dead. Ideally, the estimations of the transition probabilities and the transition intensities rely on longitudinal data; however, most of the national surveys of disability are based on cross-sectional data measuring the disabled status of an individual at one point in time. This chapter aims to propose a generic method of the estimation of the transition probabilities when the model allows recovery from disability using the UK cross-sectional data. The disability prevalence rates are modelled by taking into consideration the effect of age and time. Under some plausible assumptions concerning the probabilities of death between disabled and non-disabled people and the estimated prevalence rates of disability are used to decompose the survival probabilities in each state.

**Keywords:** cross-sectional data, disability, multiple state model, transition probabilities, working-age people

## 6.1 Introduction

Public social spending, which comprises 21% of GDP in 2016 on average across the OECD, is mostly spent on cash benefits related to old age and survivor pensions, incapacity benefits, unemployment, family cash benefits and other social benefits (OECD, 2017c). On average, cash income support for the working-age population amounted to 4.4% of GDP in 2013, consisting of 1.8% for disability benefits, 1.3% for family cash benefits, 1% for unemployment benefits and 0.3% for other social cash support (OECD, 2014). Notably, for the working-age population, the fiscal cost of incapacity benefit or disability insurance – defined as a periodic income, usually weekly or monthly, paid to an individual who is unable to work due to illness or disablement – has been increasing in several countries, such as Australia, Belgium, France, Iceland, Netherlands, and the United States, due to a substantial growth in disability beneficiary rates (OECD, 2017c).

Disability and poor health conditions lead to a decline in labour force participation. Many workers leave the labour market permanently due to health problems or disability, while there are few people with reduced work capacity who remain employed (Jones, 2008; OECD, 2010; Webber and Bjelland, 2015). Very few recipients of disability benefits return to the labour market, even if they have a significant remaining work capacity (OECD, 2009). In the late 2000s, only around 40% of disabled people in OECD countries were employed, and unemployment rates of disabled individuals doubled those of people with no disability (OECD, 2010). At the same time, the high level of unemployment among the disabled population as well as the increasing/larger number of individuals who are receiving long-term sickness and disability benefits raises serious concerns about the sustainability of the public finance of such benefits (Bell and Smith, 2004; McVicar, 2008).

A thorough understanding of the transition of an individual into and out of a disability state and the accurate estimation of the probability of becoming and remaining disabled are the essential information for the government to design the provision of a

disability benefit programme, to determine the demand for such programme and to project public expenditure on incapacity benefits.

The logical concept used to describe the transition of disability risk is commonly provided by a multiple state model<sup>1</sup> with relevant states of active (or healthy), disabled (or invalid) and dead (Haberman and Pitacco, 1999; Pitacco, 2014). Although the model typically relies on longitudinal data, most of the national surveys of disability or poor health conditions are cross-sectional data measuring the disabled/invalid status of a person at one point in time. Also, the data required for the estimation of the transition rates are often missing.

Several researchers have recently shown how to derive transition rates across active and disabled states by using disability prevalence rates from cross-sectional data in order to overcome the limited data for the use of multiple state models. Rickayzen and Walsh (2002), Leung (2004), Leung (2006) and Hariyanto *et al.* (2014) have identified the functional forms for the one-year deterioration probabilities, i.e. the probabilities of moving to any worse disability level state. The parameters for each function type have been chosen to replicate the observed prevalence rates closely while assuming of a stationary population structure<sup>2</sup>. Nuttall *et al.* (1994) suggested a multiple state model of health among the elderly considering three states – healthy, disabled and dead – with no transition from the disabled to the healthy state. The disability incidence rates were calculated from the disability prevalence rates and disabled mortality rates. By using the disability prevalence rates, Albarran *et al.* (2005) computed the probabilities of transition, of survival and of death for the ageing population under the active and disabled states. Also, they employed the annual national probabilities of death to decompose the subgroup-specific probabilities of death between disabled and non-disabled people under some plausible assumptions regarding the relative risk of mortality for each group of individuals.

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<sup>1</sup> Estimations of the transition probabilities and the intensities require the total number of transitions from one state to another (e.g. active to disabled, disabled to active, active to dead and disabled to dead), time at transition occurrences and the exposure to risk in each state.

<sup>2</sup> The age structure, mortality and birth are constant. The rate of variation between birth and mortality is also constant. Therefore, the number of population, of births and of deaths of any single age are the fraction of the number of population at an initial age (United Nations, 1968).

The abovementioned recent studies have mostly modelled disability rates among the elderly, whereas this chapter aims to investigate the evolution among working-age people. We develop a generic estimation method for calculating the transition probabilities in a one-year multiple state model based on disability prevalence rates, hence our method is an extension of Albarran *et al.*'s (2005) modelling. Our method is applied to the UK working-age population using the cross-sectional Labour Force Survey (LFS) to identify employment circumstances and disability prevalence. We then model the disability prevalence rates and the recovery rates from disability to an active state taking into consideration the effect of age and time trends.

Following this introduction, the chapter is structured as follows; Section 6.2 first describes the LFS dataset used to estimate gender- and age-specific disability prevalence and recovery rates and then reports the probabilities of death within one year provided by the Human Mortality Database. Section 6.3 describes our multiple state model and the multiple logistic regression models to estimate disability prevalence rates and the one-year recovery rates. Section 6.4 the estimated disability rates, one-year recovery rates and transition probabilities are illustrated. Lastly, Section 6.5 provides conclusions and additional comments.

## 6.2 Data description

The UK LFS is a quarterly survey of the employment circumstances of the UK working-age population, aged 16–59 for women and 16–64 for men<sup>3</sup>. This survey<sup>4</sup> contains self-reported disability data incorporating two definitions of disability: the Disability Discrimination Act<sup>5</sup> (DDA) and the work-limiting disabled. The former

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<sup>3</sup> Until April 2010, the state pension age in the UK was 60 for women and 65 for men.

<sup>4</sup> There are a few national surveys on disability in the UK. For example, in 1986 the Office of Population Censuses and Surveys (OPCS) classified disabled children and adults according to ten degrees of disability. However, this dataset is out of date and given that it was carried out only in a single year is unable to illustrate trends in disability. The Understanding Society panel, wave 1-6, 2009-2015 is one of longitudinal study covering the questions on self-reported longstanding illness or disability and activity limiting condition; however, at the time we are conducting this research only three consecutive waves have been released.

<sup>5</sup> The Disability Discrimination Act (1995) (DDA), which protects disabled people from discrimination, was repealed and replaced by the Equality Act 2010, except in Northern Ireland where this Act is still applied.

applies to any person that currently has a long-term health problem or disability and whose impairments have a substantial and long-term adverse effect<sup>6</sup> on his/her ability to undertake normal day-to-day activities<sup>7</sup>. The latter applies to any work-limiting disabled individual who has a long-term health problem or disability relating specifically to working life and whose impairments affect either the kind or amount of work he/she might do.

The LFS surveys any respondent every three months for five consecutive quarters. This allows us to have a one-year observation of transitions among the different states. The LFS provides information on the individual's labour force status, i.e. employed, unemployed or economically inactive. The overall sample size of the cross-sectional LFS dataset over the period 1999–2011 consists of 576,402 people, of which 288,576 are males aged 16–64 and 287,826 are females aged 16–59. In each dataset, we use the given person-weight variable to gross up the survey estimates to population totals. This sampling weight is based on the number of similar people in the whole population in the particular time of the survey and controlling for age and sex. We estimate the total number of people in the working-age population and the disabled population as shown in Table 6.1<sup>8</sup>.

In this paper, we link self-assessed disabled people with the labour force status – unemployed or economically inactive – as a proxy for the number of disabled individuals who are entitled to receive incapacity benefits (state ‘Disabled’ in Figure 6.1). The remaining individuals, i.e. non-disabled people and employed disabled people, act as a proxy for the number of non-recipients of disability/incapacity benefits (state ‘Active/Non-disabled’ in Figure 6.1). We use the cross-sectional dataset of each first quarter (January–March) over the period 1999–2011 to model trends in disability

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<sup>6</sup> See Equality Act 2010: Guidance on matters to be taken into account in determining questions relating to the definition of disability, Section B: Substantial, p. 14–26.

<sup>7</sup> See Equality Act 2010: Guidance on matters to be taken into account in determining questions relating to the definition of disability, Section D: Normal day-to-day activities, p. 34–47.

<sup>8</sup> The estimator for the number of individuals in the population is the sum of person-weight provided in the LFS dataset. It can be expressed as  $\hat{Y} = \sum_{j=1}^n w_j$ ; where  $w_j$  is a sampling weight for the  $j$ -th sampled individual from the population,  $j = 1, 2, \dots, n$  and  $n$  is the number of observations in the sample.

prevalence rates, while we estimate one-year recovery rates for disabled people using the status information drawn from interviews in the 1<sup>st</sup> and 5<sup>th</sup> quarter<sup>9</sup>.

**Table 6.1:** Number of disabled working-age population estimates and disability prevalence rates by gender and year 1999–2011

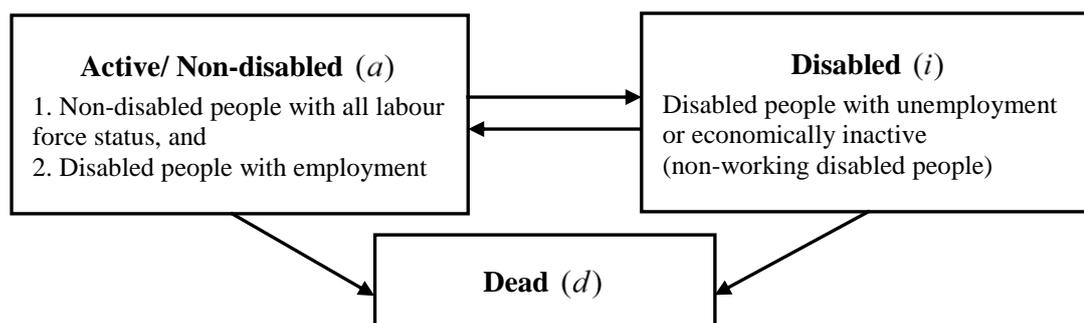
Year	Male			Female		
	Disabled Population	Total Population <sup>10</sup>	Disability rate (%)	Disabled Population	Total Population	Disability rate (%)
1999	1,776,477	18,323,457	9.6951	1,722,695	17,213,707	10.0077
2000	1,772,596	18,422,980	9.6217	1,803,039	17,327,492	10.4057
2001	1,797,411	18,556,952	9.6859	1,786,135	17,462,581	10.2284
2002	1,880,029	18,680,082	10.0643	1,802,317	17,539,222	10.2759
2003	1,778,020	18,818,881	9.4481	1,888,851	17,668,803	10.6903
2004	1,789,503	18,946,537	9.4450	1,831,128	17,798,479	10.2881
2005	1,800,498	19,145,553	9.4043	1,790,341	17,954,150	9.9717
2006	1,812,026	19,339,180	9.3697	1,825,118	18,107,852	10.0792
2007	1,849,885	19,532,406	9.4708	1,836,278	18,189,036	10.0955
2008	1,870,487	19,704,044	9.4929	1,737,931	18,256,260	9.5196
2009	1,842,044	19,814,587	9.2964	1,811,686	18,331,005	9.8832
2010	1,951,609	19,910,234	9.8020	1,851,813	18,408,702	10.0594
2011	2,088,107	19,955,266	10.4639	1,816,924	18,241,314	9.9605

Source: Own calculation based on the LFS dataset

In the following subsection, we clarify the characteristics of the datasets, including the disability prevalence rates, the recovery rates and the one-year probabilities of death of the working-age population that are used in this study.

<sup>9</sup> We merge all five-quarter longitudinal datasets over the period 1999–2011 since the sample size for disabled people within a year is relatively small. As a result, the modelling of gender- and age-specific recovery rates (explained in Section 6.2.2) ignores time effects and the rates remain constant over time.

<sup>10</sup> The total number of working-age population estimates in each year are approximately equal to the number of population estimates provided by the Human Mortality Database (HMD).

**Figure 6.1:** Three-state model of working-age people

### 6.2.1 Disability prevalence rates of the working-age population

The prevalence rate of disability at age  $x$  is computed as the total number of disabled individuals aged  $x$  divided by the total population of age  $x$ . On average, disability prevalence rates have remained quite constant (between 9–10%) for overall men and women during the period of analysis and the rates of women are higher than men (see Table 6.1).

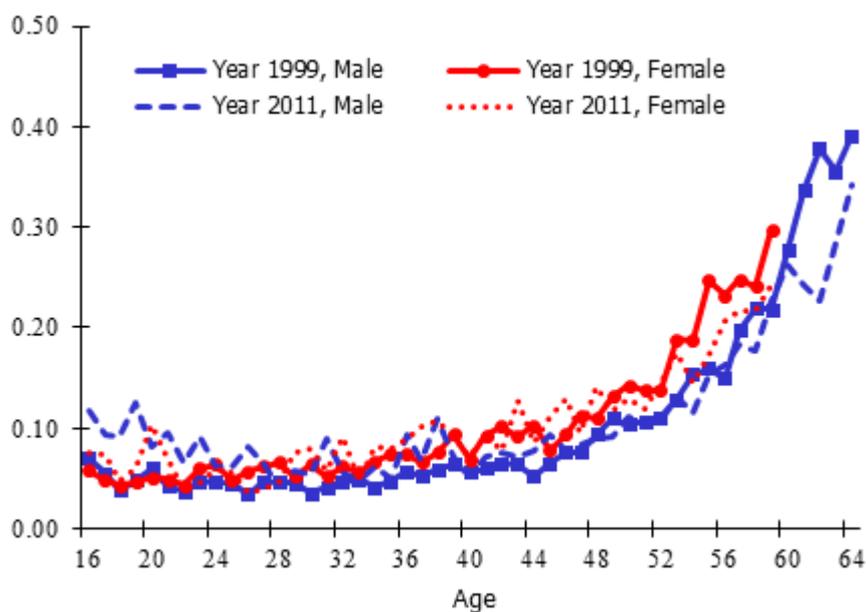
As shown in Figure 6.2, there is a noticeable age pattern in the disability rates, with lower rates among the young individuals and an increase in older ages. In early adulthood, aged 16–25, the disability prevalence rates for males are slightly larger than for females, whereas the prevalence rates tend to be larger among women during the middle age, i.e. around age 40. As a result, the disability prevalence rates are associated with age and gender. The time effect might also have an influence on changes in rates, although the trends in disability rates are quite unclear over time.

### 6.2.2 One-year recovery rates of the working-age population

The one-year recovery rate from the disabled state at age  $x$  is represented by the ratio between the total number of disabled population aged  $x$  transferring to the active state over one year and the total number of the disabled population of age  $x$ . The recovery rates, as shown in Figure 6.3, decline with age, from around 4% to 0.5% for males and

females; however, it is unclear whether there are gender differences in the recovery rates.

**Figure 6.2:** Observed disability prevalence rates by gender in 1999 and 2011



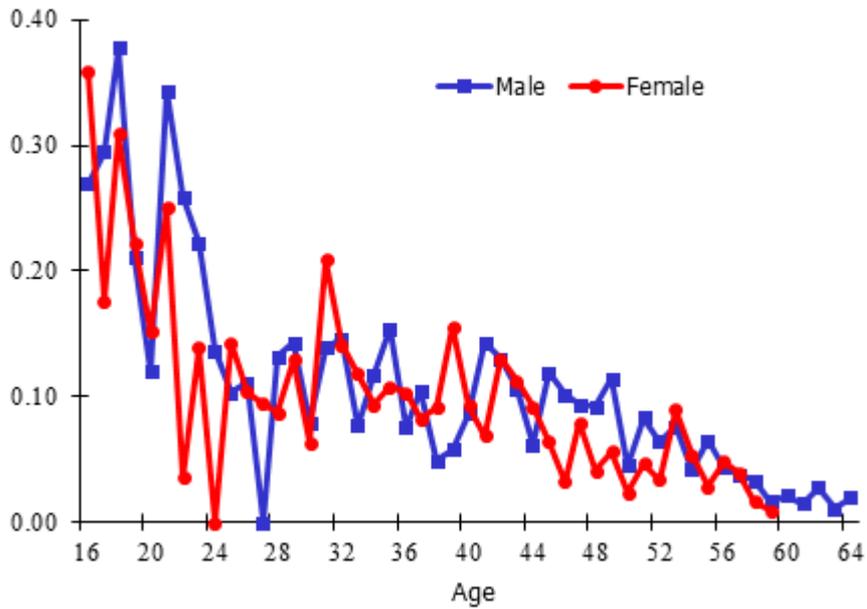
Source: Own calculation based on the LFS dataset

### 6.2.3 One-year probabilities of death

The annual probabilities of death for the whole population will be used to decompose the probabilities of death among people with and without disability under the multiple state model explained in the next section. The death probabilities for males, as shown in Figure 6.4, are higher than females while those of both sexes have dropped gradually at all ages over the period 1999–2011<sup>11</sup>.

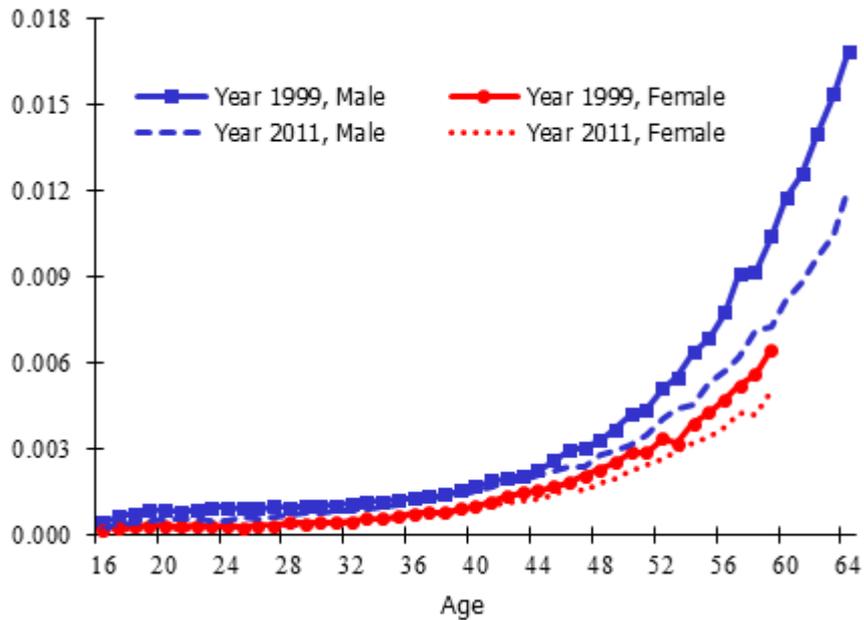
<sup>11</sup> The mortality improvement arises from economic development, progress in health technology, better access to health care services, rising living standards, improved lifestyles and a shift in the leading causes of death and illness from the infectious and parasitic diseases to non-communicable diseases and chronic conditions, especially cancers and diseases of the circulatory system (Howse, 2006; Soubotina and Shera, 2000; WHO, 2011).

**Figure 6.3:** Observed one-year recovery rates among men and women, 1999–2011



Source: Own calculation based on the LFS dataset

**Figure 6.4:** Age-specific probabilities of death ( $q_x$ ) among men and women in 1999 and 2011



Source: Human Mortality Database (HMD)

## 6.3 Model specification

This section describes a discrete-time multiple state model to compute for each of the three states, particularly the following two types of probabilities: those associated with remaining in the same state and the other related to transition between states. Our model allows for recovery from a disabled to an active state by extending Albarran *et al.*'s (2005) approach, which introduced some assumptions about the relative mortality ratio among disabled and non-disabled people to decompose the probability of death in any state.

### 6.3.1 The multiple state model for working-age disabled people

In the three-state model of disability, as shown in Figure 6.1, the possible transitions are as follows:

- i) disablement, i.e. transition from an 'active' to a 'disabled' state,  $a \Rightarrow i$ ;
- ii) recovery, i.e. transition from a 'disabled' to an 'active' state,  $i \Rightarrow a$ ;
- iii) death of an active individual, i.e. transition from an 'active' to a 'dead' state,  $a \Rightarrow d$ ;
- iv) death of a disabled person, i.e. transition from a 'disabled' to a 'dead' state,  $i \Rightarrow d$ .

The actuarial notations of one-year transition probabilities and the equations used to estimate the transition probabilities and probabilities in any state are included in the following subsections.

#### 6.3.1.1 One-year transition probabilities

We apply a discrete-time of three-state disability model in a one-year period according to Haberman and Pitacco (1999) and Pitacco (2014). We also assume that, except the possible death of an individual, no more than one transition occurs during one particular year. The fundamental relations of one-year transition probabilities related

to an active individual and a disabled individual aged  $x$  are explained in the following notations (see more details in Appendix E.1):

$$p_x^{jj} + p_x^{jk} = p_x^j \quad (6.1)$$

$$q_x^{jj} + q_x^{jk} = q_x^j \quad (6.2)$$

$$p_x^j + q_x^j = 1 \quad (6.3)$$

$$p_x^{jk} + q_x^{jk} = w_x^{jk} \quad (6.4)$$

$$p_x^{jj} + q_x^{jj} = 1 - w_x^{jk} \quad (6.5)$$

where  $p_x^{jk}$  denotes the probability that a person aged  $x$  in a state  $j$  is alive in a state  $k$  at age  $x+1$ ;

$q_x^{jk}$  denotes the probability that a person aged  $x$  in a state  $j$  dies within one year in a state  $k$ ;

$p_x^j$  denotes the probability that a person aged  $x$  in a state  $j$  is alive at age  $x+1$ ;

$q_x^j$  denotes the probability that a person aged  $x$  in a state  $j$  dies within one year;

$w_x^{jk}$  denotes the probability that a person aged  $x$  in a state  $j$  moves to a state  $k$ ;

$j, k$  represent any state of  $a = \text{active}$  and  $i = \text{disabled}$ ,  $j \neq k$ .

As we assume that there is no more than one transition occurring during one year, apart from the possible death, consequently the  $p_x^{aa}$  and  $p_x^{ii}$  represent the probabilities of remaining active and disabled, respectively, from age  $x$  to  $x+1$ .

Furthermore, the probability of becoming disabled within one year, equivalently to  $w_x^{ai}$ , is unknown while the probability of recovery from a disabled to an active state within one year ( $w_x^{ia}$ ) is represented by the estimated one-year recovery rate from the logistic regression model in Section 6.2.

### 6.3.1.2 Estimating the survival and transition probabilities

In this subsection, we explain how to estimate the transition probabilities and the probabilities of remaining in the same state over one year, thereby extending Albarran *et al.*'s (2005) approach. Due to the lack of information on the probabilities of death across subpopulation, Albarran *et al.* (2005) disaggregated the probability of dying for the national population at age  $x$  ( $q_x$ ) into the probabilities of dying for the disabled and the non-disabled people. The assumptions about the relative risk<sup>12</sup> of death among subgroups are then made to approximate the related probabilities of dying and transition between the active and the disabled population.

Initially, the probability of death for the entire population aged  $x$  ( $q_x$ ) is decomposed into the probability of death for the non-disabled people ( $q_x^a$ ) and the probability of death for the disabled people ( $q_x^i$ ) by the weighted average with the proportion of the disabled and the non-disabled population, respectively (Majer *et al.*, 2013) that can be expressed as:

$$q_x = (1 - v_x)q_x^a + v_x q_x^i = (1 - v_x)(q_x^{aa} + q_x^{ai}) + v_x(q_x^{ii} + q_x^{ia}) \quad (6.6)$$

The gender- and age-specific probabilities of death for the UK working-age population are obtained from the Human Mortality Database<sup>13</sup>. The proportion of the disabled people at any age  $x$  is measured by the age-specific prevalence rate of disability ( $v_x$ ) which is modelled by the multiple logistic regression explained in Section 6.2.

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<sup>12</sup> The relative risk (also called as risk ratio,  $RR$ ) of death is treated as the relative mortality risk of disabled people versus non-disabled people:  $RR = q_x^i / q_x^a$  where  $q_x^i$  and  $q_x^a$  denote the probability of death for the disabled and the non-disabled people at age  $x$ , respectively.

<sup>13</sup> The Human Mortality Database (2017) is available in [www.mortality.org](http://www.mortality.org).

Also, the probability that an individual aged  $x$  survives up to age  $x+1$  ( $p_x$ ) is identified as:

$$p_x = (1 - v_x)(p_x^{aa} + p_x^{ai}) + v_x(p_x^{ii} + p_x^{ia}) \quad (6.7)$$

We then make three assumptions regarding the risk ratio between the probabilities of death for the disabled and non-disabled people. The three common assumptions are defined as follows:

**Assumption 1:**  $q_x^{ai} = k_1 w_x^{ai} q_x^{ii} ; 0 < k_1 \leq 1$

**Assumption 2:**  $q_x^{ia} = k_2 w_x^{ia} q_x^{aa} ; 0 \leq k_2 \leq 1$

**Assumption 3:**  $q_x^{aa} = k_3 q_x^{ii} ; 0 < k_3 \leq 1$

According to Albarran *et al.* (2005), we follow their Assumption 1 in term of the age distribution of becoming disabled. For Assumption 2, we establish additionally the ratio between the two death probabilities of  $q_x^{ia}$  and of  $q_x^{aa}$  as a function of the age distribution of recovery. Because of the work of Albarran *et al.* (2005) focusing on the elderly, they assume the ratio between  $q_x^{aa}$  relative to  $q_x^{ii}$  is a function of age and the gap of both probabilities tends to increase with age among the old-age group. However, they point out that this may not be true for the whole population. Majer *et al.* (2013) also found that there was no significant age interaction or time trend in the Cox proportional hazard ratios between the death of the Dutch non-disabled and disabled populations, with the constant ratio of 0.54 and 0.58 for men and women, respectively. As a result, in Assumption 3, we suppose that the relative ratios for the mortality risk among working-age people with non-disability and disability are constant over age.

The probabilities of death for the disabled persons are generally higher than those for the non-disabled people, i.e.  $q_x^{ii} > q_x^{aa}$ , meaning that the relative mortality ratios between non-disabled versus disabled people are lower than 1. Also, the probabilities of dying in the different state are likely to be lower than the probabilities of death in

the same state, i.e.  $q_x^{ii} > q_x^{ai}$  and  $q_x^{aa} > q_x^{ia}$ . Therefore, the range of  $k_1$ ,  $k_2$ , and  $k_3$  is set between 0 and 1.

Next, the assumption about a stationary population is required in which the number of disabled people at age  $x+1$  is equal to the number of disabled people aged  $x$  surviving in the same disabled state at age  $x+1$  plus the number of active people aged  $x$  surviving in the disabled state at age  $x+1$ . It is expressed as follows:

$$v_{x+1}P_x = v_x P_x^{ii} + (1-v_x)P_x^{ai} \quad (6.8)$$

Substituting the expression (6.5) and Assumption 1 in the above expression (6.8), we obtain the relationship:

$$v_{x+1}(1-q_x) = v_x(1-w_x^{ia} - q_x^{ii}) + (1-v_x)(w_x^{ai} - k_1 w_x^{ai} q_x^{ii}) \quad (6.9)$$

which yields the probability of becoming disabled between age  $x$  and  $x+1$ :

$$w_x^{ai} = \frac{v_{x+1}(1-q_x) - v_x(1-w_x^{ia} - q_x^{ii})}{(1-v_x)(1-k_1 q_x^{ii})} \quad (6.10)$$

Substituting Assumption 1, 2 and 3 in the expression (6.6), the probability that a disabled person aged  $x$  dies within one year while he/she is still being disabled is

$$q_x^{ii} = \frac{q_x}{(1-v_x)k_3 + (1-v_x)k_1 w_x^{ai} + v_x + v_x k_2 k_3 w_x^{ia}} \quad (6.11)$$

Finally, replacing the  $w_x^{ai}$  from the expression (6.10) into (6.11), a quadratic equation of  $q_x^{ii}$  can be expressed as

$$A(q_x^{ii})^2 + Bq_x^{ii} - q_x = 0 \quad (6.12)$$

where 
$$A = -k_1 k_3 \left[ (1 - v_x) + v_x k_1 k_2 w_x^{ia} \right]$$

and 
$$B = k_3 \left( 1 - v_x + v_x k_2 w_x^{ia} \right) + k_1 v_{x+1} (1 - q_x) + k_1 \left( q_x - v_x + v_x w_x^{ia} \right) + v_x$$

The equation admits two real positive solutions. However, we choose the unique solution that lies in the (0, 1) interval, as shown in Appendix E.2.

Substituting the known values of  $q_x$ ,  $v_x$  and  $w_x^{ia}$  with the different values of  $k_1$ ,  $k_2$  and  $k_3$  in the solution of equation (6.12), we obtain the probability that a disabled person dies while he/she is in a disabled state ( $q_x^{ii}$ ). We then compute the probability of becoming disabled ( $w_x^{ai}$ ) following the equation (6.10) and the probability of death in any state  $q_x^{ai}$ ,  $q_x^{ia}$  and  $q_x^{aa}$  from Assumption 1, 2 and 3.

### 6.3.2 Multiple logistic regression model

The LFS dataset contain a binary outcome indicator of the disability event occurrence (i.e. active or disabled status) and of the case of recovery from a disabled to an active status (i.e. non-recovery or recovery status). The logistic regression is a popular model for the binary dependent variables that allows us to estimate the probability of the event of interest (De Jong and Heller, 2008; Frees, 2009; Hosmer *et al.*, 2013; Guillen, 2014).

We apply the logistic regression models to capture the occurrence of disability and recovery events separately for males and females by taking into account the effect of age as a polynomial function (Renshaw and Haberman, 1995; Fong *et al.*, 2015) and of time trends (Renshaw and Haberman, 2000). The estimations of disability prevalence rates and one-year recovery rates are explained in the following subsection.

### 6.3.2.1 Estimating the disability prevalence rates, $v_x$

The logistic regression with age and time trend as the predictor variables is applied for estimating the gender- and age-specific disability prevalence rates of the working-age population in each calendar year over the period 1999–2011. The binary outcome of the event that the  $n$ -th person is being disabled,  $y_n$ , is defined as follows:

$$y_n = \begin{cases} 1 & \text{if the } n\text{-th person is disabled with probability } v_n \\ 0 & \text{if the } n\text{-th person is non-disabled with probability } 1-v_n \end{cases}$$

The logistic regression model of disability prevalence rates is based on a polynomial of age with degree 4 and a time trend, as shown in the next equation below<sup>14</sup>. The model is analysed for males and females separately.

The logistic regression of estimation disability prevalence rates is defined as

$$\text{logit}(v_{n,t}) = \ln\left(\frac{v_{n,t}}{1-v_{n,t}}\right) = \alpha + \beta_1 \text{age} + \beta_2 \text{age}^2 + \beta_3 \text{age}^3 + \beta_4 \text{age}^4 + \beta_5 t \quad (6.13)$$

where  $v_{n,t}$  represents the disability prevalence rate of the  $n$ -th person in calendar year  $t$ ,  $\text{age}$  denotes the age of an individual and  $t$  is the calendar year, i.e. 0, 1, ..., 12 corresponding to the year between 1999–2011.

The fitted gender- and age-specific disability prevalence rate at age  $x$  in each calendar year over the period 1999–2011 is expressed as follows:

$$\hat{v}_{x,t} = \frac{\exp(\hat{\alpha} + \hat{\beta}_1 \text{age} + \hat{\beta}_2 \text{age}^2 + \hat{\beta}_3 \text{age}^3 + \hat{\beta}_4 \text{age}^4 + \hat{\beta}_5 t)}{1 + \exp(\hat{\alpha} + \hat{\beta}_1 \text{age} + \hat{\beta}_2 \text{age}^2 + \hat{\beta}_3 \text{age}^3 + \hat{\beta}_4 \text{age}^4 + \hat{\beta}_5 t)} \quad (6.14)$$

<sup>14</sup> Different polynomial forms have been carried out but we only show results for the best fit to the data, see the model selection in Appendix E.3.

### 6.3.2.2 Estimating the one-year recovery rates, $w_x^{ia}$

The binary outcome of the event that the  $n$ -th disabled person recovers to an active state over a one-year period,  $z_n$ , is defined as follows:

$$z_n = \begin{cases} 1 & \text{if the } n\text{-th disabled person recovers to an active state with probability } w_n^{ia} \\ 0 & \text{if the } n\text{-th disabled person is still disabled with probability } 1-w_n^{ia} \end{cases}$$

The logistic regression model is explained by gender and a quadratic function of age using the following equation<sup>15</sup>:

$$\text{logit}(w_n^{ia}) = \ln\left(\frac{w_n^{ia}}{1-w_n^{ia}}\right) = \alpha + \beta_1 \text{gender} + \beta_2 \text{age}^2 \quad (6.15)$$

where *gender* is a dummy variable with 1 for males and 2 for females and *age* is the age of the  $n$ -th disabled individual.

The constant estimated gender-specific one-year recovery rate at age  $x$  over the period 1999–2011 is expressed as

$$\hat{w}_x^{ia} = \frac{\exp(\hat{\alpha} + \hat{\beta}_1 \text{gender} + \hat{\beta}_2 \text{age}^2)}{1 + \exp(\hat{\alpha} + \hat{\beta}_1 \text{gender} + \hat{\beta}_2 \text{age}^2)} \quad (6.16)$$

The logistic regression of both equations (6.13) and (6.15) are fitted to the data by using maximum likelihood methods to obtain the estimates of parameters, i.e. the intercept ( $\alpha$ ) and coefficients ( $\beta$ ). Then, we compute the fitted disability prevalence rates from the equation (6.14) and the fitted one-year recovery rates from the equation (6.16) by substituting the estimated parameters  $\alpha$  and  $\beta$ , which are illustrated in the following section.

<sup>15</sup> Different polynomial forms have been carried out but the model with gender and degree 2 of age as predictors is the best fit to the data, see the model selection in Appendix E.4.

## 6.4 Results

In this section, we discuss the results of the estimated gender- and age-specific disability prevalence rates and the estimated one-year recovery rates over the period of 1999–2011. All estimated rates are included in the one-year multiple state model to generate the transition probabilities in each state, i.e. active, disabled and dead. The probabilities of death among each state are also computed, based on the assumptions about the relative ratio of mortality between the disabled and non-disabled people.

### 6.4.1 The estimated disability prevalence rates, $v_x$

The results of the estimates for the unknown parameters for men and women are shown in Table 6.2. In the logistic regression model, parameters are interpreted in terms of logit rather than directly in the response variable. Then, these estimated parameters are calculated following the equation (6.14) to produce the fitted disability prevalence rates by age and gender in each calendar year.

As shown in Table 6.2, all parameters are statistically significant at the 90% confidence level and both models give the lowest AIC values (see Appendix E.3 for the six model variants). The prevalence rates of disability rise with age and are higher for women than men. Conversely, the young men aged 18–27 have higher disability prevalence rates than the young women. The trends in disability rates among men and women have dropped slightly over time due to the negative value of the coefficient of  $t$ .

In Figure 6.5, the fitted disability rates of young men aged 16–28 in 1999 decreased from 0.0795 to 0.0483, whereas the rates of men aged 29–64 increased from 0.0486 to 0.3723. For females, the fitted rates were lower, i.e. ranging between 0.0565–0.2808 for the age interval 16–59. The disability rates in 2011 slightly decreased from 1999 for both sexes. The fitted rates have the same trend as the disability living allowance – the social welfare for the UK disabled people – claimant rates. However, for all age groups the fitted rates from our model produce higher values than the rates of claimants

since beneficiaries who are entitled to receive benefits have to meet strict conditions<sup>16</sup>. As a result, the number of benefit recipients is likely to be lower than the number of people with self-reported disability.

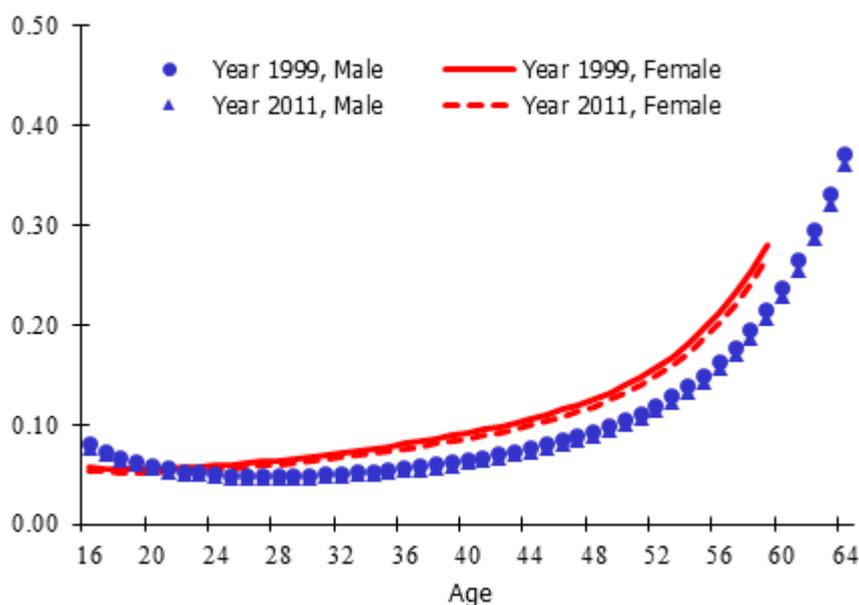
**Table 6.2:** The logistic regression model of disability prevalence rates

Parameter	Estimate	Std.Error	z value	Pr >  z
<b>Male</b>				
$\alpha$ (intercept)	1.9380	0.5292	3.6600	0.0000***
$\beta_1$ (age)	-0.4964	0.0615	-8.0700	0.0000***
$\beta_2$ (age <sup>2</sup> )	0.0179	0.0025	7.2000	0.0000***
$\beta_3$ (age <sup>3</sup> /1000)	-0.2790	0.0421	-6.6300	0.0000***
$\beta_4$ (age <sup>4</sup> /10000)	0.0174	0.0025	6.8200	0.0000***
$\beta_5$ (t)	-0.0038	0.0019	-2.0400	0.0420**
<b>Female</b>				
$\alpha$ (intercept)	-1.1506	0.6586	-1.7500	0.0810*
$\beta_1$ (age)	-0.2339	0.0792	-2.9500	0.0030***
$\beta_2$ (age <sup>2</sup> )	0.0111	0.0034	3.3000	0.0010***
$\beta_3$ (age <sup>3</sup> /1000)	-0.2000	0.0001	-3.4800	0.0000***
$\beta_4$ (age <sup>4</sup> /10000)	0.0154	0.0004	3.9400	0.0000***
$\beta_5$ (t)	-0.0057	0.0018	-3.1200	0.0020**

Source: Own illustration using the LFS dataset and the logistic regression in equation (6.13)

Note: \* $p$  – value < 0.10; \*\* $p$  – value < 0.05; \*\*\* $p$  – value < 0.01

<sup>16</sup> See the rates of claimants receiving Disability Living Allowance (DLA) from the Department for Work & Pensions (DWP) statistics tabulation tool over the period 2002-2011. The values of the disability rates differ due to the different nature of both datasets, i.e. while LFS is a self-assessed dataset, the claimants of DWP are examined by an independent healthcare professional. Also, under the DWP, the claimants must have a long-term health condition or disability and face difficulties with ‘daily living’ or getting around. These difficulties must be longer than 3 months and are expected to last at least 9 months.

**Figure 6.5:** Estimated disability prevalence rates by age and gender in 1999 and 2011

Source: Own illustration from the logistic regression model in equation (6.14)

### 6.4.2 The estimated one-year recovery rates, $w_x^{ia}$

As shown in Table 6.3, the variables age and gender are statistically significant to model the one-year recovery rates for disabled people and the model gives the lowest AIC values (see Appendix E.4 for the six model variants). We can see that the one-year recovery rates gradually decrease with the quadratic form of ages. The coefficient of gender variable is negative, which means that more men recover their health and get a job during the course of one year than women. This is not surprising since most women with disabilities encounter barriers in entering the labour market and often experience employment disadvantages, such as inequality in hiring, promotion standards and payment (O'Reilly, 2007). As a result, many women do not desire to return to work.

The fitted recovery rates over one year, as shown in Figure 6.6, decrease from 0.2071 to 0.0194 for men and from 0.1707 to 0.0230 for women. Our fitted rates are consistent with the one-year claim duration recovery rates provided by the Society of Actuaries: 2008 Long Term Disability Experience Study Report. The report gathers and analyses

historical industry data on long-term disability claims of the US insurance companies between 1997 and 2006. The analysis shows that, on average, the rates decreased with increasing age, 0.1973 for under the age of 25, and dropped to 0.0306 for the 60–64 age band. The fitted rates from our model for males and females are approximate to these experience rates.

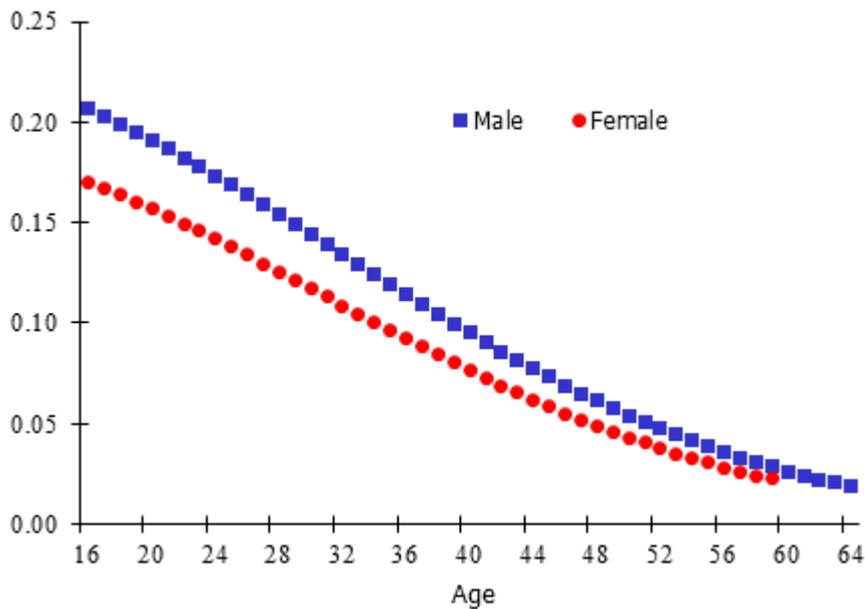
**Table 6.3:** The logistic regression model of one-year recovery rates

Parameter	Estimate	Std.Error	z value	Pr >  z
$\alpha$ (intercept)	-1.1705	0.1071	-10.9300	0.0000***
$\beta_1$ (gender)	-0.2379	0.0959	-2.4800	0.0130**
$\beta_2$ (age <sup>2</sup> )	-0.0007	0.0000	-15.7100	0.0000***

Source: Own illustration using the LFS dataset and the logistic regression in equation (6.15)

Note: \* $p$ -value < 0.10; \*\* $p$ -value < 0.05; \*\*\* $p$ -value < 0.01

**Figure 6.6:** Estimated one-year recovery rates by age and gender over 1999–2011



Source: Own illustration from the logistic regression model in equation (6.16)

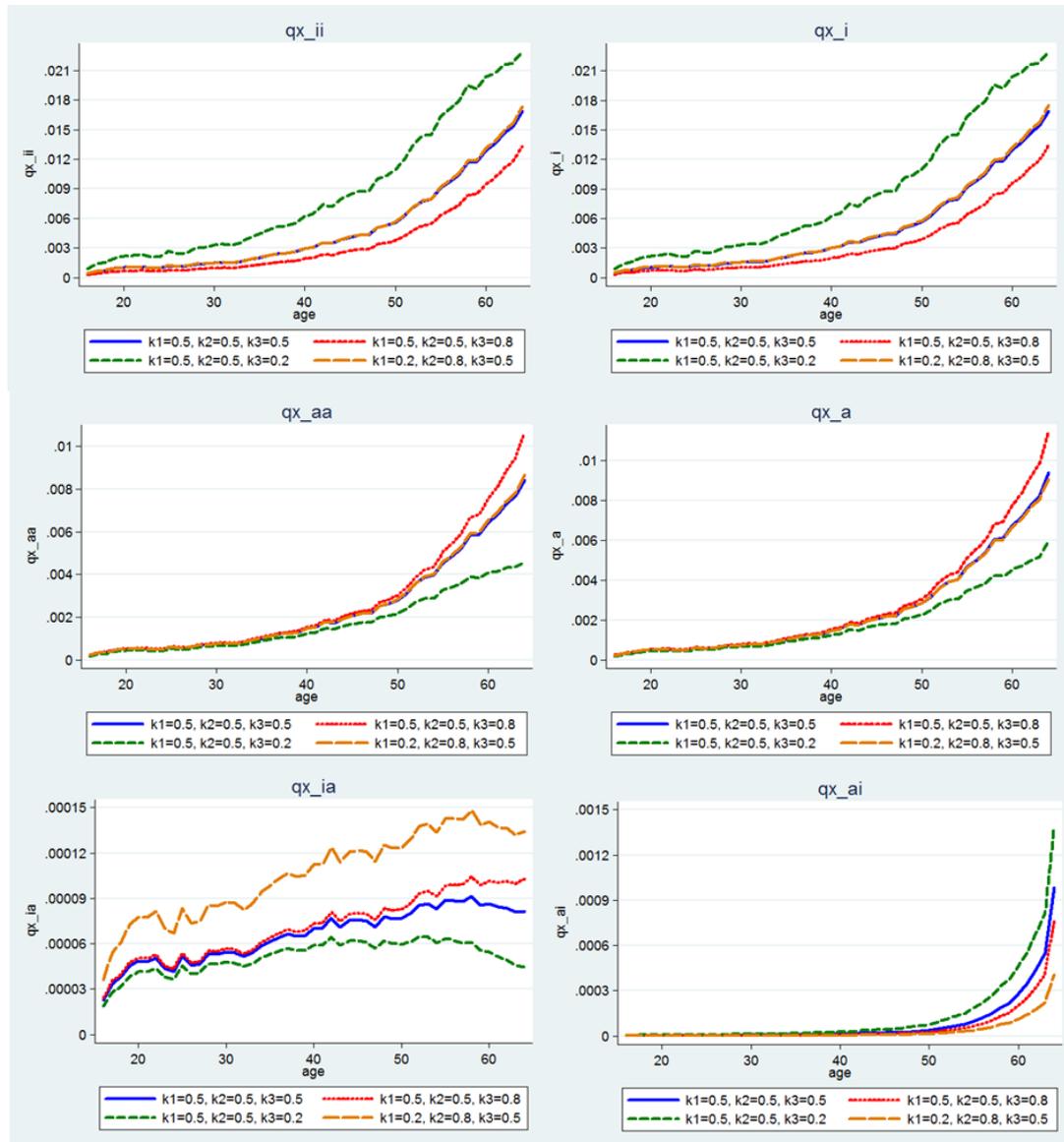
### 6.4.3 The estimated annual survival and transition probabilities

Taking into account the one-year multiple state model to estimate the transition probabilities and the probabilities of remaining in the same state, we employ the annual probabilities of death for the general population ( $q_x$ ) obtained from the HMD and the estimated disability prevalence rates ( $v_x$ ) and one-year recovery rates ( $w_x^{ia}$ ) from the logistic regression models. Then, we calculate the probabilities of becoming disabled ( $w_x^{ai}$ ), the probabilities of death in any state ( $q_x^{ai}, q_x^{ia}, q_x^{ii}$  and  $q_x^{aa}$ ) and the probabilities of remaining in the same state ( $p_x^{ii}$  and  $p_x^{aa}$ ). In this paper, we illustrate only a selected year of 2011, to perform the results in accordance with the various values of  $k_1, k_2$  and  $k_3$  from Assumption 1, 2 and 3.

The probabilities of death in any state with the different values of  $k_1 = k_2 = k_3$  and  $k_1 \neq k_2 \neq k_3$  are shown in Figure 6.7. The graphic of the probabilities of disabled people aged  $x$  dying in a disabled state ( $q_x^{ii}$ ) shows that the value of  $k_3$ , which is equivalent to the relative mortality risk ratio between the non-disabled and disabled people, is negatively correlated with the value of  $q_x^{ii}$ . The lower value of  $k_3$  produces the higher value of  $q_x^{ii}$ ; for example, the value of  $q_x^{ii}$  under the scenario of the lowest  $k_3 = 0.2$  together with  $k_1 = 0.5$  and  $k_2 = 0.5$  is higher than the other scenarios. Conversely, there is a positive relationship between the value of  $k_3$  and the probability of death of a non-disabled person. The higher value of  $k_3$  gives the lower  $q_x^{aa}$ .

In the case of changes in the values of  $k_1$  and  $k_2$  while keeping the same value of  $k_3$  at 0.5, the values of  $q_x^{ii}$  and  $q_x^{aa}$  are almost unchanged; for example, the results given  $k_1 = 0.5, k_2 = 0.5, k_3 = 0.5$  against  $k_1 = 0.2, k_2 = 0.8, k_3 = 0.5$  are very similar. Thus, the  $k_1$  and  $k_2$  have very minor effect on the probabilities of death in the same state  $q_x^{ii}$  and  $q_x^{aa}$ .

**Figure 6.7:** Estimates of annual probabilities of death with any various scenarios of  $k_1, k_2$  and  $k_3$  for males in 2011



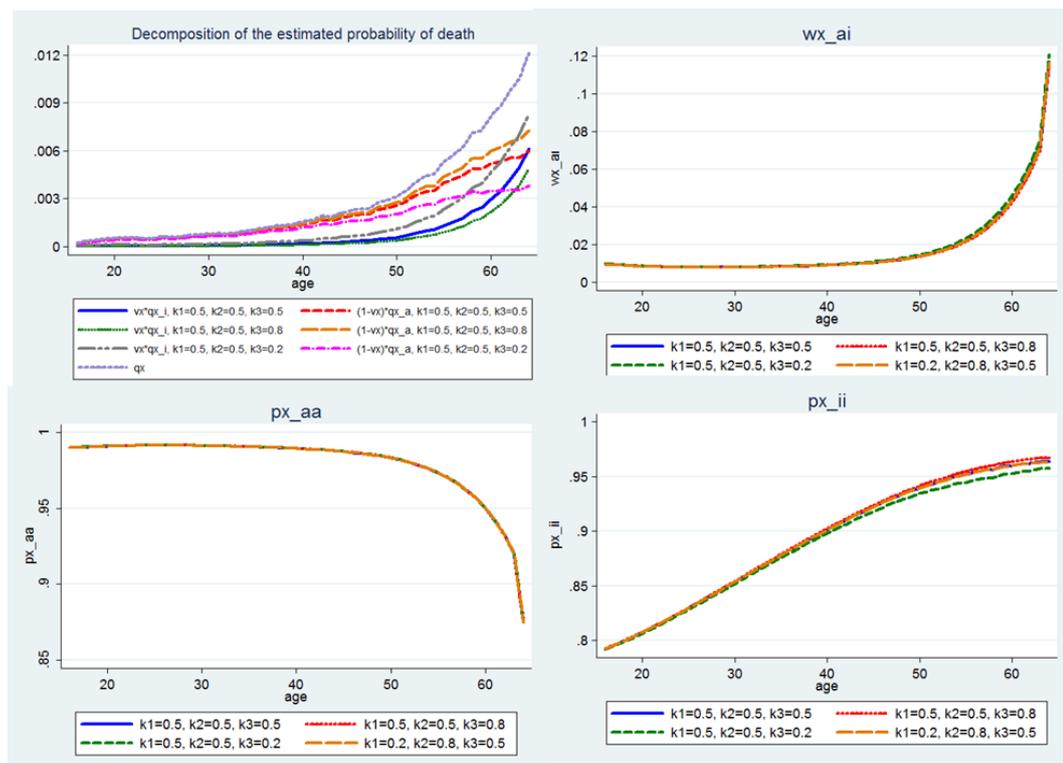
Source: Own calculation based on the one-year multiple state model

Additionally, the values of  $q_x^i$  in all scenarios are almost exactly equal to the values of  $q_x^{ii}$ , whereas the values of  $q_x^{ia}$  are extremely small, i.e. nearly zero. It means the probability that a disabled person dies in the same state ( $q_x^{ii}$ ) is likely to be the main component of the probability of dying of disabled people ( $q_x^i$ ). As expected, in the

case of an active individual, the probability of dying for active people aged  $x$  ( $q_x^a$ ) is also mostly determined by the probability of dying when individuals are still in the active state ( $q_x^{aa}$ ).

Figure 6.8 plots the decomposition of the estimated probability of death between the contribution of  $v_x q_x^i$  and  $(1-v_x)q_x^a$ , the probabilities of becoming disabled ( $w_x^{ai}$ ) and the probabilities of surviving in the same state  $p_x^{aa}$  and  $p_x^{ii}$  with the different scenarios of  $k_1, k_2$  and  $k_3$ . The graphic of the decomposition of the estimated probability of death shows that the value of  $q_x$  is largely affected by the mortality of the active people  $(1-v_x)q_x^a$  with the higher value of  $k_3$ . For example, the value of  $k_3 = 0.8$  produces the highest value of  $(1-v_x)q_x^a$  and the lowest value of  $v_x q_x^i$ .

**Figure 6.8:** Decomposition of the estimated probability of death, the probability of becoming disabled and the probability of surviving in the same state with various scenarios of  $k_1, k_2$  and  $k_3$  among males in 2011



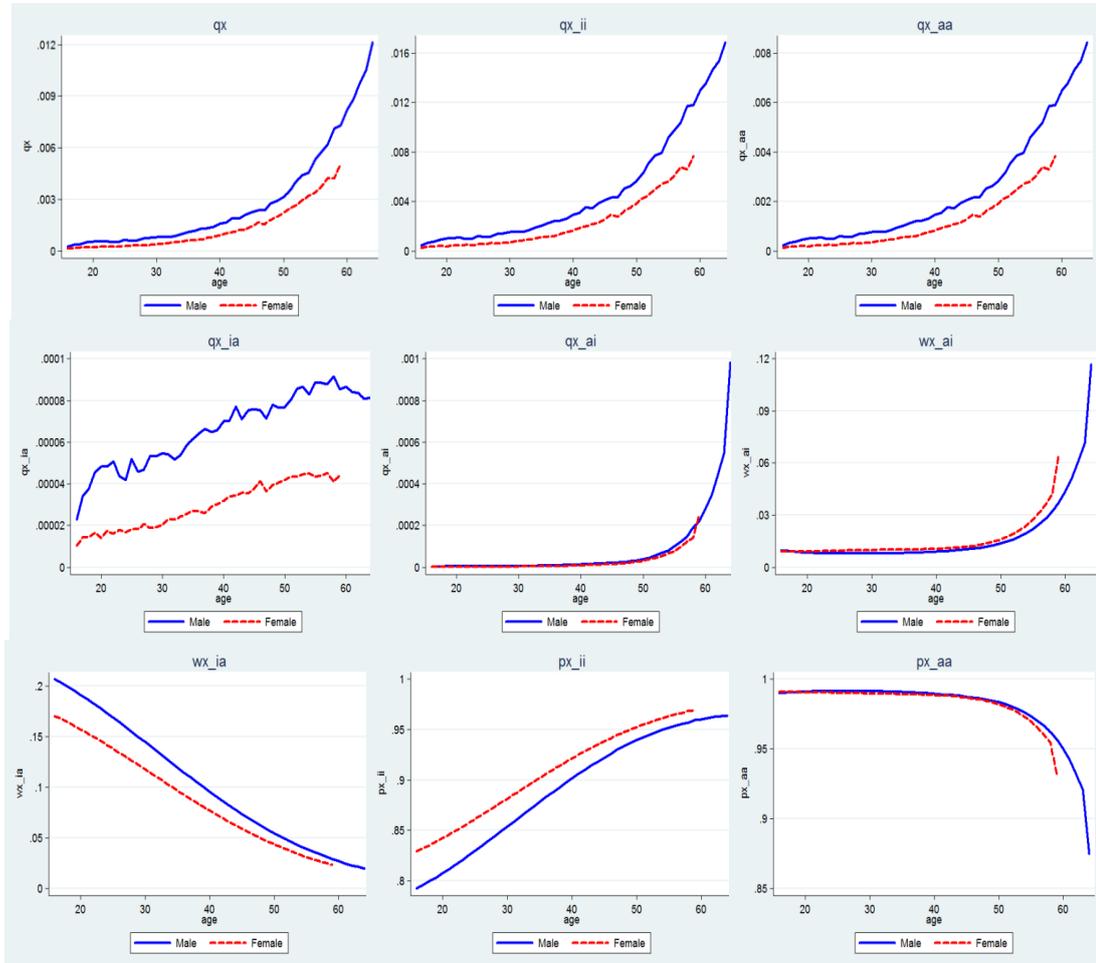
Source: Own calculation based on the one-year multiple state model

Furthermore, the probability of becoming disabled within one year ( $w_x^{ai}$ ) increases with age and steeply rises over the age of 50, in contrast to the probability of  $p_x^{aa}$  (see Figure 6.8). The value of  $p_x^{ii}$  also rises continuously with age and is associated adversely with the probability of recovering from a disabled to an active state within one year ( $w_x^{ia}$ ).

Regarding the differential mortality across gender, the probabilities of death in any state of a person aged  $x$ , i.e.  $q_x^{ii}, q_x^{aa}, q_x^{ia}$  and  $q_x^{ai}$  for men are greater than women, as shown in Figure 6.9. The probabilities of death in the same state  $q_x^{ii}$  and  $q_x^{aa}$  for both genders have the same pattern as the probabilities of death for the general population ( $q_x$ ) that are increasing with age, but the disabled people have a higher probability of death than the general population and the active people. Moreover, it is still rare for anyone to die in a different state within one year of a transition; as a result, the probabilities that the non-disabled and disabled people die in the different state  $q_x^{ia}$  and  $q_x^{ai}$  are almost zero.

The probabilities of becoming disabled ( $w_x^{ai}$ ) are increasing with age and higher among females than males. On the other hand, the probabilities of recovering from a disabled to an active state ( $w_x^{ia}$ ) have been decreasing with age and are lower for females than males. We also compare the probability of surviving in the same state  $p_x^{ii}$  and  $p_x^{aa}$  for both genders. The probabilities that the disabled people are still being disabled ( $p_x^{ii}$ ) rise with age and the values are higher for females than males. In contrast, the probabilities of the active people being in an active state ( $p_x^{aa}$ ) decrease with age and the active males have more chance to stay in the same state than females.

**Figure 6.9:** Estimates of annual probabilities in any state with  $k_1 = k_2 = k_3 = 0.5$  by age and gender in 2011



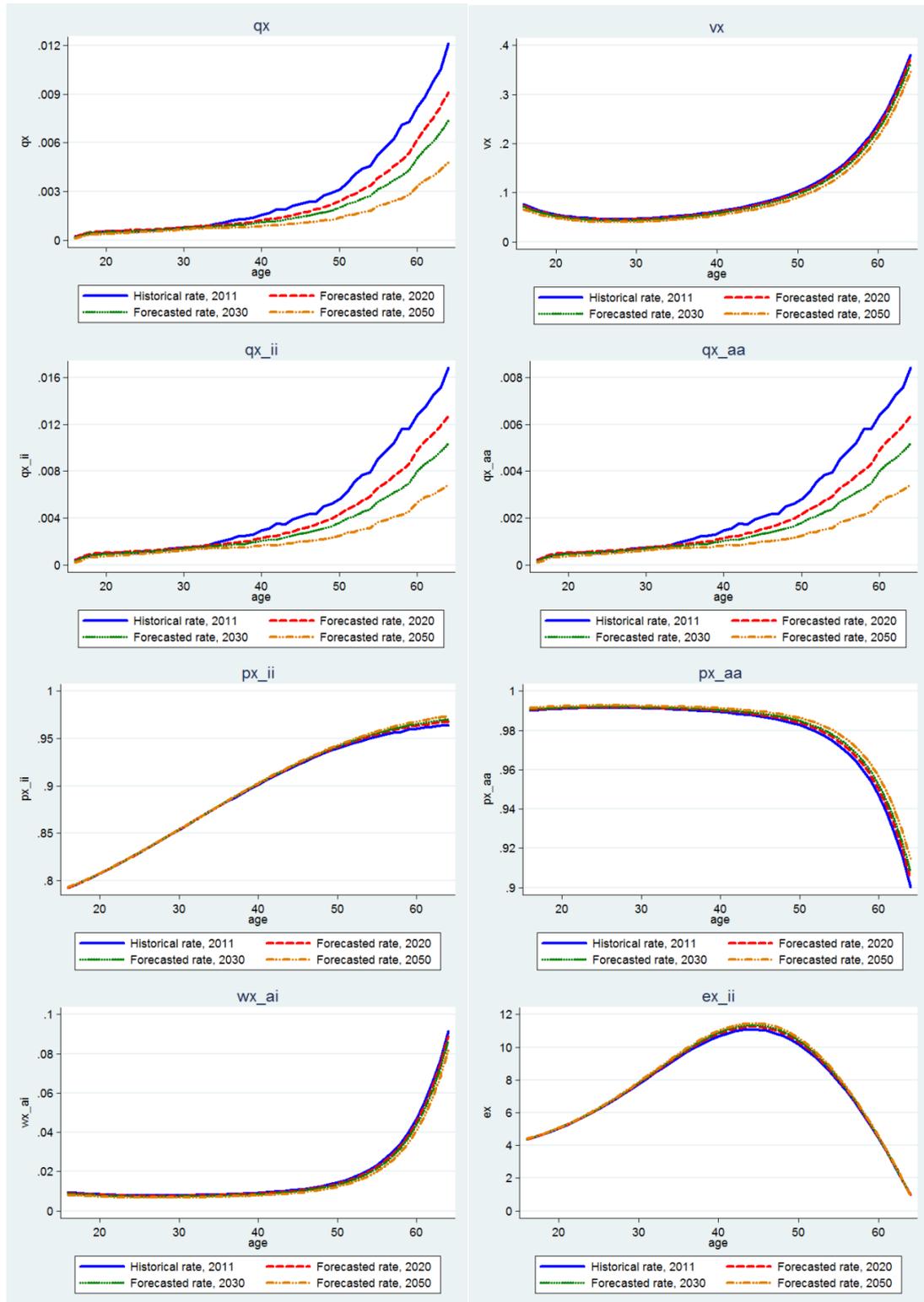
Source: Own calculation based on the one-year multiple state model

Figure 6.10 shows the probability forecasts in any state in 2020, 2030 and 2050 with the scenario of  $k_1 = k_2 = k_3 = 0.5$ . We use the Lee-Carter model to forecast the national mortality rates  $m_x$  and then approximate the probabilities of death  $q_x$ <sup>17</sup>. The disability prevalence rates and all transition probabilities are also projected.

<sup>17</sup> See Lee and Carter (1992) and following Renshaw and Haberman (2003) and Wilmoth *et al.* (2007), we fix  $f_x = \frac{1}{2}$  for all single ages. Hence, the approximation of  $q_{x,t}$  is expressed as

$$q_{x,t} \cong \frac{m_{x,t}}{1 + (1 - f_x)m_{x,t}} \text{ for } x = 16, 17, \dots, 64 \text{ in the calendar year } t = 2020, 2030 \text{ and } 2050.$$

**Figure 6.10:** The projections of transition probabilities in 2020, 2030 and 2050 compared with 2011 in the case of males with the  $k_1 = k_2 = k_3 = 0.5$



Source: Own calculation based on the one-year multiple state model

Due to the improvement of mortality in the future, the probabilities of death among the disabled and active people have also been decreasing over time. Moreover, the trends in the disability prevalence rates would fall since the sign of time effect parameter in the logistic regression model is negative. The drop in the mortality and disability prevalence rates would bring about the fall in the probabilities of becoming disabled and the rise in the probabilities of being in the same state in the future. Also, the expected lifetime (or average number of years) of the disabled people in a disabled state ( $e_x^{ii}$ ) before reaching the retirement age of 65, as shown in Figure 6.10, increases at younger ages and slightly falls at older ages until retirement.

## 6.5 Conclusions

This paper proposes the one-year discrete time multiple state model for the working-age disabled people using the self-reported cross-sectional disability data. We also allow for the recovery from a disabled to an active state in the model, whereas several previous researches focused on the elderly and does not consider their recovery. The disability prevalence rates, the probabilities of death for the national population and the assumptions made on the relative mortality ratio between non-disabled and disabled individuals are used to estimate the probabilities of dying, of surviving and of transition among states.

The estimated gender- and age-specific disability prevalence rates representing the probability of being disabled increase with age and are greater for women than men. On the other hand, the one-year recovery rates as a proxy of the probability of recovery from a disabled to an active state decrease with age and men recover their health and get back to work at a greater rate than women. The probabilities of becoming disabled are almost equal for younger ages and then rise rapidly at older ages. Moreover, the size of the relative mortality ratio among non-disabled and disabled people remaining in the same state is the main determinant of the probabilities of dying in the same state  $q_x^{ii}$  and  $q_x^{aa}$ . In contrast, the probabilities of death in the different state hardly occur – the values of  $q_x^{ai}$  and  $q_x^{ia}$  are almost zero. Consequently, the probabilities of death

among each subgroup  $q_x^i$  and  $q_x^a$  would be approximated by the probability of dying in the same state.

The disability prevalence rates in the future are predicted to drop by 6% in 2030. The fall in the probability of becoming disabled by 7% would reduce the number of new claimants of the incapacity benefits, which implies the fiscal spending on the social insurance to support income for these unemployed disabled people would decrease as well. Additionally, the government would spend the budget to pay the incapacity benefits for the working-age population, on average, for 7.6 years per individual, measured by the expected lifetime of disabled people without employment. In 2030, the average number of year is predicted to increase to 7.7 years due to the effect of mortality improvement.

The model framework presented in this paper is applicable only when the disability prevalence rates are available. However, the disability rates might be replaced with the other prevalence measures of long-term health problems, such as activities living daily (ADL) or instrumental activities of daily living (IADL). Our proposed method could be applied to project the size of the different groups of population; active, disabled and dead, and also to evaluate the demand for the incapacity benefits. In a future study, we will focus on using these estimated transition probabilities to measure the future cost of government spending on disability benefits.

In addition, a more realistic model could be built using a semi-Markov process if the longitudinal data providing the time spent in any state are available – our study is unable working on this model because of a lack of such longitudinal data. Under the semi-Markov model, disability duration effects – the duration of stay in a certain state has an effect on the occurrence of future transitions between states – of recovery and mortality are taken into account. Particularly, the probabilities of both recovery and dying for a disabled person decrease with increasing duration of disability and we would apply the parametric method described by Haberman and Pitacco (1999) to estimate transition intensities. Also, the steps of estimating the cumulative transition intensities, the transition probability matrix and the probability in any state could follow the work of Christiansen (2012).

# CHAPTER 7

## Conclusion

This thesis mainly is based on the major aspects of the old-age benefits, especially the new innovative pension scheme, known as the notional (non-financial) defined contribution (NDC). The scheme is currently approaching its adulthood, and it is time to investigate lessons learned and to explore important issues that need to be addressed to enhance the existing scheme.

Sweden is a pioneer country where the idea of the NDC public pension model has been introduced in 1994. Also, among the countries with NDCs, an automatic balancing mechanism with the calculation of a financial solvency indicator that emerges from an actuarial balance sheet has only been uniquely found in Sweden. Chapter 2 describes the experience of the Swedish NDC pension system during the period of 2007-2015 through its accounting method, with special attention to the balance ratio under the economic and financial crises. The solvency of the pension system dropped and the balance ratio was below one – pension liabilities are greater than assets – in the post-2008 recession and as a consequence, the automatic balancing mechanism was triggered and the amount of pension was reduced in 2010, 2011 and 2014.

The policy responses have been executed as follows: firstly, the smoothed buffer fund – the average of the buffer fund as of 31 December in the last three years – has been used in the balance ratio rather than using the value of the buffer fund as of 31 December in a specific year. The Swedish Pensions Agency also proposed that the effect of the solvency deficit on the pensions should be smoothed. Considering all

sources of solvency volatility – not only the volatility resulting from the value of the buffer fund – would lead to more efficient smoothing. As of the reduction in the pensions under the notional model, the second policy response was the cut of taxes on pensions to counteract the net effect of the negative indexation. However, this policy response implies that one of main objective of NDC design – to safeguard government finances from the development of the public pension plan – was not achieved. Lastly, the volatility of the solvency measure, i.e. the balance ratio, has motivated the Swedish agency analysing the reasons for the unanticipated level of volatility. The results revealed that the three-year smoothing of the income index caused a delay with respect to the development of the contributions of the plan that resulted in a significant real volatility in the ratio of the contribution assets to the pension liabilities.

The Swedish agency also suggests that the (projected) change in income in the coming year relative to the current one should be used instead of the change in income in the current year relative to the previous one to eliminate the volatility caused by the indexation. The difference between the projected and the actual income growth should affect the indexation of the following year as well. This solution would completely eliminate the avoidable solvency volatility caused by the delay in the indexation relative to the contribution development. With the aim of the further reduction in volatility, the policymaker pointed out that balancing should only come into effect if the balance ratio has been below the unity for the three consecutive years and that in balancing, the new rule, in which the indexation is reduced by only one-third of the adjustment necessary to achieve 100 percent solvency ratio, was introduced in 2015 and the new (damped) balance has been applied from 2016 onwards.

In addition, Chapter 3 explores the main channels of communication via the annual report on the solvency of the public system and the orange envelope with information on the individual accumulated capital and forecasts of expected benefits. The main challenge for the Swedish Pensions Agency is to know how the participants use the information provided and how well it communicates information about the pension system. At the same time, the new notional system puts additional demands on participants about how the level of benefits varies according to the number of years of contributions and the retirement age. Projections of future pension benefits and how

the amount of benefits changes with retirement ages are necessary information for contributors in order to make good decisions about a number of years at work and savings.

For the members' perception, understanding of the information received has been continuously increasing since the orange envelope was introduced. However, the confidence on both the pension system and the pension system administration gets worse when the automatic balancing mechanism is triggered that might cause from a lack of the knowledge on how the system works and why the activation of the mechanism is needed the solvency of the system is worsened.

Chapter 4 and 5 of the thesis focus on the annuity divisors used to convert an accumulated notional capital into a lifelong annuity at retirement age. Under the annuitisation divisor rules, all NDC countries share the same principle with respect to the use of gender-neutral annuity divisor. This introduces an intra-generational income redistribution when heterogeneity in mortality exists within the population. Chapter 4 quantifies the redistribution effects of using the two types of unisex (average) divisor; the traditional demographic annuity divisor and the economic annuity divisor, across the Swedish subpopulations assessed through gender and educational attainment.

The alternative economic divisors that are used to calculate the pension liabilities of the Swedish NDC system are based on the composition of the population. With respect to a particular gender or level of education, the values of economic divisors tend to be high if the pension system has a large size of population with low-mortality risk. Moreover, when the combination of gender and level of education is taken into consideration, the economic divisors are mainly driven by the longevity effect belonging to gender rather than educational attainment.

The concept of money's worth for participation in the pension system on a lifetime basis is used to measure the income redistribution. Numerical results strongly confirm that the expected transfer of wealth away from high-mortality risk groups to low-mortality risk groups. The subgroup-specific present value ratio also shows that all subgroups of women and high-educated men get welfare gain from the use of both

types of unisex divisors, i.e. the PVRs are greater than 1, as those of people are, in general, expected to live longer than the average population.

The other consequence on the redistribution occurring from the use of economic divisor instead of the demographic divisor is that when the value of economic divisor changes, the amount of initial pension and the PVRs would change in the opposite direction with the same relative percentage of change compared to the case of using the demographic annuity divisor for all subgroups.

Furthermore, if there is no mortality differential across subpopulations, the pension with the inclusion of the survivor dividend would create an actuarially fair pension system. Interestingly, the survivor dividend produces a trade-off between the expected transfer of wealth between men and women at the different retirement ages. The PVRs for all subgroups of women increase if the retirement is delayed while the PVRs for men decrease when the retirement age increases. The results also indicate that the distribution of the survivor dividend would lessen the effect of mortality heterogeneity on the income transfer.

Chapter 5 develops a complex actuarial model for computing the annuity divisors of a notional pension scheme that provides both old-age and survivors' benefits. The multiple state framework incorporates the survivors' pension for the widow and the orphan together with the old-age benefits and involves the probabilities that the pension benefits will be paid to the retiree and all surviving members of her family, i.e. a spouse and children. The direct method, that works on the mean values of the variables rather than their distribution functions and follows all possible trajectories that a person would occupy during her lifetime and its probability is known, is employed to compute the actuarial present values of the multiple life contingent annuities. This simulation technique is relatively simple and less time-consuming.

The values of unisex annuity divisor of the proposed model decrease with age due to the decline in the life expectancy of both pensioner and her spouse. Also, the effect of longevity risk implies a gradual increase in the values of annuity divisor over the last two decades. The comparative results of the annuity divisors under the current Italian

rules also show that the values established by law are lower than our own results using more updated data. In this case, it would lead to the current retirees receiving more amount of pension than they should actually earn. The underestimated annuity divisors would cause an increase in the pension liabilities of the system that would deteriorate the finances of the public pension scheme.

Moreover, the findings highlight the fact that the annuity divisors of the pension benefits covering multiple beneficiaries are significantly greater than the divisors of the annuity covering only a single beneficiary. From the results, we confirm that the values of divisor under the Italian rules – considering the probabilities of payments for both a pensioner and her spouse – are lower than those values obtained by the direct method – considering the probabilities of payments for a pensioner, a spouse and children, when the annuities of both cases are payable annually at the same moment in time set in the middle of the year. This means that the current retirees are benefitting from the Italian rule and receiving more amount of pension than the corresponding values under the direct method.

The actuarial values of the unitary pension benefits achieved in the numerical example endorse the fact that the proposed model works well and is close to the reality of the social insurance. The integration of the old-age and survivors' benefits to determine the initial pension would enhance the political attractiveness of the pension design. In a similar way, it is necessary to be aware of the social acceptability of the new rule along with the drop in the pension payments.

Apart from the studies relevant to the old-age pension, Chapter 6 proposes a generic method for estimating the one-year transition probabilities among the working-age non-disabled and disabled population by using the self-reported cross-sectional disability data. The estimated prevalence rates of disabilities, the probabilities of death for the general population and the assumptions made on the relative mortality ratio between active and disabled people are used to obtain the probabilities of death, of survival and of transition among the states of active and disabled.

The main findings show that the disability prevalence rates increase with age and the rates for women are higher than men. Similarly, the probabilities of becoming disabled are increasing with age and higher for women than men, whereas the probabilities of recovery from a disabled to an active state are decreasing when rising age and men recover their health and get back to work sooner than women. Furthermore, the probabilities that the disabled people are alive in a disabled state rise with age and the values are higher for women than men. In contrast, the probabilities of active people being alive in an active state decrease with age and the active men have more chance to be healthy than women.

In regard to the differential mortality across subgroups, the probabilities of death in any state illustrate that the values for men are greater than women. In particular, the probabilities that the active and disabled people die in the same state have the same pattern as the probabilities of death for the general population – the probabilities of death increase with age – but the disabled people have the probability of death higher than the general population and the active people. There is also a very small chance for everyone dying in the different state within one year, as a result, the probabilities that the non-disabled and disabled people die in the different state are almost zero.

Although the model framework proposed in Chapter 6 is applicable when the cross-sectional prevalence rates of disability are available, a possible extension could be the use of other indicators measuring the long-term health problems, such as the activities living daily (ADL) or instrumental activities of daily living (IADL). Also, a more realistic model could be built by applying a semi-Markov process in which disability duration effects of recovery and mortality are taken into account and then the transition intensities, the transition probability matrix and the probability in any state would be estimated by following the work of Christiansen (2012).

# APPENDIX A

## Appendices to Chapter 2

### A.1 Balance sheets and income statements in Swedish kronor

The balance sheets and income statements from 2007–2015 are shown in Swedish kronor in Table A.1 and A.2, respectively.

**Table A.1:** Balance sheets, December 31, 2007–2015

Item	2007	2008	2009	2010	2011	2012	2013	2014	2015
<b>Assets (SKr billions)</b>									
Fund assets	898	707	827	895	873	958	1,058	1,184	1,230
Contribution asset	6,116	6,477	6,362	6,575	6,828	6,915	7,123	7,380	7,457
Total assets	7,014	7,184	7,189	7,469	7,700	7,873	8,180	8,565	8,688
<b>Liabilities and results brought forward (SKr billions)</b>									
Opening results brought forward	100	18	-243	-323	103	157	-80	127	423
Net income or loss for the year	-82	-261	-79	425	54	-237	207	296	-252
Closing results brought forward	18	-243	-323	103	157	-80	127	423	171
Pension liability	6,996	7,428	7,512	7,367	7,543	7,952	8,053	8,141	8,517
Total liabilities and results brought forward <sup>a</sup>	7,014	7,184	7,189	7,469	7,700	7,873	8,180	8,565	8,688

*Source:* Swedish Pensions Agency (2008–2016)

<sup>a</sup> Total liabilities and result brought forward is the summation of “Opening results brought forward”, “Net income or loss for the year” and “Pension liability”.

**Table A.2:** Income statements, 2007–2015

Item	2007	2008	2009	2010	2011	2012	2013	2014	2015
<b>Change in fund assets (SKr billions)</b>									
Pension contributions	190	203	203	205	216	222	227	236	246
Pension disbursements	-186	-199	-217	-220	-220	-236	-254	-255	-265
Return on funded capital	38	-194	136	85	-17	101	128	148	67
Administrative costs	-2	-1	-2	-2	-2	-2	-2	-2	-2
Total	41	-191	120	68	-22	85	100	127	46
<b>Change in contribution assets (SKr billions)</b>									
Value of change in contribution revenue	193	395	-115	232	255	120	215	266	333
Value of change in turnover duration	-22	-33	-1	-19	-2	-33	-6	-8	-256
Total	171	361	-115	213	253	87	208	257	77
<b>Change in pension liability<sup>a</sup> (SKr billions)</b>									
New pension credits and ATP points	-194	-218	-214	-215	-207	-228	-242	-230	-246
Pension disbursements	186	199	217	220	220	236	254	255	265
Indexation or change in value	-268	-385	-64	165	-175	-403	-96	-92	-378
Value of change in life expectancy	-17	-27	-23	-25	-14	-13	-16	-20	-15
Inheritance gains arising	10	11	11	11	11	11	12	12	12
Inheritance gains distributed	-11	-12	-13	-13	-12	-13	-14	-14	-14
Deduction for administrative costs	2	1	1	1	1	1	1	2	1
Total	-293	-431	-84	145	-177	-409	-101	-88	-375
Net income or loss for the year (SKr billions)	-82	-261	-79	425	54	-237	207	296	-252

Source: Swedish Pensions Agency (2008–2016)

Note: ATP = Allmän tilläggspension.

<sup>a</sup> A negative item (-) increases the pension liability, and a positive item ( ) decreases it, by the amount shown.

# **APPENDIX B**

## **Appendices to Chapter 3**

### **B.1 The orange envelope**

Demo Person

SWEDISH  
PENSIONS AGENCY

# Annual Statement 2014

## Your National Public Pension

According to our forecast, this is how much you will receive as national public pension per month before tax. The amount may vary depending on when you decide to retire.

age 61	age 65	age 68 and 3 month	age 70
SEK 10 300	SEK 13 100	SEK 16 000	SEK 18 500

## Do you have a pension from different sources?

In addition to the national public pension, most employees also have a pension from their employer. Some also have private pension savings.

National Public Pension

+

Occupational pension

+

Private pension

=

## Your entire pension



Log in and see your entire pension

[www.pensionsmyndigheten.se/B3](http://www.pensionsmyndigheten.se/B3)

Use electronic identification or your personal code

27346

2014

## You have earned this much towards your National Public Pension

### Your Pension Credits

Changes during 2013 in SEK	Income pension	Premium pension	
Value 2012-12-31	854 596	106 942	
Pension credit for 2012	57 264	8 947	
From deceased contributors	603	560	
Administration and fund fees	- 277	- 985*	
Change in value	-10 382	8 709**	
Value 2013-12-31	901 804	124 173	<b>Totally earned to the national public pension</b>
			<b>SEK 1 025 977</b>

\* Including SEK 716 discount on fund fee for 2012.

\*\* Including SEK 135 as interest on your pension credit for 2012.

### Your Premium Pension

Premium pension account 2013-12-31	Value, SEK	Change in value, per cent	Fund fee, per cent	Chosen allocation, per cent	Current allocation, per cent
Equity Fund Sverige	50 626	22	0,29	40	41
Equity Fund Global	31 156	22	0,51	25	25
Interest Fund Sverige	27 863	3	0,13	25	22
Generation Fund	14 528	13	0,20	10	12
Total	124 173	17	0,30	100	100
<i>The average pension saver</i>		21	0,31		

**Mutual Fund Fee.** Keep in mind that high fees mean worse performance for your savings.

**Fund transfers.** In order to increase safety, all fund transfers, from 20th February 2014, take place with electronic identification or Mobile BankID. You can also switch funds using a form that you order from the Swedish Pensions Agency and which will be sent to your registered address.

# 2014

## How much will you get per month?

### Forecast for your National Public Pension

Retirement age	age 61	age 65	age 68 and 3 month	age 70
Amount SEK/month	10 300	13 100	16 000	18 500

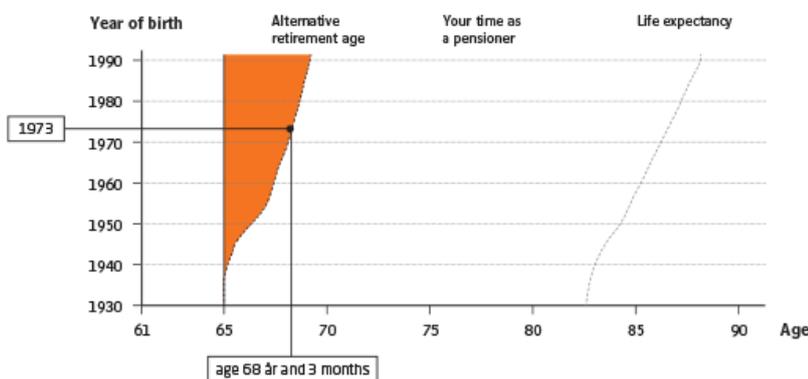
Your national public pension from age 65 (SEK 13 100 per month before tax) is estimated at SEK 9 800 in Income pension and SEK 3 300 in premium pension. The pension will be paid out for the rest of your life.

**We calculated as follows.** The forecast is based on the SEK 1 025 977 you have earned towards your national public pension so far and your annual income until you retire. We have assumed that you will have the same pensionable income per year as in 2012, that is SEK 303 300.

The forecast is calculated in today's value. This means that you can compare the amounts in the forecast with your current earnings. The forecast is developed in accordance with the pension industry forecast standard. Read more on [www.pensionsmyndigheten.se/prognosstandard](http://www.pensionsmyndigheten.se/prognosstandard).

**Why 68 years and 3 months?** The life expectancy in Sweden is rising. You, who were born in 1973 need to work until the age of 68 years and 3 months to receive the same pension amount you would have received at age 65 if life expectancy had remained unchanged. Your pension is calculated as your account value divided by the average remaining life expectancy of your age class.

**When is the best time for you to retire?** At [www.pensionsmyndigheten.se/B3](http://www.pensionsmyndigheten.se/B3), you can obtain forecasts that also include your occupational pension and possible private pension. The forecasts make it easier for you to plan and make the right decisions about your future. The forecasts are generated by Minpension.se, a collaboration between the Swedish Pensions Agency and the private pension companies.



2013-12-05

## Decision about your Pension Credits

The decision regarding your pension credits concerns 2012 since it is based on your latest established declared income.

Pension credit for income pension		Pension credit for premium pension		Your total pension credits 2012
SEK 57 264	+	SEK 8 947	=	SEK 66 211

### Basis for calculation of your pension credits

Pensionable income:	SEK	303 300
Pensionable amount:		
child years	SEK	54 600
This provides a pension basis of:	SEK	357 900

### To request a reconsideration of the decision

The regulations that are the basis for the decision are to be found in chapters 59–61 of the Social Insurance Code (2010:110). If you want the decision to be reconsidered, please write to the Pensionsmyndigheten, Box 304, 301 08 Halmstad. Indicate the decision that you want reconsidered, how you want it changed and why. Write also your name, Swedish personal ID number, address and telephone number. If you engage a legal representative you must enclose an original power of attorney. Swedish Pensions Agency must receive the letter at the latest on 31st December 2014 or, if you have not been informed before 1st November 2014, within two months from the day you receive notice of the decision.

Demo Person (eng)  
Vägen 14  
123 45 Landsorten

### Contact information

Swedish Pensions Agency, [www.pensionsmyndigheten.se](http://www.pensionsmyndigheten.se), customer service 0771-776776

④

You can also visit our service offices, see [www.pensionsmyndigheten.se/servicekontor](http://www.pensionsmyndigheten.se/servicekontor)

# APPENDIX C

## Appendices to Chapter 4

### C.1 Modelling and forecasting mortality rates across subpopulations using the common factor model

The extension of the Lee-Carter model, the so-called common factor model, proposed by Li and Lee (2005) can be transformed into the relative approach associated with the reference (national) population and the subpopulation introduced by Haberman *et al.* (2014).

The mortality of the reference population following the Lee-Carter Model can be expressed as follows:

$$\log m_{x,t} = \alpha_x + \beta_x K_t + \varepsilon_{x,t} \quad (\text{C.1.1})$$

where the central mortality rate at age  $x$  in calendar year  $t$  for the reference population,  $m_{x,t}$ , is computed as  $m_{x,t} = D_{x,t}/E_{x,t}$ . We denote  $D_{x,t}$  as the number of deaths of the reference population aged  $x$  who die in year  $t$  before they reach age  $x+1$  and the number of person-years of exposure to risk,  $E_{x,t}$ , is estimated by the mid-year population at age  $x$  in calendar year  $t$ , equivalently to the average number of individuals aged  $x$  over a year. The parameter  $\alpha_x$  describes the average age-specific mortality pattern. The variable  $K_t$  is a time-varying mortality index representing the overall change in the level of mortality in year  $t$  and  $\beta_x$  measures the deviation of the age-specific mortality level when the time index  $K_t$  changes.

Given the reference population model and the common factor model with the same parameter  $\beta_x$  and  $K_t$  for all population, the mortality level differences of the subpopulation  $i$  compared to the reference population for age group  $x$  can be determined in the form of  $\alpha_x^i$  as follows:

$$\log m_{x,t}^i - \log m_{x,t} = \alpha_x^i \quad (\text{C.1.2})$$

Substituting the mortality of reference population from the equation (C.1.1) in the equation (C.1.2), the mortality of the subpopulation  $i$  is then specified through<sup>1</sup>

$$\log m_{x,t}^i = \alpha_x + \alpha_x^i + \beta_x K_t \quad (\text{C.1.3})$$

with constraints  $\sum_{x \in X} \beta_x = 1$ ,  $\sum_{t \in T} K_t = 0$  and  $\sum_{i \in I} \alpha_x^i = 0$  for all  $x \in X$

where  $m_{x,t}^i$  is the central mortality rate at age  $x$ ,  $x \in X := \{x_1, \dots, x_k\}$ , in calendar year  $t$ ,  $t \in T := \{t_1, \dots, t_n\}$ , for the subpopulation  $i$ ,  $i \in I := \{i_1, \dots, i_s\}$ , and  $m_{x,t}^i = D_{x,t}^i / E_{x,t}^i$ . The age-specific mortality level in the subgroup  $i$  is  $\alpha_x + \alpha_x^i$ . The parameter  $\beta_x$  represents the age-specific pattern of mortality change for all subpopulations and  $K_t$  is a single-period index driving the mortality trend for all subpopulations. Also, the subgroup  $i$  is  $L$  for low education,  $M$  for medium education and  $H$  for high education, respectively.

We fit the common factor model presented in the equation (C.1.3) for unisex, male and female groups separately by the method of maximum likelihood. The resulting parameter estimates are shown graphically in Figure C.1. Then, the single-period mortality index  $K_t$  is forecasted by the univariate autoregressive integrated moving average (ARIMA) processes and is used to generate the projected life tables (see Appendix C.3 for more details of the forecast).

<sup>1</sup> See Villegas and Haberman (2014) in Equation (10).

In order to evaluate the best model that fits the historical data, we use the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Root Mean Square Error (RMSE). The estimated parameters, their standard errors and the values of goodness-of-fit statistics for unisex and both genders appear in Table C.1. The  $t$ -statistics of all parameters are statistically significant and the residual plots guide us towards retaining the chosen models. Thus, the series of index  $K_t$  for all population (unisex) and female are modelled by an ARIMA (0,1,1) process:

$$K_t = K_{t-1} + \lambda + \varepsilon_t + \theta_1 \varepsilon_{t-1} \quad (\text{C.1.4})$$

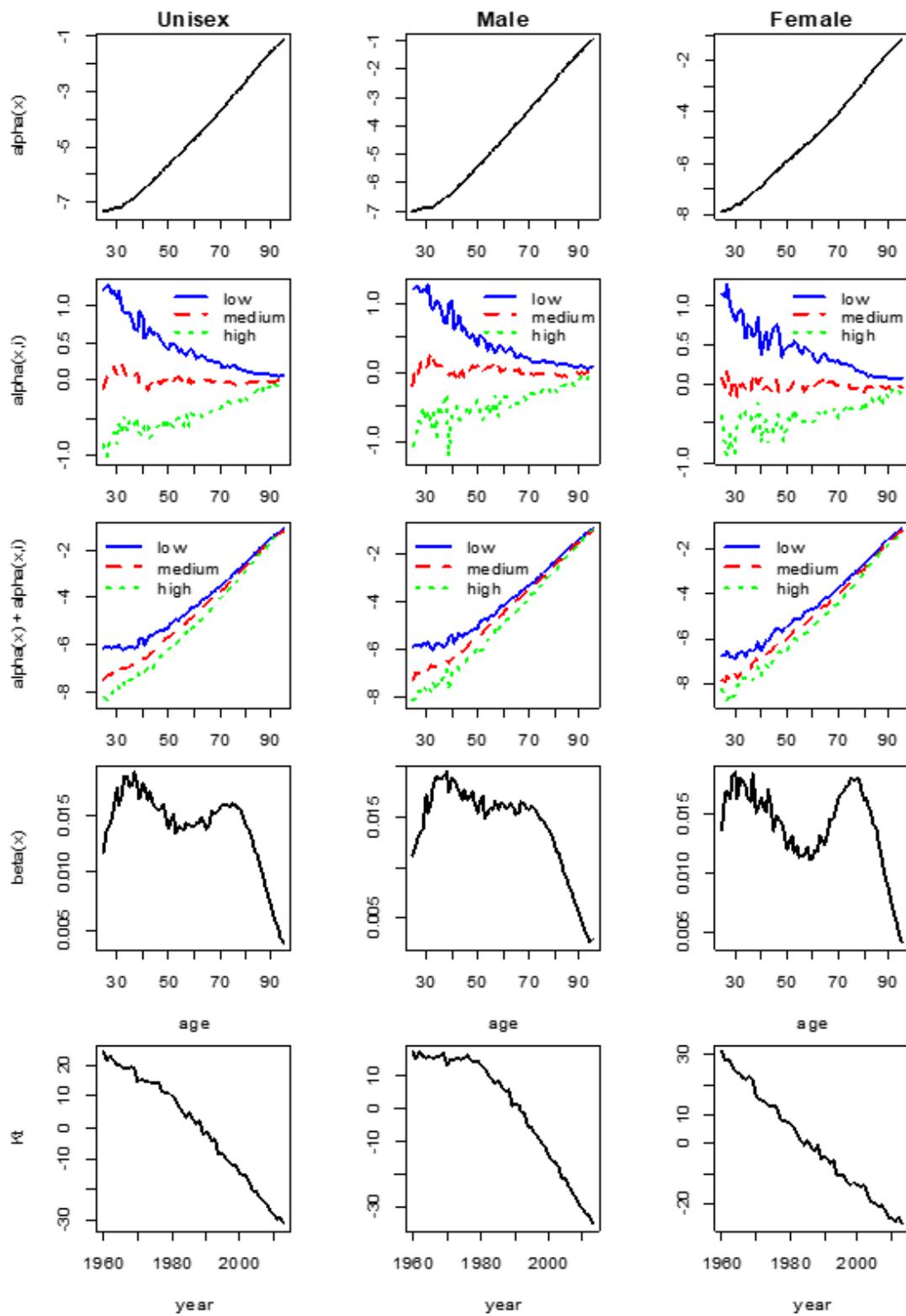
for male, the model is as an ARIMA (1,1,2) process:

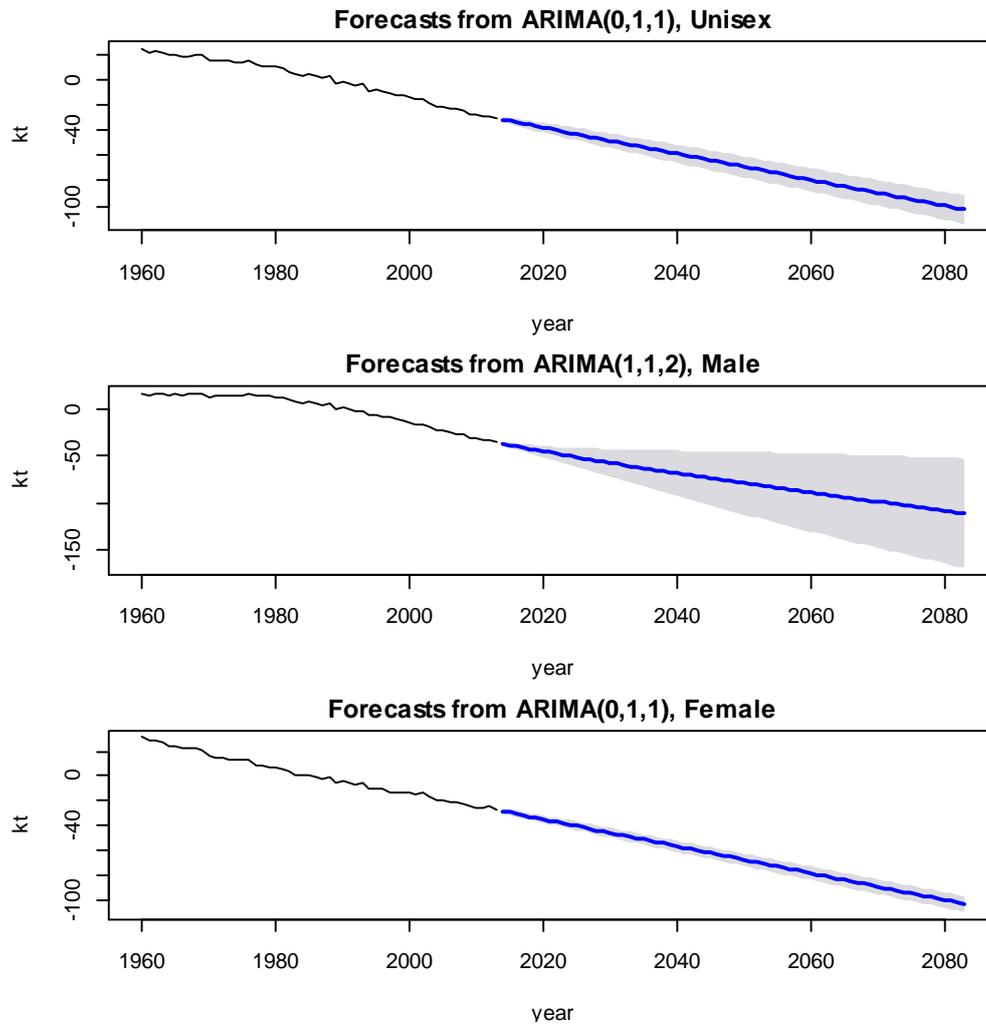
$$K_t = K_{t-1} + \lambda + \varepsilon_t + \phi_1 (K_{t-1} - K_{t-2}) + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} \quad (\text{C.1.5})$$

where the constant  $\lambda$  indicates the average annual change of  $K_t$ , the  $\theta_j$  represents the  $j$ -th moving average parameter, the  $\phi_j$  is the  $j$ -th autoregressive and the error term  $\varepsilon_t$  is normally distributed with zero mean and variance  $\sigma^2$ .

**Table C.1:** Parameter estimates of the ARIMA ( $p, d, q$ ) model

Variable	Unisex ARIMA(0,1,1)			Male ARIMA(1,1,2)			Female ARIMA(0,1,1)		
	Coeff.	Std. Error	t-Stat	Coeff.	Std. Error	t-Stat	Coeff.	Std. Error	t-Stat
$\lambda$	-1.032	0.092	-11.234	-0.924	0.467	-1.982	-1.084	0.053	-20.641
$\theta_1$	-0.473	0.100	-4.711	-1.429	0.121	-11.833	-0.724	0.102	-7.121
$\theta_2$	-	-	-	0.609	0.092	6.624	-	-	-
$\phi_1$	-	-	-	0.954	0.051	18.840	-	-	-
$\sigma^2$	1.615			1.403			1.795		
log-likelihood	-87.01			-82.90			-90.05		
AIC	180.01			175.79			186.11		
BIC	185.92			185.64			192.02		
RMSE	1.235			1.128			1.302		

**Figure C.1:** Parameter estimates of the common factor model for unisex, male and female

**Figure C.2:** Forecasts of the mortality index  $K_t$  with 95% confident interval

Next, we use the ARIMA(0,1,1) for unisex and female and the ARIMA(1,1,2) for male to generate the forecasts of the mortality index  $K_t$  during the period 2014-2083. Figure C.2 shows the past values of  $K_t$  along with the projected values and the associated 95% confident intervals for unisex, male and female respectively.

We then use the estimated parameters from the common factor model in the equation (C.1.3) to compute the estimated gender- and age-specific mortality rates by subgroup  $i$  in 2013 as the following formula:

$$\hat{m}_{x,2013}^i = \exp\left(\hat{\alpha}_x + \hat{\alpha}_x^i + \hat{\beta}_x \hat{K}_{2013}\right) \quad (\text{C.1.6})$$

Also, given the above estimated mortality rates  $\hat{m}_{x,2013}^i$  and the resulting forecasts of  $\{\tilde{K}_{2013+s} : s = 1, 2, \dots, 70\}$ , we calculate the forecasted mortality rates for subpopulation  $i$  as:

$$\tilde{m}_{x,2013+s}^i = \hat{m}_{x,2013}^i \exp\left\{\hat{\beta}_x \left(\tilde{K}_{2013+s} - \hat{K}_{2013}\right)\right\} \quad (\text{C.1.7})$$

The projected mortality rates  $m_{x,t}^i$  are converted into  $q_{x,t}^i$ , i.e. the probability of an individual aged  $x$  in the calendar year  $t$  and subgroup  $i$  dying within one year (also namely the one-year probability of death). These mortality rates are used to construct the life tables.

Let  $f_x$  be the average number of years lived within the age interval  $[x, x+1)$  for people dying at that age. Following Renshaw and Haberman (2003) and Wilmoth *et al.* (2007), we fix  $f_x = \frac{1}{2}$  for all single ages. Hence, the approximation of  $q_{x,t}^i$  is expressed as:

$$q_{x,t}^i \cong \frac{m_{x,t}^i}{1 + (1 - f_x)m_{x,t}^i} \quad (\text{C.1.8})$$

for  $x = 25, 26, \dots, 94$  and  $q_{95,t}^i = 1$  in the calendar year  $t = 2013, 2014, \dots, 2083$ .

The elements of the matrix of  $q_{x,t}^i$  whose rows correspond to ages and columns to calendar years can be simplified as in Table C.2 (Pitacco *et al.*, 2009). For a diagonal arrangement,

$$q_{25,t}^i, q_{26,t+1}^i, \dots, q_{25+s,t+s}^i, \dots \quad (\text{C.1.9})$$

corresponding to a sequence of cohort life tables, with each table referring to the cohort born in year  $t - 25$ . We use the values of  $\{q_{25,2013}^i, q_{26,2014}^i, \dots, q_{95,2083}^i\}$  to construct the

subgroup-specific projected cohort life tables of an individual aged 25 in 2013 (equivalently to who was born in 1988).

**Table C.2:** One-year probabilities of death in a dynamic context

	$t$	$t+1$	...	$t+70$
25	$q_{25,t}^i$	$q_{25,t+1}^i$	...	$q_{25,t+70}^i$
26	$q_{26,t}^i$	$q_{26,t+1}^i$	...	$q_{26,t+70}^i$
...	...	...	...	...
$x$	$q_{x,t}^i$	$q_{x,t+1}^i$	...	$q_{x,t+70}^i$
$x+1$	$q_{x+1,t}^i$	$q_{x+1,t+1}^i$	...	$q_{x+1,t+70}^i$
...	...	...	...	...
95	$q_{95,t}^i$	$q_{95,t+1}^i$	...	$q_{95,t+70}^i$

Let  $p_x^i$  be the probability that an individual in subgroup  $i$  survives from age  $x$  to  $x+1$ , to complete the subgroup-specific life table calculations. Therefore,

$$p_x^i = 1 - q_x^i \quad (\text{C.1.10})$$

for all ages  $x$ .

Let the radix of the life table be  $l_{25}^i = 100,000$ . Then, the number of survivors at age  $x$  is

$$l_{x,t}^i = l_{25}^i \cdot \prod_{h=25}^{x-1} p_h^i \quad (\text{C.1.11})$$

and the distribution of deaths by age in the life table population is

$$d_x^i = l_x^i \cdot q_x^i \quad (\text{C.1.12})$$

We finally construct the projected cohort subgroup-specific life tables for the people who were born in 1988. The unisex life table is used to compute the unisex (average)

annuity divisors and the gender- and educational-specific life tables are for the calculation of the money's worth ratio.

## C.2 Modelling and forecasting mortality rates across subpopulations using the Joint- $K$ model

The Joint- $K$  model proposed by Carter and Lee (1992) assumes that mortality rates of all subpopulations are jointly driven by a single period index  $K_t$ , but the age-specific mortality pattern and the age-specific responses to changes in the level of mortality are subgroup-specific. The model is given by

$$\log m_{xt}^i = \alpha_x^i + \beta_x^i K_t \quad (\text{C.2.1})$$

Also, given the reference population model, the mortality of subpopulation  $i$  can be rewritten as

$$\log m_{xt}^i = \alpha_x + \alpha_x^i + (\beta_x + \beta_x^i) K_t \quad (\text{C.2.2})$$

According to the Three-way Lee-Carter model introduced by Russolillo *et al.* (2011), a new component  $\lambda^i$  capturing the variability in the improvement rates of the subpopulations was proposed. Then, the Joint- $K$  model in the equation (C.2.2) can be reparametersied as follows:

$$\log m_{xt}^i = \alpha_x + \alpha_x^i + \beta_x \lambda^i K_t \quad (\text{C.2.3})$$

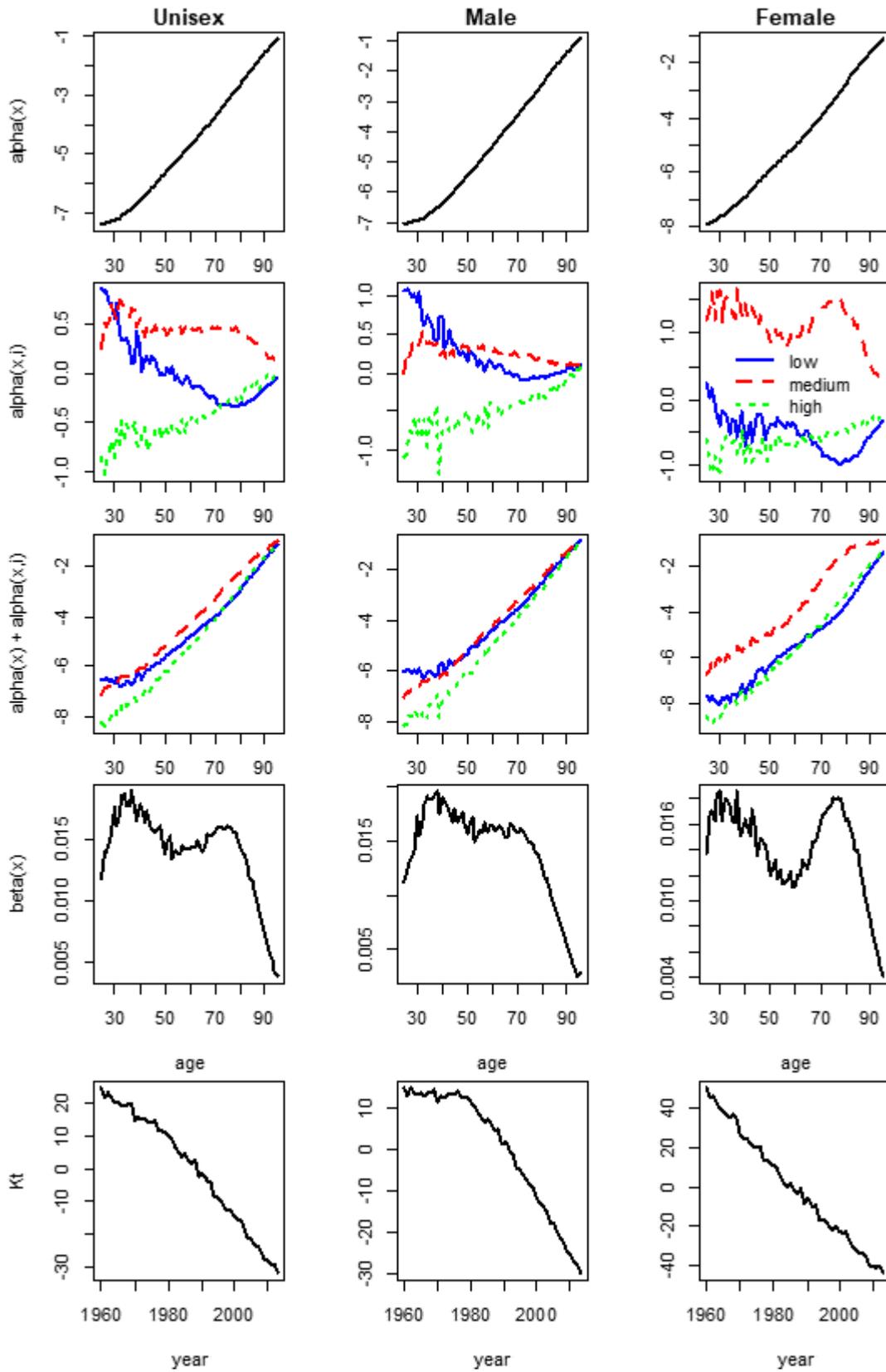
with constraints  $\sum_{x \in X} \beta_x = 1$ ,  $\sum_{t \in T} K_t = 0$ ,  $\sum_{i \in I} \alpha_x^i = 0$  for all  $x \in X$  and  $\frac{1}{S} \sum_{i \in I} \lambda^i = 1$

where  $\alpha_x + \alpha_x^i$  describes the general age shape of the age-specific death rates  $m_{xt}^i$  in the subgroup  $i$ . If the improvement differentials in mortality  $\lambda^i > 1$  ( $\lambda^i < 1$ ), it means

that mortality rate in subgroup  $i$  improves at a faster (slower) than mortality in general population.

However, the results of the parameter estimates, especially the age-specific mortality pattern for subpopulation  $i$ ,  $\alpha_x + \alpha_x^i$  as shown in Figure C.3, are not reasonable – mortality rates for medium-educated people are found to be greater than those of low education. This inconsistency might be caused for the use of a very short time period of our available data on education-specific mortality.

**Figure C.3:** Parameter estimates of the Joint- $K$  model for unisex, male and female



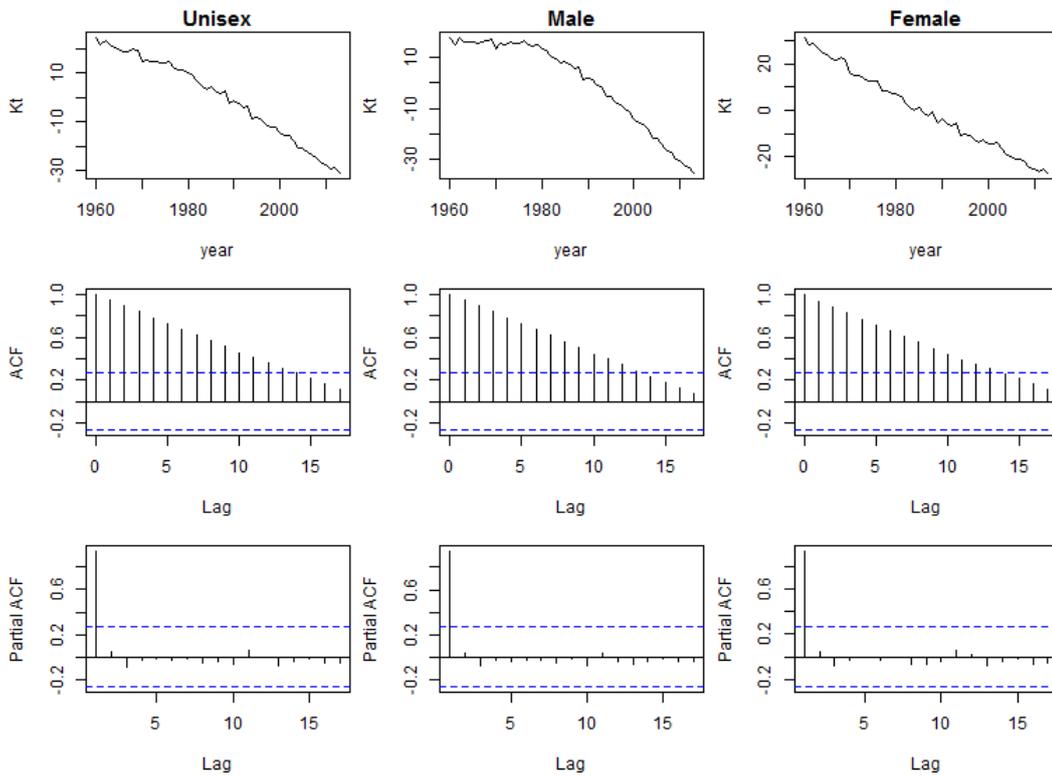
### C.3 Time series of the mortality time index $K_t$

We forecast the period indexes by using the univariate autoregressive integrated moving average (ARIMA) processes to project age-specific mortality rates for three different groups of education to construct the subgroup-specific life tables. The procedure of modelling that fits our observed data involves an iterative five-stage process: data preparation, model selection, parameters estimation, model diagnostics and forecasting (Makridakis *et al.*, 1998) described as follows:

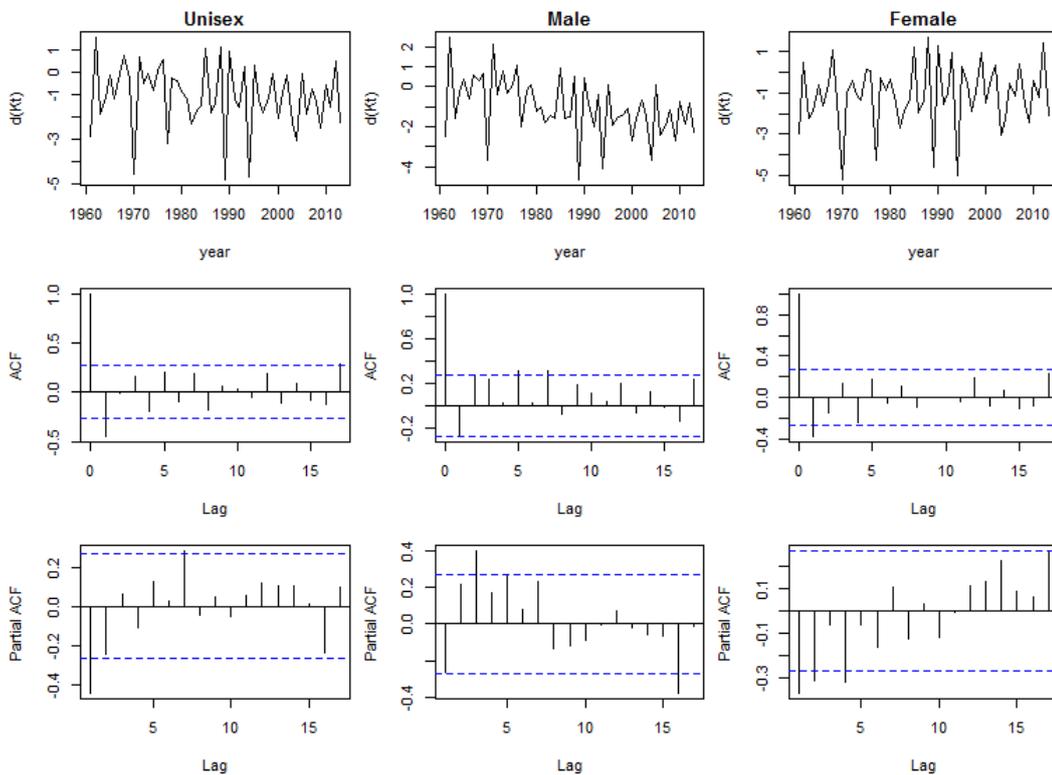
#### 1) Data preparation

We first analyse the general pattern of the time series  $K_t$  for unisex and both genders as illustrated in Figure C.4. There are noticeable downward trends for all types of population indicating that the mortality has deteriorated gradually over the years. We also consider the autocorrelation function (ACF) and the partial autocorrelation function (PACF) that display a typical pattern for a nonstationary process. The ACF plots of three groups are slowly decaying to zero. For the PACF plot, we observe a significant spike at only lag 1 and cutting off suddenly thereafter for all groups. Since the ARIMA model requires the stationary time series data (i.e. a constant mean, variance and autocorrelation through time), we then transform the data to obtain the stationary series by taking differences between consecutive observations. The number of differencing the time series to achieve stationary is reflected in the  $d$  parameter. In our analysis, the first differencing,  $d = 1$ , seems to eliminate the nonstationary in mean. Our differenced data become stationary as exhibited in Figure C.5.

**Figure C.4:** Time series of  $K_t$



**Figure C.5:** Differenced series of  $K_t$



## 2) Model selection (Identification)

Once the series are stationary, we determine the order of the autoregressive  $AR(p)$  and/or moving average  $MA(q)$  by looking at the autocorrelation function (ACF) and partial autocorrelation (PACF) plots of the differenced series as shown in Figure C.5. For the series of unisex group, the ACF plot of the differenced data displays a negative spike at only lag 1 and no significant spikes thereafter. There is negative spike at lag 1 and then tailing off in the PACF plot, which might indicate an  $MA(1)$  process. Hence, we fit the  $ARIMA(0,1,1)$  with a constant. The ACF and PACF plots for males are unclear to identify the  $p$  and  $q$  parameters. Similarly, for females, there is a negative insignificant spike at lag 1 in the ACF but the PACF plot shows the negative insignificant spike at lag 1, 2 and 4. Therefore, we are unable to decide what appropriate  $AR(p)$  and/or  $MA(q)$  would be. As a result, we modelled nine models based on the combinations of the order  $p$  and  $q$  varying between 0 and 2.

**Table C.3:** Goodness of fit test statistics of the  $ARIMA(p,1,q)$

Model	Unisex			Male			Female		
	AIC	BIC	RMSE	AIC	BIC	RMSE	AIC	BIC	RMSE
ARIMA(0,1,0)	191.3	195.2	1.403	193.0	197.0	1.426	200.7	204.7	1.534
ARIMA(0,1,1)	<b>180.0</b>	<b>185.9</b>	<b>1.235</b>	-			<b>186.1</b>	<b>192.0</b>	<b>1.302</b>
ARIMA(0,1,2)	-			185.9	193.8	1.280	-		
ARIMA(1,1,0)	181.4	187.3	1.251	-			194.9	200.8	1.422
ARIMA(1,1,2)	-			<b>175.8</b>	<b>185.6</b>	<b>1.128</b>	-		
ARIMA(2,1,0)	-			-			191.5	199.4	1.349
ARIMA(2,1,1)	-			179.5	189.4	1.173	-		

We use the Akaike's Information Criterion (AIC) and the Bayesian Information Criterion (BIC) to decide the best model. The models are ranked according to their AIC and BIC and the one having the lowest information criterion value is the best. Table C.3 shows the values of AIC and BIC of the  $ARIMA(p,1,q)$  with the significance of the parameter estimates for three groups of population. Thus, we

choose the ARIMA(0,1,1) with constant for unisex and female and the ARIMA(1,1,2) for male.

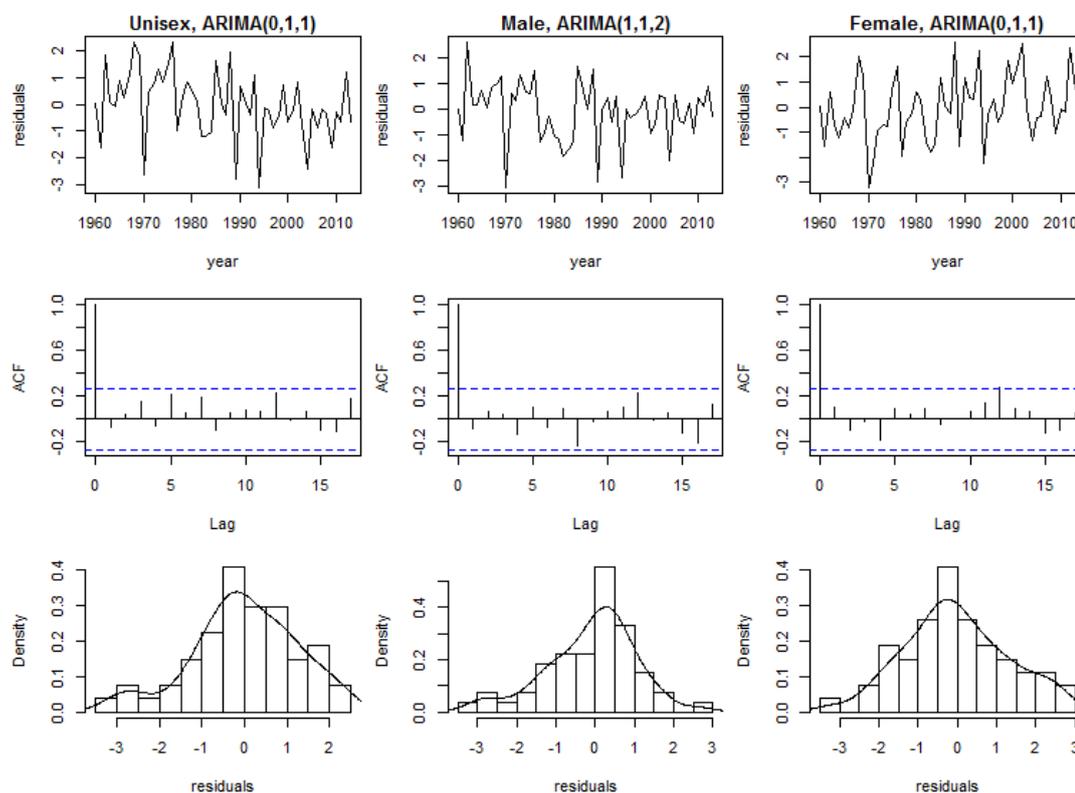
### 3) Parameters estimation

We fit the chosen model to our time series data to obtain the parameter estimates. All estimated coefficients are statistically significant, as indicated from the t-statistics in Table C.1 (see Appendix C.1). The series of mortality index  $K_t$  for all population (unisex) and female are modelled by an ARIMA (0,1,1) process and an ARIMA (1,1,2) process for males.

### 4) Model diagnostics

The evaluation of the model aims to verify that the chosen model is adequate. If the model is found to be inadequate, it is necessary to identify an alternative model. Ideally, the chosen model should extract all systematic information from the data and the part of data unexplained by the model (i.e. the residuals) should be small. These checks are usually based on the residuals of the model. One assumption of the ARIMA model is that the residuals of the model should be white noise. We can check the properties of the residuals by plotting the ACF of the residuals. If all residual autocorrelations fall inside the 95% confidence bounds, it indicates the residuals to be randomness. The normality of the residual can also be checked by considering the normal probability plot. The residuals seem to be a white noise as shown in Figure C.6. Also, the ACF plots of the residuals show all correlations within the threshold limits and all three histograms seem to be symmetric distribution. This indicates that the residuals are behaving like white noise. Additionally, we consider the Root Mean Square Error (RMSE) to measure the accuracy of the chosen model.

The lower RMSE values represent the smaller percentage of error produced by the forecasting model. We find that the chosen models, ARIMA(0,1,1) for all population (unisex), ARIMA(1,1,2) for male and ARIMA(0,1,1) for female, are the ones with the lowest RMSE as shown in Table C.3. We can conclude that these ARIMA models perform the best fit with our given data.

**Figure C.6:** Residuals of the ARIMA ( $p, d, q$ ) model

### 5) Projected subgroup-specific life tables

Next, we use the ARIMA(0,1,1) for unisex and female groups and the ARIMA(1,1,2) for male to generate the forecasts of the mortality index  $K_t$  during the period 2014-2083 (see Figure C.2 in Appendix C.1).

## C.4 Age-earnings profile

In the analysis of income redistribution, we require the salary histories of an individual by age, gender and level of education to compute the individual's contributions, but there are no relevant administrative data available. However, the Eurostat Database provides the cross-sectional average annual income for the Swedish population by age-group, gender and level of education as shown in Table C.4. We then estimate the age-earnings profile for males and females based on the method of linear interpolation.

Due to lack of data on income by age-group among level of education, we assume the education-specific income for each age is fixed as the percentage of the average gender-specific income. For example, the age-earnings income for low-educated males, for medium-educated males, for high-educated males is as 92%, 99% and 108% of the average income for all ages of males, respectively.

**Table C.4:** The average annual income of the Swedish people in 2013 by gender, age-group and educational attainment (in EUR)

<b>Age-group</b>	<b>Men</b>		<b>Women</b>	
age 18-24	21,956		21,330	
age 25-49	27,804		27,451	
age 50-64	33,231		32,921	
age over 64	24,490		20,620	
All ages	28,323		27,993	
<b>Educational attainment levels</b>	<b>in EUR</b>	<b>% of all ages</b>	<b>in EUR</b>	<b>% of all ages</b>
Low education (L)	25,978	92	21,723	78
Medium education (M)	27,917	99	26,830	96
High education (H)	30,501	108	30,328	108

# APPENDIX D

## Appendices to Chapter 5

### D.1 The expected annual payments for different types of the pension recipient

#### A. The case of the pensioner

Recalling the formula (5.15), the expected life annuity of 1 monetary unit covering the old-age and survivor benefits paid to a pensioner aged  $x_r$  in the middle of year  $t$  can be decomposed as:

For  $t = 0$ ;

$$Y^r(x_r, t) = p_{x_r+t}^{rr} + \frac{1}{2} \left[ \begin{aligned} & \left( p_{x_r+t}^{rf} + p_{x_r+t}^{rs} + p_{x_r+t}^{rc} + p_{x_r+t}^{re} \right) \\ & + \left( FB \cdot p_{x_r+t}^{rf} + SB \cdot p_{x_r+t}^{rs} + CB \cdot p_{x_r+t}^{rc} \right) \end{aligned} \right] \quad (D.1.1)$$

For  $t = 1, 2, 3, \dots, w - x_r$ ;

$$Y^r(x_r, t) = {}_t p_{x_r+t}^{rr} \left[ \begin{aligned} & p_{x_r+t}^{rr} + \frac{1}{2} \left( p_{x_r+t}^{rf} + p_{x_r+t}^{rs} + p_{x_r+t}^{rc} + p_{x_r+t}^{re} \right) \\ & + \frac{1}{2} \left( FB \cdot p_{x_r+t}^{rf} + SB \cdot p_{x_r+t}^{rs} + CB \cdot p_{x_r+t}^{rc} \right) \end{aligned} \right] \\ + {}_{t-1} p_{x_r}^{rr} \left[ \begin{aligned} & FB \cdot p_{x_r+t-1}^{rf} \cdot EPV_{x_s+t}^f + SB \cdot p_{x_r+t-1}^{rs} \cdot EPV_{x_s+t}^s \\ & + CB \cdot p_{x_r+t-1}^{rc} \cdot EPV_{x_c+t}^c \end{aligned} \right] \quad (D.1.2)$$

We then demonstrate the relations of the expected annual payment at a particular age and time from the above equation (D.1.1) and (D.1.2).

At  $t=0$ ;

$$Y^r(x_r, 0) = p_{x_r}^{rr} + \frac{1}{2} \left( p_{x_r}^{rf} + p_{x_r}^{rs} + p_{x_r}^{rc} + p_{x_r}^{re} \right) + \frac{1}{2} \left( FB \cdot p_{x_r}^{rf} + SB \cdot p_{x_r}^{rs} + CB \cdot p_{x_r}^{rc} \right)$$

$t=1$ ;

$$Y^r(x_r, 1) = p_{x_r}^{rr} \left[ p_{x_{r+1}}^{rr} + \frac{1}{2} \left( p_{x_{r+1}}^{rf} + p_{x_{r+1}}^{rs} + p_{x_{r+1}}^{rc} + p_{x_{r+1}}^{re} \right) \right] \\ + \frac{1}{2} \left( FB \cdot p_{x_{r+1}}^{rf} + SB \cdot p_{x_{r+1}}^{rs} + CB \cdot p_{x_{r+1}}^{rc} \right) \\ + FB \cdot p_{x_r}^{rf} \cdot EPV_{x_{s+1}}^f + SB \cdot p_{x_r}^{rs} \cdot EPV_{x_{s+1}}^s + CB \cdot p_{x_r}^{rc} \cdot EPV_{x_c+1}^c$$

$t=2$ ;

$$Y^r(x_r, 2) = p_{x_r}^{rr} \cdot p_{x_{r+1}}^{rr} \left[ p_{x_{r+2}}^{rr} + \frac{1}{2} \left( p_{x_{r+2}}^{rf} + p_{x_{r+2}}^{rs} + p_{x_{r+2}}^{rc} + p_{x_{r+2}}^{re} \right) \right] \\ + \frac{1}{2} \left( FB \cdot p_{x_{r+2}}^{rf} + SB \cdot p_{x_{r+2}}^{rs} + CB \cdot p_{x_{r+2}}^{rc} \right) \\ + p_{x_r}^{rr} \left[ FB \cdot p_{x_{r+1}}^{rf} \cdot EPV_{x_{s+2}}^f + SB \cdot p_{x_{r+1}}^{rs} \cdot EPV_{x_{s+2}}^s + CB \cdot p_{x_{r+1}}^{rc} \cdot EPV_{x_c+2}^c \right]$$

$$Y^r(x_r, 2) = p_{x_r}^{rr} \cdot Y^r(x_r + 1, 1)$$

$t=3$ ;

$$Y^r(x_r, 3) = {}_3p_{x_r}^{rr} \left[ p_{x_{r+3}}^{rr} + \frac{1}{2} \left( p_{x_{r+3}}^{rf} + p_{x_{r+3}}^{rs} + p_{x_{r+3}}^{rc} + p_{x_{r+3}}^{re} \right) \right] \\ + \frac{1}{2} \left( FB \cdot p_{x_{r+3}}^{rf} + SB \cdot p_{x_{r+3}}^{rs} + CB \cdot p_{x_{r+3}}^{rc} \right) \\ + {}_2p_{x_r}^{rr} \left[ FB \cdot p_{x_{r+2}}^{rf} \cdot EPV_{x_{s+3}}^f + SB \cdot p_{x_{r+2}}^{rs} \cdot EPV_{x_{s+3}}^s + CB \cdot p_{x_{r+2}}^{rc} \cdot EPV_{x_c+3}^c \right]$$

$$Y^r(x_r, 3) = p_{x_r}^{rr} \cdot Y^r(x_r + 1, 2) = p_{x_r}^{rr} \cdot p_{x_{r+1}}^{rr} Y^r(x_r + 2, 1)$$

⋮

$$t = w - x_r;$$

$$Y^r(x_r, w - x_r) = p_{x_r}^{rr} \cdot Y^r(x_r + 1, w - x_r - 1)$$

Thus, the actuarial value of the entire annuity pension for a person retired at age  $x_r$  expressed in the formula (5.15) is equivalent to

$$EPV_{x_r} = \sum_{t=0}^{w-x_r} Y^r(x_r, t) \cdot v^{(t+0.5)} \quad (\text{D.1.3})$$

## B. The case of the spouse

Recalling the formula (5.16), the expected life annuity of 1 monetary unit for a spouse aged  $x_s + t$  in the middle of the year  $h$  is given by

For  $h = 0, 1, 2, \dots, w - x_s + t$  and  $t = 0, 1, 2, \dots, w - x_r$

$$Y^s(x_s + t, h) = {}_h p_{x_s+t}^{ss} \left[ p_{x_s+t+h}^{ss} + \frac{1}{2} p_{x_s+t+h}^{se} \right] \quad (\text{D.1.4})$$

Let consider the relations of the expected annual payments at a particular time from the above formula (D.1.4)

$$\text{At } t = 0; \quad Y^s(x_s + t, 0) = p_{x_s+t}^{ss} + \frac{1}{2} p_{x_s+t}^{se}$$

$$t = 1; \quad Y^s(x_s + t, 1) = p_{x_s+t}^{ss} \left[ p_{x_s+t+1}^{ss} + \frac{1}{2} p_{x_s+t+1}^{se} \right] = p_{x_s+t}^{ss} Y^s(x_s + t + 1, 0)$$

$$t = 2; \quad Y^s(x_s + t, 2) = {}_2 p_{x_s+t}^{ss} \left[ p_{x_s+t+2}^{ss} + \frac{1}{2} p_{x_s+t+2}^{se} \right] = p_{x_s+t}^{ss} Y^s(x_s + t + 1, 1)$$

$$t = 3; \quad Y^s(x_s + t, 3) = {}_3 p_{x_s+t}^{ss} \left[ p_{x_s+t+3}^{ss} + \frac{1}{2} p_{x_s+t+3}^{se} \right] = p_{x_s+t}^{ss} Y^s(x_s + t + 1, 2)$$

⋮

$$t = w - (x_s + t); \quad Y^s(x_s + t, w - x_s + t) = p_{x_s+t}^{ss} Y^s(x_s + t + 1, w - 1 - x_s + t)$$

We also illustrate the backward recursion from the maximum reachable age  $w$  to some particular age as follows;

$$\text{Age } x_s + t = w; \quad Y^s(w, 0) = p_w^{ss} + \frac{1}{2} p_w^{se}$$

$$\text{Age } x_s + t = w - 1; \quad Y^s(w - 1, 0) = p_{w-1}^{ss} + \frac{1}{2} p_{w-1}^{se}$$

$$Y^s(w - 1, 1) = p_{w-1}^{ss} \left[ p_w^{ss} + \frac{1}{2} p_w^{se} \right] = p_{w-1}^{ss} Y^s(w, 0)$$

$$\text{Age } x_s + t = w - 2; \quad Y^s(w - 2, 0) = p_{w-2}^{ss} + \frac{1}{2} p_{w-2}^{se}$$

$$Y^s(w - 2, 1) = p_{w-2}^{ss} \left[ p_{w-1}^{ss} + \frac{1}{2} p_{w-1}^{se} \right] = p_{w-2}^{ss} Y^s(w - 1, 0)$$

$$Y^s(w - 2, 2) = p_{w-2}^{ss} \cdot p_{w-1}^{ss} \left[ p_w^{ss} + \frac{1}{2} p_w^{se} \right] = p_{w-2}^{ss} Y^s(w - 1, 1)$$

$$\text{Age } x_s + t = w - 3; \quad Y^s(w - 3, 0) = p_{w-3}^{ss} + \frac{1}{2} p_{w-3}^{se}$$

$$Y^s(w - 3, 1) = p_{w-3}^{ss} \left[ p_{w-2}^{ss} + \frac{1}{2} p_{w-2}^{se} \right] = p_{w-3}^{ss} Y^s(w - 2, 0)$$

$$Y^s(w - 3, 2) = p_{w-3}^{ss} \cdot p_{w-2}^{ss} \left[ p_{w-1}^{ss} + \frac{1}{2} p_{w-1}^{se} \right] = p_{w-3}^{ss} Y^s(w - 2, 1)$$

$$Y^s(w - 3, 3) = p_{w-3}^{ss} \cdot p_{w-2}^{ss} \cdot p_{w-1}^{ss} \left[ p_w^{ss} + \frac{1}{2} p_w^{se} \right] = p_{w-3}^{ss} Y^s(w - 2, 2)$$

Then, the actuarial value of the lifetime annuity for a spouse aged  $x_s + t$  is the summation of the discounted annual payments from any time  $h = 0, 1, 2, \dots, w - x_s + t$

to time 0 with the value of  $v = \frac{1}{1+r'}$  where  $r'$  is the annual rate of interest, that can be expressed as:

$$EPV_{x_s+t}^s = \sum_{h=0}^{w-x_s+t} Y^s(x_s+t, h) \cdot v^{(h+0.5)} \tag{D.1.5}$$

The following triangular matrix presents all pension payments to the surviving spouse that we construct to calculate the expected present value of the survivor pensions for the spouse.

**Table D.1:** The algorithm path of the expected annual pension for a surviving spouse

Age	Time ( $h$ )				
	0	1	...	$w-1-(x_s+t)$	$w-(x_s+t)$
$\vdots$			...		
$x_s+t$	$Y^s(x_s+t,0)$	$Y^s(x_s+t,1)$	...	$Y^s(x_s+t, w-1-x_s-t)$	$Y^s(x_s+t, w-x_s-t)$
$x_s+t+1$	$Y^s(x_s+t+1,0)$	$Y^s(x_s+t+1,1)$	...	$Y^s(x_s+t+1, w-1-x_s-t)$	
$\vdots$	$\vdots$	$\vdots$	$\ddots$		
$w-2$	$Y^s(w-2,0)$	$Y^s(w-2,1)$	$Y^s(w-2,2)$		
$w-1$	$Y^s(w-1,0)$	$Y^s(w-1,1)$			
$w$	$Y^s(w,0)$				

### C. The case of the complete family

In this case, we suppose the main recipient of the complete family is the spouse aged  $x_s+t$ , conditional on that the age of children is  $x_c+t$ . The annuity is payable annually until the children have reached the maximum age,  $M$ . Recalling the formula (5.17), the annuity of 1 monetary unit of the complete family benefits paid to the recipient aged  $x_s+t$  in the middle of year  $h$  can be decomposed as:

For  $h = 0$ ;

$$Y^s(x_s + t, h) = p_{x_s+t}^{ff} + \frac{1}{2} p_{x_s+t}^{fc} + \frac{1}{2} CB \cdot p_{x_s+t}^{fc} \quad (D.1.6)$$

For  $h = 1, 2, 3, \dots, M - 1 - (x_c + t)$ ;

$$Y^s(x_s + t, h) = {}_h p_{x_s+t}^{ff} \left[ p_{x_s+t+h}^{ff} + \frac{1}{2} p_{x_s+t+h}^{fc} + \frac{1}{2} CB \cdot p_{x_s+t+h}^{fc} \right] \\ + CB \cdot \sum_{k=0}^{h-1} {}_k p_{x_s+t}^{ff} \cdot p_{x_s+t+k}^{fc} \quad (D.1.7)$$

For  $h = M - (x_c + t)$ ;

$$Y^s(x_s + t, h) = {}_h p_{x_s+t}^{ff} \left[ \frac{1}{2} p_{x_s+t+h}^{ff} + \frac{1}{4} p_{x_s+t+h}^{fc} + \frac{1}{4} CB \cdot p_{x_s+t+h}^{fc} \right. \\ \left. + \frac{1}{2} SB \cdot p_{x_s+t+h}^{fs} + \frac{1}{4} SB \cdot p_{x_s+t+h}^{fe} \right] \\ + \frac{1}{2} CB \cdot \sum_{k=0}^{h-1} {}_k p_{x_s+t}^{ff} \cdot p_{x_s+t+k}^{fc} + SB \cdot {}_h p_{x_s+t}^{ff} \cdot p_{x_s+t+h}^{fs} \cdot EPV_{x_s+t+h}^s \quad (D.1.8)$$

Thus, the actuarial value of the lifetime annuity for the complete family is given by

$$EPV_{x_s+t}^f = \sum_{h=0}^{M-(x_c+t)} Y^f(x_s + t, h) \cdot v^{(h+0.5)} \quad (D.1.9)$$

We do the backward recursive by beginning with the last row – the maximum age of children,  $M$  – to construct the elements of the annual pension shown in Table D.2.

$$Y^f(M, 0) = Y^f(x_s + M - x_c, 0)$$

$$= \frac{1}{2} p_{x_s+M-x_c}^{fs} (1 + SB) + \frac{1}{4} \left[ p_{x_s+M-x_c}^{fc} + CB \cdot p_{x_s+M-x_c}^{fc} + SB \cdot p_{x_s+M-x_c}^{fe} \right]$$

$$= \frac{1}{2} p_{x_s+M-x_c}^{fs} (1+SB) + \frac{1}{4} p_{x_s+M-x_c}^{fe} [1+CB+SB]$$

where  $p_{x_s+M-x_c}^{fc} = p_{x_s+M-x_c}^{fe}$  represents the probability of dying of the spouse aged  $x_s + M - x_c$ .

**Table D.2:** The algorithm path of the expected annual pension for a complete family

Children age	Time (h)				
	0	1	...	$M-1-(x_c+t)$	$M-(x_c+t)$
⋮			...		
$x_c+t$	$Y^f(x_s+t,0)$	$Y^f(x_s+t,1)$	...	$Y^f(x_s+t,M-1-x_c-t)$	$Y^f(x_s+t,M-x_c-t)$
$x_c+t+1$	$Y^f(x_s+t+1,0)$	$Y^f(x_s+t+1,1)$	...	$Y^f(x_s+t+1,M-1-x_c-t)$	
⋮	⋮	⋮	⋮		
$M-2$	$Y^f(M-2,0)$	$Y^f(M-2,1)$	$Y^f(M-2,2)$		
$M-1$	$Y^f(M-1,0)$	$Y^f(M-1,1)$			
$M$	$Y^f(M,0)$				

Some elements of the row  $M-1$  and  $M-2$  are illustrated as examples:

$$Y^f(M-1,0) = Y^f(x_s+M-1-x_c,0) = p_{x_s+M-1-x_c}^{ff} + \frac{1}{2} p_{x_s+M-1-x_c}^{fc} (1+CB)$$

$$Y^f(M-1,1) = Y^f(x_s+M-1-x_c,1) = p_{x_s+M-1-x_c}^{ff} \cdot Y^f(M,0) + \frac{1}{2} CB \cdot p_{x_s+M-1-x_c}^{fc}$$

$$Y^f(M-2,1) = Y^f(x_s+M-2-x_c,1) = p_{x_s+M-2-x_c}^{ff} \cdot Y^f(M-1,0) + \frac{1}{2} CB \cdot p_{x_s+M-2-x_c}^{fc}$$

Then, we get the general formulae and  $k = 0, 1, 2, \dots, x_c + t$ :

For  $h = 0$ ;

$$Y^s(M-k, h) = p_{x_s+M-k-x_c}^{ff} + \frac{1}{2} p_{x_s+M-k-x_c}^{fc} + \frac{1}{2} CB \cdot p_{x_s+M-k-x_c}^{fc}$$

For  $h = 1, 2, 3, \dots, M-1-(x_c+t)$ ;

$$Y^f(M-k, h) = p_{x_s+M-k-x_c}^{ff} \cdot Y^f(M-k+1, h-1) + CB \cdot p_{x_s+M-k-x_c}^{fc}$$

For  $h = M-(x_c+t)$ ;

$$Y^f(M-k, h) = p_{x_s+M-k-x_c}^{ff} \cdot Y^f(M-k+1, h-1) + \frac{1}{2} CB \cdot p_{x_s+M-k-x_c}^{fc}$$

## **D.2 Comparison of the unisex annuity divisors for the different types of pension benefits payable in the middle of the year**

As the annuities of the two methods – the Italian rule and the direct method – are payable annually in the different time point which is at beginning of each year for the Italian rule and in the mid-year for the direct method, when we compare the annuity divisors between two types it might not reflect the differences. We then show the example values of the annuity divisors for the case of only old-age pension and the old-age combined with survivors' benefits from the Italian rule and our proposed model in which those life annuities are payable annually in the mid-year according to the following formulae:

• For the old-age pension, the actuarial value of a life annuity of 1 monetary unit per year payable annually in the middle of the year throughout the lifetime of the pensioner now aged  $x_r$  is given by

$$EPV_{x_r}^{\text{Old-age}} = \sum_{t=0}^{w-x_r} {}_{t+0.5}P_{x_r} \cdot v^{(t+0.5)} \quad (\text{D.2.1})$$

Under the assumption of a uniform distribution of deaths (UDD), for integer  $x_r$  and for  $t = 0, 1, 2, \dots$  we can express  ${}_{t+0.5}P_{x_r}$  as

$${}_{t+0.5}P_{x_r} = \prod_{h=0}^{t-1} P_{x_r+h} \cdot 0.5 P_{x_r+t} = \prod_{h=0}^{t-1} P_{x_r+h} \cdot (1 - 0.5q_{x_r+t}) = \frac{0.5(l_{x_r+t} + l_{x_r+t+1})}{l_{x_r}} \quad (\text{D.2.2})$$

Substituting (D.2.2) in (D.2.1) and thus

$$EPV_{x_r}^{\text{Old-age}} = \sum_{t=0}^{w-x_r} \frac{0.5(l_{x_r+t} + l_{x_r+t+1})}{l_{x_r}} \cdot v^{(t+0.5)} \quad (\text{D.2.3})$$

The unisex annuity divisors for the old-age pension is

$$\overline{EPV}_{x_r}^{\text{Old-age}} = \frac{EPV_{x_r, \text{male}}^{\text{Old-age}} + EPV_{x_r, \text{female}}^{\text{Old-age}}}{2} \quad (\text{D.2.4})$$

• For the case of the Italian rule, the expected present value of a life annuity for the old-age and survivor benefits payable yearly in the mid-year can be modified from the formulae in Section 5.2 as follows:

$$EPV_{x_r, g}^{\text{Italian}} = EPV_{x_r, g}^{\text{Old-age}} + EPV_{x_r, g}^{\text{Survivor}} \quad (\text{D.2.5})$$

$$EPV_{x_r, g}^{\text{Survivor}} = \eta \delta_g \sum_{t=0}^{w-x_r} \frac{0.5(l_{x_r+t, g} + l_{x_r+t+1, g})}{l_{x_r, g}} q_{x_r+t+0.5, g} \cdot v^{(t+0.5)} \theta_{x+t, g} \cdot a_{x+t+1.5, g}^{\text{spouse}} \quad (\text{D.2.6})$$

$$a_{x+t+1.5, g'}^{\text{spouse}} = \sum_{\tau=1}^{w-x-t+\varepsilon_g} \frac{0.5(l_{x+t+\tau-\varepsilon_g, g'} + l_{x+t+1+\tau-\varepsilon_g, g'})}{l_{x+t+1-\varepsilon_g, g'}} \cdot v^\tau \quad (\text{D.2.7})$$

$$\text{Thus, } \overline{EPV}_{x_r}^{\text{Italian}} = \frac{EPV_{x_r, \text{male}}^{\text{Italian}} + EPV_{x_r, \text{female}}^{\text{Italian}}}{2} \quad (\text{D.2.8})$$

$$\text{where } q_{x_r+t+0.5} = 1 - p_{x_r+t+0.5} = 1 - \frac{l_{x_r+t+1.5}}{l_{x_r+t+0.5}} = 1 - \frac{(l_{x_r+t+1} + l_{x_r+t+2})}{(l_{x_r+t} + l_{x_r+t+1})} = \frac{l_{x_r+t} - l_{x_r+t+2}}{l_{x_r+t} + l_{x_r+t+1}}$$

under the UDD assumption;

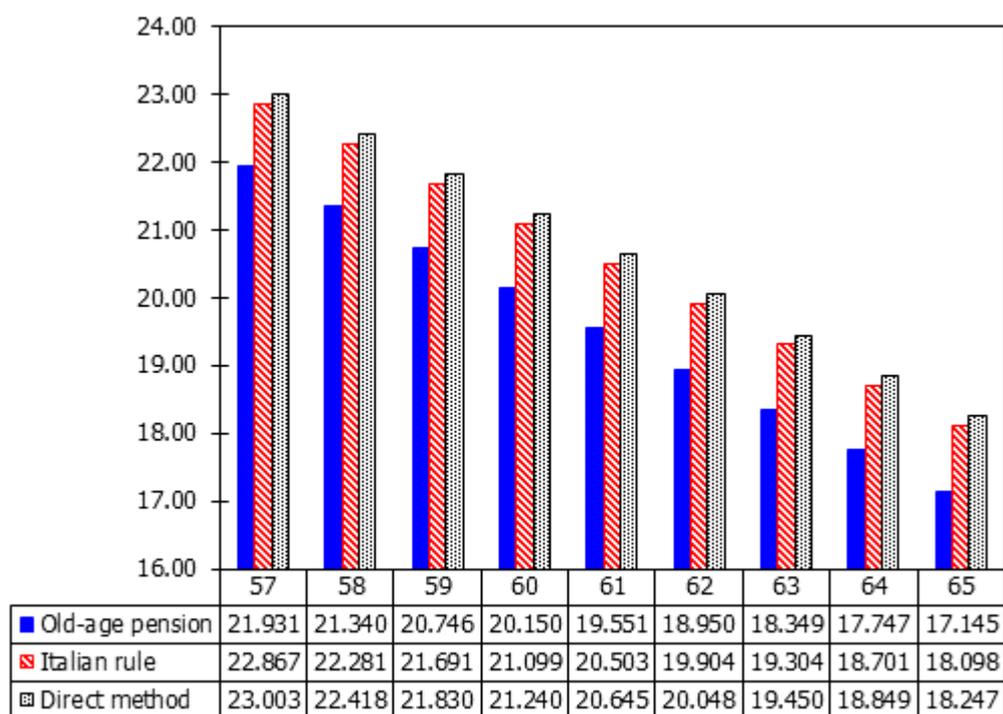
$g$  denotes the gender of the pensioner with  $g = \{male(m), female(f)\}$  and  $g'$  is the gender of his/her spouse,  $g' = \{f \text{ if } g = m, m \text{ if } g = f\}$ .

- For our proposed model by using the direct method in the computation, the annuity divisors are calculated according to the formula (5.15) in Section 5.3 and the unisex divisor is

$$\overline{EPV}_{x_r} = \frac{EPV_{x_r, \text{male}} + EPV_{x_r, \text{female}}}{2} \quad (\text{D.2.9})$$

The average (unisex) annuity divisors of the three cases in a selected year of 2015 are shown in Figure D.1. Clearly, the divisors of the life annuity of the old-age pension (only pensioner) are lower than those values of the Italian rule covering two lives (pensioner and surviving spouse) and the direct method involving more than two lives (pensioner, spouse and children), respectively. The annuity divisors of the direct method are higher than those values of the Italian rule with a range of 0.60-0.82%, just only 0.14-0.15 units, at different ages of retirement while the gap between the divisors of the direct method and the old-age pension is between 1.07 and 1.10 units, around 4.9-6.4%.

**Figure D.1:** Average annuity divisors of the pension paid in the mid-year of the old-age pension, the Italian rule and the direct method in a selected year of 2015



# APPENDIX E

## Appendices to Chapter 6

### E.1 One-year transition probabilities

- **An active individual**

The fundamental relations of one-year probabilities and transition probabilities of an active individual age  $x$  are defined as follows:

$$p_x^{aa} + p_x^{ai} = p_x^a \quad (\text{E.1.1})$$

$$q_x^{aa} + q_x^{ai} = q_x^a \quad (\text{E.1.2})$$

$$p_x^a + q_x^a = 1 \quad (\text{E.1.3})$$

$$p_x^{ai} + q_x^{ai} = w_x^{ai} \quad (\text{E.1.4})$$

$$p_x^{aa} + q_x^{aa} = 1 - w_x^{ai} \quad (\text{E.1.5})$$

where  $p_x^{aa}$  is the probability that an active person aged  $x$  is alive in an active state at age  $x+1$ ;

$q_x^{aa}$  is the probability that an active person aged  $x$  dies in an active state at age  $x+1$ ;

$p_x^{ai}$  is the probability that an active person aged  $x$  is alive in a disabled state at age  $x+1$ ;

$q_x^{ai}$  is the probability that an active person aged  $x$  dies in a disabled state at age  $x+1$ ;

$p_x^a$  is the probability that an active person aged  $x$  is alive at age  $x+1$ ;

$q_x^a$  is the probability that an active person aged  $x$  dies within one year;

$w_x^{ai}$  is the probability that an active person aged  $x$  becomes disabled within one year.

- **A disabled individual**

The one-year conditional probabilities related to a disabled individual age  $x$  are hold in the following relations:

$$p_x^{ii} + p_x^{ia} = p_x^i \quad (\text{E.1.6})$$

$$q_x^{ii} + q_x^{ia} = q_x^i \quad (\text{E.1.7})$$

$$p_x^i + q_x^i = 1 \quad (\text{E.1.8})$$

$$p_x^{ia} + q_x^{ia} = w_x^{ja} \quad (\text{E.1.9})$$

$$p_x^{ii} + q_x^{ii} = 1 - w_x^{ja} \quad (\text{E.1.10})$$

where  $p_x^{ii}$  is the probability that a disabled person aged  $x$  is alive in a disabled state at age  $x+1$ ;

$q_x^{ii}$  is the probability that a disabled person aged  $x$  dies in a disabled state at age  $x+1$ ;

$p_x^{ia}$  is the probability that a disabled person aged  $x$  is alive in an active state at age  $x+1$ ;

$q_x^{ia}$  is the probability that a disabled person aged  $x$  dies in an active state at age  $x+1$ ;

$p_x^i$  is the probability that a disabled person aged  $x$  is alive at age  $x+1$ ;

$q_x^i$  is the probability that a disabled person aged  $x$  dies within one year;

$w_x^{ja}$  is the probability that a disabled person aged  $x$  recover to an active state within one year.

## E.2 The quadratic equation of $q_x^{ii}$

The parabolic function of  $q_x^{ii}$  is expressed as:

$$A(q_x^{ii})^2 + Bq_x^{ii} - q_x = 0 \quad (\text{E.2.1})$$

where  $A = -k_1 k_3 \left[ (1 - v_x) + v_x k_1 k_2 w_x^{ia} \right]$

and  $B = k_3 \left( 1 - v_x + v_x k_2 w_x^{ia} \right) + k_1 v_{x+1} (1 - q_x) + k_1 \left( q_x - v_x + v_x w_x^{ia} \right) + v_x$

Following the quadratic formula to solve the equation (E.2.1), there are two real solutions with the positive values. However, we obtain the unique solution that exists the (0,1) interval as follows:

$$q_x^{ii} = \frac{-B + \sqrt{B^2 + 4Aq_x}}{2A} \quad (\text{E.2.2})$$

### E.3 Model selection for the disability prevalence rates

We compare all possible model variants for males and females separately. Table E.1 illustrates the parameter estimates and AIC values for six model variants.

**Table E.1:** Model selection for disability prevalence rates: parameter estimates and goodness-of-fit test

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6 <sup>a</sup>
<b>Male</b>						
$\alpha$ (intercept)	-2.2383***	-4.3688***	-1.1546***	-1.1299***	-1.4001***	<b>1.9380***</b>
$\beta_1$ (age)		0.0496***	-0.1208***	-0.1209***	-0.0972***	<b>-0.4964***</b>
$\beta_2$ (age <sup>2</sup> )			0.0020***	0.0020***	0.0014***	<b>0.0179***</b>
$\beta_3$ (age <sup>3</sup> /1000)					0.0050*	<b>-0.2791***</b>
$\beta_4$ (age <sup>4</sup> /1000)						<b>0.0017***</b>
$\beta_5$ (t)				-0.0037**	-0.0037**	<b>-0.0038**</b>
AIC	189,561	177,769	174,377	174,369	174,359	<b>174,325</b>
<b>Female</b>						
$\alpha$ (intercept)	-2.1851***	-3.9831***	-2.4081***	-2.3712***	-3.5759***	<b>-1.1506*</b>
$\beta_1$ (age)		0.0448***	-0.0430***	-0.0432***	0.0657***	<b>-0.2339***</b>
$\beta_2$ (age <sup>2</sup> )			0.0011***	0.0011***	-0.0019***	<b>0.0111***</b>
$\beta_3$ (age <sup>3</sup> /1000)					0.0259***	<b>-0.2103***</b>
$\beta_4$ (age <sup>4</sup> /1000)						<b>0.0015***</b>
$\beta_5$ (t)				-0.0057***	-0.0056***	<b>-0.0057***</b>
AIC	191,443	183,891	183,148	183,143	183,087	<b>183,082</b>

Source: Own source using the LFS dataset and the logistic regression in the equation (6.13)

Note: <sup>a</sup> is the model selected for the estimation of the disability prevalence rates.

AIC denotes the Akaike Information Criterion.

\* $p$  – value < 0.10; \*\* $p$  – value < 0.05; \*\*\* $p$  – value < 0.01

## E.4 Model selection for the one-year recovery rates

We compare all six possible model variants as shown in Table E.2 that exhibits the parameter estimates and AIC value for each model.

**Table E.2:** Model selection for one-year recovery rates: parameter estimates and goodness-of-fit test

Parameter	Model 1	Model 2	Model 3	Model 4 <sup>a</sup>	Model 5	Model 6
$\alpha$ (intercept)	-2.4785***	-0.3389**	-0.8770**	<b>-1.1705***</b>	-0.7225*	-1.0419***
$\beta_1$ (gender)	-0.0523	-0.1979**	-0.2252**	<b>-0.2379**</b>	-0.2253**	-0.2389**
$\beta_2$ (age)		-0.0514***	-0.0177		-0.0191	
$\beta_3$ (age <sup>2</sup> )			-0.0004*	<b>-0.0007***</b>	-0.0004*	-0.0007***
time					-0.0206	-0.0202
AIC	5286	4898	4895	<b>4894</b>	4893	4892

Source: Own source using the LFS dataset and the logistic regression in the equation (6.15)

Note: <sup>a</sup> is the model selected for the estimation of the disability prevalence rates.

AIC denotes the Akaike Information Criterion.

\* $p$ -value < 0.10; \*\* $p$ -value < 0.05; \*\*\* $p$ -value < 0.01

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